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Tuning particle settling in fluids with magnetic fields

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A magnetic field is generated to modify the effective gravity acting on settling particles in a laboratory 8 experiment. When applied to a magnetized spherical particle settling in water-glycerol mixtures, the magnetic 9 field produces a vertical force that counteracts the gravitational field, hence allowing for the magnetic tuning 10 of the settling properties of the particle. While doing so, the spin of the particle around the direction 11 perpendicular to the applied magnetic field is blocked, thus allowing spin solely around the direction of 12 the magnetic field. This method of magnetic modification of the effective gravity is tested on the settling 13 of spherical magnets in quiescent fluids over Galileo numbers in the range [100, 300], and a fixed particle 14 density of 8200 kg/m^3 . The results obtained by varying the Galileo number via the magnetic modification 15 of effective gravity are compared to those obtained with non-magnetic spheres when the Galileo number is 16 modified by varying the fluid's viscosity. We show that the same taxonomy of settling regimes, with nearly 17 identical geometrical properties (in terms of planarity and obliqueness) of the trajectories are recovered. In 18 addition to proving that it is possible to magnetically tame the settling of particles in fluids preserving the 19 features of the non-magnetic case, this also reveals that blocking the spin of the particles does not produce 20 any significant effect on its settling properties in a quiescent fluid. This novel experimental methodology 21 opens new possibilities to experimentally explore many other subtle aspects of the coupling between settling 22 particles and fluids (for instance, to disentangle the effects of rotation, inertia, and/or anisotropy of the 23 particles) in more complex situations including the case of turbulent flows. 24

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25 I. INTRODUCTION

It is of the utmost difficulty to reduce or suppress the $^{\rm 57}$ 26 effect of gravity in a laboratory on Earth. In the context $^{\scriptscriptstyle 58}$ 27 of particle-laden flows research, only a handful of very 28 particular situations allow for the suppression of gravity, 60 29 such as the use of neutrally buoyant particles. This pre-30 vents the exploration of crucial particle-fluid mechanisms 62 31 including inertial effects, which are related to particle-to- $^{\rm 63}$ 32 fluid density ratio. The means available to do experimen- $^{\rm 64}$ 33 tal research in a low gravity environment while preserv- $^{\rm 65}$ 34 ing the capacity to explore inertial effects due to density $^{\rm 66}$ 35 differences are expensive, scarce, and lack repeatability. ⁶⁷ 36 The only options are drop towers, parabolic flights, or 68 37 space experiments in the International Space Station¹⁻³. ⁶⁹ 38 In this article, we present a method to compensate the 70 39 gravity acting on a particle in a flow by the application 71 40 of a magnetic induction (the terms "magnetic field" and 72 41 "magnetic induction" will be used interchangeably in this $^{\rm 73}$ 42 article). With this purpose and as a proof of concept, we ⁷⁴ 43 revisit the problem of the settling of spherical particles in 75 44 a quiescent fluid⁴⁻⁹ while the effective vertical force expe-⁷⁶ 45 rienced by the particles is magnetically varied. Relevant 77 46 research is present in the literature and a brief summary 47 follows. 48

the technique of high-gradient magnetic separation that allows the sorting of sample components with different magnetic susceptibilities 13-15. In parallel, the Magnetic Resonance Imaging community developed the technical aspects to achieve an arbitrary magnetic field profile in a laboratory (or a hospital) with the use of $coils^{16,17}$. These are apart from the study of particles in conductive fluids under the influence of external magnetic inductions, which are central to a number of industrial situations, such as clean metal production¹⁸. The profiles of the external magnetic induction needed to obtain a constant vertical force that can counteract gravity in a number of scenarios such as liquid helium or oxygen were also studied $^{19-22}$. With these tools in hand, some progress has also been made in the particular situations that interest this article: paramagnetic/ferromagnetic or permanently magnetized particles in a weak diamagnetic liquid (i.e., water) subjected to a *weak* external magnetic induction environment (thus no liquid magnetisation occurs). Some studies explored the effects of a homogeneous magnetic induction on one or more particles in non-magnetic fluids $^{23-27}$. In these studies, the main focus was exploring the role of particle-particle interactions on their coupling with the fluid, using a homogeneous magnetic field as a way to tune interactions between particles.

In this article, we focus on the control of the settling of single magnetized spherical particles in a quiescent flow, using an externally applied magnetic field to modify the vertical force acting on the particles. The rich dynamics of single spheres settling in quiescent flows has been extensively explored using numerical^{6,7,28,29} and experi-

⁶ France.

mental tools^{4,5,8,30}. The settling of spheres in a quiescent₁₃₆ 86

viscous fluid is controlled by two non-dimensional param-137 87

eters: (i) the particle-to-fluid density ratio Γ , and (ii) the 88 Galileo number Ga = $U_q d_p / \nu = \sqrt{|\Gamma - 1|gd_p} d_p / \nu$, with 89 U_g the buoyancy velocity, d_p the particle diameter, g the 90

local acceleration of gravity, and ν the kinematic viscos-

91 ity. The diversity of settling regimes has been represented¹³⁸ 92 in the Γ – Ga parameter space^{7,8,30}. As Ga increases, the¹³⁹ 93 trajectory dynamics exhibit bifurcations between differ-140 94 ent regimes such as rectilinear, steady oblique, oscillating $^{\scriptscriptstyle 141}$ 95 oblique, rotating, or chaotic (see fig. 5). Note that, for a^{142} 96 given density ratio Γ , the Galileo number may be changed¹⁴³ 97 either by changing the particle's diameter or the fluid vis-144 98 cosity. This problem thus offers an appealing framework¹⁴⁵ 99 to validate a magnetic method to tune the effective ver-¹⁴⁶ 100 tical force experienced by a particle. In this article, we¹⁴⁷ 101 explore how the Γ – Ga parameter space is modified when¹⁴⁸ 102 an additional constant magnetic force is applied to the¹⁴⁹ 103 particles. Bifurcations between regimes and properties¹⁵⁰ 104 of the trajectory collapse when using a corrected Galileo¹⁵¹ 105 number $Ga(\tilde{g})$ that considers the effective gravity \tilde{g} in¹⁵² 106 its computation. In order to explore $\mathrm{Ga}(\tilde{\mathrm{g}})$ ranging from 153 107 100 to 280, we varied not only the magnetic vertical force¹⁵⁴ 108 but also the viscosity of the fluid. 109

The article is organized as follows. We first present 110 the basic theoretical layout of the magnetic gravity com-111 pensation method and experimental design in Section II. 112 The results of the sedimentation of magnetic particles 113 with modified gravity are described in Section III. Then, 114 we discuss conclusions and perspectives of this work in $_{158}$ 115 Section IV. Finally, the Appendix presents further de-116 tails on the theoretical foundation of the method, its val-117 idation, and further experimental details. 118 161

П. **GRAVITY TUNING USING MAGNETIC FORCES** 119

General Principles Α. 120

The method introduced in this article consists on the₁₆₈ 121 application of a constant vertical magnetic force to a 122 magnetized particle in order to modify the effective verti-123 cal force it experiences. The forces applied are the mag-124 netic force $\mathbf{F}^{\hat{M}}$, the weight, and the fluid force $\mathbf{F}^{\text{fluid}}$ ex-169 125 erted by the fluid to the particle. Similarly, a magnetic $_{170}$ 126 torque \mathbf{T}^{M} adds up to the fluid torque $\mathbf{T}^{\text{fluid}}$. In this₁₇₁ 127 section, we discuss the features of the externally applied $_{172}$ 128 magnetic field required to modify the vertical force on_{173} 129 small spherical permanent magnets. 174 130 The magnetic torque and force acting on a particle₁₇₅ 131 with a magnetic dipole \mathbf{M} in the presence of an external₁₇₆ 132 magnetic induction **B** read: 133 177

$$\mathbf{F}^M = \nabla(\mathbf{M} \cdot \mathbf{B}). \tag{1}$$

$$\mathbf{T}^M = -\mathbf{M} \times \mathbf{B}.$$
 (2)¹⁷⁶

These equations indicate that the magnetic force and 179 134 torque can be varied independently by the gradient and 180 135

magnitude of B, respectively. We focus on the modification of the vertical force along the \hat{z} axis, which reads:

$$\mathbf{F} \cdot \hat{\mathbf{z}} = m\tilde{g} = m[g + M\nabla_z(B)\cos(\psi)/m], \qquad (3)$$

where the effective gravity \tilde{g} was defined, $B = |\mathbf{B}|$, and ψ is the angle between the vectors **B** and **M**. See the full equation of motion in App. A1. The method aims at reaching a constant value of \tilde{g} over the settling of the small permanent dipole and requires: (1) a constant ψ angle and (2) a linear evolution of the vertical component of the magnetic field $B_z = G_z \ z + B_0$, where the gradient G_z sets the amplitude of the vertical magnetic force and B_0 is an offset value. Condition (1) is met when the magnetic torque exceeds the hydrodynamic torque, a condition which is always met in our configuration, as discussed below. A limitation of the method comes from the existence of a radial force, due to the radial component of the magnetic field in cylindrical coordinates, imposed by the divergence-free nature of magnetic fields. Indeed, condition (2) imposes the following magnetic field evolution (see App. A 2):

$$\mathbf{B}(r,z) = (G_z \ z + B_0) \ \hat{\mathbf{z}} + (-G_z/2 \ r) \ \hat{\mathbf{r}}.$$
 (4)

The relative intensity of the undesired radial drift is controlled by both the value of the axial gradient G_z and the offset B_0 (see App. A 2 for a full derivation of the force). In order to minimize spurious effects of magnetic radial drift, we have designed our experiment choosing values of G_z and B_0 (see Table III) so that the ratio between the radial and axial magnetic forces within the measurement volume (see Eq. E7) remains smaller than $5 \cdot 10^{-2}$.

Conversely, the magnitude of the torque can be quantitatively estimated. Given the expression of the linear magnetic field given by Eq. (4), the ratio of the magnetic torque over the hydrodynamic torque reads (see App. A 2 for complete derivation):

$$T^M/T^{\text{fluid}} = \frac{|\mathbf{M}||\mathbf{B}|}{1/64C_\omega d_p^5 \pi \rho_f \omega^2},\tag{5}$$

where d_p is the particle diameter, $|\mathbf{M}|$ its magnetic moment, ω the particle angular velocity, C_{ω} the rotational drag coefficient, and ρ_f the fluid density. The value of the torque ratio is of the order of 10^{11} , based on the typ-ical values used in our experiment³¹. This implies that the magnetic dipole aligns with the magnetic field virtually instantaneously and the particle rotation is therefore only possible around the direction of the magnetic field.

Β. **Experimental Setup**

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The experiments are performed in a transparent PMMA water tank with a square cross-section of $170 \times$



FIG. 1. Experimental setup. Two cameras image the particles²³⁰ inside the water tank. Six circular coils produce a magnetic²³¹ induction used to compensate gravity on magnetic particles.²³² The origin of coordinates is located at the geometrical center²³³ of the water tank, which coincides with the middle distance²³⁴ between coils 1 and 6.

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 170 mm^2 and a height of 710 mm, which is sketched in₂₃₈ 181 Fig. 1. The tank is filled with different mixtures of $pure_{239}$ 182 glycerol (Sigma-Aldrich W252506-25KG-K) and distilled₂₄₀ 183 water, ranging from 0% to 40% glycerol concentration.₂₄₁ 184 Viscosity is measured with a rheometer Kinexus $ultra+_{242}$ 185 from Malvern industries with a maximum uncertainty of_{243} 186 0.6%. The kinematic viscosity ν ranges from 1×10^{-6} to₂₄₄ 187 1×10^{-3} m²/s. Furthermore, an air-conditioning system₂₄₅ 188 keeps a constant room temperature of $(22\pm0.6)^{\circ}\mathrm{C}$ yield-_{^{246}} 189 ing a 2% uncertainty on the precise value of the viscosity. $_{\rm 247}$ 190 191 248

A 150 mm region of fluid above and below the visu- $_{249}$ 192 alization volume is set to ensure both the disappearance₂₅₀ 193 of any initial condition imposed on the particles when₂₅₂ 194 released and the effects of the tank's base. Furthermore, 253 195 a minimum distance of 20 mm between the tank walls₂₅₄ 196 and the particles is maintained. Using available correla-255 197 tions³², the settling velocity hindering due to wall effects₂₅₆</sub> 198 is estimated to be lower than 3%, thus neglected. 199 257

To record the trajectory of the particles, two high- $_{258}$ speed cameras model fps1000, from The Slow Motion $_{259}$ Company, image the water tank with a resolution of $_{260}$ $720 \times 1280 \text{ px}^2$ and 2300 Hz. Backlight illumination is $_{261}$ used, as represented by the dark blue rectangles in Fig. 1.262

These dual recordings allow the implementation of a 4D-LPT (Lagrangian Particle Tracking resolved in time and in three dimensions) 33 . This method accurately tracks particles with a precision of approximately $90\mu m$. This level of precision is determined by assessing the disparity between rays during the stereo-matching process between the two cameras. It is important to note that the experimental noise affecting particle position is short-term in nature. This noise is effectively mitigated due to the temporal redundancy achieved through oversampling at a high frame rate of 2300 Hz. In order to reduce experimental noise (due to inevitable particle detection errors in the Lagrangian particle tracking treatment³⁴), the raw trajectories are smoothed by convolution with a Gaussian kernel of width $\Sigma = 12$ frames; acting as a low-pass filter with a cut-off frequency $f_c = 2300 \text{ Hz}/\Sigma = 192 \text{ Hz}$. Consequently, the uncertainty associated with instantaneous velocity along these trajectories is less than 4 mm/s^{33} . Moreover, when the velocity is averaged over a specific trajectory, the associated uncertainty of the mean velocity decreases to a few hundred microns per second.

The particles are spherical permanent neodymium magnets with a mass density $\rho_p = 8200 \text{ kg/m}^3$, a diameter $d_p = 1 \text{ mm}$, and an average arithmetic roughness $Ra = \frac{1}{L_x L_y} \int_0^{L_x} \int_0^{L_y} |z(x,y) - \overline{z(x,y)}| dxdy = 15 \ \mu\text{m}$, where (x, y) is the local planar surface and L_x and L_y are

where (x, y) is the local planar surface and L_x and L_y are the distances over which the height z(x, y) and its average over the measurement area $\overline{z(x, y)}$ are obtained. These measurements are performed with a Scanning Electron Microscope (SEM) model ZEISS SUPRA 55 VP, over an area of 200 × 500 µm². The surface roughness of particles has been proven to influence the boundary layer, therefore modifying several aspects of the dynamics^{35–37}. Note that the small values $Ra/d_p < 1.5 \times 10^{-2}$ are not expected to notably modify the dynamics³⁶. Particle dimensions and shapes were measured using a microscope with a precision of 10 µm. No significant deviation from the spherical shape or the documented diameter were measured. The weight of one hundred particles was measured with a precision of 1×10^{-3} g. The result was divided by one hundred to obtain the average mass of a single particle: $m_p = 4.3 \times 10^{-3}$ g with a precision of 1×10^{-4} g.

The ranges of non-dimensional numbers reached are Ga = [100, 280] and Γ = [6.8, 8.2]. Note that the water density is different at each viscosity and therefore Γ varies as well. The magnetic moment $|\mathbf{M}|$ is computed using Eq. E8 and results in $|\mathbf{M}| = 4.96 \times 10^{-4} \text{ Am}^2$. The external magnetic induction is created from a six-coil system shown in Fig. 1 and detailed below. While a larger number of coils would produce a magnetic field closer to the linear profile needed, the choice of six coils resulted in the compromise between quality of magnetic field (measured in Sec. II C) and the need for optical access. The coils were wound from copper wires of 0.5 mm (coils 2 to 5) and 1.5 mm (coils 1 and 6) in diameter. The coils are placed at vertical distances Z_i ($i \in [1, 6]$) from the origin of coordinates, set at the middle distance between coils

TABLE I. Characteristics of the coils used for the magnetic compensation of gravity. Columns show the following parameters for each coil used: effective number of turns N, radius R, and cross section σ . Each coil is represented by a number as in Fig. 1.

	1	2	3	4	5	6
Ν	965	103	450	452	101	969
R (cm)	16.3	22.1	15.6	15.4	21.8	16.1
$\sigma ~({\rm cm}^2)$	15.6	3.9	3.6	3.6	3.9	15.6

1 and 6. Details on the position of coils and the current $^{\scriptscriptstyle 300}$ 263 301 imposed are available in App. B. 264

Magnetic Field Generation С. 265

The system of coils generating the magnetic induction₃₀₆ 266 is described in detail in what follows. Each coil has a₃₀₇ 267 radius R_i and a current I_i . In order to simplify compu-₃₀₈ 268 tations, each coil is modeled as a single loop of radius $R_{i^{309}}$ 269 with an effective current $N_i I_i$ (N_i is the effective number₃₁₀ 270 of turns). The value of N_i is determined from a non-₃₁₁ 271 linear fit of experimental measurements of the magnetic 272 312

induction using a Bell 7030 Teslameter by: 273

$$B_i(z) = 2\pi 10^{-7} \frac{R_i^2 N_i I_i}{(z^2 + R_i^2)^{3/2}}, \qquad (6)_{_{315}}^{^{314}},$$

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where z is the distance from the coil *i*'s individual geo-³¹⁷ 274 metric center. 275

The coils' effective parameters are presented in Table I. $^{\scriptscriptstyle 318}$ 276 The uncertainties on N_i and R_i by the aforementioned³¹⁹ 277 method are estimated to be 4% and 3%, respectively,³²⁰ 278 while the coils' cross-section (σ) uncertainty is 0.4 cm² 279 Note that three sets of identical coils were used (namely³²² 280 1-6, 2-5, and 3-4), and the uncertainties of their effective $\frac{3^{23}}{2^{23}}$ 281 parameters are identical. 282

The six circular coaxial coils used to generate the re-283 quired magnetic induction (Eq. 4) are sketched in Fig. 1. $\frac{^{326}}{^{327}}$ 284 The coils are modelled as infinitesimal current loops,²²⁷ 285 therefore the theoretical axial magnetic induction at the 286 320 six-coil system axis reads: 287 330

$$\mathbf{B} \cdot \hat{\mathbf{z}} = 2\pi 10^{-7} \sum_{i=1}^{6} \frac{R_i^2 N_i I_i}{((z+Z_i)^2 + R_i^2)^{1.5}}.$$
 (7)³³²
³³¹
³³¹
(7)³³²
³³³
³³⁴

In order to set the magnetic field presented in Eq. 4 in the₃₃₅ 288 laboratory, a nonlinear least squares fit of Eq. 7 to the336 289 axial component of Eq. 4: $B_z = G_z z + B_0$ is performed.³³⁷ 290 The fit's fixed parameters are: N_i , R_i , G_z , B_0 , and $Z_{fit^{338}}$ 291 the range at which the fit is performed . The outputs are:339 292 I_i the current in each coil, and Z_i the distance between₃₄₀ 293 the coil *i*'s geometrical center and the origin of coordi- $_{341}$ 294 nates z = 0 (located at the middle distance between coils₃₄₂ 295 1 and 6). This method to solve equations numerically₃₄₃ 296 is not affected by the fact that there are more variables₃₄₄ 297

TABLE II. Details of the two external magnetic inductions implemented here: Case g_0 and $g^* = 0.65$.

	Nonlinear fits		Measurements		
	Case g_0	Case $g^* = 0.65$	Case g_0	Case $g^* = 0.65$	
G_z (G/m)	0	-290	0 ± 20	-286 ± 25	
B_0 (G)	20	26	20 ± 1	22 ± 1	
$Z_{fit} (\mathrm{mm})$	(-150, 50)	(-100, 0)	×	×	

than equations, as the variables are modified at random until the coefficient of determination (R^2) is minimized.

Note that there is no radial dependence on Eq. 7. The magnetic field outside the axis is estimated by Maxwells' equations in App. A 2 (see Fig. A.1). This theoretical prediction was qualitatively confirmed with a Teslameter.

Two sets of values for G_z , B_0 , and Z_{fit} were chosen:

1. Case g_0 : homogeneous vertical magnetic induction.

In this case $G_z = 0 \& B \neq 0$, yielding no net magnetic force and only impacting the particle's rotation through the blocking effect of the magnetic torque previously discussed.

2. Cases g^* : constant gradient vertical magnetic induction.

In this case $G_z \neq 0 \& B \neq 0$, yielding both a net magnetic force (used to compensate gravity) and a net magnetic torque, resulting in the rotationblocking effect, respectively.

The two cases differ on the magnetic force magnitude (proportional to G_z) that they impose on the particles: whereas Cases g^* block the rotation and apply a force, Case g_0 blocks the rotation but the applied force is negligible. The size of the fit window Z_{fit} needed to be shortened for Cases q^* to minimize inhomogeneities in the magnetic force. The specific values of these two sets of input parameters in the nonlinear fit to Eq. 7 are presented in Table II, alongside the corresponding magnetic induction measurements discussed below. Details about coils' parameters for both cases are detailed in App. B.

Note that new cases are achieved by the multiplication of the base currents in each coil given by the solutions of Cases g_0 and $g^* = 0.65$.

Fig. 2 presents the profiles of axial magnetic induction B_z (first row) and its gradient $\nabla_z B$ (second row) obtained from the nonlinear fit (black lines), and measurements performed with a Teslameter (blue crosses), versus distance to the origin. Finally, the blue shade represents the region (Z_{fit}) where the fit was performed. Note that because the magnetic field is measured at the axis, the equation fitted represents the center of a current loop $\mathbf{B} = B_z \hat{\mathbf{z}}$.

It can be seen that, for Case g_0 , the magnetic induction measurements (Fig. 2(a)) overlap with the simulations, whereas the magnetic induction gradient (Fig. 2(c)) presents an average difference of 10%, with maximum



FIG. 2. B_z and its gradient computed from the axial magnetic induction measurements (blue points) and simulations (black₃₇₉ line) against the distance to the origin z = 0, for Cases g_0 and $g^* = 0.65$. The insets present a zoomed version of the plots. The blue area shows the z-range Z_{fit} over which the magnetic³⁸¹ field is optimized. Finally, a horizontal arrow denotes the³⁸² rightwards direction of gravity.

386 values of 30% that occur near the extremes of Z_{fit} . On₃₈₇ 345 the other hand, Cases g^* have a 10% discrepancy on₃₈₈ 346 the magnetic induction gradient $\nabla_z B$ (Fig. 2(d)) and in₃₈₉ 347 the magnetic field (Fig. 2(b)). Additionally, note that₃₉₀ 348 the radial-to-axial force ratio (Eq. E7) takes the follow-₃₉₁ 349 ing maximum value for Cases g_0 and g^* : 1×10^{-2} and g_{392} 350 1.5×10^{-1} , respectively. In the case of the 1 mm spherical₃₀₃ 351 permanent magnets that are studied here, these inhomo-394 352 geneities in the magnetic induction gradient produce a_{395} 353 5% and 3% of typical variability in the effective gravity $_{\tt 396}$ 354 value for Cases g^* and g_0 , respectively. 355 397

The presence of oscillations in the magnetic field can³⁹⁸ 356 be explained as the interference between the higher har-³⁹⁹ 357 monics that compose the total magnetic induction of⁴⁰⁰ 358 each coil¹⁹. The interaction between these higher har-401 359 monics can be modified considerably by small errors in⁴⁰² 360 the coil positioning. To explore this idea, Fig. 3 shows⁴⁰³ 361 the theoretical magnetic induction (a) and gradient (b)⁴⁰⁴ 362 for Case $q^* = 0.65$, evaluated at different coil 1 posi-405 363 tions (Z_1) : its original position (black), 5 mm downwards⁴⁰⁶ 364 (red), and 5 mm upwards (magenta). While the magnetic⁴⁰⁷ 365 inductions are indistinguishable (Fig. 3 (a)), the gradi-408 366 ents show clear differences (Fig. 3 (b)): Within the mea-367 surement region (blue shade) a difference of up to 3% is 368 present. Note that the coil positioning error is estimated⁴⁰⁹ 369 to be 5 mm; this value includes the approximation of the⁴¹⁰ 370 coils by an infinitesimal loop at its geometrical centre 371 and the 2 mm precision in positioning due to the tools₄₁₁ 372 used. These factors are hypothesized to produce the dif-412 373

ference between the experimental measurements and the simulated values encountered in Fig. 2.



FIG. 3. Theoretical magnetic field (a) and gradient (b) for Case $g^* = 0.65$, evaluated at different coil 1 vertical positions. The original coil position (black); 5 mm downwards (red); and 5 mm upwards (magenta) are shown.

D. Data Sets

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The camera-tank distance A was varied to obtain two different measurement volume heights h (see Fig. 1): h = 100 mm and h = 200 mm which correspond to the ranges Z_{fit} . This translates into different maximum particle non-dimensional trajectory lengths $l_{max}^* = h/d_p =$ 100 & 200 (recall that d_p is 1 mm for all cases), while the measured volume has a 150×150 mm² transverse section.

The experimental procedure is as follows: Initially, the tank is filled with a water-glycerol mixture and 24 hours are allowed to pass. During this time, the temperature at various locations within the bulk of the fluid stabilizes to a temperature difference of less than 0.6°C. This was determined by a movable temperature probe that was used to measure the temperature in the fluid at different locations. Subsequently, a standard calibration process for the 4D-LPT system is executed³³. Once calibration is complete, the magnetic field is activated and spherical particles are released from a plastic tube with a 5 mm diameter positioned at the center of the tank and 5 cm above the region of interest. The particles are carefully introduced one by one into the tube where they settle due to gravitational forces. To ensure that the fluid remains undisturbed between successive particle releases, a minimum waiting time of 120 seconds is observed. This interval is chosen to be at least 12 times the viscous relaxation time, denoted as $\tau = d_p^2/\nu$. The specific value of the viscous time varies across experimental cases and a maximum waiting time value of $1 \times 10^3 \tau$ is achieved in the more viscous case.

III. SINGLE MAGNETIC SPHERE SETTLING AT FIXED ORIENTATION AND MODIFIED GRAVITY

In this section the local gravitational pull is reduced via the magnetic gravity compensation method presented

previously. It is proven that the Galileo number (and 413 hence the settling regime) of spherical magnetic particles 414 can be magnetically tuned. Furthermore, these results 415 confirm the validity of the magnetic gravity compensa-416 tion method. This is the first step towards the deploy-417 ment of a global strategy to experimentally explore the 418 influence of gravity in particle-fluid interactions. 419

Α. Parameters explored 420

The results of an experimental study on spherical 421 metallic particles settling in a quiescent flow at moder-422 ate Reynolds numbers performed by our group⁸ are used 423 as reference data to be compared against the present 424 measurements. Note that those experiments were per-425 formed in the water tank presented in Fig. 1 and the467 426 non-magnetic particles used had $\rho_p = 7950 \text{ kg/m}^3$, which 468 427 is comparable to the density of our magnetic particles. 469 428 Recall that the particles are affected in two ways due₄₇₀ 429 to the presence of the external magnetic inductions pro-471 430 duced here: First, as the applied magnetic field is mostly₄₇₂ 431 vertical on the region of interest, particle rotation is par-473 432 tially blocked and only allowed around the vertical axis.474 433 Second, the spatial profile of the imposed field is specifi-475 434 cally tailored to be as close as possible to a homogeneous476 435 vertical gradient field in the region of interest, hence par-477 436 ticles experience an almost constant magnetic force which₄₇₈ 437 counteracts the gravitational force yielding different ef-479 438 fective gravity values \tilde{q} . 439 480

In the sequel, the following nomenclature will be used₄₈₁ 440 to refer to the different experiments: 441 482

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• Case $\emptyset B$ 442 Reference case with non-magnetic particles⁸. 443

• Case q_0 444

Uniform magnetic induction (spin blocking effect).487 445 This magnetic induction profile was presented in 446 Fig. 2(a)-(c). 447

• Cases q^* 448

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Uniform magnetic induction gradient (spin block-488 449 ing and modified gravity). The magnetic induction₄₈₉</sub> 450 profile was presented in Fig. 2(b)-(d). 451 490

As detailed in Section IID, the visualization volume in 452 Case g^* has a maximum non-dimensional height $l^*_{max} =$ 453 100, whereas for Case $g_0 l_{max}^* = 200$. This difference⁴⁹¹ 454 is due to the finite size of the coils: It is possible to 455 produce a homogeneous magnetic induction (Case q_0) in₄₉₂ 456 a larger region of space compared to the production of a493 457 homogeneous magnetic induction gradient (Cases g^*). 494 458 For the Cases g^* , Table III summarizes the different⁴⁹⁵ 459 effective gravities \tilde{g} explored (details about the estima-496 460 tion of \tilde{q} are given in the next subsection). The effective₄₉₇ 461 gravity will be given from now on in non-dimensional⁴⁹⁸ 462 form $g^* = \tilde{g}/g$. In practice, changing the effective grav-499 463 ity is simply achieved by multiplying the currents in the 500464 original configuration $(g^* = 0.65)$ by a constant value. 501

TABLE III. Different magnetic induction gradients applied and regimes of effective gravity \tilde{g} explored. The columns present the five different variants of the magnetic field in the Cases q^* . The rows show the non-dimensional gravities q^* defined as the ratio between the effective gravity \tilde{g} and the usual gravity acceleration $g = 9.8 \text{ m/s}^2$, and the values of $\nabla_z B$ evaluated at r = 0 (i.e., the coils' axis) denoted G_z .

$g^* = \tilde{g}/g$	G_z (G/m)	B_0 (G)
0.43 ± 0.02	-476 ± 42	37 ± 1
0.65 ± 0.02	-286 ± 25	22 ± 1
0.77 ± 0.02	-191 ± 16	15 ± 1
0.80 ± 0.02	-171 ± 15	13 ± 1
0.90 ± 0.02	-95 ± 8	7 ± 1

The exploration of settling regimes is performed by independently changing the effective gravity and the fluid viscosity according to the following protocol. For a given value of fluid viscosity, all the previous values of \tilde{q} are applied to vary Galileo number. In this way, with a fixed viscosity, the Galileo number can be modified by varying the effective gravity between 65% and 100% of its value at $q = 9.8 \text{ m/s}^2$. The range of Ga values explored is [100, 280].

In the remainder of this article, non-dimensional parameters are denoted by a superscript asterisk. Spatial variables are normalized by particle diameter $x^* = x/d_n$, velocities are normalized by the buoyancy velocity $v^* =$ $v/U_g = v/\sqrt{|\Gamma - 1|\tilde{g}d_p}$, and time is normalized by the response time of the particles $\tau_q = d_p/U_q$.

Finally, the trajectory angle is defined as the ensemble average of the angle between a linear fit to each trajectory and the vertical. The trajectory planarity is quantified by the ratio of eigenvalues λ_2/λ_1 (with $\lambda_1 \geq \lambda_2$) of the non-dimensional perpendicular (to gravity) velocity correlation matrix, defined as^{38} :

$$\langle \mathbf{v}_{\perp}^{*} \; \mathbf{v}_{\perp}^{*\mathrm{T}} \rangle = \begin{bmatrix} \langle v_{x}^{*2} \rangle & \langle v_{x}^{*} v_{y}^{*} \rangle \\ \langle v_{y}^{*} v_{x}^{*} \rangle & \langle v_{y}^{*2} \rangle \end{bmatrix}, \tag{8}$$

with $v^* = v/U_g$. Perfect planar (non-planar) trajectories yield $\lambda_2/\lambda_1=1$ (=0), while the fractions represent intermediate cases.

Terminal Velocity & Effective Gravity Homogeneity В.

It is important to verify that the particles reach a terminal velocity, and that there is no global deviation caused by the magnetic induction gradient inhomogeneities measured in Section IIC. In this sense, Fig. 4 presents some examples of settling velocity, represented by particle Reynolds numbers $(Re_p = v_s d_p / \nu, \text{ with } v_s$ the velocity component parallel to gravity), for various amplitudes of the applied magnetic gradient in Cases q^* , which correspond to various values of the effective gravity. Two examples of Case g_0 at two different values of



FIG. 4. Particle Reynolds number ($Re_p = v_s d_p / \nu$, with v_s^{543} the velocity component parallel to gravity) versus the non-544 dimensional distance to origin z^* (panel a) and versus the⁵⁴⁵ Galileo number computed with the corresponding effective546 gravity (panel b), for all the variants of external magnetic547 fields studied. (a) Representative particle Reynolds number₅₄₈ from independent realizations for the following cases: g_0 at₅₄₉ ${\rm Ga}(\tilde{g})=235\pm7$ (red-violet dashed line) and ${\rm Ga}(\tilde{g})=80\pm5_{_{550}}$ (yellow dashed line); $g^* = 0.43$ at $Ga(\tilde{g}) = 105 \pm 5$ (blue⁵⁵⁰ line); $g^* = 0.65$ at $Ga(\tilde{g}) = 129 \pm 5$ (purple line); $g^* = 0.77_{res}^{551}$ at $Ga(\tilde{g}) = 178 \pm 5$ (red line); $g^* = 0.80$ at $Ga(\tilde{g}) = 143 \pm 5^{552}$ (green line); and $g^* = 0.90$ at $\operatorname{Ga}(\tilde{g}) = 193 \pm 7$ (cyan line). (b)⁵⁵³ Particle Reynolds number versus Galileo number, alongside⁵⁵⁴ the theoretical settling velocity computed from the correlation⁵⁵⁵ presented in Cabrera-Booman et al.⁸ and the reference data⁵⁵⁶ of Case $\emptyset B^8$. 557

Ga (80 and 235) are also shown to illustrate the sole ef- 560 502 fect of rotation blockage in a particle in the Rectilinear⁵⁶¹ 503 (Ga=80) and Planar Rotating (Ga=235) regimes. Note⁵⁶² 504 that panel (a) presents single realizations showing the 505 evolution of vertical velocities versus distance to the ori-506 gin for individual particles at given values of g^* and Ga,⁵⁶³ 507 while points for the terminal velocity shown in panel (b)⁵⁶⁴ 508 are obtained from the ensemble and spatial average of⁵⁶⁵ 509 tens of drops. 510

Fig. 4(a) shows the particle Reynolds number versus 567 511 the non-dimensional distance to origin z^* , for all the vari-512 ants of external magnetic fields studied. Note that the 569 513 different examples shown here were not obtained with the ⁵⁷⁰ 514 same fluid viscosity, therefore, the corresponding particle $^{\scriptscriptstyle 571}$ 515 Reynolds numbers are not ordered by g^* but by $Ga(\tilde{g})$.⁵⁷² 516 573 There is a well defined terminal velocity (i.e., the particle 517 Reynolds number is constant) for all magnetic field cases 518 except for Case $g^* = 0.43$ (which corresponds to the case 519 of highest amplitude of applied magnetic field). In that 520 case, Re_p varies as much as 10% along the particle tra-⁵⁷⁴ 521 jectory. This variation is of less than 2% in the other⁵⁷⁵ 522 cases, which is in the range of the fluctuations reported 523 in the literature⁹ and in the non-magnetic experimental 524 data from Case $\emptyset B^8$. This indicates that (except for the 525 largest applied field, case $q^* = 0.43$) the applied mag-576 526 netic field gradient effectively compensates gravity ho-577 527 mogeneously, and that the rotation blocking effect does578 528 not prevent particles from attaining a well defined termi-579 529 nal velocity. A possible explanation for the lesser quality₅₈₀ 530

of Case $g^* = 0.43$ could be related to finite size effects of the coils which were not taken into account in the design of the magnetic field linear profiles and which may enhance the magnetic gradient inhomogeneity.

Fig. 4(b) shows the ensemble and space average of the measured particle Reynolds numbers versus a Galileo number computed using the effective gravity. The marker size represents each data point uncertainty. Black solid circles present the data from the reference $case^8$ and a solid line presents the Re_p vs Ga correlation law proposed by Cabrera-Booman *et al.*⁸, which was inspired from a previous study by Brown *et al.*³⁹ Note that this correlation has been shown to underestimate the terminal velocity of high density ratio particles⁸ by between 5% and 10%, as it can be seen in the figure. We can see that the terminal particle Reynolds numbers of all cases consistently follow the same trend, in good agreement with the proposed correlation. Only the point at $q^* = 0.43$ shows a slight deviation from the global trend, confirming the lesser quality of the magnetically modified gravity in that case.

Overall, it is then concluded that measurements up to $g^* = 0.65$ can be expected to behave analogously to non-magnetic spheres settling at modified gravity (this claim will be confirmed below).

Note regarding the estimation of the effective gravity. The fact that particles reach a terminal velocity allows for the definition of a constant effective gravity, and determining its precise value is crucial. Recall the equation linking \tilde{g} with the experimental parameters (see Section II A):

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$$\tilde{g} = g - |\mathbf{M}|\nabla_z|\mathbf{B}|/m. \tag{9}$$

As discussed previously, the value of M can be computed from the manufacturer's data and $\nabla_z B$ was measured. The values of \tilde{g} can then be computed. The method to obtain the effective gravity given by Eq. 9 and detailed above is used throughout the manuscript. However, another way to compute the effective gravity is by the measurement of terminal settling velocity with no external magnetic induction applied $v_{s,0}$ and comparing it to that of particles settling in the same flow but with its gravity modified to $v_{s,M}$. The particle settling velocity can be calculated as:

$$v_s = \sqrt{\frac{mg}{\frac{\pi}{8}C_D(Re_p)d_p^2\rho_f}}.$$
 (10)

The ratio $m/(\frac{\pi}{8}d_p^2\rho_f)$ is identical because the same particles are used, therefore:

$$g^* = \frac{\tilde{g}}{9.8 \ m/s^2} = \frac{v_{s,M}^2 C_D(Re_{p,M})}{v_{s,0}^2 C_D(Re_{p,0})},\tag{11}$$

with $Re_{p,0} = v_{s,0}d_p/\nu$, and $Re_{p,M} = v_{s,M}d_p/\nu$. Then, using usual drag correlations^{8,39}, it is possible to extract \tilde{g} for all the Cases g^* studied. Values of \tilde{g} that overlap with the computed ones are independently obtained from particle terminal settling velocities.

581 C. Path instability Results

The different path instabilities of a single spherical 582 magnetic particle settling in a quiescent flow are pre-583 sented here. The dynamics are controlled by Galileo 584 number (Ga), which varies as the square root of gravity. 585 The goal is to test whether the different settling regimes 586 which have been reported in previous numerical^{6,7} and 587 experimental^{4,5,8,30} studies (where Ga is classically varied 588 by changing the particle diameter, the particle-to-fluid 589 density ratio, or the viscosity) are consistently recovered 590 when Ga is varied by magnetically modifying the gravity. 591 To further clarify the possible impact of rotation block-592 age alone, we also consider Case q_0 (homogeneous applied 593 magnetic field) where Ga is classically varied. Measure-636 594 ments in Cases g^* and g_0 are compared to the reference 595 non-magnetic case⁸. First, we qualitatively show the tax-638 596 onomy of the different type of trajectories observed when $_{630}$ 597 spanning Ga in the magnetic case. Then, the trajectory₆₄₀ 598 angle and planarity are quantitatively compared to the $_{641}$ 599 experimental⁸ and numerical⁷ data in the non-magnetic₆₄₂ 600 reference scenario Case $\emptyset B$. 601

⁶⁰² It is shown that by magnetically changing gravity, the⁶⁴⁴ ⁶⁰³ path instability in action can be tuned and that all the⁶⁴⁵ ⁶⁰⁴ classical regimes are recovered in the same range of Ga⁶⁴⁶ ⁶⁰⁵ and with the same trajectory properties than for the ref-⁶⁴⁷ ⁶⁰⁶ erence non-magnetic case.

⁶⁰⁷ The state-of-the-art Γ - Ga parameters space taken₆₄₉ ⁶⁰⁸ from Cabrera-Booman *et al.*, alongside the data points₆₅₀ ⁶⁰⁹ explored in the present study are presented in Fig. 5.

It is worth stressing that, as revealed in previous non- $_{652}$ 610 magnetic experimental studies (see for instance the re-653 611 cent work by Raaghav et al.³⁰ and Cabrera-Booman et_{654} 612 $al.^8$) the boundaries between regimes in this space of pa- $_{655}$ 613 rameters are not always sharp. Some regimes exist in a₆₅₆ 614 narrow range of Ga and some regions have been shown₆₅₇ 615 to exhibit multi-stability (i.e., several regimes have been₆₅₈ 616 reported in similar regions of parameters). This may also₆₅₉ 617 be affected by the uncertainty, typically of the order of_{660} 618 a few percentage points, with which the Galileo num-661 619 ber is determined. This is mostly due to uncertainties $_{662}$ 620 of the viscosity in the classical non-magnetic case, with $_{663}$ 621 the addition in the present case of the small variability $_{\rm 664}$ 622 of effective gravity modification due to the 5% variability $_{\scriptscriptstyle 665}$ 623 of the magnetic gradient across the measurement volume $_{\rm 666}$ 624 previously discussed. 625 667

627 1. Trajectories Geometry

Fig. 6 presents some representative 3D trajectories⁶⁷² (first column) alongside a top view (second column). All⁶⁷³ of these trajectories belong to Case g_0 (rotation block-⁶⁷⁴ age and normal gravity), as those of Case g^* present the⁶⁷⁵ same dynamics but with shorter trajectories. Each sub-⁶⁷⁶ panel presents results with Ga in the main four regimes⁶⁷⁷ previously identified: ⁶⁷⁸

 $_{635}$ (1) Steady Oblique – Ga = {153, 158}. Fig. 6(a) 679

Particle-to-fluid density ratio

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FIG. 5. Particle-to-fluid density ratio (Γ) – Galileo number (Ga) space of parameters. Alongside the data points measured in this study taken from Cabrera-Booman *et al.*⁸

presents some trajectories in this regime. Although some trajectories show slight deviations from perfect planarity (the value of the planarity parameter $\sqrt{\lambda_2/\lambda_1}$ will be shown later to be of the order of 0.2 versus 0.05 for Case $\emptyset B^8$), the trajectories are overall planar and have a well-defined angle with the vertical that, after centering, form a cone in 3D space.

(2) Oblique Oscillating – Ga = 206 (Fig. 6(b)). Trajectories at Ga = 206 would be expected to be in the Oblique Oscillating regime according to numerical simulations⁷. Fig. 6(b) presents some oblique oscillating planar trajectories while others are not perfectly planar and resemble the rotating regime *a priori* expected at slightly higher values of Ga.

The trajectories observed at this Galileo number share properties compatible with both the oblique oscillating and the planar or rotating regimes (Fig. 6(c)). As explained at the beginning of this section, it is likely that, considering the 5% uncertainty in Ga, and the vicinity with the frontier to the Planar or Rotating Regime, these measurements could be attributed to either the oblique oscillating or the planar or rotating regime.

(3) Planar or Rotating – Ga = {213, 217}. Fig. 6(c) presents some trajectories in this regime. They are composed of weakly non-planar trajectories (black) and some helicoids with diameter $D \approx 10$ and pitch $P \approx 500$ (equivalent to those of Case $\emptyset B$). Note that these values are non-dimensionalized by the particle diameter d_p . A bi-stable region is predicted by numerical simulations and has been observed experimentally with non-magnetic particles⁸. It is also confirmed for the case of magnetic spheres for this range of Ga. Apart from the effects on angle and planarity (discussed in Case $\emptyset B$), this subpanel makes explicit the presence of two regimes at these Ga numbers: Helicoid-like trajectories and oblique non-planar ones.

(4) Chaotic – Ga = 275. Finally, the Chaotic regime correctly matches the characteristics found for Case $\emptyset B$: All trajectories are different, non-planar, and oblique.

The taxonomy and range of Ga for which these different trajectories are observed for Cases g^* (when the particle rotation is blocked and there is an effective gravity) are globally indistinguishable from those found in the reference case $\emptyset B$ of non-magnetic particles and those just

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FIG. 6. Representative 3D trajectories, alongside a top view, for results with Ga in the main four regimes presented in the introduction and in Fig.5. Sub-panel (a) – Ga = {153, 158}, Steady Oblique regime. Sub-panel (b) – Ga = 206, Oblique Oscillating regime. Sub-panel (c) – Ga = {213, 217}, Planar or Rotating regime. Sub-panel (d) – Ga = 275, Chaotic regime. All these trajectories belong to Case g_0 .

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described for Case g_0 . This supports the hypothesis that 696 680 rotation blockage and the addition of a magnetic force do⁶⁹⁷ 681 not affect the regimes that a settling particle undergoes698 682 in a fluid at rest³⁸. Furthermore, this supports the idea699 683 that the magnetic method to modify the effective grav-700 684 ity presented here indeed modifies gravity without any₇₀₁ 685 spurious effect. This is further developed in the com-702 686 ing sections, using quantitative indicators for trajectory₇₀₃ 687 planarity and obliqueness in the different regimes. 689 704

690 2. Angle and Planarity of trajectories

⁶⁹¹ Fig. 7 presents the trajectories' angle and planarity⁷⁰⁹ versus Ga number, alongside the reference measurements⁷¹⁰ $\emptyset B$ from our group⁸ (empty circles). Additionally, the⁷¹¹ vertical dashed lines show the onsets for the different₇₁₂ settling regimes: Rectilinear, Steady Oblique, Oblique₇₁₃ Oscillating, Planar or Rotating, and Chaotic. In particular, Fig. 7(b) and Fig. 7(d) present the measurements against a Galileo number $Ga(\tilde{g})$ that was calculated with the effective gravity value \tilde{g} , whereas Fig. 7(a) and Fig. 7(c) show the measurements as a function of Ga(q) number based on the actual non-perturbed gravity, i.e., $\tilde{g} = 9.8 \text{ m/s}^2$. Red crosses denote the measurements from Case g_0 , for which only the particle rotation is blocked and no net magnetic force modifying the effective gravity exists. The empty circles are the non-magnetic reference data of Case $\emptyset B^8$, and the rest of the markers are the different configurations of Cases q^* and q_0 previously defined, a magnetic field is applied. Note that in the reference case $\emptyset B$ and Case g_0 , the points keep the same abscissa between rows because gravity is not modified.

For both the trajectory planarity and angle, it is observed that the data points collapse into a single trend



FIG. 7. Trajectory angles with the vertical (a-b) and planarity (c-d) versus Ga number, alongside the reference measurements of Case $\emptyset B$ (empty circles)⁸. Additionally, vertical dashed-lines and color bars⁸ show the onsets for the different regimes presented previously following the nomenclature introduced in Cabrera-Booman et al.⁸ and Fig. 5. The bottom row shows measurements against a Galileo number that was calculated with the corrected gravity value \tilde{g} . The top row presents measurements as a function of a Ga number computed assuming that the gravity did not change with the application of the external magnetic induction (i.e., $\tilde{g} = 9.8 \text{ m/s}^2$). Finally, different markers and colors are used to distinguish the data points as denoted in the legend.

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when the corrected $Ga(\tilde{q})$ is used. This is consistent with₇₃₇ 714 the reference data case $\emptyset B$, including the transitions be-715 tween settling regimes, whose detailed description has 716 been reported in Cabrera-Booman et al⁸. It can be seen⁷³⁸ 717 that Case g_0 , for which no magnetic modification to Ga is⁷³⁹ 718 applied, presents an identical behaviour as the reference 740 719 Case $\emptyset B$. This implies that there is no measurable effect⁷⁴¹ 720 of the particle rotation blockage on the trajectory $angle^{742}$ 721 or planarity. The uniform magnetic gradient strategy al-⁷⁴³ 722 lows exploration of Galileo number effects and settling⁷⁴⁴ 723 regimes by simply varying the amplitude of the applied⁷⁴⁵ 724 field (and thus of its gradient); which is equivalent. in⁷⁴⁶ 725 terms of variations of Ga, to viscosity and particle diam-⁷⁴⁷ 726 eter modifications. 727 Note that the single point with $\tilde{g} = (4.2 \pm 0.2) \text{ m/s}^2 \text{ and}^{749}$ 728 Ga = 103 (light blue circle), which corresponds to the⁷⁵⁰ 729 strongest magnetic gradient applied (Case $g^* \approx 0.43$), is⁷⁵¹ 730 off the trend in Fig. 7(b). This is due to the spurious 752 731 radial magnetic force which, as already pointed out $\mathrm{in}^{^{753}}$ 732

⁷³² Facture integration for the fully neglected for such strong⁷⁵⁴
 ⁷³⁴ effective gravity modifications. As a consequence, tra-755
 ⁷³⁵ jectories in this range of effective gravity acquire a small₇₅₆
 ⁷³⁶ radial drift and tend to become more oblique. 757

IV. CONCLUSIONS

This article presents both a magnetic method to modify the gravity on particles in a laboratory and experimental studies on the settling of single spheres in a quiescent flow with modified gravity and blocked spin. A magnetic method to modify gravity is developed, validated, and tested. Its theoretical foundation is presented, including details on how the addition of an external magnetic induction can produce a vertical force that counteracts the gravitational force on a magnetic particle. This experimental method also blocks the perpendicular-tomagnetic field spin of magnetic particles. The homogeneity of the theoretically derived magnetic field needed to compensate gravity is detailed, showing a high level of homogeneity. The coils' positions and currents to produce the desired magnetic field are obtained from a nonlinear fit, and the resulting field measured in the real world is compared to the theoretical field finding good agreement.

The settling of spherical magnets in a quiescent flow is studied and compared to a reference non-magnetic sphere settling measured by our group⁸ as both a final validation of the method and a study on the influence of particle

spinning on the particle trajectories' dynamics. The lat-817 759 ter consists of the analysis of the dynamics of spheres in⁸¹⁸ 760 the parameter space Γ – Ga, with a particle-to-fluid den-819 761 sity ratio $\Gamma = 8.2$ and Galileo numbers $Ga \in [100, 280]_{.820}$ 762 The terminal velocity of particles is discussed for each ef-821 763 fective gravity value, showing that particles achieve a ho-822 764 mogeneous terminal velocity thus implying that the mag-823 765 netic method modifies gravity homogeneously. Trajecto-824 766 ries in 3D that match benchmark results without gravity⁸²⁵ 767 are shown for each regime in the path instability param-826 768 eter space. The results on trajectory angle showed no₈₂₇ 769 difference between magnetic and non-magnetic cases, im-828 770 plying that the method to compensate gravity performs⁸²⁹ 771 well and particle spinning is not relevant for that aspect⁸³⁰ 772 of the dynamics. On the other hand, trajectory planarity⁸³¹ 773 presents minimal differences in the planar or rotating re-832 774 gion of the parameter space, although the present mea-833 775 surements are not sufficient to conclude whether there is_{834} 776 a rotation blockage effect. 777 835

A novel experimental method to compensate gravity⁸³⁶ 778 on magnetic particles in a fluid has been demonstrated⁸³⁷ 779 to compensate gravity down to a homogeneous value of⁸³⁸ 780 6.37 m/s^2 , or 65% of its full value, in the measurement⁸³⁹ 781 volume without inducing drift or any other spurious ef-840 782 fect on the particle dynamics. Note that this value can be⁸⁴¹ 783 further reduced by changing, for instance, the particles.⁸⁴² 784 Although not discussed here, this experimental technique⁸⁴³ 785 can be minimally modified to increase the gravitational⁸⁴⁴ 786 pull. There is a considerable value in the method as it⁸⁴⁵ 787 allows low-gravity experimentation in a laboratory bench⁸⁴⁶ 788 that otherwise would require substantial funding and fa-847 789 cilities such as parabolic flights, International Space Sta-848 790 tion, or drop towers. 791

This method has great potential for particle-laden tur-792 bulent flows where the aim is to disentangle the role of⁸⁵¹ 793 particle inertia and gravity on particle coupling with the $^{\rm 852}$ 794 surrounding fluid. This requires the modification of the $^{\tt 853}$ 795 settling properties of the particles independent of the in- $^{\rm 854}$ 796 ertial couplings with the fluid. This question is impor- $^{\tt 855}$ 797 tant for unveiling the mechanisms at play during turbu-798 lent transport of inertial particles $^{40},$ where inertial effects 857 799 (such as dynamic filtering⁴¹ and preferential concentra-800 tion^{42,43}) are generally parameterized by the particles'⁸⁵⁹ 801 Stokes number $St = \tau_p / \tau_\eta$ (with τ_p the particle viscous⁸⁶⁰ 802 relaxation time and τ_{η} the turbulence dissipation time)⁸⁶¹ 803 interplay with particle settling. These effects can, for in-804 stance, be parameterized by the settling velocity number 863 805 $Sv = \tau_p g/u_{\rm rms}$ (sometimes referred as Rouse number),⁸⁶⁴ 806 where g is the acceleration of gravity and $u_{\rm rms}$ is the tur-807 bulent fluctuating velocity. Exploring the role of inertia 808 in experiments by varying the Stokes number at fixed⁸⁶⁵ 809 turbulent conditions (*i.e.*, for fixed τ_{η} and $u_{\rm rms}$) requires 810 variation of the particles' relaxation time τ_p , hence in-⁸⁶⁶ 811 evitably changing their settling velocity number and the 812 settling properties. Being able to experimentally modify₈₆₇ 813 the effective gravity experienced by the particles woulds68 814 give a unique and simple way to truly explore settling⁸⁶⁹ 815 velocity number effects at fixed Stokes number. 870 816

This method restricts the particle rotation, only allowing for spin in the direction of the magnetic field. On one hand, this is a strong constraint for finite-sized particles where rotational and translational dynamics might be coupled 44,45 . On the other hand, blocking the rotation is also a unique opportunity offered by the magnetic method to disentangle the effects of particle rotation and translation on the global dynamics. Additionally, because the rotational and translational particle dynamics are decoupled in the context of the equation of motion for point particles proposed by Maxey, Riley, and Gatignol^{46,47} (MRG), the particle rotation blockage effect generated by this method does not affect its ability to disentangle the role of particle inertia and gravity. The usual rule-of-thumb for finite size effects (i.e., deviations from the MRG framework) suggests that they appear for particles with diameter $d_p > 5\eta^{48}$ (with the Kolmogorov scale $\eta = (\nu^3/\epsilon)^{1/4}$). Therefore, one can expect the 1 mm particles used here to behave like point particles in flows where $\eta > 200 \ \mu m$. Moreover, magnets with sizes of the order of 300 μ m are available, therefore allowing for η to be lowered and still be in the MRG framework.

Magnetic anisotropic particles can be considered an infinitesimal dipole with constant magnetic moment Mat their center. The fluid torque will be different than in the spherical case described here. For instance, the particle might not align with the magnetic field instanta*neously.* At this time, the smallest spherical permanent magnets have a diameter of 0.5 mm, that in the case of pure water and pure glycerol would result in $Re \approx 90$ and $Re \approx 1 \times 10^{-1}$, respectively. The size of ferromagnetic particles can be as low as 1 μ m and their application to this method is detailed in App. C. The use of this method to study collective effects will be complex as inter-particle forces will become dominant when the particles are close. The method will certainly work if the particles are dispersed enough, or encapsulated in large enough non magnetic shells (for instance, wax-encapsulation as done by De La Rosa et al.²⁷). However, investigating collective dynamics (such as turbulent preferential concentration) in the presence of additional interparticle interactions is interesting. For instance, droplets in clouds or powders in industrial processes may be charged so that hydrodynamic couplings compete then with interparticle interactions. Although magnetic and electrostatic interactions are different, studying the modification of turbulent clustering in presence of magnetic interactions may still lead to interesting and relevant new discoveries.

Appendix A: Gravity Compensation Theory

1. Equations of Motion

When applying this method to a particle in a fluid the equations of motion need to include hydrodynamical effects. Neglecting added mass and history forces^{46,47}, the fluid adds drag³⁹, torque⁴⁹, and buoyancy effects yielding

⁸⁷¹ the following equations of motion:

$$\mathbf{F} = (m_p - \mathbf{V}\rho_f)g \ \hat{\mathbf{z}} + \mathbf{M} \cdot \nabla \mathbf{B} - \frac{1}{8}C_D \pi d_p^2 \rho_f \mathbf{v} |\mathbf{v}|, \quad (E1)$$
$$\mathbf{T} = -1/64C_\omega \rho_f \boldsymbol{\omega} |\boldsymbol{\omega}| d_p^5 - \mathbf{M} \times \mathbf{B}, \quad (E2)$$

with fluid density ρ_f , particle volume V, kinematic viscosity ν , translational (C_D) and rotational (C_{ω}) drag coefficients, particle velocity \mathbf{v} , and angular velocity $\boldsymbol{\omega}$. Finally, note that if the Reynolds number is low (typically below order one⁵⁰, i.e., the Stokes regime) the fluid drag and torque are simpler:

$$\begin{aligned} \mathbf{F} &= (m_p - V\rho_f)g \; \hat{\mathbf{z}} - \nabla (\mathbf{M} \cdot \mathbf{B}) - 3\pi d_p \eta \mathbf{v}, \\ \mathbf{T} &= \pi \eta d_p^3 \omega - \mathbf{M} \times \mathbf{B}. \end{aligned}$$

878 2. Magnetic Field Derivation

To homogeneously compensate gravity, the magnetic force on gravity's direction $(\mathbf{F}^M \cdot \hat{\mathbf{z}})$ needs to be a constant independent of z, here denoted G_z . Alongside the previous condition, the external magnetic field **B** has to⁹⁰³ be a solution of Maxwell's equations, leading to the following set of equations: 904

$$\nabla_z \mathbf{B} = G_z, \tag{E3}_{906}$$

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$$\nabla \cdot \mathbf{B} = 0, \tag{E4}^{907}$$

$$\nabla \times \mathbf{B} = 0. \tag{E5}_{909}^{908}$$

The present work focuses on axisymmetric solutions⁹¹⁰ where a linear magnetic induction in $\hat{\mathbf{z}}$ can be proposed,⁹¹¹ resulting in: $\mathbf{B}(r, z) = B_r(r) \hat{\mathbf{r}} + (G_z \ z + B_0) \hat{\mathbf{z}}$, in cylin-⁹¹² drical coordinates. B_r can be obtained by solving Eq. E4,⁹¹³ leading to the following magnetic field induction: ⁹¹⁵

$$\mathbf{B}(r,z) = (-G_z/2 \ r) \ \hat{\mathbf{r}} + (G_z \ z + B_0) \ \hat{\mathbf{z}}.$$

This magnetic induction respects the irrotational condi-⁹¹⁸ tion (Eq. E5), whereas Eq. E3 is exactly satisfied only at⁹¹⁹ r = 0. The latter is an unavoidable consequence of the⁹²⁰ solenoidal nature of magnetic fields. A dependence on⁹²¹ the distance to the system axis (r) and the position on⁹²² the axis (z) are then present in the forces acting on the⁹²³ particle:

$$F_{z}^{M}(r,z) = M \frac{\partial B}{\partial z} = M \frac{(G_{z}z + B_{0})G_{z}}{\sqrt{(G_{z})^{2}/4r^{2} + (G_{z}z + B_{0})^{2}}},$$

$$F_{r}^{M}(r,z) = M \frac{\partial B}{\partial r} = M \frac{r (G_{z})^{2}/4}{\sqrt{(G_{z})^{2}/4r^{2} + (G_{z}z + B_{0})^{2}}}.$$
(E6)

Note that $F_r^M(r \to 0) = 0$ and $F_z^M(r \to 0) = MG_{z.928}$ Therefore, gravity can be fully compensated at r = 0.929without any radial force present. Note that this is not.930 in conflict with Earnshaw's theorem¹⁰ because the equi.931 librium is not stable, i.e., the Laplacian of the magnetic.932 energy is not zero. 933



FIG. A.1. Contour plot of the axial (a) and radial (b) component of the theoretical magnetic force, normalized by the axial force at z=0: $F_z^M(r,z)/F_z^M(0,0)$ and $1-F_r^M(r,z)/F_z^M(0,0)$, respectively.

3. Magnetic Field Homogeneity

Fig. A.1 presents contour plots of $F_z^M(r, z)/F_z^M(0, 0)$ and $1-F_r^M(r, z)/F_z^M(0, 0)$. Note that the normalization chosen is $F_z^M(0, 0) = MG_z$. Values of $G_z = -250$ G/m, $B_0 = 26$ G, $z \in [-150, 50]$ mm and $M = 4.96 \times 10^{-8}$ G⁻¹m²s⁻² were used to compute the forces from Equations E6, as these are typical magnitudes for the present experimental setup.

The axial component of the force F_z^M has a weak dependence on z and r, as quantified in Fig. A.1(a): A maximum axial force variation of 20% is achieved at z = 50 mm and r = 100 mm. At $z \in [-150, 0] \text{ mm}$ and $r \in [0, 20]$ mm, the ranges used in this work, the axial magnetic force has fluctuations below 2%. On the other hand, the radial force F_r^M has a stronger dependence on r and z (see Fig.A.1(b)). When r = 100 mm and z = 50, the radial force becomes as high as 30% of the reference axial force at the center $F_z^M(0,0)$. At the ranges $z \in [-150,0]$ mm and $r \in [0,50]$ mm the maximum value of radial force is reduced to 10% of its axial counterpart.

The relative magnitude of the axial and radial forces can be calculated:

$$\frac{F_r^M(r,z)}{F_z^M(r,z)} = \frac{1}{4} \frac{G_z r}{G_z z + B_0}.$$
 (E7)

As the aspiration is to solely counteract gravity, a radial force is not desired and the latter ratio needs to be minimized. There are two ways to achieve it: Keep r small compared to $(z + B_0/G_z)$; and/or have the largest possible value for B_0 . The latter approach is ideal because it allows a larger volume (r-z) where the axial force is homogeneous and the radial forces are small. Note that it translates to more current on the coils $(B \propto I)$ and,

936 Appendix B: Coils' Input Parameters

The coils' positions (Z_i) and currents (I_i) given by the₉₇₄ 937 fit for both cases are presented in Table IV. Note that 938 for Case g_0 only four coils are used as this case does not_{orb} 939 require more coils to achieve better homogeneity. 940 976 For the Cases g^* , the coils are not powered symmet-₉₇₇ 941 rically because of the need to create a gradient in the $_{_{978}}$ 942 magnetic field. To do so, the coils in the region with the $_{a79}$ 943 highest magnetic field need to have a larger current. It_{aso} 944 is also important to produce a linear profile of $magnetic_{_{981}}$ 945 field around a non-zero values in order to avoid the reori- $_{_{982}}$ 946 entation of the particle: This would happen because the 947 magnetic field would vanish and reverse direction, forcing 948 the magnetic moment of the particle to re-align. 949 083

TABLE IV. Coils' positions and currents given by the fit⁹⁸⁴ method for both Cases g_0 and $g^* = 0.65$. The coils' names⁹⁸⁵ follow the nomenclature presented in Fig.1.

$\frac{Z}{\text{Coil 1}}$	(cm) i	$I(A) \qquad Z$	(cm) 1	r (A) 9
Coil 1			()	· (++)
	X	X 2	26.5	2.28
Coil 2 1	4.2	4.24 2	24.5 -	-1.77 9
Coil 3	12	0.20	12	1.70
Coil 4	-12	0.16	0	0.52
Coil 5 -1	14.2	4.46 -	24.5 -	-3.06
Coil 6	X	× -	26.5 -	-1.16

952 Appendix C: Particle Material Discussion

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Equation 1 can be rewritten if one specifies the parti-953 cle magnetic properties: in the ferromagnetic, paramag- $_{992}^{991}$ 954 netic, or diamagnetic particle cases $\mathbf{M} \propto \mathbf{B}$, whereas for \mathbf{g}_{qqg} 955 a permanent magnet (with $B = |\mathbf{B}|$ below its coercive₉₉₄ 956 field strength) $M = |\mathbf{M}|$ is constant and $\mathbf{F}^M = \mathbf{M} \cdot \nabla \mathbf{B}^{.995}$ 957 This work focuses on the latter particle case because mag-996 958 netic moment values are at least two orders of magnitude $\frac{3}{998}$ 959 larger. This translates into lower external magnetic in-960 duction intensities (i.e., less power or smaller coils) to... 961 achieve a certain magnetic force. 1001 962

In particular, the magnetic moment M of a perma¹⁰⁰² nent magnet can be computed, if one assumes that the¹⁰⁰³ magnetic dipolar moment is dominant, in the following⁰⁰⁵ manner:

$$M = \frac{B_{\rm res}V}{\mu_0}, \qquad ({\rm E8})^{1008}_{1009}$$

where V is the volume of the magnet, μ_0 the vacuum⁰¹¹ magnetic permeability (note that $\mu_0 \approx \mu_{\text{water}}$) and $B_{\text{res}_{1012}}^{1012}$ is the remnant magnetic flux density (for the particles₁₀₁₄ used here $B_{\text{res}} = 1.192$ T), in other words the magnet's₀₁₅ magnetic flux density when the external coercive field strength is zero.

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Availability of data and materials

Data sets generated during the current study are available from the corresponding author on request.

Ethical Approval

Not applicable.

Conflict of Interest

No conflict of interest.

REFERENCES

990

- ¹Karl Cardin, Christophe Josserand, and Raúl Bayoán Cal. Droplet capture in a fiber array. *Phys. Rev. Fluids*, 8:043601, Apr 2023.
- ²W. Hwang and J.K. Eaton. Turbulence attenuation by small particles in the absence of gravity. *International Journal of Multiphase Flow*, 32(12):1386–1396, 2006.
- ³T Fallon and C B Rogers. Turbulence-induced preferential concentration of solid particles in microgravity conditions. *Experiments in Fluids*, 33(2):233–241, 2002.
- ⁴M. Horowitz and C. H. K. Williamson. The effect of reynolds number on the dynamics and wakes of freely rising and falling spheres. *Journal of Fluid Mechanics*, 651:251–294, 2010.
- ⁵C.H.J. Veldhuis and A. Biesheuvel. An experimental study of the regimes of motion of spheres falling or ascending freely in a newtonian fluid. *International journal of multiphase flow*, 33(10):1074–1087, 2007.
- ⁶G. Bouchet, M. Mebarek, and J. Dušek. Hydrodynamic forces acting on a rigid fixed sphere in early transitional regimes. *European Journal of Mechanics - B/Fluids*, 25(3):321–336, 2006.
- ⁷W. Zhou and J. Dusek. Chaotic states and order in the chaos of the paths of freely falling and ascending spheres. *International Journal of Multiphase Flow*, 75:205–223, 2015.
- ⁸F. Cabrera-Booman, N. Plihon, and M. Bourgoin. Path instabilities and drag in the settling of single spheres. *International Journal of Multiphase Flow*, 171:104664, 2024.

- ⁹O. J. I. Kramer, P. J. de Moel, S. K. R. Raaghav, E. T. Baars₁₀₈₅ 1016
- W. H. van Vugt, W.-P. Breugem, J. T. Padding, and J. P. van der086 1017 Hoek. Can terminal settling velocity and drag of natural parti+087 1018
- cles in water ever be predicted accurately? Drinking Water 088 1019 Engineering and Science, 14(1):53-71, 2021. 1089 1020 ¹⁰Earnshaw. S. Trans. Camb. Phil. Soc., page 97–112. 1021 1090
- ¹¹E Beaugnon and R Tournier. Levitation of organic materials¹⁰⁹¹ 1022 1023 Nature, 349(6309):470, 1991. 1092
- ¹²M. D. Simon and A. K. Geim. Diamagnetic levitation: Flying093 1024 frogs and floating magnets (invited). Journal of Applied Physics,1094 1025 87(9):6200-6204, 2000. 1026 1095
- ¹³Magnetic separation techniques in sample preparation for biolog₁₀₉₆ 1027 ical analysis: A review. Journal of Pharmaceutical and Biomedio97 1028 ical Analysis, 101:84-101, 2014. JPBA Reviews 2014. 1029 1098
- ¹⁴H. Okada, H. Okuyama, M. Uda, and N. Hirota. Removal of₀₉₉ 1030 aerosol by magnetic separation. IEEE Transactions on Applied100 1031 Superconductivity, 16(2):1084–1087, 2006. 1101 1032
- $^{15}\mathrm{Separation}$ of mixed waste plastics via magnetic levitation. Waster02 1033 Management, 76:46-54, 2018. 1034 1103
- ¹⁶Gradient coil design: A review of methods. Magnetic Resonance104 1035 Imaging, 11(7):903-920, 1993. 1036 1105
- ¹⁷S.S. Hidalgo-Tobon. Theory of gradient coil design methods for 106 1037 magnetic resonance imaging. Concepts in Magnetic Resonance107 1038 Part A, 36A(4):223-242. 1108 1039
- ¹⁸ Jun-Hua Pan, Nian-Mei Zhang, and Ming-Jiu Ni. Instability and 109 1040 transition of a vertical ascension or fall of a free sphere affected by110 1041 1042 a vertical magnetic field. Journal of Fluid Mechanics, 859:33-481111 2019.1043 1112
- ¹⁹Mailfert, Alain, Beysens, Daniel, Chatain, Denis, and Lorin¹¹³ 1044 Clément. Magnetic compensation of gravity in fluids: perfort114 1045 mance and constraints. Eur. Phys. J. Appl. Phys., 71(1):109021115 1046 2015.1047 1116
- ²⁰D Chatain, D Beysens, K Madet, V Nikolayev, and A Mailfertini 1048 1049 Study of fluid behaviour under gravity compensated by a magina netic field. Microgravity - Science and Technology, (3):196-1991119 1050
- ²¹Clément Lorin, Alain Mailfert, Christian Jeandey, and 120 1051 Philippe J. Masson. Perfect magnetic compensation of gravity121 1052 along a vertical axis. Journal of Applied Physics, 113(14):1439091122 1053 2013.1054 1123
- $^{22}\mathrm{V}$ S Nikolayev, D Chatain, D Beysens, and G Pichavant. Mag_{124} 1055 netic Gravity Compensation. Microgravity Science and Technol+125 1056 ogy, 23(2):113-122, 2011. 1057 1126
- ²³F Box, E Han, C R Tipton, and T Mullin. On the motion of₁₂₇ 1058 linked spheres in a Stokes flow. Experiments in Fluids, 58(4):29,128 1059 2017.1060 1129
- ²⁴Jérémy Vessaire, Nicolas Plihon, Romain Volk, and Mickaël₁₃₀ 1061 Bourgoin. Sedimentation of a suspension of paramagnetic partial 1062 ticles in an external magnetic field. Phys. Rev. E, 102:0231011132 1063 Aug 2020. 1064 1133
- ²⁵Eric E. Keaveny and Martin R. Maxey. Spiral swimming134 1065 of an artificial micro-swimmer. Journal of Fluid Mechanics,135 1066 598:293-319, 2008. 1067 1136
- $^{26}\mathrm{V.}$ Kumaran. Rheology of a suspension of conducting particles 137 1068 in a magnetic field. Journal of Fluid Mechanics, 871:139-185,138 1069 20191139 1070
- $^{27}\mathrm{H.}$ M. De La Rosa Zambrano, G. Verhille, and P. Le Gal. Fragi140 1071 mentation of magnetic particle aggregates in turbulence. Phys1141 1072 Rev.s Fluids, 3:084605, Aug 2018. 1073 1142
- ²⁸Mathieu Jenny, Gilles Bouchet, and Jan Dusek. Nonvertical as±143 1074 cension or fall of a free sphere in a newtonian fluid. Physics of 144 1075 Fluids, 15(1):L9-L12, 2003. 1076 1145
- ²⁹M. Jenny, J. Dusek, and G. Bouchet. Instabilities and transi+146 1077 tion of a sphere falling or ascending freely in a newtonian fluid1147 1078 Journal of Fluid Mechanics, 508:201-239, 2004. 1079 1148
- ³⁰Shravan K.R. Raaghav, Christian Poelma, and Wim-Paul149 1080 Breugem. Path instabilities of a freely rising or falling sphere. 1081 International Journal of Multiphase Flow, 153:104111, 2022.
- 1082
- $^{31}\mathrm{Values}$ of $G_z~=~-250$ G/m, $B_0~=~26$ G, z~=~100 mm, and 1083 $M = 4.96 \times 10^{-8} \text{ G}^{-1} \text{m}^2 \text{s}^{-2}$ 1084

- $^{32}\mathrm{R.P}$ Chhabra, S Agarwal, and K Chaudhary. A note on wall effect on the terminal falling velocity of a sphere in quiescent newtonian media in cylindrical tubes. Powder Technology, 129(1):53-58, 2003.
- $^{33}\mathrm{M}.$ Bourgoin and S. G. Huisman. Using ray-traversal for 3d particle matching in the context of particle tracking velocimetry in fluid mechanics. Rev. Sci. Instr., 91(8):085105, 2020.
- $^{34}\mathrm{Nicholas}$ T Ouellette, Haitao Xu, and Eberhard Bodenschatz. A quantitaive study of three-dimensional Lagrangian particle tracking algorithms. Experiments in Fluids, 39:722, 2005.
- ³⁵Roger I. Tanner and Shaocong Dai. Particle roughness and rheology in noncolloidal suspensions. Journal of Rheology, 60(4):809-818. 2016.
- $^{36}\mathrm{Yu}$ Zhao and Robert H. Davis. Interaction of sedimenting spheres with multiple surface roughness scales. Journal of Fluid Mechanics, 492:101-129, 2003.
- ³⁷Indresh Rampall, Jeffrey R. Smart, and David T. Lighton. The influence of surface roughness on the particle-pair distribution function of dilute suspensions of non-colloidal spheres in simple shear flow. Journal of Fluid Mechanics, 339:1-24, 1997.
- ³⁸Wei Zhou. Instabilités de trajectoires de spheres, ellipsoides et bulles. PhD thesis, 2016.
- ³⁹Phillip P. Brown and Desmond F. Lawler. Sphere drag and settling velocity revisited. Journal of Environmental Engineering, 129(3):222-231, 2003.
- ⁴⁰Luca Brandt and Filippo Coletti. Particle-Laden Turbulence: Progress and Perspectives. Annual Review of Fluid Mechanics, 54(1):159-189, 2022.
- ⁴¹C. M. Tchen. Mean value and correlation problems connected with the motion of small particles suspended in a turbulent fluid. PhD thesis, TU Delft, 1947.
- $^{42}\mathrm{Kyle}$ D. Squires and John K. Eaton. Preferential concentration of particles by turbulence. Physics of Fluids A: Fluid Dynamics, 3(5):1169-1178, 1991.
- ⁴³F. Falkinhoff, M. Obligado, M. Bourgoin, and P. D. Mininni. Preferential concentration of free-falling heavy particles in turbulence. Phys. Rev. Lett., 125:064504, Aug 2020.
- ⁴⁴Robert Zimmermann, Yoann Gasteuil, Mickael Bourgoin, Romain Volk, Alain Pumir, and Jean-François Pinton. Rotational intermittency and turbulence induced lift experienced by large particles in a turbulent flow. Physical review letters, 106(15):154501, 2011.
- ⁴⁵Varghese Mathai, Matthijs WM Neut, Erwin P van der Poel, and Chao Sun. Translational and rotational dynamics of a large buoyant sphere in turbulence. Experiments in fluids, 57:1-10, 2016.
- ⁴⁶Martin R. Maxey and James J. Riley. Equation of motion for a small rigid sphere in a nonuniform flow. The Physics of Fluids, 26(4):883-889, 1983.
- ⁴⁷R. Gatignol. The faxén formulae for a rigid particle in an unsteady non-uniform stokes flow. J. Méc. Theor. Appl., 1(143), 1983.
- ⁴⁸Haitao Xu and Eberhard Bodenschatz. Motion of inertial particles with size larger than kolmogorov scale in turbulent flows. Physica D: Nonlinear Phenomena, 237(14):2095-2100, 2008. Euler Equations: 250 Years On.
- ⁴⁹Nikolay Lukerchenko, Yury Kvurt, Alexander Kharlamov, Zdenek Chara, and Pavel Vlasak. Experimental evaluation of the drag force and drag torque acting on a rotating spherical particle moving in fluid. Journal of Hydrology and Hydromechanics, 56:88-94, 01 2008.
- ⁵⁰F. Cabrera, M. Z. Sheikh, B. Mehlig, N. Plihon, M. Bourgoin, A. Pumir, and A. Naso. Experimental validation of fluid inertia models for a cylinder settling in a quiescent flow. Phys. Rev. Fluids, 7:024301, Feb 2022.