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1	Numerical investigation of spreading time in droplet impact with
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ABSTRACT

Spreading time, the time that an impacting droplet attains the maximum wetting area on a 26 solid surface, plays a critical role in many engineering applications particularly where heat 27 transfer or chemical reactions are involved. Although the impact dynamics of a droplet 28 significantly differ across the different spreading regimes depending on various collision 29 parameters, it still remains unclear (i) how the spreading time changes for each spreading 30 regime, and (ii) how the target curvature can affect the spreading time. In the present study, 31 32 the spreading time during droplet impact on a spherical target is systematically studied at the 33 three different spreading regimes for a wide range of impact parameters (Weber number, 34 equilibrium contact angle, Ohnesorge number and droplet-to-target size ratio). The changes of spreading time depending on the impact parameters and underlying physical mechanisms 35 are analyzed in detail at the level of three different spreading regimes. Our results show that 36 the spreading time, proper time scales, dominant impact parameters and associated physical 37 38 behaviors all significantly and non-linearly change across the three spreading regimes. An improved prediction model for the spreading time is also proposed for each regime, which is 39 now based on only the controllable variables and has an explicit form. The effect of target 40 curvature on the spreading time is further investigated, and finally, a data-driven prediction 41 42 model is proposed to represent the complicated and non-linear nature of the spreading time broadly across the three spreading regimes. 43

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Keywords: droplet impact; spreading time; multiphase flow simulation; prediction model;
data-driven analysis

47 I. INTRODUCTION

48

49 Collisions of droplets with solid surfaces can be found not only in a broad range of modern technologies but also in everyday life [1,2]. Since droplet collision phenomena can 50 play crucial roles in the performance of diverse engineering applications such as spray 51 cooling and chilling [3], spray drying [4], wet-scrubbers [5], fluid catalytic cracking (FCC) 52 [6], trickle-bed reactors [7], microfluidic-based encapsulation of biomaterials [8] and tablet 53 coating [9], to name a few examples, their collision processes have naturally been of great 54 interest across various fields of chemical, mechanical, pharmaceutical, food and bio 55 industries [1,9-11]. In general, the collision behavior is dominated by a complicated 56 competition and interaction among inertial, viscous, and capillary forces (sometimes 57 58 including the gravitational force as well) at very small length and time scales [12,13].

59 For droplet impact problems with a solid surface much effort has focused on 60 investigating the maximum spreading extent (normally characterized by the maximum spreading diameter) [14-17] and the spreading time [18-21]. The former measures "how 61 much a droplet can spread over a target surface", while the latter quantifies "how long it takes 62 to reach the maximum spreading state". Both quantities usually play critical roles in 63 determining process performance and efficiency particularly in the context of controlling the 64 droplet deposition process [1,12,19]. For example, the maximum spreading extent of a 65 catalyst particle significantly affects the catalyst efficiency as well as the system performance 66 67 of trickle-bed reactors [7] and the spreading time also plays an important role in collision processes where heat transfer (e.g., droplet solidification) [14] and/or chemical reactions 68 (e.g., cracking reaction in the FCC system) [22] are involved. 69

70

After Chandra and Avedisian's work [23] which has initiated a series of energy-balance-

based theoretical approaches for estimating the maximum spreading diameter of droplets 71 numerous studies have been devoted to the spreading dynamics of droplets on a solid surface. 72 Based on the concept of the energy-balance between the collision instant and the maximum 73 74 spreading state Pasandideh-Fard et al. [24] proposed more refined length and time scales to 75 evaluate the energy loss in the boundary layer where the viscous dissipation occurs and their 76 approach has indeed provided a cornerstone for many later theoretical energy-balance-based 77 approaches [25,26]. This type of analysis was also widely utilized for estimating the rebound criteria of bouncing droplets on a non-wettable surface [25] as well as the maximum 78 79 spreading extent [23-26]. Afterward, Roisman [27] considered the full Navier-Stokes equations to estimate the flow field inside the droplet and proposed a semi-empirical model 80 using the experimentally fitted data. Later, his model was experimentally demonstrated to be 81 one of the most accurate models for the maximum spreading extent for a wide range of 82 Weber number and surface wettability [14]. Wilderman et al. [17] presented an interesting 83 84 analysis for high-speed drop impact problems. They showed that roughly one-half of the initial (impact) kinetic energy can be converted into surface energy owing to a universal head 85 loss regardless of detailed impact conditions and energy dissipation mechanisms. More 86 87 recently some other sources causing extra energy loss during the spreading stage of droplets have been found to affect the maximal spreading extent, e.g., residual kinetic energy in the 88 form of vortical flow inside a droplet [16] or energy loss due to 'interface relaxation' near a 89 solid surface [18]. Although there is still a debate on the maximum spreading extent [28], in 90 general, scaling laws of We^{1/4} derived from momentum conservation [16] and We^{1/2} derived 91 from energy conservation [29] have been experimentally shown to hold for the inviscid 92 regime (for less viscous droplets) whereas a Re^{1/5} rule [30] has worked well for a viscous 93 regime (for highly viscous droplets). 94



There have also been several attempts to examine spreading dynamics for droplet

collision on curved surfaces such as a spherical target. Bakshi et al. [31] experimentally 96 97 showed that the film thickness at the collision center undergoes three different phases: (i) 98 initial drop-deformation, (ii) inertia-dominated and (iii) viscosity-dominated. The 99 dimensionless film thickness profiles were collapsed on a single curve for the first and second 100 phases. Malgarinos et al. [32] used volume-of-fluid (VOF) simulation to study the problem of 101 drop collision on spherical targets. They investigated the boundary separating the coating 102 from the rebound regime, the film thickness, and the wetted area. Afterward, they further 103 performed extensive VOF simulations to study drop impact on a spherical particle including 104 phase change and reaction phenomena in a high temperature environment ($T \ge 800$ K) [33]. 105 Zhang et al. presented two-dimensional [34] and three-dimensional [35] Lattice-Boltzmann simulations. They examined the effects of Weber and Reynolds numbers on the film thickness 106 107 at the collision center, and their simulation results reproduced Bakshi et al.'s [31] experimental observations well. Liang et al. [36] experimentally studied drops impacting on 108 109 wetted spherical targets, and they showed the drop-to-target curvature ratio played a critical role in not only the spreading characteristics but also determining the splashing threshold. 110 Mitra et al. [37], Liu et al., [38], and Khurana et al. [39] proposed energy-balance-based 111 112 prediction models for evaluating the spreading extent on a spherical surface and their models reproduced their experimental observations well. Yoon and Shin [40] also proposed scaling 113 114 laws and an empirical correlation for the maximum spreading diameter of a droplet colliding with a spherical surface for a wide range of liquid viscosity. More recently, Yoon *et al.* [41] 115 proposed a data-driven prediction model for the maximum spreading that can be applied to 116 117 both a flat surface and a spherical surface. Although these studies reviewed above have provided useful relations to estimate the maximum spreading extent as well as have notably 118 shed light on our understanding of the physical mechanisms of spreading droplets, they have 119 mainly focused on the maximum spreading extent itself rather than on the spreading time. 120

We now review studies in the context of the spreading time which has been relatively 121 less well-explored compared to the maximum spreading extent. Although it has long been 122 understood that the contact time of a droplet is limited by the Rayleigh oscillation time [42] 123 which is a function of the drop's density, diameter and surface tension coefficient, the energy-124 balance-based approaches, at the very early stage, simply approximated the spreading time τ 125 ignoring the effect of surface tension. Chandra and Avedisian [23] considered τ to be $\tau = D_d / D_d$ 126 V_{ini} , where D_{d} and V_{ini} are the initial droplet diameter and the impact velocity, respectively. 127 This is indeed the advective characteristic time scale which has normally been used to 128 nondimensionalize the physical time scale [31], and also can be interpreted as the required 129 time for a droplet to reach zero height from its initial diameter, D_d , at a constant impact 130 velocity V_{ini} . Pasandideh-Fard *et al.* [24] modified the spreading time to be $\tau = 8D_d / 3V_{ini}$ 131 based on a simple geometric assumption and mass conservation and their model has been 132 widely used in many other theoretical studies [26,43]. Recently, Antonini and Amirfazli [14] 133 experimentally showed that the spreading time τ is considerably affected by surface 134 wettability. Lee *et al.* [44] also presented significant effects of liquid properties on τ . They 135 replaced the initial drop diameter (D_d) in Pasandideh-Fard *et al.*'s model [24] with D_m (where 136 $D_{\rm m}$ is the maximum spreading diameter), suggesting an empirical model, $\tau = b(D_{\rm m}/V_{\rm ini})$, 137 where b is the ratio of the surface tension coefficient between a working droplet and a water 138 droplet. Huang and Chen [18] and Wang et al. [45] also adopted a similar spreading time as 139 the model of Lee et al. [44]. Lin et al. [19] presented a remarkable experimental study for the 140 spreading time and systematically analyzed the effects of various collision parameters (e.g., 141 142 impact velocity, liquid properties and surface wettability) on τ for a very wide range of collision conditions. They also proposed a new empirical model for the spreading time as a 143 function of Weber number, i.e., $\tau^* = 0.92 \text{We}^{-0.43}$. Here, τ^* is the rescaled spreading time using 144 a modified capillary time scale $(\rho D_m^{3/8}\sigma)^{0.5}$ based on the maximal spreading diameter, D_m , 145

146 where ρ and σ are the density of a droplet and the surface tension coefficient, respectively. Du 147 *et al.* [21] also proposed a similar empirical model ($\tau^* = 1.47$ We^{-0.44}) as Lin *et al.*'s model 148 [19]. More recently, Aksoy *et al.* [20] showed the effect of liquid viscosity on the spreading 149 time and proposed an empirical model as a function of both Weber and Reynolds number, i.e., 150 $\tau^* = 2$ We^{-0.45}Re^{-0.09}.

Even considering the numerous studies heretofore undertaken, it still remains difficult to comprehensively understand how the spreading time of a droplet can be affected by various collision parameters for a wide range of impact parameters. In particular, one can find significant knowledge gaps as follows:

155

It is well-known that drop spreading dynamics vary significantly across the 156 (i) different spreading regimes. For example, a drop's physical behavior and its 157 maximum spreading extent change considerably across inviscid and viscous 158 regimes [16,29,30]. However, it is still unclear how the spreading time (τ) 159 changes for different spreading regimes, because the existing studies have usually 160 studied the global behavior of spreading time at a general level, not focused on 161 162 detailed changes and associated physical mechanisms at the level of each different spreading regime. 163

164

165 (ii) Although the significant effects from impact parameters (e.g., We, Re, surface 166 wettability, etc.) on the spreading time have been evidently demonstrated by 167 many experiments [14,18-20,44], it is not easy yet to model the spreading time 168 using those impact parameters. All recent scaling laws (or empirical models) 169 reviewed above [18-21,44,45] have still been a function of the maximum 170 spreading diameter (D_m) which is usually unknown. Since the effects of impact

171parameters are already reflected in D_m [14-17,25,27], D_m cannot be considered as172a controllable independent variable to model τ resulting in the limitation of direct173"physical interpretation" of relations between the impact parameters and the174spreading time. In addition, the existing models [18-21,44,45] cannot be175explicitly solved due to the multiple unknown variables (i.e., the maximum176spreading diameter and the spreading time) leading to inconvenience for users.

177

178 (iii) The effect of surface curvature on the spreading time has not been systematically
179 investigated so far. Although a few recent numerical studies [46,47] have
180 demonstrated that the surface curvature can affect the spreading time of the
181 droplet collision system, it is still unclear that how the spreading time changes
182 depending on the surface curvature for different spreading regimes. In addition,
183 to the best of our knowledge, there has not yet been a prediction model for the
184 spreading time which can consider the effect of surface curvature.

185

In this study, we expand our previous work [41] which examined the maximum 186 187 spreading extent, now focusing on the systematic investigation of the spreading time which is another important design parameter in the droplet collision system. Based on our verified 188 computational framework, effects of important impact parameters (i.e., collision velocity, 189 surface wettability, liquid viscosity, surface tension and surface curvature) on the spreading 190 time are quantified for a wide range of impact parameters. In particular, detailed changes in 191 192 the spreading time and associated underlying mechanisms are analyzed for three different spreading regimes to better understand the complicated physical behaviors. A new prediction 193 model for the spreading time using only controllable independent variables is presented for 194 each regime, and a data-driven prediction model for the entire spreading regime is also 195

196 proposed broadly covering all ranges of impact parameters considered in the present study.

197 The remainder of the current paper is organized as follows: Section II provides a brief 198 introduction of the simulation methods used herein. Section III quantitatively and 199 systematically examines the spreading time of a droplet collision system. Physical analysis 200 and prediction models are also presented in this section. The major findings are summarized 201 in Section IV.

202

203 II. COMPUTATIONAL FORMULATION

204

Since we primarily focus on understanding the physical characteristics of the spreading 205 time, and have utilized the same computational framework as in our previous studies 206 [40,41,48,49], we provide here a brief introduction of our numerical methods used in this 207 study rather than a fully detailed explanation. For more algorithmic details and relevant 208 techniques on our simulation methods, and for our various benchmark tests including grid 209 convergence characteristics, readers can refer to our previous work [40,41,48,49]. Note also 210 that our computational framework has been extensively applied to diverse droplet collision 211 cases for various types of solid surface including flat substrates [49], spherical objects 212 [40,41,48], cylindrical targets [50] and has also been streamlined (using an adaptive-mesh-213 214 refinement approach [51]) and parallelized [52].

For incompressible flows the governing equations can be applied to all three phases (solid, liquid and gas) and are solved on a fixed Cartesian (Eulerian) grid as a single-field formulation:

- 218
- $\nabla \cdot \mathbf{u} = 0 \tag{1}$

221
$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right] = -\nabla P + \rho \mathbf{g} + \nabla \cdot \mu \left(\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}} \right) + \mathbf{F}$$
(2)

222

where **u**, *P* and **g** are the velocity vector, the pressure, and the gravitational acceleration, respectively. ρ is the density and μ is the viscosity. The surface tension force **F** is considered only at the phase (gas-liquid) interface and can be modeled by the following hybrid representation based on the continuum-surface-force (CSF) formulation [53]:

- 227
- 228
- 229

where $\kappa_{\rm H}$ is the curvature field. Note that *I* is the indicator function which has the characteristics of the Heaviside function, varying from 0 (zero) in one phase (liquid droplet) to 1 (one) in the other phase (ambient air).

 $\mathbf{F} = \sigma \kappa_{\rm H} \nabla I$

(3)

The physical properties of both phases (i.e., the ambient air and the liquid droplet) can also be assigned by using the indicator function *I* as follows:

235

$$\rho = \rho_{\rm d} + (\rho_{\rm a} - \rho_{\rm d})I \tag{4}$$

237

238 $\mu = \mu_{\rm d} + (\mu_{\rm a} - \mu_{\rm d})I$ (5)

239

Here, the subscript "d" denotes droplet, whereas "a" denotes air. Note that this common approach has been broadly applied to various multiphase-flow simulations and more detailed techniques dealing with the surface tension force and the physical properties near the gasliquid phase interface can be found in Unverdi and Tryggvason [54] and Brackbill *et al.* [53]. Our dynamic contact angle modeling technique is given in Appendix A, and further details of the discretization of the governing equations, time-stepping, and other solution techniques used in the present study can also be found in Yoon *et al.* [41] and Yoon and Shin [48].

247 One of the most essential algorithms for this type of multiphase-flow simulation is an interface tracking method. Here we use the level contour reconstruction method (LCRM) [55-248 57] to track the gas-liquid phase interface. The LCRM is a hybrid method benefitting from 249 two of the most well-established and popular interface tracking methods, i.e., level set [58] 250 and front tracking [54]. In the LCRM the phase interface is basically represented by using 251 252 (Lagrangian) moving marker elements (lines in 2D simulations and triangles in 3D 253 simulations, as in the original front tracking method) but the moving elements are re-meshed using the distance function which is the key feature of the level set method. Such a 254 reconstruction procedure can allow us to avoid the algorithmic complexity of dealing with the 255 connectivity among the moving interface elements which is the most well-known 256 257 shortcoming of the original front tracking method. Note that each moving element can always be implicitly (naturally) interconnected without any further artificial connection technique 258 because the reconstruction procedure is performed at each boundary between cells (i.e., at 259 260 cell faces) on the fixed Cartesian grid. Note also that the LCRM retains the tracking capability of the original front tracking method (i.e., accurate representation of the phase 261 interface using moving markers) while avoiding the complex algorithmic difficulties in 262 handling the topology changes (e.g., deformation, pinch-off or coalescence of the phase 263 interface). For more details on the LCRM readers can refer to Shin and Juric [55,56]. 264

Figure 1 illustrates the simulation geometry and boundary conditions used in this study. All simulations are performed in a (two-dimensional) axi-symmetric domain. The axisymmetric boundary condition is set for the left boundary whereas open boundary conditions are set for the right, upper and lower boundaries. The lengths of the physical domain are set

as $R_L = 7.5$ (in the radial r direction) and $Z_L = 20$ (in the axial z direction) times the droplet 269 radius, respectively, which are sufficiently large compared to the size of the droplet. Note that 270 271 near the splashing threshold [59,60] the droplet rim may exhibit considerable asymmetric behavior along the circumferential direction (e.g., fingering or cusp of lamella [12,13]). 272 273 Therefore, we set the simulation conditions sufficiently below the splashing threshold 274 (specific conditions will be described later) to ensure that our axi-symmetric formulation can 275 be a reasonable approach to reducing the necessary computational resources [41,51]. A droplet impacts onto a stationary spherical target at an initial impact velocity, (V_{ini}) and D_s is 276 277 the diameter of the spherical target where the subscript "s" denotes the target surface.

278

279 III. RESULTS AND DISCUSSION

280

The spreading dynamics of a droplet on a solid surface is primarily dominated by a 281 complicated time-and-space dependent interplay among inertial, viscous, and capillary effects 282 283 [12,13] and to characterize their relative importance, three non-dimensional variables, i.e., the Weber number (We = $\rho_d V_{ini}^2 D_d / \sigma$), the Reynolds number (Re = $\rho_d V_{ini} D_d / \mu_d$) and the 284 Ohnesorge number $[Oh = \mu_d/(\rho_d \sigma D_d)^{0.5}]$ have usually been used. Because Oh can also be 285 286 represented by using We and Re (i.e., $Oh = We^{0.5}/Re$) the spreading dynamics can basically be characterized by two of these numbers. Here we use We and Oh as impact parameters with 287 two other variables, i.e., the equilibrium contact angle (θ_{eqi}) and the droplet-to-target size 288 ratio (Ω) which characterize surface wettability and the curvature of the target surface, 289 respectively. 290

Three characteristic time scales are considered, i.e., the advective time, $t_a = D_d/V_{ini}$ [30,61], the capillary time, $t_c = (\rho D_d^3/8\sigma)^{0.5}$ [16,62], and the viscous time. $t_v = \mu_d D_d/\sigma$ [16,62]. The physical time scale, *t*, can be non-dimensionalized by the three characteristic time scales above and the non-dimensional advective, capillary, and viscous time scales are denoted by $t_a^* (t_a^* = t / t_a), t_c^* (t_c^* = t / t_c), t_v^* (t_v^* = t / t_v)$, respectively.

To sufficiently cover the broad range of impact parameters, and to systematically 296 examine the effects of impact parameters on the spreading time, we consider a total of 2400 297 collision cases. The droplet diameter and the density are fixed at $D_d = 2 \text{ mm}$ and $\rho_d = 998.2$ 298 kg/m³, respectively. The physical properties of the gas phase (ambient air) and the surface 299 tension coefficient are also fixed ($\rho_a = 1.2 \text{ kg/m}^3$, $\mu_a = 0.000018 \text{ N s/m}^2$, and $\sigma = 0.0728 \text{ N/m}$). 300 Four collision parameters, i.e., We, Oh, θ_{eqi} and Ω are varied by controlling the impact 301 302 velocity (V_{ini}), droplet viscosity (μ_d), equilibrium contact angle (θ_{eqi}) and the diameter of the target (D_s) , respectively. The splashing threshold is usually described by We and Oh [59,63], 303 and splashing can be observed more easily on a small spherical target compared to that on a 304 flat substrate [63]. Since splashing can occur near We ~ 150 - 160 if $\Omega \sim 1/4 - 1/2$ (for 305 millimetric water drops, Oh ~ 0.0026) [48,63], we consider a range of Weber number below 306 307 110 ($1 \le We \le 110$) to ensure that the collision outcomes would not fall into the splashing region. Therefore, a total of 12 cases of We (i.e., We = 1, 2, 4, 6, 8, 10, 20, 30, 50, 70, 90, and 308 110) are selected (the initial impact velocity V_{ini} is varied from 0.190 to 2.003 m/s). 309

310 Also, we considered a broad range of Oh number ($0.0013 \le Oh \le 0.7869$), a total of 10 cases of Oh (i.e., Oh = 0.0013, 0.0026, 0.0052, 0.0104, 0.0262, 0.0525, 0.1049, 0.2620, 311 0.5246 and 0.7869) are selected (the droplet viscosity μ_d is varied from 0.0005 to 0.3 N s/m²). 312 Note that such a range of droplet viscosity considered herein corresponds to a range of 0.5 -313 300 times the viscosity of a water droplet which covers a range of Oh of most practical 314 interest among diverse engineering fields, e.g., gasoline (Oh ~ 0.0018), water (Oh ~ 0.0026), 315 ethyl alcohol (Oh ~ 0.0063), squalene (Oh ~ 0.05), and printable ink (Oh ~ 0.1–1.0) with the 316 given droplet diameter (2 mm) [64,65]. 5 cases of θ_{eqi} (i.e., $\theta_{eqi} = 20^{\circ}$, 55°, 90°, 125° and 317 318 160°) are also selected to cover a wide range of surface wettability from hydrophilic to

319 (super) hydrophobic surfaces.

The impact physics on a spherical target is practically identical to that on a flat substrate 320 if a droplet collides onto a sufficiently large spherical target which has a diameter 10 times 321 larger than the droplet (i.e., $\Omega \leq 1/10$) [38,47]. Note that this assumption has been 322 323 experimentally [38] and numerically [40,41] confirmed. Conversely, collision dynamics can 324 differ greatly on a sufficiently small spherical target because a target can be fully coated by the droplet. In this case the typical maximum spreading state cannot be observed and full 325 coating can usually be observed where $\Omega \ge 1/1.5$ [39,48]. Therefore, we here examine the 326 327 effect of target size between those two thresholds $(1/10 \le \Omega \le 1/1.5)$ and 4 cases of the droplet-to-target size ratio (i.e., $\Omega = 1/10, 1/4, 1/3, 1/2$) are selected. 328

As a result, a total of 2400 collision cases (12 Weber numbers, 10 Ohnesorge numbers, 5 surface wettabilities and 4 droplet-to-target size ratios) are considered in the current study. This large dataset will also be used to train a deep neural network to formulate a data-driven prediction model as a non-linear regressor as well as to systematically examine the effects of 4 collision parameters (We, Oh, θ_{eqi} and Ω) on the physical behavior of the spreading time. For more details of the simulation settings readers can also refer to our previous study [40,41].

336

337 A. Review of typical spreading dynamics

The schematic diagram in Figure 2 summarizes the different spreading regimes for the droplet-solid collision system depending on its physical behavior. If a droplet collides onto a solid surface with sufficiently high inertia, splashing can occur where tiny re-atomized droplets are generated [12,13]. It has been found that higher We and lower Oh normally promote splashing [59] but the detailed threshold for the onset of splashing can also be affected by many other collision environments, e.g., surface roughness [63], target curvature [63] and ambient gas pressure [60], etc. Conversely, if a droplet impacts onto a solid surface with very low inertia compared to surface tension, i.e., at very low Weber number (We \leq 1), a droplet can behave like an elastic ball, sometimes bouncing off a solid surface numerous times [66]. This region is the so-called elastic regime.

Between those two thresholds (i.e., above the elastic threshold and under the splashing 348 threshold), in general, a droplet spreads over a solid surface and then reaches a maximum 349 spreading state. Afterward, the droplet can retract and is eventually deposited on a surface (or 350 can also bounce off a surface depending on surface hydrophobicity) [12,13]. Such a 351 352 spreading regime can typically be categorized into the following three sub-regimes depending on the underlying physical mechanisms: (i) capillary-driven spreading at relatively low 353 Weber number [41,61], (ii) inertia-driven but capillary-limited (inviscid) at high We and low 354 Oh [16,40,62] and (iii) inertia-driven but viscous-limited at high We and high Oh [16,40,62]. 355

If We is sufficiently small (i.e., We on the order of 10^{0}), spreading dynamics is primarily 356 driven by capillary effects acting on the three-phase triple contact line [61,67,68]. At such a 357 358 capillary-driven spreading regime (i.e., regime 1 in Fig.2, also denoted as "CD regime" hereafter), surface wettability and the associated wetting nature can play more significant 359 360 roles in the spreading process rather than inertial effects due to the low Weber number [18,41]. At a very early stage of contact the apparent contact angle (θ_{ap}) is nearly 180° which 361 is far from its equilibrium contact angle (θ_{eqi}). Therefore, the capillary force which can be 362 scaled as $F_c \sim \sigma R_w(\theta_{ap} - \theta_{eqi})$ (where R_w is the contact radius) starts to act near the contact line 363 and drive the spreading process. Note that this capillary effect can make a droplet spread over 364 365 a solid surface even if the initial collision velocity is zero and it has also been experimentally [15] and numerically [41] confirmed that the maximum spreading extent of millimetric 366 367 droplets can be larger than the initial droplet diameter in cases of zero impact velocity. In 368 Fig.2 the CD regime is indicated by the blue region.

If We is sufficiently large [i.e., We $\geq O(10^1)$] but Oh is small [i.e., Oh $\leq O(10^{-2})$], the 369 spreading process is primarily governed by inertial effects but is terminated by the capillary 370 371 limit [16,40,62]. At this inertia-driven but capillary-limited (inviscid) regime (i.e., regime 2 in Fig.2, also denoted as "IC regime" hereafter), the effects of surface wettability and viscous 372 373 dissipation play minor roles in the spreading process, due to the high We and low Oh numbers. In other words, a droplet can spread over a surface until the interfacial deformation 374 is no longer available. The spreading dynamics in the IC regime can also be characterized by 375 the ejection of a thin liquid sheet (i.e., jetting lamella) at the initial stage [69], and 376 377 considerable deformation of the leading edge outer rim can be expected during the subsequent spreading stage [27,40,70]. The maximum spreading extent can be scaled by the 378 Weber number, and $We^{1/4}$ [16,40,62] and $We^{1/2}$ [29] rules have been shown to work well. In 379 Fig.2 the IC regime is indicated by the red region. 380

On the other hand, if both We and Oh are sufficiently large [i.e., We $\geq O(10^1)$ and Oh \geq 381 382 $O(10^{-2})$], the spreading process is primarily governed by inertial effects but is terminated by the viscous limit [16,40,62]. At this inertia-driven but viscous-limited spreading regime (i.e., 383 regime 3 in Fig.2, also denoted as "IV regime" hereafter), the effect of viscous dissipation 384 385 plays a significant role whereas the effect of surface wettability is still negligible due to the high We and Oh numbers. In other words, a droplet can spread over a surface until all the 386 initial impact (kinetic) energy is dissipated by viscous damping. No lamella ejection is 387 expected in this viscous regime [19] owing to the strong viscous resistance which hinders the 388 interfacial deformation. The maximum spreading extent can be scaled by the Reynolds 389 number and a Re^{1/5} rule has been found to hold well [16,29,40,62]. In Fig.2 the IV regime is 390 indicated by the green region. 391

392 Note that, in the present study, we focus on these three spreading regimes (i.e., CD, IC, 393 and IV regimes). The other collision regimes (elastic regime and splashing regime) are beyond the current scope due to their different collision phenomena and different area ofinterest among the scientific communities.

Figure 3 depicts the typical spreading processes of droplets for the three different 396 spreading regimes summarized in Fig.2 above. The morphological evolutions of spreading 397 droplets for the CD, IC, and IV regimes are presented in Fig.3(a), (b), and (c), respectively. 398 Note that the (non-dimensional) advective time scale (t_a^*) is used for all cases in Fig.3, since 399 here we simply focus on describing the typical spreading processes rather than addressing 400 their detailed physical interpretations. Note also that the collision cases where $\Omega = 1/10$ 401 which can reasonably be regarded as a flat substrate [38,47] are considered in Fig.3 due to its 402 generality. As seen in Fig.3(a), the capillary effect acting near the contact line drives the 403 404 spreading process for the CD regime. The upper-part of the droplet can nearly maintain its spherical shape owing to the low Weber number (low impact velocity), whereas a capillary 405 wave is formed above the contact line due to the local interfacial curvature [see $t_a^* = 0.34$ in 406 Fig.3(a)] [18,19,68]. As the capillary waves propagate to the central part of the droplet, the 407 droplet shows a staircase-like pyramidal structure [18,19,68] without significant interface 408 deformation (0.34 $\leq t_a^* \leq 0.50$). After the maximum spreading state, the oscillatory motion of 409 410 the central part is observed in the vertical direction and then the droplet reaches an equilibrium state ($0.50 \le t_a^* \le 5.00$). 411

In the IC regime, as seen in Fig.3(b), a thin liquid sheet (lamella) is ejected above the solid surface due to the strong inertial effect overwhelming the surface tension without significant viscous resistance [see $t_a^* = 0.25$ in Fig.3(b)]. The droplet rapidly spreads over the surface and the thin lamella continues consuming liquid mass, forming a thick rim at the leading edge of the lamella ($0.80 \le t_a^* \le 1.74$) [27,40,61,70]. As the lamella spreads on a surface, the leading edge of the lamella is detached from the surface due to its strong radial motion at $t_a^* = 0.80$. After the maximum spreading state, the droplet starts its retraction 419 motion, driven by the capillary effect, to reduce the increased surface energy during the 420 spreading stage $(1.74 \le t_a^* \le 8.00)$.

In the IV regime, as explained above, no notable lamella ejection is observed [19]. The interfacial deformation is significantly hindered by the strong viscous damping. After the maximum spreading state ($t_a^* = 0.98$), the central part of the droplet slightly and slowly rises again, reaching the equilibrium state, but no meaningful recoiling motion is seen in terms of its wetted area. Note that all these results showing the evolutions of the droplet morphologies for the three spreading regimes depicted in Fig.3 are consistent with the experimental observations of Lin *et al.* [19].

Figure 4 quantifies the temporal evolution of the spreading extent characterized by the 428 non-dimensional spreading diameter β^* ($\beta^* = \beta/D_d$ where β denotes the arc length of the 429 droplet in contact with a solid target, see the inset in Fig.4) of the droplets for the three 430 collision cases shown in Fig.3. Note that, for the CD and the IC regimes, detecting the 431 spreading time with the maximum value of β^* can be straightforward because the droplet 432 starts its recoiling motion soon after the maximum spreading state (see black solid line and 433 blue dashed line). The corresponding time for the maximum value of β^* is visually evident. 434 Conversely, as shown in Fig.3(c), usually no obvious recoiling stage occurs for the IV regime 435 due to the high viscous resistance (see red dotted line in Fig.4). In this case, detecting the 436 spreading time can be quite confusing and even a very small disturbance in β^* can lead to a 437 critical bias for measuring the spreading time. Therefore, to avoid such a complexity, the time 438 when a droplet attains 95 % of the maximum value of β^* is defined as the spreading time as 439 440 marked by three triangles in Fig.4 (see also green arrow). Note that, hereafter, the nondimensional forms of the spreading time are denoted by τ_a (if the advective time scale is 441 used), τ_c (if the capillary time scale is used) and τ_v (if the viscous time scale is used). 442

444 **B. Validations**

In the above subsection, we addressed the typical spreading dynamics for the three different spreading regimes. Before we start analyzing the spreading time for each regime in more detail in this subsection, we validate our simulation method by comparisons with existing experimental results to check its capability for simulating the droplet collision phenomena in terms of the spreading time.

Figure 5 compares our simulation results with the experimental data of Mitra et al. [2] 450 451 and Banitabaei and Amirfazli [71] focusing on the collision behavior on spherical targets for a broad range of We (We = 0.9 – 155.5). First, Fig.5(a) depicts the temporal variation of β^* 452 and the morphological evolution of the droplet for the low Weber number case (We = 0.9). 453 The insets in Fig.5(a) shows the interfacial shapes captured by the experiment (on the right 454 side of each panel) [2] and by the current simulation (on the left side of each panel) at three 455 different time instants ($t_a^* = 0.24, 0.47, and 0.71$). As seen, our simulation result shows a 456 good agreement with the experimental observation. Fig.5(b) depicts the temporal variation of 457 the non-dimensional film thickness, $h^* [h^* = h/D_d$, where h is the liquid film thickness 458 measured on the impact center, see the inset on the lower-left corner in Fig.5(b)] and the 459 morphological evolutions of the droplet for the high Weber number case (We = 155.5). The 460 other insets in Fig.5(b) show the interfacial shapes captured by the experiment [71] and by 461 the current simulation at four different time instants ($t_a^* = 0.30, 0.60, 1.05$ and 1.95). Again, 462 our simulation result shows a good agreement with the experimental observation. Since the 463 liquid film thickness is one of the most sensitive parameters to the mesh size in this type of 464 numerical simulations [72], we here present our grid-convergence test result as well in 465 Fig.5(b). As seen in the dashed gray box, our simulated h^* almost converges at 32 CPR (cells 466 per radius) as in our previous study [48,51]. Overall, our simulation result shows an excellent 467 agreement with the experimental result [71] in terms of both qualitative and quantitative 468

469 comparisons. Note also that all simulation cases presented in the current study have been470 performed with 64 CPR to be on the safe side.

In figure 6 we further validate our simulation results focusing on the quantification of the 471 spreading time for a broad range of We and Oh. Fig.6(a) depicts the non-dimensional 472 spreading time, τ_a , for the low Oh case (Oh = 0.0026 for a water drop). As seen, our 473 simulation result is consistent with the experimentally measured data of Huang and Chen 474 [18]. A small deviation can be attributed to the difference of the definition of spreading time 475 (i.e., we defined the spreading time as the time that a droplet attains 95% of the maximum 476 spreading extent. See Fig.4 and its explanation). In Fig.6(b), we compare the simulated τ_a 477 with the existing numerical result [21] and empirical correlation [19] for the high Oh case 478 (Oh = 0.62 in the case where the droplet is 240 times the viscosity of a water drop). Our 479 result again shows an excellent agreement with the existing results [19,21]. 480

Further validation test results for drop impact problems are also extensively provided in 481 Appendix B, which were presented in our previous study. From Figs. 5 - 6 and Appendix B, 482 we can conclude that our simulation methods used in this study can reasonably reproduce the 483 experimental data for a wide range of collision parameters in the context of both qualitative 484 485 and quantitative comparisons. Note that the current numerical framework has been widely applied to various droplet-solid collision problems and has been thoroughly validated against 486 many existing experimental data [40,41,48,49,51]. Note also that the impact parameters 487 considered in our current and previous validation tests reasonably cover the practical 488 conditions used in the current study ($0 \le We \le 110, 20^\circ \le \theta_{eqi} \le 160^\circ, 0.0013 \le Oh \le 0.7869$ 489 490 and $1/10 \le \Omega \le 1/2$).

491

492 C. Effects of collision parameters on the spreading time and prediction
 493 models

We now analyze the spreading time for the three different spreading regimes addressed above, i.e., the capillary-driven (CD), the inertia-driven but capillary-limited (IC) and the inertia-driven but viscous-limited (IV) regimes in more detail and present a prediction model for each regime. Note that the collision cases for a flat surface ($\Omega = 1/10$) are chosen first for demonstrating our results owing to its generality. Then we further extend our analysis considering the effect of the droplet-to-target size ratio (Ω).

We first examine the spreading time for the CD regime (We $\leq 10^{1}$). At this low We region the spreading dynamics is primarily governed by its wetting nature and the associated capillary effect where surface wettability plays a dominant role. Therefore, the capillary time scale, t_{c}^{*} , can be an appropriate time scale for analyzing physical characteristics of the spreading time in this regime [16,19].

Figure 7(a), (b), and (c) depict the effects of We, θ_{eqi} and Oh on the spreading profiles, 505 respectively. As seen in Fig.7(a), the spreading time (τ_c) basically decreases as We increases 506 whereas the maximum value of β^* (denoted as β^*_{max} hereafter) increases with We. This result 507 is natural because higher spreading velocity can be attained at the high We cases [19], thus a 508 shorter spreading time is needed. Note that this is also consistent with the experimental 509 observation of Lin *et al.* [19]. For the cases shown in Fig.7(a), τ_c , where We = 10 ($\tau_c = 0.69$), 510 is about 21 % shorter compared to that for the case where We = 2 ($\tau_c = 0.87$). In Fig.7(b), as 511 expected, the dominant effect of the surface wettability is seen. The droplet can spread for a 512 longer time over the surface as θ_{eqi} decreases (i.e., as the surface becomes hydrophilic) and 513 can attain a larger spreading extent as well owing to the strongly wettable nature of the 514 515 hydrophilic surface. This result is also consistent with the existing experimental data [19,43]. For the cases shown in Fig.7(b), τ_c , where $\theta_{eqi} = 20^\circ$ ($\tau_c = 1.38$), is almost double the case 516 where $\theta_{eqi} = 160^{\circ} (\tau_{c} = 0.70)$. 517



It is interesting to observe the effect of Oh on τ_c . As depicted in Fig.7(c), for relatively

low Oh cases (Oh ≤ 0.0104 , see 4 dashed lines), no meaningful effect of Oh is seen because 519 the spreading process is primarily driven by the capillary effect and is also terminated by the 520 capillary limit due to the negligible role of the viscous resistance at such low Oh numbers 521 [16,40]. Note also that this trend is consistent with the existing experimental observation of 522 Lee et al. [15] who showed that there was no notable difference of the spreading time 523 between a water droplet (Oh ~ 0.0026) and a glycerol mixture (Oh ~ 0.0263, about 10 times 524 the viscosity of a water droplet) under the given surface conditions ($\theta_{eqi} = 52^{\circ} - 61^{\circ}$) in the 525 low We region (We < 2.5). Conversely, the spreading profiles start to be affected by Oh as Oh 526 further increases above $Oh \ge 0.0262$ [see solid lines in Fig.7(c)] and the recoiling stages 527 disappear for much higher Oh cases (Oh ≥ 0.1049) owing to the significant viscous damping. 528 The interesting point is that no notable difference of τ_c can be found even for higher Oh cases 529 (τ_c is almost constant for the entire range of Oh). We found that this is because Oh primarily 530 affects the spreading velocity, not the spreading time, for the higher Oh region. To check the 531 effect of Oh on the spreading velocity in more detail, in Fig.7(d) and (e), we plot the non-532 dimensional spreading velocity V_s^* (defined as $V_s^* = \beta_{max}^* / \tau_c$) and the non-dimensional 533 maximum spreading extent β^*_{max} for the cases shown in Fig.7(c), respectively. As seen, V_s^* is 534 nearly constant for the low Oh region (roughly where $Oh \leq 0.01$), whereas it significantly 535 decreases as Oh increases in the high Oh region (Oh ≥ 0.05). Since the effect of Oh on β^*_{max} 536 (this result in Fig.7(e) is also consistent with the existing experimental result [16]) evidently 537 shows exactly the same trend as the effect of Oh on V_s^* , τ_c can be considered almost 538 independent of Oh for the entire range of Oh. 539

It is also interesting to compare the spreading mechanism for this CD regime with the retraction mechanism for high-speed impact cases. In fact, both processes (i.e., spreading of low-speed impact and retraction of high-speed impact) are presumably based on the same physical mechanism, i.e., capillary-driven motion under viscous resistance. In the high-speed

impact case, at the beginning of the retraction stage (i.e., right after the maximum spreading 544 state), the droplet shows a severely deformed interfacial shape and its motion is almost 545 546 stopped. Then we begin to see the capillary-driven recoiling motion under viscous resistance, to reduce its increased surface energy caused by surface deformation during the former 547 548 spreading stage [62]. Since the spreading stage in the CD regime is also driven by the 549 capillary effect under viscous resistance, one can expect that there should essentially and 550 naturally be consistent physical characteristics between the spreading behavior of the lowspeed impact in the CD regime and the retraction behavior of the high-speed impact cases. 551 552 We found that the spreading velocity in the CD regime analyzed in Fig.7(d) demonstrates the same trend as the retraction characteristics which were experimentally observed by Bartolo et 553 al. [62]. They showed that the retraction rate of a droplet is not affected by Oh where Oh <554 0.05, whereas it is strongly reduced as Oh increases where Oh > 0.05. Their result shows an 555 excellent agreement with our findings described in Figs.7(c) and (d). 556

In Fig.7 above, we observed the obvious effects of We and θ_{eqi} on τ_c , whereas the effect of Oh can be negligible. Therefore, τ_c in the CD regime can be properly represented by We and θ_{eqi} and we propose a prediction model:

560

561

$$\tau_{\rm c} = (1.66 - 0.31\theta_{\rm eqi}) \,\mathrm{We}^{-0.24} \tag{6}$$

562

In figure 8, a total of 600 τ_c data are plotted with the prediction model in Eq.(6). As seen, all τ_c data for a range of We \leq 10 can nearly be collapsed on the single curve (see black solid line), regardless of Oh. Above the threshold of We ~ 10 (i.e., beyond the CD regime) τ_c cannot be properly represented by Eq.(6).

567 We now examine the spreading time for the IC regime [We $\ge O(10^1)$ and Oh $\le O(10^{-2})$]. 568 At this high We but low Oh region, the spreading dynamics is primarily dominated by the strong inertial effect but terminated by the capillary limit [16,40,62]. Therefore, the capillary time scale, t_c^* , can still be applicable for this regime [16,19].

Figure 9(a) depicts the effect of We on the spreading profiles. τ_c first decreases as We 571 increases for the CD regime (see 5 dashed lines) as shown in Fig.7(a) above, but it increases 572 again if We further increases (We ≥ 30) as the collision dynamics sufficiently enter the IC 573 regime (see 6 solid lines). This somewhat interesting result can be attributed to the severe 574 deformation of the leading edge of the lamella and its relaxation phenomena [40]. As the 575 droplet spreads over the surface the leading edge of the lamella continues deforming. The tip 576 of the lamella is detached from the surface, and it also continues growing as a rim by 577 consuming the liquid mass from the central region of the droplet [27,70]. Note that such a 578 deformed rim plays an important role in determining the maximum wetting area [70] since 579 the volume of the rim is not small [27] and the maximum wetting area is attained during the 580 relaxation period of the rim [40]. Therefore, a more deformed shape at higher We leads to the 581 longer relaxation time of the rim and eventually the longer spreading time. Fan et al. [73] also 582 experimentally showed that the spreading time of a water droplet on a spherical target first 583 decreases as We increases, then it again increases with We if We \geq 36.5, showing an 584 agreement with our results in Fig.9(a). However, the authors think it would be ideal if our 585 physical interpretation of the underlying mechanism shown in Fig.(b) could also be 586 confirmed by additional experiments. 587

Fig.9(b) illustrates such a deformation and relaxation process of the rim in more detail. For the relatively low We case (We = 20), as depicted on the left side of Fig.9(b), the droplet continues its spreading process until it reaches the maximum spreading state and no significant deformation of the lamella edge is observed because the spreading dynamics does not fully enter the IC regime yet. Conversely, for the high We case where We = 110 [see the right side of Fig.9(b)], a severe deformation of the rim detached from the surface is clearly captured at $t_c^* = 0.60\tau_c$ (see black line). Then the rim begins its relaxation to reduce the surface energy and eventually reaches the maximum spreading state (see pink line). Note that the relaxation period takes quite a long time (about 40 % of τ_c in this case) eventually leading to the delayed τ_c .

Fig.9(c) and (d) show the effects of θ_{eqi} and Oh on the spreading profiles, respectively. 598 As seen, no meaningful effect of θ_{eqi} on τ_c is observed, because the effect of the surface 599 wettability is overwhelmed by the strong inertial effect during most of the spreading stage 600 [27,41]. Although the surface wettability plays a non-negligible role in the spreading profiles 601 602 near the maximum spreading state and after τ_c (i.e., during the retraction stage) where the spreading velocity and the associated inertial effect are at a minimal level, the effect of θ_{eqi} on 603 $\tau_{\rm c}$ is almost negligible until the droplet reaches nearly the maximum spreading state. For this 604 case τ_c can be considered to be a constant value regardless of the surface wettability. The 605 effect of Oh on τ_c is also not meaningful if Oh is sufficiently small. As seen in Fig.9(d), τ_c is 606 not affected by Oh if Oh < 10^{-2} (see 3 solid lines, i.e., at IC regime), whereas τ_c decreases as 607 Oh increases if $Oh > 10^{-2}$ (see 3 dashed lines, i.e., beyond the IC regime). This result is also 608 consistent with the existing result of Zhu et al. [46] who demonstrated that the spreading 609 610 dynamics is independent of Oh for low Oh, whereas it is strongly affected by Oh for high Oh under similar We conditions as the present study (We ≥ 25). Their proposed threshold (Oh ~ 611 0.008) is also consistent with the current observation (Oh ~ 0.01). 612

In Fig.9 above, we observed the obvious effect of We on τ_c , whereas the effects of θ_{eqi} and Oh can be negligible. Therefore, τ_c in the IC regime can be properly represented by We and we propose a prediction model:

- 617 $\tau_{\rm c} = 0.31 {\rm We}^{0.18}$ (7)
- 618

In figure 10, a total of 150 τ_c data are plotted with the prediction model in Eq.(7). As seen, all τ_c data for a range of We > 30 can nearly be collapsed on the single curve (see black solid line) regardless of θ_{eqi} and Oh.

We now examine the spreading time for the IV regime [We $\ge O(10^1)$ and $Oh \ge O(10^{-2})$]. At this high We and Oh region the spreading dynamics is primarily dominated by the strong inertial effect but terminated by the viscous limit [16,40,62]. Therefore, the viscous time scale, t_v^* , would be the proper scale in this regime.

Figure 11(a), (b), and (c) depict the effects of We, θ_{eqi} , and Oh on the spreading profiles, 626 627 respectively. As seen in Fig.11(a), the spreading time, τ_v , naturally decreases as We increases due to the higher spreading velocity [19]. The delayed spreading time shown in the IC regime 628 (caused by the relaxation period of the deformed lamella rim) is not observed in this IV 629 regime because the interfacial deformation is significantly restricted by the strong viscous 630 damping. No meaningful effect of θ_{eqi} on τ_v is seen [see Fig.11(b)] due to its minor role 631 compared to the strong inertial and viscous effects. Conversely, as expected, the dominant 632 effect of Oh on τ_v is obviously detected in Fig.11(c). Since the spreading stage is terminated 633 much earlier for higher Oh cases under the given We due to increased viscous resistance (i.e., 634 635 the initial kinetic energy is more quickly dissipated [44]), τ_v is significantly reduced for higher Oh conditions. Note that the effect of Oh on τ_v observed in Fig.11(c) is also consistent 636 with the existing experimental observation [19,44]. 637

In Fig.11 above, we observed the obvious effects of We and Oh on τ_v , whereas the effect of θ_{eqi} can still be negligible. Therefore, τ_v in the IV regime can be properly represented by We and Oh, and we propose a prediction model:

- 642 $\tau_{\rm u} = 0.3 \mathrm{Oh}^{-1.13} \mathrm{We}^{-0.22}$ (8)
- 643

In figure 12, a total of 250 τ_v data are plotted with the prediction model in Eq.(8). As seen, all τ_v data for a range of We > 30 can nearly be collapsed on the single curve (see black solid line), regardless of θ_{eqi} .

647

648 **D. Effect of target curvature and data-driven prediction model**

In the above subsection, we addressed the effects of the three collision parameters (We, 649 θ_{eqi} , and Oh) on the spreading time for the three different spreading regimes (CD, IC, and IV 650 651 regimes) and a prediction model was presented for each regime for the cases where $\Omega = 1/10$ (for a flat surface). We now expand our analysis considering the surface curvature which has 652 been relatively less explored. The effect of the target curvature characterized by the droplet-653 to-target size ratio (Ω) on the spreading time and a new prediction model for the spreading 654 time which can be applied to the entire range of the 4 collision parameters will be presented 655 in this subsection. 656

Figure 13(a), (b), and (c) depict the effects of Ω on the spreading profiles for the CD, the 657 IC, and the IV regime, respectively. In the CD regime, as seen in Fig.13(a), τ_c increases with 658 Ω (on the smaller target). This behavior is attributed to the reduced capillary force acting near 659 the contact line for the smaller targets. As described above, at this low We level, the dominant 660 force driving the spreading process is the capillary force F_c [which can be scaled as $F_c \sim$ 661 $\sigma R_{\rm w}(\theta_{\rm ap} - \theta_{\rm eqi})$] formed at the initial stage of contact. At the instant that a droplet contacts a 662 solid surface, its apparent contact angle (θ_{ap}) is almost close to 180° and the difference 663 between the apparent contact angle and the equilibrium contact angle results in the capillary 664 force F_c on the contact line. Since such an apparent contact angle at the initial contact stage is 665 formed at a lower level on the smaller target [see θ_{ap2} and yellow dashed line in the zoom-in 666 of the dashed black box, in Fig.13(d)] compared to the bigger target (see θ_{ap1}), $\theta_{ap} - \theta_{eqi}$ and 667 $F_{\rm c}$ are reduced on the smaller targets eventually leading to lower spreading velocity and 668

longer spreading time. For the cases shown in Fig.13(a), τ_c where $\Omega = 1/2$ ($\tau_c = 0.99$) is about 21 % longer compared to that for the flat surface ($\Omega = 1/10$, $\tau_c = 0.82$).

As seen in Fig.13(b), the effect of Ω on τ_c is apparently observed also in the IC regime, 671 i.e., τ_c increases on the smaller targets. Note that the trend of the effect of Ω on τ_c in the IC 672 regime is also consistent with many existing studies [40,46,47] and this behavior is attributed 673 to the reduced lamella deformation on the smaller target [40]. As already shown in Fig.3(b) 674 above, a thin lamella sheet is ejected at the early stage of spreading. Although the motion of 675 bulk liquid is primarily dominated by inviscid inertial effects [16,40,62], the dynamics in the 676 677 vicinity of the lamella jet is locally but considerably affected by viscous effects [20,69]. Since the smaller targets provide more space in the vertical direction as in an expanding channel 678 flow for the lamella sheet being ejected [i.e., higher ζ is provided on smaller targets as 679 illustrated in the zoom-in of the dashed red box in Fig.13(d)] the local velocity of the radially 680 expanding lamella jet is reduced on the smaller targets. Therefore, the lamella jet develops 681 682 less sharply, having a more rounded and bluff shape [40,69]. This allows the droplet to expend its kinetic energy more efficiently for the spreading itself rather than the rim 683 deformation detached from the surface eventually leading to the longer spreading time as 684 685 well as the longer spreading extent compared to those on the larger targets (under the same kinetic energy level). A more detailed physical mechanism for the reduced lamella 686 deformation on smaller targets can be found in the existing numerical study [40]. For the 687 cases shown in Fig.13(b), τ_c , where $\Omega = 1/2$ ($\tau_c = 0.80$) is about 45 % longer compared to that 688 for the flat surface ($\Omega = 1/10$, $\tau_c = 0.55$). 689

690 The effect of Ω on τ_v in the IV regime is depicted in Fig.13(c). As seen, no meaningful 691 effect of Ω on τ_v is observed for this highly viscous regime. Although the maximum 692 spreading extent is slightly increased on the smaller targets due to the reduced energy 693 dissipation [40], τ_v is almost constant in terms of its practical measurement.

In figure 14, we refine the regime map (shown in Fig.2 above) to summarize the major 694 findings presented so far, including the details of the physical behaviors of the spreading 695 696 time. The underlying physical mechanism governing the spreading process, the characteristic time scale, the effects of various collision parameters on the spreading time and the 697 prediction model (for a flat surface) are also presented for each regime. Note that the effects 698 of 4 collision parameters, i.e., We, θ_{eqi} , Oh, and Ω are indicated by 4 different markers, i.e., 699 700 circles, squares, triangles and inverted triangles, respectively. If a certain parameter has a positive (or negative) effect on the spreading time, its effect is indicated by a red (or blue) 701 702 marker, whereas an empty marker is used if its effect is negligible. For example, if the spreading time increases with We, the effect of We is marked by a filled red circle. 703 Conversely, the effect of We is marked by a filled blue circle if the spreading time decreases 704 as We increases. 705

Fig.14 highlights the complex nature of the spreading time owing to the different physical mechanism for each regime. The effects of the impact parameters (We, θ_{eqi} , Oh and Ω) also play different roles. For instance, the effect of We is positively related to the spreading time for the CD and the IV regime, whereas it is negatively related to the spreading time for the IC regime. In particular, none of the 4 impact parameters shows a consistent effect across the three spreading regimes, which highlights the significant nonlinearity and complexity of the spreading time. The characteristic time scales are also different.

Therefore, formulating a single correlation for the spreading time as a broad cross-over across the three different spreading regimes is quite difficult since it should cover all the different physical mechanisms, different characteristic time scales, and different effects of the collision parameters. In addition, if the effect of the target curvature (i.e., the effect of Ω) on the spreading time were to be considered together, the problem would be much more complicated. Rather than formulating a single correlation, therefore, we consider here a datadriven approach to model such complex and non-linear behavior of the spreading time acrossall the spreading regimes.

Recently, data-driven approaches have been broadly applied to model complicated fluid phenomena such as turbulent flows [74] or multiphase flow systems [75]. For droplet impact problems as well, the maximum spreading extent [41], the splashing threshold [76], and the lamella area [77] have successfully been modeled using data-driven techniques, based on their powerful ability to correlate non-linear relations among large datasets and reduce the level of complexity.

The spreading time can basically be modeled as a function of 4 impact parameters asfollows:

- 729
- 730

$$\tau_{\rm a} = f(\text{We, Oh}, \theta_{\rm eqi}, \Omega) \tag{9}$$

731

732 Here, the advective time scale is applied for our data-driven prediction model, in order to treat our dataset in a consistent manner. f is the non-linear and complicated (unknown) 733 function between inputs (4 impact parameters) and output (τ_a) data which is now modeled by 734 735 using an artificial intelligence technique. The multilayer neural network, also called the multilayer perceptron (MLP) [78], is utilized as a nonlinear regressor to model the function f. 736 Figure 15 illustrates the schematic diagram of the typical MLP structure. The MLP consists of 737 the input layer (marked by red nodes), the multiple hidden layers (marked by green nodes) 738 and the output layer (marked by blue nodes). Each layer usually consists of a number of 739 740 nodes (also called neurons), whereas the last output layer has only one output node. The 4 input features (We, Oh, θ_{eqi} and Ω) are provided from the input layer to the first hidden layer 741 and then linear combinations of the input features are calculated on each node. Such linear 742 combinations are nonlinearized by using an activation function and then are delivered to the 743

next hidden layer. Such an operation is called a feedforward procedure and is applied to all
the subsequent layers. The last output layer has no nonlinearization operation and provides
the final output value.

A total of 2400 impact cases (12 Weber numbers, 10 Ohnesorge numbers, 5 surface 747 wettabilities, and 4 droplet-to-target size ratios) are utilized to set the dataset. The dataset is 748 randomly split into 3 parts: the training dataset (70 %, 1680 data points), the validation 749 dataset (15 %, 360 data points) and the test dataset (15 %, 360 data points). The training 750 dataset and the test dataset are used to train our MLP model and finally test the accuracy of 751 752 the trained model, respectively, whereas the validation dataset is utilized to check if overfitting occurs during the training process. More detailed information on our MLP 753 modeling and its training procedure can be found in the Appendix C. 754

Figure 16 compares the spreading time predicted by our data-driven model with the true 755 values (measured values from our numerical simulations). 360 data points from the test 756 dataset are plotted for comparison (see black circles). As can be seen, τ_a predicted by our 757 758 MLP model demonstrates a good agreement with the true dataset, showing that the mean square error (MSE) is on the order of O(10⁻⁴) (MSE = 5.23×10^{-4}). Note that, except for only 759 a few cases, all the tested cases fall inside a deviation range of \pm 15%. Note that all the test 760 data samples shown in Fig.16 have not been used for the training process of our MLP model. 761 To the best of our knowledge, our prediction model is the first data-driven model for the 762 763 spreading time of the droplet collision system which can be applied for a curved spherical 764 target. As one of the most universal models, the current model can cover a very broad range of impact parameters across not only the three spreading regimes (i.e., CD, IC, IV regimes 765 and the transition area among them as well), but also a wide range of droplet-to-target size 766 767 ratio (from a flat substrate to a small spherical target).

768

Last but not least, the authors would like to clarify that there is no physics in the current

data-driven prediction model, which can indeed be understood as providing both pros and 769 770 cons [75]. The fact that such a data-driven approach is not fully based on a physical 771 background might be a reason for which it has been less attractive to some scientific communities compared to conventional approaches supported by strong physical principles 772 [79,80]. However, both approaches share identical goals, i.e., modeling complicated 773 774 phenomena which usually need to consider many multi-dimensional variables [80,81]. The 775 advantages of data-driven approaches can also be that users would not need to consider physics models alone for those complex system because data-driven models basically don't 776 777 rely on constraints attributed to physics [75]. This characteristic allows them to work as a useful non-linear regressor. For example, the current data-driven prediction model can predict 778 the spreading time even for transition regimes marked in the gray zones in Fig.14, which is 779 apparently very difficult to predict using Eqs. 6 - 8 above due to their non-linear nature. This 780 clearly shows the advantage of data-driven approaches at least in the context of engineering 781 782 applications and practical usefulness.

783

784 IV. CONCLUSION

785

In this study we quantitatively and systematically investigate the spreading time of droplet impact onto a spherical target for a wide range of impact parameters ($0 \le We \le 110, 20^\circ \le \theta_{eqi} \le 160^\circ, 0.0013 \le Oh \le 0.7869$ and $1/10 \le \Omega \le 1/2$). A total of 2400 collision cases are simulated and are used to better understand the physical behavior of the spreading time. This large simulation dataset is also used for training a data-driven prediction model as well as for the derivation of prediction models.

For the three different spreading regimes, i.e., the capillary-driven (CD), the inertia-

driven but capillary-limited (IC) and the inertia-driven but viscous-limited (IV) regimes, the 793 effects of the impact parameters on the spreading time are analyzed in detail. Not only the 794 795 spreading time, but also proper time scales, dominant impact parameters and associated 796 physical behaviors all significantly and non-linearly change across the three spreading 797 regimes. The spreading time is primarily dominated by the We and θ_{eqi} in the CD regime 798 whereas only the Weber number plays a meaningful role in the IC regime. In the IV regime it 799 is mainly affected by We and Oh. An improved prediction model is also presented for each regime. To the best of our knowledge, these proposed prediction models (Eqs. 6-8) are the 800 801 first models for the spreading time using only the controllable independent variables (e.g., We, Oh and θ_{eqi}) in explicit form which can provide more straightforward solutions compared 802 to the existing models [18-21,44,45]. 803

The effect of target curvature on the spreading time is also analyzed in detail for the three different spreading regimes, showing complicated trends due to its non-linear nature. Finally, a data-driven prediction model is proposed to predict the spreading time broadly across the three different spreading regimes. Our proposed data-driven model shows good prediction capability for the full ranges of impact parameters considered in the present study and also shows a good agreement with our simulation dataset.

The limitation of the current study is as follows: (i) As in the existing models, prediction 810 models (Eqs. 6-8) are still not derived from physical background. Further efforts are needed 811 to combine "physics-based models" which could be helpful in understanding such 812 complicated behavior within the existing empirical results. (ii) Users need to be careful in 813 using the proposed data-driven model. Although our numerical methods have thoroughly 814 been validated, the number of current data samples (2400 cases) is still insufficient to model 815 the spreading time for the entire spreading regime compared to other common data-driven 816 models trained using "big-data". Our numerical data used for the training are also confined to 817

the current impact conditions ($0 \le We \le 110, 20^\circ \le \theta_{eqi} \le 160^\circ, 0.0013 \le Oh \le 0.7869$ and 818 $1/10 \le \Omega \le 1/2$). Broader ranges of impact parameters, and more precise and abundant data 819 820 samples can improve the applicability and accuracy of the current model. (iii) While the current axi-symmetric simulation code is sufficiently validated, full three-dimensional 821 822 simulations are still essential to better understand those complicated physical phenomena. In 823 particular, impact dynamics at high We and low Oh where the liquid sheets and rim can be 824 highly deformed still need to be captured more precisely. We are currently working on these issues. 825

826

APPENDIX A: Details of the dynamic contact angle

- 828 modeling
- 829

The dynamic contact angle θ_{dyn} is basically modeled as a function of the equilibrium contact angle and the velocity of the contact line, as in well-known previous numerical studies on drop impact problems [82,83]. In the present study, we use the model of Yokoi *et al.* [83]:

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835
$$\theta_{dyn} \left(U_{CL} \right) = \begin{cases} \min \left[\theta_{eqi} + \left(\frac{Ca}{q_1} \right)^{1/3}, \theta_{mda} \right], & \text{if } U_{CL} \ge 0 \text{ (for spreading)} \\ \max \left[\theta_{eqi} + \left(\frac{Ca}{q_2} \right)^{1/3}, \theta_{mdr} \right], & \text{if } U_{CL} \le 0 \text{ (for receding).} \end{cases}$$
(A1)

836

837 Ca is the capillary number (Ca = $\mu U_{CL}/\sigma$), where U_{CL} is the velocity of the contact line. 838 Contact angle hysteresis is represented by the difference between the allowable minimum 839 (θ_{mdr}) and maximum (θ_{mda}) contact angles, and the dynamic contact angle (θ_{dyn}) can always 840 be maintained between θ_{mdr} and θ_{mda} . q_1 and q_2 are experimentally measured constants, and 841 the same values as in Yokoi *et al.* [83] $(q_1 = 9.0 \times 10^{-9} \text{ and } q_2 = 9.0 \times 10^{-8})$ are applied to the 842 current study.

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APPENDIX B: Validation tests in our previous studies

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We here introduce our validation test results presented in our previous studies. Figure 846 17(a) compares the maximum spreading extent β^*_{max} computed by our simulation with Mao 847 et al. [25]'s semi-empirical model which is well-known as one of the most accurate models 848 that can be applied for a wide range of drop viscosity [84]. β^*_{max} shows generally good 849 850 agreement with the existing model of Mao *et al.* [25] for a wide range of We number ($30 \le$ We \leq 90) and Oh number (0.0013 \leq Oh \leq 0.7869). Note that a discrepancy shown at the very 851 low Oh region (Oh ≤ 0.0026) is attributed to the limitation of shape approximation in Mao *et* 852 al.'s model [25], which overestimates β^*_{max} if the viscosity is very low and the surface tension 853 force plays an important role [25,84]. Therefore, in figure 17(b), we compare our simulated 854 855 β^*_{max} again with the well-known viscous-free scaling law proposed by Clanet *et al.* [16]. As can be seen, for cases with very low Oh number (Oh = 0.0026), our simulation result shows 856 857 an excellent agreement with the existing scaling law [16].

In figure 17(c), we further compare our simulated time-dependent non-dimensional spreading diameter β^* with three existing experimental results to see if our numerical framework can simulate the impact phenomena well on spherical targets. The solid lines in Fig. 17(c) depict the simulation results, whereas the red squares, blue circles, and black crosses show the experimental results from Mitra *et al.* [2], Liu *et al.* [38], and Khurana *et al.* [39], respectively. Our simulation results also show good agreement with the existing experimental observations in the literature, in the context of dynamic spreading characteristics. Overall, we concluded that our simulation framework can simulate the drop impact phenomena on both flat and spherical targets well for a wide range of collision parameters. Note again that these validation tests shown in figure 17 are not new tests, but were already presented in our previous study [40].

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APPENDIX C: Details of MLP training

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872 The output value on the n^{th} node in the m^{th} layer is:

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874
$$a_n^m = g\left(\sum_{k=1}^{N^{m-1}} w_{nl}^m a_l^{m-1} + b_n^m\right)$$
(C1)

875

Here, *N* stands for the number of nodes of each layer. The function *g* denotes the nonlinear activation function. w_{nl} is the weight between the *l*th node of the previous layer and the *n*th node of the current layer whereas *b* is the bias.

The weights (w) and biases (b) are adjustable coefficients and are first initialized as random variables and zero, respectively. w and b are continuously updated during the training process, minimizing the loss function based on the back-propagation algorithm [85]. In the current study, the mean square error (MSE) is used as a loss function:

883

MSE =
$$\frac{1}{M} \sum_{k=1}^{M} (\tau_a - \tau_{a(p)})^2$$
 (C2)

885

884

886 where M is the number of data points considered and $\tau_{a(p)}$ is the predicted value of the

spreading time using the trained MLP model.

888 Three hyperparameters (i.e., the activation function, the number of nodes at each hidden 889 layer and the number of hidden layers) can usually be selected by the user. We chose the 890 common rectified linear unit function (generally called ReLU) as our activation function 891 owing to its well-known capability for deep neural networks [86]. The ReLU can be written 892 as:

- 893
- 894

 $g(x) = \operatorname{ReLU}(x) = \max(x, 0)$

(C3)

895

Two other hyperparameters are set by manual search because the current dataset is relatively 896 very simple. Finally, a 4×30 (4 hidden layers and 30 neurons in each hidden layer) 897 architecture is chosen as the MLP network. The ADAM optimization procedure [87,88] is 898 used for the MLP training and the computational implementation for the MLP training is 899 900 performed using the open-source libraries Keras [89] and TensorFlow [90]. For more details on the deep learning architecture and its training techniques readers can refer to Nielsen [91]. 901 The training of our MLP model is carried out until the validation MSE (measured from 902 the validation dataset) reaches a steady state. After 3300 training epochs the validation MSE 903 converges to a steady state of about 4.62×10^{-4} (see Fig.18). 904

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912 **DECLARATIONS**

913

914 Ethical Approval

915 Not applicable.

916

917 **Conflict of interest**

918 The authors declare that they have no conflict of interest.

919

920 Authors' contributions

921 Ikroh Yoon led conceptualization, data curation, formal analysis, investigation, validation, 922 visualization, and writing (draft). Seungwon Shin led funding acquisition, methodology, 923 resources, and software, and supported conceptualization, data curation, formal analysis, 924 investigation, validation, visualization, and writing (draft). Damir Juric and Jalel Chergui 925 supported conceptualization, data curation, formal analysis, funding acquisition, 926 investigation, methodology, resources, software, validation, visualization, and writing (draft). 927 All authors equally contributed to reviewing/editing the manuscript.

928

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936 Data availability

937	The data that support the findings of this study are available from the corresponding author, I.
938	Yoon, upon reasonable request.
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Fig. 2. Schematic regime-map for droplet spreading dynamics. Three different spreading regimes, i.e., the capillary-driven (CD), the inertia-driven but capillary-limited (IC), and the inertia-driven but viscous-limited (IV) regimes are marked by the blue, red and green areas, respectively.



Fig. 3. Typical evolution of droplet interfacial morphology for the three different spreading regimes. (a) CD regime (We = 4, $\theta_{eqi} = 90^{\circ}$ and Oh = 0.0026). (b) IC regime (We = 110, $\theta_{eqi} =$ 90° and Oh = 0.0026). (c) IV regime (We = 110, $\theta_{eqi} = 90^{\circ}$ and Oh = 0.5246). In these cases, 1336 $\Omega = 1/10$ for generality.



Fig. 4. Typical evolution of the non-dimensional spreading extent, β^* , for the three different spreading regimes. All collision conditions for each regime are the same as Fig.3. The inset depicts the schematic for measuring β^* . The spreading times (the time that a droplet attains 95 % of the maximum value of β^*) are also marked by the triangles.



Fig. 5. Benchmark tests. (a) Evolution of the non-dimensional spreading extent, β^* , for a 1384 droplet spreading on a small spherical target at low We (We = 0.9, $\theta_{eqi} = 85^{\circ}$, Oh = 0.0026 1385 and $\Omega = 0.83$). The insets compare the interfacial shapes between the experiment [2] and the 1386 current simulation for 3 different time instants ($t_a^* = 0.24, 0.47, and 0.71$). Reproduced with 1387 permission from S. Mitra et al., "Interactions in droplet and particle system of near unity size 1388 1389 ratio," Chem. Eng. Sci. 170, 154-175 (2017). Copyright 2017 Elsevier, Ltd. (b) Evolution of the non-dimensional film thickness, h^* , for a droplet spreading on a small spherical target at 1390 high We (We = 155.5, $\theta_{eqi} = 97^{\circ}$, Oh = 0.0026 and $\Omega = 1.7$) simulated by using 4 different 1391 grid resolutions (8, 16, 32 and 64 CPR). The insets compare the interfacial shapes between 1392 the experiment [71] and the current simulation for 4 different time instants ($t_a^* = 0.30, 0.60$, 1393 1.05 and 1.95), whereas the other inset in the lower-left corner depicts the schematic for 1394 measuring h. Reproduced with permission from S. A. Banitabaei and A. Amirfazli, "Droplet 1395 impact onto a solid sphere: Effect of wettability and impact velocity," Phys. Fluids 29, 1396 062111 (2017). Copyright 2017 AIP Publishing LLC. 1397



Fig. 6. Benchmark tests. (a) Non-dimensional advective spreading time, τ_a , at low Oh (Oh = 1402 0.0026). The existing experimental result [18] is marked by black squares. (b) τ_a at high Oh 1403 (Oh = 0.62). The existing results from the numerical simulation [21] and the empirical 1404 correlation [19] are marked by black squares and red triangles, respectively.



Fig. 7. Effect of the impact parameters on τ_c for the CD regime. (a) Effect of We (We = 2–10, 1413 $\theta_{eqi} = 90^\circ$ and Oh = 0.0026). (b) Effect of θ_{eqi} (We = 2, $\theta_{eqi} = 20^\circ - 160^\circ$ and Oh = 0.0026). (c) 1414 Effect of Oh (We = 2, $\theta_{eqi} = 90^\circ$ and Oh = 0.0013–0.7869). (d) Effect of Oh on the non-1415 dimensional spreading velocity, V_s^* . (e) Effect of Oh on the non-dimensional maximum 1416 spreading extent, β^*_{max} . For (d) and (e), all collision conditions are the same as Fig.7(c).



Fig. 8. A total of 600 τ_c data (We = 1–110, $\theta_{eqi} = 20^{\circ}-160^{\circ}$ and Oh = 0.0013–0.7869). The proposed prediction model, i.e., Eq.(6) for the CD regime is indicated by the black solid line. The green dashed line depicts the threshold for the CD regime.





Fig. 9. Effect of the impact parameters on τ_c for the IC regime. (a) Effect of We (We = 20– 1443 110, $\theta_{eqi} = 125^{\circ}$ and Oh = 0.0026). (b) Effect of We on the interfacial evolution of a droplet 1444 and its lamella deformation (We = 20 and 110, $\theta_{eqi} = 20^{\circ}$ and Oh = 0.0026). (c) Effect of θ_{eqi} 1445 (We = 90, $\theta_{eqi} = 20^{\circ}-160^{\circ}$ and Oh = 0.0026). (d) Effect of Oh (We = 70, $\theta_{eqi} = 90^{\circ}$ and Oh = 1446 0.0013–0.0525).



Fig. 10. A total of 150 τ_c data (We = 4–110, $\theta_{eqi} = 20^{\circ}-160^{\circ}$ and Oh = 0.0013–0.0052). The proposed prediction model, i.e., Eq.(7), for the IC regime is indicated by the black solid line. The green dashed line depicts the threshold for the IC regime.



Fig. 11. Effect of the impact parameters on τ_v for the IV regime. (a) Effect of We (We = 30– 1479 110, $\theta_{eqi} = 20^\circ$ and Oh = 0.5246). (c) Effect of θ_{eqi} (We = 70, $\theta_{eqi} = 20^\circ-160^\circ$ and Oh = 1480 0.5246). (d) Effect of Oh (We = 70, $\theta_{eqi} = 90^\circ$ and Oh = 0.0525–0.7869).



1493 **Fig. 12.** A total of 250 τ_c data (We = 4–110, $\theta_{eqi} = 20^{\circ}-160^{\circ}$ and Oh = 0.0525–0.7869). The 1494 proposed prediction model, i.e., Eq.(8), for the IV regime is indicated by the black solid line. 1495 The green dashed line depicts the threshold for the IV regime.



Fig. 13. (a) Effect of Ω on τ_c for the CD regime (We = 4, $\theta_{eqi} = 90^\circ$, and Oh = 0.0026). (b) Effect of Ω on τ_c for the IC regime (We = 50, $\theta_{eqi} = 90^\circ$ and Oh = 0.0026). (c) Effect of Ω on τ_v for the IV regime (We = 50, $\theta_{eqi} = 90^\circ$ and Oh = 0.5246). (d) Schematics of the initial apparent contact angles (on the left side) and the lamella characteristics (on the right side).



Fig. 14. Refined regime-map of the droplet spreading mechanism. Three different spreading regimes, i.e., CD, IC, and IV regimes with the transition area are depicted. The physical mechanism driving the spreading process, the characteristic time scale, the effects from 4 impact parameters (We, θ_{eqi} , Oh, and Ω) and the prediction models are also presented for each regime.





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1560	Fig. 15. Schematic diagram of the multilayer perceptron (MLP) used in the current study.
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Fig. 16. Comparison of the spreading time, τ_a , between predicted data by using the datadriven (MLP) model and true (measured) data obtained by our simulations. The deviation range of $\pm 15\%$ is marked by the two dashed-black lines.



Fig. 17. Validation comparisons between our simulation results and existing experimental 1618 results. (a) Comparison with Mao et al.'s [25] semi-empirical model for droplet impact with a 1619 flat surface $(30 \le We \le 90, 0.0013 \le Oh \le 0.7869)$. (b) Comparison with Clanet *et al.*'s [16] 1620 empirical scaling law for droplet impact with a flat surface ($30 \le We \le 110$, Oh = 0.0026). (c) 1621 comparisons with the experimental data of Mitra *et al.* (red squares; We = 0.9, $\Omega = 0.83$, $\theta_{eqi} =$ 1622 85°, Oh = 0.0024) [2], Liu *et al.* (blue circles; We = 19.2, $\Omega = 0.15$, $\theta_{eqi} = 93^{\circ}$, Oh = 0.0022) 1623 [38], and Khurana *et al.* (black crosses; We = 35.3, $\Omega = 0.60$, $\theta_{eqi} = 85^{\circ}$, Oh = 0.0022) [39] for 1624 1625 droplet impact with a spherical target. Reproduced with permission from I. Yoon and S. Shin [40], "Maximal spreading of droplet during collision on particle: effects of liquid viscosity 1626 and surface curvature," Phys. Fluids 33, 083310 (2021). Copyright 2021 AIP Publishing 1627 1628 LLC.



Fig. 18. Training errors (MSE) vs. training epochs for training and validation datasets. The training is terminated at 3300 training epochs where the MSE for the validation dataset (see black solid line) is sufficiently converged. The error for the validation dataset is 4.62×10^4 .



