

ccDice: A Topology-Aware Dice Score Based on Connected Components

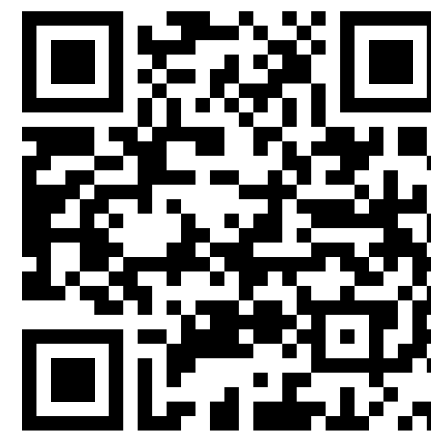
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Introduction

Recently, **topology** is being increasingly considered in image segmentation to improve the segmentation of objects presenting challenging shapes. In this work, we introduce ccDice (connected component Dice), a **topological metric** that generalises the popular Dice score but acts at the **scale of connected components**. **ccDice is a simple, explainable, normalized and low-computational topological metric.**

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Conclusion

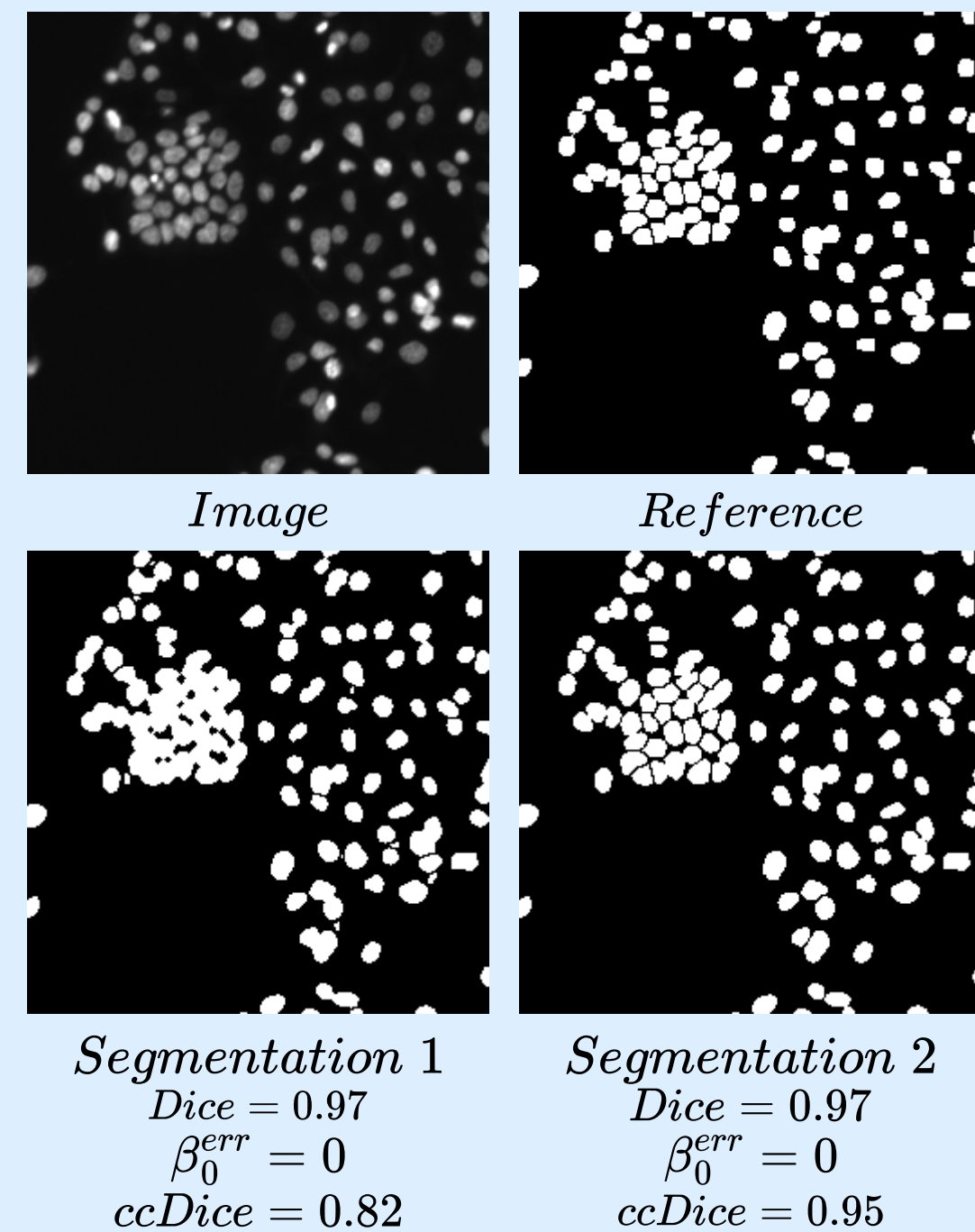
The proposed **ccDice** offers a viable alternative to the Betti matching error for **evaluating the topological properties of segmentations and incorporating spatial awareness**. It has the distinct advantages of being **normalized and computationally efficient**. Additionally, ccDice extends the widely-used Dice metric, highlighting its theoretical robustness. Future work will focus on proposing a differentiable ccDice-based loss function.

1. Context

Medical image segmentation involve objects with **complex geometries and shapes** which need to be preserved during segmentation.

For such objects, **standard metrics** (like Dice or Hausdorff distance) are often inadequate.

Topological metrics or loss functions allow to correctly measure the topological structure of segmentations.



2. Limitation of current topological metrics

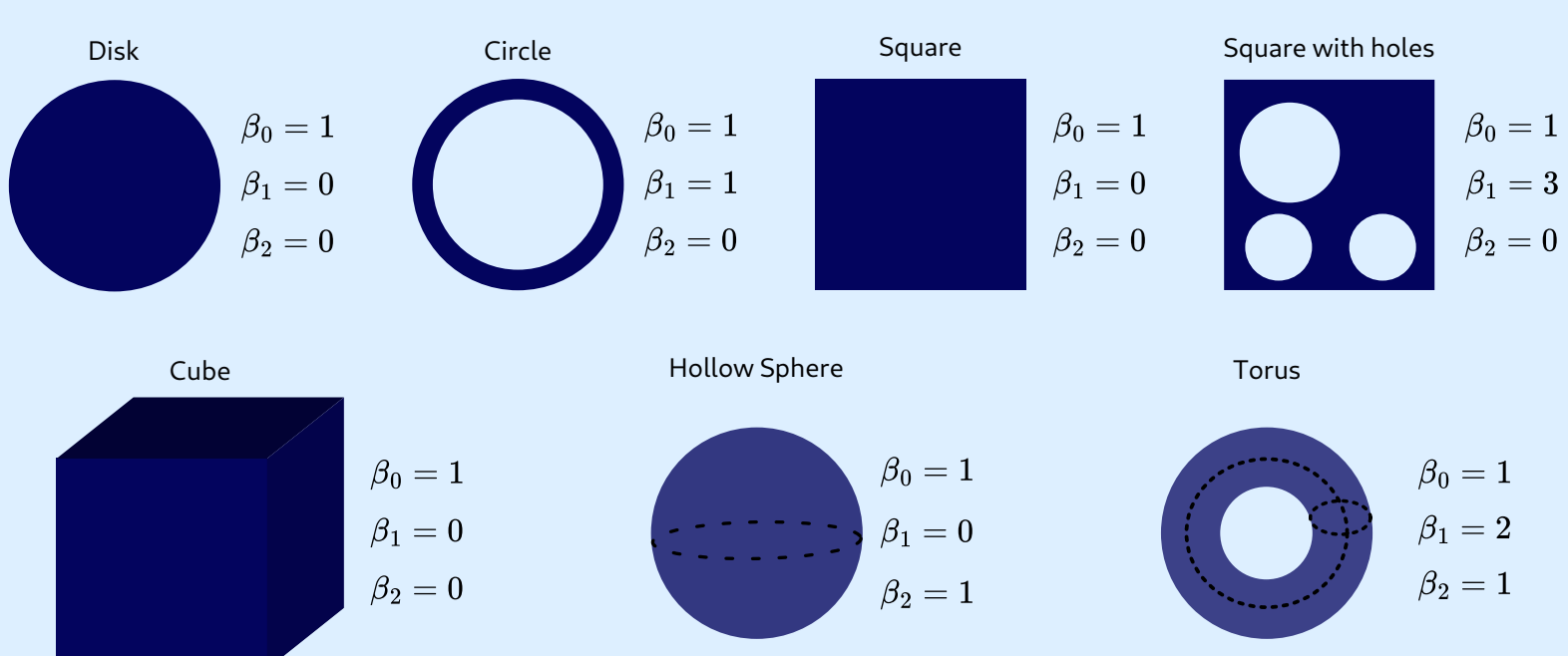
Betti numbers [1]:

β_0 : number of connected components.

β_1 : number of tunnels.

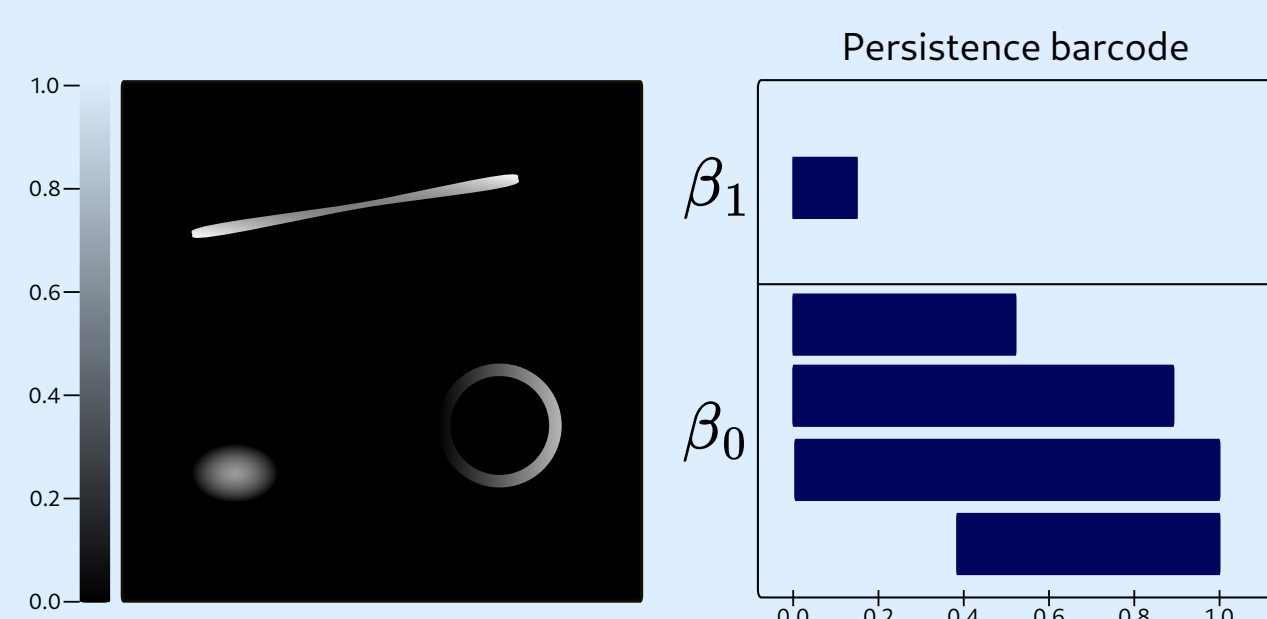
β_2 : number of cavities.

Limitation: not spatially correlated.



Betti matching [2]: spatially match the topological objects thanks to the persistence barcodes of the respective images.

Limitation: not normalized and expensive to compute.



ccDice overcomes these limitations by **being spatially correlated, normalized and fast to compute.**

3. Dice from (mis)matching

Let S and G be two binary images that represent a segmentation result and a ground truth.

The **Dice score** is classically defined by:

$$Dice(S, G) = \frac{2|S \cap G|}{|S| + |G|} = \frac{2 \cdot tp(S, G)}{2 \cdot tp(S, G) + fp(S, G) + fn(S, G)}$$

Alternatively to these definitions we propose to view the Dice score as a combination of the number of **matching pixels** m and **mismatching pixels** \bar{m} .

$$Dice(S, G) = \frac{m(S, G) + m(G, S)}{m(S, G) + \bar{m}(S, G) + m(G, S) + \bar{m}(G, S)} \quad (1)$$

where:

$$\bar{m}(S, G) = |S \setminus G| = fp(S, G) = fn(G, S) = |S| - m(S, G)$$

$$m(S, G) = m(G, S) = |S \cap G| = tp(S, G) = tp(G, S)$$

4. Matching for connected components

We need to extend the notion of Dice **from points comparison to connected components (CCs) comparison** and then to generalize the notions of **(mis)matching**.

A connected component of S matches a connected component of G if their **overlap is significant** with respect to the size of X .

We define $\varphi_{S,G}^\lambda : \mathcal{C}[S] \rightarrow \mathcal{C}[G]$ the **matching function** of connected components from S to G where $\lambda \in (0, 1]$ is the parameter controlling the required degree of overlap.

We define the **embedding score** $\varepsilon(X, Y)$ as a function quantifying the degree of overlap.

$$\varepsilon(X, Y) = \frac{|X \cap Y|}{X} \in [0, 1]$$

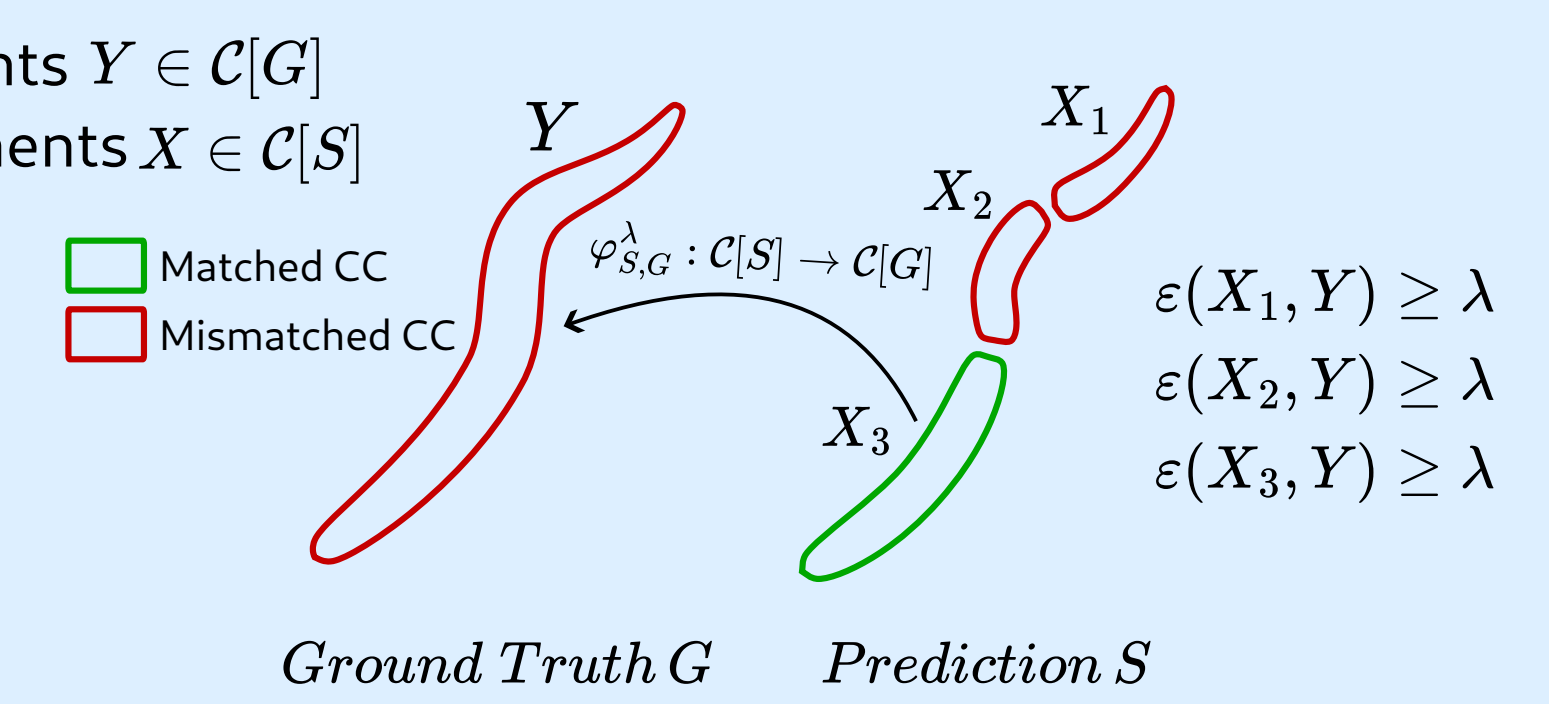
Then, the matching $\varphi_{S,G}^\lambda$ is defined such that for any $X \in \mathcal{C}[S]$, we have:

$$\varepsilon(X, \varphi_{S,G}^\lambda(X)) \geq \lambda.$$

5. From Dice to ccDice

With this definition each connected components $Y \in \mathcal{C}[G]$ can be associated to many connected components $X \in \mathcal{C}[S]$

To avoid this we force $\varphi_{S,G}^\lambda$ to be **injective**.



When $\varphi_{S,G}^\lambda$ is built we can simply define the number of matching μ and mistaching pixels $\bar{\mu}$ as:

$$\mu(S, G) = |\{X \in \mathcal{C}[S] \mid \exists Y \in \mathcal{C}[G], Y = \varphi_{S,G}^\lambda(X)\}|$$

$$\bar{\mu}(S, G) = |\{X \in \mathcal{C}[S] \mid \forall Y \in \mathcal{C}[G], Y \neq \varphi_{S,G}^\lambda(X)\}|$$

We can now define the notion of ccDice by substituting m and \bar{m} by μ and $\bar{\mu}$ in equation (1).

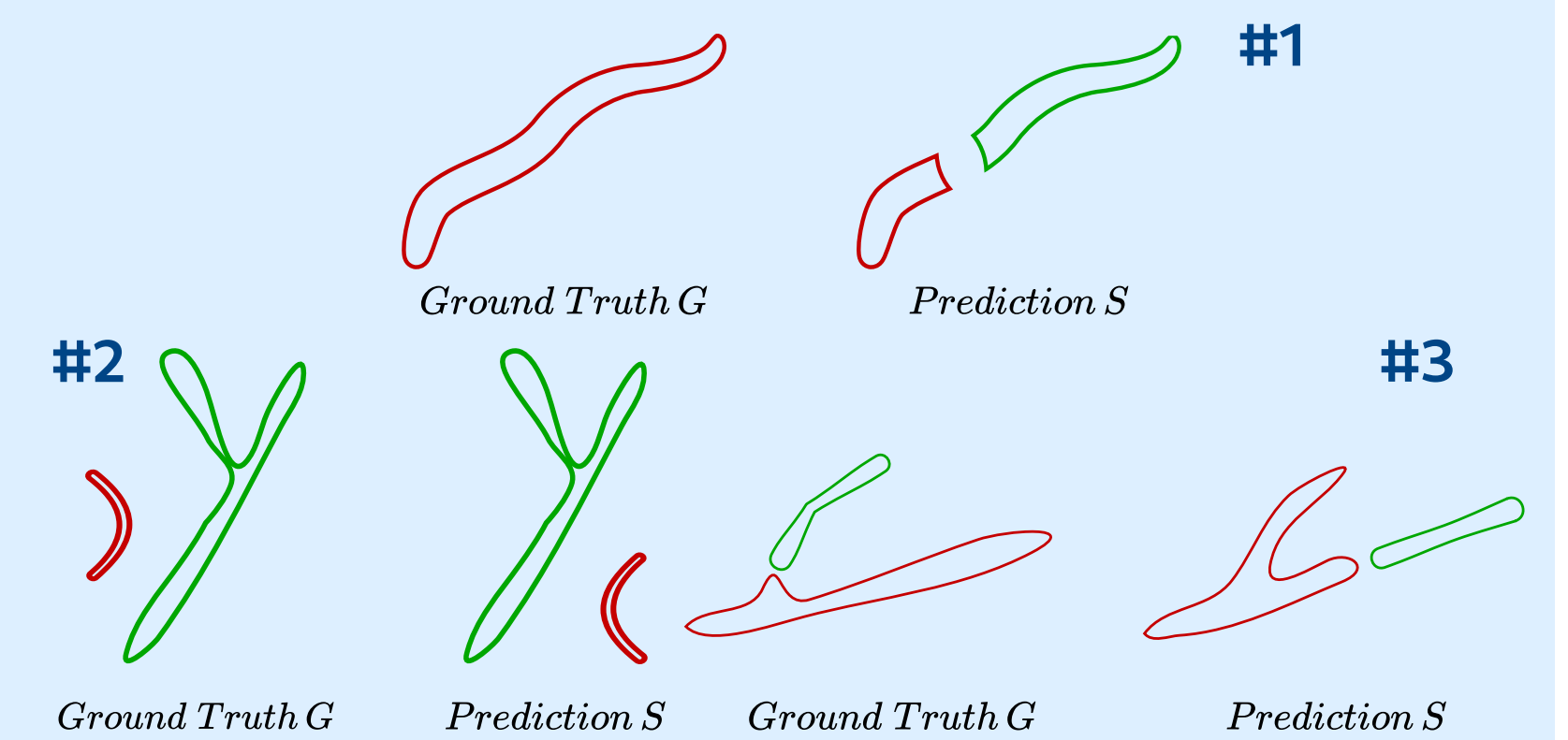
$$ccDice(S, G) = \frac{\mu(S, G) + \mu(G, S)}{\mu(S, G) + \bar{\mu}(S, G) + \mu(G, S) + \bar{\mu}(G, S)}$$

6. Experiments

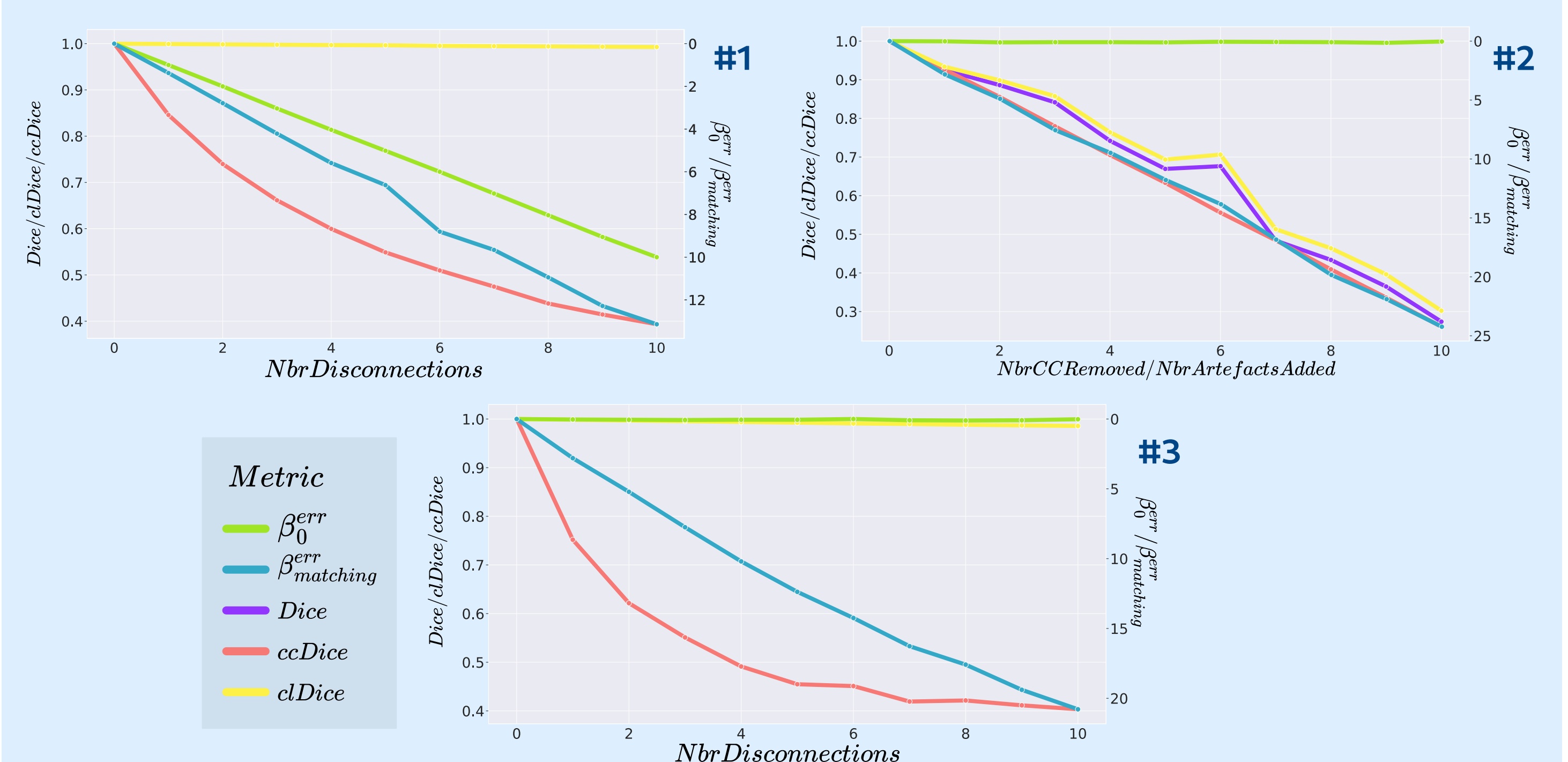
Experiment #1: Study the behavior of metrics with an increasing number of disconnections in the segmentation.

Experiment #2: Study the behavior of the metrics when the number of CCs remains constant while the pixel-wise overlap of G and S progressively decreases.

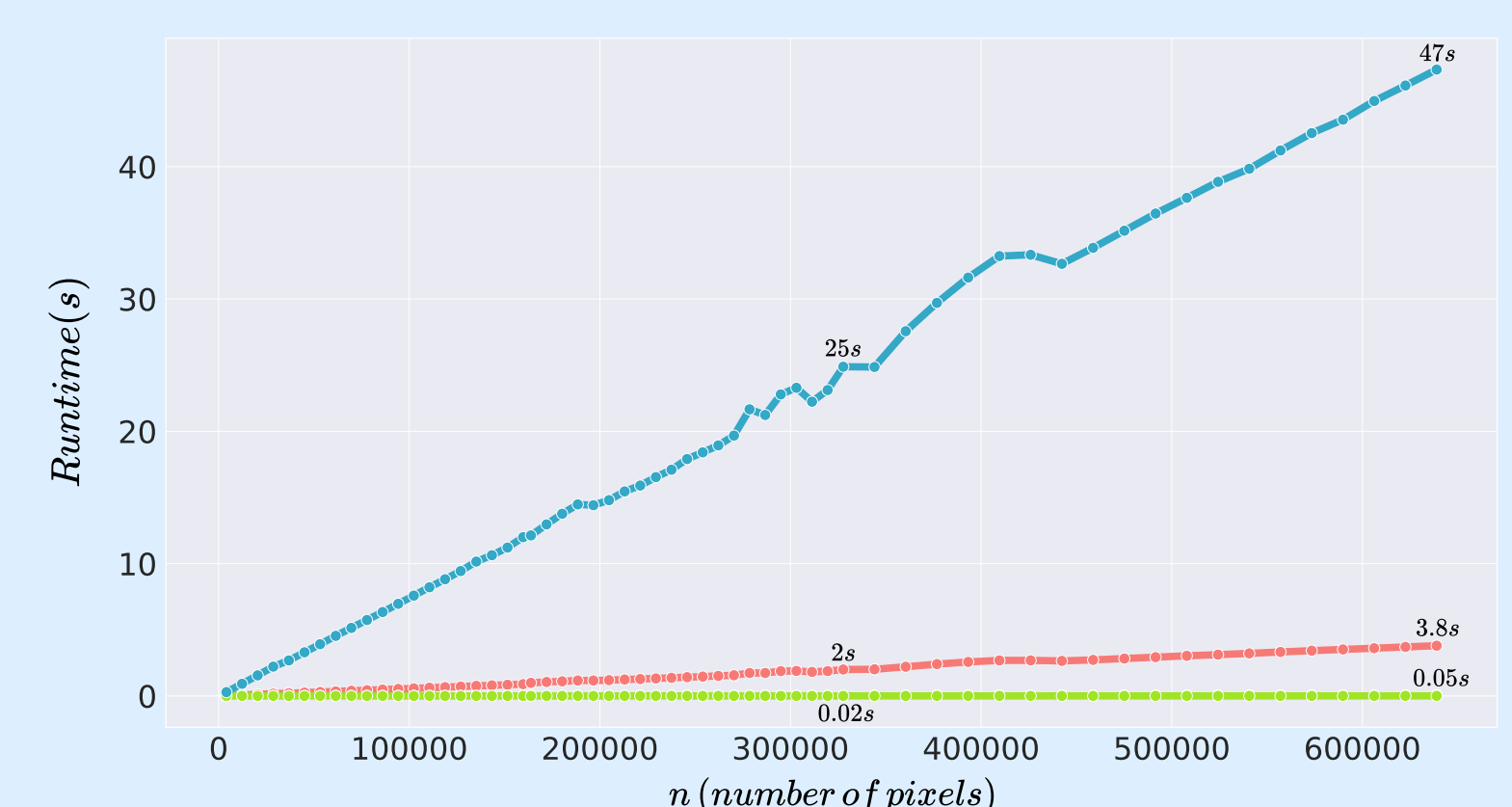
Experiment #3: Study the behavior of the metrics when the number of CCs and pixelwise overlap remain constant between S and G , but with distinct CCs.



7. Results



Only **ccDice and Betti matching** exhibit **correct behavior** in the three experiments.



ccDice is **significantly less computationally expensive** than Betti matching.

8. References

- [1] Edelsbrunner, Herbert, and John L. Harer. Computational topology: an introduction. American Mathematical Society, 2022.
- [2] Stucki, Nico, et al. "Topologically faithful image segmentation via induced matching of persistence barcodes." International Conference on Machine Learning. PMLR, 2023.