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FRACTIONAL THERMAL MODEL OF THE LUNGS USING HAVRILIAK-NEGAMI FUNCTION

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ABSTRACT

This paper is about fractional system identification of a thermal model of the lungs. Usually, during open-heart surgery, an extracorporeal circulation (ECC) is carried out on the patient. In order to plug the artificial heart/lung machine on the blood stream, the lungs are disconnected from the circulatory system. This may result in postoperative respiratory complications. A method to protect the lungs has been developed by surgeon and anesthetist. It is called: bronchial hypothermia. The aim is to cool the organ in order to slow down its deterioration. Unfortunately the thermal properties of the lungs are not well-known yet. Mathematical models are useful and needed in order to improve the knowledge of these organs. As proved by several previous works, fractional models are especially appropriate to model thermal systems (model capacity, accuracy) and the dynamic of fractal systems. Thus, fractional models of the lungs have been determined using time domain system identification with the Havriliak-Negami function. A comparison with integer order models was also carried out. The aim of this paper is to present the results of this study.

INTRODUCTION

Most of the time during open-heart surgery, it is necessary to stop the heart from beating to allow the surgeon to operate

properly. This is possible thanks to artificial heart/lung device. This device is called extracorporeal circulation (ECC). In order to connect the machine with the circulatory system, the heart and lungs are excluded from the blood stream. The lack of blood in the lungs during ECC, is recognized as a potential cause of postoperative pulmonary dysfunction. Thus, anesthetists are developing methods to protect the lungs. One of these methods is to cool down the lungs by insufflating fresh gas inside it. This method has shown results but is based on the assumption that the thermal insulation of the lungs increases during ECC. This assumption has not been verified yet. A possible solution to verify it, is to design a thermal model of the lungs during ECC and during standard condition, and to compare both models. This paper presents the first step of this study. Different models of thermal transfer inside the lungs are identified.

Fractional transfer functions are especially appropriate to model thermal [1] [2] [3] and biological systems [4] [5] and they are useful in system identification because they allow a few number of parameter (parametric capacity of the model) [6]. Among the fractional functions, the Havriliak-Negami function is not often used, though it showed interesting results in system identification [7].

This paper is composed of four parts. First, the approximations used for time simulation of fractional operator and Havriliak-Negami function are recalled. The second part

deals with the experimental protocol. The results of identification are presented in the third part. Finally, a conclusion considers the prospect.

TIME DOMAIN SIMULATION OF FRACTIONAL SYSTEMS

This part, recalls mathematical definition of non integer derivative, and then usual methods to simulate fractional systems.

Non integer derivative

The non integer derivative concept of any order (non integer),

$$\frac{d^\nu f(t)}{dt^\nu} \quad (1)$$

was defined in the 19th century by Riemann and Liouville. Fractional derivative of a function $f(t)$ is defined by the order derivative ($[.]$ is the floor operator) of a fractional order integral [8] :

$$\frac{d^\nu f(t)}{dt^\nu} = \frac{d^{[\nu]} f(t)}{dt^{[\nu]}} \quad (2)$$

, the Euler's function Γ is defined by :

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \quad (3)$$

The Laplace Transform of the order derivative (with $+$) of a function relaxed at the initial instant is such as [9]:

$$(4)$$

Frequency-bounded integro-differential operator synthesis

Because of the natural limits of a physical system and of the input/output signals (e.g. Shannon frequency and input signals spectrum), fractional operators s^ν are usually approximated with high rational order transfer function, over a frequency range $[\omega_A, \omega_B]$. Thus, the integro-differential operator and the rational transfer function have the same fractional behavior over this frequency range. The most commonly used approximation was suggested by Oustaloup [8]. It consists in a recursive distribution of poles and zeros over frequency range $[\omega_A, \omega_B]$. Such a method naturally induces a degradation of the approximation around the frequencies ω_A and ω_B . This side-effect is known as "boundary effect". The approximation error is decreased over ω_B by widening the frequency range over which the s^ν operator is synthesized, to $[\omega_A, \omega_B]$, where

$$\omega_B = \omega_A \cdot 10^{2/n} \quad (5)$$

Boundary effect is considerably reduced by choosing [8]. The approximation of the integro-differential

operator over the $[\omega_A, \omega_B]$ frequency range is given by the relation :

$$\frac{d^\nu f(s)}{ds^\nu} \approx \frac{C_0 \prod_{i=1}^n (s + \sigma_i)}{\prod_{j=1}^n (s + \omega_{zj})} \quad (6)$$

where n is an integer which is the number of pole-zero pair and where C_0 , and pulsations σ_i and ω_{zj} are given by :

$$\sigma_i = \omega_A \cdot 10^{(i-1)/n} \quad (7)$$

$$\omega_{zj} = \omega_B \cdot 10^{(j-1)/n} \quad (8)$$

$$C_0 = \prod_{i=1}^n \sigma_i \quad (9)$$

ω_0 is calculated in order to obtain a unitary gain at the frequency $1 \text{ rad} \cdot \text{s}^{-1}$.

Real parameters α and η are defined by the integro-differential order ν , such as

$$\nu = \alpha + \eta \quad (10)$$

Approximation and time simulation of Havriliak-Negami function

In this paper, the Havriliak-Negami function [10] [11]:

$$(11)$$

is used for identification and synthesized for $\nu_2 \in \mathbb{R}$, and $\frac{2}{\nu_2}$. The method used for the synthesis and time simulation of the Havriliak-Negami function was developed by Sommaccal in [7]. Because the details of the approximation are a bit complex, this part presents only the general idea. For further explanations the reader should refer to [7].

It is shown that the function (11) can be written as :

$$(12)$$

with

$$(13)$$

and

$$(14)$$

where n is the number of s-roots of function H_N .

Function F_1 is a product of classical Davidson-Cole functions which can be approximated using a recursive distribution of pole-zeros as presented above in the previous part of the paper. On the other hand, $\kappa(s)$ is approximated using a non recursive distribution of pole-zeros. Thus the Havriliak-Negami function is approximated on a frequency range as a product of pole-zeros.

EXPERIMENTAL PROTOCOL

Dr Youssef Abdelmoumen and his team are specialized in open-heart surgery. They provide the thermal data acquired on sheep. The experiment was carried out at the PTIB platform of Xavier Arnozan Hospital.

The general aim of this work is to study the variation of thermal characteristics of the lungs during extracorporeal circulation. Thus, the medical team recorded on a sheep the temperature of the lung during open-heart surgery before the ECC and during the ECC.

Lungs have a tree-like structure. From the trachea they are subdivided in series of bronchial tubes as shown in Fig. 1. The temperature inside the lungs is measured at the entry of the first three consecutive bronchial tubes (T_1, T_2, T_3). The temperature of the body T_c is also measured. All these temperature are measured thanks to GI type thermocouple. These thermocouples are linked to a National Instruments PXI interface. A special software was designed with LABVIEW® and load into the PXI in order to acquire and record the signals.

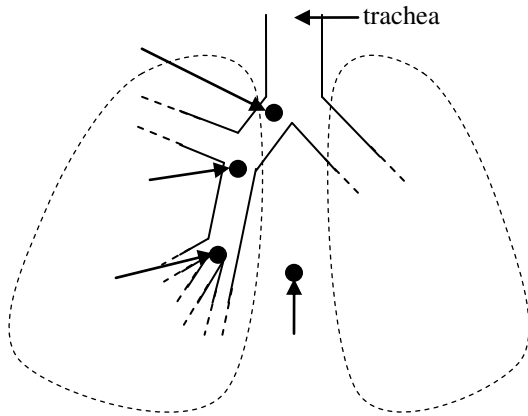


Figure 1 Rough diagram of lungs, the black dots (●) show the approximate location of the thermal sensors

The measures are limited to these three locations because of the difficulty to accurately plant a sensor deeper into the lung.

SYSTEM IDENTIFICATION

If ones want to study the variation of the thermal characteristics of the lungs during ECC, then one can identify two different models of the thermal transfer in the lungs, during normal condition of the organ and during ECC, and finally compare both models. However this paper presents only the first step which is the result of the identification of the models.

In order, to simplify the problem and to limits the effect of the body temperature, the temperatures T_1, T_2 and T_3 are normalized by T_c . Thus, two models are identified:

$$\text{---} \quad (15)$$

and

$$\text{---} \quad (16)$$

where

$$(17)$$

The transfer function chosen for and is

$$(18)$$

it is identified with output error model. The vector of parameter is optimized using the non linear simplex optimization algorithm [12] [13].

The data are sampled at , thus the Havriliak-Negami function is approximated over the frequency range:

$$(19)$$

The data are also divided in two parts, one used to identify the model parameters (identification data) and the other to validate the value of the parameters (validation data).

The results of identification are shown on Fig. 2, 3, 4, 5, 6, 7, 8 and 9. On these figures the red curves are the model response, the blue curves are the input signals (ΔT_k), and the black curves are the system response (ΔT_{k+1}). One can see that the model response is close to the system response on every diagram which means our model is able to reproduce accurately the dynamic behavior of the system.

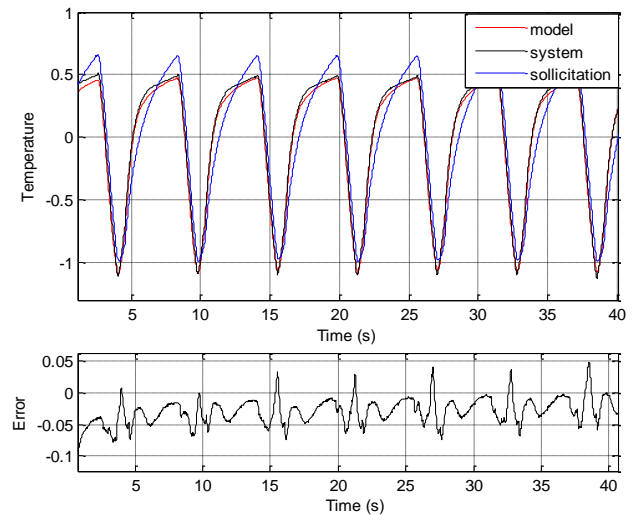


Figure 2 System identification of and modeling error, before ECC.

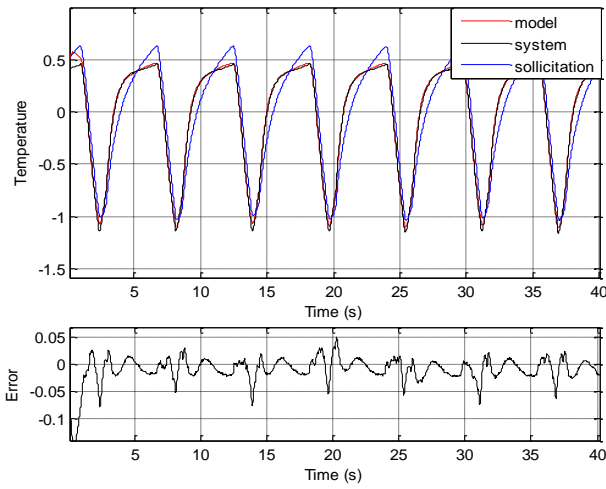


Figure 3 Validation data and modeling error of before ECC.

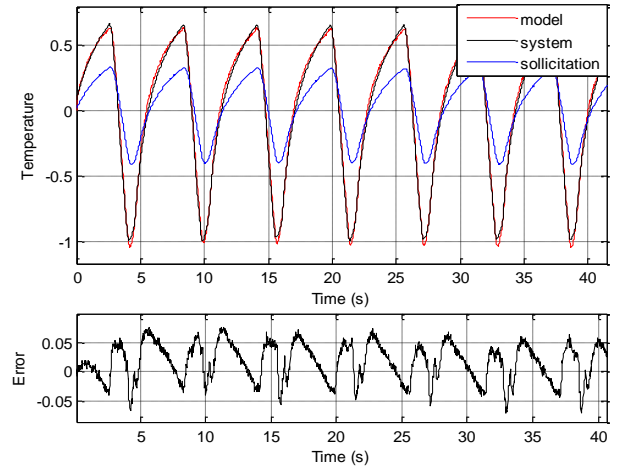


Figure 6 System identification of and modeling error, before ECC.

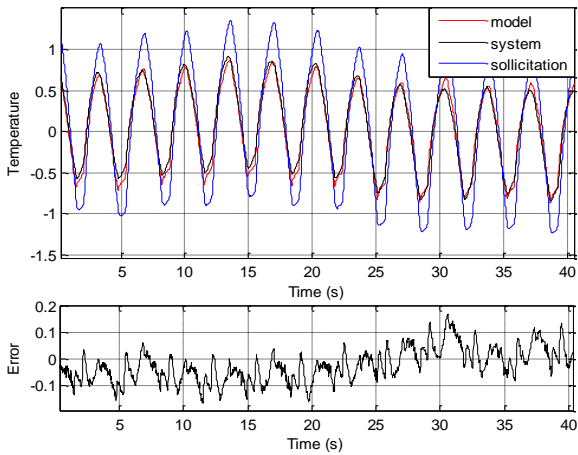


Figure 4 System identification of and modeling error, during ECC.

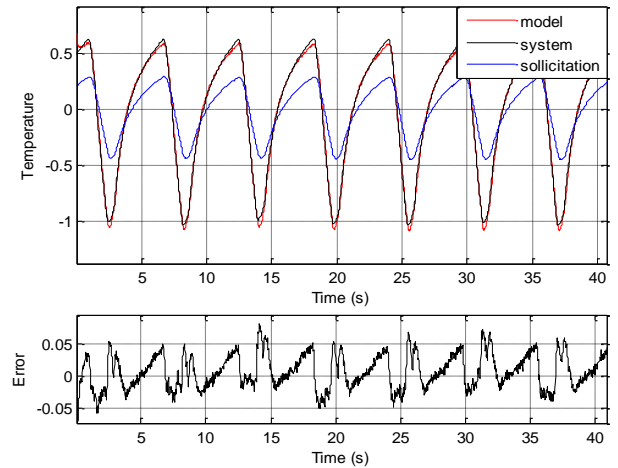


Figure 7 Validation data and modeling error of before ECC.

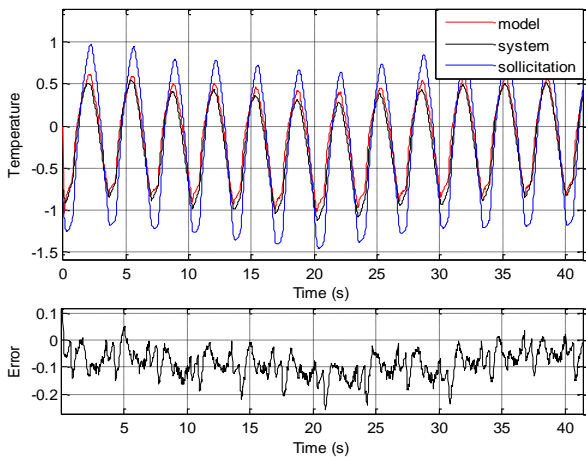


Figure 5 Validation data and modeling error of during ECC.

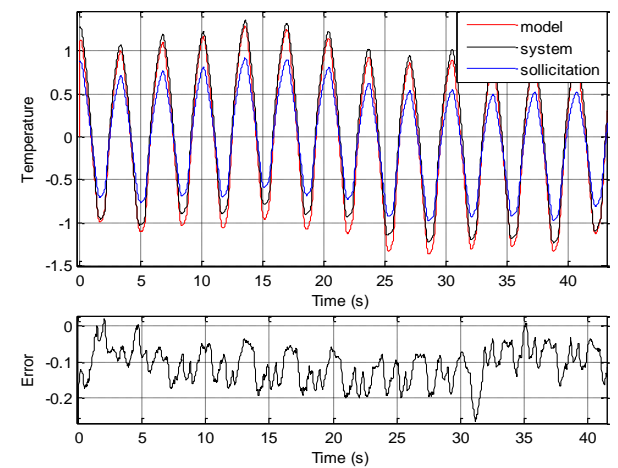


Figure 8 System identification of and modeling error, during ECC.

CONCLUSION

Four models of bronchial tubes thermal transfer are identified considering two different conditions ie with or without ECC. A significant variations in the parameters of the models is observed. For now, these variations are not linked with a physical meaning.

A comparison between an integer model and a fractional model is proposed, which shows the interest of the latter.

The main focus of the followings works will be to link the parameter of the model with a physical or physiological meaning.

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