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# ccDice: A Topology-Aware Dice Score Based on Connected Components

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## Introduction

Recently, **topology** is being increasingly considered in image segmentation to improve the segmentation of objects presenting challenging shapes. In this work, we introduce ccDice (connected component Dice), a **topological metric** that generalises the popular Dice score but acts at the **scale of connected components**. **ccDice is a simple, explainable, normalized and low-computational topological metric.**

## Personal page



## Conclusion

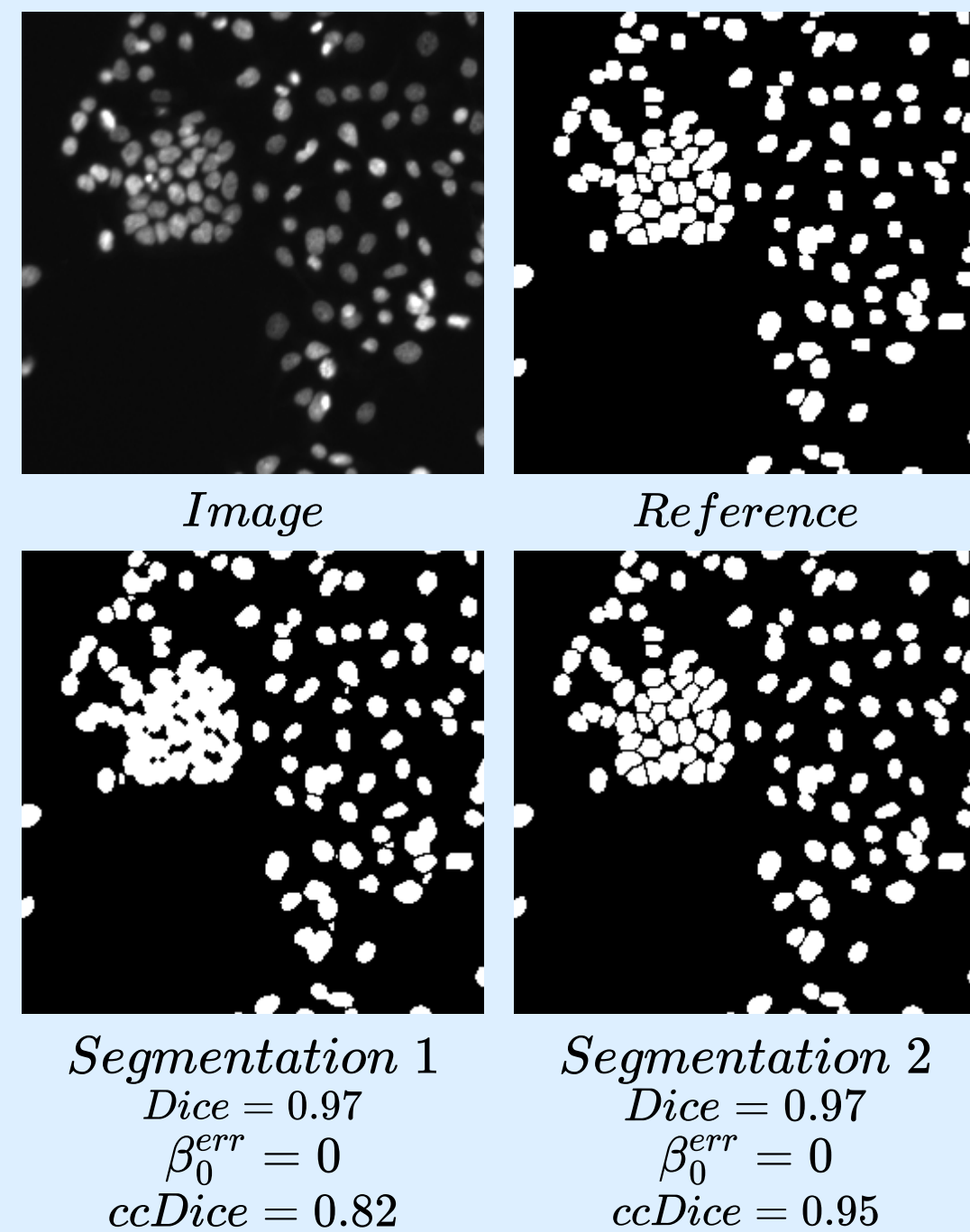
The proposed **ccDice** offers a viable alternative to the Betti matching error for **evaluating the topological properties of segmentations and incorporating spatial awareness**. It has the distinct advantages of being **normalized and computationally efficient**. Additionally, ccDice extends the widely-used Dice metric, highlighting its theoretical robustness. Future work will focus on proposing a differentiable ccDice-based loss function.

## 1. Context

**Medical image segmentation** involve objects with **complex geometries and shapes** which need to be preserved during segmentation.

For such objects, **standard metrics** (like Dice or Hausdorff distance) are often inadequate.

**Topological** metrics or loss functions allow to correctly measure the topological structure of segmentations.

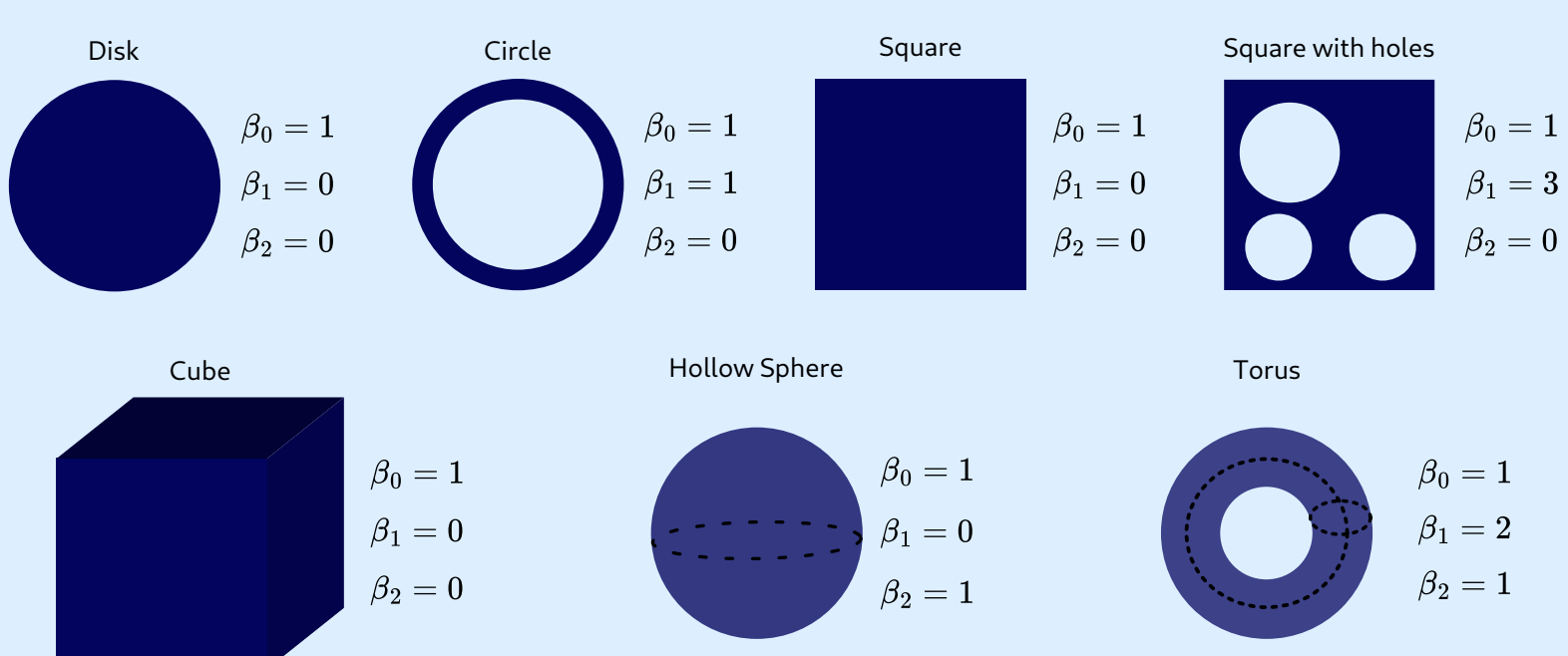


## 2. Limitation of current topological metrics

### Betti numbers [1]:

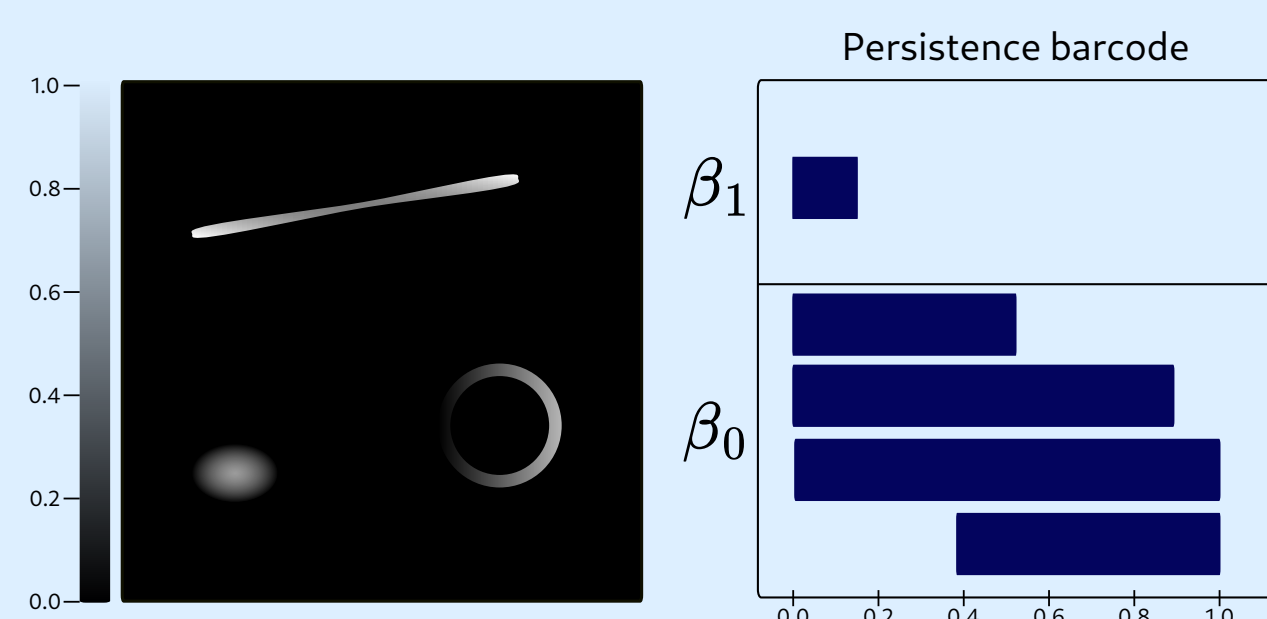
$\beta_0$ : number of connected components.  
 $\beta_1$ : number of tunnels.  
 $\beta_2$ : number of cavities.

**Limitation: not spatially correlated.**



**Betti matching [2]:** spatially match the topological objects thanks to the persistence barcodes of the respective images.

**Limitation: not normalized and expensive to compute.**



ccDice overcomes these limitations by **being spatially correlated, normalized and fast to compute.**

## 3. Dice from (mis)matching

Let  $S$  and  $G$  be two binary images that represent a segmentation result and a ground truth.

The **Dice score** is classically defined by:

$$Dice(S, G) = \frac{2|S \cap G|}{|S| + |G|} = \frac{2 \cdot tp(S, G)}{2 \cdot tp(S, G) + fp(S, G) + fn(S, G)}$$

Alternatively to these definitions we propose to view the Dice score as a combination of the number of **matching pixels**  $m$  and **mismatching pixels**  $\bar{m}$ .

$$Dice(S, G) = \frac{m(S, G) + m(G, S)}{m(S, G) + \bar{m}(S, G) + m(G, S) + \bar{m}(G, S)} \quad (1)$$

where:

$$\bar{m}(S, G) = |S \setminus G| = fp(S, G) = fn(G, S) = |S| - m(S, G)$$

$$m(S, G) = m(G, S) = |S \cap G| = tp(S, G) = tp(G, S)$$

## 4. Matching for connected components

We need to extend the notion of Dice **from points comparison to connected components (CCs) comparison** and then to generalize the notions of **(mis)matching**.

A connected component of  $S$  matches a connected component of  $G$  if their **overlap is significant** with respect to the size of  $X$ .

We define  $\varphi_{S,G}^\lambda : \mathcal{C}[S] \rightarrow \mathcal{C}[G]$  the **matching function** of connected components from  $S$  to  $G$  where  $\lambda \in (0, 1]$  is the parameter controlling the required degree of overlap.

We define the **embedding score**  $\varepsilon(X, Y)$  as a function quantifying the degree of overlap.

$$\varepsilon(X, Y) = \frac{|X \cap Y|}{X} \in [0, 1]$$

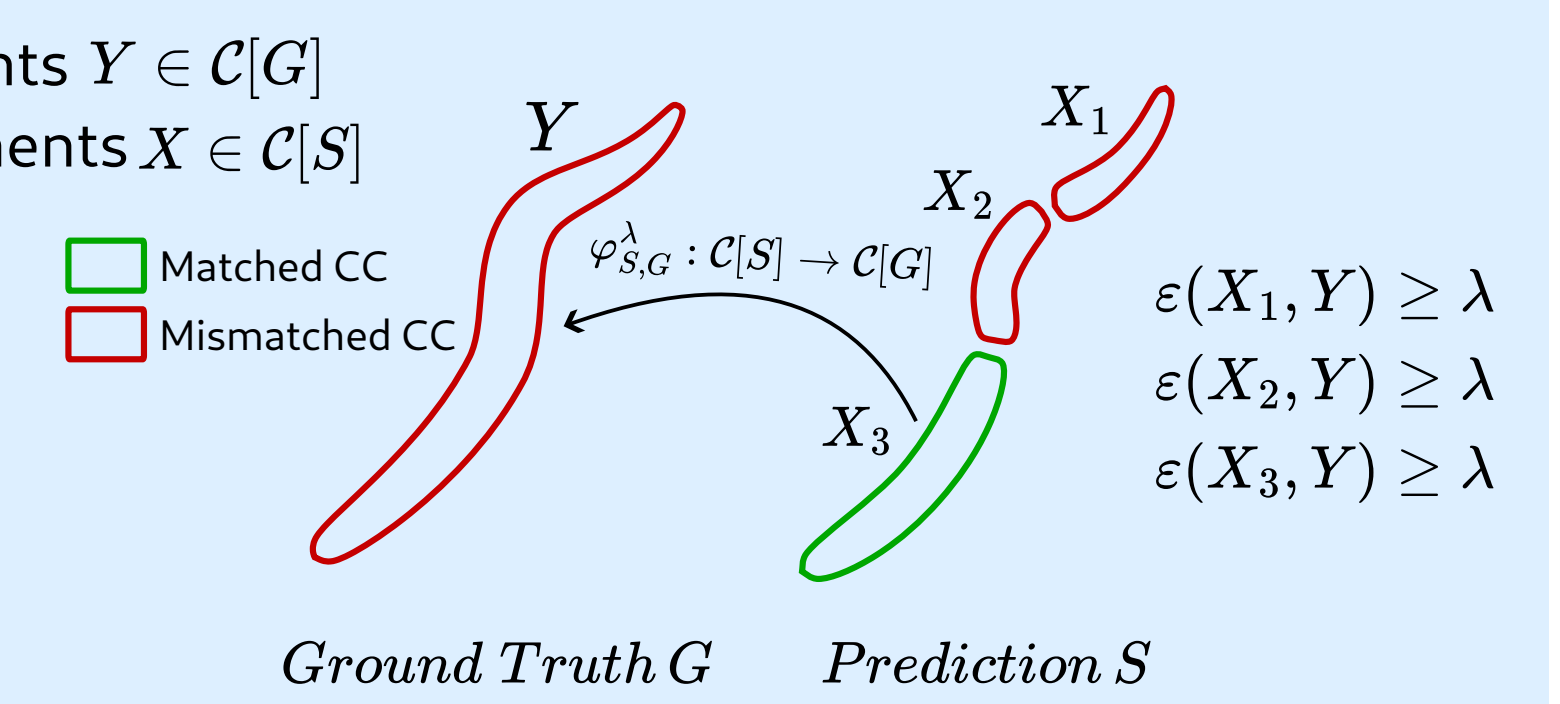
Then, the matching  $\varphi_{S,G}^\lambda$  is defined such that for any  $X \in \mathcal{C}[S]$ , we have:

$$\varepsilon(X, \varphi_{S,G}^\lambda(X)) \geq \lambda.$$

## 5. From Dice to ccDice

With this definition each connected components  $Y \in \mathcal{C}[G]$  can be associated to many connected components  $X \in \mathcal{C}[S]$

To avoid this we force  $\varphi_{S,G}^\lambda$  to be **injective**.



When  $\varphi_{S,G}^\lambda$  is built we can simply define the number of matching  $\mu$  and mistaching pixels  $\bar{\mu}$  as:

$$\mu(S, G) = |\{X \in \mathcal{C}[S] \mid \exists Y \in \mathcal{C}[G], Y = \varphi_{S,G}^\lambda(X)\}|$$

$$\bar{\mu}(S, G) = |\{X \in \mathcal{C}[S] \mid \forall Y \in \mathcal{C}[G], Y \neq \varphi_{S,G}^\lambda(X)\}|$$

We can now define the notion of ccDice by substituting  $m$  and  $\bar{m}$  by  $\mu$  and  $\bar{\mu}$  in equation (1).

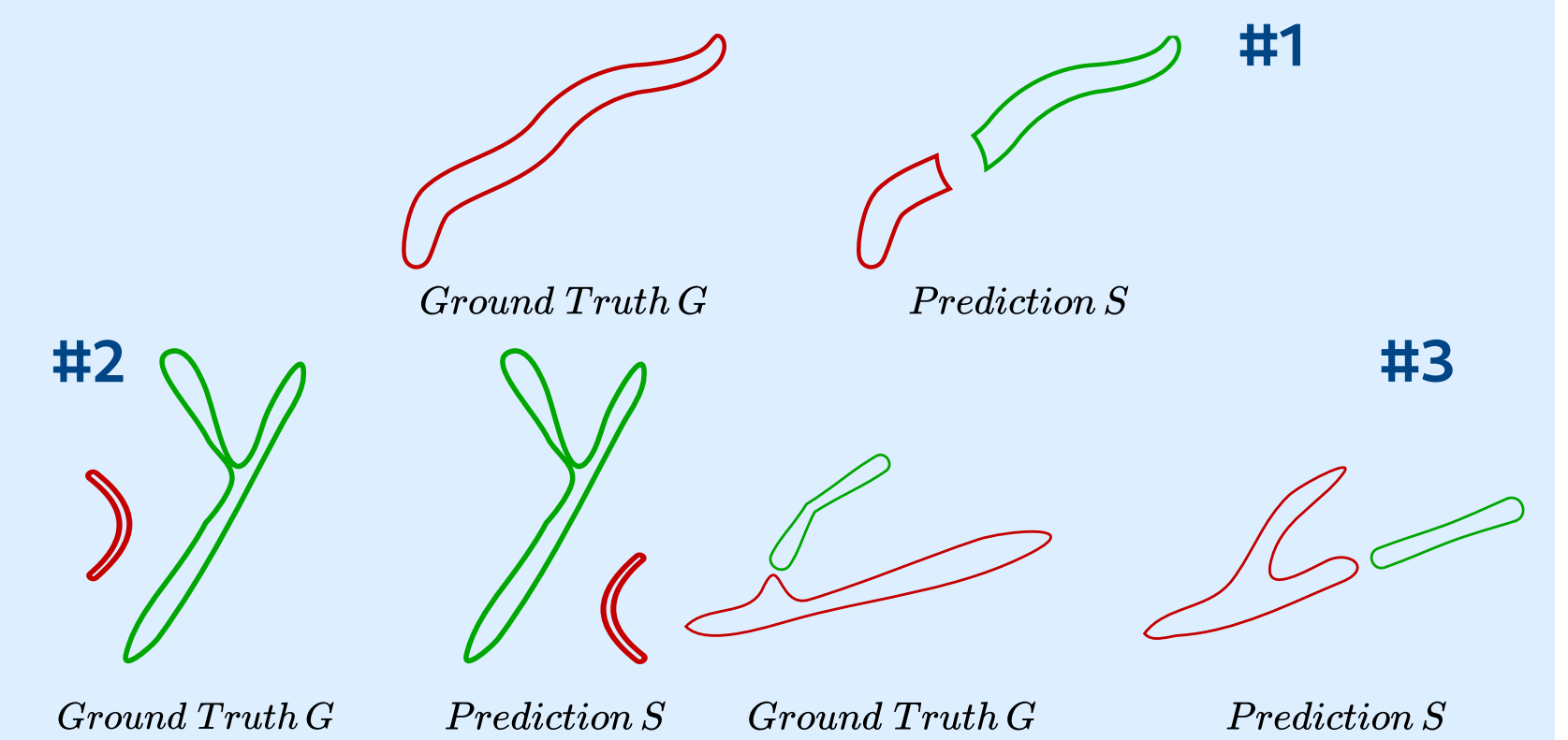
$$ccDice(S, G) = \frac{\mu(S, G) + \mu(G, S)}{\mu(S, G) + \bar{\mu}(S, G) + \mu(G, S) + \bar{\mu}(G, S)}$$

## 6. Experiments

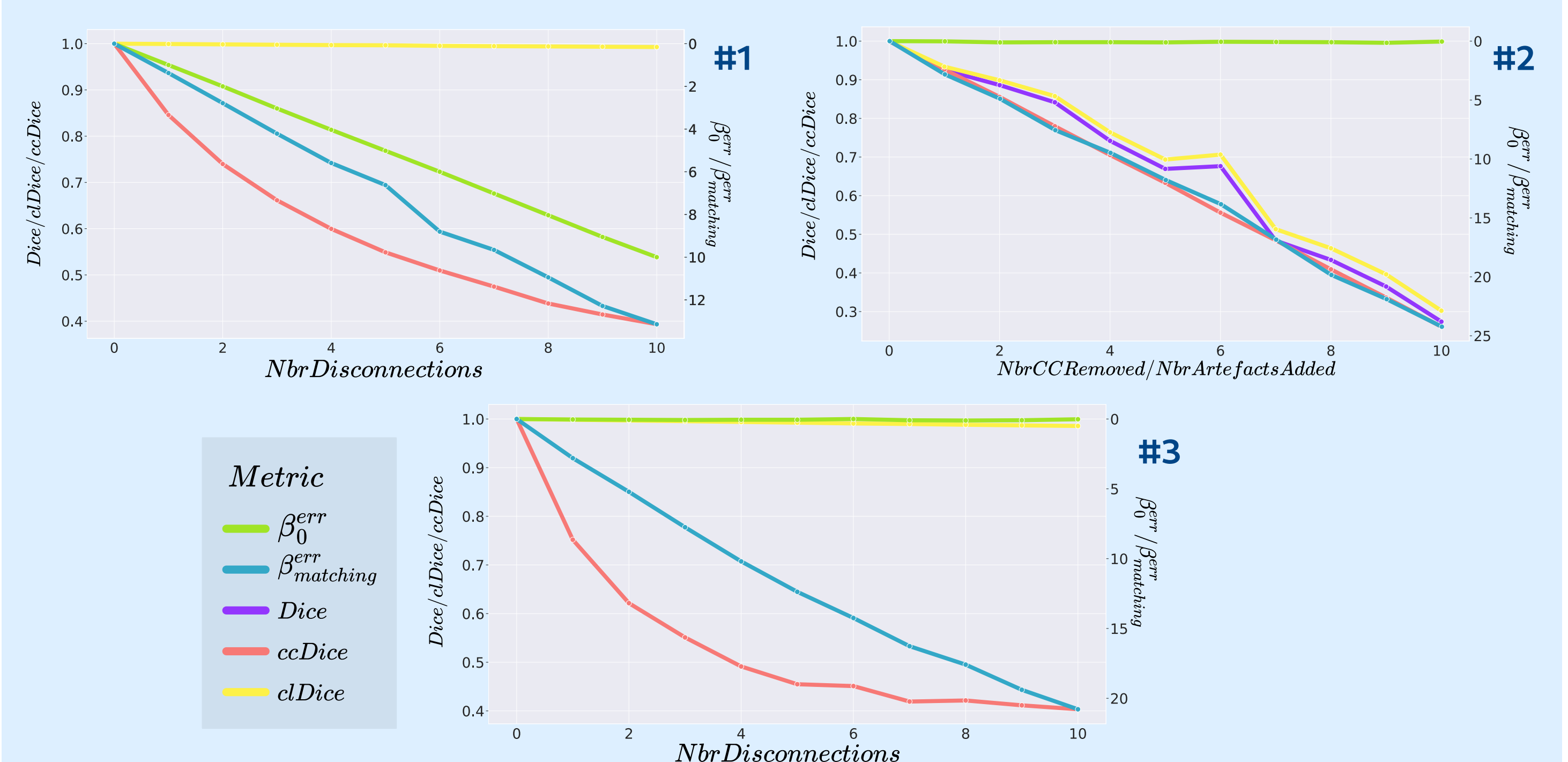
**Experiment #1:** Study the behavior of metrics with an increasing number of disconnections in the segmentation.

**Experiment #2:** Study the behavior of the metrics when the number of CCs remains constant while the pixel-wise overlap of  $G$  and  $S$  progressively decreases.

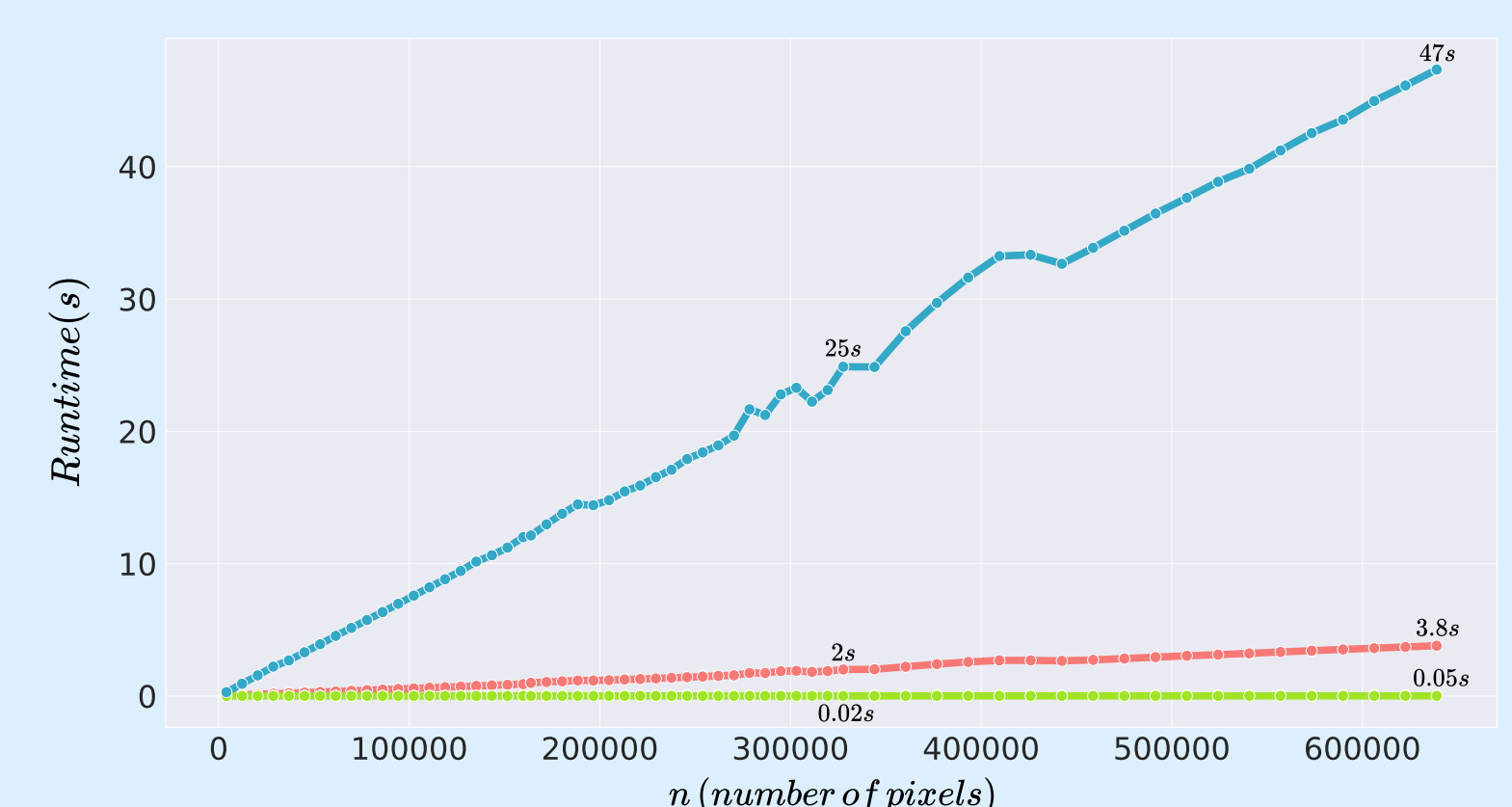
**Experiment #3:** Study the behavior of the metrics when the number of CCs and pixelwise overlap remain constant between  $S$  and  $G$ , but with distinct CCs.



## 7. Results



Only **ccDice and Betti matching** exhibit **correct behavior** in the three experiments.



ccDice is **significantly less computationally expensive** than Betti matching.

## 8. References

- [1] Edelsbrunner, Herbert, and John L. Harer. Computational topology: an introduction. American Mathematical Society, 2022.
- [2] Stucki, Nico, et al. "Topologically faithful image segmentation via induced matching of persistence barcodes." International Conference on Machine Learning. PMLR, 2023.