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REAL UNREALS: the first part of the eponym book

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REAL UNREALS

*Spatiotemporal concepts
in the sight of constructivist requirement*

I owe a debt of gratitude to H el ene Fonvieille, who has never failed to faithfully and courageously examine my work,

to Francis Campos for his constant support and sound advice, and for his impromptu editing, many years ago, of a little booklet in which, in a still very imperfect form, the demonstration of the forwards and backwards movement of a rotating cylinder experiment was beginning to take shape,

to Michel Grosmann, whose path the spatiotemporal dispersion of this brochure crossed in Strasbourg, and who contacted me to suggest I do a thesis in physics,

to Robert Vidal, with whom I had begun a mathematics thesis on algorithms for decomposing the 'very large' natural integers into products of prime factors, a project that faded away at the same time as the question of the foundations of spatiotemporal notions began to devour almost my entire life,

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to Pierre Stratonovitch, specialist in the Python language, for his help with the *program to simulate the illusion of escape from distant celestial bodies in a Bruno universe*,

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to Catherine Jouglet for putting up with me during the almost thirty-five years it took to write this book, in its many successive versions...

INTRODUCTION

For some physicists, the philosophy of physics does not have a good image. From their point of view, ‘understanding’ reality is not at all what physics is about. It is an undertaking with no real scope, since there are so many things we will never understand, starting with why there is a real rather than nothing. It is also a subjective undertaking, since what is deemed enlightening by some is not necessarily so for others. It is deceptive, pointless, irrelevant entertainment. The only thing that counts, according to this point of view, is to build models that give a finely accurate description of reality; only this point is verifiable and has scientific value.

Admittedly, this view is to some extent tenable, but it can also be seen as hopelessly nihilistic, or as a fall-back position. Another point of view, which joins the topic of this book, is that physics is not intended to be merely a corpus whose formulas, though possibly working perfectly, are in no way corollary to any ‘understanding’ of reality, and have no relevance except through the magic of a fortuitous, accidental symbolism that it is pointless to try to elucidate.

Between these two approaches, we will always choose the second. What is more, we are betting that physics *begins* with philosophy, that the latter is its indispensable midwife, and that if we forget it, we will have every chance of going astray. A rather striking current example will illustrate this theme. It will indeed be *demonstrated* in this book, within a mathematical framework deriving from the closest possible philosophical work on the foundations of spatiotemporal notions, that a flat, homogeneous, isotropic universe, obeying the cosmological principle, subject to the simplest laws of chance, — and last but not least characteristic — stable, i.e. without expansion or contraction, is a universe that generates the illusion it is expanding. The phenomenon takes place according to a form that coincides

with the observable and, in particular, shows *the same* so-called ‘acceleration’ of this illusion of expansion.

We are therefore faced with two possible explanations for the red shift in the spectra of distant bodies: that of real expansion and that of optical illusion. These two explanations cannot be considered as having the same value. The first is based on a practice where tinkering and approximation are legitimate customs, and where, depending on the problem at hand, cosmological constants and energies of various colours are added to the pot in a haphazard fashion. The latter, on the other hand, is based on a rigorous approach, and achieves an exact representation of the same appearances without ever needing to introduce any artifacts whatsoever.

Physics, that science dedicated to the study of the real, if ever there were one, has throughout its history cultivated a lot of chimeras, from the crystal spheres on which the stars were set, to phlogiston, the horror of the vacuum and many others. It is because reality, this infinitely mysterious continent, forever impossible to understand in its entirety, is nonetheless crossed by paths, lines of understanding which we may not see if we do not look for them with all the necessary rigor. They are then almost automatically replaced by irrelevant metaphysical entities. So, until we understand that it is the same phenomenon that makes the apple fall from the tree and the Moon not fall to Earth, we can only believe in the existence of two different contexts, two different ‘realities’, the one below and the one above, and consequently construct a ‘real’ whose foreground is occupied by two chimeras leaning on each other. Their central, crucial position in the world they generate puts them in a good position to multiply like Fibonacci rabbits. Every physics concept will have to be interpreted, or rather married to one and other of these two chimeras. This will inevitably give rise to a tribe of unicorns and thirteen-legged snails, whose necessary cross-breeding will create a pretty menagerie of the absurd.

Luckily, or rather unluckily, by inventing suitable unicorns for each new difficulty, such a system can long find ways to survive. When Ptolemy's system runs out of steam due to the accumulation of discrepancies between the trajectories it predicts and those observed, it gets out of trouble by creating supplementary chimeras, the equants, these second-order circular motions added to the planets' primary circular motion. With such methods, this system could probably get away with difficulties for a very long time, offering a concordance with reality converging towards perfection by fitting artefacts of order $n + 1$ with artefacts of order n . It was neither mathematics nor physics that revoked the chimeras of Ptolemy's system, but philosophy — the first and sovereign philosophy based on these two disciplines.

All the eras of physics have cultivated their own chimeras. Ours, or rather the one now drawing to a close, has added its own to the procession. In this book, we will methodically interrogate presumed candidates for this role: space, time and space-time, considered as having a physical reality and not as mere grids that our intelligence constructs and superposes onto reality.

FIRST PART: REAL UNREALS

*The problem of the real and the unreal
in spatiotemporal notions*

1 — CHOSISM VERSUS NON-CHOSISM

1.1 — Mathematics and fairy tales

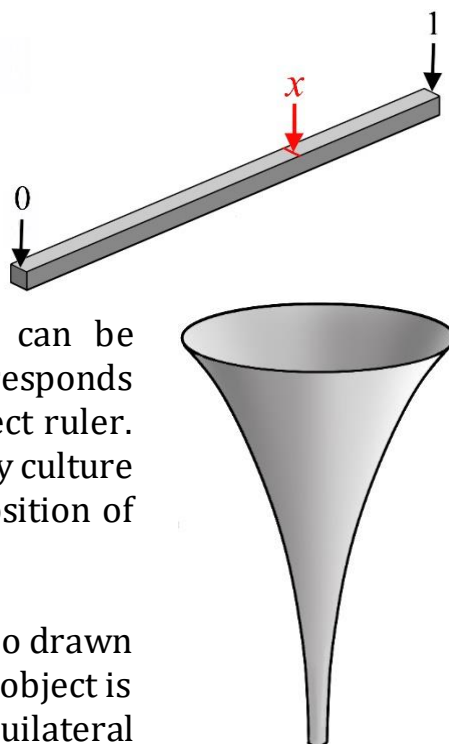
1.1.1 — Three tales

The same vertigo flooded with terror that sometimes seizes human beings in front of the starry sky should also seize them when they look at those numbers we call 'real'.

Each of them — or almost all of them — has a writing of infinite length, which would therefore go infinitely further, if we could unroll it straight out into space, than the most distant visible celestial bodies. I say 'almost' because we have to exclude decimals, whose writing is not infinite. However, this is a negligible case, since they form a set the measurement of which is zero. Leaving aside this particular case, each of these numbers, considered as a code, contains within itself more — infinitely more — information than would be needed to encode everything that humanity has so far produced that can be digitized — texts, images, music, films and so on. This encoding would take up so little space in the unlimited sequence of decimals that there would be enough left over to encode in the same way everything digitizable that all civilizations in the universe have produced, even if there were an infinite number of them.

The number x which encodes all the cultural productions of the entire universe, can be chosen from the interval $]0, 1[$. It therefore corresponds to a single graduation on a geometrically perfect ruler. All the knowledge, in the broadest sense, of every culture in the universe can be encoded by the exact position of this graduation on the ruler.

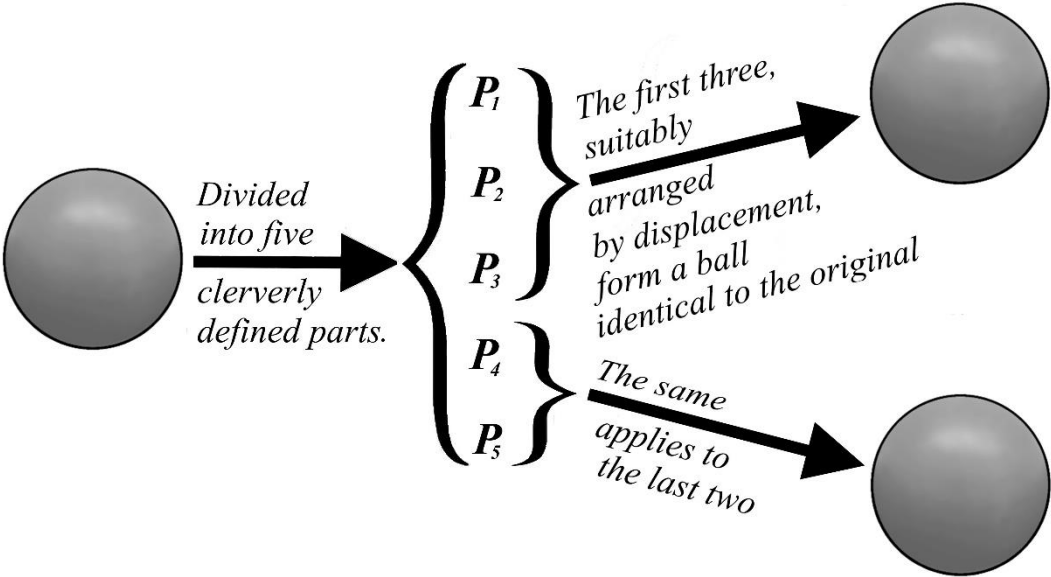
Another fairy tale is that of the funnel, also drawn from the 'popular repertoire' of geometry. This object is generated by the rotation of the arc of an equilateral



hyperbola $y = f(x) = \frac{1}{x+1}$ for $x \geq 0$ around the x axis. Its surface is infinite, while its volume is finite.

A finite amount of paint poured into it suffices to fill it and nevertheless covers an infinite surface.

Another example, this time from the high-level repertoire of geometry, is Banach and Tarski's paradox.¹ A ball can be cut into five parts which, when properly reassembled, form two balls identical to the first.



1.1.2 — Unreality of the real numbers

A simple line on a real ruler will never have the infinite thinness it has in the mathematical fairy tale. It will be at least the thickness of an atom, about one angstrom, or 10^{-10} meters. If the ruler measures one meter, this line, far from being able to potentially encode all the knowledge of all the cultures in the universe, will at best encode the ten digits of a single telephone number.

Similarly, the funnel, whose diameter tends towards zero when its distance from the opening tends towards infinity, is not constructible.

In Banach and Tarski's paradox, the term 'part' is misleading, evoking something analogous to what breaking or cutting an object produces. This is in no way the case. The 'pieces' are unthinkable laces, interpenetrating everywhere, themselves unions of infinite uncountable numbers of equally

unthinkable laces selected thanks to the axiom of choice. This certainly assures a mathematical existence to the constructed object, but goes no further and gives no information as to what it may be. We do not, nor will we ever, know how these laces are made. Even if we did, their infinite thinness would obviously make their physical construction impossible. Banach and Tarski's theorem in no way allows for the multiplication of effective balls, as Jesus is said to have done with the loaves.

These three mathematically irreproachable fairy tales are built on the same resource: the infinite smallness of geometric points, or, what is the same thing, the infinite exactitude of real numbers.

They show us that mathematics, albeit wielded according to the rules of the art, easily fabricates the unreal when it uses the full power of real numbers to plunge into the infinitely small.

There is something a little strange, even despairing, in this observation. Mathematics has been born, along with arithmetical and geometry, out of the need and desire to approach reality as rigorously as possible. From that point on, it followed the straightest path possible, ruthlessly pruning out all doubtful reasoning, and now, despite this faultless path guided by impeccable intentions, it is capable of telling fairy tales — exactly the opposite of what it set out to do.

I use the term 'large scale' here and in other places according to its usual meaning, and not the mathematical one, which is the opposite. Mathematically speaking, a large scale is what makes it possible to scale microscopic domains to our size, whereas in everyday life a large scale is the attribute of immense territories.

1.2 — The unreality of the infinitely small

As long as knowledge of the infinitely small remains non-existent, no problem arises. It is only natural to believe that the property of divisibility observed at macroscopic and microscopic scales persists indefinitely — and therefore that the infinitely small is infinitely large. This is what Pascal proclaims, for example, in his famous meditation on the two infinities — another fairy tale.

It is interesting to note that the concept of space does not appear: it is the real, concrete object that is

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infinitely divisible: the body of the cheese mite, part by smaller part nested within the previous one, until it contains an entire universe, itself infinitely divisible.

The definitive proof of the existence of atoms and more generally of the corpuscular aspect of the infinitely small should logically overturn or chip away at this paradigm. But this is not the case. The presumed reality of the infinitely small, indefinitely divisible, finds survival solutions.

I see various reasons for this:

—The principle that nature is written in mathematical language is a powerful one. It has given many proofs of its effectiveness; and these proofs all, or almost all, invoke differential calculus. In particular, instantaneous velocity and instantaneous acceleration are the founding concepts of modern physics. How can we consider infinitely small variations in the quantity "position" to be meaningless, when their emergence in the theorization of reality has proved so fertile?

— From a mathematical point of view, there is no other relevant choice than that of real numbers. It is certainly possible to construct systems of numbers that "refuse" to tend towards zero, but these pose insurmountable problems: loss of the notion of limit, continuity, derivation and so on. Once we unreservedly admit that nature is written in mathematical language, it is hard to escape the conclusion that these inescapable real numbers must correspond to some reality.

— If the only instrument for measuring lengths were this prime instrument that is the measuring ruler, we would not be able to give meaning to lengths smaller than the size of an atom. Fortunately, relay instruments based on various radiations have been discovered that allow geometry to be extrapolated to the subatomic world. We can thus give meaning to lengths considerably smaller than that of the atom, for example the size of the nucleus. This reinforces the hypothesis of the infinite divisibility of reality.

— The concept of *space* has taken hold in the time between Blaise Pascal and Jean Perrin, whose book *Les atoms*² signed the death of anti-atomist conceptions still vigorously alive in scientific circles in 1908. This concept was born by Descartes as a simple mathematical object: the set of possible coordinates of a point. It was chosified by Newton to become a real physical entity that is both container and background of the world. It does not matter then that the corpuscular aspect of the "infinitely small" conflicts with the postulate of its divisibility to infinity, since *space* is there, which can assume this divisibility now largely unhooked from reality.

Nevertheless, other considerations condemn this outcome. To "see" into the "infinitely small", we need to use wavelengths on the scale of what we want to observe. Their use requires a quantity of energy inversely proportional to them, and which therefore tends towards infinity when the dimensions of what we want to observe tend towards zero. There is thus a size below which all observation becomes definitively impossible: the Planck length, around 10^{-35} metres.

Admittedly, this is immensely small compared to the size of an atom, but it is not zero. Two theoretical points whose distance is less than Planck's length are two points that no observation can separate. Yet geometry separates them — and separates them, moreover, by an abyss where Pascal could fit, to the smallest detail, as many billions of galaxies as he fancied. On the magic rule where mathematical enchantment would have us encode all the knowledge in the world, the finest line we could draw, by a possible sophisticated process going beyond the granularity of the atom, will never allow us, whatever the process, to encode more than 35 characters: three telephone numbers.

Similarly, there is a duration, Planck's duration, such that two instants separated by a duration less than or equal to it are, in any case, physically indistinguishable. Consequently, the property extends to space-time: there is a definitive limit to our ability to separate the theoretical local instants that make it up.

The infinite divisibility of reality no longer has any real territory in which to take refuge. It is nothing more than a figment of the imagination. And yet, real numbers continue imperturbably to construct an infinitely divisible "real".

This is not without damage.

— While we did not know Planck's length and duration, theoretical points and instants, although non-existent in their infinite thinness, could nonetheless be seen as the limits of actual objects. They didn't belong to reality, of course, but were nonetheless part of its "adherence". This link has been broken: these theoretical objects can no longer be designated by convergent sequences of actual objects. They are definitively too small to be reached in any way. They are forever swallowed up in the unreal. This poses a problem because they are the prime logical objects of physics: without a point or a local instant, we can do nothing.

— We identify space, time or space-time with affine or differential varieties. However, the varieties are infinitely separable, whereas the former are not. Varieties are made up of elements that we can theoretically put our finger on, whereas space, time and space-time are bordered on the infinitely

small side by a region that is definitively out of reach. Effective space, time and space-time cannot therefore coincide with the affine or differential varieties with which we identify them.

— This lack of coincidence can certainly be said to be so tenuous that it can almost always be neglected, and this is certainly true in many cases: 10^{-35} meters or 10^{-44} seconds really is not that much. It is equally true, however, from the point of view of mathematical accounting, that the neglected is infinitely larger than the non-neglected: the infinity of decimals compared to a short handful of them. Pascal's paradox persists, except that it is no longer in the real that he accommodates the infinitely large present in the infinitely small, but in the unreal engendered by the use of real numbers. The trouble with this unreal is that it is impossible to evacuate, given the inescapable mathematical necessity of using real numbers.

1.3 — Space, time, space-time: real or unreal?

1.3.1 — Non-chosist hypothesis and chosist hypothesis

The conclusion seems inexorable:

Space, time, space-time, defined as sets of infinitely small elements, do not belong to reality. They are mathematical constructions, grids that we place on reality, but which are not part of it.

They are not things.

Are they then representations of things? Are there things, still called space, time or space-time which, according to what we have just seen, are impossible to mathematize perfectly, but which, in themselves, exist? Or are these entities nothing more than unreal grids?

Let's give a name to both terms of this alternative.

The hypothesis of the grid devoid of physical reality is the relativist hypothesis, in the original sense of the word. However, we prefer — see below why — the locution non-chosist. The hypothesis that, on the contrary, space, time, space-time have a physical reality is the chosist hypothesis.

1.3.2 — A point of vocabulary

The words 'relativist' and 'relativity' cover a motley array of meanings.

a) 'Relativist' in the first sense is the opposite of what we call 'chosist'. According to this meaning, relativistic conceptions of space and time consider these entities to have no existence in themselves, but to be merely constructs by which we order reality.

b) Obeying the principle of relativity. In relativistic physics in this sense, all Galilean spaces are essentially identical, none is privileged, and the laws of physics are the same with respect to each of them. Relativistic physics in the sense of a) is not necessarily relativistic in the sense of b): while not being things, different Galilean spaces can have different properties depending on their situation relative to the distant Sky.

c) In line with the theory of special relativity. Thus, the 'relativistic' form is the form of special relativity. The term is doubly unfortunate:

— Galilean relativity, which is relativistic in senses a) and b), does not lead to the same form;

— non-relativistic theories (e.g., admitting an absolute space and time identical to Newton's) can have Lorentzian kinematics and thus a largely (or totally) 'relativistic' form.

d) Refers to phenomena or velocities obeying the ‘relativistic’ form in the sense of c) and characterized by values significantly different from those that would be given by the Galilean relativity form. If we regard special relativity as a geometrical theory and not as a physical theory, this use of the word is irrelevant, as ‘relativistic’ effects exist even at very low speeds.

To avoid this confusion, we will say

— of a theory that does not admit the physical reality of time, space or space-time, and is therefore relativistic in the first sense of the word, that it is **non-choisist**;

— of a theory admitting the validity of the principle of relativity, and which is therefore relativistic in the second sense of the word, that it is **relativistic**;

— of a theory having a Lorentzian form, and which is therefore relativistic in the third sense of the word, that it is **Lorentzian**.

1.3.3 — The flat frame

We study the question of the reality or unreality of space-time edifices in the ‘flat’ frame, the ‘intergalactic or interstellar desert’, which plays a central role in the construction of notions of space and time. Indeed, gravitational forces modify space and time in the choisist hypothesis, or the behaviour of the instruments in the non-choisist hypothesis. The non-flat frame is everywhere singular and under the dependence of local conditions each time different. If we do not want to see the theory of space and time — the *kinematics* — splintered into an infinite number of unconnected theories of local scope, we need a universal framework relative to which all these local occurrences can be constructed in their difference from it. This framework cannot be anything other than the only universal we have, that of the intergalactic desert, which is also that of special relativity.

The flat frame, the prerequisite for all the others, is also both a limiting case and a particular case of a more general theory, just as special relativity is before general relativity. It is therefore logically deducible from this general theory, and is thus in the position of a logical guarantor in relation to it: any lack of consistency in a flat-frame theory would undermine the consistency of the general theory.

1.4 — Immediate arguments in favour of the unreality of space, time or space-time

— **Space is a cultural construct.** It is a late notion that began to take hold with Descartes and his Cartesian coordinates. This first avatar is merely an analytical convenience, not part of the fabric of the world.

It was only later that Newton, analysing the experiment of water spinning in a bucket and hollowing out at its centre, formulated the highly debatable conclusion that it proves the existence of absolute motion and therefore of absolute space.

Space (or space-time) has thus become inscribed in the collective conscious and unconscious as a necessary container and background to the world. It is imagined and felt with such intensity that it has become difficult for us to understand how antiquity and other cultures could have ignored it and lived in a universe content to exist in itself, without being contained within a space or space-time.

— **Is real the cause of a real phenomenon.** This is the principle Newton applies when he looks for a real cause of the actual hollowing of the water in the centre of the bucket.

— **But is not real, in principle, the overabundant cause.** This is known as the principle of economy, parsimony, minimality or even Ockham's razor principle. Distant celestial bodies, viewed from a point in the universe, appear to be mutually 'interdependent' and generate a sphere of fixed bodies exactly in place to assume the role of cause of the hollowing. Thus it is to them that an action exactly in line with them should be attributed, and not to any Newtonian space.

Similarly, the chosist point of view does not dispense with the need for geometric instruments and clocks. Its panel of 'real' things contains space-time, 'objects' and events. For such a complete description of reality, the non-chosist panel contains only the same 'objects' and events. From this point of view, space-time is overabundant and therefore cannot be real.

Is 'real' what can be reached by experiment.

While an experiment on a piece of iron is an experiment on a piece of iron, an experiment on space, time or space-time is never anything other than an experiment on objects, events, instruments. Neither space nor time nor space-time can be reached by experiment.

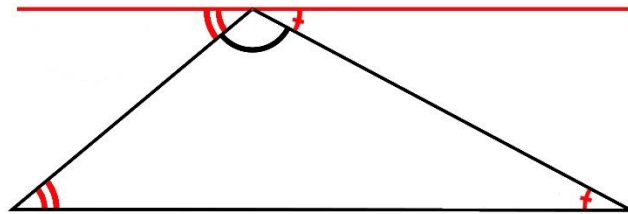
— A viewpoint or theory afflicted with internal contradictions is suspect of unreality. Thus the discordance between the infinite thinness of theoretical points and instants and the impossibility of separating their concrete counterparts as soon as they are close enough can make us doubt the reality of spatiotemporal edifice. Whereas if they are no more than empty grids laid over reality with the means we have, the embarrassment disappears.

— Qualitative leaps in explanatory scope testify to a better approach to the real. Newton, reducing the fall of bodies and the motion of planets to the same law of universal gravitation, unveils a real that is not necessarily definitive, but at any rate 'more real' than the Aristotelian paradigm it replaces. If we believe that reality cannot be understood in its entirety — it would be a foolish ambition — but rather traversed by a network of lines of understanding, then we must believe that progress in the explanatory domain bears witness to progress in the approach to reality. When it comes to the reality or unreality of spatiotemporal edifices, we will see the second hypothesis brings explanatory gain on several points.

1.5 — Euclid's property

1.5.1 — The Euclidean miracle

Let us don our cosmic hermit costume and set up our laboratory in a sidereal desert, where we postulate that geometry is universal. Let us then do a little experiment: draw a triangle, transfer its angles to one of the vertices, in the same plane, and evaluate the angle thus obtained.



We've known since Beltrami and Klein that it can take on an infinite number of values without logical contradiction. One of them, however, generates a particular geometry, simpler than the others and independent of the scale at which we consider the world. This value, singular for that reason, is also singular because it 'just falls': it is the flat angle. If this value is reached, the geometry is Euclidean. If not, it is not.

This value is just an isolated point among the infinite possibilities. If we were to draw it in a lottery, the probability of it emerging would be zero. Yet it is the one we get.

(Of course, we do not have the means to go and do our experiments in intergalactic or interstellar deserts, but we have the strongest reasons to believe that the geometry of the flat frame that reigns there is remarkably Euclidean. The proof of this is that on our Earth, admittedly massive, but not to the point of significantly bending the flat frame, we concretely do not use any other — even to build that very-high-precision object that is a modern telescope).

Too extraordinary coincidences are too miraculous to be fortuitous. Euclid's property, true even though it may seem infinitely improbable, is certainly not accidental. It should therefore be possible to explain it.

But the explanation cannot be where it cannot be. The assertion that space has no existence 'in itself', that it is nothing more than an empty grid laid over reality, means that attributing any intrinsic quality to it is nonsense. The 'properties of space' can be nothing other than those of the empty grid.

This grid is the unlimited virtual extension of a memofrom body at Galilean rest — i.e. an elastic body in restricted use, hypothetically becoming identical to itself again from one position of Galilean rest to another, this

identity being that defined by the experiment of durable superposition. If space, then, is Euclidean, it is because memoform matter at Galilean rest is.

1.5.2 — Euclidean property and properties of matter

We are not concerned here with the biologist's matter, which is alive, nor concerned with the chemist's matter, nor with the physicist's, which has been stripped of most of its attributes but retains a mass, a density, a temperature, a conductivity, a state, solid, liquid, gaseous, plasma, etc. We are concerned here with the geometer's matter, a minimal variant of the physicist's matter, which in its final stripping retains

- elastic solidity: during experiments, bodies and instruments undergo only temporary changes that have disappeared when a new experiment is started;

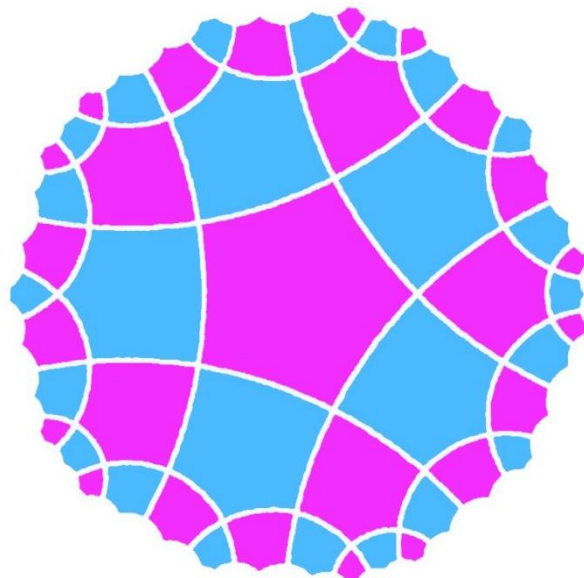
- infinite divisibility, impossible to eliminate if we want to have an object that can be used in mathematics;

- homogeneity, in a fairly general way;

- isotropy or anisotropy, depending on its own constitution and the conditions to which it is subjected;

- continuity;

- whether it is Euclidean or not, and, if it is not, a parameter characterising its curvature, for example, if its geometry is hyperbolic, the length of the side of a regular pentagon with five right angles;



The geometer's matter builds edifices that can be mathematized, but whose relevance fades away at the subatomic scale. Granular matter, on the other hand, would build relevant edifices if it could, but cannot for lack of mathematical efficiency.

What we have to do, in the circumstances, is to nail together these two matters which are both deficient each in its own way. That is to express in the geometer's matter, the positions of the atoms, and in particular those of their centre. These centres are the geometrical nodes of a certain generally nonregular paving which is both generator and witness of the geometry the geometer's matter must be equipped with.

Because of the cosmological principle and what we observe around us, almost the entire geometric extent of the universe consists of cosmic desert. Since this desert is the same everywhere, its curvature is constant. So, its geometry is homogeneous.

The homogeneous geometries behave locally like Euclidean geometry, and up to a fairly respectable distance from the solar system, Euclidean geometry cannot be defaulted or perhaps only insignificantly. We conclude that this region is in the sufficiently small range around us where the geometry of the desert is indistinguishable from Euclidean geometry, and that we have to look further afield to find out what the curvature of the cosmic desert's geometry is. This is not that easy, because the further you look, the less you can see. The current answer³ resembles the one that might have been given by Pythia: "The universe is sufficiently flat for the question of its non-platitude not to be decided".

This reasoning, considered from the non-choisist angle, overlooks an essential point: space is an entity without physical reality constructed with imaginary copies of actual matter. The homogeneity of memoform matter at Galilean rest, that these copies reproduce, means that the geometry of the cosmic desert that they construct has the same curvature everywhere, regardless of the concrete construction that generates it.

We know the geometric structures that tessellate space in Euclidean and non-Euclidean geometries. The angles of a triangle whose sides have a given size are specific to a given geometry, and so are the tessellations. For example, there is a regular tessellation of hyperbolic space and a regular tessellation of Euclidean space, both of which have six equal edges extending from each vertex and carried by three perpendicular lines in pairs. However, the faces of the regular polyhedrons that make up these tessellations are pentagons in the first case and squares in the second. The difference between these two types of tessellations is obvious and becomes more pronounced as

the number of atoms increases. For crystalline structures on our scale, the difference can be described as 'considerable'. The two objects do not belong to the same world and each cannot be represented in the world of the other. In the general case of amorphous crystallisations produced by the same atoms, their meshes would diverge by the same amount, and the bodies to which they belong would also diverge by a 'considerable' gap.

So, while the chosist paradigm looks towards the infinitely large to determine what is the curvature of the geometry that governs sidereal deserts, the non-chosist paradigm looks towards the 'infinitely' small, towards the way in which matter is constructed by assembling its atoms. According to the chosist point of view, the infinite smallness of the world on our scale compared to the immensity of the cosmic expanse explains why its geometry is indistinguishable from Euclidean geometry. According to the non-chosist point of view, the world on our scale is no longer infinitely small compared to the structure that holds the key to the problem, but infinitely large. It thus hypertrophies the possible non-Euclidean character of the geometry at the scale of the infinitely small. As it remains Euclidean on our scale and even on much larger scales, we can be sure that it is still Euclidean, with remarkable precision, on the scale of the atom. Crystallography has also enabled us to draw up a list of crystalline structures, all of which, without a single exception, are Euclidean. As they cannot exist in non-Euclidean geometry, this proves that flat geometry is not non-Euclidean.

However, is it exactly Euclidean, or only approximately Euclidean? If our answer to this question is based solely on measurements of angles and lengths, which are fraught with uncertainty, we will never have a formal answer, but only more or less strong presumptions. On the other hand, since geometry requires axioms, if we accept Euclid's first four queries, which are as admirably plausible as very little constraining...

Query 1: Through two distinct points passes one and only one straight line.

Query 2: A segment can be extended indefinitely into a longer segment.

Query 3: Given two distinct points, there exists a circle having the first as its centre and passing through the second.

Query 4: All right angles (i.e., dividing a flat angle into two equal angles) are equal.

... then flat geometry is either exactly Euclidean, or exactly non-Euclidean, and since we know it cannot be non-Euclidean, it is exactly Euclidean.

A small detail: Euclid's four demands don't apply to elliptic geometry, but only to Euclidean or hyperbolic geometry. This does not invalidate the reasoning, since we know from examining crystal structures that flat geometry is not elliptical.

The non-choisist point of view shows that the geometry of the flat frame is neither hyperbolic nor elliptic; and that if we accept Euclid's first four postulates, then it is exactly Euclidean.

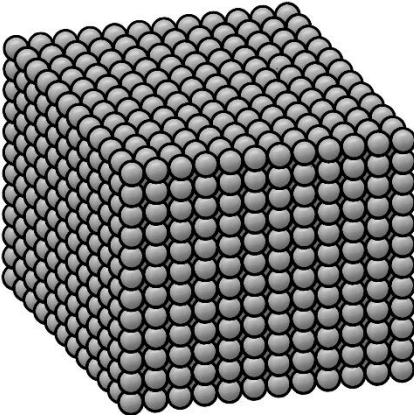
Of course, this result in no way invalidates the logical independence of the fifth postulate from the first four. The solidity of non-Euclidean geometries remains unaffected. This demonstration uses non-mathematical levers: the physical unreality of space-time and the examination of actual crystal structures.

1.5.3 — A temptative of explanation

Euclidean structures can be expanded or contracted while preserving their architecture, since their angles remain unchanged. This is impossible with non-Euclidean structures, whose angles are modified by this operation. For the same change in atomic size due to a gain or loss of energy, Euclidean structures are the least deformed of all, i.e. those whose deformations consume the least energy. They are therefore points of stable equilibrium for matter by the same law that causes water in a container, whether hot or cold, to stabilize horizontally, i.e. at the lowest possible energy level. Non-Euclidean structures, on the other hand, are points of unstable equilibrium, and it is conceivable that a few fugitive germs may be created, since no space is there to impose its mould.

Much the same thing can be said, but in a different mode. Non-Euclidean structures are objects whose architecture makes them more rigid than Euclidean in terms of their ability to contract or expand. They are made of an elastic material that cannot, as such, produce rigid structures. Their existence, subject to two contradictory requirements, is not viable. It can only be fleeting. The only perennial crystalline structures are therefore Euclidean.

Today, the Euclidean character of the flat frame is explained by the expansion of the universe. But this, as we shall see, is based on an erroneous argument.



1.6 — The three-dimensionality of space

1.6.1 — A quick overview of the existing literature

Many authors have answered this problem. Graig Callender in his article *An Answer in Search of a Question: 'Proofs' of the Tri-Dimensionality of Space* provides an interesting critical overview of these productions⁴. I rely on his work.

The first, it seems, is given by Kant, who observes in his book *Pensées sur la véritable estimation des forces vives...*, in 1746, that the three-dimensionality of space appears to derive from the law of universal gravitation inversely proportional to the square of distance.

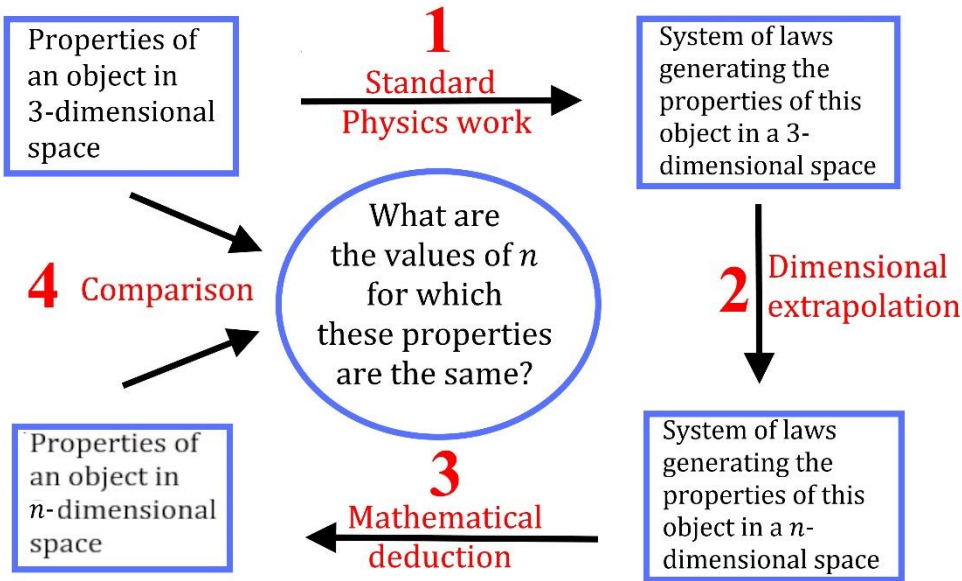
Ehrenfest, in 1918, showed that planetary orbits can only be stable in dimension three⁵. All demonstrations are based on assumptions, and Ehrenfest makes the assumption that in a space of dimension n the law of universal attraction is in $1/r^{n-1}$, property derived from Gauss's law. However natural this law may seem — it corresponds to a total gravitational influence at distance r independent of r and therefore inversely proportional to the 'surface' of a hypersphere with this radius — it nevertheless has its weak points.

— It legislates gravitation in a universe where the number of geometric dimensions is postulated to be other than three, i.e. a universe in which we have not only never had the slightest experiment, but which does not even exist. So, what do we know about the laws of physics that would prevail in that inexistent world? The answer to this objection is to extend to this universe the laws that prevail in our own three-dimensional universe, to show that they are not viable. In this case, however, the extension is not unequivocal. Indeed (as Burgbacher et al. show for the hydrogen atom, but their reasoning extends to planetary orbits⁶), if we derive the force of attraction from a potential energy in $-1/r$ that n may or may not be equal to three, the orbits are stable. In dimension three, both constructions amount to the same thing, so why choose one over the other?

— We might also wonder about the law he chooses: the force of universal attraction. Why this law? Does it hold — apart from the historical aspect — the central role in physics that should hold a law capable of determining the dimension of space? The reasoning certainly also works with the hydrogen atom and the electromagnetic interaction, but why not the strong interaction?

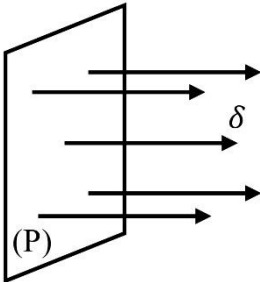
The argument of the stability of planetary orbits or of the hydrogen atom is not the only one to be used. There are various phenomena whose laws place constraints on the number of dimensions in space: for example, the diffraction of neutrons in a crystal⁷, the proper propagation of electromagnetic waves, the spectrum of the atom, the possibility of two atoms or molecules having a non-zero probability of meeting when each follows a Brownian trajectory, and thus of forming polymers⁸.

All these arguments have a common structure:



The fragile point is obviously 2, that of ‘dimensional extrapolation’. We have seen that it is not unequivocal when the object is matter considered from the angle of universal gravitation. It is to be feared that this is not a singular case, and that the dimensional extrapolation of a law is often non-univocal.

Take, for example, the propagation of a plane wave. The wavefront is a plane (P) to which the direction of propagation δ is perpendicular.



How can we extrapolate this situation to a space of, say, four dimensions? Is it the dimension of δ which becomes two, or that of (P) which becomes three? Both solutions are *a priori* admissible, and in either case, we will have no trouble mathematizing them. How do we know which is the right one? And are we not then wondering about the sex of angels, since the case does not exist anyway?



Dayantis' Brownian trajectories argument seems to me to escape this objection somewhat. The law under consideration is that of a point object following an erratic trajectory whose steps all have the same length, with successive directions drawn at random independently of the preceding ones and uniformly in the (compact) set of possible directions. If the dimension of space is greater than three, the (fractal) dimensions of the trajectories are too small for two objects to have a non-zero probability of meeting, and thus of aggregating to form polymers — and thus generating life. The conclusion is given by the anthropic argument: since we exist and are alive, the dimension of space is three.

The demonstration, while ingenious and instructive, only works if we assume the objects, atoms or molecules, to be infinitely punctual, whereas this is not actually the case. As soon as we give them a non-zero size — which dimensional extrapolation obliges — and therefore a non-zero hypervolume, a random distribution of these objects inside a hypercube provides whatever the dimension a non-zero probability that two of them are adjacent.

1.6.2 — Dimensionality of extended matter, dimensionality of space, external and internal dimensionality of the atom

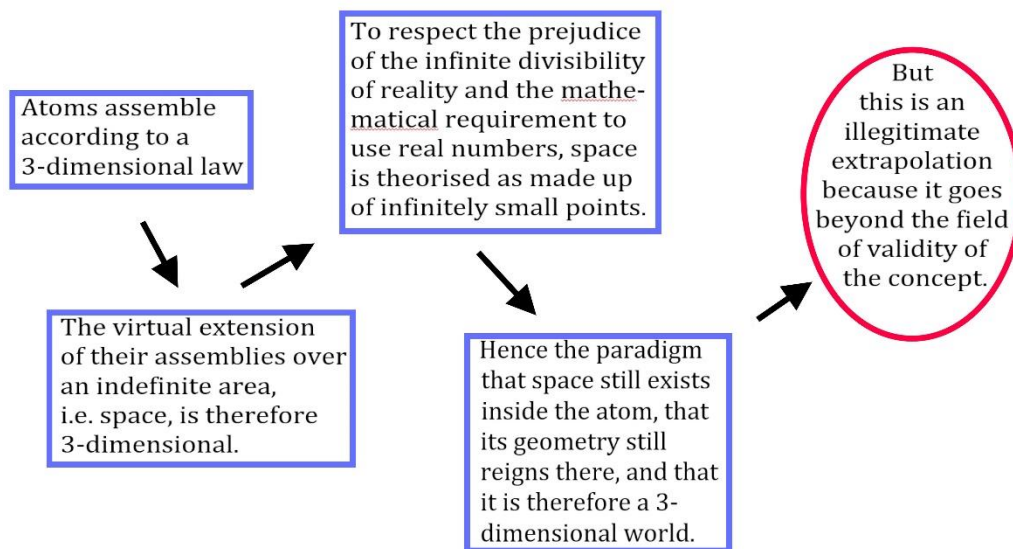
None of these attempts at explanation is entirely satisfactory. Perhaps this is because they are sought where they cannot be. None of them, indeed, adopts the non-chosist point of view, which would begin by observing that 'space', having no real existence of its own, cannot have any dimensions whatsoever. It can only be three-dimensional by inheritance. And this inheritance can only come from that which generates it, matter, of which it is the virtual extension.

The question of why ‘space’ is three-dimensional — or why ‘space-time’ is four-dimensional — is therefore the question of why matter at Galilean rest is three-dimensional.

But then again, matter is made of atoms, and if it is three-dimensional, it is because atoms, when they assemble, do so in three-dimensional structures.

However, it would be going too far to deduce that they are three-dimensional objects. Just as we can pave a plane with tiny three-dimensional balls and thus obtain a surface on a macroscopic scale, we can pave a three-dimensional Euclidean space with objects of infinitesimal size and arbitrary dimension, which does not even have to be the same for all of them. All you have to do is draw in a Euclidean space of dimension n a space of dimension three and pave it.

Today's physics seems to need these ‘folded’ dimensions. String theory (or rather theories), for example, inserts its particles into spaces with 10 or 11 or even 26 dimensions. If we think of space as an ‘in itself’, a ‘thing’, they enter the scene like a rabbit out of a conjurer's hat. If, on the other hand, we think of space as having no reality, then their appearance is no magic exercise. If space is not a ‘thing’, its only realm of validity begins at the scale of atomic assemblies. Below that, it can be anything.



Consequently, nothing obliges us to force us to think of the atom and the subatomic world in general, as enclosed in a three-dimensional straitjacket. Nevertheless, the only concrete view we can take of it is necessarily inserted into the three-dimensional context in which we are and where our experimental devices are located.

As a result, the atom and the subatomic world are geometrically ambivalent. They have an *external* geometry, which we reach from the three-dimensional context via *relay instruments*, rays of various natures with which we extrapolate to the subatomic world the macroscopic geometry, which itself is built on that *prime instrument* that is the memofrom body. This is obviously not without result, as all physics since the end of the nineteenth century testifies. We may also think that this extrapolation, even if it is efficient, even if we have no other way of ‘entering’ the atom, provides us with a framework burdened with a certain lack of legitimacy. It sets up a hiatus between this projected three-dimensional geometry and the *internal real* geometry of the subatomic world. The emergence in today's thinking of ‘folded’ dimensions seems to confirm this point of view, at the same time as lending credence to the thesis of the unreality of space.

It seems, then, that we are in a position to propose an explanation of the three-dimensionality of space that is rather different from those listed above:

1) It is not space that is three-dimensional, but the matter that generates it.

2) Atoms are not *a priori* intrinsically three-dimensional objects. They can be considered

— either as intrinsically dimensional n depending perhaps on their nature,

— or as being of indeterminate dimension, within the framework of a geometry less constraining than if its dimension were fixed, but nonetheless sufficient for us to write the laws governing their behaviour. The ‘dimension’ parameter is quite secondary in geometry. The notions of alignment and distance do not depend on it. The only difference is that, as the dimension increases, more different figures can be drawn. The system of axioms proposed later in the chapter *An axiomatization of geometry based on instrument and experiment* comprises 21 axioms, the last of which fixes the dimension of space. By removing it, we obtain an axiomatization of a Euclidean space of unknown dimension greater than or equal to 2.

3) In any case, the world of the atom is governed by a combinatorial of quantum numbers that determine the structure of its electron cloud and, in particular, that of its outermost layer. This combinatorial system, in itself, offers no evidence of being attached to one dimensionality rather than another. The number of balls of radius R that can be placed around a ball of

the same radius, tangent to it, grows very rapidly with the dimension of space⁹. In dimension 5 it is already greater than or equal to 40. In dimension 4 it is equal to 24. These values exceed the possibilities of the valence layer of atoms determined by quantum combinatorics. In dimension 2, this number is 6, which, on the contrary, is insufficient. That leaves 3.

4) Extended matter is therefore three-dimensional. Let (E) be a 'space' constructed around a location O, in the sense of a virtual extension of a memiform body in O, and M a point of 'space', in the second sense of a set of possible positions of infinitesimal body in the cosmos. Stretch a long string between these two points and it will have a definite trace in space at O. Each point M can thus be described by a direction from O in three-dimensional space (E) and by a distance, the length of string it takes to reach it. The 'space' — in both senses of the word - is therefore three-dimensional.

This demonstration — or rather 'monstration' — escapes the flaw of haphazardly extrapolating one physical law from dimension 3 to another. In fact, it puts its finger on laws — those of quantum combinatorics that construct electronic layers — which it justifies as not having to be considered as bound to a particular dimension, and shows that these laws construct a three-dimensional universe.

The non-choisist hypothesis makes it possible to give the problem of the three-dimensionality of space an explanation free from the defect of dimensional extrapolation. It also justifies the existence of 'folded' dimensions.

1.7 — Lorentz contraction and the slowing down of moving clocks

Here again, the driving force is the same: it is absurd to attribute any action whatsoever on moving rulers and clocks to a space and time — or to a space-time — that would have no physical reality. The explanation of these **Lorentzian** phenomena can only be sought where it can be found, in the properties of matter.

We know today — and we didn't know when the theory of special relativity was born — that the atom can be likened to a stable vibratory system possessing a certain expanse. At its various points, physical variables take on periodic values, which are from one point to another synchronized by electromagnetic interactions, in other words by light.

Let (E_{is}) be an isotropic Galilean space, i.e. considered to be the same in all directions. Let us consider a stationary atom in this space. It is traversed in all directions by electromagnetic interactions which, as a consequence of the isotropy of (E_{is}) , are all moving at the same speed, which we call C . The time taken by light to travel back and forth or in more or less complex closed cycles between the various singular points, nodes or wombs, of its wave system are part of its rhythmic architecture, and are therefore in tune with it.

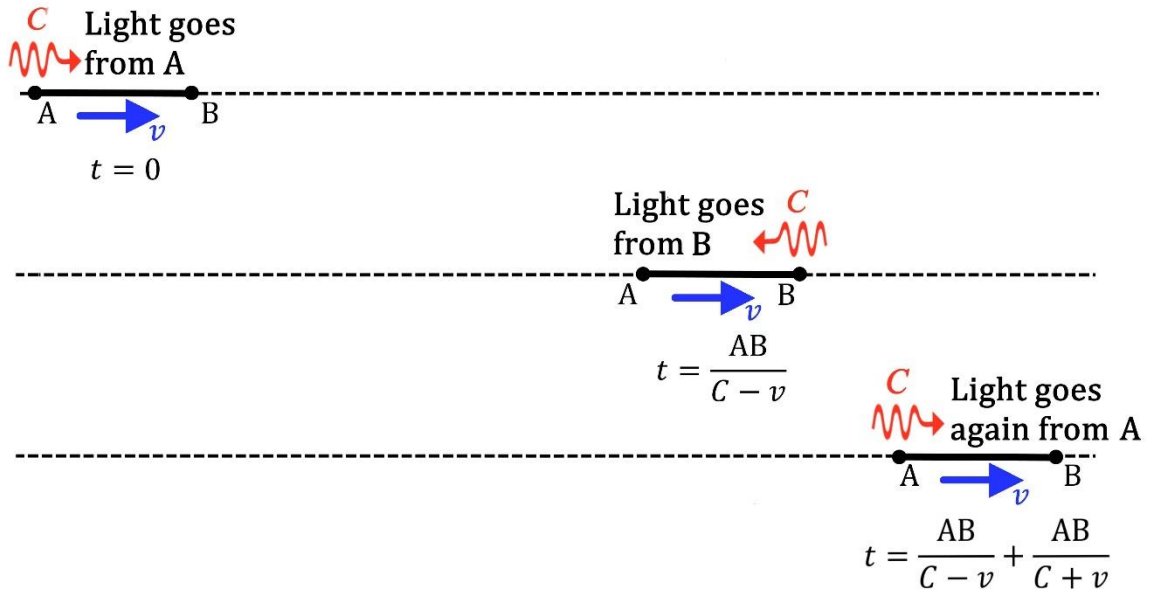
A **Galilean** space is the set of 'events' (in the sense given to this word by special relativity) whose three spatial coordinates are eternally invariant relative to a given Galilean reference frame. A Galilean frame of reference defines a single Galilean space, and any Galilean space can be equipped with an infinite number of Galilean frames of reference.

Suppose ABA is an elementary cycle, traversed in time T . We therefore have $AB = C \frac{T}{2}$.

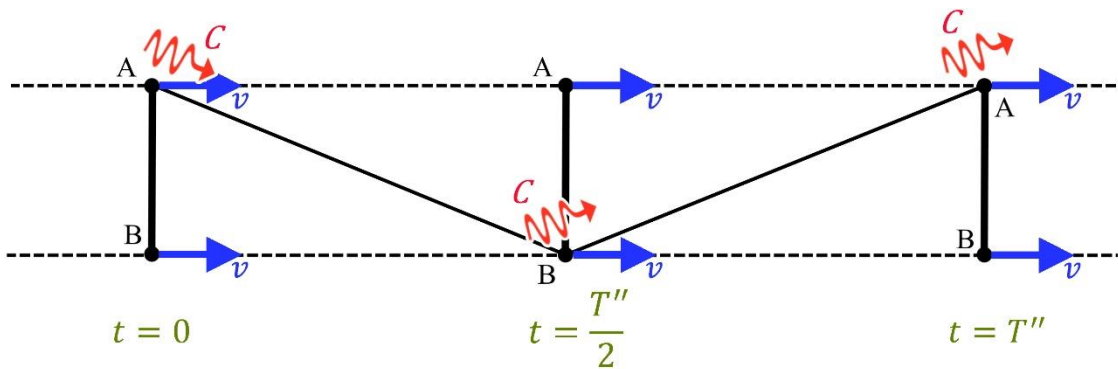
Consider this same atom moving at speed v relative to (E_{is}) . Assume that all its geometric dimensions relative to (E_{is}) remain invariable.

When AB is collinear with \vec{v} , the time relative to (E_{is}) of the ABA path becomes

$$T' = \frac{AB}{C + v} + \frac{AB}{C - v} = C \frac{T}{2} \times \frac{2C}{C^2 - v^2} = \frac{T}{1 - \frac{v^2}{C^2}}$$



When AB is perpendicular to \vec{v} the objective time T'' of the path is by Pythagoras' theorem, $(C \frac{T''}{2})^2 = AB^2 + (v \frac{T''}{2})^2$ hence $T'' = \frac{4(C \frac{T}{2})^2}{C^2 - v^2}$, thus $T'' = \frac{T}{\sqrt{1 - \frac{v^2}{C^2}}}$.



When \vec{AB} makes with \vec{v} an angle other than 0 or $\frac{\pi}{2}$ the intermediate value theorem assures us that the ABA cycle duration takes all possible values between $\frac{T}{\sqrt{1 - \frac{v^2}{C^2}}}$ and $\frac{T}{1 - \frac{v^2}{C^2}}$.

Thus, if the dimensions relative to (E_{is}) of the atom were to remain invariant to change in its velocity, the internal electromagnetic cycles, which

are necessarily tuned to one another for a stable rhythmic structure to exist, would see their durations dispersed. The atom would not retain its architecture and would cease to exist. Therefore, its dimensions must change when its speed changes. This change in dimensions, in addition to ensuring the atom's survival, means that the durations of the cycles performed by light in its domain also vary.

The dimensions and rhythms of matter relative to an isotropic Galilean space are not the same when it is stationary or in motion relative to this space.

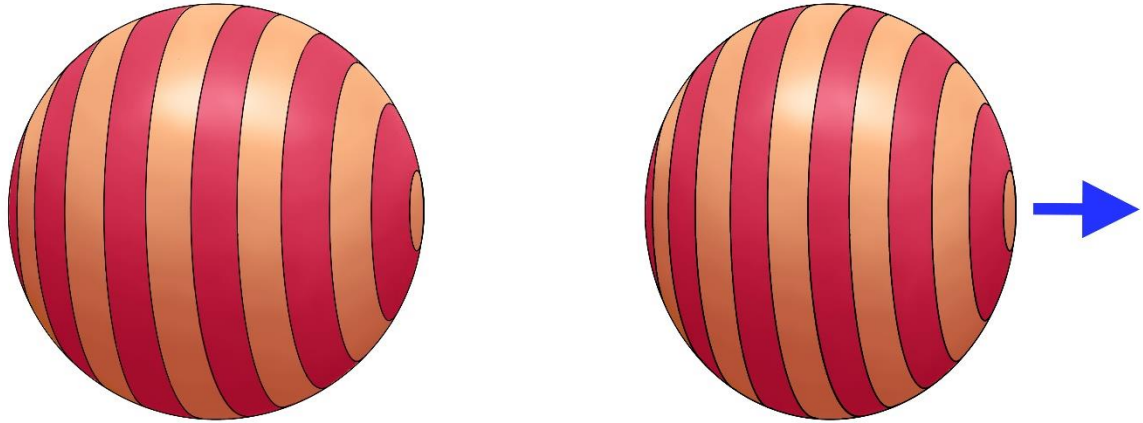
The following result is demonstrated in the appendix:

The Lorentzian shortening of lengths is perfectly sufficient to preserve the wave structure of matter at rest.

This allows us to determine, in the general case, the deformations of matter in uniform translation that preserve the wave structure of atoms. The law that governs them obeys a symmetry of rotation around any straight line collinear with the speed of translation, and is everywhere the same along such a straight line. This leaves room only for homogeneous dilations and contractions. By rotational symmetry, the planes perpendicular to this line are globally invariant. Since the round-trips of light along these planes must be matched by the round-trips of light along the straight lines collinear with the speed of translation, one determines the other, and vice versa. The value of dilation or contraction colinear with the speed of translation therefore determines at most one solution to the problem. The homothetic of a solution is a solution, and we know of one, the Lorentz contraction. Its homothetics cover all possible dilations or contractions colinear with the speed of translation, and thus form the set of solutions to the problem.

Of all these homothetical Lorentz contractions that preserve the wave structure of matter, the question remains which is the right one. Experiment tells us that it is the Lorentz contraction itself.

We can try to explain it. The rebalancing that matter undergoes to preserve its wave structure is carried out as economically as possible. When an atom is in motion relative to an isotropic space, the successive slices that can be cut into it perpendicular to its velocity do not have to be modified in their transverse extent to preserve their wave architecture. Rebalancing is therefore limited to axial modifications.



What applies to an atom applies to any block of matter and to all particles that can be assimilated to periodic systems of a certain expanse, whose internal synchronization takes place at the speed of light.

The non-relativistic hypothesis, by reducing the properties of space-time to those of matter, explains Lorentz's contraction and the slowing down of the rhythm of matter in motion.

1.8 — The invariance of the speed of light

If the reference Galilean space is isotropic space (E_{is}), light respects this isotropy and therefore has the same speed relative to it in all directions. Let (E) be another Galilean space, moving at speed \vec{v} relative to (E_{is}). Let (B) be a ball of intrinsic radius R and center O, stationary in (E), intrinsically perfect in the sense that each of its rays has the same length as an alignment of N identical atoms. It is flattened when considered from (E_{is}). However, when viewed from (E), which is generated by virtual memoform matter moving at speed \vec{v} relative to (E_{is}), and whose geometric instruments immobile relative to it, are contracted in the same ratio and direction, it is a geometrically perfect ball. For any point M on its surface, the OMO light cycles have the same duration. If we provide (E) with light-simultaneity, which attributes the same duration to the OM and MO paths, the speed of light relative to (E) is the same in all directions.

This is easily calculated along a round trip perpendicular to \vec{v} in (E). Relative to (E_{is}), its length is $\frac{2R}{\sqrt{1-\frac{v^2}{c^2}}}$, its relative duration is $\frac{2R}{c\sqrt{1-\frac{v^2}{c^2}}}$ as the clocks in (E), based on the rhythm of matter, beat $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ times slower than those of (E), the duration relative to (E) of this round trip of length $2R$ in (E) is $\frac{2R}{c}$.

The non-choisist hypothesis explains the invariance of the speed of light relative to all Galilean spaces when measured with instruments immobile relative to this space.

1.9 —The impossibility of exceeding the speed of light

Let us call ‘Lorentzian’ an object identifiable with a stable vibratory system where the local values of certain variables are dependent on those taken in other places by interactions moving at the speed of light. Such is the case with atoms. However, as other particles manifest this same slowing-down as they do, we have no reason to believe that the category of objects structured in this way is limited to atoms.

Let us consider a Lorentzian object (LO) in motion relative to (E_0) , and A and B two of its locations in which such synchronized variables beat. If (LO) is to retain its vibratory structure and continue to exist, light from A must be able to reach B, and vice versa, which is only possible if the velocities of these points in (E_0) are lower than that of light.

The non-chosist hypothesis explains the impossibility of matter in motion exceeding the speed of light.

This impossibility is based — once again — on considerations different from those stipulated by special relativity. It derives from the properties of matter, whereas special relativity makes it a property of space-time. It applies only to Lorentzian objects, and it could be that there are non-Lorentzian objects that can travel faster than light, whereas in special relativity this hypothesis would contradict the causal link.

2 — IS THE FAR SKY THE GREATEST TROMPE-L'OEIL?

2.1 — The zero degree of rotation problem

Whatever the arguments in its favour, the non-choisist hypothesis seems to run up against a difficulty. Since it considers each space as strictly relative to the body that generates it, it equips the universe with a 'jumble' of spaces to which nothing absolute seems to can be attached. But this is not the case, since there is a universal — and therefore absolute — zero degree of rotation.

In the choisist framework, the existence of this zero does not raise any difficulty, since the universe is thought of as being inserted 'into' the 'space' or 'space-time' of which this zero is one of the attributes. In the non-choisist framework, however, this absolute space disappears and with it the explanation it provides. The impossibility of providing an explanation would ruin the non-choisist hypothesis. Let us show that this is not the case.

Let us place ourselves in a non-choisist universe. Space, time and space-time are what is returned back to us by instruments used according to 'legitimate' protocols that, for the moment, we do not specify.

Let (O) be a body in O, generating a space (E). The velocities of bodies depend heavily on the choice of (O), since replacing (O) by (O') rotating on itself in O adds to the velocity of a body in M the vector quantity $\vec{\omega} \wedge \overrightarrow{OM}$, which in general tends towards infinity when M tends towards infinity. Nevertheless, this replacement introduces no change anywhere, except in O itself, which is perfectly negligible on the scale of the universe. Since the notions of distance, duration and simultaneity depend on the physical behaviour of the instruments in the entire universe, which themselves depend on the state of the universe, this replacement introduces no significant change in the notions of distance, duration and simultaneity in the universe we consider. In this universe where we do not yet have a zero degree of rotation, the velocities of bodies relative to the spaces generated by other bodies are of little relevance. The relevant velocities are the algebraic distances between bodies, in other words the time derivatives of the distances between bodies. By virtue of the homogeneity of the universe, we can find a universal protocol for defining them.

The **cosmological principle** ensures that the universe, considered on a sufficiently large scale, is in every place more or less the same — homogeneous — and all the better for the larger the scale. We have made implicit use of this by asserting that the sidereal desert is universal. This principle allows us to state that

The probability law giving the variation of the distance between two bodies (A) and (B) in a lapse of time τ depends only on the distance AB.

Let $O_0 = O, O_1, O_2, \dots, O_n$ be bodies forming a regular alignment of pitch D . The elongation speed of $O_k O_{k+1}$ obeys for all values of k the same probabilistic law depending on parameter D . Let us call \bar{v}_D its average and σ_D its standard deviation. We will note $v_D = \bar{v}_D \pm \sigma_D$

Lengthening OO_k during a short time τ is the cumulative elongation of lengths $OO_1, O_1 O_2, \dots, O_{n-1} O_n$ at the instants at which they are considered.

In the case of Galilean kinematics, relative elongation is

$$\frac{\Delta OO_n}{OO_n} = \frac{\Delta OO_1 + \Delta O_1 O_2 + \dots + \Delta O_{n-1} O_n}{nD}$$

Random variables $\frac{\Delta O_{k-1} O_k}{D}$ obey the same probability law, and we postulate that they are 'largely independent', i.e. sufficiently independent for their sum to be the object of a global compensation as it is the case when they are exactly independent. We do not require that the failure of exact compensation be proportional to \sqrt{n} but only that it be negligible compared to n and nevertheless tends towards infinity with n .

$$\begin{aligned} \frac{\Delta OO_n}{OO_n} &= \frac{(\bar{v}_D \tau \pm \sigma_D \tau) + (\bar{v}_D \tau \pm \sigma_D \tau) + \dots + (\bar{v}_D \tau \pm \sigma_D \tau)}{nD} \tau \\ &= \frac{\bar{v}_D \tau}{D} \pm \frac{(\pm \sigma_D \pm \sigma_D \pm \dots \pm \sigma_D) \tau}{nD} = \frac{\bar{v}_D \tau}{D} \pm \frac{o(n) \tau}{nD} = \frac{\bar{v}_D \tau}{D} \pm \frac{\varepsilon(n) \tau}{D} \end{aligned}$$

où

In this formula $\varepsilon(n)$ tends to zero when n tends to infinity, and $\frac{\bar{v}_D}{D}$ is a universal constant, positive if the universe is expanding, negative if it is contracting, and zero if it is stable. This formula teaches us that, in the context of Galilean kinematics, when the length of an alignment of bodies tends towards infinity, its relative variation over a unit of time tends towards a

certain constant characteristic of the universe under study. The same is trivially true when the kinematics is Lorentzian, since the lengths of the alignments are not bounded, whereas the velocities of the bodies are.

In both cases, the postulates of homogeneity and isotropy mean that, given two bodies (A) and (B) far apart, a regular graduation can always be placed between them with sufficient precision. These postulates also mean that this graduation obeys the principle of relativity: it is "the same" from A to B as from B to A. There are only two relativistic kinematics of one-dimensional affine space, the Galilean and the Lorentzian ones, thus it follows that :

In a non-choisist universe obeying the cosmological principle, the relative variation in a unit of time in the distance separating two bodies tends towards a universal constant when this distance tends towards infinity.

We postulate that the space-time framework of this universe is reasonable enough for the following statement to be inferred from the one above:

In a non-choisist universe obeying the cosmological principle, the relative variations in one unit of time of the distances separating three bodies tend towards the same universal constant when the smallest of these distances tends towards infinity.

Let us consider, in a non-choisist universe obeying the cosmological principle, triangles whose vertices are celestial bodies. The lengths of their sides are such that the relative variations in one unit of time of their ratios tend towards zero when the smallest of these lengths tends towards infinity. We showed in Chapter 1 that the non-choisist frame must be Euclidean. Consequently, the variation in one unit of time of the angles of these triangles tends towards zero when the length of their smallest side tends towards infinity. Thus, using the language of naïve non-standard analysis, it follows that:

In a non-choisist universe obeying the cosmological principle, a triangle of which the vertices are infinitely far apart celestial bodies, has its angles invariant in time. There is therefore a sphere of fixed stars.

2.2 - 'Synchronous' sphere of fixed stars and 'Observed' sphere of fixed stars

We are not out of the woods yet, for the sphere of fixed stars derived from the cosmological principle, the 'synchronous sphere of fixed stars' exists today, at this very instant. We would have to be sitting in 'God's chair' to be able to embrace it with our eyes. In fact the sphere we see is made up of 'fossil' images extracted from the spheres of the fixed stars of the past, whose light reaches us with a greater delay the further away from us the bodies that emitted it were.

We can understand how the observed sphere of the fixed stars is formed only by postulating certain properties of light propagation.

Let us install in O a body generating a space relative to which the synchronous sphere of fixed stars is immobile. Our universe is massively made up of desert expanses, so that O is generally far enough away from any other body to be considered as being subject to 'no action'. We can therefore consider it as a Galilean object, as is the space it generates. Let us assume that light travels relatively to it in a straight line at an invariable speed. Since the 'infinitely distant' bodies form an invariable and immobile sphere of fixed objects in this space, the light coming from them to us follows straight lines of invariable direction: we, who are not in God's chair, nevertheless see, a few billion years later, the 'infinitely distant' bodies forming a mutually interdependent whole. This is the observed sphere of the fixed stars.



The non-choisist paradigm explains the existence of a sphere of fixed bodies and an 'absolute' zero degree of rotation. These two absolutes are only apparently so, and their existence does not contradict the non-choisist hypothesis that space, time and space-time are not real, but are merely grids that we project onto the real world.

2.3 — Illusion of expansion in a stable non-chosist universe

2.3.1 — Distant bodies fleeing away: the seemingly limp ‘traditional’ reasoning is wrong

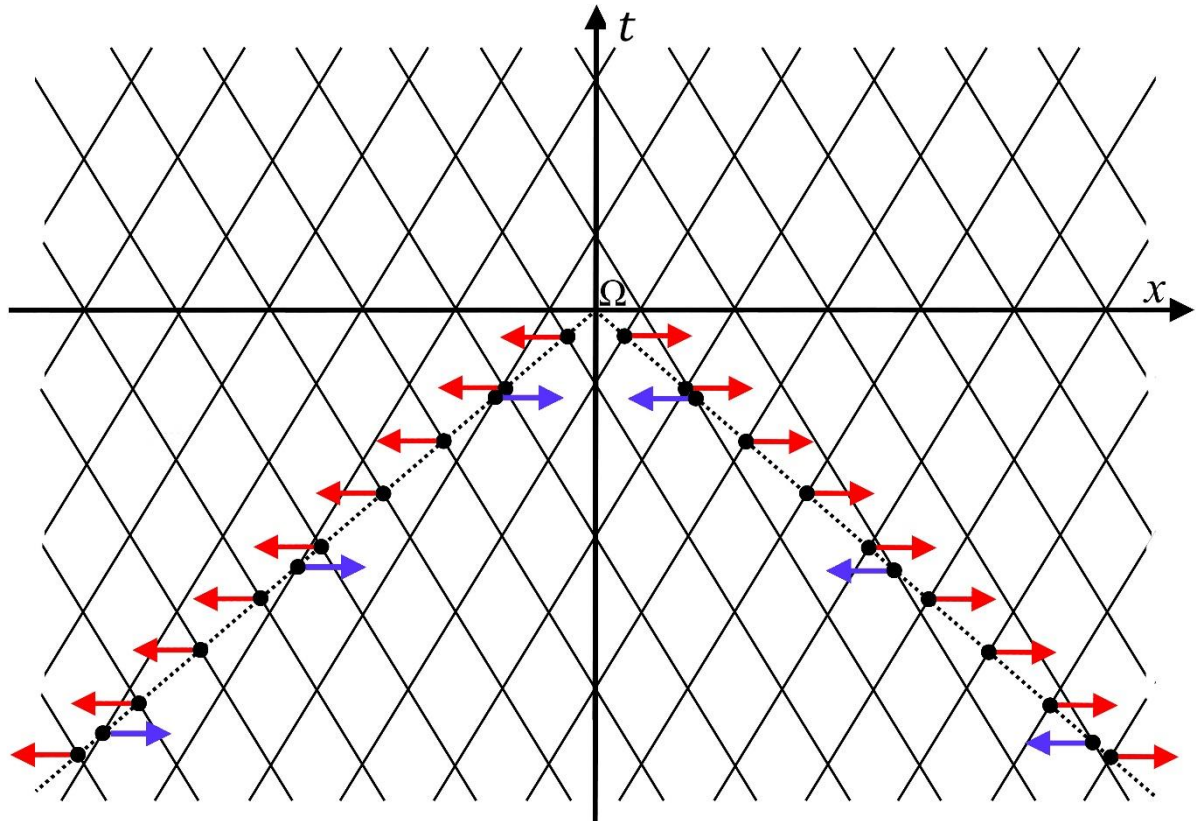
The ‘traditional’ reasoning for inferring the expansion of the universe from the observed escape of distant celestial bodies is as follows:

Let us assume that the universe is stable in time, and therefore neither expanding nor contracting. What we see at distance D from us is the image of a past that took place at the instant $-D/c$. At that instant, as the universe is stable, there were as many bodies moving away from us at distance D as there were moving towards us. So, what we see, when we look at distance D , is half of the bodies moving away, half of the bodies moving towards us. Since this is true whatever the distance D , we must see in the sky as many bodies moving away as approaching. But this is not the case, since we see more bodies moving away than moving towards us at any sufficiently large horizon D . This means that our universe is not stable, but expanding.

To test this reasoning, let us consider a very simple model of the universe, reduced to a straight line on which all bodies move at the same arithmetic speed v in one direction or the other. These two families are made up of regularly spaced bodies. (The fact that this universe is of dimension one does not prevent the bodies from crossing each other, nor does it prevent us from seeing through them).

In a plane related to the reference frame (Ω, x, t) where x is the space variable and t that of time, let us trace the curves representing the movements of bodies relative to a location O in this universe. These are two families of parallel, regularly-spaced straight lines with slopes $\frac{1}{v}$ and $-\frac{1}{v}$

Let us also trace, in dotted lines, the cone of light arriving at O at time 0 . It is formed by two half-lines coming from Ω with slopes $\frac{1}{c}$ and $-\frac{1}{c}$. Its intersections with the preceding curves show how the celestial bodies are seen from O , at instant 0 : in red those seen moving away, in blue those seen moving closer.



An observer in Ω sees more bodies moving away than approaching, even though exactly as many are doing one as the other. The above reasoning is therefore incorrect.

Indeed, it has a bias. Given a distance D the probability that a celestial body exists at distance D from O is zero. What has to be considered at a given instant are not the bodies at distance D from O , but those whose distance from O is between D and $D + dD$. This set is indeed made up of as many approaching bodies as there are moving away. However, when we observe from O the bodies between these two distances, we are not dealing with a set of this type, since what we see at horizon $D + dD$ is older than what we see at horizon D .

2.3.2 — Qualitative analysis

In an invariant universe obeying the cosmological principle and endowed with a thing-space, in other words an absolute space, the velocities of bodies are on average everywhere and always the same *relative to this absolute space*. Whereas in an invariant non-choisist universe obeying the same principle, where there is therefore no thing-space, they are on average everywhere the same *relative to bodies in their neighbourhood*.

In the first hypothesis, the velocities of distant bodies relative to absolute space are the same as those of nearby bodies: they are "small".

In the second hypothesis, similar to what happens when you toss a coin indefinitely, the velocities result from an accumulation in which they compensate each other only imperfectly. The lack of compensation is the speed of these bodies relative to the first one. When their distance tends towards infinity, these arithmetic speeds, whether they are those of bodies moving away or towards us, tend in probability towards infinity if the kinematics is Galilean, towards C if it is Lorentzian.

This difference between the two cases has sufficiently important effects for us to give a name to the universe model concerned. In homage to the man who persisted to the stake in asserting what he believed to be true, we will call universe of Bruno a universe which is flat, infinite, stable (without expansion nor contraction), non choisist, and obeys the cosmological principle. In particular, on a sufficiently large scale, it is homogeneous and isotropic.

'Isotropic' means: among all historicised Galilean points intersecting at any local instant there is one (and only one) relative to which the universe is 'the same' in all directions.

Furthermore, we attribute to a Bruno universe the property of being random, in that sense that the reasonings made assuming it to be random, analogous for example to those that can be made with a gas, give relevant results.

Christopher Conselice¹⁰ and al. have shown that the number of galaxies in the observable universe, previously estimated at 200 billion, must be multiplied by ten by adding a very large number of small galaxies. In a section devoted to Olbers' paradox, the authors show that the number of galaxies is such that any point in the sky almost certainly belongs to a distant galaxy.

In a Bruno universe, homogeneity implies that all spherical rings of the same thickness r absorb the same proportion p of the luminous flux that passes through them towards their centre. They therefore let through the proportion $1 - p$, and a ring of thickness nr will let through the proportion

$(1 - p)^n$, which tends towards zero when n tends towards infinity. On this point, a Bruno universe is in accordance with recent observations by Conselice and al.

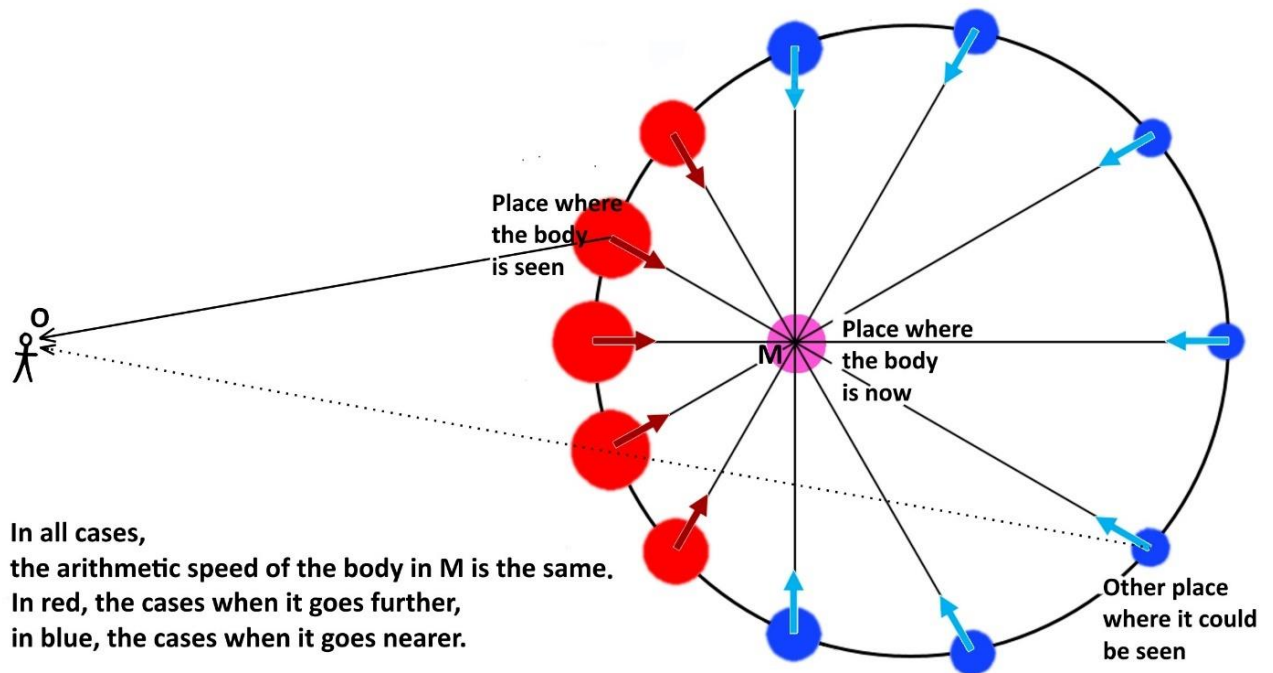
Let us consider, in a Bruno universe, a location O marked by a Galilean body at rest, such that the Galilean space (E) it generates is isotropic at O . At time $t = 0$ of the chronology of this space, a generic celestial body (K) is located at a point M at distance D from O . What an observer at O sees of it at this instant is not what it is at this instant, since light does not propagate at infinite speed.

Taking as reference the Galilean space that is tangent to the motion of (K) at time $t = 0$ and running time backwards, the particular circumstances of (K) 's path modify its speed, which is zero at time $t = 0$, according to a probability law that can reasonably be assumed to be spherically symmetrical, and whose expected value should therefore be zero. The 'mean' position of (K) , at the instant the observer in O sees it, should therefore be about that which it would have if its path were uniform. Let us call it P . We have

$$\frac{PM}{PO} = \frac{v}{C}$$

As a result of a classical geometric theorem, for a given arithmetic speed v , the set of points P is the sphere of diameter $[UV]$, where U and V are the points that divide $[OM]$ in the ratio $\frac{v}{C}$. The range (O, M, U, V) is harmonic and the points of tangency of the tangents from O to this sphere lie in the plane perpendicular at M to (OM) .

In the non-chosist hypothesis that we are now assuming, when D tends towards infinity, the velocities of bodies at this horizon, whatever their direction, tend to become almost everywhere infinitely large if the kinematics is Galilean, infinitely close to C if it is Lorentzian. The distance MP is then of the same order of magnitude as the distance OM .



Whether moving towards or away, bodies currently at M and having the same arithmetic velocity in all possible directions are seen at very different places. Those who approach us today are seen considerably farther away than where they are today, those who move away considerably closer.

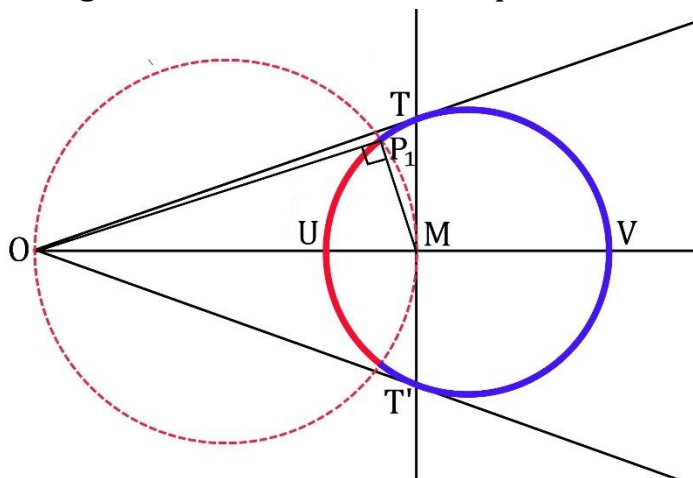
— This phenomenon increases the apparent size of bodies moving away from us, and decreases that of those moving towards us. It makes the former more visible than the latter.

— Bodies moving away are seen through a smaller slice of the universe, which is not perfectly transparent (interstellar and intergalactic mediums, and in places totally opaque (planet, stars, ...)). As a result, their brightness is less attenuated than that of approaching bodies.

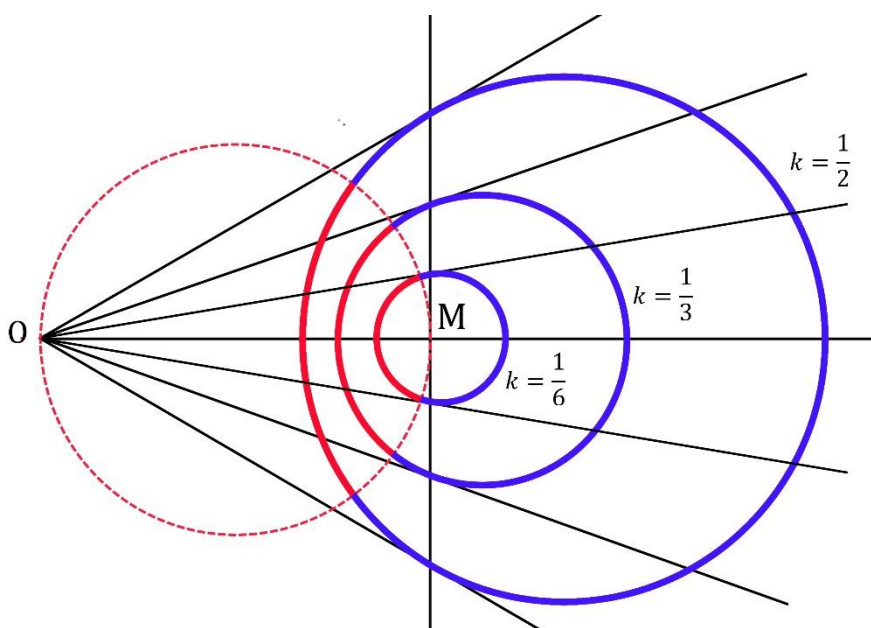
— Bodies moving away, being seen closer than approaching ones, are more likely to obscure them than to be obscured by them. A red body hides more blues than a blue hides reds.

— These effects are all the more intense the further you look. At the limit — we will show below — we see only bodies moving away. The universe appears to be expanding, when it is not.

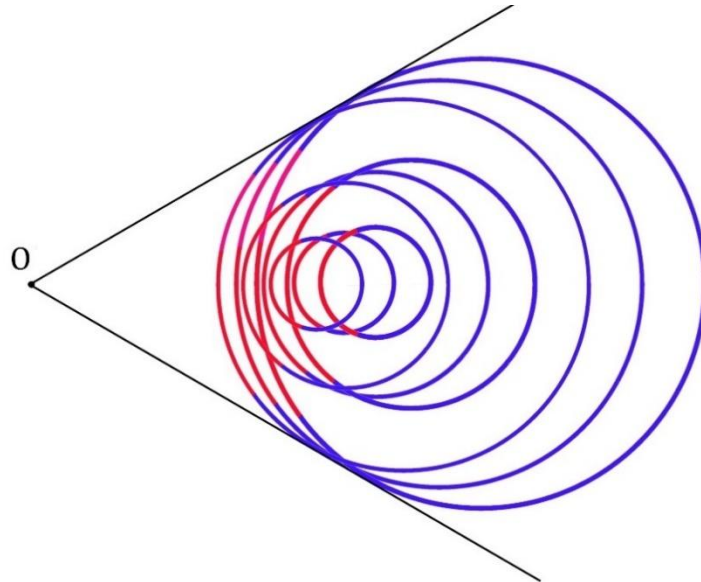
Let P_1 be a point separating the 'red' set where the body is seen moving away from the 'blue' set where it is seen approaching. This point is such that the angle $\widehat{OP_1M}$ is straight. It is therefore on the sphere of diameter $[OM]$.



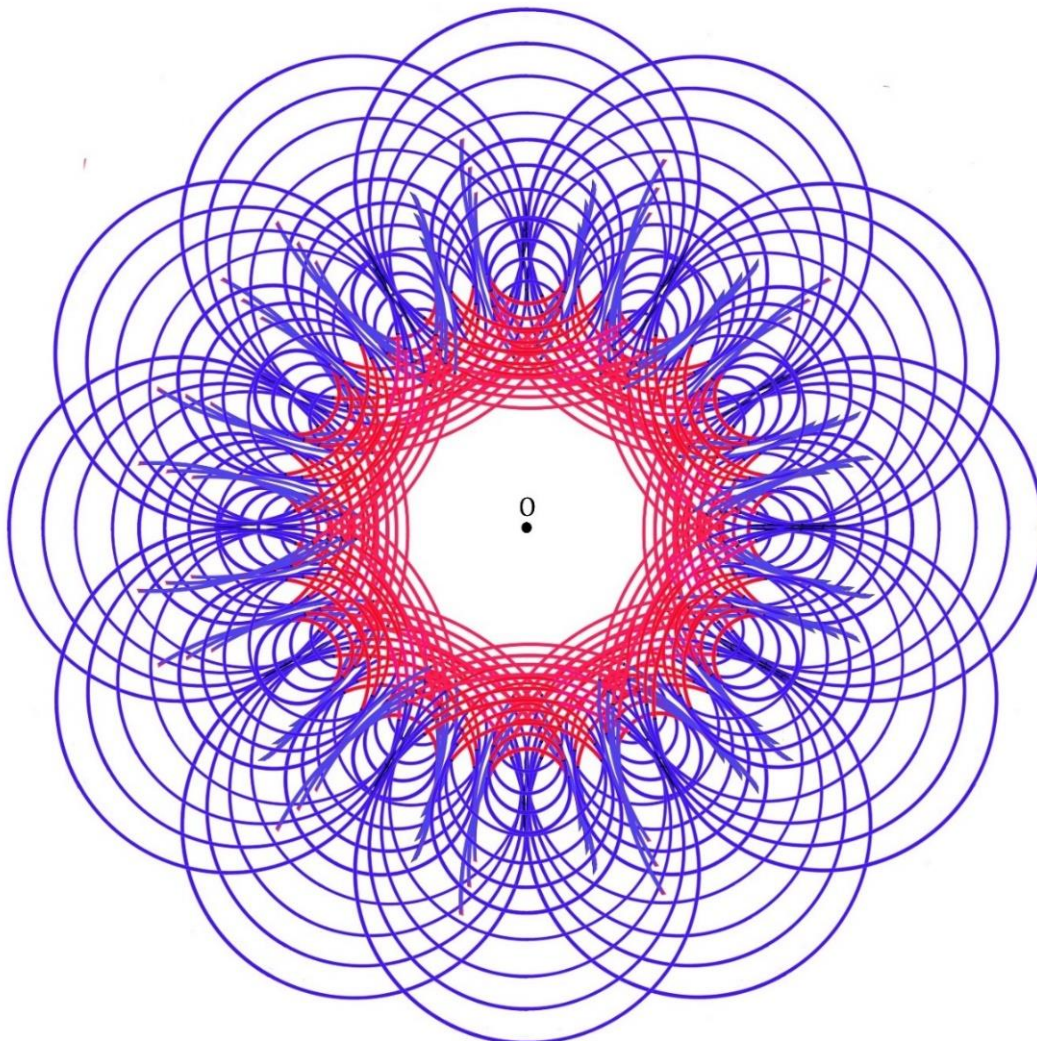
For the same point M , we can construct the sphere of points P for different values of $k = \frac{v}{c}$.



We can also vary the distance $D = OM$ by dragging the point M on the line (OM) . As for a given k the harmonic range (O, M, U, V) which determines the figure, is transformed into a homothetic harmonic range, the figure obtained, for this D and this k is homothetic to the initial figure.



Finally, we can rotate the previous configuration around 0 in any possible way.



2.3.3 — Conclusion

The reasoning above is a mixture of "monstrations" and demonstrations, which does not produce a true demonstration in the mathematical sense of the term. But if the point is to convince, the monstration can have the same force as the demonstration.

Let us examine again the traditional reasoning by which we deduce the expansion of the universe from the fact that from a certain distance we see all the bodies moving away:

Let us assume that the universe is stable in time, and therefore neither expanding nor contracting. What we see at distance D from us is the image of a past that took place at the instant $- D/C$. At that instant, as the universe is stable, there were as many bodies moving away from us at distance D as there were moving towards us. So what we see, when we look at distance D , is half of the bodies moving away, half of the bodies moving towards us.

The figures above, drawn in a stable universe, where there are on average at distance D as many bodies moving away as there are moving towards, show us that this assertion is erroneous and that, on the contrary, we see more bodies moving away than moving towards.

This does not obviate the need for a proper demonstration, and this book will provide one. Before doing so, we have to study other questions, which are linked to those concerning the premises on which the above monstration is based.

— The replacement of bodies moving at non-constant effective velocities by bodies moving at uniform velocities becomes less and less relevant when it takes place over greater and greater distances.

— We have reasoned in a global framework, whose constructivist paradigm obliges us to deny reality 'in itself'; this framework must thus be *constructed*, and its construction can only be achieved by assembling local frameworks — which are themselves to be constructed. We will see that the latter must be the local isotropic Galilean spaces, the assembly of which is made difficult by the fact that they drift relative to each other. Two rulers or two clocks that are stationary relative to each other, do not move at the same speed relative to their respective local isotropic spaces, and therefore do not behave in the same way.

— We attribute an invariant speed C to light relative to any Galilean space, but we will show that the Lorentzian deformations of rulers and clocks must be considered as objective when they are relative to the isotropic space.

This invariance must therefore be considered as the effect of an instrumental bias that must be taken into consideration

Our work on space-time issues from a constructivist angle has only just begun, and we have many points to explore before we can approach this problem in a solid manner.

At the end of this overview, we conclude that the non-chosist paradigm has enough arguments in its favour for us to adopt it, at least in this book.

We shall therefore adopt it and, since space-time expanses are then constructed notions, we shall begin by giving a rigorous construction of them in the non-chosist framework.

¹ An excellent and lively presentation of the Banach-Tarski theorem can be found on YouTube under the title Deux (deux?) minutes pour le théorème de Banach-Tarski, by El Jj - it lasts about thirty minutes.

² This book did not immediately find its final form. Originally a series of articles on Brownian motion published in 1907-1909, Jean Perrin summarised them in 1909 under the title *Mouvement Brownien et réalité moléculaire* in the *Annales de Chimie et de Physique*, followed by the book *Les Atomes* in 1913, which has been reprinted many times since.

³ G. Hinshaw, D. Larson, E. Komatsu, D. N. Spergel, C. L. Bennett et al., *Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results*, Supplement Series, The American Astronomical Society, vol. 208, n° 2, octobre 2013, p. 1-25, article n° 1.

⁴ *Studies in History & Philosophy of Modern Physics*, March 2005, pp. 113-136.

⁵ P. Ehrenfest, In what way does it become manifest in the fundamental laws of physics that space has three dimensions? *KNAW, Proceedings*, 20 I, 918, pp. 200-209.

⁶ F. Burgbacher and C. Lammerzähl and A. Macias, *Is there a stable hydrogen atom in higher dimensions?* *Journal of mathematical physics*, vol.40, num. 2, pp. 625-634, 1999.

⁷ Caruso, F. and Xavier, R.M., On the Physical Problem of Spatial Dimensions: An Alternative Procedure to Stability Arguments, *Fundamenta Scientiae* 8, 1987, pp. 73-91.

⁸ Dayantis, J., *Pourquoi l'espace dans lequel nous vivons a trois dimensions et ne peut en avoir davantage ?* *Annales de la Fondation Louis de Broglie*, Volume 32, n° 4, 2007, pp. 513-517.

⁹ Mittelman, Hans D., Vallentin, Frank (2009), *High accuracy semidefinite programming bounds for kissing numbers.*, *Experimental Mathematics* 19: 174-178, arXiv :0902.1105.

O. R. Musin (2003), *The problem of the twenty-five spheres*, *Russ. Math. Surv.* 58 (4), 794-795.

¹⁰ Christopher J. Conselice, Aaron Wilkinson, Kenneth Duncan et Alice Mortlock, *The evolution of galaxy number density at $z < 8$ and its implications*, *The Astrophysical Journal*, oct. 2016.