



ccDice: A Topology-Aware Dice Score Based on Connected Components

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Odyssée Merveille²

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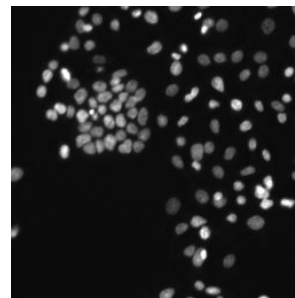
²CREATIS; CNRS (UMR 5220); INSERM (U1294); INSA Lyon; Université de Lyon, France

Context

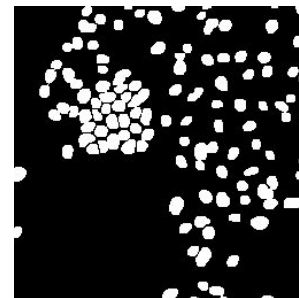
Medical image segmentation involves objects with **complex geometries and shapes** which need to be preserved during segmentation.

For such objects, **standard metrics** (like Dice or Hausdorff distance) are often **inadequate**.

Topological metrics or loss functions allow to correctly measure the topological structure of segmentations.

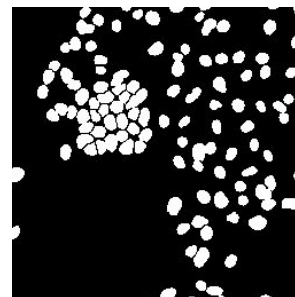


Image

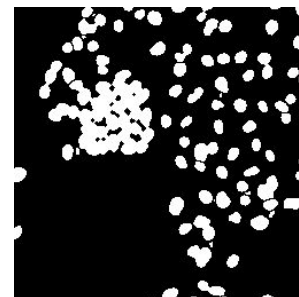


Prediction 1

$Dice = 0.97$



GT

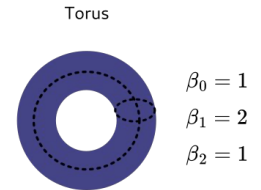
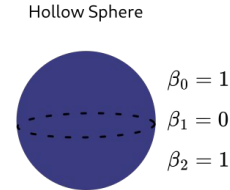
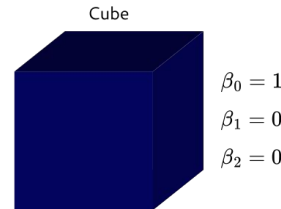
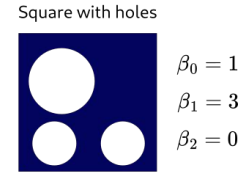
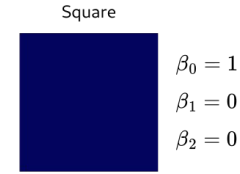
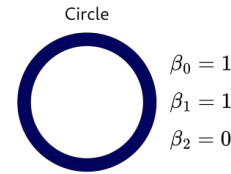
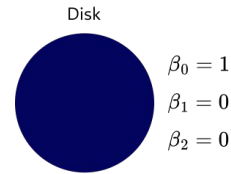


Prediction 2

$Dice = 0.97$

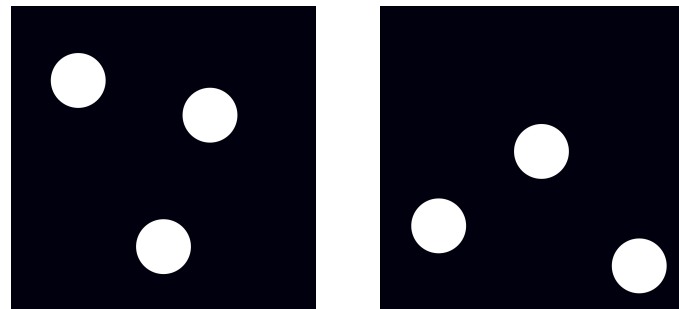
Limitations of the current topological metrics

- **Betti numbers:**
 - β_0 : number of connected components.
 - β_1 : number of holes(2D)/tunnels(3D).
 - β_2 : number of cavities.
- **Limitation: not spatially correlated.**



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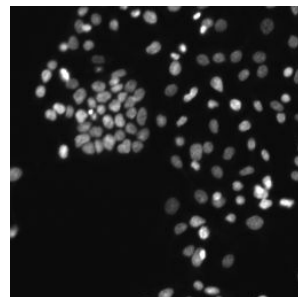
Reference

Prediction

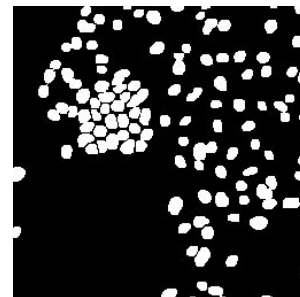
$$\beta_0^{err} = |\beta_0^{pred} - \beta_0^{ref}| = 0$$

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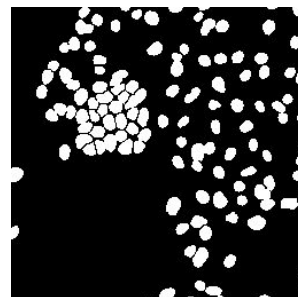
Image



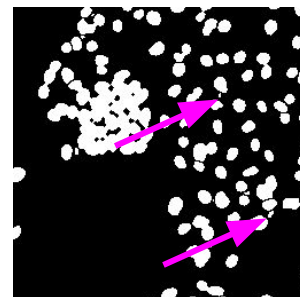
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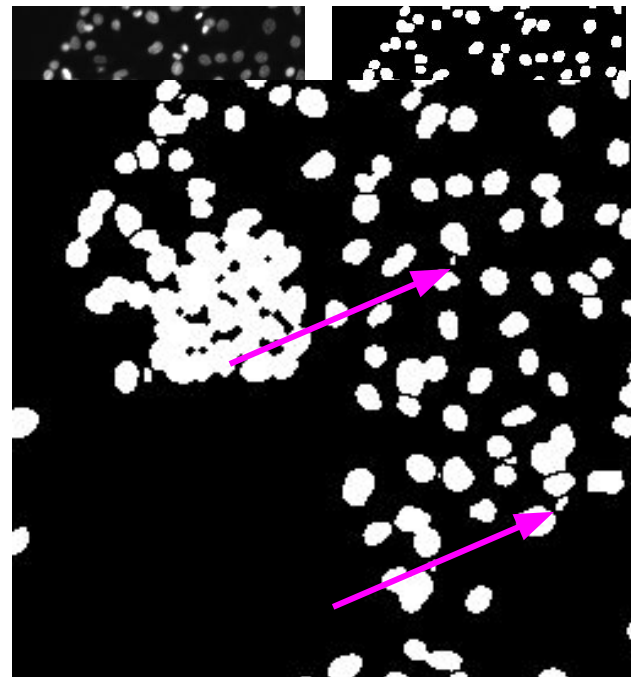
Prediction 2

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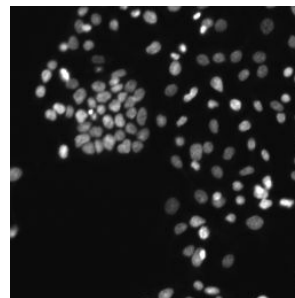
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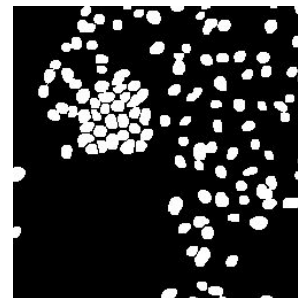
Prediction 2

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 - β_0 : number of connected components.
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- **Limitation: not spatially correlated.**
- **Betti matching (Stucki et al. 2022)**
 - spatially match the topological objects thanks to the persistence barcodes of the respective images.
- **Limitation: not normalized and expensive to compute.**



Image

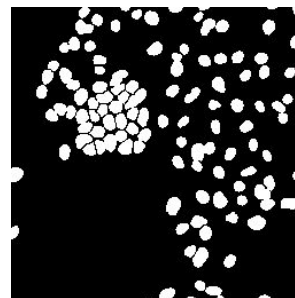


Prediction 1

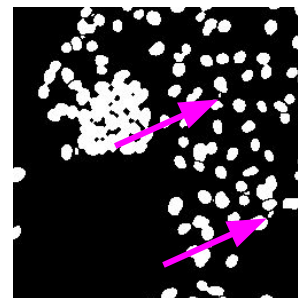
$$Dice = 0.97$$

$$\beta_0^{err} = 0$$

$$\beta_{match}^{err} = 0$$



GT



Prediction 2

$$Dice = 0.97$$

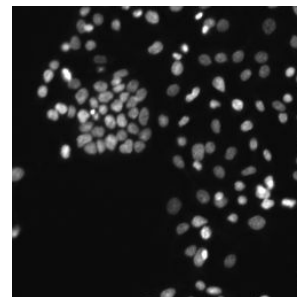
$$\beta_0^{err} = 0$$

$$\beta_{match}^{err} = 26$$

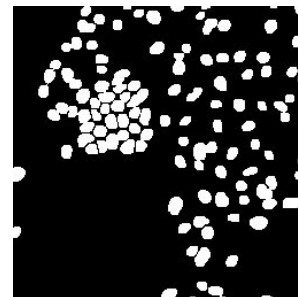
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ccDice overcomes these limitations by being **spatially correlated**, **normalized** and **fast to compute**.

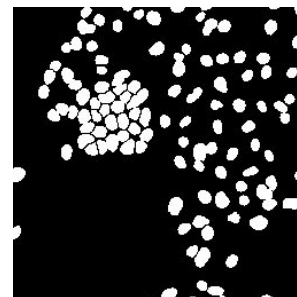


Image

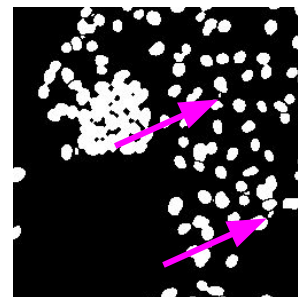


Prediction 1

$$\begin{aligned}
 \text{Dice} &= 0.97 \\
 \beta_0^{\text{err}} &= 0 \\
 \beta_{\text{match}}^{\text{err}} &= 0 \\
 \text{ccDice} &= 0.95
 \end{aligned}$$



GT



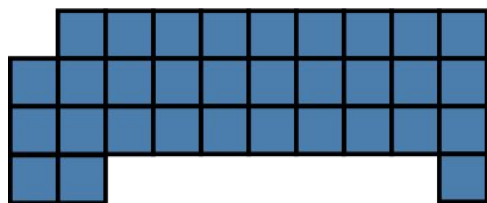
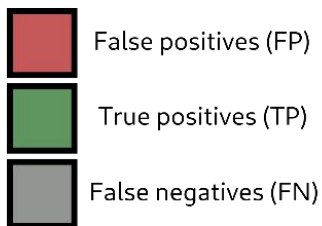
Prediction 2

$$\begin{aligned}
 \text{Dice} &= 0.97 \\
 \beta_0^{\text{err}} &= 0 \\
 \beta_{\text{match}}^{\text{err}} &= 26 \\
 \text{ccDice} &= 0.82
 \end{aligned}$$

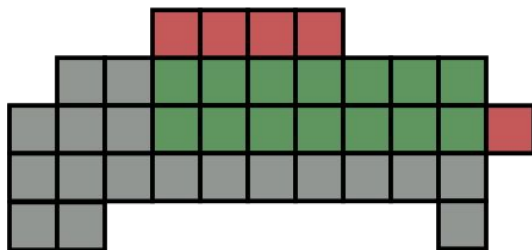
From Dice to ccDice

Let S, G be two binary images representing a segmentation and a ground truth.

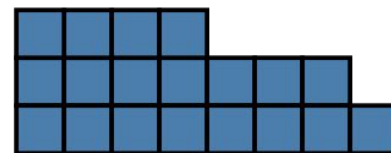
$$Dice(S, G) = \frac{2 \cdot tp(S, G)}{2 \cdot tp(S, G) + fp(S, G) + fn(S, G)}$$



Ground Truth G



Intersection

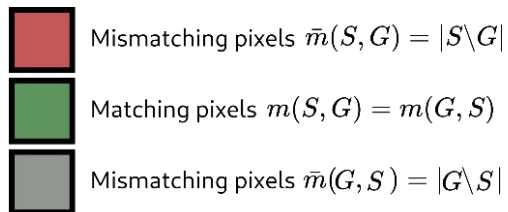


Prediction S

From Dice to ccDice

We can define *Dice* in terms of number of matching m and mismatching \bar{m} pixels.

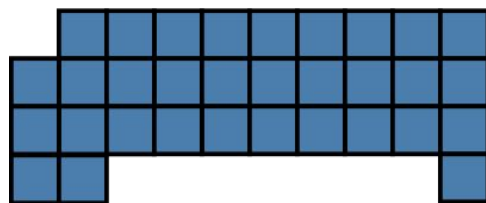
$$Dice(S, G) = \frac{m(S, G) + m(G, S)}{m(S, G) + \bar{m}(S, G) + m(G, S) + \bar{m}(G, S)}$$



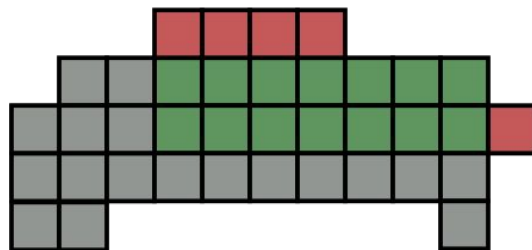
with :

$$m(S, G) = m(G, S) = |S \cap G|$$

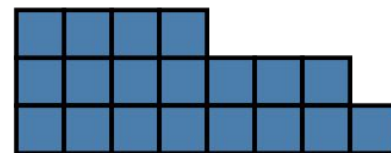
$$\bar{m}(S, G) = |S \setminus G|$$



Ground Truth G



Intersection



Prediction S

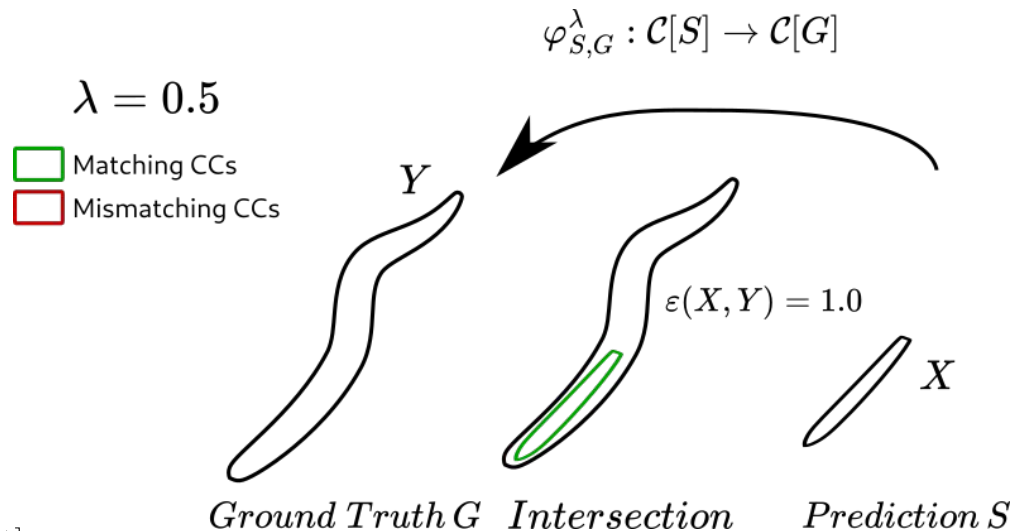
How to define (mis)matching for connected components?

First case

- S and G : segmentation and ground truth.
- $\mathcal{C}[G]$ and $\mathcal{C}[S]$ sets of CCs
- $X \in \mathcal{C}[S] \quad Y \in \mathcal{C}[G]$

- We define $\varphi_{S,G}^\lambda : \mathcal{C}[S] \rightarrow \mathcal{C}[G]$ the **matching function** of CCs.
- **Embedding score** $\mathcal{E}(X, Y) = \frac{|X \cap Y|}{|X|} \in [0, 1]$
- Then the **matching function** $\varphi_{S,G}^\lambda : \mathcal{C}[S] \rightarrow \mathcal{C}[G]$ is defined for an $X \in \mathcal{C}[S]$ such that:

$$\mathcal{E}(X, \varphi_{S,G}^\lambda(X)) \geq \lambda$$

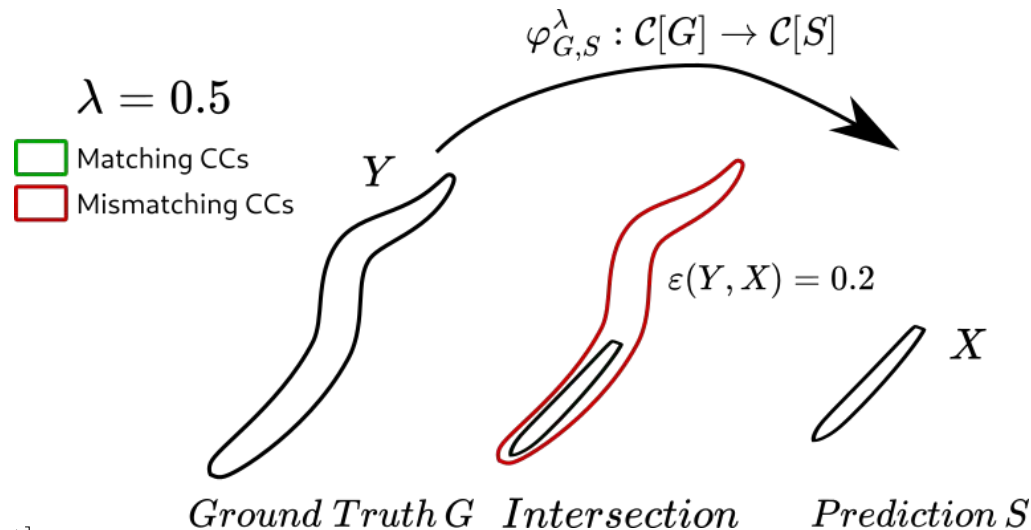


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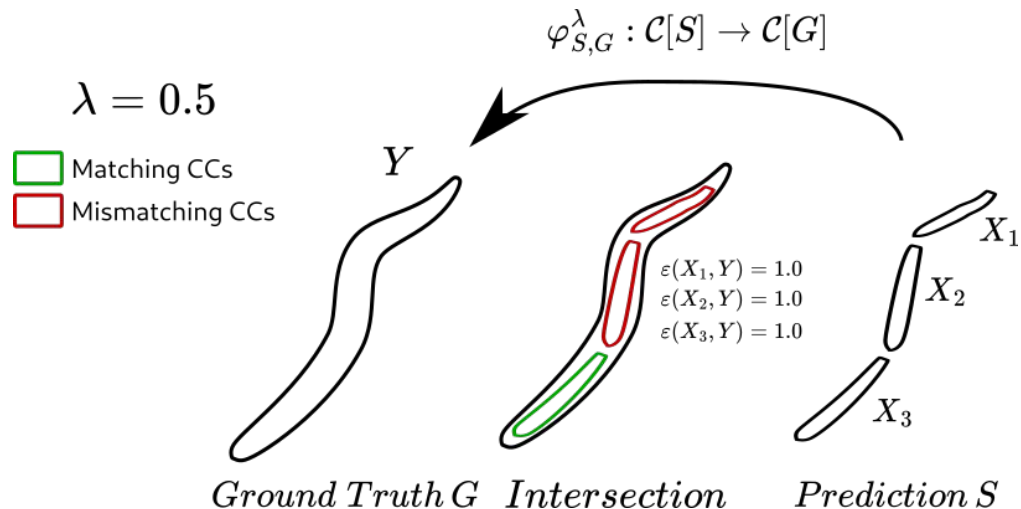


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Second case

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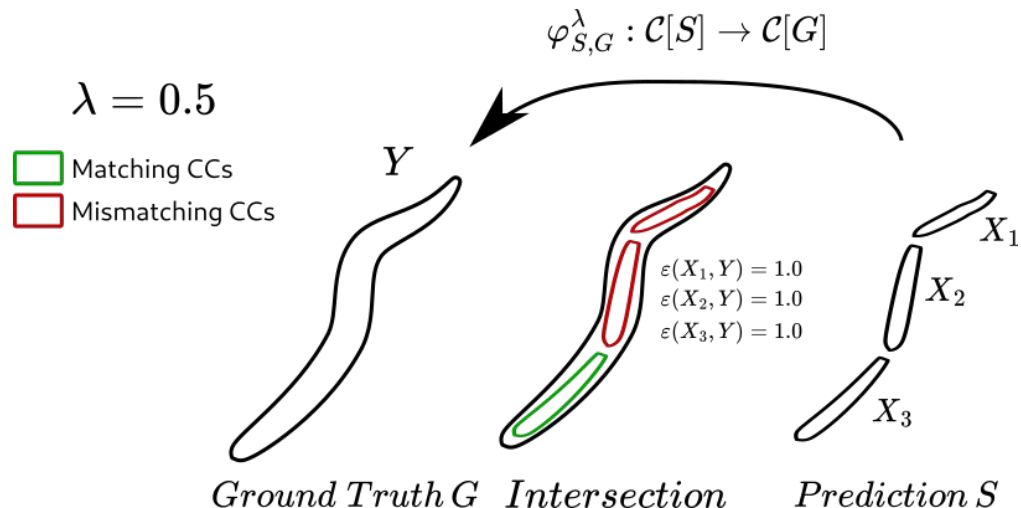
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- Note:
 - we force $\varphi_{S,G}^\lambda$ being **injective**.

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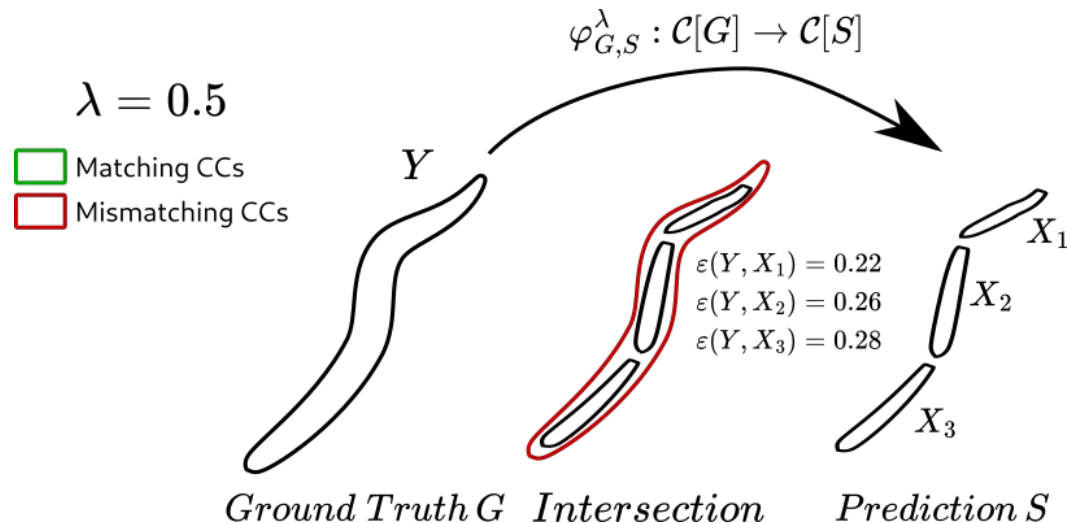
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- Note:
 - we force $\varphi_{S,G}^\lambda$ being **injective**.

From Dice to ccDice: matching for connected components

Based on the introduced notions, we can now define formally the number of matching μ and mismatching $\bar{\mu}$ connected components.

$$\mu(S, G) = |\{X \in \mathcal{C}[S] \mid \exists Y \in \mathcal{C}[G], Y = \varphi_{S,G}^\lambda(X)\}|$$
$$\bar{\mu}(S, G) = |\{X \in \mathcal{C}[S] \mid \forall Y \in \mathcal{C}[G], Y \neq \varphi_{S,G}^\lambda(X)\}|$$

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Then we can define ccDice (connected component Dice) from the previous definition of Dice.

From:

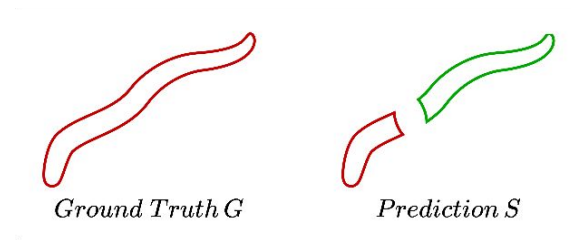
$$Dice(S, G) = \frac{m(S, G) + m(G, S)}{m(S, G) + \bar{m}(S, G) + m(G, S) + \bar{m}(G, S)}$$

To:

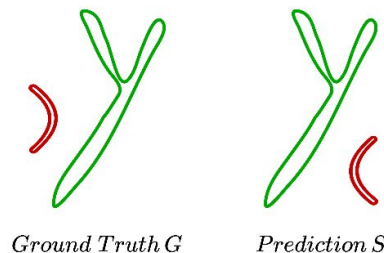
$$ccDice(S, G) = \frac{\mu(S, G) + \mu(G, S)}{\mu(S, G) + \bar{\mu}(S, G) + \mu(G, S) + \bar{\mu}(G, S)}$$

Experiments

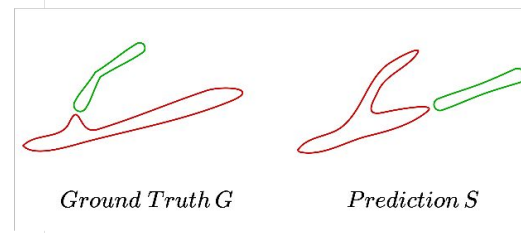
- Experiment #1: Study the behaviour of metrics with an **increasing number of disconnections in the segmentation**.
- Experiment #2: Investigate the metrics when the connected components **are not spatially coherent and not overlapped**.
- Experiment #3: Analyse the metrics when the connected components **are not spatially coherent and overlapped**.
- **Goal:** compare ccDice to other metrics.



Experiment #1



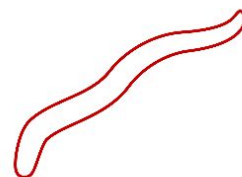
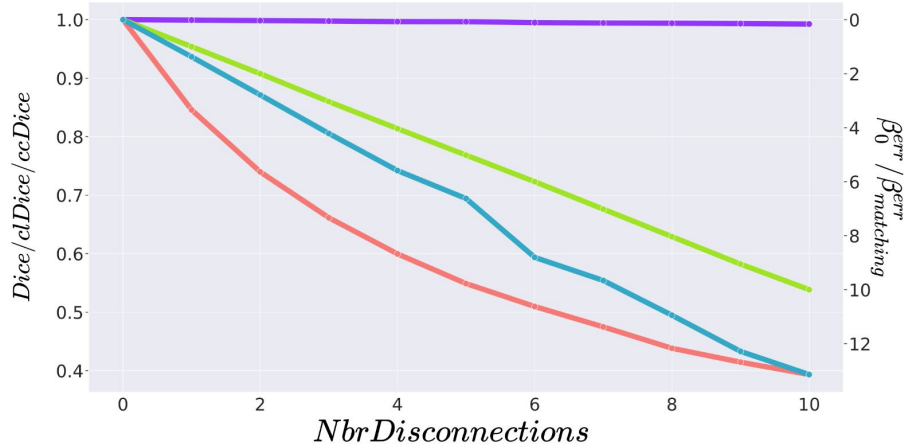
Experiment #2



Experiment #3

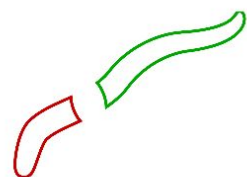
Results: experiment #1

- Experiment #1: Study the behaviour of metrics with an **increasing number of disconnections** in the segmentation.



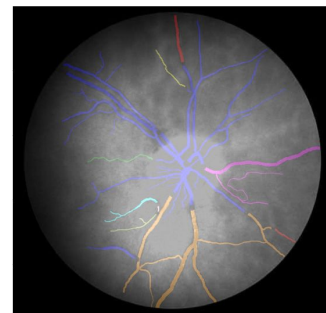
Ground Truth G

G



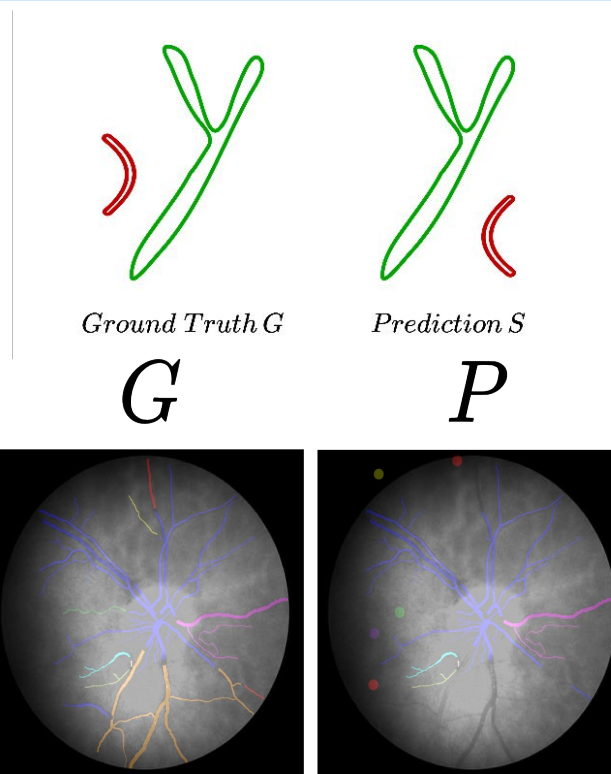
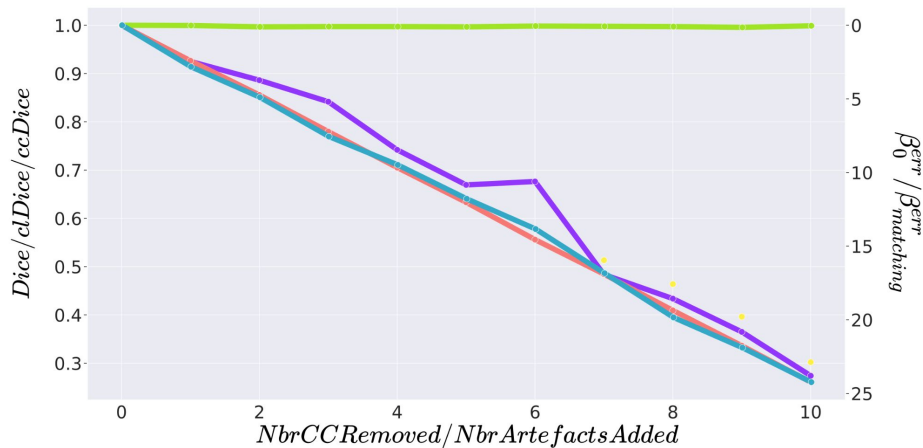
Prediction S

P



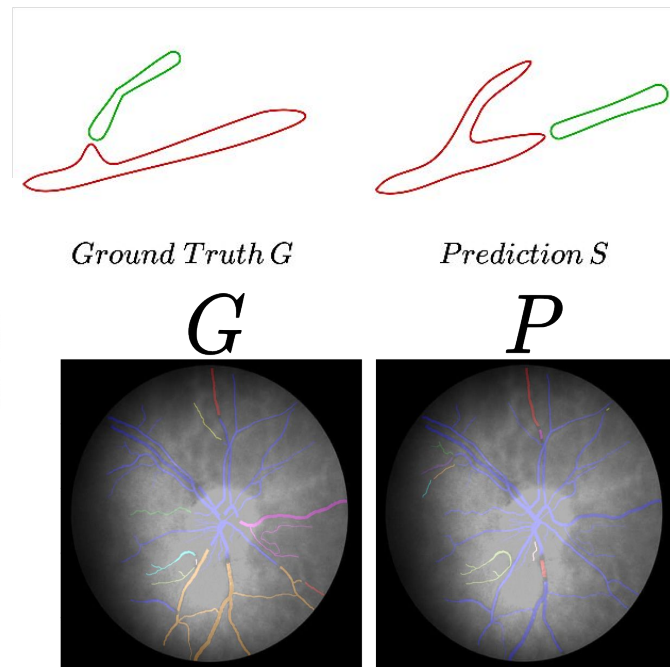
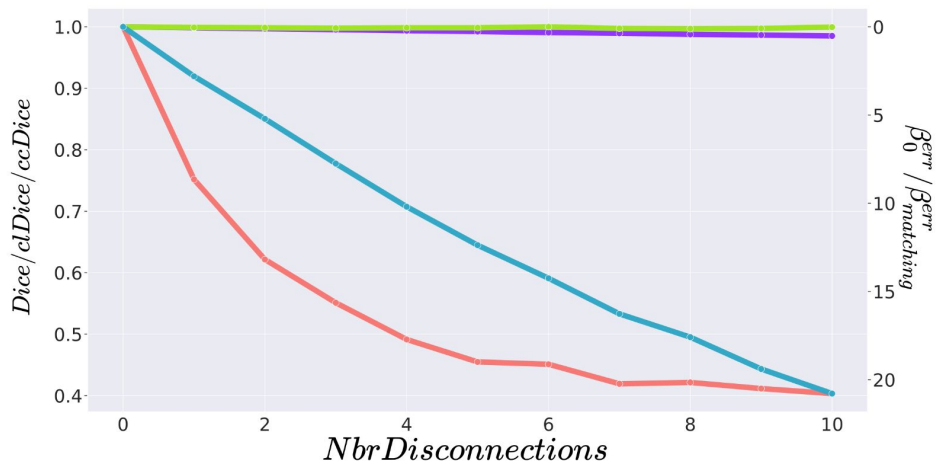
Results: experiment #2

- Experiment #2: Investigate the metrics when the connected components **are not spatially coherent and not overlapped**.



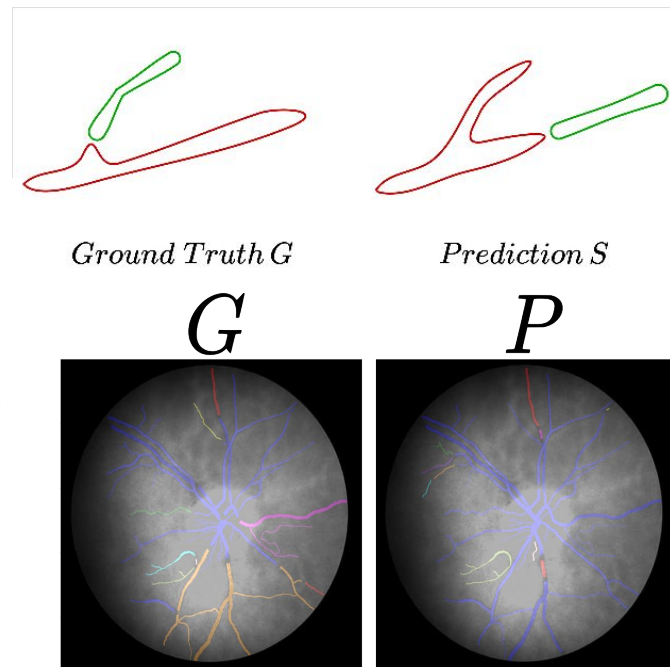
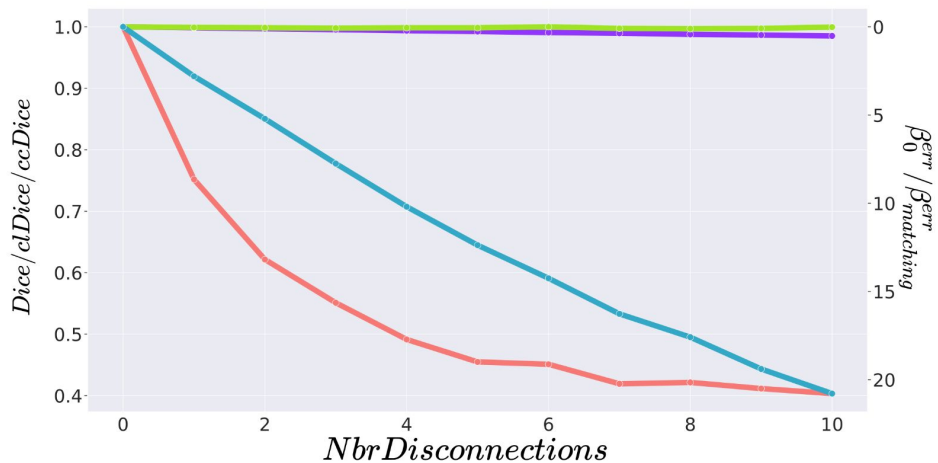
Results: experiment #3

- Experiment #3: Analyse the metrics when the connected components **are not spatially coherent and overlapped**.



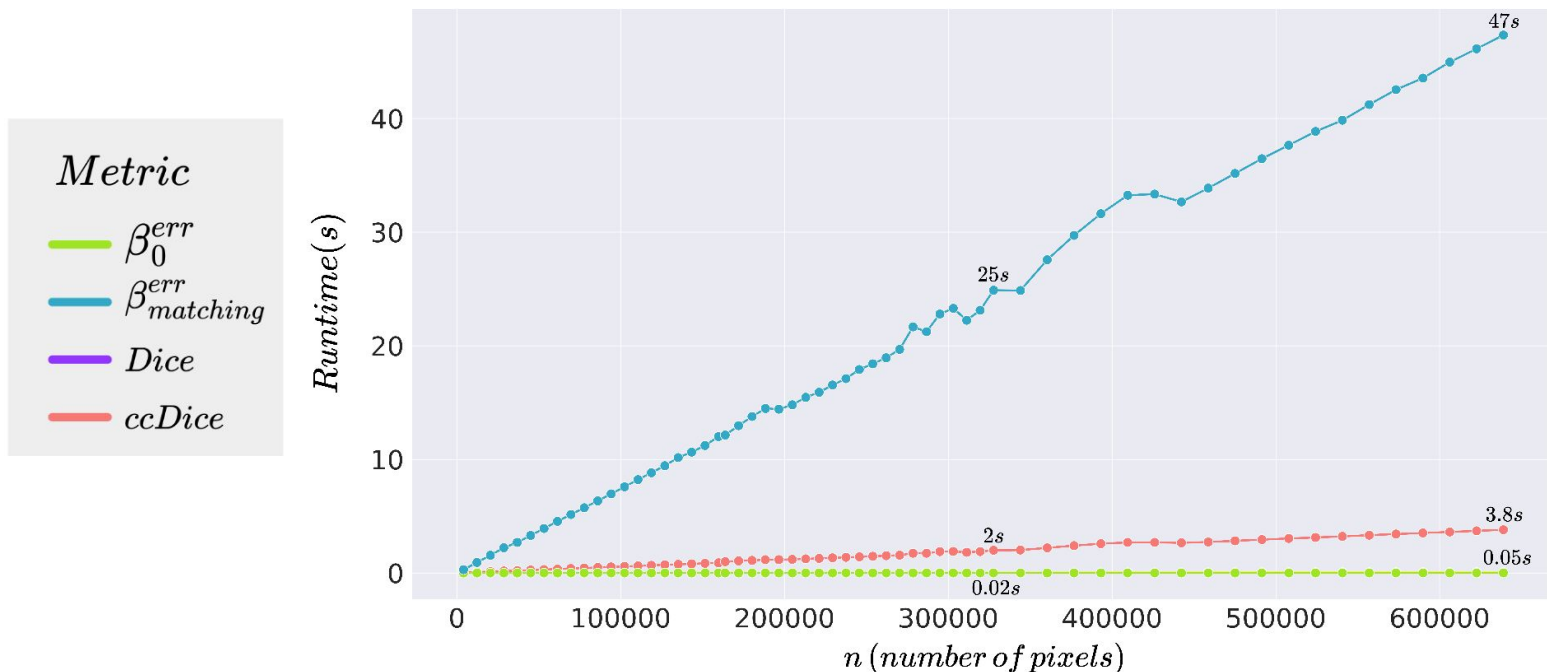
Results: experiment #3

- Experiment #3: Analyse the metrics when the connected components **are not spatially coherent and overlapped**.



- Only **ccDice** and **Betti matching** exhibit **correct behavior** in the three experiments.

Runtime analysis: average runtime for one image



- ccDice is **significantly less computationally expensive** than Betti matching.

Conclusion

- We proposed a new metric called **ccDice** based on the spatial matching of connected components.
- We evaluated its behaviour on three representative experiments

	β_{match}^{err}	<i>ccDice</i>
Spatial coherence	✓	✓
Computation time	✗	✓
Normalisation	✗	✓
Implementation	2D	2D/3D
Homology groups	All	In 2D : All In 3D : first and last
Differentiability	✓	✗

Paper



Contact

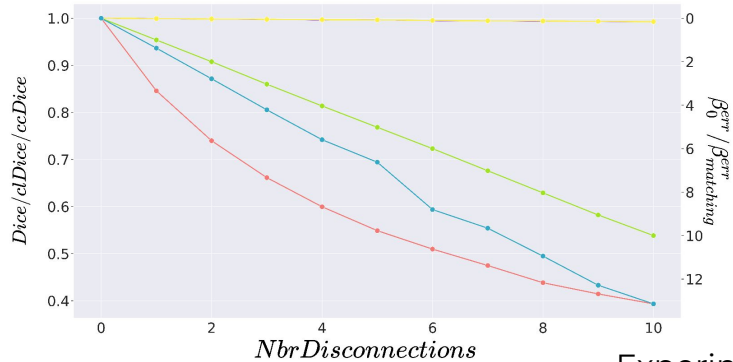


Thank you for your attention !

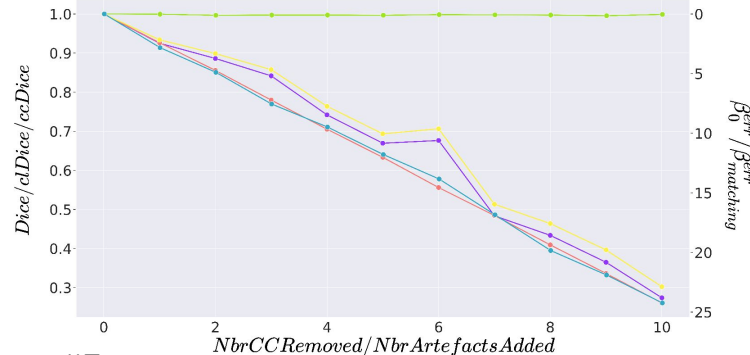
Backup slides

Results

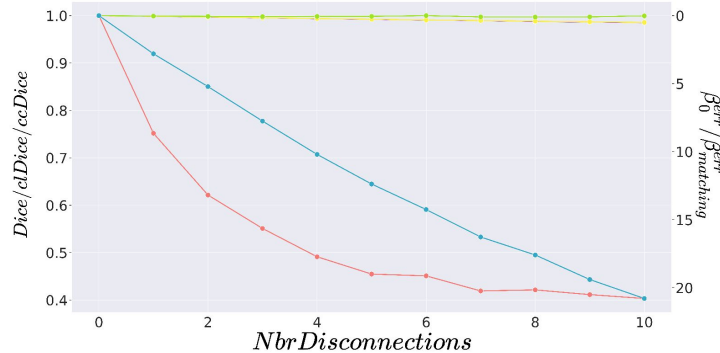
Experiment #1



Experiment #2



Experiment #3



Metric

- β_0^{err}
- $\beta_{matching}^{err}$
- *Dice*
- *ccDice*
- *clDice*

Algorithm ccDice

Algorithm 1: Compute ccDice

Input: $S, G \subseteq \Omega$
Input: $\lambda \in (0, 1]$
Output: ccDice $\in [0, 1]$

- 1 **Build** $\mathcal{C}[S]$
- 2 **Build** $\mathcal{C}[G]$
- 3 $\mu(S, G) := \text{Compute Matching}(\mathcal{C}[S], \mathcal{C}[G], \lambda)$
- 4 $\mu(G, S) := \text{Compute Matching}(\mathcal{C}[G], \mathcal{C}[S], \lambda)$
- 5 ccDice := $(\mu(S, G) + \mu(G, S)) / (|\mathcal{C}[S]| + |\mathcal{C}[G]|)$

Algorithm 2: Compute matching

Input: $\mathcal{C}[S] = \{X_i\}_{i=1}^t$
Input: $\mathcal{C}[G] = \{Y_j\}_{j=1}^u$
Input: $\lambda \in (0, 1]$
Output: $\mu(S, G)$

- 1 **Build** $\{\varepsilon_{i,j}\}_{(i,j) \in \llbracket 1,t \rrbracket \times \llbracket 1,u \rrbracket}$
- 2 **Sort** $E = \{(i, j) \mid \varepsilon_{i,j} \geq \lambda\}$ by decreasing values of $\varepsilon_{i,j}$
- 3 $\mu(S, G) := 0$
- 4 **foreach** $(i, j) \in E$ (sorted) **do**
- 5 **if** (i, \star) and (\star, j) are not discarded **then**
- 6 $\mu(S, G) := \mu(S, G) + 1$
- 7 **Discard** (i, \star)
- 8 **Discard** (\star, j)

How to define (mis)matching for connected components ?

First case

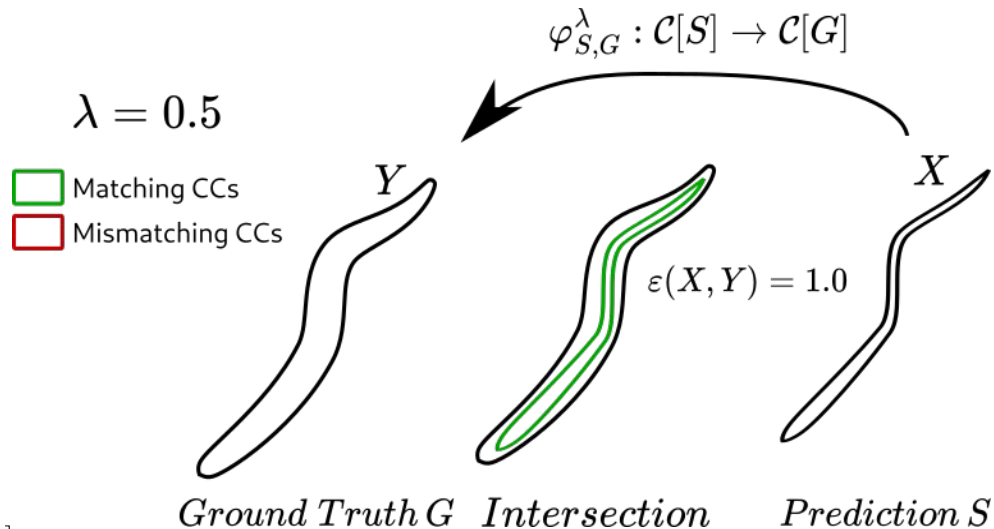
- S and G : segmentation and ground truth.
- $\mathcal{C}[G]$ and $\mathcal{C}[S]$ sets of CCs
- $X \in \mathcal{C}[S]$ $Y \in \mathcal{C}[G]$

- We define $\varphi_{S,G}^\lambda : \mathcal{C}[S] \rightarrow \mathcal{C}[G]$ the **matching function** of CCs.

- **Embedding score** $\mathcal{E}(X, Y) = \frac{|X \cap Y|}{|X|} \in [0, 1]$

- Then the **matching function** $\varphi_{S,G}^\lambda : \mathcal{C}[S] \rightarrow \mathcal{C}[G]$ is defined for an $X \in \mathcal{C}[S]$ such that:

$$\mathcal{E}(X, \varphi_{S,G}^\lambda(X)) \geq \lambda$$



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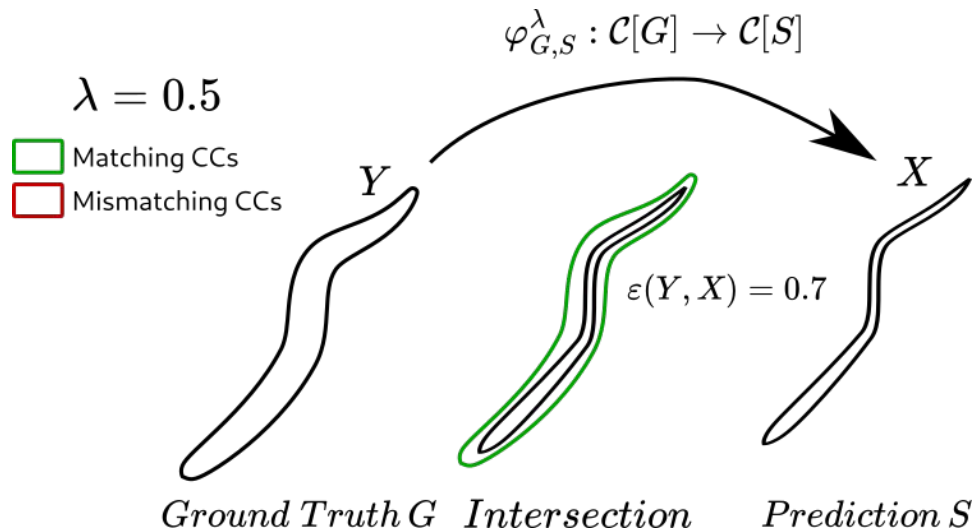
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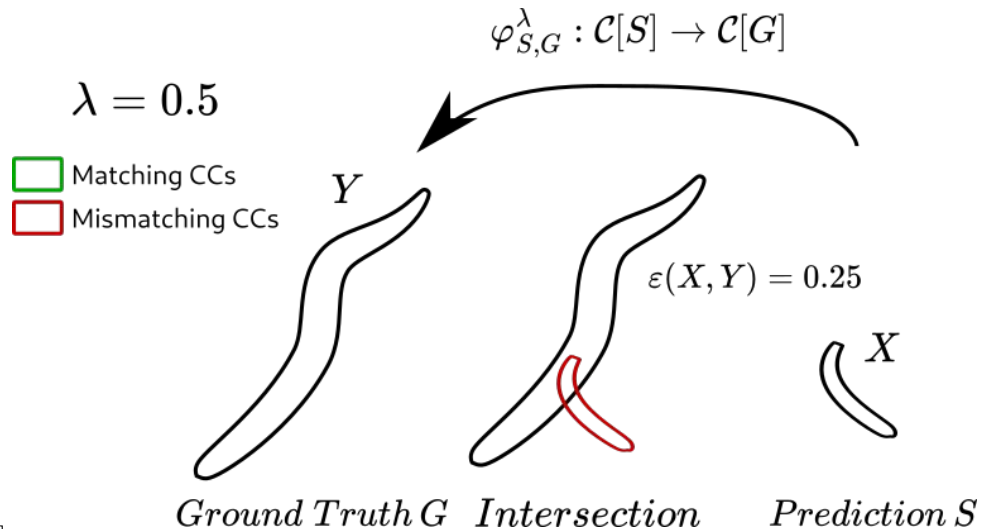
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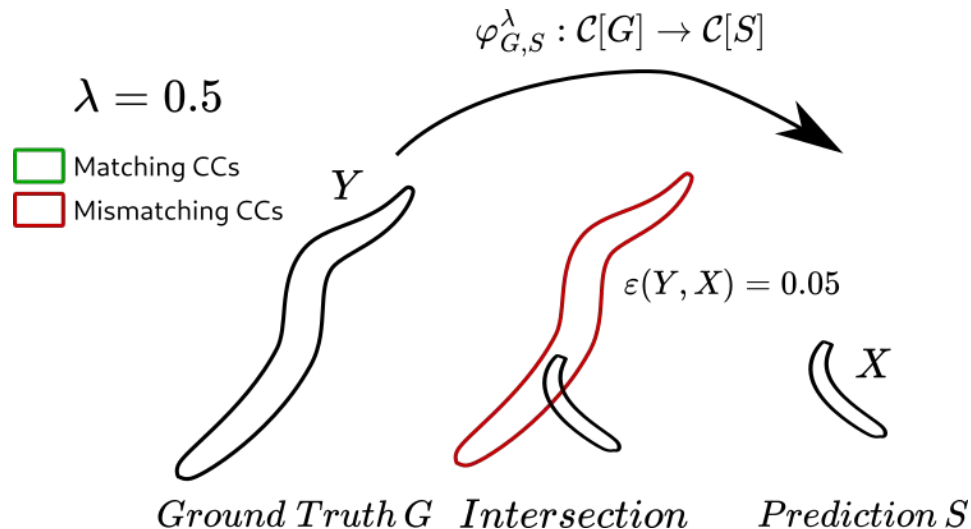
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Persistence barcode

