Plant-Order Saturated Output-Feedback Regional Controller Synthesis With Sign-Indefinite Quadratic Forms

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***Abstract*— This note provides tools to design a dynamic plant-order output-feedback controller for linear plants subject to saturating input with measurable output. Based on Linear Matrix Inequalities (LMIs), together with appropriate transformations and sector conditions, the proposed solution exploits sign-indefinite quadratic forms to define a locally positive definite piece-wise Lyapunov function providing non-ellipsoidal estimates of the closed-loop basin of attraction. With guaranteed local exponential stability, methods to ensure a prescribed local exponential convergence rate and to maximize the estimates of the region of attraction are also given.**

***Index Terms*— Stability of nonlinear systems, LMIs, Lyapunov methods, Saturation**

I. Introduction

In the recent decades, the physical or technological limitations of actuators have been progressively more considered to construct control laws systems subject to input saturation. Indeed, neglecting the existence of these intrinsic properties may produce poor performance or undesirable behaviors of dynamic controlled closed-loop systems, including overshoot, slow convergence or even, in some cases, stability loss. This is especially critical in systems with fast dynamics, such as aircrafts and rockets [16]. Restrictive solutions preventing the actuators to reach saturation bounds, called low-gain approaches, may limit the system operation to such an extent that poor performance or instability is induced.

Under this scenario, different constructive control methods considering actuator limitations in linear systems subject to input saturation have been developed in the last years; some of them based on Linear Matrix Inequalities (LMIs) stemming from quadratic Lyapunov functions and adequate sector conditions (see, for example, [17], [19] and references therein). It is also worth to mention that the direct feedback controller design, which considers the actuator saturation during the synthesis of both dynamic output feedback controller and anti-windup compensation, requires the controller to have the same order as the plant [15] and the addition of a static anti-windup term to allow performing factorizations in [15] and transformations in [3], [6] leading to convex LMI-based conditions.

Motivated by the discussions in [17, Example 3.4] and [19, Section 4.4.1.1] suggesting that the use of common quadratic Lyapunov functions may reveal some numerical conservatism (see also the parallel approaches in [8], [9] and [18]), the current paper complements the existing dynamic output feedback controller synthesis results by proposing the use of the piece-wise nonquadratic Lyapunov Function encompassing a sign-indefinite quadratic form already employed in [14] to address the stability analysis problem, and in [11] and [13] to synthesize an anti-windup gain guaranteeing local and global exponential stability, respectively. For regional stabilization, this paper proposes a dynamic output feedback controller design procedure extending the quadratic designs in [5] and [6] by using sign-indefinite quadratic forms. This leads to sufficient and less conservative LMI conditions ensuring regional exponential stability of the origin of the closed-loop system, while considering the size of the basin of attraction as a primary performance indicator [7], [14]. Moreover, sufficient conditions are also given to guarantee a desired convergence rate, understood as a local property that holds only when the input saturation is not active. Some preliminary results in the direction of this paper are presented in [12], where the global stabilization problem is addressed by proposing a global design that is only applicable to exponentially stable plants, whereas the construction presented here is applicable to any stabilizable and detectable plant.

This paper is organized as follows. Section II introduces the controller synthesis problem under consideration, together with the definitions of the plant and the dynamic controller. The main results obtained leveraging the sign-indefinite quadratic forms are presented in Section III. Section IV shows two numerical examples and, finally, Section V reports some concluding comments.

Notation: M^{T} is the transpose of the matrix M and $M^{-\mathsf{T}}$ the transpose of the invertible matrix M^{-1}. Define $\mathrm{He}(M) = M + M^{\mathsf{T}}$. $\mathbb{R}^m$ is the Euclidean space of dimensions m. Let $\mathbb{D}^m$ ($\mathbb{D}^m_{>0}$) be the set of diagonal (positive-definite) matrices of dimensions $m \times m$ and $\mathbb{S}^m$ ($\mathbb{S}^m_{>0}$) the set of symmetric (positive-definite) matrices of dimensions $m \times m$. Given any matrix $M \in \mathbb{R}^{m\times m}$, $\lambda(M)$ is the set of eigenvalues of M, $\lambda_{\min}(M)$ the minimum eigenvalue of M and $\lambda_{\max}(M)$ is the maximum eigenvalue of M. Finally, I_m is the identity matrix of dimensions $m \times m$ and 0 is the null matrix of appropriate dimensions.

II. Preliminary definitions

Consider the linear plant subject to input saturation

$$\left|\begin{array}{rcl} \dot{x}_p &=& A_p x_p + B_p \mathrm{sat}\,(u) \\ y &=& C_p x_p \end{array}\right. , \tag{1}$$

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with state $x_p \in \mathbb{R}^n$ and output $y \in \mathbb{R}^m$, while denoting by $u \mapsto \text{sat}(u)$ the decentralized symmetric saturation function, with components

$$\text{sat}_i(u_i) := \max\{-\bar{u}_i, \min\{\bar{u}_i, u_i\}\}$$

for all $i = 1, \cdots, m$, where $\bar{u}_i > 0$ is the symmetric saturation limit on the input u_i, with u_i and $\bar{u}_i$ being the ith entry of the vectors u and $\bar{u}$, respectively. Introduce also the deadzone function $\text{dz}(u) := u - \text{sat}(u)$. Assuming that (1) is stabilizable and detectable (a necessary condition for output feedback stabilizability), the goal of this paper is to design a dynamic output feedback controller with anti-windup compensation

$$\left| \begin{array}{rcl} \dot{x}_c &=& A_c x_c + B_c y + E_c \text{dz}(u) \\ u &=& C_c x_c + D_c y \end{array} \right. , \tag{2}$$

which has the same order as the plant, i.e. with state $x_c \in \mathbb{R}^n$ and output $u \in \mathbb{R}^m$. Controller (2) is defined in such a way that the anti-windup compensation is performed by the term $E_c\text{dz}(u)$. With these definitions, let A_c, B_c, C_c, D_c and E_c be the controller state-space model and anti-windup gain matrices to be synthesized.

Define also the extended state vector $x := \begin{bmatrix} x_p^\mathsf{T} & x_c^\mathsf{T} \end{bmatrix}^\mathsf{T}$, $x \in \mathbb{R}^{2n}$, and combine (1) and (2) to write the closed-loop dynamics as

$$\left| \begin{array}{rcl} \dot{x} &=& Ax + B\text{dz}(u) \\ u &=& Cx \end{array} \right. , \tag{3}$$

with

$$\left[\begin{array}{c|c} A & B \\ \hline C & — \end{array}\right] = \left[\begin{array}{cc|c} A_p + B_p D_c C_p & B_p C_c & -B_p \\ B_c C_p & A_c & E_c \\ \hline D_c C_p & C_c & — \end{array}\right].$$

This paper proposes sufficient and convex LMI design conditions for (2) that ensure regional exponential stability of the origin of (3). As a second aim, this note determines the sufficient LMI conditions for guaranteeing a prescribed spectral abscissa $-\alpha < 0$ for matrix A in (3), which produces a guaranteed local convergence rate of α in the closed-loop response for any initial state in the basin of attraction of the origin when the input saturation is inactive. In the same way, sufficient LMI conditions limiting the spectral radius of A to a prescribed radius $\rho > \alpha$ are given to improve the numerical behavior of the convex optimization.

III. Sign-Indefinite Nonquadratic Certificates

Construct the extended state vector

$$\eta = \begin{bmatrix} x^\mathsf{T} & \text{dz}(Cx)^\mathsf{T} \end{bmatrix}^\mathsf{T}.$$

so that, according to [14], V is a locally Lipschitz sign-indefinite quadratic form defined as

$$V(x) := \eta^\mathsf{T} P \eta = \eta^\mathsf{T} \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^\mathsf{T} & P_{22} \end{bmatrix} \eta, \tag{4}$$

where $P_{11} = P_{11}^\mathsf{T} > 0$ and $P_{22} = P_{22}^\mathsf{T}$ may be sign-indefinite. Notice that having P with nonpositive P_{22} may still ensure local positive-definiteness and Lipschitz continuity for V in (4) whenever P_{11} is positive definite, as it was shown in [14, Proposition 1]. As a consequence, V encompasses a broad class of Lyapunov functions since positive-definiteness of P is not imperative, in contrast with the results in [6] and [3], where positive-definiteness of P is actually required.

Following a similar procedure to [11], characterizing the basin of attraction of the origin for (3) requires defining the function $x \to h(x)$ as

$$h(x) := H_1 x + H_2 \text{dz}(Cx) \tag{5}$$

with $H_1 \in \mathbb{R}^{m \times 2n}$, $H_2 \in \mathbb{R}^{m \times m}$ being both arbitrary design parameters. With h in (5), introduce the subset

$$\mathcal{S}_h := \{x \in \mathbb{R}^n : |h(x)|_\infty \leq 1\} \tag{6}$$

and impose the condition

$$V(x) \geq |h(x)|_\infty^2 \tag{7}$$

for all $x \in \mathcal{S}_h$ so that the Lyapunov function candidate

$$W(x) := \left\{ \begin{array}{ll} V(x) & \text{if } x \in \mathcal{S}_h \\ 1 & \text{otherwise} \end{array} \right. , \tag{8}$$

is continuous and Lipschitz. Moreover, introducing the open sublevel set

$$\mathcal{S}(W) := \{x \in \mathbb{R}^n : W(x) < 1\}, \tag{9}$$

it holds under (7) that

$$\mathcal{S}(W) = \mathcal{S}(V) \cap \mathcal{S}_h. \tag{10}$$

To parameterize P, generalizing the approach in [15], introduce the full-rank matrices $\mathbf{X}$, $\mathbf{Y}$, $\tilde{X}$, $\tilde{Y} \in \mathbb{S}_{>0}^n$, and full-rank $\mathbf{M}$, $N \in \mathbb{R}^{n \times n}$ such that

$$P_{11} = \begin{bmatrix} \mathbf{X} & \mathbf{M} \\ \mathbf{M}^\mathsf{T} & \tilde{X} \end{bmatrix}, \quad P_{11}^{-1} = \begin{bmatrix} \mathbf{Y} & N \\ N^\mathsf{T} & \tilde{Y} \end{bmatrix}. \tag{11}$$

Using (11) and the fact that $P_{11}P_{11}^{-1} = P_{11}^{-1}P_{11} = I_{2n}$, the symmetry of P_{11} is guaranteed by the identities

$$\begin{aligned} \mathbf{X}\mathbf{Y} + \mathbf{M}N^\mathsf{T} &= \mathbf{Y}\mathbf{X} + N\mathbf{M}^\mathsf{T} = I_n, \\ \mathbf{M}^\mathsf{T}\mathbf{Y} + \tilde{X}N^\mathsf{T} &= \mathbf{Y}\mathbf{M} + N\tilde{X} \ = 0. \end{aligned} \tag{12}$$

Moreover, from (12) and the invertibility of N, note that

$$\begin{aligned} \tilde{X} &= -\mathbf{M}^\mathsf{T}\mathbf{Y}N^{-\mathsf{T}} \\ &= -N^{-1}(\mathbf{Y} - \mathbf{Y}\mathbf{X}\mathbf{Y})N^{-\mathsf{T}}. \end{aligned} \tag{13}$$

Generalizing also the derivations in [15], it is possible to parameterize the controller matrices in (2) as

$$\begin{aligned} A_c &= \mathbf{M}^{-1}\big(\hat{\mathbf{A}}_c - \mathbf{X}\big(A_p + B_p D_c C_p\big)\mathbf{Y} \\ &\quad -\mathbf{M}B_c C_p \mathbf{Y} - \mathbf{X}B_p C_c N^\mathsf{T}\big)N^{-\mathsf{T}}, \\ B_c &= \mathbf{M}^{-1}\big(\hat{\mathbf{B}}_c - \mathbf{X}B_p D_c\big), \\ C_c &= \big(\hat{\mathbf{C}}_c - D_c C_p \mathbf{Y}\big)N^{-\mathsf{T}}, \\ D_c &= \hat{\mathbf{D}}_c, \\ E_c &= \mathbf{M}^{-1}\hat{\mathbf{E}}_c\mathbf{S}^{-1} + \mathbf{M}^{-1}\mathbf{X}B_p, \end{aligned} \tag{14}$$

the remaining entries in (4) as

$$P_{12} = \begin{bmatrix} \mathbf{Y} & I_n \\ N^\mathsf{T} & 0 \end{bmatrix}^{-\mathsf{T}} \begin{bmatrix} \mathbf{Z}_p \\ \mathbf{Z}_c \end{bmatrix} \mathbf{S}^{-1}, \;\; P_{22} = \mathbf{S}^{-1}\hat{\mathbf{P}}_{22}\mathbf{S}^{-1} \quad (15)$$

and, finally, the parameters of (5) as

$$H_1 = \begin{bmatrix} \mathbf{G}_p & \mathbf{G}_c \end{bmatrix} \begin{bmatrix} \mathbf{Y} & I_n \\ N^\mathsf{T} & 0 \end{bmatrix}^{-1}, \quad H_2 = \hat{\mathbf{H}}_2\mathbf{S}^{-1}. \quad (16)$$

With these parameterizations, matrices $\hat{\mathbf{A}}_c \in \mathbb{R}^{n\times n}$, $\hat{\mathbf{B}}_c \in \mathbb{R}^{n\times m}$, $\hat{\mathbf{C}}_c \in \mathbb{R}^{m\times n}$, $\hat{\mathbf{D}}_c \in \mathbb{R}^{m\times m}$, $\hat{\mathbf{E}}_c \in \mathbb{R}^{n\times m}$, $\mathbf{Z}_p \in \mathbb{R}^{n\times m}$, $\mathbf{Z}_c \in \mathbb{R}^{n\times m}$, $\hat{\mathbf{P}}_{22} \in \mathbb{S}^m$, $\mathbf{S} \in \mathbb{D}^m_{>0}$, $\mathbf{G}_p \in \mathbb{R}^{n\times n}$, $\mathbf{G}_c \in \mathbb{R}^{n\times n}$ and $\hat{\mathbf{H}}_2 \in \mathbb{R}^{n\times m}$ are the decision variables of the convex LMI-based synthesis formulated in the next theorem.

Theorem 1: If there exist matrices $\mathbf{X} \in \mathbb{S}^n_{>0}$, $\mathbf{Y} \in \mathbb{S}^n_{>0}$, $\mathbf{Z}_p \in \mathbb{R}^{n\times m}$, $\mathbf{Z}_c \in \mathbb{R}^{n\times m}$, $\hat{\mathbf{P}}_{22} \in \mathbb{S}^m$, $\mathbf{S} \in \mathbb{D}^m_{>0}$, $\mathbf{G}_p \in \mathbb{R}^{n\times n}$, $\mathbf{G}_c \in \mathbb{R}^{n\times n}$, $\hat{\mathbf{H}}_2 \in \mathbb{R}^{n\times m}$, $\hat{\mathbf{A}}_c \in \mathbb{R}^{n\times n}$, $\hat{\mathbf{B}}_c \in \mathbb{R}^{n\times m}$, $\hat{\mathbf{C}}_c \in \mathbb{R}^{m\times n}$, $\hat{\mathbf{D}}_c \in \mathbb{R}^{m\times m}$ and $\hat{\mathbf{E}}_c \in \mathbb{R}^{n\times m}$ satisfying

$$\Psi_1 = \mathrm{He}\begin{bmatrix} \frac{1}{2}\mathbf{Y} & 0 & \mathbf{Z}_p + \mathbf{G}_p^\mathsf{T}\bar{U} - \hat{\mathbf{C}}_c^\mathsf{T} & 0 \\ I_n & \frac{1}{2}\mathbf{X} & \mathbf{Z}_c + \mathbf{G}_c^\mathsf{T}\bar{U} - C_p^\mathsf{T}\hat{\mathbf{D}}_c^\mathsf{T} & 0 \\ 0 & 0 & \frac{1}{2}\hat{\mathbf{P}}_{22} + \bar{U}\hat{\mathbf{H}}_2 + \mathbf{S} & 0 \\ \mathbf{G}_p & \mathbf{G}_c & \hat{\mathbf{H}}_2 & \frac{1}{2}I_m \end{bmatrix} > 0 \quad (17)$$

and condition (18) at the bottom of this page, then function W in (8) is Lipschitz continuous, inclusion $\mathcal{S}(W) \subset \mathcal{S}_h$ holds and the origin of (3) with the controller state-space model matrices A_c, B_c, C_c, D_c and E_c as selected in (14) is locally exponentially stable from $\mathcal{S}(W)$.

Proof: First, notice that selections (14)-(16) can be uniquely inverted as

$$\begin{aligned} \hat{\mathbf{A}}_c &= \mathbf{X}\left(A_p + B_cD_cC_p\right)\mathbf{Y} + \mathbf{M}B_cC_p\mathbf{Y} \\ &\quad + \mathbf{X}B_pC_cN^\mathsf{T} + \mathbf{M}A_cN^\mathsf{T}, \\ \hat{\mathbf{B}}_c &= \mathbf{X}B_pD_c + \mathbf{M}B_c, \quad \hat{\mathbf{C}}_c = D_cC_p\mathbf{Y} + C_cN^\mathsf{T}, \\ \hat{\mathbf{D}}_c &= D_c, \qquad\qquad \hat{\mathbf{E}}_c = \mathbf{M}E_c\mathbf{S} - \mathbf{X}B_p\mathbf{S}, \\ \begin{bmatrix} \mathbf{Z}_p \\ \mathbf{Z}_c \end{bmatrix} &= \begin{bmatrix} \mathbf{Y} & I_n \\ N^\mathsf{T} & 0 \end{bmatrix}^\mathsf{T} P_{12}\mathbf{S}, \quad \hat{\mathbf{P}}_{22} = \mathbf{S}P_{22}\mathbf{S}, \\ \begin{bmatrix} \mathbf{G}_p & \mathbf{G}_c \end{bmatrix} &= H_1 \begin{bmatrix} \mathbf{Y} & I_n \\ N^\mathsf{T} & 0 \end{bmatrix}, \qquad \hat{\mathbf{H}}_2 = H_2\mathbf{S}. \end{aligned} \quad (19)$$

Together with selection (11), introduce the invertible matrix

$$\Pi := \begin{bmatrix} \mathbf{Y} & I_n \\ N^\mathsf{T} & 0 \end{bmatrix} \quad (20)$$

that satisfies the properties

$$\Pi^\mathsf{T}P_{11} = \begin{bmatrix} I_n & 0 \\ \mathbf{X} & \mathbf{M} \end{bmatrix}, \quad \Pi^\mathsf{T}P_{11}\Pi = \begin{bmatrix} \mathbf{Y} & I_n \\ I_n & \mathbf{X} \end{bmatrix}, \quad (21)$$

which are easily verified by substitution using (12). Furthermore, observe that $W(x) > 0$ for all $x \notin \mathcal{S}_h$ and $W(x) = V(x)$ for all $x \in S_h$ from definition (8). Define the diagonal matrix $\bar{U} := \mathrm{diag}\{\bar{u}\}$ and consider the following facts proven in [7] and [4].

Local Sector Condition [7, Lemma 1]: For any $T_1 \in \mathbb{D}^m_{>0}$, it holds that for all $x \in \mathcal{S}_h$,

$$\mathrm{dz}\,(Cx)^\mathsf{T}T_1(Cx - \mathrm{dz}\,(Cx) - \bar{U}h(x)) \geq 0. \quad (22)$$

Derivative of the Deadzone [4, Fact 4]: For any $T_2 \in \mathbb{D}^m$, $T_3 \in \mathbb{D}^m$ and for all $x \in \mathbb{R}^{2n} : |u_i(x)| \neq \bar{u}_i$,

$$\mathrm{dz}\,(Cx)^\mathsf{T}T_2(C\dot{x} - \dot{\mathrm{dz}}\,(Cx)) \equiv 0, \quad (23)$$

$$\dot{\mathrm{dz}}\,(Cx)^\mathsf{T}T_3(C\dot{x} - \dot{\mathrm{dz}}\,(Cx)) \equiv 0, \quad (24)$$

where $\dot{x} = Ax + B\mathrm{dz}\,(u)$ as in (3) and $\dot{\mathrm{dz}}\,(Cx)$ denotes the time-derivative of $x \to \mathrm{dz}\,(Cx)$, which is well defined for almost all values of $x \in \mathbb{R}^{2n}$.

Now, exploiting the invertibility of Π and considering selections in (19), pre- and post-multiply Ψ_1 in (17) by

$$\begin{bmatrix} \Pi & 0 & 0 \\ 0 & \mathbf{S} & 0 \\ 0 & 0 & I_m \end{bmatrix}^{-\mathsf{T}}$$

and its transpose to obtain the matrix

$$\mathrm{He}\begin{bmatrix} \frac{1}{2}P_{11} & P_{12} + H_1^\mathsf{T}\bar{U}\mathbf{S}^{-1} - C^\mathsf{T}\mathbf{S}^{-1} & 0 \\ 0 & \frac{1}{2}P_{22} + \mathbf{S}^{-1}\bar{U}H_2 + \mathbf{S}^{-1} & 0 \\ H_1 & H_2 & \frac{1}{2}I_m \end{bmatrix},$$

which is positive-definite due to the hypothesis (17). Applying Schur complement, define

$$\bar{\Psi}_1 := P - \mathrm{He}\begin{bmatrix} 0 & C^\mathsf{T}\mathbf{S}^{-1} - H_1^\mathsf{T}\bar{U}\mathbf{S}^{-1} \\ 0 & -\mathbf{S}^{-1} - \mathbf{S}^{-1}\bar{U}H_2) \end{bmatrix} - \begin{bmatrix} H_1^\mathsf{T} \\ H_2^\mathsf{T} \end{bmatrix}\begin{bmatrix} H_1^\mathsf{T} \\ H_2^\mathsf{T} \end{bmatrix}^\mathsf{T} > 0, \quad (25)$$

which verifies that, for all $x \in \mathcal{S}_h$,

$$\begin{aligned} &V(x) - h(x)^\mathsf{T}h(x) \\ &\quad \geq V(x) - h(x)^\mathsf{T}h(x) \\ &\qquad -2\,\mathrm{dz}\,(Cx)^\mathsf{T}\mathbf{S}^{-1}(Cx - \mathrm{dz}\,(Cx) - \bar{U}h(x)) \\ &\quad = \eta^\mathsf{T}\bar{\Psi}\eta \geq \lambda_{\min}(\bar{\Psi}_1)\,|\eta|^2 \geq \lambda_{\min}(\bar{\Psi}_1)\,|x|^2, \end{aligned} \quad (26)$$

which results in $W(x) = V(x) \geq h^\mathsf{T}(x)h(x) > 0$, proving inclusion $\mathcal{S}(W) \subset \mathcal{S}_h$, and ensures that $V(x) \geq 1$

$$\Psi_2 = \mathrm{He}\begin{bmatrix} A_p\mathbf{Y} + B_p\hat{\mathbf{C}}_c & A_p + B_p\hat{\mathbf{D}}_cC_p & -B_p\mathbf{S} & 0 & 0 & \mathbf{Z}_p \\ \hat{\mathbf{A}}_c & \hat{\mathbf{X}}A_p + \hat{\mathbf{B}}_cC_p & \hat{\mathbf{E}}_c & 0 & 0 & \mathbf{Z}_c \\ \hat{\mathbf{C}}_c - \bar{U}\mathbf{G}_p & \hat{\mathbf{D}}_cC_p - \bar{U}\mathbf{G}_c & -\mathbf{S} - \bar{U}\hat{\mathbf{H}}_2 & \mathbf{Z}_p^\mathsf{T} - \hat{\mathbf{C}}_c & \mathbf{Z}_c^\mathsf{T} - \hat{\mathbf{D}}_cC_p & \hat{\mathbf{P}}_{22} \\ A_p\mathbf{Y} + B_p\hat{\mathbf{C}}_c & A_p + B_p\hat{\mathbf{D}}_cC_p & -B_p\mathbf{S} & -\mathbf{Y} & -I_n & 0 \\ \hat{\mathbf{A}}_c & \hat{\mathbf{X}}A_p + \hat{\mathbf{B}}_cC_p & \hat{\mathbf{E}}_c & -I_n & -\mathbf{X} & 0 \\ 0 & 0 & \mathbf{S} & \hat{\mathbf{C}}_c & \hat{\mathbf{D}}_cC_p & -\mathbf{S} \end{bmatrix} < 0 \quad (18)$$

in the boundary of $\mathcal{S}_h$, implying Lipschitz continuity of $W(x)$. Moreover, since $u = Cx$ is globally Lipschitz, using $|\mathrm{dz}\,(Cx)| \leq |Cx| \leq L\,|x|$, it can be stated that

$$W(x) = V(x) \leq \lambda_{\max}(P)\,|\eta|^2 \leq \lambda_{\max}(P) L\,|x|^2\,,$$

which, together with property (26), implies positive-definiteness of W for all $x \in \mathcal{S}(W)$. Besides, since $\dot{W}(x)$ coincides with $\dot{V}(x)$ for almost all $x \in \mathcal{S}(W)$, defining the extended state vector υ and matrix K as

$$\upsilon := \begin{bmatrix} x^\mathsf{T} & \mathrm{dz}\,(u)^\mathsf{T} & \dot{x}^\mathsf{T} & \dot{\mathrm{dz}}\,(u)^\mathsf{T} \end{bmatrix}^\mathsf{T},$$
$$K := \begin{bmatrix} 0 & C^\mathsf{T}\mathbf{S}^{-1} - P_{12} & P_{11} & -C^\mathsf{T}\mathbf{S}^{-1} \end{bmatrix}^\mathsf{T},$$

due to continuity of (3), using [5, Section 2.2], stability of the origin of (3) from $\mathcal{S}(W)$ can be proven by showing that $\dot{W}(x) = \dot{V}(x) < -\epsilon_v |x|^2$ for some $\epsilon_v > 0$ and for almost all $x \in \mathcal{S}(W) \subset \mathcal{S}_h$. To this end, with (19) and after some extensive derivations, pre and postmultiplying Ψ_2 in (18) by

$$\begin{bmatrix} \Pi & 0 & 0 & 0 \\ 0 & \mathbf{S} & 0 & 0 \\ 0 & 0 & \Pi & 0 \\ 0 & 0 & 0 & \mathbf{S} \end{bmatrix}^\mathsf{T}$$

and its transpose allows determining the symmetric matrix

$$\begin{aligned}
\bar{\Psi}_2 :=& \mathrm{He}\begin{bmatrix} P_{11}A & (C-\bar{U}H_1)^\mathsf{T}\mathbf{S}^{-1} & 0 & P_{12} \\ B^\mathsf{T}P_{11} & -\mathbf{S}^{-1}(I_m+\bar{U}H_2) & P_{12}^\mathsf{T} - \mathbf{S}^{-1}C & P_{22} \\ P_{11}A & P_{11}B & -P_{11} & 0 \\ 0 & \mathbf{S}^{-1} & \mathbf{S}^{-1}C & -\mathbf{S}^{-1} \end{bmatrix} \\
=& \mathrm{He}\left(\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^\mathsf{T} & P_{22} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} A & B & 0 & 0 \\ 0 & 0 & 0 & I_m \end{bmatrix}\right) \\
&+ \mathrm{He}\left(\begin{bmatrix} 0 \\ \mathbf{S}^{-1} \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} C-\bar{U}H_1 & -I_m - \bar{U}H_2 & 0 & 0 \end{bmatrix}\right) \\
&+ \mathrm{He}\left(\begin{bmatrix} 0 \\ -\mathbf{S}^{-1} \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} CA & CB & 0 & -I_m \end{bmatrix}\right) \\
&+ \mathrm{He}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathbf{S}^{-1} \end{bmatrix}\begin{bmatrix} CA & CB & 0 & -I_m \end{bmatrix}\right) \\
&+ \mathrm{He}\left(\begin{bmatrix} 0 \\ \mathbf{S}^{-1}C - P_{12}^\mathsf{T} \\ P_{11} \\ -\mathbf{S}^{-1}C \end{bmatrix}\begin{bmatrix} A & B & -I_{2n} & 0 \end{bmatrix}\right),
\end{aligned}$$

which is negative-definite thanks to hypothesis (18). Exploiting facts (22)-(24), the expression above implies

$$\begin{aligned}
\dot{V}(x) \leq \dot{V}(x) &+ 2\,\mathrm{dz}\,(Cx)^\mathsf{T}\mathbf{S}^{-1}(Cx - \mathrm{dz}\,(Cx) - \bar{U}h(x)) \\
&- 2\,\mathrm{dz}\,(Cx)^\mathsf{T}\mathbf{S}^{-1}(C\dot{x} - \dot{\mathrm{dz}}\,(Cx)) \\
&+ 2\,\dot{\mathrm{dz}}\,(Cx)^\mathsf{T}\mathbf{S}^{-1}(C\dot{x} - \dot{\mathrm{dz}}\,(Cx)) \\
&+ 2\,\upsilon^\mathsf{T}K(Ax + B\mathrm{dz}\,(Cx) - \dot{x}) = \upsilon^\mathsf{T}\bar{\Psi}_2\upsilon \qquad (27)
\end{aligned}$$

for almost all $x \in \mathcal{S}_h$. Additionally, notice that selecting $\epsilon_v = \lambda_{\min}(-\bar{\Psi}_2) > 0$,

$$-\upsilon^\mathsf{T}\bar{\Psi}_2\upsilon \geq \lambda_{\min}\left(-\bar{\Psi}_2\right)|\upsilon|^2 \geq \lambda_{\min}\left(-\bar{\Psi}_2\right)|x|^2 = \epsilon_v|x|^2$$

holds for almost all $x \in \mathcal{S}(W)$, ensuring, due to [5], regional exponential stability of the origin of (3) from $\mathcal{S}(W)$. □

Remark 1: The preliminary statement (22) requires T_1 to be positive-definite, while facts (23) and (24) admit diagonal sign-indefinite multipliers. Hence, to preserve the convexity and feasibility of (17) and (18), Theorem 1 takes $T_1 = T_3 = \mathbf{S}^{-1} > 0$ but, specifically for the examples in Section IV, selecting $T_2 = -\mathbf{S}^{-1}$ was observed to produce more voluminous estimates of the basin of attraction for (3). Moreover, since the gains T_1, T_2 and T_3 rely on the same decision variable $\mathbf{S}$, some conservatism upcoming from these selections may be expected in the design stage, but the analysis tools of [14] may be used a posteriori to recompute a less conservative nonquadratic estimate of the basin of attraction.

With Theorem 1 ensuring local exponential stability of (3), the resulting output feedback control system may have an arbitrarily slow convergence rate or arbitrarily fast dynamics. To address this fact, the next theorem allows ensuring a prescribed spectral abscissa of A smaller than or equal to $-\alpha < 0$, which leads a minimum given convergence rate α for dynamics (3) in the linear tail of the response, and a spectral radius of A smaller than a given scalar $\rho > \alpha$.

Theorem 2: Given a prescribed convergence rate $\alpha \geq 0$ and a prescribed spectral radius $\rho > \alpha$, if there exist matrices $\mathbf{X} \in \mathbb{S}^n_{>0}$, $\mathbf{Y} \in \mathbb{S}^n_{>0}$, $\mathbf{Z}_p \in \mathbb{R}^{n\times m}$, $\mathbf{Z}_c \in \mathbb{R}^{n\times m}$, $\mathbf{G}_p \in \mathbb{R}^{n\times n}$, $\mathbf{G}_c \in \mathbb{R}^{n\times n}$, $\hat{\mathbf{H}}_2 \in \mathbb{R}^{n\times m}$, $\hat{\mathbf{P}}_{22} \in \mathbb{S}^m$,

$$\Psi_3 = \mathrm{He}\begin{bmatrix} A_p\mathbf{Y} + B_p\hat{\mathbf{C}}_c + \alpha\mathbf{Y} & A_p + B_p\hat{\mathbf{D}}_cC_p + \alpha I_n & \hat{\mathbf{T}}_{pp} - \mathbf{Y} & \hat{\mathbf{T}}_{pc} - I_n \\ \hat{\mathbf{A}}_c + \alpha I_n & \mathbf{X}A_p + \hat{\mathbf{B}}_cC_p + \alpha\mathbf{X} & \hat{\mathbf{T}}_{pc}^\mathsf{T} - I_n & \hat{\mathbf{T}}_{cc} - \mathbf{X} \\ A_p\mathbf{Y} + B_p\hat{\mathbf{C}}_c + \alpha\mathbf{Y} & A_p + B_p\hat{\mathbf{D}}_cC_p + \alpha I_n & -\mathbf{Y} & -I_n \\ \hat{\mathbf{A}}_c + \alpha I_n & \mathbf{X}A_p + \hat{\mathbf{B}}_cC_p + \alpha\mathbf{X} & -I_n & -\mathbf{X} \end{bmatrix} < 0 \qquad (29)$$

$$\Psi_4 = \mathrm{He}\begin{bmatrix} -\frac{1}{2}\rho\mathbf{Y} & -\rho I_n & A_p\mathbf{Y} + B_p\hat{\mathbf{C}}_c & A_p + B_p\hat{\mathbf{D}}_cC_p \\ 0 & -\frac{1}{2}\rho\mathbf{X} & \hat{\mathbf{A}}_c & \hat{\mathbf{X}}A_p + \hat{\mathbf{B}}_cC_p \\ 0 & 0 & -\frac{1}{2}\rho\mathbf{Y} & -\rho I_n \\ 0 & 0 & 0 & -\frac{1}{2}\rho\mathbf{X} \end{bmatrix} < 0 \qquad (30)$$

$\mathbf{S} \in \mathbb{D}^m_{>0}$, $\hat{\mathbf{A}}_c \in \mathbb{R}^{n\times n}$, $\hat{\mathbf{B}}_c \in \mathbb{R}^{n\times m}$, $\hat{\mathbf{C}}_c \in \mathbb{R}^{m\times n}$, $\hat{\mathbf{D}}_c \in \mathbb{R}^{m\times m}$, $\hat{\mathbf{E}}_c \in \mathbb{R}^{n\times m}$, $\hat{\mathbf{T}}_{pp} \in \mathbb{S}^n$, $\hat{\mathbf{T}}_{pc} \in \mathbb{R}^{n\times n}$ and $\hat{\mathbf{T}}_{cc} \in \mathbb{S}^n$ such that conditions (17), (18) are satisfied and, with

$$\hat{T} = \begin{bmatrix} \hat{\mathbf{T}}_{pp} & \hat{\mathbf{T}}_{pc} \\ \hat{\mathbf{T}}_{pc}^{\mathsf{T}} & \hat{\mathbf{T}}_{cc} \end{bmatrix} > 0, \tag{28}$$

conditions (29) and (30) at the bottom of the previous page also hold, then the origin of (3) with the controller state-space model matrices A_c, B_c, C_c, D_c and E_c selected as in (14) is locally exponentially stable from $\mathcal{S}(W)$ and the eigenvalues of A in (3) have real part smaller than $-\alpha$ and a modulus smaller than ρ.

Proof: The proof of the local exponential stability of the origin of (3) from $\mathcal{S}(W)$ is proven in Theorem 1. To show that $\lambda_{\max}(A) < -\alpha$, introduce the matrix $T := \Pi^{-\mathsf{T}}\hat{T}\Pi^{-1}$, with $\hat{T}$ as defined in (28). Then, hypothesis (29) implies

$$\begin{bmatrix} \Pi & 0 \\ 0 & \Pi \end{bmatrix}^{-\mathsf{T}} \Psi_3 \begin{bmatrix} \Pi & 0 \\ 0 & \Pi \end{bmatrix}^{-1} = \mathrm{He}\begin{bmatrix} P_{11}(A+\alpha I_{2n}) & T - P_{11} \\ P_{11}(A+\alpha I_{2n}) & -P_{11} \end{bmatrix},$$

which is negative definite due to hypothesis (29) and pre- and post-multiplied by $\begin{bmatrix} x^{\mathsf{T}} & x^{\mathsf{T}}(A+\alpha I_{2n})^{\mathsf{T}} \end{bmatrix}$ and its transpose reads $x^{\mathsf{T}}\mathrm{He}(T(A+\alpha I_{2n}))x < 0$, proving that the matrix $A + \alpha I_{2n}$ is Hurwitz. Furthermore, exploiting the results of [2] and the invertibility of Π in (20), to show that the spectral radius of A is smaller than ρ, pre- and post-multiply Ψ_4 by

$$\begin{bmatrix} \Pi^{-\mathsf{T}} & 0 \\ 0 & \Pi^{-\mathsf{T}} \end{bmatrix}$$

and its transpose. This product leads to

$$\mathrm{He}\begin{bmatrix} -\frac{1}{2}\rho P_{11} & P_{11}A \\ 0 & -\frac{1}{2}\rho P_{11} \end{bmatrix}, \tag{31}$$

which is negative-definite by assumption (30) and corresponds to the characteristic LMI region of a disk of radius ρ centered on the origin, ensuring that the eigenvalues of A in (3) have modulus smaller than ρ. □

Remark 2: Following a similar approach to [17, Section 3.4.3], minimizing

$$\mathrm{trace}\{P_{11}\} = \mathrm{trace}\{\mathbf{X}\} + \mathrm{trace}\{\tilde{X}\} \tag{32}$$

indirectly maximizes the volume of $\mathcal{S}(W)$. Moreover, from the definitions in (11), it can be found that

$$\tilde{X} = \mathbf{M}^{\mathsf{T}}\left(\mathbf{X} - \mathbf{Y}^{-1}\right)^{-1}\mathbf{M}. \tag{33}$$

Therefore, introducing a scalar $\boldsymbol{\tau} > 0$, inequality $\tilde{X} \leq \boldsymbol{\tau} I_n$ can be enforced (by a Schur complement) via the LMI

$$\begin{bmatrix} \boldsymbol{\tau} I_n & \mathbf{M}^{\mathsf{T}} & 0 \\ \mathbf{M} & \mathbf{X} & I_n \\ 0 & I_n & \mathbf{Y} \end{bmatrix} \geq 0. \tag{34}$$

As a consequence, the cost (32) can be replaced with $\mathrm{trace}\{\mathbf{X}\} + \lambda_{\max}(\tilde{X}) \leq \mathrm{trace}\{\mathbf{X}\} + \boldsymbol{\tau}$ under constraint (34), which leads to the convex optimization problem

$$\begin{array}{ll} \min\limits_{\substack{\boldsymbol{\tau},\mathbf{X},\mathbf{Y},\mathbf{Z}_p,\mathbf{Z}_c,\hat{\mathbf{P}}_{22},\mathbf{S},\\ \hat{\mathbf{A}}_c,\hat{\mathbf{B}}_c,\hat{\mathbf{C}}_c,\hat{\mathbf{D}}_c,\hat{\mathbf{E}}_c,\\ \mathbf{G}_p,\mathbf{G}_c,\hat{\mathbf{H}}_2,\hat{\mathbf{T}}_{pp},\hat{\mathbf{T}}_{pc},\hat{\mathbf{T}}_{cc}}} & \mathrm{trace}\{\mathbf{X}\} + \boldsymbol{\tau}, \\ \text{Subject to} & \\ & \text{(17), (18), (29), (30).} \end{array} \tag{35}$$

Due to Theorem 2, problem (35) allows determining optimal controller matrices $A_c \in \mathbb{R}^{n\times n}$, $B_c \in \mathbb{R}^{n\times m}$, $C_c \in \mathbb{R}^{m\times n}$, $D_c \in \mathbb{R}^{m\times m}$ and $E_c \in \mathbb{R}^{n\times m}$ that maximize the volume of $\mathcal{S}(W)$ while ensuring a prescribed convergence rate α and a spectral radius smaller that ρ for matrix A in (3). Moreover, notice that whenever the optimization (35) produces any $P_{22} < 0$, the volume of $\mathcal{S}(W)$ increases since, in such a case, $\mathrm{trace}\{P_{11}\} > \mathrm{trace}\{P\}$.

IV. Numerical examples

In order to illustrate the effectiveness of the solution presented in Theorem 1 and its reduced conservativeness compared to results based on quadratic Lyapunov functions, two numerical applications are presented in this section. Additionally, for the following findings, the optimizer employed was Mosek [1], executed with YALMIP [10].

A. Balancing pointer

Consider the balancing pointer SISO example in [17, Example 3.4]. Thus let $\bar{u} = 5$, $\alpha = 0.5$, $\rho = 10$ and

$$A_p = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B_p = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad C_p = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{36}$$

For this plant, leveraging Theorem 2 and Remark 2, using selections in (11), (13) and (14), the optimizer produces a state-space model of the controller with

$$\begin{aligned} A_c &= \begin{bmatrix} -2.1075 & 0.5482 \\ 0.5258 & -2.2020 \end{bmatrix}, & B_c &= \begin{bmatrix} -2.0303 \\ 2.1958 \end{bmatrix} \cdot 10^3, \\ C_c &= \begin{bmatrix} 1.4809 & -1.6369 \end{bmatrix} \cdot 10^{-3}, & D_c &= 3.8971, \\ E_c &= \begin{bmatrix} 5.0368 & -5.3993 \end{bmatrix}^{\mathsf{T}} \cdot 10^2, \end{aligned} \tag{37}$$

and a sign-indefinite matrix P with eigenvalues $\lambda(P) = \{2.0800 \cdot 10^{-1}, 7.0724 \cdot 10^{-2}, -5.7345 \cdot 10^{-2}, 1.0860 \cdot 10^{-12}, 1.8044 \cdot 10^{-7}\}$.

Figure 1 shows, on the left, the solid red estimate $\mathcal{S}(W)$ of the basin of attraction of the controlled closed-loop system, obtained by running (35) to design the controller matrices. For the synthesis phase, $\mathcal{S}(W)$ is 27.5% larger than the quadratic estimate found with the solution of [3]. Furthermore, leveraging the results in [14], the subsequent stability analysis of the closed-loop system produces a more voluminous non-ellipsoidal set $\mathcal{S}(W)$, which is coherent with Remarks 1 and 2. The right plot of Figure 1 shows the control

$$\left[\begin{array}{c|c|c} A_c & B_c & E_c \\ \hline C_c \cdot 10^4 & D_c \cdot 10 & — \end{array}\right] = \left[\begin{array}{cc|cc|cc} -2.8947 & -0.0064 & -0.8332 & -0.0624 & 17.7281 & -0.4332 \\ 0.0046 & -2.8628 & -0.3043 & 1.4487 & -0.3013 & 3.9641 \\ \hline -0.0247 & -0.0252 & -5.7010 & 0.1093 & \multicolumn{2}{c}{\multirow{2}{*}{—}} \\ 0.6426 & 0.3646 & 0.26670 & -1.7815 & & \end{array}\right] \tag{39}$$

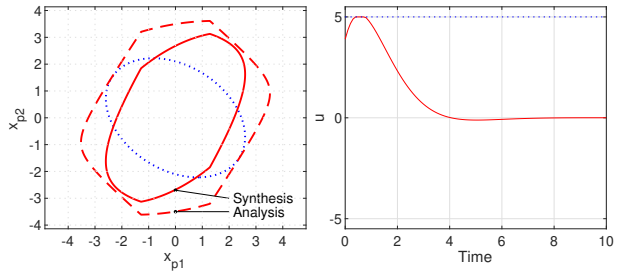

Fig. 1. Left: Set $\mathcal{S}(W)$ found for the closed loop (36), (37) in the synthesis phase (solid red) and in the analysis phase (dashed red). Right: controller output response (37) from the initial state $x_0 = [1 \;\; 3 \;\; 0 \;\; 0]^\mathsf{T}$. Quadratic estimate obtained using the solution of [3] in dotted blue.

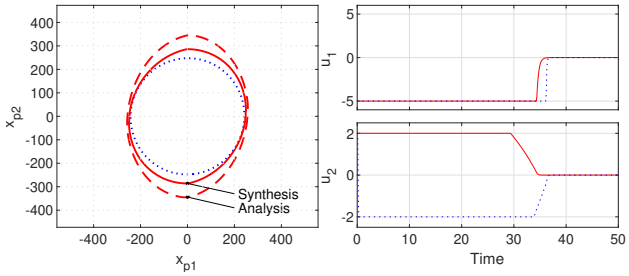

Fig. 2. Left: Set $\mathcal{S}(W)$ found for the closed loop (38), (39) in the synthesis phase (solid red) and in the analysis phase (dashed red). Right: controller output response (39) from the initial state $x_0 = [250 \;\; 0 \;\; 0 \;\; 0]^\mathsf{T}$. In dotted blue, results obtained with [17, Section 3.4.3].

output response u from the initial state $x_0 = [1 \quad 3 \quad 0 \quad 0]^\mathsf{T}$, which is observed to converge to 0 with the procedure in Remark 2 and remains saturated with the solution of [3].

B. MIMO Academic Example

Consider the MIMO plant in [7, Example 2] with

$$A_p = \begin{bmatrix} 0.1 & -0.1 \\ 0.1 & -3 \end{bmatrix}, \quad B_p = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_p = I_2, \quad (38)$$

$\bar{u} = [5 \quad 2]^\mathsf{T}$, $\alpha = 2.5$ and $\rho = 100$. In this case, exploiting the proposed stability certificates and parameterizations (11) to (14), the optimizer returns the controller state-space model matrices (39), at the bottom of the previous page. Further, the obtained matrix P has eigenvalues $\lambda(P) = \{-2.6336 \cdot 10^{-4}, 3.1426 \cdot 10^{-5}, 1.4229 \cdot 10^{-5}, -3.2326 \cdot 10^{-6}, 1.9984 \cdot 10^{-9}, 1.9913 \cdot 10^{-9}\}$.

It is worth to remark that, for this example, the approach of [3] does not find any feasible solution maximizing the estimate of the region of stability of the closed-loop dynamics. Figure 2 reports, at left, on the estimate of the basin of attraction of the controlled closed-loop system, which, in the synthesis phase, is 15.9% larger if estimated with the optimization criterion proposed in Section III, as compared to the estimate obtained with solution in [17, Section 3.4.3] using classic quadratic Lyapunov function. At the right side, the faster response of the controller output u from the initial state $x_0 = [250 \quad 0 \quad 0 \quad 0]^\mathsf{T}$ obtained with the solution in Remark 2, as compared to the dynamics resulting from [17, Section 3.4.3].

V. Concluding Comments

This paper addressed the synthesis of a dynamic output-feedback controller that guarantees regional exponential stability of the closed-loop dynamics. The in-design controller has the same order as the plant, which is not exponentially stable and is subject to input saturation. The design conditions are based on convex LMIs optimizations arising from sign-indefinite quadratic forms and coherent factorization inspired in [15], adequate variable changes and the use of three sector conditions introduced in [14]. Future works may include the external global and regional stability analysis and the discrete-time global and regional stability analysis of systems subject to input saturation using sign-indefinite quadratic forms.

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