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Review of “Mesh density and geodesic tortuosity in planar triangular tessellations devoted to fracture mechanics”

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^R Reviewer

^E Editor

Review of version 1

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1 Reviewer 1 (Anonymous)

Reviewer As stated by the authors: "the main objective of this paper is to estimate the mesh density and the geodesic tortuosity for standard triangular tessellations suitable for finite element simulations". The title of the paper indicate fracture mechanics but there is no mechanics in the paper. The words "fracture mechanics" could be replace by crack path in the title since the paper is mainly about geometry. The paper is however very interesting because it sheds new light on finite elements to depict crack path with cohesive elements. Since I am not an expert in meshing technology, I was not able to check all the assertions in the paper. It deserves publication for its originality.

Authors Answers to Reviewers 1 and 2.

Our intention when writing this article was to provide useful insights in some geometrical features statistics of planar triangular tessellations which can be used for applications we have in mind (as presented in paragraph 6.5). We understand that the title and introduction could be misleading.

However, we would like to keep the mention "fracture mechanics" in the title as this work would prove useful for the community of fracture mechanics.

We propose then to modify our introduction, focusing more on the paper content and less on the fracture mechanics framework.

Also, we propose to add a subsection (7.1 in the new version) dealing with the practical use of the mesh density estimates in fracture mechanics simulations.

2 Reviewer 2 (Anonymous)

Reviewer I have accepted to review the paper due to the presence of Fracture Mechanics in the title. This impression was even stronger when reading the introduction where a long part is devoted to Fracture Mechanics. But in fact, if a connection exists it is by the fact that using cohesive elements the crack path cannot be arbitrary. The paper is "only" focused on some estimate of the geometrical bias with different types of meshing in 2D.

Regarding those estimates, the paper seems a good one but I am not an expert on meshing. My main problem is there are no examples in the paper showing how knowing these estimates could help to correct the meshes bias and performe correct fracture mechanics simulations.

Therefore, I would recommend the paper not to be accepted in its present form. Either the authors demonstrate, in one way or another, the interest of their estimates, whatever domain it could concern (mesh generation, percolation ...), or they should present at least one meaningful application in Fracture Mechanics. At minimum the title and introduction of the paper should be focused on what is done in the paper and mention possible applications in a light manner only.

Authors Answers to Reviewers 1 and 2.

Our intention when writing this article was to provide useful insights in some geometrical features statistics of planar triangular tessellations which can be used for applications we have in mind (as presented in paragraph 6.5). We understand that the title and introduction could be misleading.

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3 Reviewer 3 (Jérémy GIRARDOT)

Reviewer This paper entitled "Mesh Density and Geodesic Tortuosity in Planar Triangular Tessellations Devoted to Fracture Mechanics" is proposing an original analysis on different type of meshes dedicated to finite element analysis with a purely 2D geometric point of view. Indeed, the authors introduce two original parameters: the geodesic tortuosity and the mesh density.

A wide parametric analysis is then processed regarding different types of meshing algorithm. A clear and relevant link is also made between this analysis and the fracture simulations using the so-called cohesive-volumetric element method (CVEM), where the final crack path is directly related to the mesh organization. As the CVEM is indeed a relevant application of this study, we should find more than one reference in the introduction. It could be also interesting to introduce other numerical techniques dedicated to fracture mechanics that are linked to the mesh, for example the wide family of the discrete element method among others.

Authors Concerning the cohesive zones models, we added references to articles of Dugdale, Barenblatt, and Tijssens. We also added three references to DEM modeling.

Reviewer It is important to mention that this parametric analysis is systematically backed up by analytic formulations that give usefull explanations of the observed tendancies.

The paper is well written and clearly fits the scope of the journal, mainly due to the originality to give some new informations on standard meshing techniques used in FEM simulation.

To my opinion, this paper can be publish in JTCAM but needs first some answers and modifications with the followings remarks :

Reviewer it is indeed very difficult to read the results of the different analysis as NO labels on x and y are given on any curves of this article (except for figures 12 and 16). It is then strongly advised to fill every curves with both x and y labels, plots and histograms with the corresponding variables

(with its proper unity if needed) ;

Authors Corrected.

Reviewer Page 7 "This strategy leads to triangle-based random meshes with edge length and triangle top angle probability density functions respectively defined by equation (14) and equation (15), see figure 5." It is strange to me to introduce these equations and figure before the next section, and do not help to understand the point of the authors ;

Authors We have deleted the sentence.

Reviewer The figure 6 illustrates the parametric analysis for the edge length pdf with several standard mesh generators. It is not clear that the authors decide to use gmsh for low values of intensity τ and Triangle for high values? As far as I know the opposite is also possible... please add some explanations in the text accordingly ;

Authors There were no particular reasons for doing so. We added the following sentence in brackets in the paragraph following Figure 6: "(efficiency indexes associated to each mesh generator have been chosen arbitrarily)".

Reviewer The tendency observed in figure 8 is interesting but not explained or discussed as it could be. Or if already known, please add some references ;

Authors We added the following paragraph:

"The dependance of the mesh density on the square root of the intensity τ is in accordance with the Vaschy-Buckingham theorem. Indeed, the intensity of the point process and the total area of the discretized domain fully describe the tessellation geometrical features statistics. Thus, an intensive feature such as the mesh density Z would only depend on the point process intensity τ . Using the Vashy-Buckingham theorem, it is then expected that the density Z - homogeneous to the inverse of a length - would be proportional to the square root of the point process intensity."

Reviewer In the same ways, some overestimations are observed between the measures and the formulations at the end of the section 6.4 but are not discussed ; please modify the content accordingly ;

Authors We added:

"Figure 14 shows that estimate (52) leads to an overestimation of the actual geodesic tortuosity. This overestimation is explained by the fact that the geodesic path is, by definition, always associated to the minimum tortuosity.

In addition, the overestimation tends to decrease as the quality index increases. This observation is in accordance with figure 9 which shows that the inclination rule is more respected by meshes having a high quality index.

Reviewer In the conclusion, a discussion should be made on the results in this work and the transtion to the 3D case.

Authors We added at the end of conclusion:

"We emphasize the fact that the results of this work are valid only for "sufficiently disordered" homogeneous planar triangular meshes. They may be improved by considering the case of inhomogeneous Poisson point processes or three-dimensional Poisson-Delaunay tesslations.

In three-dimensional tesslations, the notion of geodesic plane might be introduced as well as an associated tortuosity. Up to our knowledge, there are no known results for estimating the three-dimensional tortuosity distribution. To provide such an estimation seems to be far more challenging than providing the estimation of the two-dimensional tortuosity".

Reviewer Some minor mistakes :

page 5 : "are estimates (x) equation (14)..."

Authors Corrected.

Review of version 2

Permalink: [hal-03703549v3](https://hal.archives-ouvertes.fr/hal-03703549v3)

4 Reviewer 3 (J r my GIRARDOT)

Reviewer Thank you for this revised version, which addresses all my comments.
I agree for publication

5 Reviewer 4 (Mahmoud SHAQFA)

Reviewer Representing damage via the Cohesive Zone Models (CZM) has wide applications in the fracture mechanics literature. Indeed the problem of crack propagation is well-known to be mesh-dependent. The motivation for the presented work is crucial for many relevant problems in the literature. This work presents statistical tools for studying the geometrical features of tessellations through:

- the mesh density and
- geodesic tortuosity.

I would suggest the authors to take the following suggestions to enhance their document:

Reviewer In the introduction, the authors briefly mention the CZM taxonomy. The zero-thickness cohesive elements are comprehensively described in the paper of Snozzi and Molinari (2013) "A cohesive element model for mixed mode loading with frictional contact capability". I advise the authors to define "extrinsic" and "intrinsic" terms in the introduction as that paper did or any other relevant reference. This is important as the term "intrinsic" has been used in the first paragraph of Section 7.1 without introducing it.

Authors We added (in red) to our introduction:

"Two particular tessellations geometrical features are studied in the present work [...] between two adjacent elements of a considered tessellation. **The interface between the adjacent elements are governed by cohesive zone models which can either be described by intrinsic or extrinsic laws.**

Within intrinsic model, the CZM curve comprises two parts: an artificial elastic part with a finite initial slope and a softening part describing the failure process. They are pre-inserted in the underlying mesh of the simulation leading to inherent crack activations without any need to add additional criteria. On the other hand, the extrinsic models have infinite initial slope, i.e. no initial elastic part, and the cohesive elements are activated, or incorporated, on the fly once the crack criterion is reached."

Reviewer Can you provide a reference, if available, for the fourth paragraph when you discuss the "overestimation" of the crack length relative to the theoretical one?

Authors We added a reference to the work of Pele, K. (DOI: 10.46298/jtcam.8322)

Reviewer In Section 4.1, the authors mention a few standard tools for meshing. I would recommend checking out some of the relatively new approaches for generating meshes that surpass Gmsh and other methods: fTetWild or TetWild (Hu et al. 2018). It would be interesting to see the same tools applied to that (for the sake of discussing 3D extensions of the presented work).

Authors We understand that you propose to look at three-dimensional meshing techniques.

Thank you for the very interesting papers you have recommended. However, we have restricted our work to the plane two-dimensional cases. The 3D case is thus not studied in this paper and is let in outlooks.

Reviewer I am curious if you have considered using the "Edge Swapping" technique for enhancing the quality of meshes using the Lloyd relaxation algorithm? This behavior might infringe the duality of Voronoi but it was used successfully in the remeshing applications. Maybe check this report by NASA, Mavriplis (1982) "An Advancing Front Delaunay Triangulation Algorithm Designed for Robustness".

Authors We have not considered the edge swapping technique in this work. However, it is used by the standard meshers considered in paragraph 4 (Gmsh, Abaqus, Cast3m and Triangle). As it can be

seen in Figures 6 and 7, the use of the probability density estimate for meshes generated by using the edge swapping technique remains satisfying.

Reviewer The first line of page 10, has a typo. You should replace the word “mid” with “intermediate”. Another typo in the first paragraph on page 12, replace “caracterise” with “characterise”.

Authors Done

Reviewer The following comments should be addressed/revealed in the discussions of the results:
All the tested mesh domains are simple. What is the effect of using domains with complex outer edges on both the density and geodesic tortuosity?

Authors As stated in this paper, we only have interest here in “sufficiently disordered meshes”, typically used in the cohesive-volumetric finite element approach when the crack path is not known a priori. In those meshes, the influence of the outer edges on the overall mesh is reduced and the mesh geometry statistical description is locally “robust” (see Appendix A).

This would have been an issue if the outer edges would modify the crack orientation. But this is out of the scope of the present work focused on the study of straight crack paths.

Reviewer In Section 5 Eq. (23) the inputs on the LHS do not appear in the RHS (Z and τ).

Authors We have slightly changed the notations adding a semicolon for separating the random variable name (for instance Z) from the actual variables of a function.
Equation 23 has also been modified.

Reviewer For the shortest path problem, in Section 6.1, I understand that the problem is NP-hard. It is common to use very efficient heuristic algorithms to estimate the shortest distance, e.g. Dijkstra’s algorithm. It is relevant to benchmark your results with that as it is standard for computing geodesic distances. Also, Dijkstra’s algorithm will readily help in generalizing your approach to 3D meshes and heterogeneous material. It also might be useful to check the work of Zhang et al. (2018) “A 2D typology generator for historical masonry elements” as they propose a weighted approach for the shortest distance in heterogeneous microstructures.

Authors We want to give a practical formula for estimating the tortuosity of a path between two points of a given random triangular tessellation as function of its reduced efficiency index.

From our point of view there are two possible options for doing so:

- computing the exact shortest path of numerous meshes with distinct efficiency index using Dijkstra or A^* algorithms so as to propose formula based on a regression method;
- or estimating the density of probability of the tortuosity by making simplifying hypothesis (“the shortest path approximation”, paragraph 6.1).

We have chosen the second option in this work.

As it can be seen in Figure 15, the tortuosity probability density estimate slightly overestimate the tortuosity distribution obtained by a Monte Carlo simulation using the Dijkstra algorithm on numerous random meshes. This is explained by the fact that the “true” shortest path does not follow exactly follow the “inclination rule” (see Figure 9).

We propose to add the following sentence at the end of paragraph 6.4:

“It could have been possible to use regressions on the results of the Monte Carlo simulation to obtain estimates of the tortuosity value. However, this method comes with an additional difficulty: finding the underlying tortuosity probability density distribution.”

Reviewer I understand that you mentioned the focus of this paper is on statistical tools. However, I believe the last paragraph in Section 7, should be backed by numerical evidence. Such evidence could include simulations of randomly seeded simulations with Monte Carlo (MC) or other relevant simulation methods.

Authors A Monte Carlo simulation has been carried out and the results are displayed in Figure 15 for instance. For sake of clarity, we have modified the legend of Figure 15:

“[...] Obtained on Gmsh meshes: top: histograms of geodesic tortuosities of cracks containing distinct number of edges n computed via a Monte Carlo simulation using Dijkstra’s algorithm. [...]”.

Reviewer The initial stiffness in the intrinsic CZM approach should be dependent on the density of the mesh. Do you think this should be discussed in Section 7?

Authors It seems to us that we already have addressed this matter in equation 57, paragraph 7.1

6 Editor’s assessments (Julien RÉTHORÉ)

Editor The paper presents a statistical analysis of triangular meshes. The topic falls well within the scope of JTCAM, as such analysis is not only useful in itself but also because the problem it addresses is crucial for fracture simulations using models that are known to be mesh dependent (e.g. cohesive zone models, discrete element method, etc.). Two brief reports were initially received for the paper, but I was unable to make a decision based on them. Subsequently, two favourable, comprehensive reviews were received. A productive discussion and notable enhancements to the paper resulted from the exchange between the authors and the reviewers. I based my decision to accept the paper on these final two reports and the outcomes of this exchange.

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