

Radiative transfer: molecular physics Benjamin Godard

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(diatomic) Molecular physics

International Research Track

2023

- introduction
- energy structure
- molecular lines
- photoprocesses
- conclusion

- 1. Introduction
- 2. Energy structure
 - Born-Oppenheimer
 - Vibration / rotation
 - Additional couplings

- 3. Molecular lines
 - **Dipole transitions**
 - Quadrupole transitions
- 4. Photo-processes
- 5. Conclusions

Molecules are ubiquitous in astrophysics

<u>Outline</u>

- introduction
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In summary, in 2018

- → 204 individual molecular species
- comprised of 16 different elements
- → composed of 2 to 70 atoms
- detected from tens-cm wavelength to ultraviolet

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2 Atoms		3 Atoms		4 Atoms	5 Atoms	6 Atoms	7 Atoms
СН	СР	H ₂ O	N ₂ O	NH ₃	HC ₃ N	CH ₃ OH	CH ₃ CHO
CN	NH	HCO^+	MgCN	H ₂ CO	HCOOH	CH ₃ CN	CH ₃ CCH
CH^+	SiN	HCN	H3 ⁺	HNCO	CH ₂ NH	NH ₂ CHO	CH ₃ NH ₂
OH	SO^+	OCS	SiCN	H_2CS	NH ₂ CN	CH ₃ SH	CH ₂ CHCN
СО	CO^+	HNC	AINC	C_2H_2	H ₂ CCO	C_2H_4	HC ₅ N
H_2	HF	H ₂ S	SiNC	C ₃ N	C ₄ H	C ₅ H	C ₆ H
SiO	N_2	N_2H^+	HCP	HNCS	SiH ₄	CH ₃ NC	c-C ₂ H ₄ O
CS	\mathbf{CF}^+	C_2H	CCP	$HOCO^+$	$c-C_3H_2$	HC ₂ CHO	CH ₂ CHOH
SO	PO	SO_2	AlOH	C ₃ O	CH ₂ CN	H_2C_6	C_6H^-
SiS	O_2	HCO	H_2O^+	l-C ₃ H	C ₅	C ₅ S	CH ₃ NCO
NS	AlO	HNO	H_2Cl^+	HCNH ⁺	SiC ₄	HC ₃ NH ⁺	HC ₅ O
C_2	CN^{-}	HCS^+	KCN	H_3O^+	H ₂ CCC	C ₅ N	
NO	OH^+	HOC^+	FeCN	C ₃ S	CH_4	HC ₄ H	
HCl	SH^+	SiC ₂	HO ₂	c-C ₃ H	HCCNC	HC ₄ N	
NaCl	HCl^+	C ₂ S	TiO ₂	HC ₂ N	HNCCC	c-H ₂ C ₃ O	
AlCl	SH	C ₃	CCN	H_2CN	H_2COH^+	CH ₂ CNH	
KCl	TiO	CO ₂	SiCSi	SiC ₃	C_4H^-	$C_5 N^-$	
AlF	ArH^+	CH_2	S ₂ H	CH ₃	CNCHO	HNCHCN	
PN	NS^+	C_2O	HCS	C_3N^-	HNCNH	SiH ₃ CN	
SiC		MgNC	HSC	PH ₃	CH ₃ O		
		NH ₂	NCO	HCNO	NH_3D^+		
		NaCN		HOCN	H_2NCO^+		
				HSCN	$NCCNH^+$		
				HOOH	CH ₃ Cl		
				$1-C_3H^+$			
				HMgNC			
				HCCO			
				CNCN			

Table 2

Table 3

List of Detected Interstellar Molecules with Eight or More Atoms, Categorized by Number of Atoms, and Vertically Ordered by Detection Year

8 Atoms	9 Atoms	10 Atoms	11 Atoms	12 Atoms	13 Atoms	Fullerenes
HCOOCH ₃	CH ₃ OCH ₃	(CH ₃) ₂ CO	HC ₉ N	C ₆ H ₆	c-C ₆ H ₅ CN	C ₆₀
CH ₃ C ₃ N	CH ₃ CH ₂ OH	HO(CH ₂) ₂ OH	CH ₃ C ₆ H	n-C ₃ H ₇ CN		C_{60}^{+}
C ₇ H	CH ₃ CH ₂ CN	CH ₂ CH ₂ CHO	CH ₃ CH ₂ OCHO	i-C ₃ H ₇ CN		C ₇₀
CH ₃ COOH	HC ₇ N	CH ₃ C ₅ N	CH ₃ COOCH ₃			
H_2C_6	CH ₃ C ₄ H	CH ₃ CHCH ₂ O				
CH ₂ OHCHO	C ₈ H	CH ₃ OCH ₂ OH				
HC ₆ H	CH ₃ CONH ₂					
CH ₂ CHCHO	C_8H^-					
CH ₂ CCHCN	CH ₂ CHCH ₃					
NH ₂ CH ₂ CN	CH ₃ CH ₂ SH					
CH ₃ CHNH	HC ₇ O					
CH ₃ SiH ₃						

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HIFI Spectrum of Water and Organics in the Orion Nebula

© ESA, HEXOS and the HIFI consortium E. Bergin

Radiative transfer equation



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- absorption coefficient
- diffusion coefficient
- emissivity
- opacity
- source function $S_{
 u}$ erg s⁻¹ cm⁻² Hz⁻¹ sr⁻¹

jν

 $\alpha_{\nu} = \sum n_a s_{\nu,a}$

 $\sigma_{\nu} = \sum_{i}^{a} n_d s_{\nu,d}$

 $d\tau_{\nu} = \left[\alpha_{\nu} + \sigma_{\nu}\right] dl$

cm⁻¹

cm-1

erg s⁻¹ cm⁻³ Hz⁻¹ sr⁻¹

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through which processes do molecules interact with light?

- → absorption / emission
- → polarization
- dissociation / ionization / fluorescence

what are the properties of the microphysical processes ?

- transitions (energy structure / selection rules)
- → interaction cross sections
- products of ionization and dissociation

how to implement these processes in radiative transfer codes ?

- → what data are required ?
- → couplings expected ?
- analytical considerations & numerical methods

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Energy structure of molecules

Complexity of the task

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Complexity of the task

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complexity

symmetry

Compared to atomic energy structure

- → less symmetry, complicated electronic wavefunctions
- nuclei move, additional freedom, rotational and vibrational motions

Focus on diatomic (homonuclear or heteronuclear)

gives a strong foothold to understand molecular spectra

Stationary states and usual hypothesis

Stationary Schrödinger equation for diatomic molecules

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$$\left(-\frac{\hbar^2}{2M_A}\nabla_A^2 - \frac{\hbar^2}{2M_B}\nabla_B^2 - \frac{\hbar^2}{2m_e}\sum_{i=1}^N \nabla_i^2 + V_e - E\right)\psi(\mathbf{R}_A, \mathbf{R}_B, \mathbf{r}_i) = 0$$

$$V_e = \frac{e^2}{4\pi\epsilon_0}\left(-\sum_{i=1}^N \frac{Z_A}{r_{Ai}} - \sum_{i=1}^N \frac{Z_B}{r_{Bi}} + \sum_{i=2}^N \sum_{j=1}^{i-1} \frac{1}{r_{ij}} + \frac{Z_A Z_B}{R}\right)$$

- Usual simplifications & hypothesis
 - → Born-Oppenheimer
 - → center of mass frame

$$\psi(\mathbf{R}_A, \mathbf{R}_B, \mathbf{r}_i) = \psi_e(\mathbf{r}_i)\psi_n(\mathbf{R}_A, \mathbf{R}_B)$$

 $\psi_n(\mathbf{R}_A, \mathbf{R}_B) o \psi_n(\mathbf{R})$

e₂

 r_{B2}

r₁₂

R

A

 r_{A2}

B

→ separation vibration - rotation $\psi_n(\mathbf{R}) = \psi_v(R)\psi_r(\theta,\phi)$

Electronic structure

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Stationary Schrödinger equation for electronic wavefunction

$$\left(-\frac{\hbar^2}{2m_e}\sum_{i=1}^N \nabla_i^2 + V_e - E_e\right)\psi_e(\boldsymbol{r}_i) = 0$$

- solve at each internuclear separation R
- → gives eigenvalue $E_e(R)$
- → gives Ψ_e & quantum numbers
 without couplings
 S, Λ, g/u, +/-
- gives potential V(R) seen by nuclei
- gives equilibrium radius & dissociation energy



Electronic structure

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Labelling of electronic states

 $\longrightarrow S = \sum_{i} s_{i}$ $\longrightarrow \Lambda = \left(L = \sum_{i} l_{i} \right)_{|z|}$

→ g/u (gerade/ungerade)

+/-(parity)

 \rightarrow letter

letter^(2S+1)
$$\Lambda_{g/u}^{+/-}$$

total electronic spin (careful with paired e- in closed shells)

total angular momentum projected on z

Λ	0	1	2	3	4	•••
states	\sum	Π	Δ	Φ	Γ	•••

parity upon nuclei swap (for homonuclear only)

parity upon reflexion through nuclei plane (for Σ states only)

label states of same quantum numbers

X = ground electronic state (GES)

A, B, ... = same spin multiplicity as GES

a,b, ... = diff spin multiplicity than GES

Vibrational structure

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Stationary Schrödinger equation for nuclei vibration

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + V(R) - E_v \bigg) \psi_v(R) = 0$$

for V(R) approx by a parabolic potential $V_{\rm HO}(R) = V_0 + \frac{1}{2}k(R - R_{\rm eq})^2$ quantum harmonic oscillator

wavefunction derived from

Hermite polynomials

zero point energy $zpe = \frac{1}{2}\hbar\left(\frac{k}{u}\right)^{\frac{1}{2}}$

 $E_v = V_0 + \hbar \left(\frac{k}{\mu}\right)^{\frac{1}{2}} \left(v + \frac{1}{2}\right)$



Rotational structure

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Stationary Schrödinger equation for nuclei rotation

$$-\frac{\hbar^2}{2\mu R^2} \left[\frac{1}{\sin\theta} \frac{d}{d\theta} (\sin\theta \frac{d}{d\theta}) + \frac{1}{\sin^2\theta} \frac{d}{d\phi} \right] - E_r \right) \psi_r(\theta, \phi) = 0$$

- spherical harmonics solution $\psi_r(\theta, \phi) = Y_{J,M_J}(\theta, \phi)$
 - $Y_{J,M_J} \longrightarrow$ already seen for H atom
 - Jrotational angular momentum \rightarrow
 - projection of J onto molecular axis M_J
 - reduced mass μ
- rotational eigenvalues

$$E_r = \frac{\hbar^2}{2\mu R^2} J(J+1)$$

if rotation as rigid body

$$E_r = B_0 J (J+1)$$

- degeneracy of rotational levels $g_r = 2J + 1$
 - IRT radiative transfer

Couplings between angular momenta

---- electron angular momentum and spin, nuclei rotational motion and spin

- coupling schemes known as Hund's cases (a) and (b) most common
- \rightarrow R = rotational angular momentum

Hund's case (a)	Hund's case (b)
1 - $\Lambda = L_{ z}, \ \Sigma = S_{ z}$	1 - $\Lambda = L_{ z }$
$2 - \Lambda + \Sigma \to \Omega$	$2 - \Lambda + R \rightarrow N$
$3 - \Omega + R \rightarrow J$	$3 - N + S \rightarrow J$

→ hyperfine coupling

coupling with nuclear spin I

 $J+I\to F$

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Detailed electronic / vibrational / rotational structure

In reality, anharmonic & non rigid rotation with rot / vib couplings

- corrections terms (used in a polynomial expansion)
- derived from spectroscopic measurements of molecular lines
 - anharmonic correction, centrifugal distorsion, correction of rot constant

(1)

example for CH+

 $E(\Lambda, v, J) =$ $T_{\rm e}(\Lambda) + \omega_{\rm e}(\Lambda)\left(\nu + \frac{1}{2}\right) - \omega_{\rm e}x_{\rm e}(\Lambda)\left(\nu + \frac{1}{2}\right)^2 + \omega_{\rm e}y_{\rm e}(\Lambda)\left(\nu + \frac{1}{2}\right)^3$ $+\left[B_{\rm e}(\Lambda)-\alpha_{\rm e}(\Lambda)\left(\upsilon+\frac{1}{2}\right)+\gamma_{\rm e}(\Lambda)\left(\upsilon+\frac{1}{2}\right)^{2}+\varepsilon_{\rm e}(\Lambda)\left(\upsilon+\frac{1}{2}\right)^{3}\right]$ $\times \left(J(J+1) - \Lambda^2 \right) - \left[D_{\rm e}(\Lambda) + \beta_{\rm e}(\Lambda) \left(\upsilon + \frac{1}{2} \right) + \delta_{\rm e}(\Lambda) \left(\upsilon + \frac{1}{2} \right)^2 \right]$ $\times \left(J(J+1) - \Lambda^2\right)^2 + \left[H_{\rm e}(\Lambda) - \alpha_{\rm H_e}(\Lambda)\left(\upsilon + \frac{1}{2}\right)\right] \left(J(J+1) - \Lambda^2\right)^3$ $\pm \frac{1}{2} \left| q_{\rm e}(\Lambda) + \alpha_{q_{\rm e}}(\Lambda) \left(\upsilon + \frac{1}{2} \right) \right| \left(J(J+1) \right)$ $\pm \frac{1}{2}q_{D_{\rm e}}(\Lambda) \left(J(J+1)\right)^2,$

constant	$X^1\Sigma^+$		$A^{1}\Pi$	
T _e			2.41187262	(+4)
$\omega_{ m e}$	2.8575609	(+3)	1.864402	(+3)
$\omega_{\rm e} x_{\rm e}$	5.93179	(+1)	1.158317	(+2)
$\omega_{\rm e} y_{\rm e}$	2.2534	(-1)	2.6301	
$B_{\rm e}$	1.41774612	(+1)	1.1886774	(+1)
$\alpha_{\rm e}$	4.94739	(-1)	9.1629	(-1)
$\gamma_{ m e}$	2.4904	(-3)	-2.292	(-2)
\mathcal{E}_{e}			4.952	(-3)
$D_{\rm e}$	1.38914	(-3)	1.929606	(-3)
$\beta_{\rm e}$	-2.66	(-5)	1.07331	(-4)
$\delta_{ m e}$			-1.3123	(-5)
$H_{\rm e}$	1.2036	(-7)		
$lpha_{ m H_e}$	2.079	(-8)		
$q_{\rm e}$			4.1018	(-2)
$\alpha_{q_{\mathrm{e}}}$			-3.135	(-3)
q_{D_e}			-2.2	(-5)

Hakalla et al. (2006)

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Detailed electronic / vibrational / rotational structure

Hierarchical energy structure

<u>Outline</u>

introduction

- \rightarrow rule of thumb $E_e > E_v > E_r$
 - ladders are not separated but superimposed



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Molecular spectra

IRT - radiative transfer

Electric dipole transitions

Interaction of a molecule with EM field

- → electric dipole, magnetic dipole, electric quadrupole, ...
 - electric dipole terms $\sim 10^5 10^6$ times larger than the others

$$P\propto |oldsymbol{M}|^2$$
 with $oldsymbol{M}=\int\psi^{\prime*}oldsymbol{\mu}\psi^{\prime\prime}doldsymbol{r}$

- *M* electric dipole moment integral
- μ dipole moment operator
- symmetry properties of $\Psi_{,} \Psi_{,}$ and μ give the selection rules $\Delta S, \Delta \Lambda, \Delta v, \Delta J, ...$

IRT - radiative transfer

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Selection rules on electronic quantum numbers

for allowed electric dipole transitions

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 $\Delta S = 0$ spin conserving selection rule

 $\bullet \quad \Delta \Lambda = 0, \pm 1$ projection of orbital momentum

 $\longrightarrow \Sigma^+ \leftrightarrow \Sigma^+$ for Σ states only

 $\Sigma^- \leftrightarrow \Sigma^-$

 $g \leftrightarrow u$ for homonuclear diatomic molecules

 \Rightarrow no *dipole allowed* vibration-rotation transition

within an electronic state for homonuclear species

IRT - radiative transfer

Selection rules on vibrational quantum number

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transitions from two different electronic states

Franck-Condon principle nuclei static during elec transition transition probability proportional to wavefunctions overlap

$$\left|\int \psi_v'(v')^*\psi_v''(v'')d\boldsymbol{r}\right|^2$$

•
$$\Delta v = any$$
 no selection rule



transition within the same electronic state

 $\rightarrow \Delta v = any$ for anharmonic oscillators stronger transitions for $\Delta v = \pm 1$ (HO)

Selection rules on rotational quantum number

for allowed electric dipole transitions

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- $\Delta J = \pm 1 for \Lambda = 0 0 transitions$
 - $\Delta J = 0, \pm 1 \ (0 \nleftrightarrow 0)$ for other $\Delta \Lambda$
- → labelling: J'(J'') associated with the upper (lower) energy level $\frac{\text{name}}{\Delta J = J' - J''} \begin{vmatrix} R(J'') & Q(J'') & P(J'') \\ -1 & 0 & -1 \end{vmatrix}$
- \rightarrow for transition within the same electronic state \rightarrow regular structure



Oscillator strength and Einstein coefficients

for allowed electric dipole transitions

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•

→ the Einstein coefficient

$$A_{\Lambda'\upsilon'J'\Lambda''\upsilon''J''} = \frac{8\pi^2\nu^2e^2}{m_ec^3}\frac{2J''+1}{2J'+1}f_{\Lambda'\upsilon'J'\Lambda''\upsilon''J''}$$

→ with the oscillator strength

$$f_{\Lambda'\upsilon'J'\Lambda''\upsilon''J''} = \frac{8\pi^2 m_e\nu}{3he^2} M_{\Lambda'\upsilon'\Lambda''\upsilon''}^2 \frac{S_{\Lambda''J''}^{\Lambda'J'}}{2J''+1}$$

 $S^{\Lambda'J'}_{\Lambda''J''}$ Hönl-London factor, dependence on nuclear angular momentum

under the Born-Oppenheimer and the r-centroid approximations

Pure vibration-rotation of homonuclear diatomic molecules

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electric dipole transitions forbidden within an electronic state of homonuclear diatomic molecules (e.g. H₂, C₂, N₂, ...)

- dipole magnetic and quadrupole electric transitions
- example : H₂ quadrupole electric transitions

selection rule for pure vibration-rotation $\Delta J = 0, \pm 2 \ (0 \nleftrightarrow 0)$

labelling: J'(J') associated with the upper (lower) energy level



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Ionization / dissociation / fluorescence

Principles of photo-processes

photoionization and photodissociation processes are fundamental

- to compute radiative transfer in molecular environments
- to compute chemical composition of diffuse & dense ISM, PDRs, protoplanetary disks, cometary & exoplanetary atmosphere, ...

microphysical process

1 - excitation of any (bound or not) electronic state

 $AB + photon \rightarrow AB^*$

 $k = \int_{\lambda} \sigma(\lambda) N_{\lambda} d\lambda$

- → 2 relaxation towards a lower state branching ratios $AB^* \to AB^+ + e^- \longrightarrow \eta^i(\lambda)$ $AB^* \to A + B \longrightarrow \eta^d(\lambda)$
 - $AB^* \to AB + \text{photon} \longrightarrow \eta^e(\lambda)$

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Continuous or discret absorption

Several pathways to photodissociation



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- \rightarrow photodissociation may be a line or a continuum mechanism
- cross sections must account for all possible process
- → line processes favor self-shielding

Threshold and cross sections

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examples of cross sections



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Threshold and cross sections

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examples of cross sections



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Self-shielding process

Consider a cloud illuminated by energetic photons

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the abundance of X at any point depends on photo-processes

- intensity of the radiation field at specific wavelength
- → absorption processes from the border of the cloud
 - by dust grains - - -

- → dust shielding
- by atoms and molecules $\langle X - \rangle$ cross shielding $X - \rangle$ self-shielding

Self-shielding process

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Molecular structure and spectroscopy

- simplifications gives strong insight on molecular structure
 Born-Oppenheimer → elec orbitals & quantum numbers
 separation vibration / rotation → vib oscillator & rotational orbitals
 additional couplings → fine & hyperfine structure
- electric dipole transitions obey to stringent transition rules molecular spectra highly structured (band, branches, fine / hyperfine)
 homonuclear diatomic molecules radiate through quadrupole transitions
 - ionization / dissociation / fluorescence originate from mol transitions
 - non linear coupling between transfer and composition (e.g. self-shielding)