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# Time- and Event-triggered Communication for Multi-agent Systems – Part I: General Framework

V. S. Dolk, K. J. A. Scheres, R. Postoyan, *Senior Member, IEEE*, and W. P. M. H. Heemels, *Fellow, IEEE*

**Abstract**—We present a framework for the design of time- and event-triggered communication schemes for a broad class of multi-agent systems (MAS). The framework is general in the sense that we consider a class of heterogeneous nonlinear MAS with directed communication graphs subject to disturbances and the inevitable imperfections induced by packet-based networked communication, such as time-varying transmission delays and intervals. Under suitable conditions, the designed distributed time- or event-triggered communication schemes lead to a MAS that is dissipative with respect to a desired supply rate, and thus can be used to study various properties such as  $\mathcal{L}_p$ -gain stability, input-to-state stability and passivity. Strictly positive lower bounds on the inter-event times are ensured and robustness to non-uniform time-varying delays in terms of maximum allowable delays is guaranteed. In addition, the framework allows for the consideration of destination protocols which determine, at each transmission instant, to which (groups of) agent(s) a local output measurement is being transmitted. These destination protocols can be used to include packet loss and denial-of-service, as will be shown in the companion paper [1].

## I. INTRODUCTION

Recently, the interest in multi-agent systems (MAS) has grown significantly due to the wide range of applications. Examples include the distributed control of platoons and formations of vehicles [2], [3] and distributed state estimation in large-scale processes such as power grids and building automation [4]–[7], see also [8] for an overview. In many of the aforementioned applications, the sensor and the actuator data are sent over shared (packet-based) communication networks, which, in contrast to dedicated point-to-point links, offer many benefits in terms of flexibility, maintenance and ease of installation. In fact, in some applications, such as vehicle platooning, the use of (wireless) communication networks is unavoidable, see, *e.g.*, [9]. However, these shared communication networks also come with inevitable imperfections. For example, the internal clocks of the agents may not be synchronized. Moreover, the transmission delays in MAS are in general time-varying and unknown. Additionally, since

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broadcast data is typically not simultaneously received and processed by each connected agent, the delays are often non-uniform in the sense that the time between the transmission and receipt of a data package might vary per connected agent. Moreover, it is impossible to *continuously* transmit data through a (packet-based) communication medium, *i.e.*, the communication bandwidth is limited, in the sense that communication can only occur at some discrete points in time. Communication network artifacts include:

- (i) time-varying non-uniform transmission delays;
- (ii) asynchronous sampling/transmission instants;
- (iii) limited communication bandwidth;
- (iv) packet losses and/or denial-of-service attacks.

Given the above mentioned artifacts inherent to MAS operating over digital communication networks, there is a strong need for novel analysis tools and control algorithms for MAS that take all these imperfections into account. The majority of the available literature only considers a subset of the aforementioned network-induced artifacts [10] and does not allow a straightforward generalization to include all of them. Moreover, a framework that connects results on centralized, decentralized and distributed networked control systems (NCS) is currently missing in the literature. Below, we provide a brief overview of some of the numerous existing works on MAS that focus on one or a subset of the four above mentioned artifacts.

In, *e.g.*, [3], [11]–[14], the presence of delays in MAS is studied, however, the communication between the agents is assumed to be continuous and thus (ii)–(iv) have not been addressed. Although [14] addresses item (ii) for the specific case of distributed control of nonholonomic systems, (iii)–(iv) are not taken care of. In, *e.g.*, [15]–[19], MAS with sampled-data communication are examined in which the sampling/transmission intervals are purely based on time. To be more specific, [15] considers MAS with single-integrator dynamics subject to constant communication delays. The works [16]–[18] study MAS with double-integrator dynamics, where [17] proposes a nonlinear control law, [18] includes the presence of time-varying communication delays and [16] examines MAS with non-periodic bounded transmission intervals under uncertain communication topologies. An advantage of communication schemes in which the transmission intervals are purely based on time is the ease of implementation. These time-based communications schemes, however, typically also lead

to redundant transmissions, which might not be desirable in shared networks in which communication resources are scarce.

In order to deal with the presence of limited communication resources, *e.g.*, [19]–[32] proposed the use of event-triggered control (ETC) strategies for MAS to cope with these limited communication resources, see [10] for a recent survey. These ETC strategies aim to reduce the utilization of communication resources by letting the transmission instants depend on state or output measurements of the system. If well-designed, these schemes can guarantee desired closed-loop behavior in terms of stability and performance, see, *e.g.*, [4], [33]–[39] for more details on ETC. However, the ETC approaches presented in [19]–[25], [28] do not consider delays and the approaches in [26], [27] assume that the delays are uniform and constant. Notable exceptions that study event-triggered MAS under non-uniform time-varying delays include [29]–[31], [40]. In [30], a *periodic* ETC approach is presented that can deal with non-uniform delays. However, it is assumed that the duration of these delays are known. Hence, this approach requires that the internal clocks of all agents are perfectly synchronized. In [29], [31], it was proposed to use transmission protocols that rely on acknowledgment and permission signals in order to make sure that the information available at each individual agent is updated simultaneously. However, the latter approach might put a burden on the communication channel due to the presence of additional acknowledgment and permission signals. In addition, practically it might not be feasible in some applications to realize acknowledgment and permission signals that can be sent and received instantaneously. For this reason, [29] also proposed an ETC scheme that does not require permission signals under the assumption that disturbances are absent. In [40], a *periodic* ETC scheme is proposed for tracking control of nonlinear MAS. It is shown that the closed loop tracking error satisfies an input-to-state stability property.

In the current work, we present a systematic and general design framework for a class of possibly heterogeneous nonlinear MAS with directed communication graphs subject to disturbances and the aforementioned network-induced imperfections (i)–(iv). Hereto, we consider *both* time-triggered and event-triggered communication schemes that support the use of general holding devices with zero-order hold as a special case. The design procedure of the closed-loop system relies on an emulation-based approach, which consists of two steps. In the first step, the control system is designed such that the closed-loop system, ignoring the presence of the digital communication network, satisfies appropriate (robust versions of) dissipativity properties that express the desirable closed-loop specifications. In the second step, the event-triggering mechanisms (ETMs) – or the time-triggering mechanisms (TTMs) as a special case – are designed, exploiting the robustness of the dissipative properties, such that the desired dissipativity properties are ensured in the presence of the network-induced imperfections. If well-designed according to our novel design conditions, the proposed framework leads to MAS that are dissipative with respect to a well-chosen supply rate and thereby can be used to guarantee various properties including asymptotic stability of sets, covering properties such as asymptotic consensus,  $\mathcal{L}_p$ -gain stability, input-to-state stability

(ISS) and passivity. In fact, the framework captures situations that have not been studied in literature before even for basic *centralized* networked control schemes such as guarantees for general dissipativity properties and the use of generalized holding devices in ETC systems subject to delays, while our results also apply to distributed NCS and MAS. The MAS are guaranteed to have strictly positive lower bounds on the inter-transmission times despite the presence of disturbances. In the companion paper [1], we will show by means of relevant examples how to systematically design the ETMs (and TTMs) such that the proposed conditions are fulfilled.

Let us emphasize that the proposed schemes do *not* require clock synchronization, acknowledgment signals that need to be transmitted and received instantaneously, or exact knowledge of the sizes of the transmission delays (only an upper-bound needs to be known). In addition, we show that the concept of network scheduling protocols, which are typically used in the context of NCS (see, *e.g.*, [41]), can be used for the design of so-called *destination protocols*. These destination protocols determine, at each transmission instant, to which (groups of) agent(s) a local output measurement is being transmitted. Interestingly, as we will discuss in the companion paper [1], the proposed concept of destination protocols also allows to capture packet losses, see, *e.g.*, [29], [42]–[44], and even (malicious) denial-of-service attacks, which are intended to interfere with the communication channel, see, *e.g.*, [45]. In other words, item (iv) above is addressed in the second part of this paper, in which we also provide a full design of digital implementations of the proposed ETC schemes for MAS, leading to periodic event-triggered control schemes [46], [47] with appropriate guarantees on stability, performance and minimal-inter event-times. Two numerical case studies are included in the companion paper [1].

## II. DEFINITIONS AND PRELIMINARIES

### A. Notation

The set  $\mathbb{N}$  denotes the set of non-negative integers,  $\mathbb{N}_{>0}$  the set of all positive integers,  $\mathbb{R}$  the field of all real numbers and  $\mathbb{R}_{\geq 0}$  the set of all non-negative reals. For  $N$  vectors  $x_i \in \mathbb{R}^{n_i}$ ,  $i \in \{1, 2, \dots, N\}$ , we denote the vector obtained by stacking all vectors in one (column) vector  $x \in \mathbb{R}^n$  with  $n = \sum_{i=1}^N n_i$  by  $(x_1, x_2, \dots, x_N)$ , *i.e.*,  $(x_1, x_2, \dots, x_N) = [x_1^\top \ x_2^\top \ \dots \ x_N^\top]^\top$ . The vectors in  $\mathbb{R}^N$  whose elements are all ones or all zeros are denoted by  $\mathbf{1}_N$  and  $\mathbf{0}_N$ , respectively. Given matrices  $A_1, \dots, A_n$ , we denote by  $\text{diag}(A_1, \dots, A_n)$  the block-diagonal matrix where the main diagonal blocks consist of the matrices  $A_1$  to  $A_n$  and all other blocks are zero matrices. By  $|\cdot|$  and  $\langle \cdot, \cdot \rangle$  we denote the Euclidean norm and the usual inner product of real vectors, respectively. Moreover, for  $x \in \mathbb{R}^n$  and a given non-empty set  $\mathcal{A} \subset \mathbb{R}^n$ ,  $|x|_{\mathcal{A}} = \inf_{y \in \mathcal{A}} |x - y|$ . For two matrices  $A \in \mathbb{R}^{m \times n}$  and  $B \in \mathbb{R}^{p \times q}$ , the Kronecker product of  $A$  with  $B$  is denoted  $A \otimes B \in \mathbb{R}^{mp \times nq}$ . The interior of a set  $\mathcal{A} \subseteq \mathbb{R}^n$  is denoted by  $\text{int } \mathcal{A}$ . We consider  $\mathcal{K}_\infty$  and  $\mathcal{KL}$  functions as defined in [48, Def. 3.4, 3.38]. Given a function  $x : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$  with  $n \in \mathbb{N}_{>0}$ ,  $\|x\|_\infty := \text{ess sup } \{|x(t)| : t \geq 0\}$ , and  $x$  is said to be of class  $\mathcal{L}_\infty^n$  if  $\|x\|_\infty < \infty$ . The notation

$F : X \rightrightarrows Y$ , indicates that  $F$  is a set-valued mapping from  $X$  to  $Y$  with  $F(x) \subseteq Y$  for all  $x \in X$ . We use  $U^\circ(x; v)$  to denote the generalized directional derivative of Clarke of a locally Lipschitz function  $U$  at  $x$  in the direction  $v$ , i.e.,  $U^\circ(x; v) := \limsup_{h \rightarrow 0^+, y \rightarrow x} (U(y + hv) - U(y))/h$ , which reduces to the standard directional derivative  $\langle \nabla U(x), v \rangle$  when  $U$  is continuously differentiable; see [49] for more details. We denote the logical *and* and *or* operators as  $\wedge$  and  $\vee$ , respectively.

## B. Graph Theory Notions [50]

A *graph* is a pair  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  composed of a vertex set  $\mathcal{V}$  and a set of edges  $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ . The cardinality of  $\mathcal{V}$ , denoted by  $N \in \mathbb{N}_{>0}$ , is the number of vertices in  $\mathcal{V}$ . An ordered pair  $(i, j) \in \mathcal{E}$  with  $i, j \in \mathcal{V}$  is said to be an edge *directed* from  $i$  to  $j$ . A graph is called *undirected* if it holds that  $(i, j) \in \mathcal{E}$  if and only if  $(j, i) \in \mathcal{E}$ . Otherwise, the graph is a *directed* graph, also referred to as a *digraph*. A vertex  $j$  is said to be a *neighbor* of  $i$  if  $(j, i) \in \mathcal{E}$ . The set of neighbors of a vertex  $i$  is denoted by  $\mathcal{V}_i^{\text{in}}$  and defined as  $\mathcal{V}_i^{\text{in}} := \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$  and the set of vertices for which vertex  $i$  is a neighbor is denoted by  $\mathcal{V}_i^{\text{out}}$  and defined as  $\mathcal{V}_i^{\text{out}} := \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}\}$ . The cardinality of  $\mathcal{V}_i^{\text{out}}$ ,  $i \in \mathcal{V}$  is denoted by  $N_i$ . An edge  $(i, i) \in \mathcal{E}$  is called a *self-loop*. A *directed path* from  $i$  to  $j$  is a (finite) sequence of edges starting in  $i$  and ending at  $j$ . A digraph  $\mathcal{G}$  is connected if there exists a path between all vertices  $i, j \in \mathcal{V}$ , and it is called weakly connected if, ignoring all orientations on its edges, the resulting graph is connected.

## III. PROBLEM STATEMENT

### A. Distributed Control Configuration

We consider a collection of agents  $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$  that are possibly physically coupled and that communicate according to a weakly connected digraph  $\mathcal{G}(\mathcal{V}, \mathcal{E})$ .

The dynamics of the  $m$ -th agent  $\mathcal{A}_m$  for  $m \in \mathcal{V}$  are given by

$$\mathcal{A}_m : \begin{cases} \dot{x}_m = f_m(x, \hat{y}_m^{\text{in}}, v_m) \\ y_m = h_m(x_m), \end{cases} \quad (1)$$

where  $x_m \in \mathbb{R}^{n_{x,m}}$  represents the local state vector,  $x = (x_1, x_2, \dots, x_N) \in \mathbb{R}^{n_x}$  with  $n_x := \sum_{i \in \mathcal{V}} n_{x,i}$  the global state vector,  $v \in \mathbb{R}^{n_v}$  the disturbance or external input,  $y_m \in \mathbb{R}^{n_{y,m}}$  the output measurement available at agent  $\mathcal{A}_m$ . Moreover,  $\hat{y}_m^{\text{in}} := (\hat{y}_1^m, \hat{y}_2^m, \dots, \hat{y}_N^m) \in \mathbb{R}^{n_y}$ , where  $\hat{y}_i^m \in \mathbb{R}^{n_{y,i}}$ ,  $m, i \in \mathcal{V}$ , denotes the local estimate at agent  $\mathcal{A}_m$  of the output  $y_i$  of agent  $\mathcal{A}_i$ , see also Fig. 1, and where  $n_y := \sum_{i \in \mathcal{V}} n_{y,i}$ . Let us remark that the variable  $\hat{y}_i^m$ ,  $i \in \mathcal{V}_m^{\text{in}}$  and  $m \in \mathcal{V}$ , corresponds to the edge  $(i, m) \in \mathcal{E}$  of graph  $\mathcal{G}$ . The variables  $\hat{y}_i^m$  for which  $i \notin \mathcal{V}_m^{\text{in}}$  with  $m \in \mathcal{V}$ , i.e.,  $(i, m) \notin \mathcal{E}$ , are non-existent in practice and thus in principle redundant due to the communication topology. However, for ease of notation, we still use these variables and assume that  $\hat{y}_i^m = 0$  for all  $i \notin \mathcal{V}_m^{\text{in}}$  with  $m \in \mathcal{V}$ . Moreover, the dynamics of the local state vector  $x_m$ ,  $m \in \mathcal{V}$ , may depend on the entire state  $x$  due to possible physical couplings between agents. System (1) may represent a local plant and its local controller in closed loop, which has been designed to ensure

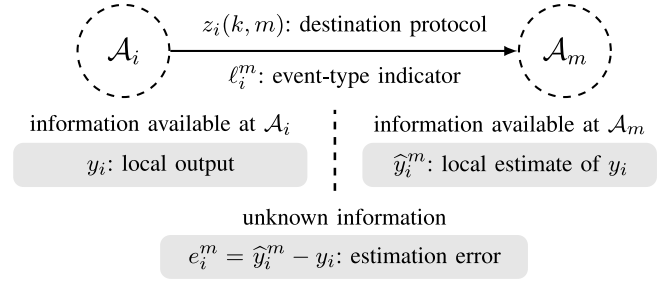


Fig. 1. Schematic representation of the control setup.

the desired control properties for the overall system with perfect, continuous communication between the agents (i.e., ignoring the imperfections induced by the digital network). We assume throughout the paper that the map  $f_m$  is continuous and the map  $h_m$  is continuously differentiable for  $m \in \mathcal{V}$ .

Due to the presence of network-induced imperfections such as time-varying transmission delays, time-varying transmission intervals and destination protocols, we typically have that  $\hat{y}_i^m \neq y_i$ , for all  $i \in \mathcal{V}$  and  $m \in \mathcal{V}_i^{\text{out}}$ . In other words, the presence of a network introduces estimation errors given by

$$e_i^m := \hat{y}_i^m - y_i, \quad \forall i \in \mathcal{V}, m \in \mathcal{V}_i^{\text{out}}. \quad (2)$$

Hence,  $e_i^m$ ,  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}_i^{\text{out}}$ , denotes the error present in the information  $\hat{y}_i^m$  available at agent  $\mathcal{A}_m$  regarding the output  $y_i$  of agent  $\mathcal{A}_i$ . For all  $i \in \mathcal{V}$  and  $m \in \mathcal{V} \setminus \mathcal{V}_i^{\text{out}}$ , i.e., for all redundant variables, we set  $e_i^m = 0$

### B. Networked Communication

As already mentioned, (packet-based) networked communication induces inherent imperfections such as the fact that data can only be transmitted at discrete instants in time (sampled-data communication) and the presence of unknown non-uniform time-varying delays. More precisely, the output  $y_i$ ,  $i \in \mathcal{V}$ , is only sampled and transmitted over the network at discrete time instants  $t_k^i$ ,  $k \in \mathbb{N}$ , that satisfy  $0 = t_0^i < t_1^i < \dots$ , for all  $i \in \mathcal{V}$ . At a transmission instant  $t_k^i$ ,  $i \in \mathcal{V}$ ,  $k \in \mathbb{N}$ , agent  $\mathcal{A}_i$  broadcasts its current output information  $y_i$  to a selection of the connected agents that is determined by a *destination protocol*. In case an agent  $\mathcal{A}_m$ ,  $m \in \mathcal{V}_i^{\text{out}}$ ,  $i \in \mathcal{V}$ , belongs to this selection at this transmission instant, the corresponding estimate  $\hat{y}_i^m$  is updated after a communication delay of  $\Delta_k^{i,m} \geq 0$  time units, i.e.,

$$\hat{y}_i^m((t_k^i + \Delta_k^{i,m})^+) = y_i(t_k^i). \quad (3)$$

In this paper, we focus on small-delay scenarios meaning that an agent only transmits after the previous broadcast information of that agent has been received by all targeted agents. In addition, we assume that the delays are bounded from above by a time-constant called the *maximum allowable delay* (MAD), as formalized next.

**Assumption 1.** For each  $i \in \mathcal{V}$ , there is a time-constant  $\tau_{\text{mad}}^i$  such that the transmission delays are bounded according to  $0 \leq \Delta_k^{i,m} \leq \tau_{\text{mad}}^i \leq t_{k+1}^i - t_k^i$ ,  $m \in \mathcal{V}_i^{\text{out}}$  for all  $k \in \mathbb{N}$ , where  $\tau_{\text{mad}}^i$  denotes the maximum allowable delay.

The *destination protocol* determines at each transmission instant  $t_k^i$ ,  $i \in \mathcal{V}$ ,  $k \in \mathbb{N}$ , the selection of the connected agents to which agent  $\mathcal{A}_i$  broadcasts its current output information  $y_i$ . The protocol determines this selection on the basis of the transmission counter  $k$ . To be more precise, the destination protocol associated with agent  $\mathcal{A}_i$  is described by the function  $z_i : \mathbb{N} \times \mathcal{V} \rightarrow \{0, 1\}$ ,  $i \in \mathcal{V}$ , which, at transmission time  $t_k^i$ ,  $k \in \mathbb{N}$ , is such that  $z_i(k, m) = 1$  when the output measurement  $y_i$  is transmitted to agent  $\mathcal{A}_m$ ,  $m \in \mathcal{V}_i^{\text{out}}$ , and  $z_i(k, m) = 0$  otherwise. When no destination protocols are implemented, *i.e.*, when  $y_i$  is sent to all neighbors of agent  $i$ ,  $z_i(k, m) = 1$  for any  $m \in \mathcal{V}_i^{\text{out}}$ ,  $i \in \mathcal{V}$ .

Given the description above, the update of the estimate  $\hat{y}_i^m$  can be expressed as

$$\hat{y}_i^m((t_k^i + \Delta_k^{i,m})^+) = z_i(k, m)y_i(t_k^i) + (1 - z_i(k, m))\hat{y}_i^m(t_k^i + \Delta_k^{i,m}), \quad (4)$$

for all  $i \in \mathcal{V}$  and  $m \in \mathcal{V}_i^{\text{out}}$ . The values of the delays  $\Delta_k^{i,m}$ ,  $i, m \in \mathcal{V}$ ,  $k \in \mathbb{N}$ , for which  $m \in \mathcal{V} \setminus \{p \in \mathcal{V}_i^{\text{out}} \mid z_i(k, p) = 1\}$  have no physical meaning since for these cases, agent  $\mathcal{A}_i$  does not transmit any information to agent  $\mathcal{A}_m$  at transmission time  $t_k^i$  due to the communication topology specified by the communication graph  $\mathcal{G}$  and/or the destination protocol given by  $z_i$ . Therefore, we simply take  $\Delta_k^{i,m} = 0$  for  $i, m \in \mathcal{V}$ ,  $k \in \mathbb{N}$  for which  $m \in \mathcal{V} \setminus \{p \in \mathcal{V}_i^{\text{out}} \mid z_i(k, p) = 1\}$ .

Interestingly, as we will show in detail in the second part [1], the function  $z_i$  can also be exploited to capture packet losses and denial-of-service attacks, which are results of independent interest.

**Remark 1.** *Destination protocols are modeled in a similar fashion as network scheduling protocols as described in [41]. As such, we can take inspiration from [41] for the design of destination protocols. For example, the destination protocols could be the well-known Sampled-Data (SD) and the Round-Robin (RR) protocol. Our framework also allows to generate new protocols that are meaningful in the context of MAS such as destination protocols that capture switching communication topologies. In fact, in Part II [1] they are used to describe Denial-of-Service attacks and packet losses.*

**Remark 2.** *In specific situations, the destination protocol  $z_i$  might also depend on other variables such as the network-induced estimation error  $e_i^m$ ,  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}_i^{\text{out}}$  (if available), provided that Condition 1 in the following is satisfied. For these cases, the results below apply mutatis mutandis.*

In time periods in which agent  $\mathcal{A}_m$  does not receive new information of agent  $\mathcal{A}_i$ , the estimate  $\hat{y}_i^m$  evolves as

$$\dot{\hat{y}}_i^m(t) = \hat{f}_i(\hat{y}_i^m(t)), \quad (5)$$

for all  $t \in (t_k^i + \Delta_k^{i,m}, t_{k+1}^i + \Delta_{k+1}^{i,m})$ , with  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}_i^{\text{out}}$ ,  $k \in \mathbb{N}$ , where  $\hat{f}_i : \mathbb{R}^{n_y} \rightarrow \mathbb{R}^{n_y}$  describes the holding device. Observe that each agent  $\mathcal{A}_m$ ,  $m \in \mathcal{V}_i^{\text{out}}$  and  $i \in \mathcal{V}$ , employs the same holding device to obtain the estimate  $\hat{y}_i^m$ . By taking  $\hat{f}_i = 0$ , zero-order hold (ZOH) devices are obtained.

### C. Time-triggered and Event-Triggered Communication

In conventional (digital) control schemes, data is typically transmitted over dedicated (wired) communication channels, often according to a fixed sampling rate. Since in NCS, the communication resources are in general shared and thus scarce, fixed sampling rates often cannot be realized. Instead, the time-intervals in between data transmissions are typically varying over time. In this paper, we consider two types of communication protocols for NCS, namely, time-triggered communication (TTC) schemes and event-triggered communication (ETC) schemes.

In TTC schemes, the transmissions instants are determined purely based on time. To make sure that the NCS satisfies the desired closed-loop stability and performance criteria, the transmission intervals per agent are assumed to be bounded by a pre-defined upper-bound referred to as the *maximum allowable transmission interval* (MATI). To be more concrete, the transmission intervals satisfy

$$d_i \leq t_{k+1}^i - t_k^i \leq \tau_{\text{mati}}^i, \quad (6)$$

where  $0 < d_i \leq \tau_{\text{mati}}^i$ ,  $i \in \mathcal{V}$ , is an arbitrary small positive constant and  $\tau_{\text{mati}}^i$  is a time-constant representing the MATI, see, *e.g.*, also [41], [51], [52]. In practice, it is useful to select  $d_i \geq \tau_{\text{mad}}^i$  to ensure that the small delay assumption is not violated when no acknowledgment signals are available.

In ETC schemes, the transmission instants are determined based on state- or output measurements, see, *e.g.*, [4], [33]–[39], [53], [54] and the references therein. In this paper, we consider *dynamic* event-generators of the form as proposed in [34] that schedule transmission instants according to

$$t_0^i = 0, t_{k+1}^i := \inf \{t \geq t_k^i + \tau_{\text{miet}}^i \mid \eta_i(t) \leq 0\}, \quad (7)$$

for  $i \in \mathcal{V}$  and  $k \in \mathbb{N}$  and where  $\eta_i \in \mathbb{R}$  is the triggering variable of agent  $\mathcal{A}_i$  that evolves according to

$$\dot{\eta}_i(t) = \Psi_i(o_i(t)) - \varphi_i(\eta_i(t)), \quad \text{when } \eta_i(t) \geq 0, \quad (8a)$$

$$\eta_i(t^+) = \eta_i^0(y_i(t)), \quad \text{when } \eta_i(t) = 0, \quad (8b)$$

where the functions  $\Psi_i : \mathbb{O}_i \rightarrow \mathbb{R}$ ,  $\varphi_i : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  and  $\eta_i^0 : \mathbb{R}^{n_y} \rightarrow \mathbb{R}_{\geq 0}$  and the time constant  $\tau_{\text{miet}}^i \in \mathbb{R}_{>0}$  are to be designed as we will discuss in Section V. The variable  $o_i \in \mathbb{O}_i$ ,  $i \in \mathcal{V}$ , represents information locally available at agent  $\mathcal{A}_i$ , such as the output  $y_i$ , the local estimates  $\hat{y}_m^i$ ,  $m \in \mathcal{V}_i^{\text{in}}$ , and the most recently transmitted output measurement of  $y_i$ . Note that the time constant  $\tau_{\text{miet}}^i \in \mathbb{R}_{>0}$  is an enforced lower bound on the inter-transmission times for agent  $\mathcal{A}_i$  referred to as the *minimum inter-event time* (MIET). Clearly, we have that  $t_{k+1}^i - t_k^i \geq \tau_{\text{miet}}^i$  and thus that Zeno-behavior is excluded in the event-triggering mechanisms (ETMs) and therefore also in the overall system. The latter property is obviously important to enable implementation of the ETC scheme in practice. In accordance with Assumption 1, we select  $\tau_{\text{miet}}^i \geq \tau_{\text{mad}}^i$ .

The ETM described by (7) and (8) is referred to as a *dynamic* ETM because the transmission instants are determined based on dynamic variables such as  $\eta_i$  in contrast to the commonly adopted *static* ETMs, which rely on a static expression dependent on the locally available information  $o_i$ , see [19], [34], [39], [55]. Employing dynamic ETMs instead of

static versions has several advantages including significantly larger average inter-event times in many cases, see [34], [39], [55] for more details. However, our results apply to both static and dynamic ETMs, as explained in Remark 3, where this case is described explicitly.

Let us emphasize that the ETM of agent  $\mathcal{A}_i$  described by (7) and (8) will *not* rely on the continuous availability of output measurements of other agents, *i.e.*,  $y_m$ ,  $m \in \mathcal{V}_i^{\text{in}}$ , and will *not* require clock synchronization, acknowledgment signals that need to be transmitted and received instantaneously or knowledge about the sizes of the transmission delays (only the upper-bounds  $\tau_{\text{mad}}^i$ ,  $i \in \mathcal{V}$ , need to be known).

#### D. Problem Formulation

Given the descriptions above, the problem considered in this paper can now be stated informally as follows:

Consider a collection of agents  $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_N$  described by (1) and a collection of maximum allowable delays  $\tau_{\text{mad}}^i$ ,  $i \in \mathcal{V}$ . Propose design conditions for the time constants  $\tau_{\text{mati}}^i (\geq \tau_{\text{mad}}^i)$  as in (6) in the context of TTC schemes, and for  $\tau_{\text{miet}}^i (\geq \tau_{\text{mad}}^i)$  and the functions  $z_i$ ,  $\Psi_i$ ,  $\varphi_i$  and  $\eta_i^0$ ,  $i \in \mathcal{V}$ , as in (7) and (8), in the context of ETC schemes, such that the resulting MAS has the desired (and to be specified) closed-loop stability, performance and robustness properties.

As we will discuss in the next section, we will consider general dissipativity properties that can reflect a broad range of control (stability, performance and robustness) objectives.

### IV. MATHEMATICAL FORMULATION OF THE MAS

In this section, we formulate the MAS with non-uniform communication delays in terms of a hybrid model [48] to facilitate the stability analysis in Section V and to formalize the problem stated at the end of Section III. The hybrid model will capture both TTC as the ETC cases in the sense that all (hybrid) trajectories resulting from the two communication schemes are solutions to the model, see also Section IV-B.

#### A. Hybrid model

A hybrid model  $\mathcal{H}(\mathcal{C}, F, \mathcal{D}, G)$  with state  $\xi$  and a disturbance  $v$  describes the system in terms of flow and jump equations

$$\begin{aligned} \dot{\xi} &\in F(\xi, v), & \text{when } \xi \in \mathcal{C}, \\ \xi^+ &\in G(\xi), & \text{when } \xi \in \mathcal{D}, \end{aligned} \quad (9)$$

where  $F$  and  $G$  denote the flow and the jump map, respectively,  $\mathcal{C}$  and  $\mathcal{D}$  the flow and the jump set, respectively, and where  $\xi^+$  denotes the updated value of  $\xi$  right after a jump, see [48] and its extension to continuous-time inputs [56] for profound definitions regarding this modeling framework.

We now explain how to write the overall system in the form of (9). To model the updates of  $e_i^m$  in (2) at update times  $t_k^i + \Delta_k^{i,m}$  in terms of (9), we first obtain from (2) and (4), that for all  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}_i^{\text{out}}$  and  $k \in \mathbb{N}$  for which  $z_i(k, m) = 1$ ,

$$\begin{aligned} e_i^m((t_k^i + \Delta_k^{i,m})^+) &= \widehat{y}_i^m((t_k^i + \Delta_k^{i,m})^+) - y_i((t_k^i + \Delta_k^{i,m})^+) \\ &= y_i(t_k^i) - y_i(t_k^i + \Delta_k^{i,m}). \end{aligned} \quad (10)$$

As in [52] and [34], to capture the updates of  $e_i^m$  in terms of jump equations, we need to distinguish between two types of jumps, namely, jumps corresponding to time instants at which an agent  $\mathcal{A}_i$ ,  $i \in \mathcal{V}$ , transmits a new measurement to (a selection of) the agent(s)  $\mathcal{A}_m$ ,  $m \in \mathcal{V}_i^{\text{out}}$  (referred to as *transmission* events/jumps), and jumps corresponding to time instants at which an agent  $\mathcal{A}_m$ ,  $m \in \mathcal{V}_i^{\text{out}}$ , receives a new measurement from agent  $\mathcal{A}_i$ ,  $i \in \mathcal{V}_m^{\text{in}}$  (referred to as *update* events/jumps). To keep track of these two jump types, we introduce auxiliary variables  $\ell_i^m \in \{0, 1\}$ , that indicate that the last transmitted value of agent  $\mathcal{A}_i$  has already been received by agent  $\mathcal{A}_m$  ( $\ell_i^m = 0$ ) or still has to be received ( $\ell_i^m = 1$ ). Hence, the *next* jump corresponds to the transmission of an output measurement by agent  $\mathcal{A}_i$ ,  $i \in \mathcal{V}$  ( $\ell_i^m = 0$  for all  $m \in \mathcal{V}$ ), or to the reception of an output measurement by agent  $\mathcal{A}_m$  from agent  $\mathcal{A}_i$  ( $\ell_i^m = 1$ ), see also Figure 1. Moreover, we introduce the memory variable  $r_i \in \mathbb{R}^{n_y}$ ,  $i \in \mathcal{V}$ , to store the value of  $y_i$  at times  $t_k^i$ ,  $k \in \mathbb{N}$ , *i.e.*, at each transmission instant  $t_k^i$ , we have that

$$r_i(t_k^{i+}) = y_i(t_k^i). \quad (11)$$

Between two successive transmissions, the variable  $r_i$  remains constant, *i.e.*,  $\dot{r}_i(t) = 0$  for all  $t \in (t_k^i, t_{k+1}^i)$ ,  $i \in \mathcal{V}$  and  $k \in \mathbb{N}$ . Given the description above, the update of  $e_i^m$ ,  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}_i^{\text{out}}$ , as described in (10), can be rewritten as

$$e_i^m((t_k^i + \Delta_k^{i,m})^+) = r_i(t_k^i + \Delta_k^{i,m}) - y_i(t_k^i + \Delta_k^{i,m}). \quad (12)$$

To secure that Assumption 1 holds and to keep track of the time elapsed since the most recent transmission, we adopt the timer variable  $\tau_i \in \mathbb{R}_{\geq 0}$ ,  $i \in \mathcal{V}$ . Variable  $\tau_i$ ,  $i \in \mathcal{V}$ , represents the time elapsed since the most recent transmission instant of agent  $\mathcal{A}_i$ , *i.e.*,  $\tau_i(t) = t - \sup\{t_k^i \mid k \in \mathbb{N}, t_k^i \leq t\}$ . Moreover, we let  $\kappa_i \in \mathbb{N}$  denote the transmission counter of agent  $\mathcal{A}_i$ , which is constant on flows and increased by 1 at each transmission jump. Observe that the state variables  $e_i^m$  and  $\ell_i^m$ ,  $i \in \mathcal{V}$ ,  $m \notin \mathcal{V}_i^{\text{out}}$ , are in principle redundant due to the communication topology. However, for consistency of notation and simplicity, we keep these variables in the hybrid model presented next and assume that they are zero. For this purpose, we define the concatenation of network induced errors corresponding to agent  $\mathcal{A}_i$ ,  $i \in \mathcal{V}$ , as

$$e_i^{\text{out}} := (\delta_i(1)e_i^1, \delta_i(2)e_i^2, \dots, \delta_i(N)e_i^N) \in \overline{\mathbb{E}}_i \subseteq \mathbb{R}^{Nn_{y,i}},$$

where  $\overline{\mathbb{E}}_i := \mathbb{E}_i(1) \times \mathbb{E}_i(2) \times \dots \times \mathbb{E}_i(N)$  with

$$\begin{aligned} \mathbb{E}_i(m) &:= \begin{cases} \mathbb{R}^{n_{y,i}}, & \text{when } m \in \mathcal{V}_i^{\text{out}}, \\ \{\mathbf{0}_{n_{y,i}}\}, & \text{otherwise,} \end{cases} \\ \delta_i(m) &:= \begin{cases} 1, & \text{when } m \in \mathcal{V}_i^{\text{out}} \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (13)$$

Hence, the function  $\delta_i$  sets the redundant elements of the vector  $e_i^{\text{out}}$  to zero.

In addition, let us introduce the following notation:  $e_i^{\text{in}} := (\delta_1(i)e_1^i, \delta_2(i)e_2^i, \dots, \delta_N(i)e_N^i) \in \mathbb{E}_i$ ,  $\mathbb{E}_i := \mathbb{E}_1(i) \times \mathbb{E}_2(i) \times \dots \times \mathbb{E}_N(i)$ ,  $e := (e_1^{\text{in}}, e_2^{\text{in}}, \dots, e_N^{\text{in}}) \in \mathbb{E}$  with  $\mathbb{E} := \mathbb{E}_1 \times \mathbb{E}_2 \times \dots \times \mathbb{E}_N$ ,  $\tau := (\tau_1, \tau_2, \dots, \tau_N) \in \mathbb{R}_{\geq 0}^N$ ,  $\kappa := (\kappa_1, \kappa_2, \dots, \kappa_N) \in \mathbb{N}^N$ ,  $r := (r_1, r_2, \dots, r_N) \in \mathbb{R}^{n_y}$ ,  $\ell := (\ell_1^1, \ell_2^1, \dots,$

$\ell_N^1, \ell_1^2, \ell_2^2, \dots, \ell_N^N \in \{0, 1\}^{N^2}$ ,  $\eta := (\eta_1, \eta_2, \dots, \eta_N) \in \mathbb{R}_{\geq 0}^N$  and  $v := (v_1, v_2, \dots, v_N) \in \mathbb{R}^{n_v}$  with  $n_v = \sum_{i \in \mathcal{N}} n_{v,i}$ .

Now that we have introduced all the necessary variables to describe the overall system, we can cast the entire MAS in the form (9). In particular, we will use a single hybrid model that captures both the time-triggered and event-triggered schemes. Indeed, as we will see, the trajectories corresponding to MAS with transmissions triggered by time, *i.e.*, that satisfy (6), and the trajectories corresponding to MAS with event-triggered transmissions, *i.e.*, that satisfy (7), will both be solutions to this ‘‘container’’ model. Hence, any (stability or performance) property derived for this model will apply to both schemes. However, some care has to be taken regarding the existence of (non-trivial) trajectories for both individual cases as showing existence for the container model does not guarantee the existence properties for the (individual) TTC and ETC schemes themselves. We discuss this in more detail in Section IV-B.

To capture both ETC and TTC-based trajectories in one hybrid model, we consider the state vector  $\xi := (x, e, \tau, \kappa, r, \ell, \eta) \in \mathbb{X}$  with  $\mathbb{X} := \{(x, e, \tau, \kappa, r, \ell, \eta) \in \mathbb{R}^{n_x} \times \mathbb{E} \times \mathbb{R}_{\geq 0}^N \times \mathbb{N}^N \times \mathbb{R}^{N n_y} \times \{0, 1\}^{N^2} \times \mathbb{R}_{\geq 0}^N \mid \forall i, m \in \mathcal{V}, ((\ell_i^m = 0) \vee (\ell_i^m = 1 \wedge \tau_i \in [0, \tau_{\text{mad}}^i]))\}$ . The flow map  $F : \mathbb{X} \times \mathbb{R}^{n_v} \rightrightarrows \mathbb{Y}$  with  $\mathbb{Y} := \mathbb{R}^{n_x} \times \mathbb{E} \times \{1\}^N \times \{0\}^N \times \{0\}^{n_y} \times \{0\}^{N^2} \times \mathbb{R}^N$ , is given by

$$F(\xi, v) := (f(x, e, v), g(x, e, v), \mathbf{1}_N, \mathbf{0}_N, \mathbf{0}_{n_y}, \mathbf{0}_{N^2}, \Psi(y, \hat{y}, e, \tau) - \varphi(\eta)). \quad (14)$$

Based on (1), we define  $f(x, e, v) := (f_1(x, h_1^{\text{in}}(x) + e_1^{\text{in}}, v_1), f_2(x, h_2^{\text{in}}(x) + e_2^{\text{in}}, v_2), \dots, f_N(x, h_N^{\text{in}}(x) + e_N^{\text{in}}, v_N))$ , where  $h_i^{\text{in}}(x) = (\delta_1(i)h_1(x_1), \delta_2(i)h_2(x_2), \dots, \delta_N(i)h_N(x_N))$  with  $h_i(x_i)$  and  $\delta_i(m)$  as in (1) and (13), respectively. According to (2), we have that  $\dot{e}_i^m = \hat{y}_i^m - \dot{y}_i$ . By combining the latter with (1) and (5), we obtain that  $g(x, e, v) := (g_1^1(x, e, v), g_2^1(x, e, v), \dots, g_N^1(x, e, v), g_1^2(x, e, v), g_2^2(x, e, v), \dots, g_N^N(x, e, v))$ , where

$$g_i^m(x, e, v) := \delta_i(m)(\hat{f}_i(y_i + e_i^m) - f_{y,i}(x, e_i^{\text{in}}, v)), \quad (15)$$

$$f_{y,i}(x, e_i^{\text{in}}, v) := \frac{\partial h_i(x_i)}{\partial x_i} f_i(x, h_i^{\text{in}}(x) + e_i^{\text{in}}, v_i). \quad (16)$$

Observe that  $g_i^m$  is such that  $g_i^m(x, e, v) = \delta_i(m)e_i^m$  and that  $f_{y,i}$  is such that  $f_{y,i}(x, e_i^{\text{in}}, v) = \dot{y}_i$ . The functions  $\Psi(y, \hat{y}, e, \tau) = (\Psi_1(o_1), \Psi_2(o_2), \dots, \Psi_N(o_N))$  with  $\Psi_i$ ,  $i \in \mathcal{V}$ , as in (8), are defined in Section V together with the local variables  $o_i \in \mathbb{O}_i$  (collecting the information available at agent  $\mathcal{A}_i$  to base the triggering on). Furthermore,  $\varphi(\eta) = (\varphi_1(\eta_1), \varphi_2(\eta_2), \dots, \varphi_N(\eta_N))$  for any  $\eta \in \mathbb{R}_{\geq 0}^N$ , with  $\varphi_i \in \mathcal{K}_{\infty}$ , are to be designed.

The flow set is given by

$$\mathcal{C} := \mathbb{X} \quad (17)$$

in correspondence with the triggering condition in (7) in the sense that flow is only possible when  $\eta_i \geq 0$  for all  $i \in \mathcal{V}$ .

To describe the jump map  $G : \mathbb{X} \rightrightarrows \mathbb{X}$ , we define  $\Gamma_i$  as a  $N \times N$  matrix of which the  $ii$ -th (diagonal) entry is equal to one and all other entries are zero,  $\tilde{\Gamma}_i(\kappa_i) := Z_i(\kappa_i) \otimes \Gamma_i$  with

$$Z_i(\kappa_i) := \text{diag}(z_i(\kappa_i, 1), z_i(\kappa_i, 2), \dots, z_i(\kappa_i, N)), \quad (18)$$

$\Gamma_{i,m} := \Gamma_m \otimes \Gamma_i$  and  $\Lambda_i := \text{diag}(\mathbf{0}_{n_{y,1}}, \mathbf{0}_{n_{y,2}}, \dots, \mathbf{0}_{n_{y,i-1}}, \mathbf{1}_{n_{y,i}}, \mathbf{0}_{n_{y,i+1}}, \dots, \mathbf{0}_{n_{y,N}})$ . Note that  $\Gamma_{i,m}e = (0, 0, \dots, e_i^m, 0, \dots, 0)$  and  $\Lambda_i y = (0, 0, \dots, y_i, 0, \dots, 0)$ . Given these definitions, we obtain the jump map  $G(\xi) = \bigcup_{i \in \mathcal{V}} \bigcup_{m \in \mathcal{V}} G_{i,m}(\xi)$ , where

$$G_{i,m}(\xi) := \begin{cases} \{G_i^0(\xi)\}, & \text{if } \xi \in \mathcal{D}_i \wedge \ell_i^m = 0 \ \forall m \in \mathcal{V}_i^{\text{out}} \\ \{G_{i,m}^1(\xi)\}, & \text{if } \xi \in \mathcal{D}_i \wedge \ell_i^m = 1 \\ \emptyset, & \text{if } \xi \notin \mathcal{D}_i \end{cases}$$

with

$$G_i^0(\xi) := (x, e, (I_N - \Gamma_i)\tau, \kappa + \Gamma_i \mathbf{1}_N, \Lambda_i y + (I_{n_y} - \Lambda_i)r, \ell + \tilde{\Gamma}_i(\kappa_i) \mathbf{1}_{N^2}, \Gamma_i \eta_i^0(y_i) + (I - \Gamma_i)\eta), \quad (19)$$

$$G_{i,m}^1(\xi) := (x, (\Gamma_{i,m} \otimes I_{n_y})(\mathbf{1}_N \otimes (r - y) - e) + e, \tau, \kappa, r, \ell - \Gamma_{i,m} \mathbf{1}_{N^2}, \eta). \quad (20)$$

The function  $G_i^0$ ,  $i \in \mathcal{V}$ , describes the jump of the state  $\xi$  at *transmission* events of agent  $\mathcal{A}_i$ . Observe that for this case,  $r_i$  is set to  $y_i$  for all  $i \in \mathcal{V}$  and all  $m \in \mathcal{V}_i^{\text{out}}$  for which  $z_i(\kappa_i, m) = 1$ , as described in (11). Moreover,  $\ell_i^{m^+} = 1$ , for all  $m \in \mathcal{V}_i^{\text{out}}$  for which  $z_i(\kappa_i, m) = 1$ . The function  $G_{i,m}^1$ ,  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}_i^{\text{out}}$ , describes the jump of the state  $\xi$  at *update* events at agent  $\mathcal{A}_m$  due to the reception of a packet from agent  $\mathcal{A}_i$ . Indeed,  $e_i^m$  is set to  $r_i - y_i$  as described in (12). In addition, we have that  $\ell_i^m$  is set back to 0. Note that to complete the description of the jump map we have to define also the sets  $\mathcal{D}_i$ , which we will do next.

The jump set  $\mathcal{D} \subseteq \mathbb{X}$  is given by  $\mathcal{D} := \bigcup_{i \in \mathcal{V}} \mathcal{D}_i$ , where

$$\mathcal{D}_i := \left\{ \xi \in \mathbb{X} \mid (\ell_i^m = 1 \text{ for some } m \in \mathcal{V}_i^{\text{out}}) \text{ or } (\tau_i \geq d_i \text{ and } \ell_i^m = 0 \text{ for all } m \in \mathcal{V}_i^{\text{out}}) \right\}. \quad (21)$$

By construction, a jump is enforced when  $\tau_i = \tau_{\text{mad}}^i$  and  $\ell_i^m = 1$ , in line with Assumption 1. By means of (14)-(21), we define the hybrid model  $\mathcal{H} = (\mathcal{C}, F, \mathcal{D}, G)$  of the form (9).

It is important to note that the hybrid system  $\mathcal{H}$  is defined such that an agent  $\mathcal{A}_i$ ,  $i \in \mathcal{V}$ , can only schedule a next broadcast if the information of the previous broadcast of agent  $\mathcal{A}_i$  is received by *all* targeted agents (which requires  $\ell_i^m = 0$  for all  $m \in \mathcal{V}_i^{\text{out}}$ ), which is in accordance with Assumption 1. Also observe that the hybrid system cannot exhibit Zeno behavior as  $d_i$  is strictly positive (albeit arbitrarily small).

## B. TTC and ETC as special cases

As already remarked, for analysis purposes the hybrid system  $\mathcal{H}$  admits both TTC and ETC-based trajectories of the MAS. To obtain a hybrid model containing either only the TTC trajectories or only the ETC trajectories, one can restrict the flow set  $\mathcal{C}$  and the jump set  $\mathcal{D}$ , respectively.

For the TTC case, we specify

$$\mathcal{C}^{\text{TTC}} := \{\xi \in \mathbb{X} \mid \forall i \in \mathcal{V} \ \tau_i \in [0, \tau_{\text{miet}}^i]\} \quad (22)$$

with the value  $\tau_{\text{miet}}^i$  taken equal to  $\tau_{\text{mati}}^i$  in (6). Note that  $\mathcal{C}^{\text{TTC}} \subset \mathcal{C}$ . We claim now that the hybrid model  $\mathcal{H}^{\text{TTC}}$  given by the quadruple  $(\mathcal{C}^{\text{TTC}}, F, \mathcal{D}, G)$  admits all possible TTC

trajectories (and no other ones). Instrumental in this claim is that the functions  $\Psi_i : \mathbb{O}_i \rightarrow \mathbb{R}$ ,  $\varphi_i : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  and  $\eta_i^0 : \mathbb{R}^{n_v} \rightarrow \mathbb{R}_{\geq 0}$ , and  $\tau_{\text{miet}}^i$  in (7)-(8) will be designed (see (45) below) such that  $\eta_i \geq 0$  as long as  $\tau_i \in [0, \tau_{\text{miet}}^i]$ . In words, after a transmission of  $\mathcal{A}_i$ , the triggering variable  $\eta_i$  remains nonnegative for at least  $\tau_{\text{miet}}^i$  time units. Hence, before  $\tau_{\text{miet}}^i$  time units have elapsed  $\eta_i$  does not trigger events<sup>1</sup>. Given this observation, it is clear that indeed the hybrid model  $\mathcal{H}^{\text{TTC}}$  admits all possible TTC trajectories (and no other ones). Obviously, solutions to  $\mathcal{H}^{\text{TTC}}$  are also solutions to  $\mathcal{H}$  as  $\mathcal{C}^{\text{TTC}} \subset \mathcal{C}$ .

For the ETC case, we define

$$\mathcal{D}_i^{\text{ETC}} := \left\{ \xi \in \mathbb{X} \mid (\ell_i^m = 1 \text{ for some } m \in \mathcal{V}_i^{\text{out}}) \text{ or } (\tau_i \geq \tau_{\text{miet}}^i \text{ and } \ell_i^m = 0 \text{ for all } m \in \mathcal{V}_i^{\text{out}}) \right\}. \quad (23)$$

By taking  $d_i < \tau_{\text{miet}}^i$  (which is always possible as  $d_i$  is an arbitrarily small positive constant), we see that  $\mathcal{D}_i^{\text{ETC}} \subseteq \mathcal{D}_i$ . By taking the jump set  $\mathcal{D}^{\text{ETC}} := \bigcup_{i \in \mathcal{V}} \mathcal{D}_i^{\text{ETC}}$ , we have that the hybrid model  $\mathcal{H}^{\text{ETC}}$  given by  $(\mathcal{C}, F, \mathcal{D}^{\text{ETC}}, G)$  admits all possible ETC trajectories (and no other ones). As for the ETC, solutions to  $\mathcal{H}^{\text{ETC}}$  are also solutions to  $\mathcal{H}$  as  $\mathcal{D}^{\text{ETC}} \subset \mathcal{D}$ .

We would like to emphasize that the restrictions above do not depend on the destination protocols, hence the destination protocols are present in both setups.

**Remark 3.** *The event-triggering condition presented in (7) can be modified to a static event-triggered condition by taking*

$$t_0^i = 0, t_{k+1}^i := \inf \{ t > t_k^i + \tau_{\text{miet}}^i \mid \Psi_i(o_i) \leq 0 \}.$$

*This modification does not affect the stability and/or performance guarantees as given in Theorem 1 below.*

### C. Mathematical problem formulation

To specify the various and general stability and performance properties that can be studied using our framework, we adopt the following definitions that use the concepts of *hybrid arc* and *hybrid time domains*, see [48] and its extension to continuous-time inputs [56] for more details.

**Definition 1.** *A hybrid arc  $\xi$  is a solution to  $\mathcal{H}$  for a given locally essentially bounded and Lebesgue measurable signal  $v : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{n_v}$ , if*

- (i) *for all  $j \in \mathbb{N}$  such that  $I^j := \{t \in \mathbb{R}_{\geq 0} \mid (t, j) \in \text{dom } \xi\}$  has nonempty interior, it holds that  $\dot{\xi}(t, j) \in F(\xi(t, j), v(t))$  for almost all  $t \in \text{int } I^j$  and  $\xi(t, j) \in \mathcal{C}$  for all  $t \in \text{int } I^j$ ;*
- (ii) *for all  $(t, j) \in \text{dom } \xi$  such that  $(t, j+1) \in \text{dom } \xi$ ,  $\xi(t, j) \in \mathcal{D}$  and  $\xi(t, j+1) \in G(\xi(t, j))$ .*

**Remark 4.** *Although there are two notions of solutions presented in [56], namely everywhere (e) and almost everywhere (ae) solutions, in our case these solutions coincide, as the maps  $\mathcal{C}$  and  $\mathcal{D}$  only depend on the state  $\xi$ , which is absolutely continuous between jumps.*

<sup>1</sup>Alternatively, for the TTC one could also remove the  $\eta$  variables and their dynamics from the model  $\mathcal{H}^{\text{TTC}}$ .

**Definition 2.** *The hybrid system  $\mathcal{H}$  is said to be persistently flowing if all maximal solutions  $\xi$  have unbounded domains in the  $t$ -direction, i.e.,  $\sup_t \text{dom } \xi = \infty$ .*

The following definition of dissipativity is close in nature to the one used in [57].

**Definition 3.** *The system  $\mathcal{H}$  is said to be  $(\mathcal{S}_1, \mathcal{S}_2)$ -flow-dissipative with respect to a supply rate  $\tilde{s} : \mathbb{X} \times \mathbb{R}^{n_v} \rightarrow \mathbb{R}$  and non-empty closed sets  $\mathcal{S}_1, \mathcal{S}_2 \subseteq \mathbb{X}$ , for short  $(\tilde{s}, \mathcal{S}_1, \mathcal{S}_2)$ -flow-dissipative, if there exists a locally Lipschitz function  $U : \mathbb{X} \rightarrow \mathbb{R}_{\geq 0}$ , referred to as the storage function, such that*

$$1) \text{ there exist } \mathcal{K}_{\infty}\text{-functions } \underline{\alpha}, \bar{\alpha} \text{ such that for all } \xi \in \mathbb{X}, \quad \underline{\alpha}(|\xi|_{\mathcal{S}_1}) \leq U(\xi) \leq \bar{\alpha}(|\xi|_{\mathcal{S}_2}), \quad (24a)$$

$$2) \text{ for all } \xi \in \mathcal{C} \text{ and for all } v \in \mathbb{R}^{n_v} \text{ and } f \in F(\xi, v) \quad U^\circ(\xi; f) \leq \tilde{s}(\xi, v), \quad (24b)$$

$$3) \text{ for all } \xi \in \mathcal{D} \text{ and all } g \in G(\xi) \quad U(g) - U(\xi) \leq 0. \quad (24c)$$

In essence, the sandwich bounds in (24a) are not required for general dissipativity notions, as these only depend on the *supply rate* and *storage function*, however, the sandwich bounds are often useful for stability analysis. Indeed, using (24b) and (24c) we find that

$$U(\xi(t, j)) \leq U(\xi(0, 0)) + \int_0^t \tilde{s}(\xi(\tau, \iota(\tau)), v(\tau)) d\tau, \quad (25)$$

where  $\iota(\tau) := \max\{j \in \mathbb{N} \mid (\tau, j) \in \text{dom } \xi\}$ , which can be used to establish, e.g., passivity. Using (24a) and (25) we find

$$\underline{\alpha}(|\xi(t, j)|_{\mathcal{S}_1}) \leq \bar{\alpha}(|\xi(0, 0)|_{\mathcal{S}_2}) + \int_0^t \tilde{s}(\xi(\tau, \iota(\tau)), v(\tau)) d\tau. \quad (26)$$

As we will discuss after Theorem 1, in general it is useful to be able to work with such a two-measure dissipativity notion (like in [58]), where  $\mathcal{S}_1 \supseteq \mathcal{S}_2$ . Of course, in some cases  $\mathcal{S}_1 = \mathcal{S}_2$  can be taken.

Our problem can now be formalized as follows.

**Problem 1.** *Given the MAS represented by  $\mathcal{H}(\mathcal{C}, F, \mathcal{D}, G)$  with data  $F, G, \mathcal{C}$  and  $\mathcal{D}$  as described in (14)-(21), and the functions  $z_i, i \in \mathcal{V}$ , as in (4). Provide design conditions for the time-constants  $\tau_{\text{mad}}^i, \tau_{\text{miet}}^i \in \mathbb{R}_{> 0}$  with  $\tau_{\text{miet}}^i \geq \tau_{\text{mad}}^i$  and the functions  $\Psi_i, \varphi_i$  and  $\eta_i^0$  as in (7) and (8), for  $i \in \mathcal{V}$ , such that, under Assumption 1, the system  $\mathcal{H}$  (which captures the behavior of both  $\mathcal{H}^{\text{TTC}}$  and  $\mathcal{H}^{\text{ETC}}$ ) is persistently flowing and  $(\tilde{s}, \mathcal{S}_1, \mathcal{S}_2)$ -flow-dissipative for sets  $\mathcal{S}_1, \mathcal{S}_2 \subset \mathbb{X}$  with  $\mathcal{S}_1 \supseteq \mathcal{S}_2$ , and a given supply rate  $\tilde{s} : \mathbb{X} \times \mathbb{R}^{n_v} \rightarrow \mathbb{R}$  of the form*

$$\tilde{s}(\xi, v) := s(x, e, v) - \sigma(\eta), \quad (27)$$

where  $s : \mathbb{R}^{n_x} \times \mathbb{R}^{N n_y} \times \mathbb{R}^{n_v} \rightarrow \mathbb{R}$ ,  $\xi \in \mathbb{X}$ ,  $v \in \mathbb{R}^{n_v}$  and  $\varphi$  as in (14). The supply rate  $\tilde{s}$  and the set  $\mathcal{A}$  capture the desired stability, performance and robustness requirements.

As shown in, for example, [59]–[61], the dissipation inequalities in (24) allow to consider various important system properties such as asymptotic stability, input-to-state stability,  $\mathcal{L}_p$ -stability with  $p \in [1, \infty)$  and passivity, from a unified



point of view. Hence, Problem 1 captures a wide range of relevant multi-agent (but also centralized) control problems addressed in the literature including output-regulation problems (of which the consensus-seeking problem is a particular case) and vehicle-platooning problems (in which  $\mathcal{L}_p$ -contractivity,  $p \in [1, \infty)$ , is of interest as a string stability guarantee). In the companion paper [1], we demonstrate the generality of this framework through various case studies.

## V. DESIGN CONDITIONS AND MAIN RESULT

In this section, we present design conditions for the time constants  $\tau_{\text{miet}}^i$ ,  $\tau_{\text{mad}}^i$  and the functions  $\Psi_i$ ,  $\varphi_i$  and  $\eta_i^0$ ,  $i \in \mathcal{V}$ . The desired dissipativity property is then established.

### A. Design of the destination protocols

As already mentioned in Remark 1, the destination protocols considered in this paper have a strong connection to the network scheduling protocols as described in [41], [51]. As such, we are inspired by the conditions on network protocols as introduced in [41], [51] for the design of suitable destination protocols. Thereto, let  $\bar{z}_i(k, e_i^{\text{out}}) := ((1 - z_i(k, 1))e_i^1, (1 - z_i(k, 2))e_i^2, \dots, (1 - z_i(k, N))e_i^N)$  for all  $e_i^{\text{out}} \in \mathbb{E}_i$  and  $k \in \mathbb{N}$ . Consider the following condition.

**Condition 1.** ([41], [51]) *The destination protocol  $z_i : \mathbb{N} \times \mathcal{V} \rightarrow \{0, 1\}$ ,  $i \in \mathcal{V}$ , is uniformly globally exponentially stable (UGES) in the sense that there exists a Lyapunov function  $W_i : \mathbb{N} \times \mathbb{R}^{Nn_y} \rightarrow \mathbb{R}_{\geq 0}$  that is locally Lipschitz in its second argument such that for all  $k \in \mathbb{N}$  and all  $e_i^{\text{out}} \in \mathbb{E}_i$*

$$\underline{\alpha}_{W,i} |e_i^{\text{out}}| \leq W_i(k, e_i^{\text{out}}) \leq \bar{\alpha}_{W,i} |e_i^{\text{out}}| \quad (28a)$$

$$W_i(k+1, \bar{z}_i(k, e_i^{\text{out}})) \leq \lambda_i W_i(k, e_i^{\text{out}}) \quad (28b)$$

for constants  $0 < \underline{\alpha}_{W,i} \leq \bar{\alpha}_{W,i}$  and  $0 < \lambda_i < 1$ .

As shown in [41], the condition above is equivalent to the UGES property of the time-varying discrete-time system

$$q(k+1) = \bar{z}_i(k, q(k)), \quad k \in \mathbb{N}, \quad (29)$$

where  $q \in \mathbb{E}_i$ . Hence, Condition 1 is only related to the destination protocol and not to the other dynamics of the system. In fact, in case the system described by (29) is UGES for a given protocol  $z_i$ , the corresponding Lyapunov function  $W_i$  satisfying (28a) and (28b) can be specified as

$$W_i(k, e_i^{\text{out}}) = \sqrt{\sum_{m=k}^{\infty} |\psi(m, k, e_i^{\text{out}})|^2}, \quad (30)$$

where  $\psi(m, k, e_i^{\text{out}})$  denotes the solution of (29) at discrete time instant  $m \in \mathbb{N}_{\geq k}$  with initial condition  $e_i^{\text{out}} \in \mathbb{E}_i$  and initial time  $k \in \mathbb{N}$ , see also [41, Proposition 3].

The well-known Round-Robin (RR) protocol but also many other protocols are known to be UGES protocols as well, see also [41], [52]. For the RR protocol, the function  $W_{i,RR}$  in (30) that satisfies (28a) with  $\underline{\alpha}_{W,i,RR} = 1$  and  $\bar{\alpha}_{W,i,RR} = \sqrt{N_i}$ , and (28b) with  $\lambda_{i,RR} = \sqrt{(N_i - 1)/N_i}$ , where we recall that  $N_i$  is equal to the cardinality of  $\mathcal{V}_i^{\text{out}}$ , see [41] for more details. For the sampled-data (SD) protocol, corresponding to the case that at each transmission instant an agent  $\mathcal{A}_i$ ,  $i \in \mathcal{V}$ , broadcasts its output measurement to all connected

agents  $\mathcal{A}_m$ ,  $m \in \mathcal{V}_i^{\text{out}}$ , we can take  $W_i(k, e_i^{\text{out}}) = |e_i^{\text{out}}|$ ,  $\underline{\alpha}_{W,i} = \bar{\alpha}_{W,i} = 1$  and any  $\lambda_i \in (0, 1)$ .

We also consider the following condition.

**Condition 2.** ([34], [52]) *For almost all  $e_i^{\text{out}} \in \mathbb{E}_i$  and all  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}_i^{\text{out}}$  and  $k \in \mathbb{N}$*

$$\left| \frac{\partial W_i(k, e_i^{\text{out}})}{\partial e_i^{\text{out}}} \right| \leq c_i \quad (31)$$

for some constant  $c_i > 0$ .

Condition 2 is also only related to the destination protocol  $z_i$ . For the RR and SD protocol, we can take  $c_{i,RR} = \sqrt{N_i}$  and  $c_{i,SD} = 1$ , respectively.

**Remark 5.** *As the conditions on the destination protocols are independent of the system dynamics, the resilient protocols that can cope with packet loss or Denial-of-Service, which are presented in Part II [1], can be designed and implemented without any modifications to the framework as presented here.*

**Condition 3.** *For each  $i \in \mathcal{V}$ , there exist functions  $H_{e,i} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_v} \rightarrow \mathbb{R}_{\geq 0}$  and  $H_{y,i} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_v} \rightarrow \mathbb{R}_{\geq 0}$  and constants  $L_{1,i}, L_{2,i} > 0$  such that for all  $m \in \mathcal{V}_i^{\text{out}}$ ,  $x \in \mathbb{R}^{n_x}$ ,  $e \in \mathbb{R}^{Nn_y}$  and  $v \in \mathbb{R}^{n_v}$ , it holds that*

$$c_i |g_i^m(x, e, v)| \leq H_{e,i}(x, e_i^m, v) + L_{1,i} |e_i^m| + L_{2,i} |e_i^e|, \quad (32)$$

$$c_i |f_{y,i}(x, e_i^m, v)| \leq H_{y,i}(x, e_i^m, v) + L_{2,i} |e_i^e| \quad (33)$$

with the functions  $g_i^m$  and  $f_{y,i}$  as in (15) and (16), and the constant  $c_i > 0$  as in (31), respectively.

Inequality (32) is related to  $e_i^m$  and inequality (33) is related to  $y_i$ . Constant  $L_{2,i}$  can be taken equal to zero for all  $i \in \mathcal{V}$ , if  $\mathcal{E}$  does not contain self-loops.

### B. Lower-bounds on the minimum inter-event times and maximum allowable delays

To obtain the lower-bounds on the minimum inter-event times  $\tau_{\text{miet}}^i$  and the maximum allowable delay  $\tau_{\text{mad}}^i$  for each agent  $\mathcal{A}_i$ ,  $i \in \mathcal{V}$ , we first characterize the influence of the transmission errors  $e_i^m$ ,  $m \in \mathcal{V}_i^{\text{out}}$ , on the state  $x$  and the desired stability/performance property by means of the following condition.

**Condition 4.** *There exist a locally Lipschitz function  $V : \mathbb{R}^{n_x} \rightarrow \mathbb{R}_{\geq 0}$  and a non-empty closed set  $\mathcal{X} \subseteq \mathbb{R}^{n_x}$ ,  $\mathcal{K}_{\infty}$ -functions  $\underline{\alpha}_V \leq \bar{\alpha}_V$ , continuous functions  $\varsigma_i : \mathbb{R}^{n_y,i} \times \mathbb{R}^{n_y} \rightarrow \mathbb{R}_{\geq 0}$ , constants  $\mu_i, \gamma_{l,i} > 0$ ,  $l \in \{0, 1\}$ ,  $i \in \mathcal{V}$ , such that for all  $x \in \mathbb{R}^{n_x}$*

$$\underline{\alpha}_V(|x|_{\mathcal{X}}) \leq V(x) \leq \bar{\alpha}_V(|x|_{\mathcal{X}}), \quad (34)$$

and for all  $x \in \mathbb{R}^{n_x}$ ,  $e \in \mathbb{R}^{Nn_y}$ ,  $v \in \mathbb{R}^{n_v}$ ,  $\kappa \in \mathbb{N}^N$ ,  $\ell \in \{0, 1\}^{N^2}$ ,

$$V^\circ(x; f(x, e, v)) \leq s(x, e, v) + \sum_{i \in \mathcal{V}} \left( -\varsigma_i(y_i, \hat{y}_i^{\text{in}}) - \mu_i N_i H_i^2(x, e_i^{\text{in}}, v, \ell_i) + \gamma_{\ell(\ell_i), i}^2 W_i^2(\kappa_i, e_i^{\text{out}}) \right) \quad (35)$$

with  $\ell_i := (\ell_i^1, \ell_i^2, \dots, \ell_i^N)$  and  $s(x, e, v)$  from (27), and the function  $H_i : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_v} \times \{0, 1\}^N \rightarrow \mathbb{R}_{\geq 0}$  given by

$$H_i(x, e_i^{\text{in}}, v, \ell_i) := \max\{H_{e,i}(x, e_i^{\text{in}}, v), \tilde{\ell}(\ell_i) H_{y,i}(x, e_i^{\text{in}}, v)\}, \quad (36)$$

with the functions  $H_{e,i}$  and  $H_{y,i}$  as in (32) and (33), and where the function  $\tilde{\ell} : \{0, 1\}^N \rightarrow \{0, 1\}$  is given by

$$\tilde{\ell}(\ell_i) := \begin{cases} 0, & \text{when } \sum_{m \in \mathcal{V}} \ell_i^m = 0 \\ 1, & \text{when } \sum_{m \in \mathcal{V}} \ell_i^m > 0. \end{cases} \quad (37)$$

In essence, (35) constitutes a condition guaranteeing an  $\mathcal{L}_2$ -gain property from  $(W_1, W_2, \dots, W_N)$  to  $(H_1, H_2, \dots, H_N)$  for the system  $\dot{x} = f(x, e, v)$ . The constants  $\gamma_{l,i}$ ,  $l \in \{0, 1\}$ , resulting from this inequality indicate the influence of the transmission errors  $e_i^m$ ,  $m \in \mathcal{V}_i^{\text{out}}$ , on the state  $x$  and the desired stability/performance property (captured by  $s$  and  $\mathcal{X}$ ). When  $\tilde{\ell}(\ell_i) = 0$ ,  $\ell_i \in \{0, 1\}^N$ ,  $i \in \mathcal{V}$ , the most recent broadcast information by agent  $\mathcal{A}_i$  has been received by all targeted agents. Otherwise, i.e., when  $\tilde{\ell}(\ell_i) = 1$ , some (or all) of the targeted agents still need to receive the most recent broadcast information of agent  $\mathcal{A}_i$ . Condition 4 may be ensured at the first step of emulation when designing a local controller for each agent  $m \in \mathcal{V}$ .

**Remark 6.** In the TTC case, the function  $\varsigma_i$  is redundant, and, in that case, it may be taken as  $\varsigma_i = 0$  for all  $i \in \mathcal{V}$ .

The constants  $\gamma_{l,i}$ ,  $l \in \{0, 1\}$ , as in Condition 4, are used to determine  $\tau_{\text{miet}}^i$  and  $\tau_{\text{mad}}^i$ ,  $i \in \mathcal{V}$ , via the following condition.

**Condition 5.** There exist positive real constants  $\tau_{\text{miet}}^i$  and  $\tau_{\text{mad}}^i$ ,  $i \in \mathcal{V}$ , with  $\tau_{\text{miet}}^i \geq \tau_{\text{mad}}^i$  satisfying

$$\tilde{\gamma}_i(0)\phi_{0,i}(\tau_{\text{miet}}^i) \geq \lambda_i^2 \tilde{\gamma}_i(1)\phi_{1,i}(0), \quad (38)$$

$$\tilde{\gamma}_i(1)\phi_{1,i}(\tau_i) \geq \tilde{\gamma}_i(0)\phi_{0,i}(\tau_i), \text{ for all } \tau_i \in [0, \tau_{\text{mad}}^i], \quad (39)$$

where  $\phi_{l,i}$ ,  $l \in \{0, 1\}$ , evolves according to

$$\frac{d}{d\tau_i} \phi_{l,i} = - \left( 2\tilde{L}_i(l)\phi_{l,i} + \tilde{\gamma}_i(l) \left( \frac{1}{\epsilon_i \mu_i} \phi_{l,i}^2 + 1 \right) \right), \quad (40)$$

for some fixed initial conditions  $\phi_{l,i}(0)$ ,  $l \in \{0, 1\}$ , that satisfy  $\tilde{\gamma}_i(1)\phi_{1,i}(0) \geq \tilde{\gamma}_i(0)\phi_{0,i}(0) > \lambda_i^2 \tilde{\gamma}_i(1)\phi_{1,i}(0) > 0$ , where, for each  $i \in \mathcal{V}$  and  $l \in \{0, 1\}$ , the functions  $\tilde{L}_i : \{0, 1\} \rightarrow \mathbb{R}_{\geq 0}$  and  $\tilde{\gamma}_i : \{0, 1\} \rightarrow \mathbb{R}_{\geq 0}$  are given by

$$\tilde{L}_i(l) := \alpha_{W,i}^{-1} \zeta_i^{-l} \sqrt{N_i} \max \{L_{1,i}, L_{2,i}\}, \quad (41)$$

$$\tilde{\gamma}_i(l) := \zeta_i^{-l} \gamma_{l,i} \quad (42)$$

with

$$\zeta_i := \frac{\lambda_i \alpha_{W,i}}{\bar{\alpha}_{W,i}} \quad (43)$$

(note that  $\zeta_i < 1$ ) and where  $\gamma_{l,i}$  satisfies Condition 4 and with  $\lambda_i$  as in (28b). The constant  $\epsilon_i \in (0, 1]$ ,  $i \in \mathcal{V}$ , is a tuning parameter.

Conditions 4 and 5 can be obtained systematically for many systems. In a similar fashion as in [52], the result above leads to intuitive trade-off curves between  $\tau_{\text{mad}}^i$  and  $\tau_{\text{miet}}^i$ , which can be used to find appropriate values for  $\lambda_i$ ,  $\phi_{0,i}(0)$  and  $\phi_{1,i}(0)$ ,  $i \in \mathcal{V}$ , see, [52, Section VI].

**Remark 7.** Observe from (40) that  $\frac{d}{d\tau_i} \phi_{l,i}(\tau_i) < 0$ ,  $l \in \{0, 1\}$ , for  $\tau_i \in [0, \tau_{\text{miet}}^i]$ , (since  $\phi_{l,i}(\tau_i) > 0$  for  $\tau_i \in [0, \tau_{\text{miet}}^i]$ ), and that a larger  $\gamma_{l,i}$  leads to faster decay of  $\phi_{l,i}$ . By combining (38) and (39) with the latter fact, we can see that larger  $\gamma_{l,i}$  leads to less favorable  $(\tau_{\text{mad}}^i, \tau_{\text{miet}}^i)$ -combinations.

As the constants  $\gamma_{l,i}$  depend on the system dynamics and the robustness properties of the controller, there is an inherent upper bound on the MIET/MAD, which essentially depends on how fast the network-induced error  $e_i^m$  can grow. Of course, a well-designed holding function may counteract this phenomenon, i.e., the holding function may reduce the maximum growth rate of  $e_i^m$ , which results in more favorable (larger)  $(\tau_{\text{mad}}^i, \tau_{\text{miet}}^i)$ -combinations, which will be illustrated in the second part [1].

**Remark 8.** Obtaining suitable constants such that Conditions 3-5 are satisfied is problem and system-dependent, and there may exist settings for which global information is necessary to obtain them. However, in some cases, these constants can be obtained systematically and in a decentralized manner, as illustrated in the examples in Part II [1].

### C. Event-triggering mechanism design

To facilitate ETM design, consider the following condition.

**Condition 6.** There exist continuous functions  $\underline{H}_i : \mathbb{R}^{n_y} \times \mathbb{R}^{n_y} \rightarrow \mathbb{R}_{\geq 0}$ ,  $i \in \mathcal{V}$ , that satisfy for all  $x \in \mathbb{R}^{n_x}$ ,  $e \in \mathbb{R}^{Nn_y}$ ,  $v \in \mathbb{R}^{n_v}$ ,  $\kappa \in \mathbb{N}^N$ ,  $\ell \in \{0, 1\}^{N^2}$ ,

$$\underline{H}_i(y_i, \hat{y}_i^{\text{in}}) \leq H_i(x, e_i^{\text{in}}, v, \ell_i) \quad (44)$$

for functions  $H_i$  as in (36).

Condition 6 is trivially satisfied by taking  $\underline{H}_i(y_i, \hat{y}_i^{\text{in}}) = 0$  for all  $i \in \mathcal{V}$ ,  $y_i \in \mathbb{R}^{n_{y,i}}$  and  $\hat{y}_i^{\text{in}} \in \mathbb{R}^{n_y}$  as the function  $H_i$  is non-negative. However, the introduction of the function  $\underline{H}_i$  is an important feature that allows to capture ETM designs for relevant systems such as MAS with single-integrator dynamics, as we will show in the case study in Part II [1].

The dynamics of the triggering variables  $\eta_i$ ,  $i \in \mathcal{V}$ , which, according to (7), are used to determine the transmission instants, are defined by the functions  $\Psi_i : \mathbb{O}_i \rightarrow \mathbb{R}$  and  $\eta_i^0 : \mathbb{O}_i \rightarrow \mathbb{R}_{\geq 0}$ ,  $i \in \mathcal{V}$ . The local variables  $o_i$  at agent  $\mathcal{A}_i$  are given by  $o_i = (y_i, \hat{y}_i^{\text{in}}, \eta_i, e_i^{\text{out}}) \in \mathbb{O}_i$  with  $\mathbb{O}_i := \mathbb{R}^{n_y} \times \mathbb{R}^{Nn_y} \times \mathbb{R} \times \mathbb{R}^{Nn_y}$ . For all  $\xi \in \mathbb{X}$ , the functions  $\Psi_i$  and  $\eta_i^0$ ,  $i \in \mathcal{V}$ , as in (8), are given by

$$\Psi_i(o_i) := \varsigma_i(y_i, \hat{y}_i^{\text{in}}) + (1 - \epsilon_i) \mu_i N_i \underline{H}_i^2(y_i, \hat{y}_i^{\text{in}}) - (1 - \omega_i(\tau_i)) \tilde{\gamma}_i \sum_{m \in \mathcal{V}_i^{\text{out}}} |e_i^m|^2, \quad (45)$$

$$\eta_i^0(y_i) := \epsilon_\eta^0 \sum_{m \in \mathcal{V}_i^{\text{out}}} |e_i^m|^2 \quad (46)$$

with  $\epsilon_\eta^0 := \alpha_{W,i}^2 (\tilde{\gamma}_i(0)\phi_{0,i}(\tau_{\text{miet}}^i) - \tilde{\gamma}_i(1)\phi_{1,i}(0)\lambda_i^2)$ , where

$$\omega_i(\tau_i) := \begin{cases} \{1\}, & \text{for } \tau_i \in [0, \tau_{\text{miet}}^i) \\ [0, 1], & \text{for } \tau_i = \tau_{\text{miet}}^i, \\ \{0\}, & \text{for } \tau_i > \tau_{\text{miet}}^i, \end{cases} \quad (47)$$

$$\tilde{\gamma}_i := \bar{\alpha}_{W,i} \tilde{\gamma}_i(0) \times$$

$$\left( 2\tilde{L}_i(0)\phi_{0,i}(\tau_{\text{miet}}^i) + \tilde{\gamma}_i(0) \left( 1 + \frac{1}{\epsilon_i \mu_i} \phi_{0,i}^2(\tau_{\text{miet}}^i) \right) \right). \quad (48)$$

Constants  $\lambda_i$ ,  $\epsilon_i$  and the function  $\varsigma_i$ ,  $i \in \mathcal{V}$ , are part of the ETM described by (7), (8), (45) and (46). Hence, the design of  $\lambda_i$  and  $\varsigma_i$  have a significant influence on the average inter-event times generated by this ETM.

The triggering rules presented above are essentially compensating the destabilizing (increasing) part of the supply rate corresponding to the network-induced errors by exploiting the stabilizing (decreasing) parts in the supply rate. To be specific, the destabilizing parts are characterized by the functions  $W_i$  and the stabilizing parts are characterized by the functions  $\varsigma_i$ . These two parts are filtered through the auxiliary variables  $\eta_i$ , to ensure that, “on average,” the system is dissipative. To prevent Zeno behavior, we employ time-regularization, which is achieved through the maps  $\omega_i$ . The inclusion of the maps  $\omega_i$  ensures that we do not trigger for at least  $\tau_{\text{miet}}^i$  time-units.

**Remark 9.** *Note that the conditions above can always be satisfied in case of stabilizable and detectable LTI systems, in which case an LMI can be constructed to systematically guarantee that the main conditions regarding the system dynamics are satisfied and to find the corresponding parameters. Additionally, the results also hold for a large class of nonlinear systems. One such nonlinear example is presented in Part II [1]. Moreover, the application of the presented framework can be extended further, if, instead of global non-practical properties, semi-global practical properties are of interest. Such derivations can be obtained by taking inspiration from, e.g., [62], thereby applying to a large class of locally Lipschitz systems. In any case, we focus here on a general representation and framework, and, therefore, the results may require some tailoring to incorporate specific problem settings.*

A crucial comment has to be made on the availability of the information  $e_i^{\text{out}} = (e_i^1, e_i^2, \dots, e_i^N)$  at agent  $\mathcal{A}_i$ . As mentioned before, due to the presence of *unknown* time-varying delays, the estimate  $\hat{y}_i^m$  of output  $y_i$ ,  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}^{\text{out}}$  (available at agent  $\mathcal{A}_m$ ) is *not* available at agent  $\mathcal{A}_i$ . So, agent  $\mathcal{A}_i$  does not always precisely know which estimate agent  $\mathcal{A}_m$  is using for its output  $y_i$  and, consequently, the network-induced error  $e_i^m = \hat{y}_i^m - y_i$ ,  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}^{\text{out}}$ , see (2), is sometimes not precisely known for agent  $\mathcal{A}_i$ . However, note that due to  $(1 - \omega_i(\tau_i))$  being zero for  $\tau_i \in [0, \tau_{\text{miet}}^i)$ , the information  $e_i^{\text{out}}$  is *not* always needed in the trigger dynamics. In particular, for the first  $\tau_{\text{miet}}^i$  time units after a transmission by agent  $\mathcal{A}_i$ , agent  $\mathcal{A}_i$  does not require this information. *Only* when  $\tau_{\text{miet}}^i$  time units have elapsed after a transmission by agent  $\mathcal{A}_i$ ,  $e_i^m$ , (and thus  $\hat{y}_i^m$ ) has to be known. This condition can be satisfied in various relevant cases. Let us shortly elaborate on this.

In case zero-order hold (ZOH) devices are used in (5), *i.e.*,  $\hat{f}_i = 0$ ,  $i \in \mathcal{V}$ ,  $\hat{y}_i^m$  is known at agent  $\mathcal{A}_i$  when  $\tau_{\text{miet}}^i$  time units have elapsed after a transmission. Indeed, due to Assumption 1, after  $\tau_{\text{mad}}^i$  time units all transmissions of  $y_i$  have arrived at the respective agents due to the small-delay assumption, and since  $\tau_{\text{mad}}^i \leq \tau_{\text{miet}}^i$ , and the use of ZOH, agent  $\mathcal{A}_i$  knows that  $\hat{y}_i^m = r_i$ , the last transmitted value of  $y_i$ . Note that the exact values of the delays need not be known in this case, and, hence, can be unknown.

In case of *known* delays (and thus with absence of delays as a special case), agent  $\mathcal{A}_i$ , knows exactly when its transmitted value  $y_i(t_k^i)$  (stored in  $r_i$ ) arrives at agent  $\mathcal{A}_m$  (namely, at time  $t_k^i + \Delta_k^{i,m}$ ), after which  $\mathcal{A}_i$  can run a copy of the holding device (5) to exactly determine  $\hat{y}_i^m$  as being used  $\mathcal{A}_m$ . Hence, in this case  $e_i^m$  can be determined exactly too.

Of course, by including synchronized clocks in the system, it is possible to *enforce* that the delays are known, even if the network-induced delay is a priori unknown. By time-stamping the information, and “updating”  $\hat{y}_i^m$  only when  $\tau_{\text{mad}}^i$  time-units have elapsed, each agent incorporates the received information at exactly the same time, making the total delay known (and constant), in which case agent  $\mathcal{A}_i$  also knows the value of  $\hat{y}_i^m$ , even when general holding devices are deployed. Such techniques have already been employed in the literature, see, *e.g.*, [63] and the references therein.

The decisive factor in the above cases is that agent  $\mathcal{A}_i$  knows the value of  $\hat{y}_i^m$  *exactly* (after  $\tau_{\text{miet}}^i$  time units have elapsed after transmission). This is an assumption implicitly used in many existing works on packet-based multi-agent control. In fact, existing works often consider absence of delays or ZOH devices. The general framework here encompasses these cases as special results, unifies them by revealing the intrinsic underlying properties and can tackle much more general cases.

Interestingly, our framework can be extended further to encompass also the case where the values of  $\hat{y}_i^m$  and thus  $e_i^m$  are not exactly known, which is the case when there are non-zero communication delays which are not known and when general holding devices are used. In this case, it is not possible to perform exact reconstruction of  $\hat{y}_i^m$  by agent  $\mathcal{A}_i$ . However, in this case it is possible that  $\mathcal{A}_i$  approximately knows  $\hat{y}_i^m$  in the sense of being able to generate a robust set-valued estimate  $\mathcal{Y}_i^m$ ,  $m \in \mathcal{V}_i^{\text{out}}$ ,  $i \in \mathcal{V}$  that satisfies the following property.

**Assumption 2.** *For each solution pair  $(\xi, v)$  to  $\mathcal{H}$  with  $v \in \mathcal{L}_{\infty}^{n_v}$ , it holds for all  $(t, j) \in \text{dom} \xi$  and for all  $i \in \mathcal{V}$  and  $m \in \mathcal{V}_i^{\text{out}}$ , that  $\hat{y}_i^m(t, j) \in \mathcal{Y}_i^m(t, j)$ .*

For instance, in the case of generalized holding functions and unknown delays, it is known that the communication delays  $\Delta_k^{i,m}$ ,  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}^{\text{out}}$ ,  $k \in \mathbb{N}$ , are bounded by  $\tau_{\text{mad}}^i$ . By exploiting this fact, it is possible to construct a set-valued estimate  $\mathcal{Y}_i^m$  that contains  $\hat{y}_i^m$ , by considering at agent  $\mathcal{A}_i$  all possible transmission delays of  $y_i(t_k^i)$  (let us parameterize them by  $d \in [0, \tau_{\text{mad}}^i]$ ) arriving at agent  $\mathcal{A}_i$  and making forward predictions  $\tilde{y}_i^m(t - t_k^i)$  using  $\tilde{y}_{i,d}^m = \hat{f}_i(\tilde{y}_{i,d}^m)$ , and  $\tilde{y}_{i,d}^m(d) = y_i(t_k^i)$  as a corresponding initial condition. The set  $\bigcup_{d \in [0, \tau_{\text{mad}}^i]} \tilde{y}_{i,d}^m(t)$  contains then  $\hat{y}_i^m$ . Of course, this induces extra computations at  $\mathcal{A}_i$ , but conceptually it is possible. Using these set-valued estimates  $\mathcal{Y}_i^m$ , we can adapt (45) and (46) to

$$\begin{aligned} \Psi_i(o_i) &:= \varsigma_i(y_i, \hat{y}_i^{\text{in}}) + (1 - \epsilon_i) \mu_i N_i \underline{H}_i^2(y_i, \hat{y}_i^{\text{in}}) \\ &\quad - (1 - \omega_i(\tau_i)) \tilde{\gamma}_i \sum_{m \in \mathcal{V}_i^{\text{out}}} \sup_{\tilde{y} \in \mathcal{Y}_i^m} |\tilde{y} - y_i|^2, \\ \eta_i^0(y_i) &:= \epsilon_\eta^0 \sum_{m \in \mathcal{V}_i^{\text{out}}} \inf_{\tilde{y} \in \mathcal{Y}_i^m} |\tilde{y} - y_i|^2, \end{aligned}$$

and still have the same stability and performance properties. Indeed, as the estimate  $\hat{y}_i^m$  of output  $y_i$ ,  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}^{\text{out}}$ , is contained in  $\mathcal{Y}_i^m$  due to Assumption 2, they are used to capture the actual transmission error  $e_i^m$ ,  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}^{\text{out}}$ , as in (2). For instance, note that  $|e_i^m|^2 \leq \sup_{\tilde{y} \in \mathcal{Y}_i^m} |\tilde{y} - y_i|^2$ . In this way, the ETMs at agent  $\mathcal{A}_i$  are indirectly based on the network-induced errors  $e_i^m = \hat{y}_i^m - y_i$ ,  $m \in \mathcal{V}_i^{\text{out}}$ . Let us add to this, that indeed, the proposed triggering mechanism does not rely on clock synchronization or acknowledgment signals as the functions  $\Psi_i$  and  $\eta_i^0$ ,  $i \in \mathcal{V}$  do not depend on  $\hat{y}_i^m$ ,  $m \in \mathcal{V}_i^{\text{out}}$ .

Note that one can always satisfy Assumption 2 by taking the (trivial) choice  $\mathcal{Y}_i^m(t, j) = \mathbb{R}^{n_{y,i}}$ , however, in this case, the triggering will become time-triggered. Recall that this indeed complies with the observation that, for the case of TTC, no conditions on knowing  $\hat{y}_i^m$  at agent  $\mathcal{A}_i$  are required due to the time-triggered communication. If, however, better estimates can be designed, which is possible in most cases, event-triggered implementations are obtained that may result in significant savings in communication compared to this time-triggered solution.

**Remark 10.** Assumption 2 reveals an important general principle for successful deployment of ETCs, namely that agent  $\mathcal{A}_i$  needs to have some idea about what estimates  $\hat{y}_i^m$  other agents are using regarding its outputs  $y_i$ . The more accurate the information, the less transmissions are needed. If no information is available, then the ETC scheme based on Assumption 2 reduces to TTC.

Interestingly, even in the TTC case our framework solves many multi-agent problems that were not even tackled before for time-triggered communication (in which the only restriction is an upper bound on the inter-transmission times).

#### D. Main result

Given the conditions and the ETM design presented above, we can now state the following result.

**Theorem 1.** Consider the system  $\mathcal{H}(\mathcal{C}, F, \mathcal{D}, G)$  with data  $\mathcal{C}$ ,  $F$ ,  $\mathcal{D}$  and  $G$  as described in (14)-(21) that satisfies Conditions 1-6 and suppose Assumption 1 holds. Then the MAS described by  $\mathcal{H}$  is  $(\tilde{s}, \mathcal{S}_1, \mathcal{S}_2)$ -flow-dissipative with the supply rate  $\tilde{s} : \mathbb{X} \times \mathbb{R}^{n_v} \rightarrow \mathbb{R}$  as given in (27) and  $\mathcal{S}_1 = \{\xi \in \mathbb{X} \mid x \in \mathcal{X}, e = 0, \eta = 0\}$ ,  $\mathcal{S}_2 = \{\xi \in \mathcal{S}_1 \mid r - y = 0\}$ . In addition, if there are no finite escape times during the flow<sup>2</sup>, then the system  $\mathcal{H}$  is persistently flowing.

The proof is provided in the Appendix. A few comments are in order. It is clear that the absence of finite escape times during flow does not directly guarantee persistence of flow. In fact, obtaining that  $\mathcal{H}$  is persistently flowing requires the exclusion of jumps outside of  $\mathcal{C} \cup \mathcal{D}$  along solutions and the satisfaction of viability conditions to guarantee that if a jump is not possible, flow is possible, see [48, Prop. 6.10] for details. In fact, these three properties would lead to completeness of all maximal solutions in the terminology of [48]. Then, the exclusion of Zeno behavior is required in order to obtain the persistence of flow. This is addressed formally in the proof of Theorem 1. Let us also remark that, depending on the problem at hand, different arguments can be invoked to prove the absence of finite escape times during flow using results for (constrained) continuous-time dynamical systems. A few typical arguments are described in the proposition below, whose proof is given in the appendix.

<sup>2</sup>Absence of finite escape times during flow is meant here in the sense that solutions to  $\mathcal{H}$  are well behaved in between jumps, in the sense that maximal solutions do not grow unbounded in finite time due to the flow dynamics, similar to case (b) in [48, Prop. 2.10] in the autonomous setting.

**Proposition 1.** Consider the system  $\mathcal{H}(\mathcal{C}, F, \mathcal{D}, G)$  in Theorem 1 with the hypotheses in Theorem 1 holding. If

- 1) the set  $\mathcal{X}$  is compact and non-empty and there exist constants  $c_V, c_W \in \mathbb{R}_{\geq 0}$  and a function  $\sigma_v \in \mathcal{K}_\infty$  such that  $s(x, e, v) \leq c_V V(x) + c_W |e|^2 + \sigma_v(|v|)$ , for all  $\xi \in \mathbb{X}$  and  $v \in \mathbb{R}^{n_v}$ , or
- 2) the functions  $f, g$  in (14) are globally Lipschitz uniformly in  $v$ , in the sense that there exist constants  $L_x, L_e, L_v \geq 0$  such that, for all  $(x_1, e_1, v_1), (x_2, e_2, v_2) \in \mathbb{R}^{n_x} \times \mathbb{E} \times \mathbb{R}^{n_v}$ , it holds that  $|f(x_1, e_1, v_1) - f(x_2, e_2, v_2)| \leq L_x |x_1 - x_2| + L_e |e_1 - e_2| + L_v |v_1 - v_2|$  and  $|g(x_1, e_1, v_1) - g(x_2, e_2, v_2)| \leq L_x |x_1 - x_2| + L_e |e_1 - e_2| + L_v |v_1 - v_2|$ , then there are no finite escape times during flow.

Lastly, we discuss the implications of  $\mathcal{S}_1 \supseteq \mathcal{S}_2$ . From the definitions of  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , we see that a term with  $|r(0, 0) - y(0, 0)|$  appears in the right-hand side of (26). This term does not affect the trajectories of the system when  $\ell_i^m(0, 0) = 0$  for all  $i, m \in \mathcal{V}$ . When  $\ell_i^m(0, 0) = 1$  for some  $i, m \in \mathcal{V}$ , however, it means that, shortly after initializing the system, a “packet” will be received by agent  $\mathcal{A}_m$  with an arbitrary update that may not reflect the actual output of the system. It is to be expected that the transient behavior of the system is affected by such a “disturbance”. In practice, however, this can be avoided by setting all  $\ell_i^m(0, 0) = 0$ . Moreover, in some cases, *i.e.*, when stabilizing the origin of the system, the functions  $\tilde{W}_i$  may be constructed such that  $\mathcal{S}_1 = \mathcal{S}_2 = \{\xi \in \mathbb{X} \mid x \in \mathcal{X}, e = 0, \eta = 0, r - y = 0\}$ , see, *e.g.*, [34]. In general, however, it may not be the case that  $r(t, j) - y(t, j) \rightarrow 0$  for  $t + j \rightarrow \infty$ , as the current framework allows, *e.g.*, limit cycles. Indeed, due to the dynamics of  $r$ ,  $r(t, j) - y(t, j) \rightarrow \mathbf{0}$  as  $t + j \rightarrow \infty$  if and only if  $f(x, 0, 0) = \mathbf{0}$  for all  $x \in \mathcal{X}$ .

## VI. CONCLUSIONS

In this work, we presented a systematic and general design framework for both *time-triggered* and (static and dynamic) *event-triggered* control strategies for a class of nonlinear MAS subject to disturbances. The proposed framework leads, under appropriate conditions, to MAS that are dissipative with respect to a desired supply rate, which can capture many relevant control problems, and that have strictly positive lower bounds on the inter-event times for each local triggering rule. Furthermore, robustness with respect to *non-uniform, possibly unknown* and *time-varying* delays is guaranteed by design. The broad applicability of the framework will be demonstrated in the companion paper [1] through several case studies and the modeling of the packet losses and denial-of-service in the destination protocols.

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## APPENDIX

*Proof of Theorem 1:* To verify the dissipativity properties of the MAS with respect to the supply rate  $\tilde{s}$  in (27), we aim to construct a storage function that satisfies (24).

For clarity of exposition, the proof is composed of six steps. At first, we present a candidate storage function and prove the bounds (24a). In the second, third and fourth step, we study the individual terms of this candidate storage function. Then, we show that the overall candidate storage function indeed satisfies (24). We complete the proof by showing that the system is persistently flowing.

**Step I. Candidate storage function.** Consider the following candidate storage function

$$U(\xi) = V(x) + \sum_{i \in \mathcal{V}} \eta_i + \sum_{i \in \mathcal{V}} \tilde{\gamma}_i(\tilde{\ell}(\ell_i)) \tilde{\phi}_{\tilde{\ell}(\ell_i), i}(\tau_i) \tilde{W}_i^2(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i), \quad (49)$$

for any  $\xi \in \mathbb{X}$ . Function  $\tilde{\ell} : \{0, 1\}^N \rightarrow \{0, 1\}$  is as in (37), and  $V$  comes from Condition 4. The function  $\phi_{l,i} : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ ,  $l \in \{0, 1\}$ ,  $i \in \mathcal{V}$ , is given by

$$\tilde{\phi}_{l,i}(\tau_i) := \begin{cases} \phi_{l,i}(\tau_i) & \text{when } \tau_i \leq \tau_{\text{miet}}^i \\ \phi_{l,i}(\tau_{\text{miet}}^i) & \text{when } \tau_i > \tau_{\text{miet}}^i \end{cases} \quad (50)$$

with  $\phi_{l,i}$  as in (40), and the function  $\tilde{W}_i : \mathbb{N} \times \{0, 1\}^N \times \mathbb{R}^{n_{y,i}} \times \mathbb{R}^{N_{y,i}} \times \mathbb{R}^{n_{y,i}} \rightarrow \mathbb{R}_{\geq 0}$  is defined as

$$\begin{aligned} \tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i) &:= \\ &\max \left\{ W_i(\kappa_i, e_i^{\text{out}} + s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)), \right. \\ &\left. \zeta_i \max_{\mathcal{S} \subset \mathcal{S}_i(\ell_i)} \left\{ W_i(\kappa_i, e_i^{\text{out}} + \sum_{l \in \mathcal{S}} Y_l s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)) \right\} \right\}, \quad (51) \end{aligned}$$

where  $Y_l := (\Gamma_l \otimes I_{n_{y,i}})$ ,  $\mathcal{S}_i(\ell_i) := \{m \in \mathcal{V}_i^{\text{out}} \mid \ell_m^i = 1\}$ , which is the set of agents that still have to receive the latest transmitted measurement by agent  $\mathcal{A}_i$ ,  $\zeta_i$  is as below (42), and

$$s_i(\ell_i, y_i, e_i^{\text{out}}, r_i) = \sum_{l \in \mathcal{S}_i(\ell_i)} Y_l (-e_i^{\text{out}} + \mathbf{1}_N \otimes (r_i - y_i)) \quad (52)$$

with the variables  $\tilde{\gamma}_i(l) \in \mathbb{R}_{\geq 0}$ ,  $l \in \{0, 1\}$ , as in (42).

Sandwich bounds (24a): It follows from (51) that

$$\tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i) \geq \zeta_i W_i(\kappa_i, e_i^{\text{out}}).$$

We derive from (28a) that  $\tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i) \geq \zeta_i \underline{\alpha}_{W,i} |e_i^{\text{out}}|$ . From (51) we also deduce that

$$\begin{aligned} \tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i) &\leq \max \left\{ \bar{\alpha}_{W,i} |e_i^{\text{out}} + s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)|, \right. \\ &\left. \bar{\alpha}_{W,i} \max_{\mathcal{S} \subset \mathcal{S}_i(\ell_i)} \left| e_i^{\text{out}} + \sum_{l \in \mathcal{S}} Y_l s_i(\ell_i, y_i, e_i^{\text{out}}, r_i) \right| \right\} \\ &\leq \bar{\alpha}_{W,i} (|e_i^{\text{out}}| + |\mathbf{1}_{N_i} \otimes (r_i - y_i)|) \\ &\leq \bar{\alpha}_{W,i} |e_i^{\text{out}}| + \bar{\alpha}_{W,i} \sqrt{N_i} |r_i - y_i|. \end{aligned}$$

We also have required bounds on  $V$  with respect to  $\mathcal{X}$  and  $\eta$  with respect to  $\{0\}$  for  $\xi \in \mathbb{X}$  as then  $\eta \geq 0$ .

**Step II. Properties of  $\tilde{W}_i$ .** Consider the following lemma.

**Lemma 1.** Consider  $\tilde{W}_i$  as in (51) with  $W_i$  satisfying Conditions 1 and 2. For each  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}_i^{\text{out}}$  and for all  $\kappa_i \in \mathbb{N}$ ,  $e_i^{\text{out}} \in \mathbb{R}^{N_{y,i}}$ ,  $r_i \in \mathbb{R}^{n_{y,i}}$  and  $0 < \lambda_i < 1$ , the function  $\tilde{W}_i$  satisfies for update events

$$\begin{aligned} \tilde{W}_i(\kappa_i, \ell_i - \Gamma_m \mathbf{1}_N, y_i, e_i^{\text{out}} + Y_m s_i(\ell_i, y_i, e_i^{\text{out}}, r_i), r_i) \\ \leq \tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i), \quad (53) \end{aligned}$$

and transmission events

$$\begin{aligned} \tilde{W}_i(\kappa_i + 1, Z_i(\kappa_i) \mathbf{1}_N, y_i, e_i^{\text{out}}, y_i) \\ \leq \lambda_i \tilde{W}_i(\kappa_i, \mathbf{0}_N, y_i, e_i^{\text{out}}, r_i) \quad (54) \end{aligned}$$

with  $Y_m = \Gamma_m \otimes I_{n_{y,i}}$  as before.

*Proof of Lemma 1: Inequality (53).* Observe that (53) with  $\tilde{W}_i$  as in (51), is equivalent to

$$\begin{aligned} &\max \left\{ \zeta_i \left[ \max_{\mathcal{S} \subset \mathcal{S}_i(\ell_i - \Gamma_m \mathbf{1}_N)} \left\{ W_i(\kappa_i, e_i^{\text{out}} + Y_m s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)) \right. \right. \right. \\ &+ \left. \left. \sum_{l \in \mathcal{S}} Y_l s_i(\ell_i - \Gamma_m \mathbf{1}_N, y_i, e_i^{\text{out}} + Y_m s_i(\ell_i, y_i, e_i^{\text{out}}, r_i), r_i) \right\} \right], \\ &W_i(\kappa_i, e_i^{\text{out}} + Y_m s_i(\ell_i, y_i, e_i^{\text{out}}, r_i) + s_i(\ell_i - \Gamma_m \mathbf{1}_N, y_i, e_i^{\text{out}} + \\ &\quad \left. Y_m s_i(\ell_i, y_i, e_i^{\text{out}}, r_i), r_i) \right\} \\ &\leq \max \left\{ \zeta_i \max_{\mathcal{S} \subset \mathcal{S}_i(\ell_i)} \left\{ W_i(\kappa_i, e_i^{\text{out}} + \sum_{l \in \mathcal{S}} Y_l s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)) \right\}, \right. \\ &\quad \left. W_i(\kappa_i, e_i^{\text{out}} + s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)) \right\} \quad (55) \end{aligned}$$

for each  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}_i^{\text{out}}$ . By means of (52) and given the fact that  $Y_l Y_m = 0$  for  $l \neq m$ , we find that

$$W_i(\kappa_i, e_i^{\text{out}} + Y_m s_i(\ell_i, y_i, e_i^{\text{out}}, r_i) + s_i(\ell_i - \Gamma_m \mathbf{1}_N, y_i, e_i^{\text{out}} + Y_m s_i(\ell_i, y_i, e_i^{\text{out}}, r_i), r_i)) = W_i(\kappa_i, e_i^{\text{out}} + s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)),$$

hence,

$$\begin{aligned} & \max_{S \subset \mathcal{S}_i(\ell_i - \Gamma_m \mathbf{1}_N)} \left\{ W_i(\kappa_i, e_i^{\text{out}} + Y_m s_i(\ell_i, y_i, e_i^{\text{out}}, r_i) \right. \\ & \left. + \sum_{l \in S} Y_l s_i(\ell_i - \Gamma_m \mathbf{1}_N, y_i, e_i^{\text{out}} + Y_m s_i(\ell_i, y_i, e_i^{\text{out}}, r_i), r_i) \right\} \\ &= \max_{S \subset \mathcal{S}_i(\ell_i - \Gamma_m \mathbf{1}_N)} \left\{ W_i(\kappa_i, e_i^{\text{out}} + \sum_{l \in S \cup \{m\}} Y_l s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)) \right\} \\ &\leq \max_{S \subset \mathcal{S}_i(\ell_i)} \left\{ W_i(\kappa_i, e_i^{\text{out}} + \sum_{l \in S} Y_l s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)) \right\}. \end{aligned}$$

By combining (55) with the above, we indeed get (53).

Inequality (54). The condition given in (54) is equivalent to

$$\begin{aligned} & \max \left\{ W_i(\kappa_i + 1, e_i^{\text{out}} + s_i(Z_i(\kappa_i) \mathbf{1}_N, y_i, e_i^{\text{out}}, y_i)), \right. \\ & \quad \zeta_i \max_{S \subset \mathcal{S}_i(Z_i(\kappa_i) \mathbf{1}_N)} \left\{ W_i(\kappa_i + 1, e_i^{\text{out}} + \right. \\ & \quad \left. \sum_{l \in S} Y_l s_i(Z_i(\kappa_i) \mathbf{1}_N, y_i, e_i^{\text{out}}, y_i)) \right\} \left. \right\} \leq \lambda_i W_i(\kappa_i, e_i^{\text{out}}) \quad (56) \end{aligned}$$

for each  $i \in \mathcal{V}$  as  $\ell_i = \mathbf{0}_N$  and thus  $S_i(\ell_i) = \emptyset$  in (51) and (52). By using the fact that, according to (52),  $s_i(Z_i(\kappa_i) \mathbf{1}_N, y_i, e_i^{\text{out}}, y_i) = -Z_i(\kappa_i) e_i^{\text{out}}$ , we find that (56) is equivalent to

$$\begin{aligned} & \max \left\{ \zeta_i \max_{S \subset \mathcal{S}_i(Z_i(\kappa_i) \mathbf{1}_N)} \left\{ W_i(\kappa_i + 1, e_i^{\text{out}} - \sum_{l \in S} Y_l e_i^{\text{out}}) \right\}, \right. \\ & \quad \left. W_i(\kappa_i + 1, h_i(\kappa_i, e_i^{\text{out}})) \right\} \leq \lambda_i W_i(\kappa_i, e_i^{\text{out}}). \quad (57) \end{aligned}$$

By combining the fact that

$$\begin{aligned} & \zeta_i \max_{S \subset \mathcal{S}_i(Z_i(\kappa_i) \mathbf{1}_N)} \left\{ W_i(\kappa_i + 1, e_i^{\text{out}} - \sum_{l \in S} Y_l e_i^{\text{out}}) \right\} \stackrel{(28a),(43)}{\leq} \\ & \lambda_i \alpha_{W,i} \max_{S \subset \mathcal{S}_i(Z_i(\kappa_i) \mathbf{1}_N)} |e_i^{\text{out}} - \sum_{l \in S} Y_l e_i^{\text{out}}| \leq \lambda_i W_i(\kappa_i, e_i^{\text{out}}) \end{aligned}$$

with (28b), we can obtain (57) and thus (54).  $\square$

**Lemma 2.** Consider  $\tilde{W}_i$  as in (51) with  $W_i$  satisfying Conditions 1 and 2 and the function  $H_i$  as in (36). Then for all  $\kappa_i \in \mathbb{N}$ ,  $\ell_i \in \{0, 1\}^N$ ,  $r_i \in \mathbb{R}^{n_{y,i}}$ ,  $x \in \mathbb{R}^{n_x}$ ,  $v \in \mathbb{R}^{n_v}$  and almost all  $e \in \mathbb{R}^{N n_y}$ , it holds that

$$\begin{aligned} & \left\langle \frac{\partial \tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i)}{\partial (e_i^{\text{out}}, y_i)}, (g_i(x, e, v), f_{y,i}(x, e_i^{\text{in}}, v)) \right\rangle \leq \\ & \sqrt{N_i} H_i(x, e_i^{\text{in}}, v, \ell_i) + \tilde{L}_i(\tilde{\ell}(\ell_i)) \tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i), \quad (58) \end{aligned}$$

where  $g_i(x, e, v) := (g_i^1(x, e, v), g_i^2(x, e, v), \dots, g_i^N(x, e, v))$  with  $g_i^m(x, e, v)$ ,  $i \in \mathcal{V}$ ,  $m \in \mathcal{V}_i^{\text{out}}$  as in (15).

*Proof of Lemma 2:* To prove Lemma 2, we consider the following two cases.

**Case 1:**  $\tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i) = W_i(\kappa_i, e_i^{\text{out}} + s_i(\ell_i, y_i, e_i^{\text{out}}, r_i))$ . For this case, we have that

$$\begin{aligned} & \left\langle \frac{\partial \tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i)}{\partial (e_i^{\text{out}}, y_i)}, (g_i, f_{y,i}) \right\rangle \\ &= \left\langle \frac{\partial W_i(\kappa_i, e_i^{\text{out}} + s_i(\ell_i, y_i, e_i^{\text{out}}, r_i))}{\partial (e_i^{\text{out}}, y_i)}, (g_i, f_{y,i}) \right\rangle \\ &\stackrel{(31)-(33),(52)}{\leq} c_i \sqrt{\sum_{m \in \mathcal{V}_i^{\text{out}} \setminus \mathcal{R}^*} |g_i^m|^2 + \sum_{m \in \mathcal{R}^*} |f_{y,i}|^2} \\ &\stackrel{(32),(33)}{\leq} \sqrt{N_i} \max \left\{ \max_{m \in \mathcal{V}_i^{\text{out}}} \{H_{e,i}(x, e_i^{\text{in}}, v) + L_{1,i} |e_i^m| \right. \\ & \quad \left. + L_{2,i} |e_i^i|\}, \tilde{\ell}(\ell_i) (H_{y,i}(x, e_i^{\text{in}}, v) + L_{2,i} |e_i^i|) \right\} \\ &\leq \sqrt{N_i} \max \{H_{e,i}(x, e_i^{\text{in}}, v), \tilde{\ell}(\ell_i) H_{y,i}(x, e_i^{\text{in}}, v)\} \\ & \quad + \sqrt{N_i} \max \{L_{1,i}, L_{2,i}\} |e_i^{\text{out}}| \\ &\stackrel{(28a),(36)}{\leq} \sqrt{N_i} H_i(x, e_i^{\text{in}}, v, \ell_i) + \max \{L_{1,i}, L_{2,i}\} \frac{\sqrt{N_i} \zeta_i}{\alpha_{W,i} \zeta_i} W_i(\kappa_i, e_i^{\text{out}}) \\ &\stackrel{(41),(43),(51)}{\leq} \sqrt{N_i} H_i(x, e_i^{\text{in}}, v, \ell_i) \\ & \quad + \tilde{L}_i(\tilde{\ell}(\ell_i)) \tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i), \quad (59) \end{aligned}$$

where we used the facts in the latter inequality that  $W_i(\kappa_i, e_i^{\text{out}}) = W_i(\kappa_i, \mathbf{0}_N, y_i, e_i^{\text{out}}, r_i)$  and thus that  $\zeta_i W_i(\kappa_i, e_i^{\text{out}}) \leq \zeta_i \max_{S \subset \mathcal{S}_i(\ell_i)} \{W_i(\kappa_i, e_i^{\text{out}} + \sum_{l \in S} Y_l s_i(\ell_i, y_i, e_i^{\text{out}}, r_i))\}$ . Moreover, recall that  $N_i$  is the cardinality of  $\mathcal{V}_i^{\text{out}}$ .

**Case 2:**  $\tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i) = \zeta_i \max_{S \subset \mathcal{S}_i(\ell_i)} \{W_i(\kappa_i, e_i^{\text{out}} + \sum_{l \in S} Y_l s_i(\ell_i, y_i, e_i^{\text{out}}, r_i))\}$  (and thus  $\ell(\ell_i) = 1$  (otherwise  $S_i(\ell_i) = \emptyset$  and thus  $s_i(\ell_i, y_i, e_i^{\text{out}}, r_i) = 0$ ). For this case, let us first define

$$S^* := \arg \max_{S \subset \mathcal{S}_i(\ell_i)} \left\{ W_i(\kappa_i, e_i^{\text{out}} + \sum_{l \in S} Y_l s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)) \right\}$$

such that

$$\begin{aligned} & \max_{S \subset \mathcal{S}_i(\ell_i)} \left\{ W_i(\kappa_i, e_i^{\text{out}} + \sum_{l \in S} Y_l s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)) \right\} = \\ & W_i(\kappa_i, e_i^{\text{out}} + \sum_{l \in S^*} Y_l s_i(\ell_i, y_i, e_i^{\text{out}}, r_i)). \quad (60) \end{aligned}$$

Using the definition above, we have that

$$\begin{aligned} & \left\langle \frac{\partial \tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i)}{\partial (e_i^{\text{out}}, y_i)}, (g_i, f_{y,i}) \right\rangle = \zeta_i \times \\ & \left\langle \frac{\partial W_i(\kappa_i, e_i^{\text{out}} + \sum_{l \in S^*} Y_l s_i(\ell_i, y_i, e_i^{\text{out}}, r_i))}{\partial (e_i^{\text{out}}, y_i)}, (g_i, f_{y,i}) \right\rangle \\ &\stackrel{(31)-(33),(52)}{\leq} \zeta_i c_i \sqrt{\sum_{m \in S^*} |g_i^m|^2 + |S^*| |f_{y,i}|^2} \\ &\stackrel{(32),(33)}{\leq} \zeta_i \sqrt{N_i} \max \left\{ \max_{m \in \mathcal{V}_i^{\text{out}}} \{H_{e,i}(x, e_i^{\text{in}}, v) + L_{1,i} |e_i^m| \right. \\ & \quad \left. + L_{2,i} |e_i^i|\}, \tilde{\ell}(\ell_i) (H_{y,i}(x, e_i^{\text{in}}, v) + L_{2,i} |e_i^i|) \right\} \stackrel{(51)}{\leq} \\ & \sqrt{N_i} H_i(x, e_i^{\text{in}}, v, \ell_i) + \tilde{L}_i(\tilde{\ell}(\ell_i)) \tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i), \quad (61) \end{aligned}$$

where we used  $\zeta_i < 1$  and the same arguments as before.

Based on (59) and (61), we can conclude that (58) is true, which completes the proof of Lemma 2.  $\square$

**Step III. Properties of  $V$ .** Consider the following lemma.

**Lemma 3.** Consider the system  $\mathcal{H}$  with data  $(C, F, D, G)$  as described in (14)-(21), the function  $V$  satisfying (35) and the

function  $H_i$  as in (36). Then for all  $x \in \mathbb{R}^{n_x}$ ,  $e \in \mathbb{R}^{Nn_y}$ ,  $r \in \mathbb{R}^{n_y}$ ,  $v \in \mathbb{R}^{n_v}$ ,  $\kappa \in \mathbb{N}^N$ ,  $\ell \in \{0, 1\}^N$ , it holds that

$$V^\circ(x; f(x, e, v)) \leq s(x, e, v) + \sum_{i=1}^N (-\varsigma_i(y_i, \hat{y}_i^{\text{in}}) - \mu_i N_i H_i^2(x, e_i^{\text{in}}, v, \ell_i) + \tilde{\gamma}_i^2(\tilde{\ell}(\ell_i)) \tilde{W}_i^2(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i)).$$

*Proof of Lemma 3:* To prove Lemma 3, based on (35), we only need to show that

$$\tilde{\gamma}_i^2(\tilde{\ell}(\ell_i)) \tilde{W}_i^2(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i) \geq \gamma_{\tilde{\ell}(\ell_i), i}^2 W_i^2(\kappa_i, e_i^{\text{out}}).$$

Recalling (42), we obtain for  $\tilde{\ell}(\ell_i) = 0$  (and thus  $\ell_i = \mathbf{0}_N$ ) and for  $\tilde{\ell}(\ell_i) = 1$  that, respectively,

$$\begin{aligned} \tilde{\gamma}_i^2(0) \tilde{W}_i^2(\kappa_i, \mathbf{0}_N, y_i, e_i^{\text{out}}, r_i) &\stackrel{(42)}{=} \gamma_{0, i}^2 \tilde{W}_i^2(\kappa_i, \mathbf{0}_N, y_i, e_i^{\text{out}}, r_i) \\ &\stackrel{(51)}{=} \gamma_{\tilde{\ell}(\ell_i), i}^2 W_i^2(\kappa_i, e_i^{\text{out}}) \end{aligned} \quad (62)$$

$$\begin{aligned} \tilde{\gamma}_i^2(1) \tilde{W}_i^2(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i) &\stackrel{(42)}{=} \gamma_{1, i}^2 \zeta_i^{-2} \tilde{W}_i^2(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i) \\ &\stackrel{(51)}{\geq} \gamma_{\tilde{\ell}(\ell_i), i}^2 W_i^2(\kappa_i, e_i^{\text{out}}) \end{aligned} \quad (63)$$

for all  $e_i^{\text{out}} \in \mathbb{R}^{Nn_{y,i}}$ ,  $r_i \in \mathbb{R}^{n_{y,i}}$  and all  $\kappa_i \in \mathbb{N}$ .  $\square$

**Step IV. Properties of  $\eta$**  As described in (8), the dynamics of  $\eta$  are governed by the functions  $\Psi_i$  and  $\eta_i^0$ ,  $i \in \mathcal{V}$ , which are given in (45) and (46), respectively. These functions are specifically designed such that the following lemma holds.

**Lemma 4.** For all  $y_i \in \mathbb{R}^{n_{y,i}}$ ,  $\hat{y}_i^{\text{in}} \in \mathbb{R}^{n_y}$ ,  $e_i^{\text{out}} \in \mathbb{R}^{Nn_{y,i}}$ ,  $\kappa_i \in \mathbb{N}$ ,  $\ell_i \in \{0, 1\}^N$ , and all  $\tau_i \in \mathbb{R}_{\geq 0}$ ,  $i \in \mathcal{V}$ , it holds that

$$\begin{aligned} \Psi_i(y_i, \hat{y}_i^{\text{in}}, \tau_i, \eta_i) &\leq \varsigma_i(y_i, \hat{y}_i^{\text{in}}) + (1 - \epsilon_i) \mu_i N_i \underline{H}_i^2(y_i, \hat{y}_i^{\text{in}}) \\ &\quad - (1 - \omega_i(\tau_i)) \gamma_{\tilde{\ell}(\ell_i), i}^2 \tilde{W}_i^2(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i) \times \\ &\quad \left( 2\tilde{L}_i(\tilde{\ell}(\ell_i)) \tilde{\phi}_{\tilde{\ell}(\ell_i), i}(\tau_i) + (\gamma_{\tilde{\ell}(\ell_i), i}^2 \tilde{\phi}_{\tilde{\ell}(\ell_i), i}^2(\tau_i) + 1) \right). \end{aligned} \quad (64)$$

Moreover, for all  $y_i \in \mathbb{R}^{n_{y,i}}$ ,  $\hat{y}_i^{\text{in}} \in \mathbb{R}^{Nn_y}$ ,  $e_i^{\text{out}} \in \mathbb{R}^{Nn_{y,i}}$ ,  $\kappa_i \in \mathbb{N}$ , and  $\tau_i \geq \tau_{\text{miet}}^i$ ,  $i \in \mathcal{V}$ , it holds that

$$\begin{aligned} \eta_i^0(y_i, \hat{y}_i^{\text{in}}, \tau_i, \eta_i) &\leq \gamma_{0, i} \tilde{\phi}_{0, i}(\tau_i) \tilde{W}_i^2(\kappa_i, \mathbf{0}_N, y_i, e_i^{\text{out}}, r_i) \\ &\quad - \tilde{\gamma}_i(1) \tilde{\phi}_{1, i}(0) \tilde{W}_i^2(\kappa_i + 1, Z_i(\kappa_i) \mathbf{1}_N, y_i, e_i^{\text{out}}, y_i). \end{aligned} \quad (65)$$

*Proof of Lemma 4:* To prove Lemma 4, let us first recall the fact that, due to Assumption 1, we have that  $\ell_i = \mathbf{0}_N$  when  $\tau_i \geq \tau_{\text{miet}}^i$  and that  $(1 - \omega_i(\tau_i)) = 0$  when  $\tau_i < \tau_{\text{miet}}^i$ . Consequently, we have that  $(1 - \omega(\tau_i)) \sum_{m \in \mathcal{V}_i^{\text{out}}} |e_i^m|^2 = (1 - \omega(\tau_i)) |e_i^{\text{out}}|^2 \stackrel{(28a)}{\geq} (1 - \omega(\tau_i)) \bar{\alpha}_{W_i}^{-1} W_i^2(\kappa_i, e_i^{\text{out}}) \stackrel{(51)}{=} (1 - \omega(\tau_i)) \bar{\alpha}_{W_i}^{-1} \tilde{W}_i^2(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i)$ , for all  $y_i \in \mathbb{R}^{n_{y,i}}$ ,  $\hat{y}_i^{\text{in}} \in \mathbb{R}^{n_y}$ ,  $e_i^{\text{out}} \in \mathbb{R}^{Nn_{y,i}}$ ,  $\kappa_i \in \mathbb{N}$ ,  $\ell_i \in \{0, 1\}^N$ , and all  $\tau_i \in \mathbb{R}_{\geq 0}$ ,  $i \in \mathcal{V}$ . By combining the latter with (45), we obtain that

$$\begin{aligned} \Psi_i(y_i, \hat{y}_i^{\text{in}}, e_i^{\text{out}}, \tau_i, \eta_i) &\leq \varsigma_i(y_i, \hat{y}_i^{\text{in}}) + (1 - \epsilon_i) \mu_i N_i \underline{H}_i^2(y_i, \hat{y}_i^{\text{in}}) \\ &\quad - (1 - \omega_i(\tau_i)) \bar{\alpha}_{W_i}^{-1} \tilde{\gamma}_i \tilde{W}_i^2(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i). \end{aligned} \quad (66)$$

From (47), (48) and (50), we obtain that for all  $\tau_i \in \mathbb{R}_{\geq 0}$

$$\begin{aligned} (1 - \omega_i(\tau_i)) \tilde{\gamma}_i &= (1 - \omega_i(\tau_i)) \bar{\alpha}_{W_i}^{-1} \tilde{\gamma}_i(\tilde{\ell}(\ell_i)) \times \\ &\quad \left( 2\tilde{L}_i(\tilde{\ell}(\ell_i)) \tilde{\phi}_{\tilde{\ell}(\ell_i), i}(\tau_i) + \tilde{\gamma}_i(\tilde{\ell}(\ell_i)) (1 + \tilde{\phi}_{\tilde{\ell}(\ell_i), i}^2(\tau_i)) \right), \end{aligned} \quad (67)$$

for all  $\tau_i \in \mathbb{R}_{\geq 0}$  (note that we again used the facts that  $\ell_i = \mathbf{0}_N$  when  $\tau_i \geq 0$  and that  $(1 - \omega_i(\tau_i)) = 0$  for  $\tau_i < \tau_{\text{miet}}^i$ ). Based on (66) and (67), we can conclude that (64) holds.

By combining the fact that  $\sum_{m \in \mathcal{V}_i^{\text{out}}} |e_i^m|^2 = |e_i^{\text{out}}|^2 \stackrel{(28a), (51)}{\leq} \bar{\alpha}_{W_i}^{-2} \tilde{W}_i^2(\kappa_i, \mathbf{0}_N, y_i, e_i^{\text{out}}, r_i)$  with (46) and (50), we obtain that

$$\begin{aligned} \eta_i^0(o_i) &\leq \left( \tilde{\gamma}_i(0) \tilde{\phi}_{0, i}(\tau_{\text{miet}}^i) - \tilde{\gamma}_i(1) \tilde{\phi}_{1, i}(0) \lambda_i^2 \right) \times \\ &\quad \tilde{W}_i^2(\kappa_i, \mathbf{0}_N, y_i, e_i^{\text{out}}, r_i). \end{aligned} \quad (68)$$

By means of (54), we can conclude that (65) holds.  $\square$

#### Step V. Validate conditions storage function

In this step, we verify that the function  $U$  as given in (49) is indeed a valid storage function for the supply rate  $s(x, e, v)$  as described in Definition 3.

**Flow Dynamics of  $U(\xi)$ :** By combining (8), (40), Lemmas 2 and 3, we obtain for all  $(\xi, v) \in \mathbb{X} \times \mathbb{R}^{n_v}$  and all  $f \in F(\xi, v)$

$$\begin{aligned} U^\circ(\xi; f) &\leq s(x, e, v) + \sum_{i=1}^N \left[ -\varsigma_i - \mu_i N_i H_i^2 \right. \\ &\quad \left. + \tilde{\gamma}_i^2(\tilde{\ell}(\ell_i)) \tilde{W}_i^2 + 2\tilde{\gamma}_i(\tilde{\ell}(\ell_i)) \tilde{\phi}_{\tilde{\ell}(\ell_i), i} \tilde{W}_i(\sqrt{N_i} H_i + \tilde{L}_i(\tilde{\ell}(\ell_i)) \tilde{W}_i) \right. \\ &\quad \left. - \omega(\tau_i) \tilde{\gamma}_i(\tilde{\ell}(\ell_i)) \tilde{W}_i^2 \times \right. \\ &\quad \left. (2\tilde{L}_i(\tilde{\ell}(\ell_i)) \tilde{\phi}_{\tilde{\ell}(\ell_i), i} + \tilde{\gamma}_i(\tilde{\ell}(\ell_i)) (\tilde{\phi}_{\tilde{\ell}(\ell_i), i}^2 + 1)) + \Psi_i - \sigma_i(\eta_i) \right], \end{aligned}$$

where we omitted the arguments of  $\tilde{W}_i(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i)$ ,  $\varsigma_i(y_i, \hat{y}_i^{\text{in}})$ ,  $H_i(x, e_i^{\text{in}}, v, \ell_i)$  and  $\Psi(o_i)$  with  $o_i = (y_i, \hat{y}_i^{\text{in}}, \tilde{y}_i, e_i^{\text{out}}, \tau_i, \eta_i)$ . By using  $2\tilde{\gamma}_i(\tilde{\ell}(\ell_i)) \tilde{\phi}_{\tilde{\ell}(\ell_i), i} \tilde{W}_i \sqrt{N_i} H_i \leq \frac{1}{\epsilon_i \mu_i} \tilde{\gamma}_i^2(\tilde{\ell}(\ell_i)) \tilde{\phi}_{\tilde{\ell}(\ell_i), i}^2 \tilde{W}_i^2 + \epsilon_i \mu_i N_i H_i^2$ , with  $\underline{H}_i$  as in (44), and by substituting (27) and (45), we obtain

$$U^\circ(\xi; f) \leq \tilde{s}(\xi, v), \quad (69)$$

for all  $f \in F(\xi, v)$ . Hence,  $U$  satisfies (24b).

**Jump Dynamics of  $U(\xi)$ :** For the jump dynamics, we need to consider the following two cases:

- Case 1 [Transmission event]: when  $\xi \in \mathcal{D}_i \wedge \sum_{m \in \mathcal{V}_i^{\text{out}}} \ell_i^m = 0$  for some  $i \in \mathcal{V}$  and  $\xi$  jumps according to  $\xi^+ = G_i^0$ , we have that

$$\begin{aligned} U(\xi^+) - U(\xi) &= \tilde{\gamma}_i(1) \tilde{\phi}_{1, i}(0) \tilde{W}_i^2(\kappa_i + 1, Z_i(\kappa_i) \mathbf{1}_N, y_i, e_i^{\text{out}}, y_i) \\ &\quad - \tilde{\gamma}_i(0) \tilde{\phi}_{0, i}(\tau_i) \tilde{W}_i^2(\kappa_i, \mathbf{0}_N, y_i, e_i^{\text{out}}, r_i) + \eta_i^0(o_i). \end{aligned} \quad (70)$$

By combining (38), (46), (54) and (65), we obtain that  $U(\xi^+) - U(\xi) \leq 0$  for all  $\xi \in \mathcal{D}_i$  with  $\sum_{m \in \mathcal{V}_i^{\text{out}}} \ell_i^m = 0$ , for some  $i \in \mathcal{V}$ .

- Case 2 [Update event]: when  $\xi \in \mathcal{D}_i \wedge \sum_{m \in \mathcal{V}_i^{\text{out}}} \ell_i^m \geq 1$  for some  $i \in \mathcal{V}$  and  $m \in \mathcal{V}_i^{\text{out}}$ , and  $\xi$  jumps according to  $\xi^+ = G_{i, m}^1(\xi)$ ,  $i \in \mathcal{V}$ , and  $\ell_i^m = 1$ , we have that

$$\begin{aligned} U(\xi^+) - U(\xi) &= \tilde{\gamma}_i(\tilde{\ell}(\ell_i - \Gamma_m \mathbf{1}_N)) \tilde{\phi}_{\tilde{\ell}(\ell_i - \Gamma_m \mathbf{1}_N), i}(\tau_i) \times \\ &\quad \tilde{W}_i^2(\kappa_i, \ell_i - \Gamma_m \mathbf{1}_N, y_i, e_i^{\text{out}} + Y_m s_i(\ell_i, y_i, e_i^{\text{out}}, r_i), r_i) \\ &\quad - \tilde{\gamma}_i(\tilde{\ell}(\ell_i)) \tilde{\phi}_{\tilde{\ell}(\ell_i), i}(\tau_i) \tilde{W}_i^2(\kappa_i, \ell_i, y_i, e_i^{\text{out}}, r_i). \end{aligned} \quad (71)$$

Based on (53), we can conclude that  $U(\xi^+) - U(\xi) \leq 0$  for all  $\xi \in \mathcal{D}_i$  with  $\ell_i^m = 1$  for  $m \in \mathcal{V}_i^{\text{out}}$ .

Based on the two cases, we conclude that  $U$  satisfies (24c).



**Step VI. Persistently flowing property.** We first prove that maximal solutions to (9) are complete. In general, when hybrid systems with continuous-time inputs are considered, *e.g.*, [56, Prop. 6] should be used to prove that maximal solutions are complete, as the results of, *e.g.*, [48, Prop. 6.10] are not applicable when inputs are present. However, in this specific setting, the external signal  $v$  enters  $F$  as in (14) via the dynamics of  $x$  and  $e$  only, as evident by the expression given in (14). Since the flow and jump maps  $\mathcal{C}$  and  $\mathcal{D}$  do not involve  $v$  and since they do not restrict  $x$  or  $e$  to any particular subset of  $\mathbb{R}^{n_x}$  or  $\mathbb{E}$ , respectively, tangent cone arguments as in [48, Prop. 6.10] can still be used in this specific context. To do so, we start by showing that for any  $\xi \in \mathcal{C} \setminus \mathcal{D}$ , there exists a neighborhood  $S$  of  $\xi$  such that for every  $\Xi \in S \cap \mathcal{C}$ ,  $F(\Xi, v) \cap T_{\mathcal{C}}(\Xi) \neq \emptyset$ , where  $T_{\mathcal{C}}(\Xi)$  is the tangent cone<sup>3</sup> to  $\mathcal{C}$  at  $\Xi$ . Let  $\xi \in \mathcal{C} \setminus \mathcal{D}$  and  $v \in \mathbb{R}^{n_v}$ . Since  $\mathcal{D}$  is closed, there exists a neighborhood of  $\xi$ , which we denote  $S$ , such that  $S \cap \mathcal{D} = \emptyset$ . Let  $\Xi = (x, e, \tau, \kappa, r, \ell, \eta) \in S \cap \mathcal{C}$ ; note that  $S \cap \mathcal{C} \neq \emptyset$  as  $\xi \in S \cap \mathcal{C}$ . Since  $\Xi \in S$ ,  $\Xi \notin \mathcal{D}$  and thus  $\ell_i^m = 0$  for all  $(i, m) \in \mathcal{V}$  in view of (21). We write  $\tilde{\Xi} = (\dot{x}, \dot{e}, \dot{\tau}, \dot{\kappa}, \dot{r}, \dot{\ell}, \dot{\eta}) \in F(\Xi, v)$  with some abuse of notation for the sake of convenience. For  $\tilde{\Xi}$  to be in  $T_{\mathcal{C}}(\Xi)$ , we need  $\dot{x} \in \mathbb{R}^{n_x}$ ,  $\dot{e} \in \mathbb{E}$ ,  $\dot{\kappa} = \mathbf{0}_N$ ,  $\dot{r} \in \mathbb{R}^{n_r}$ ,  $\dot{\ell} = \mathbf{0}_{N^2}$ , which hold in view of (14), (15) and (16). We also need  $\dot{\tau}_i \in \mathbb{R}$  when  $\tau_i > 0$  and  $\dot{\tau}_i \geq 0$  when  $\tau_i = 0$ , for all  $i \in \{1, \dots, N\}$ , which is also true in view of (14). The last conditions to verify for  $\tilde{\Xi}$  to be in  $T_{\mathcal{C}}(\Xi)$  is that  $\dot{\eta}_i \in \mathbb{R}$  when  $\eta_i > 0$ , which holds from (14) and (45), and that  $\dot{\eta}_i \geq 0$  when  $\eta_i = 0$ . To show this last condition, we notice that necessarily  $\tau_i < d_i$ , otherwise we would have  $\Xi \in \mathcal{D}_i \subset \mathcal{D}$ , which is excluded as  $\Xi \in S$ . Moreover,  $d_i < \tau_{miet}^i$  by design. Consequently,  $\omega_i(\tau_i) = 1$  in (47), and  $\dot{\eta}_i \geq 0$  when  $\eta_i = 0$  in view of (14) and (45). We have proved that  $\tilde{\Xi} \in T_{\mathcal{C}}(\Xi)$ , from which we deduce that  $F(\Xi, v) \subset T_{\mathcal{C}}(\Xi)$ . Hence, (VC) in [48, Prop. 6.10] is satisfied when  $\xi \in \mathcal{C} \setminus \mathcal{D}$ . As a result, there exists a non-trivial solution from any initial condition  $\xi$  in  $\mathcal{C} \cup \mathcal{D}$ . On the other hand, observe that  $G(\mathcal{D}) \subset \mathcal{C} \cup \mathcal{D}$ . Indeed, for all  $\xi \in \mathcal{D}$  and  $\xi^+ = (x^+, e^+, \tau^+, \kappa^+, r^+, \ell^+, \eta^+) \in G(\xi)$ , it holds that  $\kappa^+ \in \mathbb{N}^N$ ,  $\ell^+ \in \{0, 1\}^{N^2}$ ,  $\tau_i^+ = \tau_i \geq 0$  or  $\tau_i^+ = 0$  and  $\eta_i^+ = \eta_i \geq 0$  or  $\eta_i^+ = \eta_i^0 > 0$ , and thus  $\xi^+ \in \mathbb{X} = \mathcal{C}$ . Hence, case (c) in [48, Prop. 6.10] does not occur. Due to the hypothesis that there are no finite escape times during flow, also case (b) in [48, Prop. 6.10] is ruled out. As a consequence, all maximal solutions are complete. In addition, since  $d_i > 0$  and the number of agents that are receiving and transmitting information is finite, it follows that the system is persistently flowing, which completes the proof of Theorem 1.

*Proof of Proposition 1* We first consider item 1) of Proposition 1. For all  $\xi \in \mathcal{C}$ ,  $v \in \mathbb{R}^{n_v}$  and  $f \in F(\xi, v)$ , in view of (69) and Step I of the proof of Theorem 1,  $U^\circ(\xi; f) \leq \tilde{s}(\xi, v) \leq c_V V(x) + c_W |e|^2 + \sigma_v(|v|) - \varphi(\eta) \leq c_U U(\xi) + \sigma_v(|v|)$  for some  $c_U \geq 0$ . As a consequence, since  $U$  is locally Lipschitz, it holds that (see p.99 in [64]), for any solution pair  $(\xi, v)$  to  $\mathcal{H}$ , any  $i \in \mathbb{N}_{>0}$  and almost  $t \in [t_i, t_{i+1}]$  where

<sup>3</sup>The tangent cone  $T_S(x)$  to a set  $S \subset \mathbb{R}^n$  at a point  $x \in \mathbb{R}^n$  is the set of all vectors  $\omega \in \mathbb{R}^n$  for which there exist  $x_i \in S$ ,  $\tau_i > 0$ ,  $i \in \mathbb{N}$ , with  $x_i \rightarrow x$ ,  $\tau \rightarrow 0$  as  $i \rightarrow \infty$  such that  $\omega = \lim_{i \rightarrow \infty} (x_i - x)/\tau_i$  (see [48, Def. 5.12]).

$(t_i, i), (t_{i+1}, i) \in \text{dom } \xi$  and  $t_i \leq t_{i+1}$ ,

$$U(\xi(t, i)) \leq e^{c_U(t-t_i)} U(\xi(t_i, i)) + \int_{t_i}^t e^{c_U(s-t_i)} \sigma_v(|v(s)|) ds.$$

Since  $\mathcal{X}$  is compact,  $U$  is proper (*i.e.*,  $U$  bounded implies  $\xi$  is bounded), hence, we derive from the above expression that  $\xi$  cannot explode in finite time during flow.

Consider now item 2) of Proposition 1. In this case, finite escape time cannot occur because of the  $(x, e, \tau, \kappa, r, \ell)$ -component of the solutions, as the corresponding part of  $F$  is single-valued and globally Lipschitz, see [65, Chapter 1.1]. The only possible issue may come from the  $\eta$ -component. However, the  $\eta$ -component cannot explode in finite time neither during flow in view of (45) as all  $\eta_i$  are constrained to take non-negative values and  $\varphi_i$  also takes non-negative values, for all  $i \in \{1, \dots, N\}$ . Consequently, there are no finite escape times during flow.



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