Time- and event-triggered communication for multi-agent systems - Part II: Digital implementation and resilience

Koen Scheres, Victor Dolk, Michelle Chong, Romain Postoyan, Maurice Heemels

Time- and Event-triggered Communication for Multi-agent Systems – Part II: Digital Implementation and Resilience

K.J.A. Scheres, *Student Member, IEEE*, V.S. Dolk, M.S. Chong, R. Postoyan, *Senior Member, IEEE*, and W.P.M.H. Heemels, *Fellow, IEEE*

***Abstract*— We consider the design of event-triggered distributed controllers for multi-agent systems that are digitally implemented on local computation platforms and communicate over a packet-based network. Each agent is equipped with a local triggering mechanism that is only evaluated at the local sampling instants, thereby taking a periodic event-triggered approach in which the sampling intervals are allowed to vary (jitter). Moreover, the locally triggered transmissions are subject to unknown, bounded delays, and a destination protocol is locally implemented to only send the packet to a selection of the neighboring agents at each triggering instant. Building upon the framework of Part I, we present an emulation-based design of the local periodic event-triggering rules, including the maximum allowable sampling period (MASP), so that, under appropriate conditions, a general dissipativity property holds for the overall system. Interestingly, the presented digital implementation requires only minor modifications to the conditions presented in Part I. Additionally, we show how to exploit the destination protocols to ensure resilience to information loss issues such as packet losses and denial-of-service. We conclude this paper with case studies on the consensus of single integrator agents and a nonlinear stabilization problem.**

I. Introduction

The distributed control of multi-agent systems (MAS) that communicate over a packet-based network is a challenging pursuit due to the intermittent and asynchronous availability of information. In Part I [1], a unifying framework was developed to guarantee stability, robustness and performance of the MAS with both time- and event-triggered communication in the presence of network-induced artifacts such as time-varying non-uniform transmission delays as well as asynchronous transmission instants. Importantly, the designed local triggering conditions in Part I [1] are *continuously* evaluated. The objective of this Part II is to address the questions of both the digital implementation of the triggering laws developed in [1] and of their resilience to information loss, such as packet loss and denial-of-service, which may arise when communicating over a packet-based network.

We thus study the distributed control of MAS, where each agent's triggering mechanism is implemented on a local digital platform. Consequently, the sampling behavior of the digital device needs to be taken into account, primarily when fast sampling cannot be achieved, such as in low-power electronics or when evaluating the local triggering conditions at high frequency is too costly. In this case, we talk of periodic event-triggered control (PETC) [2], [3]. The challenge is then, next to the design of the control laws, to provide conditions on (i) the triggering rule, and (ii) the sequence of (local) sampling instants at which the designed triggering rule has to be evaluated, while guaranteeing the desired closed-loop system property. We concentrate on event-triggered mechanisms only in this paper, as we saw in Part I that we recover the time-triggered mechanism when no information about the other agents is exploited in the triggering rules.

Extending the framework presented in Part I [1] to PETC is far from trivial, as there exists an intricate interplay between the sequence of sampling instants of the local devices and the triggering conditions. In particular, when the triggering rule is formulated based on a triggering function becoming negative, and the triggering function is positive at the *current* sampling instance, it is unknown a priori if it *will stay* positive until the next sampling instant. Indeed, if information is received from other agents, the behavior (control input) of the local system can "instantly" change. As the times at which information is received are a priori unknown, estimating the "remaining" time until the triggering condition is violated is not trivial. Guaranteeing that the triggering condition will not be violated *before* the next sampling instant is therefore often impossible. This becomes even more involved when (a priori) unknown disturbances are present.

In literature, many works on PETC of MAS only consider linear systems, see, *e.g.*, [4]–[7]. A notable exception is [8], where a PETC algorithm for the exponential synchronization of agents with nonlinear dynamics is presented. In [8], however, the presented PETC scheme requires a specific structure in the dynamics, whereby each individual state of each agent can be controlled separately. In many practical setups this may not be feasible, *i.e.*, due to inertia when controlling the position of a mass. The recent work [9] provides important advance-

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ments on the nonlinear case with (large) unknown transmission delays while taking sampling into account, however, only the point stabilization of a single system is considered.

On the other hand, the use of (public) packet-based networks makes the MAS susceptible to information loss due to, *e.g.*, *packet loss* and *denial-of-service*. In this two-part paper, we are able to consider packet loss and denial-of-service leveraging on the generality of the frameworks of both Part I [1] and this part. These aspects are captured by the *destination protocols* that determine which other agents receive the local output measurement of each agent at each transmission instant, meaning that resilience to packet loss and denial-of-service can be considered "separately" from the design of the (periodic) triggering mechanism.

In summary, with respect to the results presented in Part I [1], we address in this work specifically the following aspects:

1) implementation on individual digital platforms, each with time-varying and asynchronous sampling intervals,
2) the exploitation of the destination protocols for resilience of the communication channel to packet loss and denial-of-service.

We do so on top of the network-induced imperfections considered in Part I [1] such as time-varying and unknown transmission delays, asynchronous transmission instants and limited communication bandwidth in the sense of communication only occurring at discrete time instances. Advantageously, the resulting periodic event-triggering mechanisms have non-zero minimal inter-event times by design and do not require clock synchronization nor acknowledgment protocols. This work generalizes the results of our preliminary conference version [10]. Compared to [10], where only sampled-data protocols where considered, we include general destination protocols, and show that these can be exploited to ensure that the MAS is resilient to packet loss and denial-of-service. Finally, we also demonstrate the generality of the framework presented in this two-part paper with two case studies on: (i) the consensus of single integrator MAS (with accompanying numerical simulations) and (ii) the stability of a coupled nonlinear system.

Details on the notation can be found in the companion paper [1, Section II-A], with the addendum that the cardinality of a finite set $\mathcal{S}$ is denoted $|\mathcal{S}|$.

II. System setup with digital implementation

A. Multi-agent systems

The details regarding the exact model can be found in [1]. For the sake of convenience, we shortly recap the main variables in Table I.

B. Digital platform

The triggering mechanism of each agent is implemented on the local digital platform, which has its own sampling times. The sequence of sampling times of agent $\mathcal{A}_i$ is denoted $\{s_n^i\}_{n\in\mathbb{N}}$, where s_n^i denotes the n^{th} local sampling instant of agent $\mathcal{A}_i$. Transmissions generated by $\mathcal{A}_i$ occur on a subset of the sampling instants, *i.e.*,

$$\{t_k^i\}_{k\in\mathbb{N}} \subseteq \{s_n^i\}_{n\in\mathbb{N}}. \tag{1}$$

TABLE I
RECAP OF VARIABLES AND FUNCTIONS USED IN THE MODEL.

Variable	*Description*
x_i	State of agent $\mathcal{A}_i$
y_i	Output of agent $\mathcal{A}_i$
t_k^i	k-th transmission of agent $\mathcal{A}_i$
$\Delta_k^{i,m}$	Delay of k-th transmission from agent $\mathcal{A}_i$ to agent $\mathcal{A}_m$
τ_i	Time elapsed since last transmission
κ_i	Transmission counter
r_i	Memory variable that stores y_i at times t_k^i
ℓ_i^m	Indicates whether $\mathcal{A}_m$ has received the last update from $\mathcal{A}_i$
η_i	Auxiliary variable for dynamic triggering
$\widehat{y}_i^m$	Estimate of output y_i at agent $\mathcal{A}_m$
e_i^m	Network induced error; $\widehat{y}_i^m - y_i$
e_i^{out} ($\widehat{y}_i^{\text{out}}$)	Collection of all nontrivial e_i^m (resp. $\widehat{y}_i^m$) with $m \in \mathcal{V}_i^{\text{out}}$
e_i^{in} ($\widehat{y}_i^{\text{in}}$)	Collection of all nontrivial e_m^i (resp. $\widehat{y}_m^i$) with $m \in \mathcal{V}_i^{\text{in}}$

We consider the general setting where the inter-sampling times satisfy, as in [3], *i.e.*,

$$0 < d_i \leqslant s_{n+1}^i - s_n^i \leqslant \tau_{\text{masp}}^i, \tag{2}$$

where d_i is an arbitrarily small but positive constant and τ_{masp}^i denotes the *maximum allowable sampling period* (MASP) for agent $\mathcal{A}_i$, $i \in \mathcal{V}$. The sampling times $\{s_n^i\}_{n\in\mathbb{N}}$ and $\{s_n^m\}_{n\in\mathbb{N}}$ of agents $\mathcal{A}_i$ and $\mathcal{A}_m$, respectively, are a priori not related for $i \neq m$. In other words, all agents operate *independently* and *asynchronously*.

Due to the agents operating asynchronously, the arrival times $t_k^i + \Delta_k^{i,m}$, $k \in \mathbb{N}$, of new information at agent $\mathcal{A}_m$ from agent $\mathcal{A}_i$ may not coincide with the sampling times $\{s_n^m\}_{n\in\mathbb{N}}$ of agent $\mathcal{A}_m$, hence information may be received in between consecutive sampling times of agent $\mathcal{A}_m$. However, the sampling-induced delay (the time between the arrival of information from agent $\mathcal{A}_i$ and the next sampling instant of agent $\mathcal{A}_m$) can be included in the total delay denoted $\overline{\Delta}_k^{i,m}$. Therefore, the total delay $\overline{\Delta}_k^{i,m}$ is equal to the combined communication delay $\Delta_k^{i,m}$, which was introduced already in Part I [1], and sampling-induced delay. Through this setup, we obtain

$$\{t_k^i + \overline{\Delta}_k^{i,m}\}_{k\in\mathbb{N}} \subseteq \{s_n^m\}_{n\in\mathbb{N}} \tag{3}$$

for all $m \in \mathcal{V}$ and $i \in \mathcal{V}_m^{\text{out}}$.

We adopt the following assumption on the total delays $\overline{\Delta}_k^{i,m}$, $k \in \mathbb{N}$.

Assumption 1-II. *For each agent $i \in \mathcal{V}$, there is a maximum allowable delay $\tau_{\text{mad}}^i \geqslant 0$ such that the delays are bounded according to $0 \leqslant \overline{\Delta}_k^{i,m} \leqslant \tau_{\text{mad}}^i \leqslant t_{k+1}^i - t_k^i$ for all $m \in \mathcal{V}_i^{out}$ and all $k \in \mathbb{N}$.*

Assumption 1-II is a "small delay" condition, which also implies that packets sent from $\mathcal{A}_i$ to $\mathcal{A}_m$, $m \in \mathcal{V}_i^{\text{out}}$, are received in the same order that they are transmitted. Note that this condition, which also includes the delays caused by sampling, essentially replaces [1, Assumption 1]. As the time instants t_k^i determine the *transmission* instants, the above condition does not rule out packet loss or denial-of-service, as the transmission instants do not necessarily coincide with the

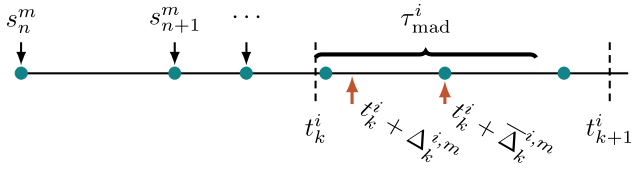

Fig. 1. Visualization of the MAD, networked delays captured by $\boldsymbol{\Delta_k^{i,m}}$ and total delays $\boldsymbol{\overline{\Delta}_k^{i,m}}$ (including the sampling-induced delays).

receival instants. Since the sampling-induced delays are never larger than the local MASP τ_{masp}^m at agent m (and can be equal to it), we have that

$$\tau_{\text{mad}}^i \geqslant \tau_{\text{masp}}^m + \Delta_k^{i,m} \text{ for all } i \in \mathcal{V}, m \in \mathcal{V}_i^{\text{out}}, k \in \mathbb{N}. \quad (4)$$

Through this particular setup, the solutions to the hybrid system that satisfy (3) become a particular subset of *all* solutions of the hybrid model presented in [1]. Hence, we can consider the same (hybrid) model. Essentially, the *total* delay $\overline{\Delta}_k^{i,m}$ "replaces" the delay $\Delta_k^{i,m}$ of [1]. By ensuring that the sampling times of each local platform are sufficiently small, the above inequalities can always be guaranteed, see also Fig. 1. Essentially, the maximum delay that the network is allowed to have will always be lower-bounded by

$$\underline{\Delta} := \min_{i \in \mathcal{N}} \left\{ \min_{m \in \mathcal{V}_i^{\text{in}}} \tau_{\text{mad}}^m - \tau_{\text{masp}}^i \right\}.$$

As long as it is ensured that $\underline{\Delta} > 0$ (which is always possible by ensuring that τ_{masp}^i is sufficiently small) and the delays in the network are less than $\underline{\Delta}$, this particular setup fits the framework presented in [1] in terms of the (total) delays τ_{mad}^i.

C. Triggering rule

Our goal is to employ dynamic event triggering rules, which rely on locally available information, namely output measurements. Due to this information only being available at the sampling instants $\{s_n^i\}_{n \in \mathbb{N}}$, the design of, *e.g.*, [11] cannot be directly used. Instead, we consider an event-triggering mechanism (ETM) where the triggering times satisfy

$$\begin{aligned} t_{k+1}^i := \inf\{t \geqslant t_k^i + \tau_{\text{dmiet}}^i \mid \\ \eta_i(t) + \nu_i(y_i(t), \widehat{y}_i^{\text{out}}(t), \tau_i(t)) \leqslant 0, t \in \{s_n^i\}_{n \in \mathbb{N}}\}, \end{aligned} \quad (5)$$

for $i \in \mathcal{V}$, $k \in \mathbb{N}$, with $t_0^i = 0$ and where $\tau_{\text{dmiet}}^i > 0$ denotes the (enforced lower bound on the) *digital minimum inter-event time* (DMIET) of agent $\mathcal{A}_i$, $\eta_i \in \mathbb{R}_{\geqslant 0}$ is the auxiliary variable mentioned earlier, $\widehat{y}_i^{\text{out}} := (\widehat{y}_i^1, \widehat{y}_i^2, \ldots, \widehat{y}_i^N)$ is the vector of estimates of the output y_i at the agents $\mathcal{A}_m$, $m \in \mathcal{V}_i^{\text{out}}$. The function $\nu_i : \mathbb{R}^{n_{y,i}} \times \mathbb{R}^{Nn_{y,i}} \times \mathbb{R}_{\geqslant 0} \rightrightarrows \mathbb{R}_{\leqslant 0}$ is to be designed. Since ν_i will be a set-valued map, the inequality in (5) can be interpreted as "should hold for all elements in $\nu_i(y_i(t), \widehat{y}_i^{\text{out}}(t), \tau_i(t))$."

The ETM (5) satisfies the constraints that arise from the usage of a digital platform, as the trigger condition in (5) only has to be evaluated at the local sampling times s_n^i, $n \in \mathbb{N}$. The triggering variable η_i generated locally by agent $\mathcal{A}_i$, $i \in \mathcal{V}$, evolves according to

$$\dot{\eta}_i = \Psi_i(\widehat{y}_i^{\text{in}}) - \varphi_i(\eta_i), \quad (6a)$$

$$\eta_i(t^+) \in \begin{cases} \{\eta_i + \varrho_i(y_i, \widehat{y}_i^{\text{out}})\}, \text{ for all } t \in \{t_k^i\}_{k \in \mathbb{N}}, \\ \{\eta_i + \nu_i(y_i, \widehat{y}_i^{\text{out}}, \tau_i)\}, \\ \qquad \text{for all } t \in \{s_n^i\}_{n \in \mathbb{N}} \setminus \{t_k^i\}_{k \in \mathbb{N}}, \end{cases} \quad (6b)$$

where the functions $\Psi_i : \mathbb{R}^{n_y} \to \mathbb{R}$, $\varrho_i : \mathbb{R}^{n_{y,i}} \times \mathbb{R}^{Nn_{y,i}} \to \mathbb{R}_{\geqslant 0}$, $\varphi_i \in \mathcal{K}_\infty$ and the constant $\tau_{\text{miet}}^i \in \mathbb{R}_{>0}$ are designed in Section III-B.

Remark 1. *It might seem unnatural that agent $\mathcal{A}_i$ has to know the estimates $\widehat{y}_i^{out}$ due to the presence of the unknown and time-varying delays. However, as also explained in [1, Section V-C], there are important cases where this information is naturally available,* e.g.*, when a zero-order-hold (ZOH) is used as a holding device, or when the (total) delays are known. In other cases, a robust set-valued estimate can be constructed in line with [1, Assumption 2], which can be used in* (1) *and* (6) mutatis mutandis.

Remark 2. *In* (6a)*, a continuous-time differential equation is used. When an (analytic) closed-form expression is available for the solutions of the pairs $(\widehat{y}_i^{in}, \eta_i)$, the solutions can be* a posteriori *computed at the sampling times. An (important) setting, where this is possible, is when a ZOH is used as a holding device,* i.e.*, when the 'external' variable $\widehat{y}_i^{in}$ is constant in between consecutive sampling times. In that case, exact discretization or numerical integration can be used to compute η_i. For instance, if $\varphi_i(\eta_i) = \alpha_i \eta_i$ with $\alpha_i \neq 0$ a constant, we obtain the exact discretization $\eta_i(s_{n+1}^i) = e^{-\alpha_i(s_{n+1}^i - s_n^i)}\eta_i((s_n^i)^+) + \alpha_i^{-1}[1 - e^{-\alpha_i(s_{n+1}^i - s_n^i)}]\Psi_i(\widehat{y}_i^{in}((s_n^i)^+))$. Hence, exact solutions to the differential equation can be obtained on a digital platform. When it is not possible to find an analytic solution to the differential equations, numerical integration techniques can be used to obtain an accurate value for η_i. We consider the dynamics of η_i as presented in* (6) *to facilitate the modeling and stability analysis later on.*

D. Objective

Given the descriptions above, the problem considered in this paper can now be stated informally as follows. Consider a collection of maximum allowable delays τ_{mad}^i, $i \in \mathcal{V}$, satisfying Assumption 1-II. Our objective is to propose design conditions for the time constants $\tau_{\text{dmiet}}^i (\geqslant \tau_{\text{mad}}^i)$, the functions z_i, Ψ_i, φ_i, ϱ_i and ν_i, $i \in \mathcal{V}$, as in (5) and (6), such that the resulting system has the desired (and to be specified) closed-loop stability, performance and robustness properties formalized in terms of suitable dissipativity properties.

III. Modified Design Conditions and Main Result

A. Modified Design Conditions

To ensure desirable performance and stability properties, several conditions are needed, which are very similar to the conditions presented in [1]. Therefore, we only highlight the key differences in the following. In particular, [1, Conditions 1-3 and 5] can be used without modifications. As the local

output y_i is not available at all times, but only at the sampling times $\{s_n^i\}_{n\in\mathbb{N}}$, the functions ς_i and $\underline{H}_i$ in [1, Conditions 4 and 6] are only allowed to depend on $\widehat{y}_i^{\text{in}}$. Thereto, we replace [1, (35)] by

$$V^\circ(x;f(x,e,v)) \leqslant s(x,e,v) + \sum_{i\in\mathcal{V}} \Big(-\varsigma_i(\widehat{y}_i^{\text{in}}) - \mu_i N_i H_i^2(x,e_i^{\text{in}},v,\ell_i) + \gamma^2_{\ell(\ell_i),i} W_i^2(\kappa_i, e_i^{\text{out}}) \Big), \quad \text{(35-II)}$$

and, similarly, we replace [1, (44)] by

$$\underline{H}_i(\widehat{y}_i^{\text{in}}) \leqslant H_i(x,e_i^{\text{in}},v,\ell_i). \quad \text{(44-II)}$$

Lastly, in order to obtain the DMIET, we introduce the following condition.

Condition 5-II. *Select $\tau^i_{\text{miet}} > 0$ and $\tau^i_{mad} > 0$, $i \in \mathcal{V}$, with $\tau^i_{\text{miet}} \geqslant \tau^i_{mad} + \tau^i_{\text{masp}}$ such that [1, Condition 5] holds. Define $\tau^i_{\text{dmiet}} > 0$ as*

$$\tau^i_{\text{dmiet}} := \tau^i_{\text{miet}} - \tau^i_{\text{masp}}. \quad (7) \qquad \square$$

Condition 5-II can always be ensured, as long as sufficiently fast sampling is available (*i.e.*, τ^i_{masp} sufficiently small). In practice, based on the constants γ_i, intuitive $(\tau^i_{\text{miet}}, \tau^i_{\text{mad}})$-curves can be generated by selecting appropriate values for λ_i, $\phi_{0,i}(0)$ and $\phi_{1,i}(0)$.

Condition 5-II is similar to [1, Condition 5] when the effect of sampling is not considered. Indeed, in the continuous-time case in [1], *i.e.*, when τ^i_{masp} approaches zero, $\tau^i_{\text{dmiet}} = \tau^i_{\text{miet}}$. Hence, if faster sampling is used, the continuous-time ETC behavior is recovered by the proposed "sampled" setup.

B. Event-triggering mechanism design

For all $\xi \in \mathbb{X}$, the function Ψ_i in (6a) is given by

$$\Psi_i(\widehat{y}_i^{\text{in}}) := \varsigma_i(\widehat{y}_i^{\text{in}}) + (1-\epsilon_i)\mu_i N_i \underline{H}_i^2(\widehat{y}_i^{\text{in}}), \quad (8)$$

where ς_i and $\underline{H}_i$ come from [1, Condition 4 and 6], respectively, where [1, (35) and (44)] are substituted by (35-II) and (44-II), respectively. The function ϱ_i is given by, for any $y_i \in \mathbb{R}^{n_{y,i}}$ and $\widehat{y}_i^{\text{out}} \in \mathbb{R}^{N n_{y,i}}$

$$\varrho_i(y_i, \widehat{y}_i^{\text{out}}) := \varepsilon_\varrho |e_i^{\text{out}}|^2 \quad (9)$$

with $\varepsilon_\varrho := \underline{\alpha}^2_{W,i}\left(\tilde{\gamma}_i(0)\phi_{0,i}(\tau^i_{\text{dmiet}} + \sigma_i) - \tilde{\gamma}_i(1)\phi_{1,i}(0)\lambda_i^2\right)$, $\phi_{l,i}$, $l \in \{0,1\}$, as in [1, (40)], $\tilde{\gamma}_i : \{0,1\} \to \mathbb{R}$ is as in [1, (42)] and $\alpha_{W,i}$ from [1, Condition 1]. Recall that σ_i is the time elapsed since the last sampling instant of agent $\mathcal{A}_i$. Finally, the function $\nu_i : \mathbb{R}^{n_y} \times \mathbb{R}_{\geqslant 0} \rightrightarrows \mathbb{R}_{\leqslant 0}$ is defined as

$$\nu_i(y_i, \widehat{y}_i^{\text{out}}, \tau_i) := (1-\omega_i(\tau_i))\tilde{\gamma}_i(0)\varepsilon_\nu |e_i^{\text{out}}|^2, \quad (10)$$

where $\varepsilon_\nu := -\underline{\alpha}^2_{W,i}\left(\phi_{0,i}(\tau^i_{\text{dmiet}}) - \phi_{0,i}(\tau^i_{\text{dmiet}} + \sigma_i)\right)$ and

$$\omega_i(\tau_i) \in \begin{cases} \{1\}, & \text{for } \tau_i \in [0, \tau^i_{\text{dmiet}}) \\ [0,1], & \text{for } \tau_i = \tau^i_{\text{dmiet}}, \\ \{0\}, & \text{for } \tau_i > \tau^i_{\text{dmiet}}. \end{cases} \quad (11)$$

Note that ν_i is single-valued for all $\tau_i \neq \tau^i_{\text{dmiet}}$, and set-valued for $\tau_i = \tau^i_{\text{dmiet}}$. Since the proof holds for *all* points in the set-valued map, in essence we can use the discontinuous version ($\omega_i(\tau_i) = 1$ if $\tau_i \leqslant \tau^i_{\text{dmiet}}$ and 0 otherwise) to verify the condition in (5).

In the proposed setup, each agent needs to know (and compute) constants ε_ϱ and ε_ν on-line due to the dependence on σ_i. If, from a computational standpoint, this is infeasible, a conservative upper-bound can be used by taking $\varepsilon_\varrho := \tilde{\gamma}_i(0)\phi_{0,i}(\tau^i_{\text{miet}}) - \tilde{\gamma}_i(1)\phi_{1,i}(0)\lambda_i^2$ and $\varepsilon_\nu := \phi_{0,i}(\tau^i_{\text{miet}}) - \phi_{0,i}(\tau^i_{\text{dmiet}})$, which can be computed a priori.

We stress that the local ETMs given by (5), (6), (8), (9) and (10), can operate fully asynchronously and no clock synchronization or acknowledgment signals are required.

C. Main result

Given the ETM design and the corresponding hybrid model of the MAS presented above, we can now state the following result. Its proof follows in a similar manner as the proof of [1, Theorem 1], and is therefore omitted.

Theorem 1. *Consider the system $\mathcal{H}(\mathcal{C}, F, \mathcal{D}, G)$ with Ψ_i, ϱ_i and ν_i given by* (8), (9) *and* (10)*, respectively. Moreover, suppose that [1, Conditions 1-6] with modifications* (35-II) *and* (44-II) *and, additionally, Condition 5-II holds. Then the digital MAS described by $\mathcal{H}$ is $(\tilde{s}, \mathcal{S}_1, \mathcal{S}_2)$-flow-dissipative with the supply rate $\tilde{s} : \mathbb{X}\times\mathbb{R}^{n_v} \to \mathbb{R}$ as given in [1,* (27)*] and $\mathcal{S}_1 = \{\xi \in \mathbb{X} \mid x \in \mathcal{X},\ e = 0,\ \eta = 0\}$, $\mathcal{S}_2 = \{\xi \in \mathcal{S}_1 \mid r - y = 0\}$. In addition, if there are no finite escape times during the flow*[1]*, then the system $\mathcal{H}$ is persistently flowing.*

Theorem 1 implies that the desired stability and/or performance properties, guaranteed by the local controllers in absence of the network, are ensured by the triggering rules when the network is taken into account.

IV. Destination Protocols for Lossy Networks

In this section, we will show how the destination protocols z_i can be exploited to capture different aspects of networked communication such as, *e.g.*, packet losses and denial-of-service. We will show that under reasonable assumptions, the resulting protocols are still UGES in the sense of [1, Condition 1] and, consequently, the results in both [1, Theorem 1] and Theorem 1 still hold even in the presence of lossy communication networks. Hence, this implies that the MAS setting with packet loss and denial-of-service can be handled also for TTC, ETC and PETC (digital implementations) by using our framework.

A. Packet losses

To model the presence of packet losses, we take the function z_i as follows, for all $i \in \mathcal{V}$, $m \in \mathcal{V}_i^{\text{out}}$, $k \in \mathbb{N}$,

$$z_i(k,m,t_k^i) = \begin{cases} 0, & \text{when packet sent at } t_k^i \text{ is lost,} \\ 1, & \text{when packet sent at } t_k^i \text{ is not lost.} \end{cases} \quad (12)$$

Observe from [1, (4)] that with this protocol function, the estimate $\widehat{y}_i^m$, $i \in \mathcal{V}$, $m \in \mathcal{V}_i^{\text{out}}$, is only updated if the

[1]The absence of finite escape times during flow is meant here in the sense that case (b) in Prop. 2.10 in [12] cannot occur.

transmission attempt of agent $\mathcal{A}_i$ at time t_k^i, $k \in \mathbb{N}$, has been successful.

Assumption 1. *The number of successive packet dropouts of transmissions from agent $\mathcal{A}_i$, $i \in \mathcal{V}$, to agent $\mathcal{A}_m$, $m \in \mathcal{V}_i^{out}$, that might occur since the last successful transmission is upper bounded by $\delta_{\max}^i$, where $\delta_{\max}^i \in \mathbb{N}$ represents the maximum allowable number of successive dropouts (MANSD).*

Assumption 1 has been used frequently [13]–[20].

Proposition 1. *Suppose Assumption 1 holds, then the protocol z_i given by* (12) *is UGES and there is a Lyapunov function W_i satisfying [1, Conditions 1 and 2].*

Proof. Recall the discrete-time system as given in [1, (29)], which is given by $q(k+1) = \bar{z}_i(k, q(k))$, $k \in \mathbb{N}$, with function z_i as (12). Let $\psi(m, k, e_i^{\text{out}})$ be the solution to q at discrete time instant $m \in \mathbb{N}_{\geqslant k}$ with initial condition $e_i^{\text{out}} \in \mathbb{E}_i$ and initial time $k \in \mathbb{N}$. We define the Lyapunov function W_i as in [1, (30)], *i.e.*, $W_i(k, e_i^{\text{out}}) = \sqrt{\sum_{m=k}^{\infty} |\psi(m, k, e_i^{\text{out}})|^2}$. The solution ψ satisfies $\psi(k, k, e_i^{\text{out}}) = |e_i^{\text{out}}| \leqslant W_i(k, e_i^{\text{out}})$, and thus that $\underline{\alpha}_{W,i} = 1$. Moreover, observe that for all $k \in \mathbb{N}$ and all $m \geqslant \delta_{\max}^i + 1$, we have that $\psi(m + k, k, e_i^{\text{out}}) = 0$. By combining the latter fact with the definition of W_i, we obtain that $W(k, e_i^{\text{out}}) \leqslant \sqrt{\delta_{\max}^i + 1}|e_i^{\text{out}}|$, and thus that $\overline{\alpha}_{W,i} = \sqrt{\delta_{\max}^i + 1}$. From the definitions of q and W_i and by recalling the facts that $\psi(k, k, e_i^{\text{out}}) = |e_i^{\text{out}}|$, we obtain that

$$W_i(k+1, \bar{z}_i(e_i^{\text{out}})) = \sqrt{\sum_{m=k}^{\infty} |\psi(m, k, e_i^{\text{out}})|^2 - |e_i^{\text{out}}|^2} \leqslant \sqrt{\tfrac{\delta_{\max}^i}{\delta_{\max}^i + 1}} W_i(k, e_i^{\text{out}}),$$

with $\bar{z}_i(k, e_i^{\text{out}}) := \big(I_{Nn_{y,i}} - Z_i(k)\big) e_i^{\text{out}}$ for all $e_i^{\text{out}} \in \mathbb{R}^{Nn_y}$, $i \in \mathcal{V}$, $m \in \mathcal{V}_i^{\text{out}}$ and all $k \in \mathbb{N}$, where we recall that $Z_i(\kappa_i) := \operatorname{diag}(z_i(\kappa_i, 1), z_i(\kappa_i, 2), \ldots, z_i(\kappa_i, N))$. Hence, [1, (28b)] holds for $\lambda_i = \sqrt{\frac{\delta_{\max}^i}{\delta_{\max}^i + 1}}$, which implies that [1, Condition 1] holds. Lastly, following similar arguments as [13, Lemma V.4], we find that [1, Condition 2] holds with $c_i = \sqrt{\delta_{\max}^i + 1}$, which completes the proof. □

Remark 3. *Note that under packet losses, the ETM described in Part I by [1,* (7), (8), (45) *and* (46)*] or in Part II by* (5), (6), (8), (9) *and* (10)*, requires acknowledgments in order to obtain $\widehat{y}_i^m$, $i \in \mathcal{V}$, $m \in \mathcal{V}_i^{out}$. Observe, however, that this acknowledgment is allowed to be delayed with $\tau_{(\text{d})\text{miet}}^i$ time units. However, the requirement for acknowledgement mechanisms can be relaxed. Indeed, inspired by [20], to construct an ETM without acknowledgements,* i.e.*, an ETM that does not depend on the number of successive packet losses δ or the transmission error e, we need to keep track of all possible values that the variable $\widehat{y}_i^{out}$ can possibly attain. For this reason, we introduce the variable*

$$\Upsilon_i = \left(\Upsilon_i^1, \Upsilon_i^2, \ldots, \Upsilon_i^{\delta_{\max}^i + 1}\right) \in \mathbb{R}^{(\delta_{\max}+1)n_{y,i}}$$

where the i-th element of this sequence corresponds to the hypothesis that $i-1$ successive dropouts have occurred since the most recent successful transmission attempt. The flow dynamics of Υ_i^q for all $i \in \mathcal{V}$ and all $q \in \tilde{\delta}$, where $\tilde{\delta} := \{1, 2, \ldots, \delta_{\max} + 1\}$, must be chosen equal to the flow dynamics of $\widehat{y}_i^{out}$, i.e.*, $\dot{\Upsilon}_i^q = \hat{f}_i(\Upsilon_i^q)$. Moreover, at each transmission instant t_k^i, $k \in \mathbb{N}$, $i \in \mathcal{V}$, the local estimates Υ_i^q, $q \in \{2, 3, \ldots, \delta_{\max} + 1\}$, is updated according to $\Upsilon_i^q(t_k^i, j+1) = \Upsilon_i^{q-1}(t_k^i, j)$, and Υ_i^1 is updated according to $\Upsilon_i^q(t_k^i, j+1) = y_i(t_k^i, j)$.*

With these variables, it is possible to construct a set-valued estimate $\mathcal{Y}_i^m$ that contains $\widehat{y}_i^m$, $i \in \mathcal{V}$, $m \in \mathcal{V}^{out}$, for all $(t, j) \in dom\ \xi$, namely, by choosing $\mathcal{Y}_i^m$ as

$$\mathcal{Y}_i^m(t, j) = \begin{cases} \mathbb{R}^{n_y}, & \text{when } \tau_i(t, j) \in [0, \tau_{\text{miet}}^i] \\ \{y \in \mathbb{R}^{n_y} \mid y = \Upsilon_i^q(t - \tau, j - \iota), \tau \in [0, \tau_{\text{mad}}^i], \\ q \in \tilde{\delta}, \iota \in \mathbb{N}, (t - \tau, j - \iota) \in dom\ \xi\}, & \text{otherw.}, \end{cases}$$

Observe that under Assumption 1-II and Assumption 1, $\mathcal{Y}_i^m$ as defined above is constructed such that, for all $i \in \mathcal{V}$ and $m \in \mathcal{V}_i^{out}$, $\widehat{y}_i^m(t, j) \in \mathcal{Y}_i^m(t, j)$ for all $(t, j) \in \operatorname{dom} \xi$, i.e.*, $\mathcal{Y}_i^m$ satisfies [1, Assumption 2]. Hence, $\mathcal{Y}_i^m$ can be used in* (9) *and* (10) mutatis mutandis.

Remark 4. *From a theoretical standpoint, the number of packets that may be lost consecutively can be arbitrarily large. However, in that case, λ_i will be close to 1, resulting in the maximum allowable delay and minimum inter-event time being close to zero. This implies that when a very large number of packets may be lost, it "should" be possible to transmit a lot of packets in a very short time. Under Assumption 1, if sufficiently many packets are transmitted, it can be ensure that at least one of them will be received. Of course, this is not desirable from a practical standpoint. Indeed, often, the sampling times, the maximum allowable delays and/or the transmission delays are lower bounded by practical constraints, which will, together with the specific system dynamics, naturally limit the number of packets that may be lost consecutively while still being able to guarantee the dissipative properties.*

B. Denial-of-service

In this section, we study MAS under *denial-of-service* (DoS). A DoS is defined as a period in time at which the communication is blocked due to an overloaded network. In case an agent $\mathcal{A}_i$, $i \in \mathcal{V}$, attempts to transmit a new measurement to agent $\mathcal{A}_m$, $m \in \mathcal{V}_i^{\text{out}}$, at transmission time t_k^i and a DoS is active, the attempt will fail and agent i cannot update $\widehat{y}_i^m$. Obviously, the latter might endanger the stability and performance of the closed-loop system. In this section we will show that DoS can be appropriately captured with the framework of destination protocols, see [1, Section V-A], after which we will show that these protocols can still be proven to be UGES in the sense of [1, Condition 1]. This ensures that the main results in Part I and II can still be directly applied.

In general, DoS can be described by a sequence of time intervals $\{H_n\}_{n \in \mathbb{N}}$, where the n-th time interval H_n, given by $H_n := \{h_n\} \cup [h_n, h_n + d_n)$, represents the n-th DoS (period). Hence, $h_n \in \mathbb{R}_{\geqslant 0}$ denotes the time instant at which the n-th DoS interval commences and $d_n \in \mathbb{R}_{\geqslant 0}$ denotes the length of the n-th DoS interval. The intervals in $\{H_n\}_{n \in \mathbb{N}}$ do not overlap, *i.e.*, $0 \leqslant h_0 \leqslant h_0 + d_0 < h_1 \leqslant h_1 + d_1 < h_2 < \ldots$.

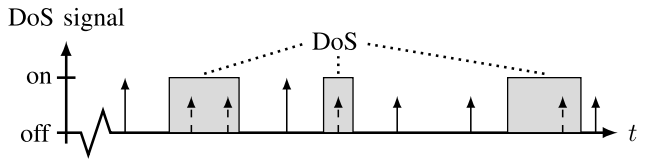

Fig. 2. Schematic representation of a sequence of DoS. The solid arrows indicate successful transmissions and the dashed arrows transmissions that are blocked due to the network not being available. The gray areas indicate the presence of a DoS.

For a given sequence $\{H_n\}_{n\in\mathbb{N}}$, the set of time instants at which a DoS is active (see Fig. 2) is defined as

$$\mathcal{T} := \bigcup_{n\in\mathbb{N}} H_n. \tag{13}$$

To capture the presence of these DoS, we consider the following protocol function, for all $i \in \mathcal{V}$, $m \in \mathcal{V}_i^{\text{out}}$, $k \in \mathbb{N}$, $\mathcal{T} \subseteq \mathbb{R}_{\geqslant 0}$ and all $t_k^i \in \mathbb{R}_{\geqslant 0}$

$$z_i(k, m, \mathcal{T}, t_k^i) = \begin{cases} 1, & \text{when } t_k^i \notin \mathcal{T} \\ 0, & \text{when } t_k^i \in \mathcal{T}. \end{cases} \tag{14}$$

Observe from [1, (4)] that indeed, $\widehat{y}_i^m$, $i \in \mathcal{V}$, $m \in \mathcal{V}_i^{\text{out}}$, is not updated in case a DoS is active at time instant t_k^i, $k \in \mathbb{N}$.

Moreover, observe that in contrast to the destination protocols as discussed in [1, Section V-A], the protocol presented in (14) explicitly depends on $\mathcal{T}$ and the sequence of transmission instants $\{t_k^i\}_{k\in\mathbb{N}}$, $i \in \mathcal{V}$.

In the context of DoS, it is reasonable to assume that the DoS cannot occur indefinitely, see also [21]–[23]. Therefore, we characterize the DoS in terms of the DoS *frequency* and the DoS *duration*. To do so, let us define the collection of times within the interval $[T_1, T_2]$, with $T_2 \geqslant T_1 \geqslant 0$, at which DoS is active as

$$\Xi(T_1, T_2) := [T_1, T_2] \cap \mathcal{T} \tag{15}$$

with $\mathcal{T}$ as in (13) and the number of DoS *off/on* transitions occurring in the interval $[T_1, T_2]$ as

$$n(T_1, T_2) := |\{n \in \mathbb{N} \mid h_n \in [T_1, T_2]\}|, \tag{16}$$

where we recall that $|\cdot|$ denotes the cardinality of a finite set. Moreover, we use $|\Xi(T_1, T_2)|$ to denote the total length of the DoS within the interval $[T_1, T_2]$.

Definition 1. [21], [24] (DoS frequency). *A given sequence of DoS $\{H_n\}_{n\in\mathbb{N}}$ is said to satisfy the* DoS frequency *constraint for a given $\tau_D \in \mathbb{R}_{>0}$, and a given $\nu \in \mathbb{R}_{\geqslant 0}$, if for all $T_1, T_2 \in \mathbb{R}_{\geqslant 0}$ with $T_2 \geqslant T_1$ it holds that $n(T_1, T_2) \leqslant \nu + \frac{T_2 - T_1}{\tau_D}$.*

Definition 2. [21] (DoS duration). *A given sequence of DoS $\{H_n\}_{n\in\mathbb{N}}$ is said to satisfy the* DoS duration *constraint for a given $T \in \mathbb{R}_{>1}$ and a given $\varsigma \in \mathbb{R}_{\geqslant 0}$, if for all $T_1, T_2 \in \mathbb{R}_{\geqslant 0}$ with $T_2 \geqslant T_1$ it holds that $|\Xi(T_1, T_2)| \leqslant \varsigma + \frac{T_2 - T_1}{T}$.*

Note that no assumptions regarding any underlying "strategy" (in case of malicious attackers) are made in Definitions 1 and 2.

To deal with the presence of DoS, we modify the ETM for ETC described by [1, (7)] to

$$t_{k+1}^i := \begin{cases} \inf\left\{t > t_k^i + \tau_{\text{miet}}^i \mid \eta_i(t) \leqslant 0\right\}, & \text{when } t_k^i \notin \mathcal{T} \\ t_k^i + \tau_{\text{miet}}^i, & \text{when } t_k^i \in \mathcal{T} \end{cases}$$

with $t_0^i = 0$, and the ETM for PETC described by (5) to

$$t_{k+1}^i := \begin{cases} \inf\{t \geqslant t_k^i + \tau_{\text{dmiet}}^i \mid t \in \{s_n^i\}_{n\in\mathbb{N}} \wedge \\ \qquad \eta_i(t) + \nu_i(y_i(t), \widehat{y}_i^{\text{out}}(t), \tau_i(t)) \leqslant 0\}, \\ \qquad\qquad \text{when } t_k^i \notin \mathcal{T}, \\ \inf\{t > t_k^i + \tau_{\text{dmiet}}^i \mid t \in \{s_n^i\}_{n\in\mathbb{N}}\}, \\ \qquad\qquad \text{when } t_k^i \in \mathcal{T}. \end{cases}$$

Observe that in case no DoS is present at a transmission instant, *i.e.*, when $t_k^i \notin \mathcal{T}$, the next transmission instant is determined in an event-based fashion, similar as in [1, (7)] for ETC and (5) for PETC. However, in case a DoS attack is present, *i.e.*, when $t_k^i \in \mathcal{T}$, which, for example, can be detected by using communication with acknowledgments, the next-time instant is scheduled after τ_{miet}^i, $i \in \mathcal{V}$, time units. The latter is necessary to probe when the network is available again. In that sense, dealing with packet losses as described in Section IV-A is different from dealing with DoS.

Proposition 2. *The protocol function z_i given by* (14) *is UGES under the DoS frequency and duration constraints with $\underline{\alpha}_{W,i} = 1$, $\overline{\alpha}_{W,i} = \sqrt{\chi + 1}$, $\lambda_i = \sqrt{\frac{\chi_i}{\chi_i + 1}}$, $c_i = \sqrt{\chi_i + 1}$ and where*

$$\chi_i := \left\lceil \left(\frac{\varsigma + \nu\tau_{\text{miet}}^i}{\tau_{\text{miet}}^i} \right) \left(1 - \frac{1}{T} - \frac{\tau_{\text{miet}}^i}{\tau_D} \right)^{-1} + 1 \right\rceil. \tag{17}$$

Proof. We show that χ_i is equal to the maximum number of consecutive transmission attempts that are blocked by the DoS. As shown in [25], the maximum time in between two successful transmissions is given by

$$d_{\max} := \left(\varsigma + \nu\tau_{\text{miet}}^i\right) \left(1 - \frac{1}{T} - \frac{\tau_{\text{miet}}^i}{\tau_D} \right)^{-1} + \tau_{\text{miet}}^i. \tag{18}$$

Given that the minimum time between two consecutive transmissions is equal to τ_{miet}^i, we indeed find that χ_i given by (17) is equal to the maximum number of consecutive transmission attempts that are blocked by the DoS. The proof can now be completed using similar arguments as in the proof of Proposition 1. □

Remark 5. *There is an intricate interplay between τ_{miet}, χ_i and λ_i, and therefore it is not possible to ensure resilience for arbitrary (constrained) DoS signals. Moreover, as these results are based on the maximum number of packets that can be lost consecutively, they are subject to some conservatism. We envision that reverse average dwell-time conditions may be used, by* e.g., *drawing inspiration from [26], to reduce conservatism by considering the "average" nature of the behavior captured by Definitions 1 and 2, but this is outside the scope of the present paper.*

C. Quantization

As the current modeling setup is very similar to *scheduling protocols*, the presented framework can be adapted to include quantization in a similar manner as [27]. However, due to the added notational complications and space limitations, we omit the treatment of quantization in this framework.

V. Case studies

A. Consensus of single integrator systems

Consider N agents with $N \in \mathbb{N}_{>0}$, where each agent has a state $x_i \in \mathbb{R}$, $i \in \mathcal{V}$, whose dynamics evolve according to $\dot{x}_i = u_i$ with $u_i \in \mathbb{R}$ the control input. The output of the system is the state x_i, *i.e.*, $y_i = x_i$. We assume that the graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ with Laplacian matrix $\mathcal{L}$ is connected and undirected, *i.e.*, $\mathcal{L}^\top = \mathcal{L}$. The control objective is for the states of all agents to asymptotically converge, *i.e.*, $\lim_{t\to\infty} |x_i(t) - x_m(t)| = 0$ for all $i, m \in \mathcal{V}$ and any initial condition. To achieve consensus, we implement the control law

$$u_i = -\sum_{m \in \mathcal{V}_i^{\text{in}}} (x_i + e_i^i - x_m - e_m^i). \tag{19}$$

We consider the Lyapunov candidate $V(x) = x^\top \mathcal{L} x$ where $x := (x_1, x_2, \ldots, x_N)$. According to [28], the derivative of this Lyapunov function can be upper-bounded as

$$\langle \nabla V(x), -\mathcal{L}x - \mathcal{L}e \rangle \leqslant \sum_{i \in \mathcal{V}} \left(-d_i z_i^2 - c_i u_i^2 + (\gamma_i^2 - \varpi_i)|e_i^i|^2 \right) \tag{20}$$

with $d_i := \delta(1 - aN_i)$, $c_i := (1-\delta)(1-aN_i)$ and $\gamma_i = \sqrt{a^{-1}N_i + \varpi_i}$, and where $\delta \in (0,1)$, $a \in (0, \frac{1}{N_i})$ and $\varpi_i > 0$ are tuning parameters. The proposition below shows the exact expressions for all the required conditions.

Proposition 3. *The system with local dynamics $\dot{x}_i = u_i$ and local controller* (19) *satisfies [1, Conditions 3, 4 and 6] with $H_i(x,e) = |u_i|$, $L_i = 0$, $s(x,e) = \sum_{i\in\mathcal{V}} \left(-d_i z_i^2 - \mu_i e_i^2\right)$, $\mathcal{X} = \{x \in \mathbb{R}^N \mid x_1 = x_2 = \ldots = x_N\}$, $\varsigma_i = 0$, $\mu_i = c_i \frac{1}{N_i}$, $\gamma_i = \sqrt{a^{-1}N_i + \varpi_i}$, and $\underline{H}_i(\widehat{y}_i^{in}) = |u_i|$.*

Sketch of Proof. Observe that $\dot{x}_i = u_i$, *i.e.*, [1, Condition 3] is satisfied by taking $H_i(x,e) = |u_i|$. Next, we obtain by substitution that the expression in [1, (35)] reduces to (20), which implies that [1, Condition 4] holds. Lastly, observe that u_i is locally available as it is based on $\widehat{y}_i^{\text{in}}$, hence we can select $\underline{H}_i(\widehat{y}_i^{\text{in}}) = H_i(x,e) = |u_i|$, which completes the sketch of proof. □

Constants τ_{dmiet}^i and τ_{mad}^i can be generated via an intuitive procedure, as described in [28]. Proposition 3 implies that asymptotic consensus is achieved with the proposed control configurations in this paper.

We simulate the results of Proposition 3 with non-uniform and time-varying transmission delays in the periodic event-triggered control framework. We consider the system presented in [28], with $N = 8$ agents which are connected as described by a graph $\mathcal{G}$ with undirected edges $(1,2)$, $(1,8)$, $(2,3)$, $(2,7)$, $(3,4)$, $(3,6)$, $(4,5)$, $(5,6)$, $(5,8)$ and $(7,8)$. We use the parameters $\delta = \varpi_i = 0.05$, $a = 0.1$ and $\epsilon_i = 0.5$ for all $i \in \mathcal{N}$. Given these tuning parameters, we obtain $\gamma_i = 4.478$ and $c_i = 0.76$ for agents $i \in \mathcal{N}$ with two neighbors (*i.e.*, $N_i = 2$, thus agents P_1, P_4, P_6 and P_7) and $\gamma_i = 5.482$ and $c_i = 0.665$ for agents $i \in \mathcal{N}$ with three neighbors (*i.e.*, $N_i = 3$, thus agents P_2, P_3, P_5 and P_8). The function $\varphi_i(\eta_i)$ is designed as $\varphi_i(\eta_i) = -\epsilon_\eta(\eta_i)$ with $\epsilon_\eta = 0.05$. We select $\lambda_i = 0.2$ for all agents, and pick $\phi_{0,i}(0) = 5$ and $\phi_{1,i} = 2$. For these values, we obtain

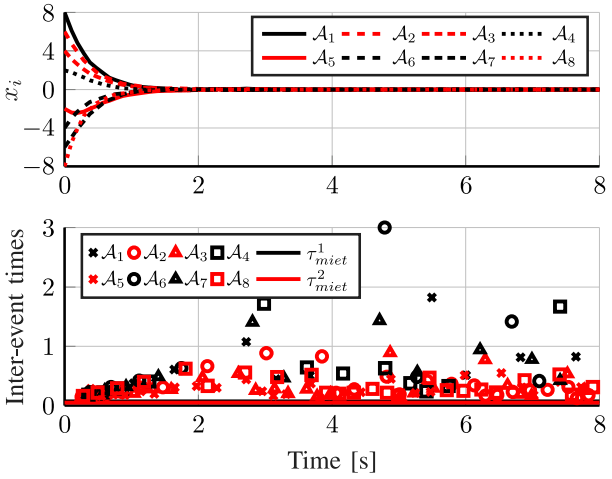

Fig. 3. States and inter-event times for the single integrator systems in Section V-A.

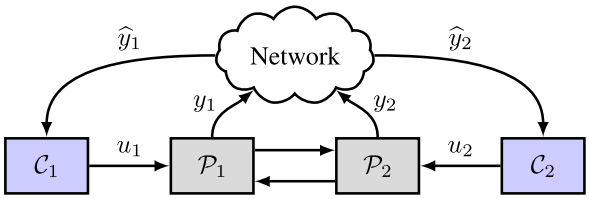

Fig. 4. Networked control setup of nonlinear example.

$(\tau_{\text{dmiet}}^i, \tau_{\text{mad}}^i) = (0.12, 0.016)$ for agents $i \in \mathcal{V}$ for which $N_i = 2$ and $(\tau_{\text{dmiet}}^i, \tau_{\text{mad}}^i) = (0.09, 0.012)$ for agents $i \in \mathcal{V}$ for which $N_i = 3$. We select $\tau_{\text{miet}}^i = 0.07$ ($\tau_{\text{miet}}^i = 0.05$) for all agents for which $N_i = 2$ ($N_i = 3$), respectively, $\tau_{\text{masp}}^i = 10^{-2}$ and $d_i = 10^{-3}$ for all $i \in \mathcal{V}$. At each sampling moment s_n^i, the next sampling moment is scheduled randomly such that $s_{n+1}^i \in [s_n^i + d_i, s_n^i + \tau_{\text{masp}}^i]$ for each $i \in \mathcal{V}$, hence the sampling of each agent is aperiodic, asynchronous and independent of the other agents. The state evolution and inter-event times are depicted in Fig. 3, confirming our main theorem.

B. Nonlinear example

We now consider the nonlinear system given by

$$\mathcal{P}_i : \dot{x}_i = x_i^2 - x_i^3 + x_{m(i)} + u_i, \ i \in \{1,2\} \tag{21}$$

with $m(i) = 3 - i$, with the controllers $\mathcal{C}_i : u_i = -2\widehat{x}_i$, $i \in \{1,2\}$, see Fig. 4. In [11], it was already shown that this system satisfies [1, Conditions 1-6] with $\underline{H}_i \equiv 0$ when a zero-order-hold is applied and by using a sampled-data protocol. We demonstrate here the benefit of applying a nonzero holding function. We therefore assume that $\dot{\widehat{x}} = -2\widehat{x}$. Thus, the closed-loop system can be described by

$$\dot{x}_i = x_i^2 - x_i^3 + x_{m(i)} - 2(x_i + e_i) \tag{22a}$$
$$\dot{e}_i = -x_i^2 + x_i^3 - x_{m(i)} \tag{22b}$$

where $i \in \{1,2\}$ and $m(i) = 3 - i$ as before. Inspired by [11], we consider the candidate Lyapunov function

$$V(x) = \sigma^2(\tfrac{\alpha}{2}x_1^2 + \tfrac{\beta}{4}x_1^4 + \tfrac{\alpha}{2}x_2^2 + \tfrac{\beta}{4}x_2^4), \tag{23}$$

where $\alpha, \beta, \varsigma \in \mathbb{R}_{>0}$. Following the same steps, we arrive at

$$\dot{V} \leqslant \left(\sum_{i=1}^{2} \left[-(\varepsilon+q)x_i^2 + (\gamma^2 - \varepsilon)e_i^2 - H_{e,i}^2 \right] + \varsigma^2 \bar{p}(x) \right) \quad (24)$$

with $\gamma = \sigma\sqrt{\alpha^2+\beta^2+\varsigma^{-2}}$ and $\bar{p}(x) := \beta x_1^3 x_2 + \beta x_1 x_2^3 + \sum_{i=1}^{2} \left[x_i^2(-\alpha + 1 + \varsigma^{-2}(10 + \varepsilon + q) + \alpha x_i + (-\alpha - 2\beta + 2\varsigma^{-2})x_i^2 + (\beta - 4\varsigma^{-2})x_i^3 + (-\beta + 1 + 2\varsigma^{-2})x_i^4) \right]$, where we used that $H_{e,i}^2 \leqslant 2(-x_i^2 + x_i^3)^2 + 2x_{m(i)}^2$. We numerically verified (using SOSTOOLS) that the parameter choices $[\alpha, \beta, \varepsilon, \varsigma, q]$ as $[3.01, 1.47, 0.01, 2.48, 0.5]$ verify that $\bar{p}(x) \leqslant 0$. Observe that these choices are the same as [11], therefore, both $p(x)$ in [11, (80)] and $\bar{p}(x)$ are negative definite for this selection of parameters, which implies that, indeed [1, Condition 4] holds with [1, (36)]. Due to the parameter choice holding for both $p(x)$ and $\bar{p}(x)$, we can relax [1, (40)] by taking $\tilde{L}_i(l) = 0$ for $l = 0$. Solving [(40)] with $\lambda_i = 0.18$, $\phi_{0,i}(0) = \lambda_i^{-1}$ and $\phi_{1,i}(0) = 2.3$, we find that $(\tau_{\text{mad}}^i, \tau_{\text{miet}}^i) = (0.01, 0.1203)$, $i \in \{1, 2\}$, which is a significant improvement *vis-à-vis* [11], where they obtained $(\tau_{\text{mad}}^i, \tau_{\text{miet}}^i) = (0.01, 0.0995)$, thereby clearly illustrating the benefits of using nonzero holding functions.

VI. Conclusion

In this work, we presented a systematic and general design framework for digitally implemented event-triggered control strategies, in the sense of the event-triggering functions only having to be verified at discrete sampling times, with even varying sampling periods, for a class of nonlinear MAS subject to disturbances. By ensuring that the conditions of the local triggering mechanisms only have to be verified at the local (asynchronous) sampling times, the proposed framework is suitable for implementation on digital platforms. We show how appropriate modifications to the results presented in Part I lead to event-triggering schemes that are dissipative with respect to a desired supply rate. With this dissipativity property, the framework can handle several relevant stability and performance properties such as asymptotic (set) stability, input-to-state stability, $\mathcal{L}_p$-stability with $p \in [1, \infty)$ and consensus and deal with both control and estimation, from a unified point of view. Robustness with respect to non-uniform, possibly unknown and time-varying delays is guaranteed by design. Moreover, we show that the destination protocols, that can be used to determine which agents get access to the network, can be used to cope with *packet loss* and *denial-of-service*, thereby enabling the study of the resilience within the proposed framework. Several case studies illustrate the general applicability of the presented framework.

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