



Radiative transfer: molecular physics

Benjamin Godard

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(diatomic) Molecular physics

International Research Track

2023

Lecture description

Outline

- introduction

- 1. Introduction

- 2. Energy structure

- Born-Oppenheimer

- Vibration / rotation

- Additional couplings

- 3. Molecular lines

- Dipole transitions

- Quadrupole transitions

- 4. Photo-processes

- 5. Conclusions

- photo-processes

- conclusion

Molecules are ubiquitous in astrophysics

Outline

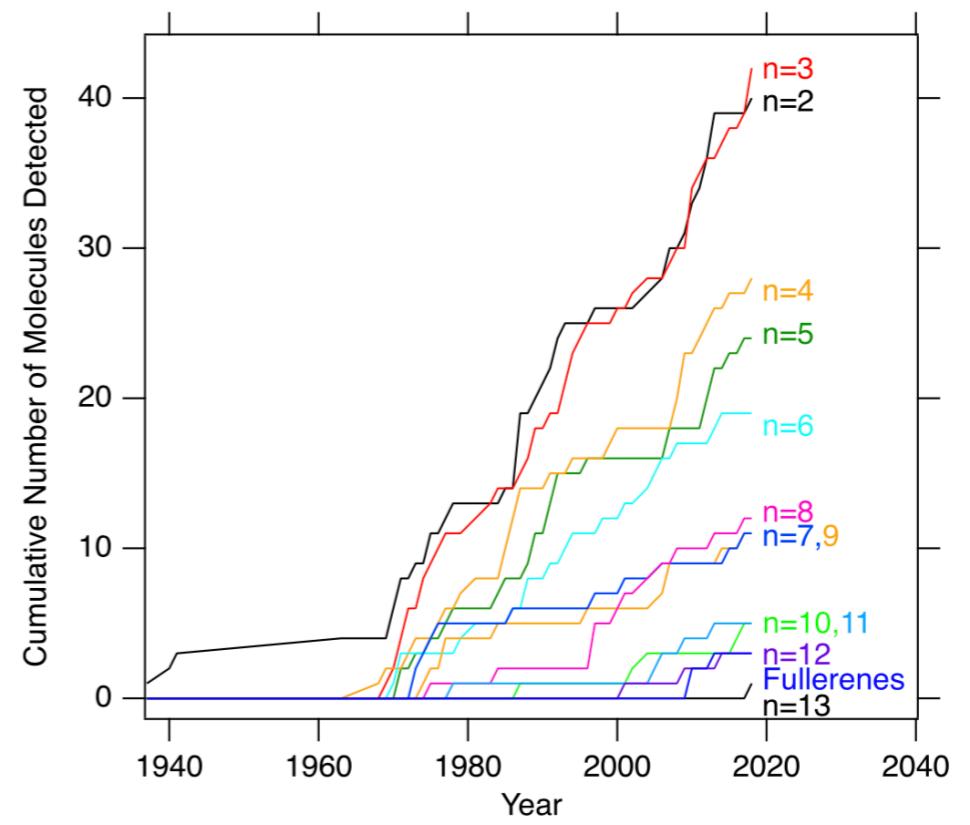
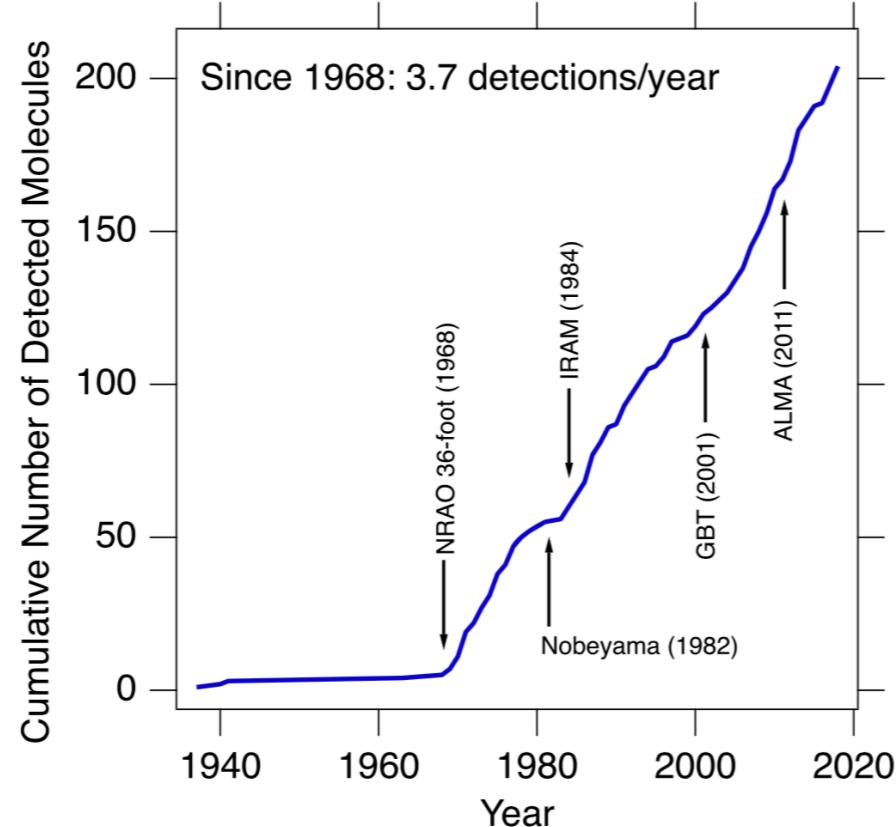
- introduction

- energy structure

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- photo-processes

- conclusion



B. McGuire (2018)

In summary, in 2018

- 204 individual molecular species
- comprised of 16 different elements
- composed of 2 to 70 atoms
- detected from tens-cm wavelength to ultraviolet

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Table 2
List of Detected Interstellar Molecules with Two to Seven Atoms, Categorized by Number of Atoms, and Vertically Ordered by Detection Year

2 Atoms	3 Atoms	4 Atoms	5 Atoms	6 Atoms	7 Atoms
CH	CP	H ₂ O	N ₂ O	NH ₃	HC ₃ N
CN	NH	HCO ⁺	MgCN	H ₂ CO	HCOOH
CH ⁺	SiN	HCN	H ₃ ⁺	HNCO	CH ₂ NH
OH	SO ⁺	OCS	SiCN	H ₂ CS	NH ₂ CN
CO	CO ⁺	HNC	AlNC	C ₂ H ₂	H ₂ CCO
H ₂	HF	H ₂ S	SiNC	C ₃ N	C ₂ H ₄
SiO	N ₂	N ₂ H ⁺	HCP	HNCS	SiH ₄
CS	CF ⁺	C ₂ H	CCP	HOCO ⁺	c-C ₃ H ₂
SO	PO	SO ₂	AlOH	C ₃ O	HC ₂ CNO
SiS	O ₂	HCO	H ₂ O ⁺	l-C ₃ H	CH ₂ CN
NS	AlO	HNO	H ₂ Cl ⁺	HCNH ⁺	H ₂ C ₆
C ₂	CN ⁻	HCS ⁺	KCN	H ₃ O ⁺	C ₆ H ⁻
NO	OH ⁺	HOC ⁺	FeCN	C ₃ S	SiH ₄
HCl	SH ⁺	SiC ₂	HO ₂	c-C ₃ H	CH ₃ NCO
NaCl	HCl ⁺	C ₂ S	TiO ₂	HC ₂ N	HC ₅ O
AlCl	SH	C ₃	CCN	H ₂ CN	c-H ₂ C ₃ O
KCl	TiO	CO ₂	SiCSi	SiC ₃	CH ₂ CNH
AlF	ArH ⁺	CH ₂	S ₂ H	C ₃ H ₃	C ₄ H ⁻
PN	NS ⁺	C ₂ O	HCS	C ₃ N ⁻	C ₅ N ⁻
SiC		MgNC	HSC	PH ₃	HNCHCN
		NH ₂	NCO	HCNO	SiH ₃ CN
		NaCN		HOCN	
				HSCN	
				HOOH	
				l-C ₃ H ⁺	
				HMgNC	
				HCCO	
				CNCN	

Table 3
List of Detected Interstellar Molecules with Eight or More Atoms, Categorized by Number of Atoms, and Vertically Ordered by Detection Year

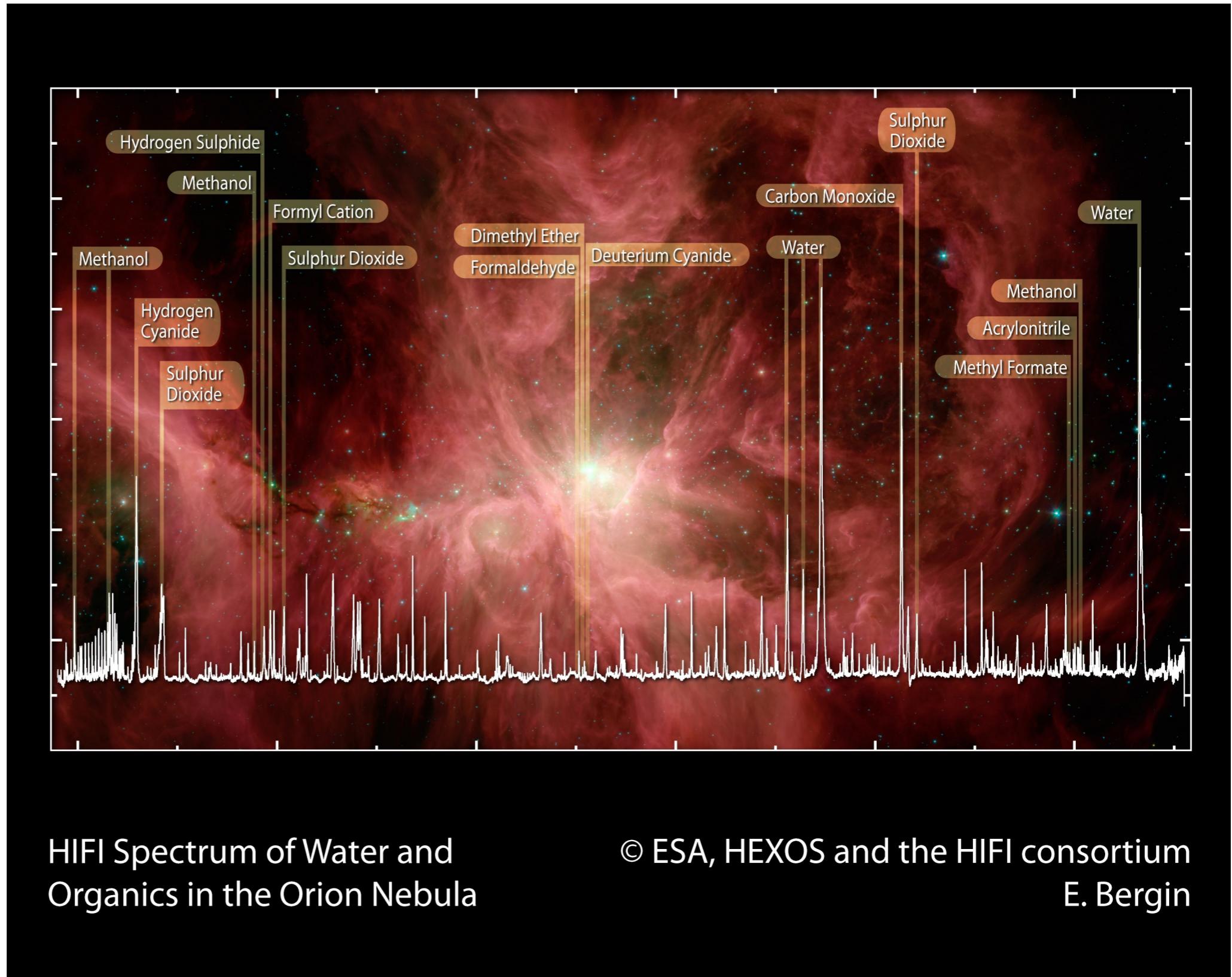
8 Atoms	9 Atoms	10 Atoms	11 Atoms	12 Atoms	13 Atoms	Fullerenes
HCOOCH ₃	CH ₃ OCH ₃	(CH ₃) ₂ CO	HC ₉ N	C ₆ H ₆	c-C ₆ H ₅ CN	C ₆₀
CH ₃ C ₃ N	CH ₃ CH ₂ OH	HO(CH ₂) ₂ OH	CH ₃ C ₆ H	n-C ₃ H ₇ CN		C ₆₀ ⁺
C ₇ H	CH ₃ CH ₂ CN	CH ₂ CH ₂ CHO	CH ₃ CH ₂ OCHO	i-C ₃ H ₇ CN		C ₇₀
CH ₃ COOH	HC ₇ N	CH ₃ C ₅ N	CH ₃ COOCH ₃			
H ₂ C ₆	CH ₃ C ₄ H	CH ₃ CHCH ₂ O				
CH ₂ OHCHO	C ₈ H	CH ₃ OCH ₂ OH				
HC ₆ H	CH ₃ CONH ₂					
CH ₂ CHCHO	C ₈ H ⁻					
CH ₂ CCHCN	CH ₂ CHCH ₃					
NH ₂ CH ₂ CN	CH ₃ CH ₂ SH					
CH ₃ CHNH	HC ₇ O					
CH ₃ SiH ₃						

B. McGuire (2018)

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Radiative transfer equation

Outline

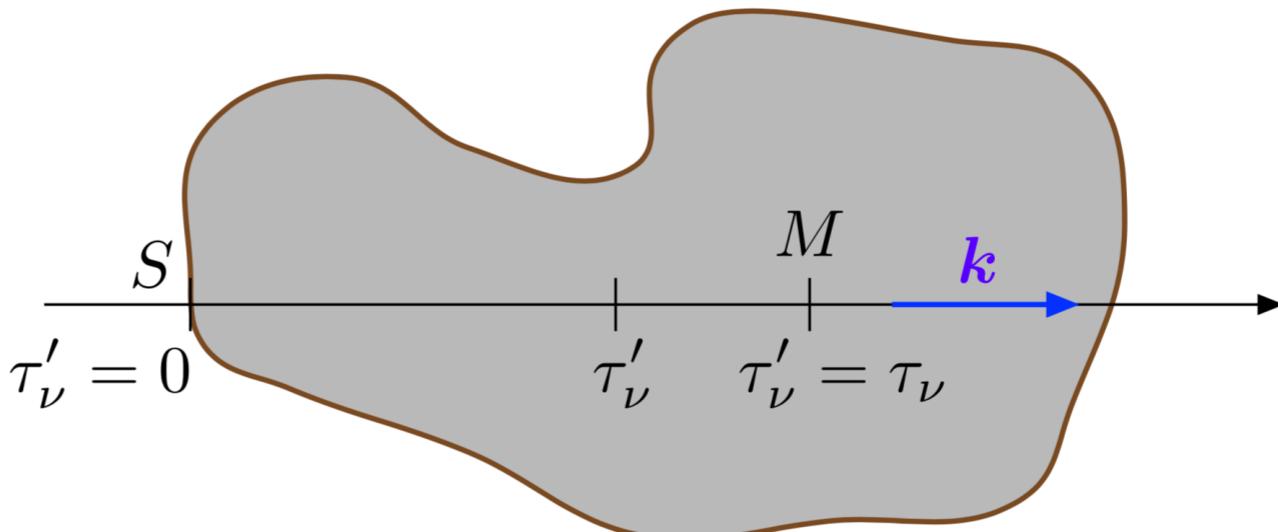
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F. Levrier - radiative transfer lecture

$$\frac{dI_\nu}{dl} = - [\alpha_\nu + \sigma_\nu] I_\nu + j_\nu + \frac{1}{4\pi} \int_{\Omega'} \tilde{\sigma}_\nu(\mathbf{k}' \rightarrow \mathbf{k}) I_\nu(\mathbf{k}') d\Omega'$$

• absorption coefficient	$\alpha_\nu = \sum_a n_a s_{\nu,a}$	cm^{-1}
• diffusion coefficient	$\sigma_\nu = \sum_d n_d s_{\nu,d}$	cm^{-1}
• emissivity	j_ν	$\text{erg s}^{-1} \text{cm}^{-3} \text{Hz}^{-1} \text{sr}^{-1}$
• opacity	$d\tau_\nu = [\alpha_\nu + \sigma_\nu] dl$	
• source function	S_ν	$\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$

Questions related to molecules in radiative transfer

Outline

• introduction

→ absorption / emission

→ polarization

→ dissociation / ionization / fluorescence

• energy
structure

what are the properties of the microphysical processes ?

→ transitions (energy structure / selection rules)

→ interaction cross sections

→ products of ionization and dissociation

• photo-
processes

how to implement these processes in radiative transfer codes ?

→ what data are required ?

→ couplings expected ?

→ analytical considerations & numerical methods

Outline

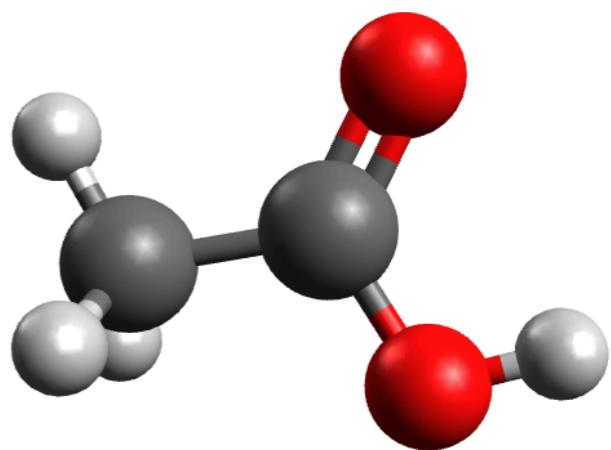
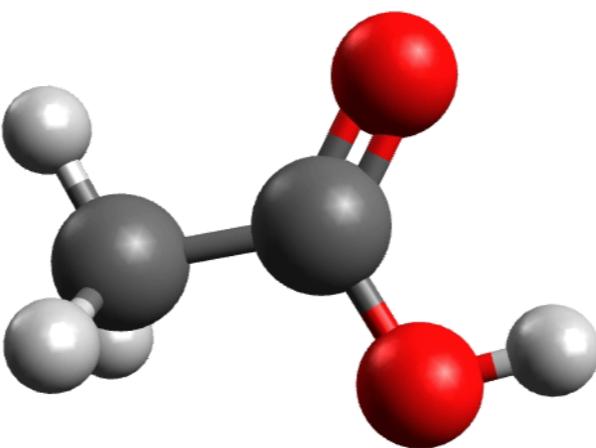
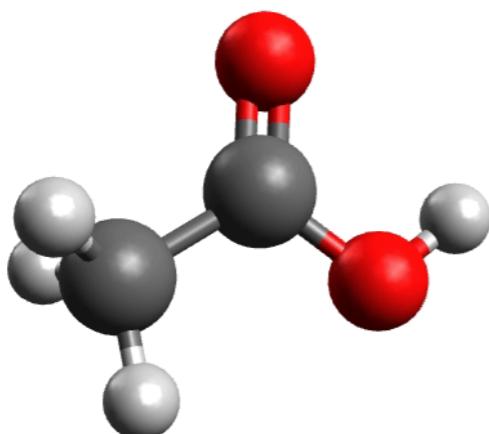
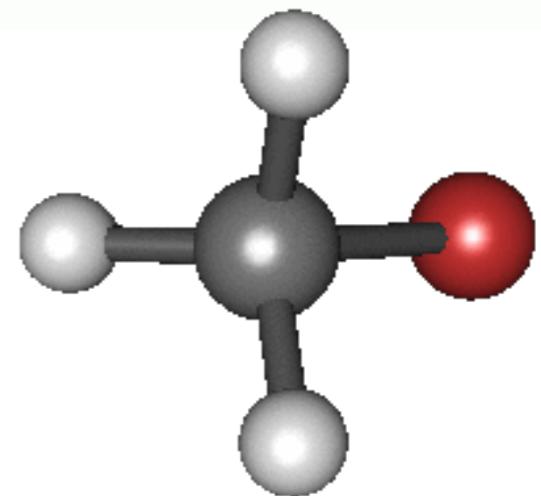
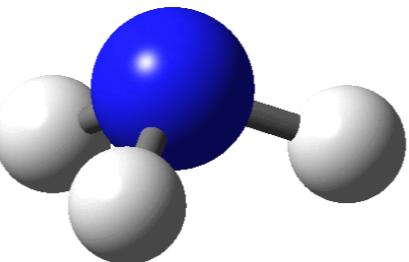
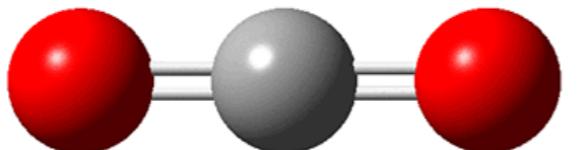
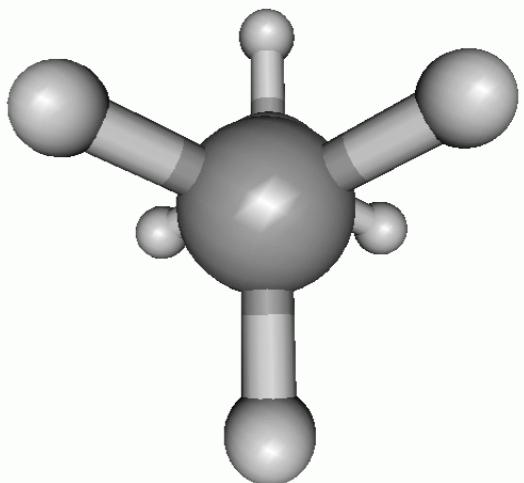
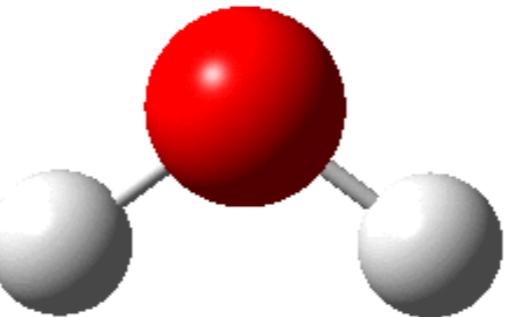
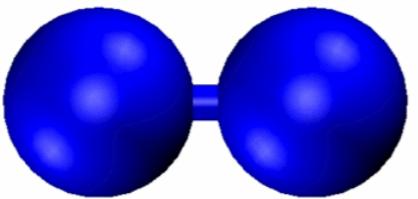
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Energy structure of molecules

Complexity of the task

Outline

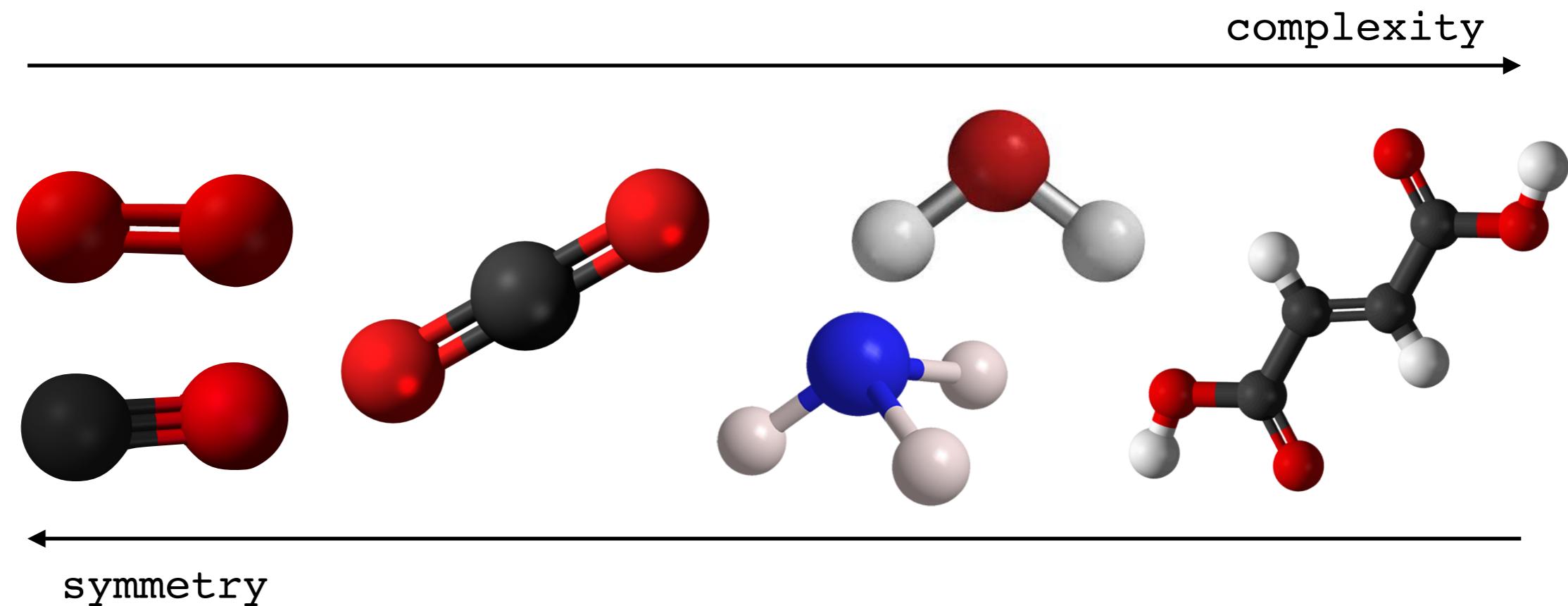
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Complexity of the task

Outline

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Compared to atomic energy structure

- less symmetry, complicated electronic wavefunctions
- nuclei move, additional freedom, rotational and vibrational motions

Focus on diatomic (homonuclear or heteronuclear)

- gives a strong foothold to understand molecular spectra

Stationary states and usual hypothesis

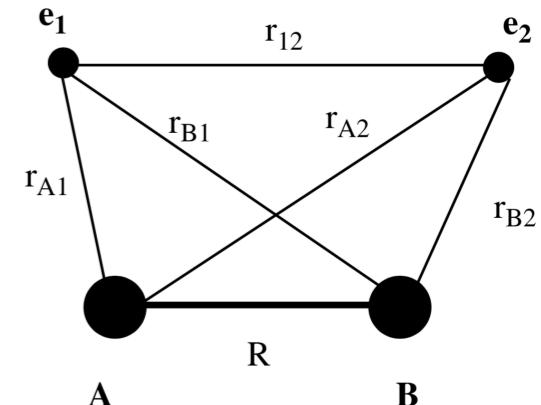
Stationary Schrödinger equation for diatomic molecules

Outline

- introduction

$$\left(-\frac{\hbar^2}{2M_A} \nabla_A^2 - \frac{\hbar^2}{2M_B} \nabla_B^2 - \frac{\hbar^2}{2m_e} \sum_{i=1}^N \nabla_i^2 + V_e - E \right) \psi(\mathbf{R}_A, \mathbf{R}_B, \mathbf{r}_i) = 0$$

$$V_e = \frac{e^2}{4\pi\epsilon_0} \left(-\sum_{i=1}^N \frac{Z_A}{r_{Ai}} - \sum_{i=1}^N \frac{Z_B}{r_{Bi}} + \sum_{i=2}^N \sum_{j=1}^{i-1} \frac{1}{r_{ij}} + \frac{Z_A Z_B}{R} \right)$$



- energy structure

- molecular lines

- photo-processes

- conclusion

Usual simplifications & hypothesis

→ Born-Oppenheimer

$$\psi(\mathbf{R}_A, \mathbf{R}_B, \mathbf{r}_i) = \psi_e(\mathbf{r}_i) \psi_n(\mathbf{R}_A, \mathbf{R}_B)$$



→ center of mass frame

$$\psi_n(\mathbf{R}_A, \mathbf{R}_B) \rightarrow \psi_n(\mathbf{R})$$



→ separation vibration - rotation $\psi_n(\mathbf{R}) = \psi_v(R) \psi_r(\theta, \phi)$



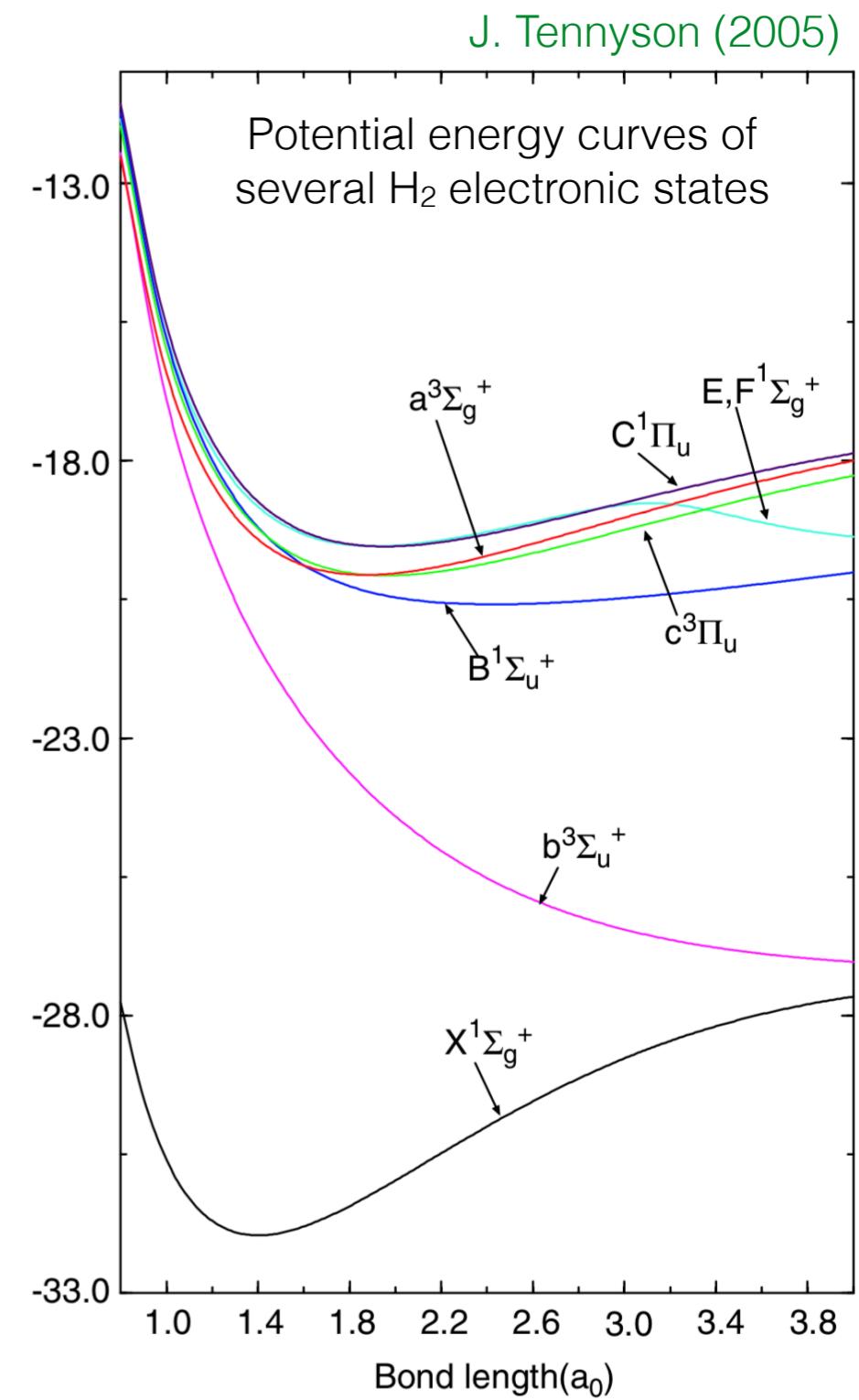
Electronic structure

Outline

- introduction
- energy structure
- molecular lines
- photo-processes
- conclusion

$$\left(-\frac{\hbar^2}{2m_e} \sum_{i=1}^N \nabla_i^2 + V_e - E_e \right) \psi_e(\mathbf{r}_i) = 0$$

- solve at each internuclear separation R
- gives eigenvalue $E_e(R)$
- gives Ψ_e & quantum numbers without couplings $S, \Lambda, g/u, +/ -$
- gives potential $V(R)$ seen by nuclei
- gives equilibrium radius & dissociation energy



Electronic structure

Outline

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Labelling of electronic states

$$\rightarrow S = \sum_i s_i$$

letter^(2S+1) $\Lambda_{g/u}^{+/-}$

total electronic spin
(careful with paired e- in closed shells)

$$\rightarrow \Lambda = \left(L = \sum_i l_i \right)_{|z}$$

total angular momentum projected on z

Λ	0	1	2	3	4	...
states	Σ	Π	Δ	Φ	Γ	...

$$\rightarrow g/u \text{ (gerade/ungerade)}$$

parity upon nuclei swap
(for homonuclear only)

$$\rightarrow +/ - \text{ (parity)}$$

parity upon reflexion through nuclei plane
(for Σ states only)

$$\rightarrow \text{letter}$$

label states of same quantum numbers

X = ground electronic state (GES)

A, B, ... = same spin multiplicity as GES

a,b, ... = diff spin multiplicity than GES

Vibrational structure

Outline

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Stationary Schrödinger equation for nuclei vibration

$$\left(-\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} + V(R) - E_v \right) \psi_v(R) = 0$$

→ for $V(R)$ approx by a parabolic potential $V_{HO}(R) = V_0 + \frac{1}{2}k(R - R_{eq})^2$

quantum harmonic oscillator

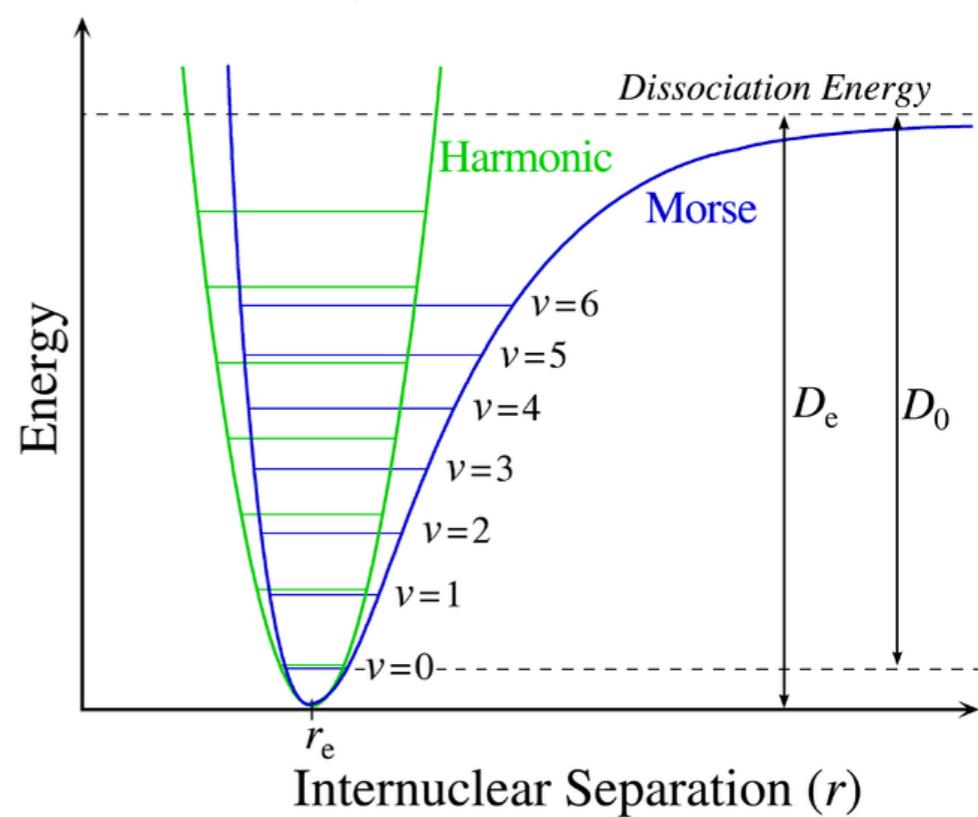
wavefunction derived from

Hermite polynomials

→ zero point energy

$$zpe = \frac{1}{2}\hbar \left(\frac{k}{\mu}\right)^{\frac{1}{2}}$$

$$E_v = V_0 + \hbar \left(\frac{k}{\mu}\right)^{\frac{1}{2}} \left(v + \frac{1}{2}\right)$$



wikipedia - Morse potential

Rotational structure

Stationary Schrödinger equation for nuclei rotation

Outline

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- energy structure

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- conclusion

$$\left(-\frac{\hbar^2}{2\mu R^2} \left[\frac{1}{\sin\theta} \frac{d}{d\theta} (\sin\theta \frac{d}{d\theta}) + \frac{1}{\sin^2\theta} \frac{d}{d\phi} \right] - E_r \right) \psi_r(\theta, \phi) = 0$$

→ spherical harmonics solution $\psi_r(\theta, \phi) = Y_{J,M_J}(\theta, \phi)$

Y_{J,M_J} → already seen for H atom

J → rotational angular momentum

M_J → projection of J onto molecular axis

μ → reduced mass

→ rotational eigenvalues $E_r = \frac{\hbar^2}{2\mu R^2} J(J+1)$

→ if rotation as rigid body $E_r = B_0 J(J+1)$

→ degeneracy of rotational levels $g_r = 2J+1$

Fine structure / hyperfine structure

Outline

- introduction
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Couplings between angular momenta

- electron angular momentum and spin, nuclei rotational motion and spin
- coupling schemes known as Hund's cases - (a) and (b) most common
- R = rotational angular momentum

Hund's case (a)	Hund's case (b)
1 - $\Lambda = L_{ z}, \Sigma = S_{ z}$	1 - $\Lambda = L_{ z}$
2 - $\Lambda + \Sigma \rightarrow \Omega$	2 - $\Lambda + R \rightarrow N$
3 - $\Omega + R \rightarrow J$	3 - $N + S \rightarrow J$

- hyperfine coupling
- coupling with nuclear spin I
- $J + I \rightarrow F$

Detailed electronic / vibrational / rotational structure

In reality, anharmonic & non rigid rotation with rot / vib couplings

Outline

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- corrections terms (used in a polynomial expansion)
- derived from spectroscopic measurements of molecular lines
- anharmonic correction, centrifugal distortion, correction of rot constant

example for CH⁺

$$E(\Lambda, v, J) =$$

$$\begin{aligned}
 & T_e(\Lambda) + \omega_e(\Lambda) \left(v + \frac{1}{2} \right) - \omega_e x_e(\Lambda) \left(v + \frac{1}{2} \right)^2 + \omega_e y_e(\Lambda) \left(v + \frac{1}{2} \right)^3 \\
 & + \left[B_e(\Lambda) - \alpha_e(\Lambda) \left(v + \frac{1}{2} \right) + \gamma_e(\Lambda) \left(v + \frac{1}{2} \right)^2 + \varepsilon_e(\Lambda) \left(v + \frac{1}{2} \right)^3 \right] \\
 & \times \left(J(J+1) - \Lambda^2 \right) - \left[D_e(\Lambda) + \beta_e(\Lambda) \left(v + \frac{1}{2} \right) + \delta_e(\Lambda) \left(v + \frac{1}{2} \right)^2 \right] \\
 & \times \left(J(J+1) - \Lambda^2 \right)^2 + \left[H_e(\Lambda) - \alpha_{H_e}(\Lambda) \left(v + \frac{1}{2} \right) \right] \left(J(J+1) - \Lambda^2 \right)^3 \\
 & \pm \frac{1}{2} \left[q_e(\Lambda) + \alpha_{q_e}(\Lambda) \left(v + \frac{1}{2} \right) \right] (J(J+1)) \\
 & \pm \frac{1}{2} q_{D_e}(\Lambda) (J(J+1))^2,
 \end{aligned} \tag{1}$$

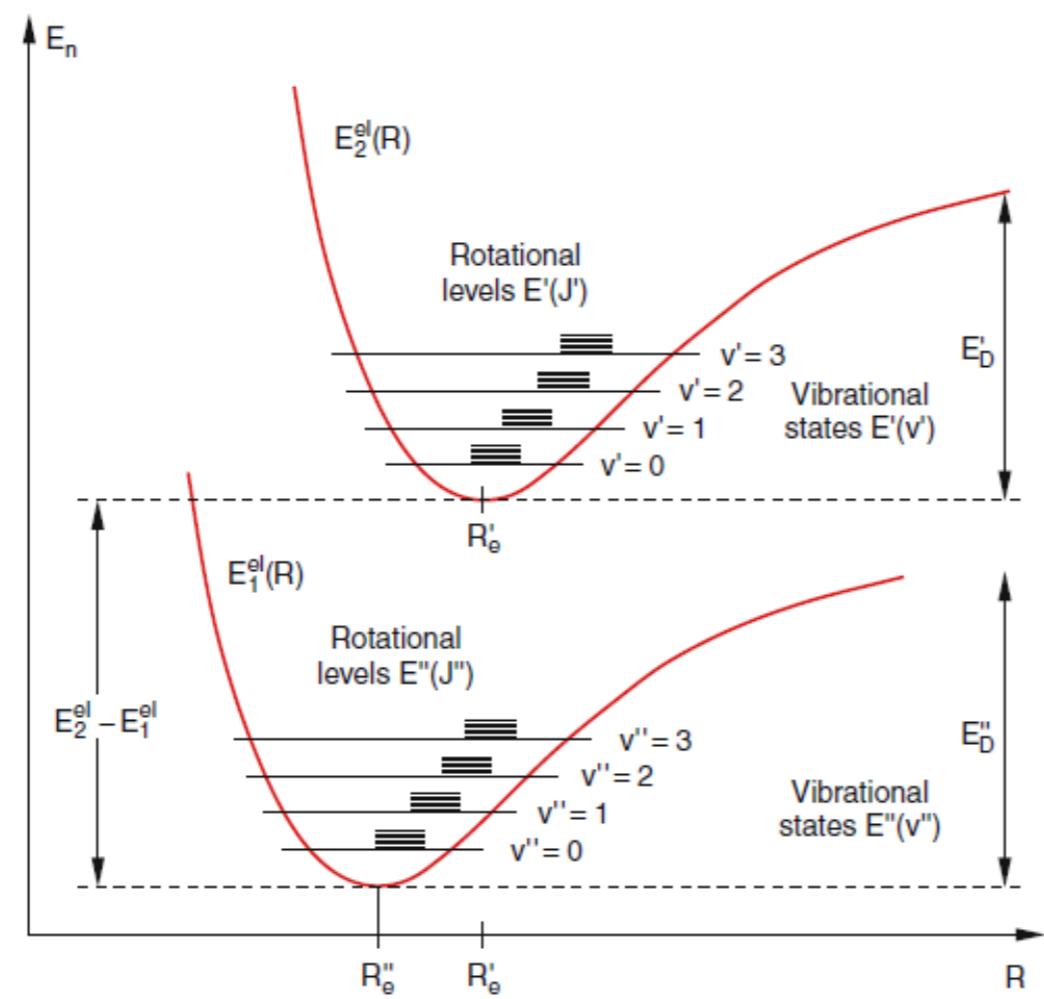
constant	$X^1\Sigma^+$	$A^1\Pi$		
T_e		2.41187262	(+4)	
ω_e	2.8575609	(+3)	1.864402	(+3)
$\omega_e x_e$	5.93179	(+1)	1.158317	(+2)
$\omega_e y_e$	2.2534	(-1)	2.6301	
B_e	1.41774612	(+1)	1.1886774	(+1)
α_e	4.94739	(-1)	9.1629	(-1)
γ_e	2.4904	(-3)	-2.292	(-2)
ε_e			4.952	(-3)
D_e	1.38914	(-3)	1.929606	(-3)
β_e	-2.66	(-5)	1.07331	(-4)
δ_e			-1.3123	(-5)
H_e	1.2036	(-7)		
α_{H_e}	2.079	(-8)		
q_e			4.1018	(-2)
α_{q_e}			-3.135	(-3)
q_{D_e}			-2.2	(-5)

Hakalla et al. (2006)

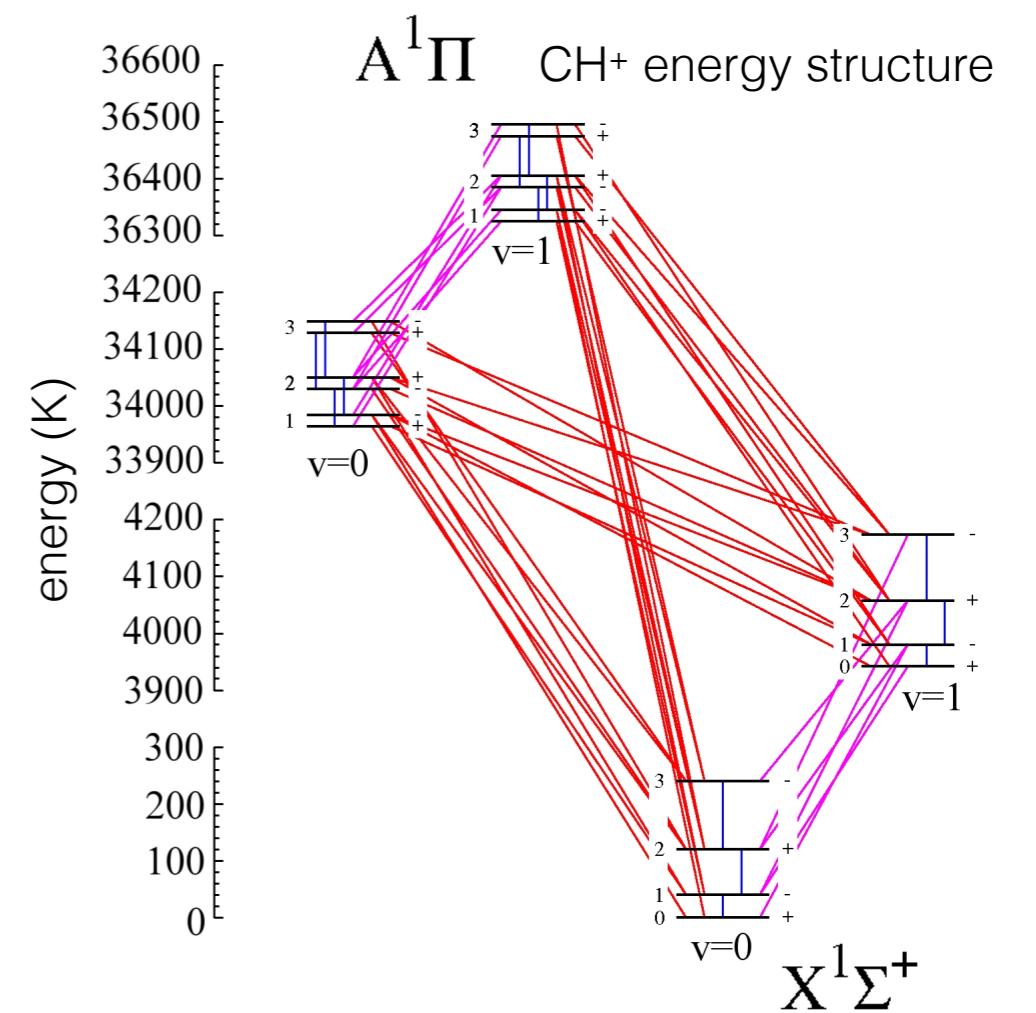
Detailed electronic / vibrational / rotational structure

Outline

- introduction
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Mihály Benedict



Godard et al. (2013)

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Molecular spectra

Electric dipole transitions

Interaction of a molecule with EM field

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- electric dipole, magnetic dipole, electric quadrupole, ...
- electric dipole terms $\sim 10^5 - 10^6$ times larger than the others
- electric dipole transition probability from Ψ'' to Ψ'

$$P \propto |\mathbf{M}|^2 \quad \text{with} \quad \mathbf{M} = \int \psi'^* \boldsymbol{\mu} \psi'' d\mathbf{r}$$

\mathbf{M} electric dipole moment integral

$\boldsymbol{\mu}$ dipole moment operator

- symmetry properties of Ψ , Ψ'' and μ give the selection rules
 $\Delta S, \Delta \Lambda, \Delta v, \Delta J, \dots$

Selection rules on electronic quantum numbers

for allowed electric dipole transitions

Outline

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→ $\Delta S = 0$ spin conserving selection rule

→ $\Delta \Lambda = 0, \pm 1$ projection of orbital momentum

→ $\Sigma^+ \leftrightarrow \Sigma^+$ for Σ states only

$\Sigma^- \leftrightarrow \Sigma^-$

→ $g \leftrightarrow u$ for homonuclear diatomic molecules

⇒ no *dipole allowed* vibration-rotation transition

within an electronic state for homonuclear species

Selection rules on vibrational quantum number

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transitions from two different electronic states

→ Franck-Condon principle

nuclei static during elec transition

transition probability proportional to
wavefunctions overlap

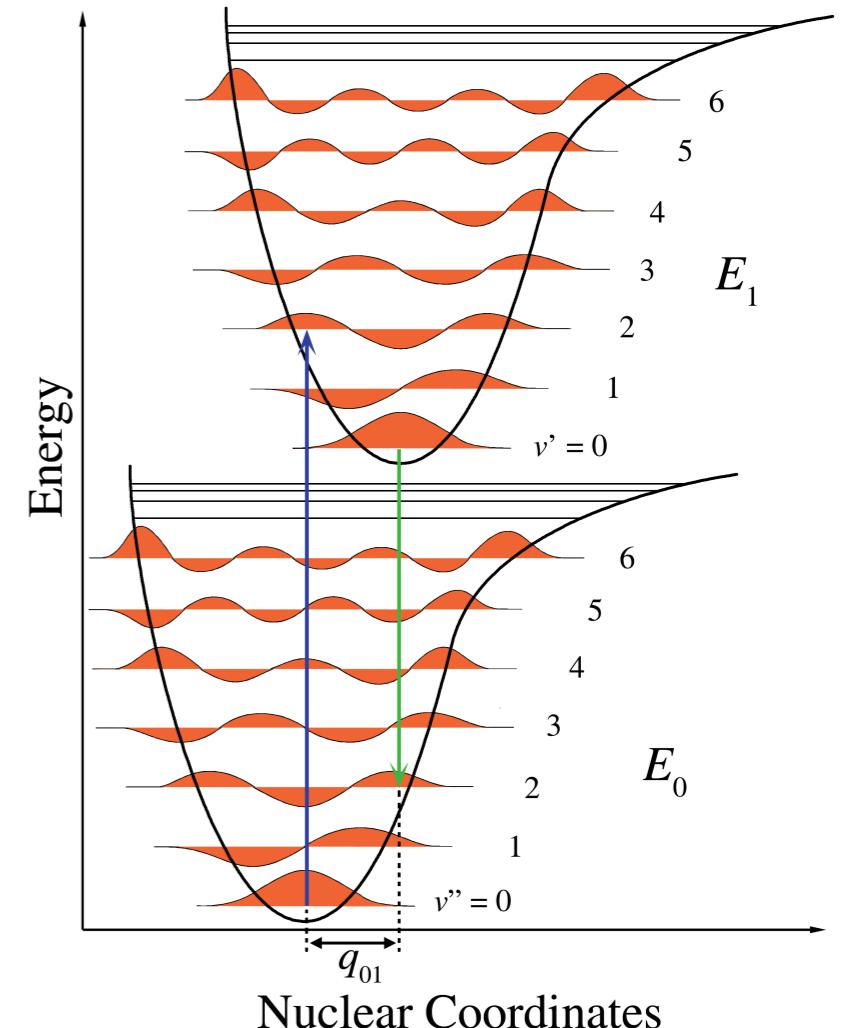
$$\left| \int \psi_v'(v')^* \psi_v''(v'') d\mathbf{r} \right|^2$$

→ $\Delta v = \text{any}$ no selection rule

transition within the same electronic state

→ $\Delta v = \text{any}$ for anharmonic oscillators

stronger transitions for $\Delta v = \pm 1$ (HO)



Selection rules on rotational quantum number

for allowed electric dipole transitions

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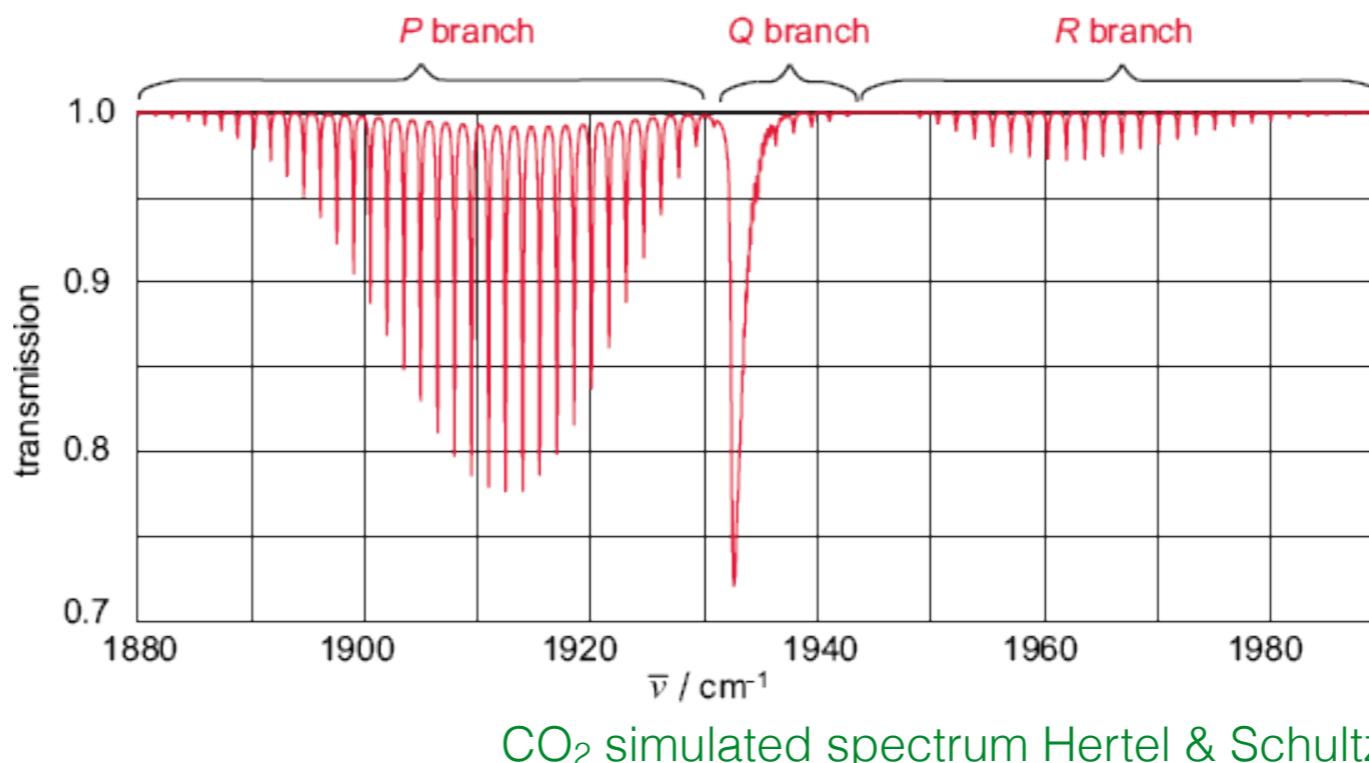
→ $\Delta J = \pm 1$ for $\Lambda = 0 - 0$ transitions

$\Delta J = 0, \pm 1$ ($0 \leftrightarrow 0$) for other $\Delta\Lambda$

→ labelling: $J'(J'')$ associated with the upper (lower) energy level

name	$R(J'')$	$Q(J'')$	$P(J'')$
$\Delta J = J' - J''$	+1	0	-1

→ for transition within the same electronic state → regular structure



Oscillator strength and Einstein coefficients

for allowed electric dipole transitions

Outline

- introduction

→ the Einstein coefficient

$$A_{\Lambda'v'J'\Lambda''v''J''} = \frac{8\pi^2\nu^2e^2}{m_e c^3} \frac{2J''+1}{2J'+1} f_{\Lambda'v'J'\Lambda''v''J''}$$

- energy structure

→ with the oscillator strength

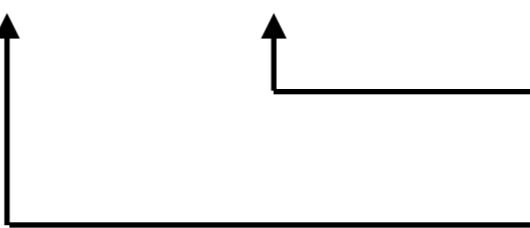
$$f_{\Lambda'v'J'\Lambda''v''J''} = \frac{8\pi^2m_e\nu}{3he^2} M_{\Lambda'v'\Lambda''v''}^2 \frac{S_{\Lambda''J''}^{\Lambda'J'}}{2J''+1}$$

$S_{\Lambda''J''}^{\Lambda'J'}$ Hönl-London factor, dependence on nuclear angular momentum

- photo-processes

→ under the Born-Oppenheimer and the r-centroid approximations

$$M_{\Lambda'v'\Lambda''v''}^2 = q_{\Lambda'v'\Lambda''v''} R_e^2(\bar{r}_{v'v''})$$



electronic transition moment

Franck-Condon factor

Pure vibration-rotation of homonuclear diatomic molecules

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electric dipole transitions forbidden within an electronic state of homonuclear diatomic molecules (e.g. H₂, C₂, N₂, ...)

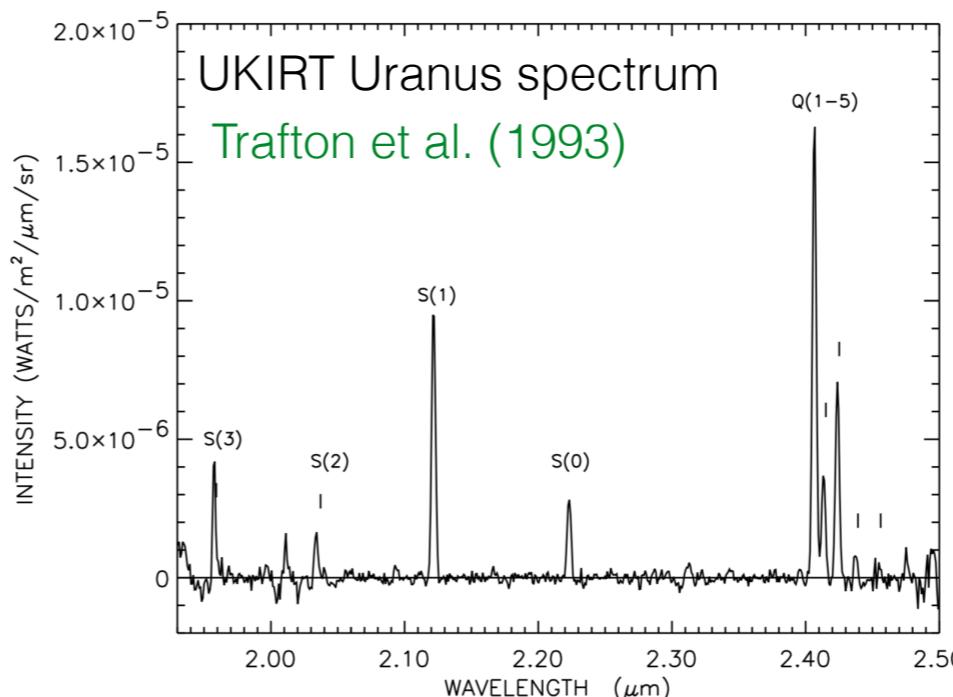
→ dipole magnetic and quadrupole electric transitions

→ example : H₂ - quadrupole electric transitions

selection rule for pure vibration-rotation $\Delta J = 0, \pm 2$ ($0 \leftrightarrow 0$)

labelling: J' (J'') associated with the upper (lower) energy level

name	$S(J'')$	$Q(J'')$	$O(J'')$
$\Delta J = J' - J''$	+2	0	-2



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Ionization / dissociation / fluorescence

Principles of photo-processes

Outline

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photoionization and photodissociation processes are fundamental

- to compute radiative transfer in molecular environments
- to compute chemical composition of diffuse & dense ISM, PDRs, protoplanetary disks, cometary & exoplanetary atmosphere, ...

microphysical process

- 1 - excitation of any (bound or not) electronic state



$$k = \int_{\lambda} \sigma(\lambda) N_{\lambda} d\lambda$$

- 2 - relaxation towards a lower state branching ratios



Continuous or discret absorption

Outline

- introduction

- energy structure

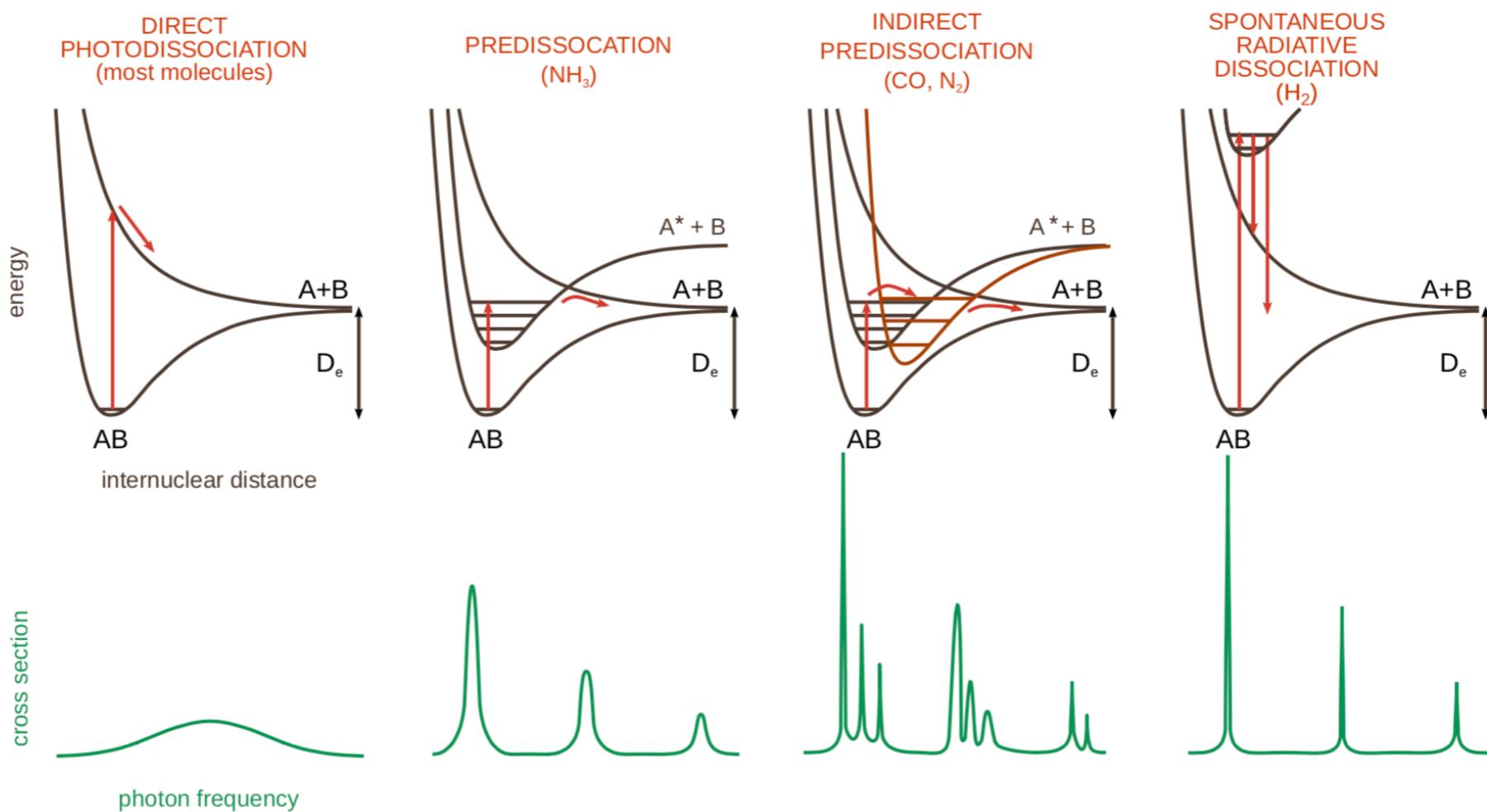
- molecular lines

- photo-processes

- conclusion

Several pathways to photodissociation

Heays et al. (2017)



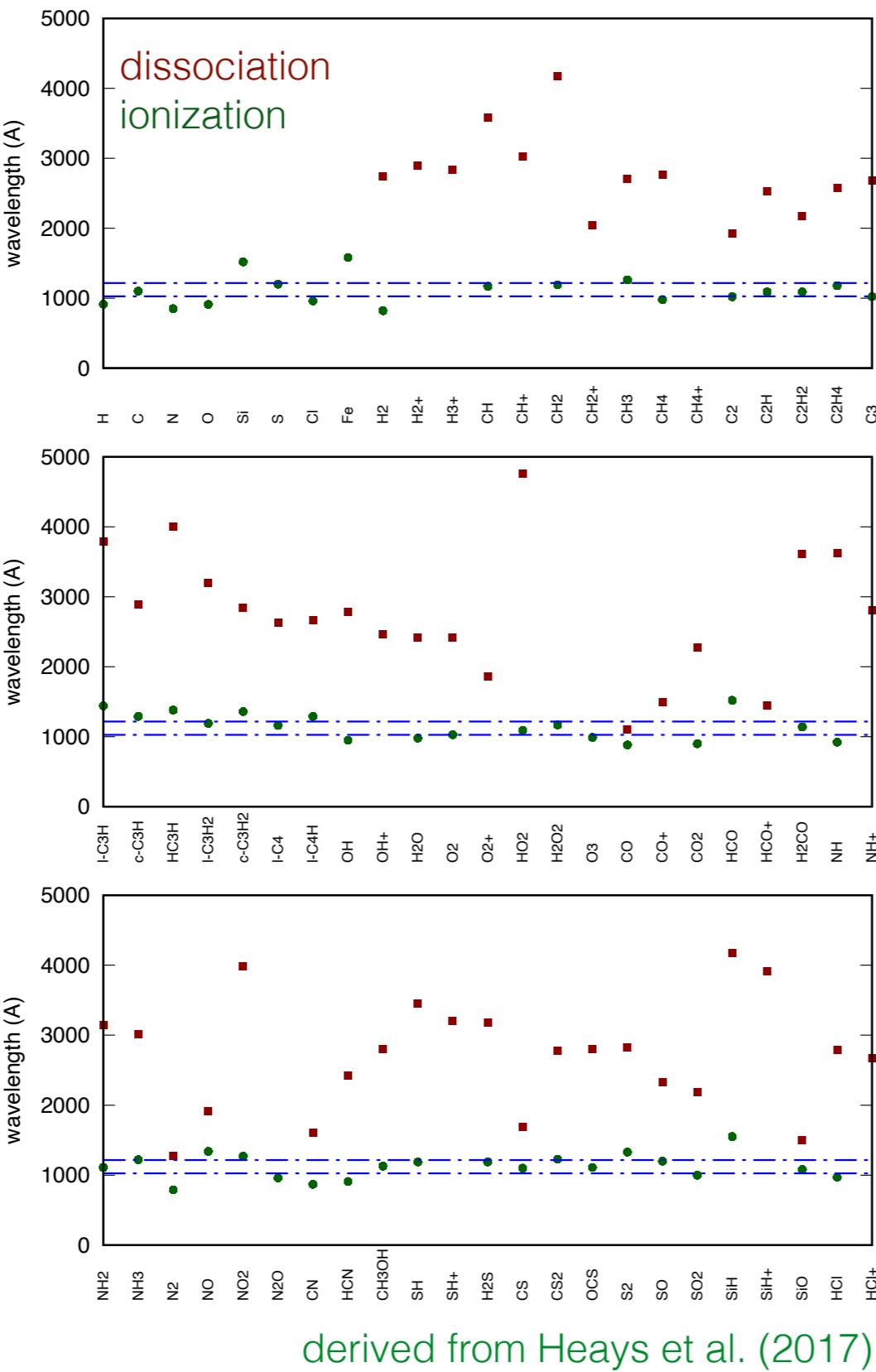
- photodissociation may be a line or a continuum mechanism
- cross sections must account for all possible process
- line processes favor self-shielding

Threshold and cross sections

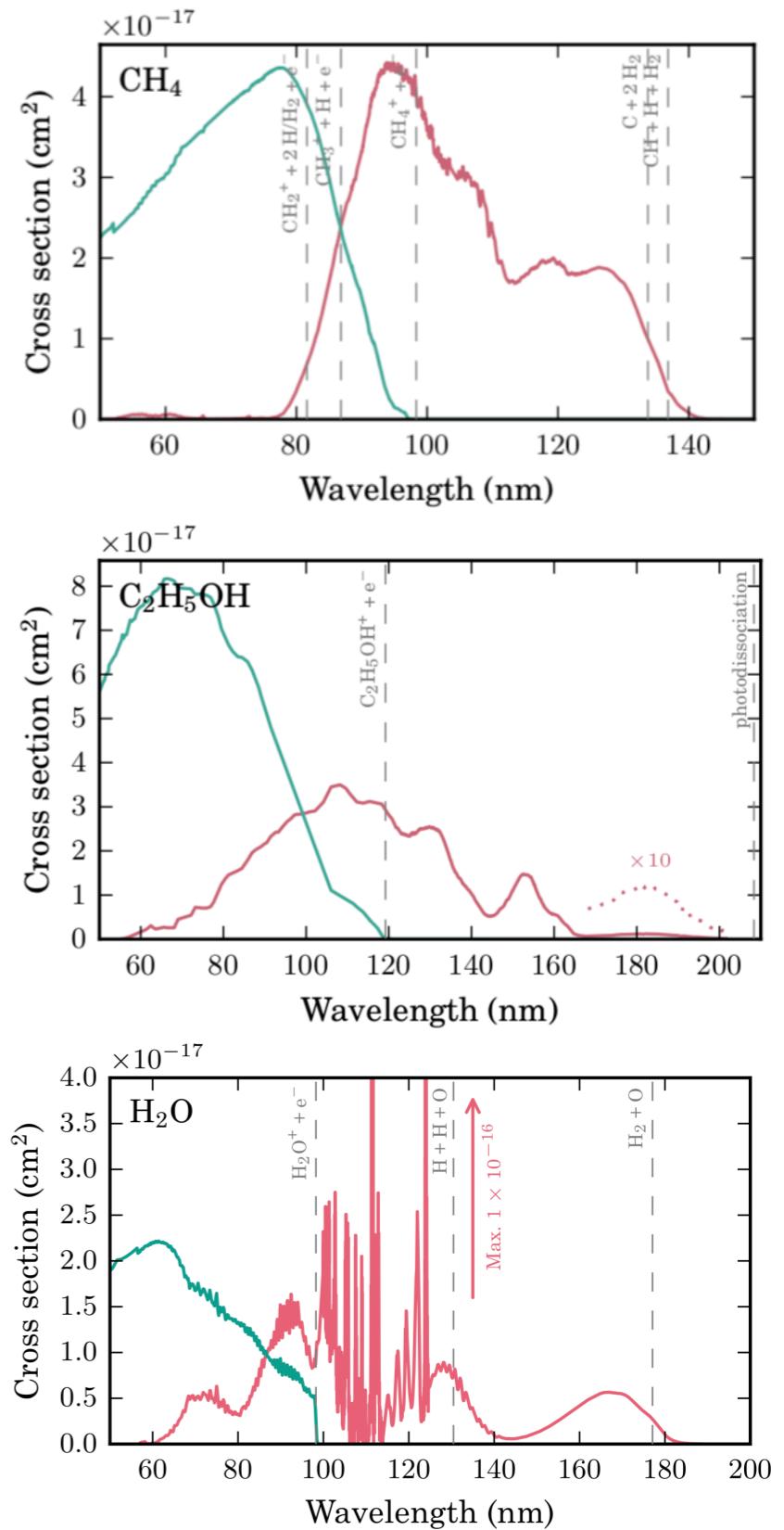
Outline

- introduction
- energy structure
- molecular lines
- photo-processes
- conclusion

photo-processes threshold



examples of cross sections

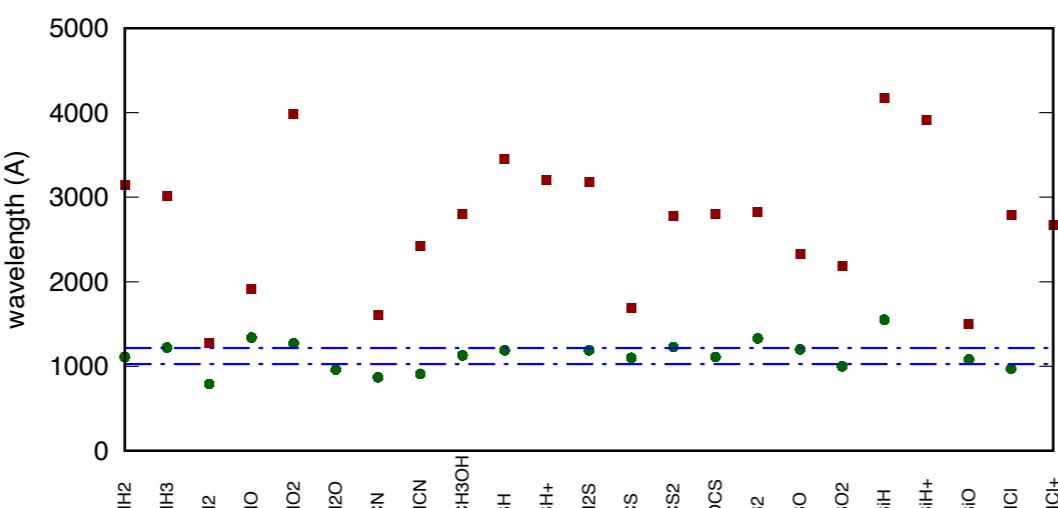
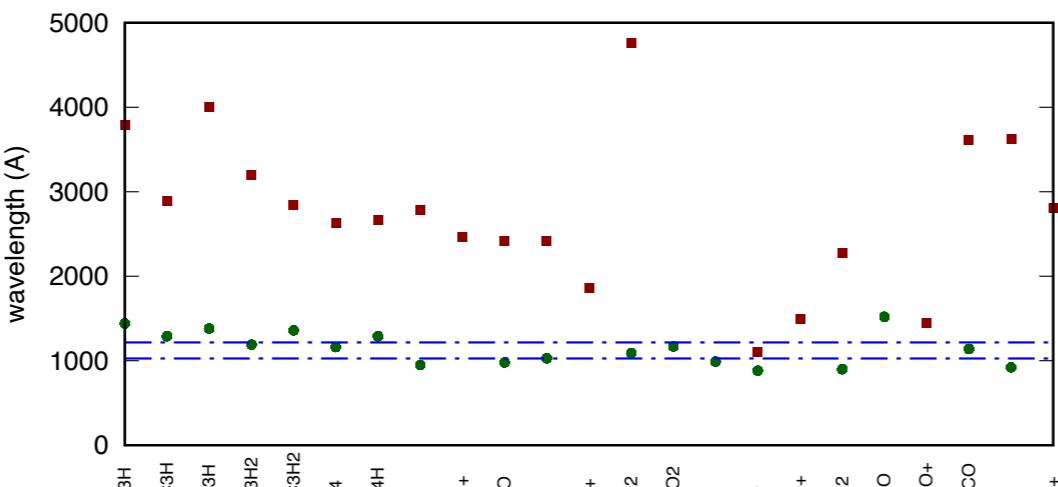
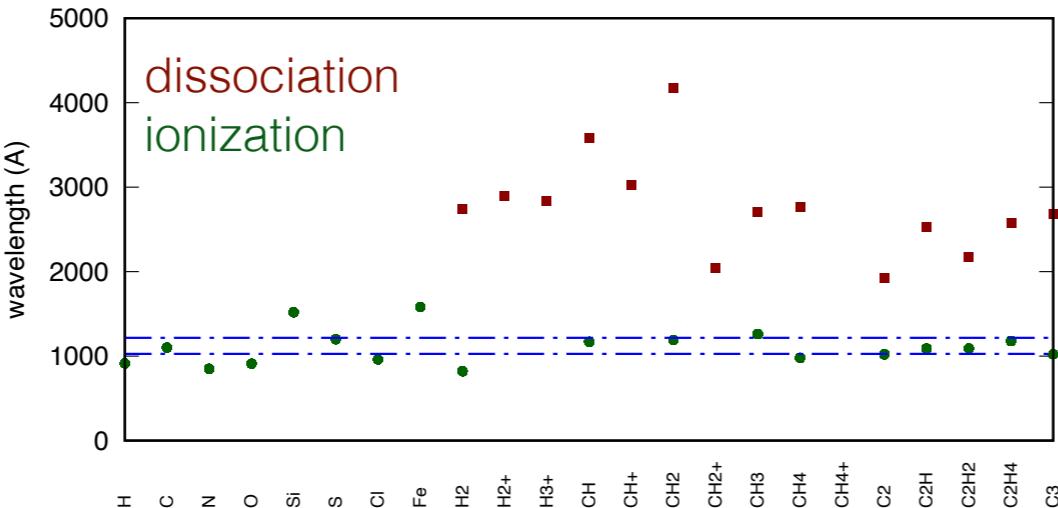


Threshold and cross sections

Outline

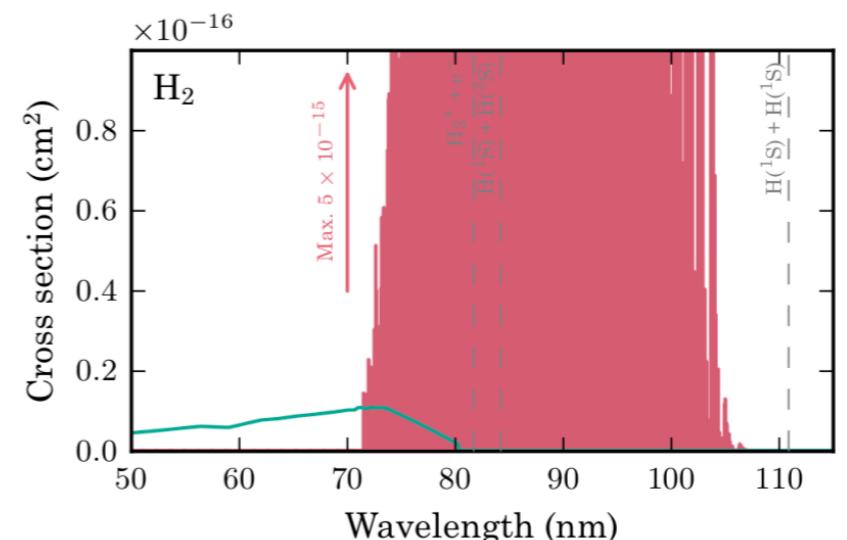
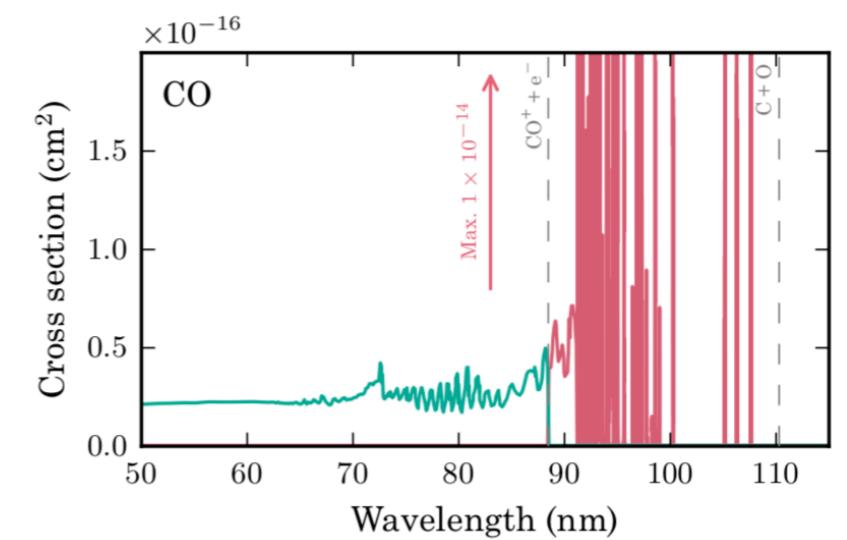
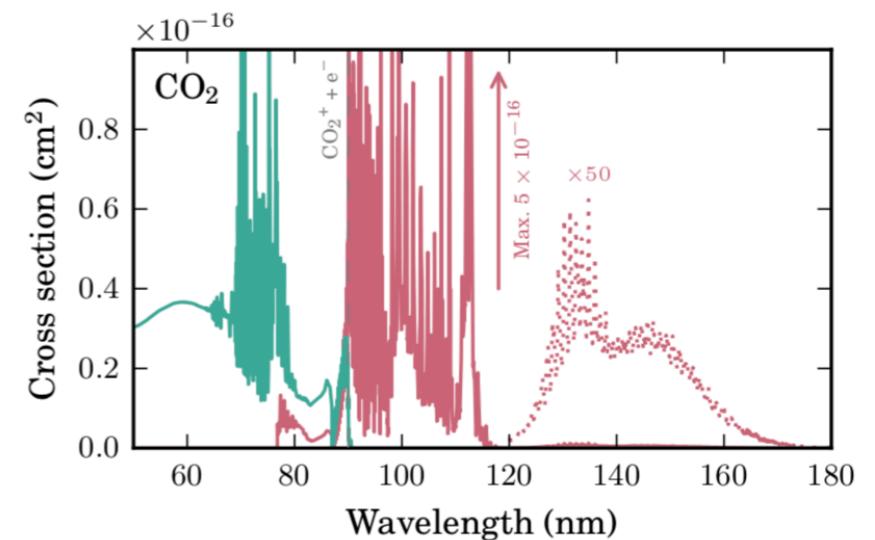
- introduction
- energy structure
- molecular lines
- photo-processes
- conclusion

photo processes threshold



derived from Heays et al. (2017)

examples of cross sections

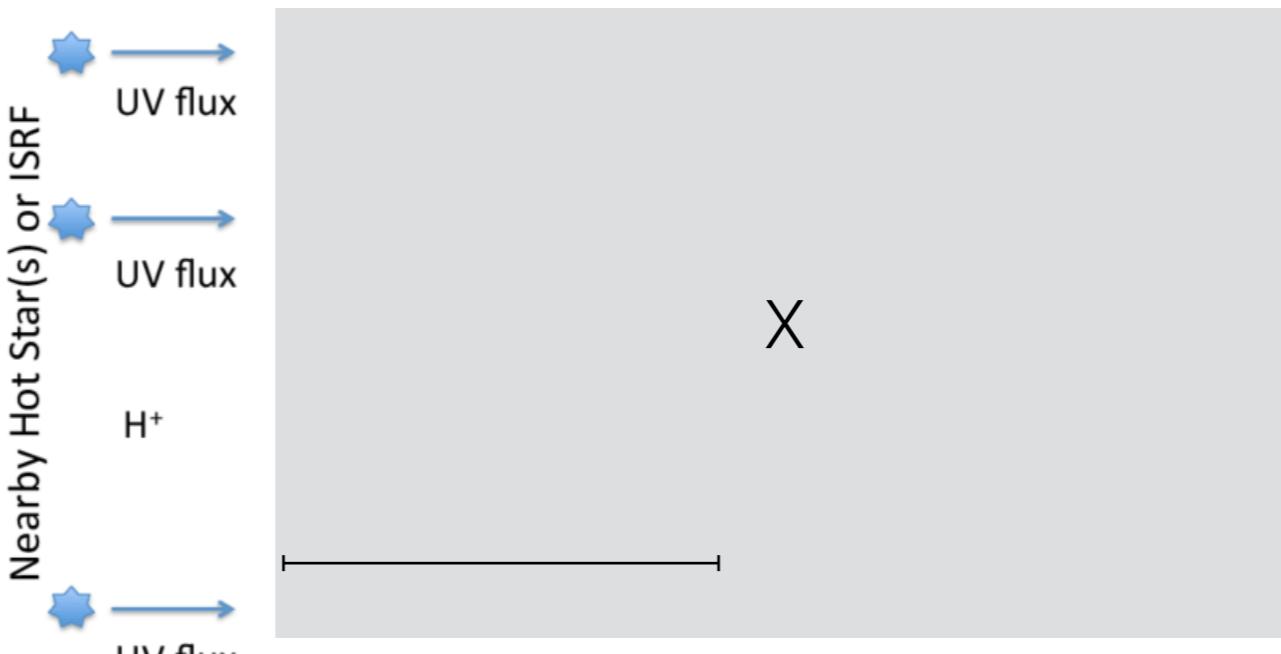


Self-shielding process

Outline

- introduction
 - energy structure
 - molecular lines
 - photo-processes
 - conclusion

Consider a cloud illuminated by energetic photons



the abundance of X at any point depends on photo-processes

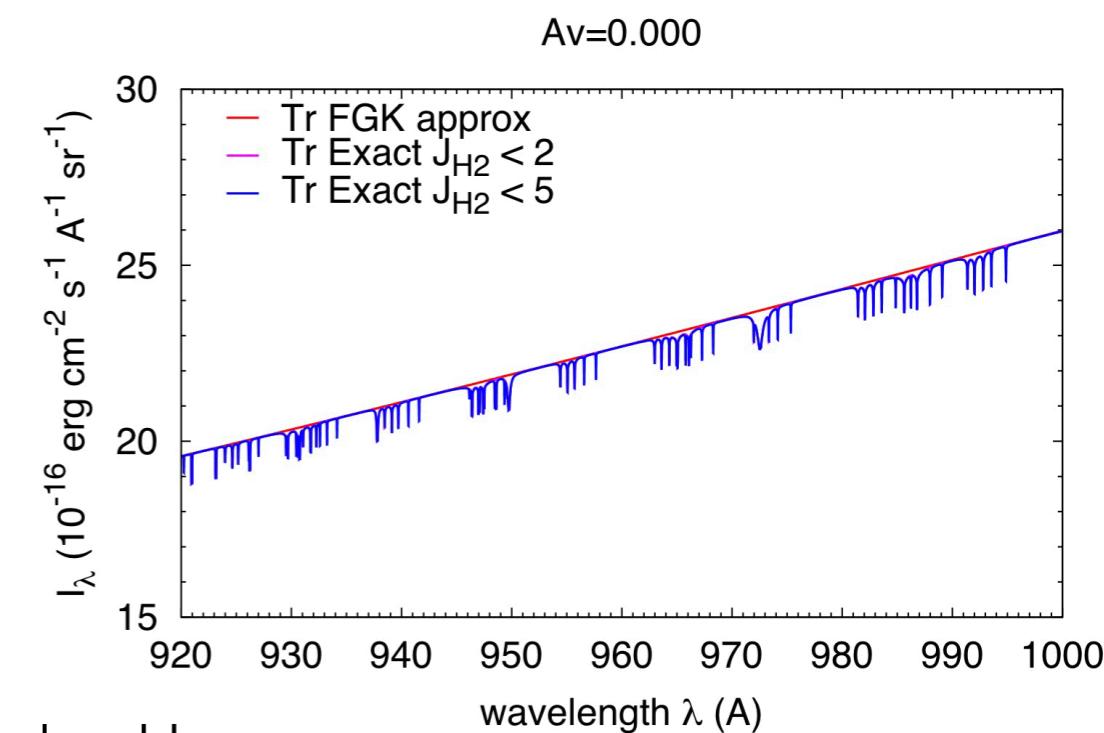
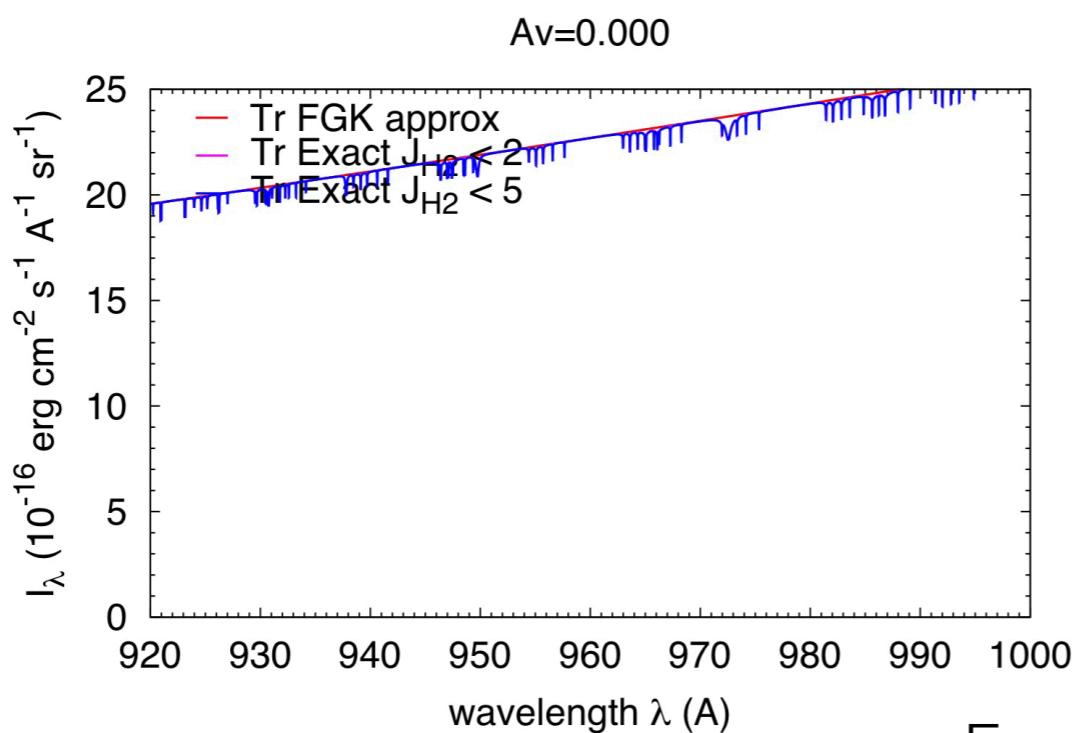
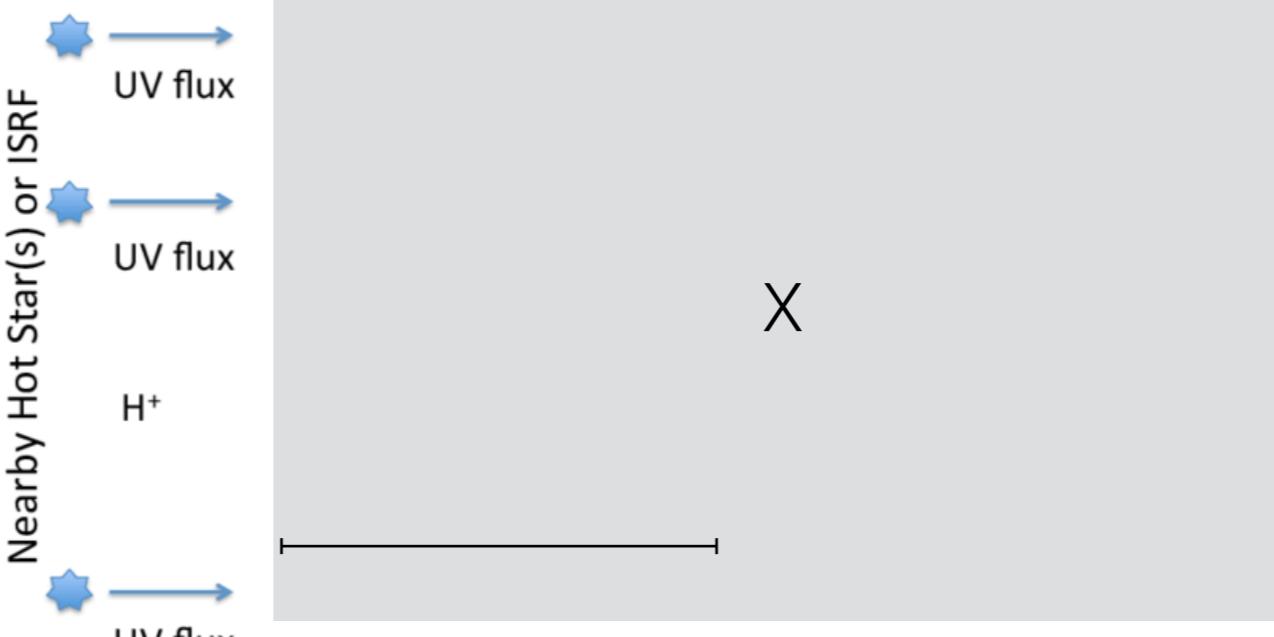
- intensity of the radiation field at specific wavelength
 - absorption processes from the border of the cloud by dust grains - - - - - - - - - → dust shielding
 - by atoms and molecules
 - Y - - - → cross shielding
 - X - - - → self-shielding

Self-shielding process

Consider a cloud illuminated by energetic photons

Outline

- introduction
- energy structure
- molecular lines
- photo-processes
- conclusion

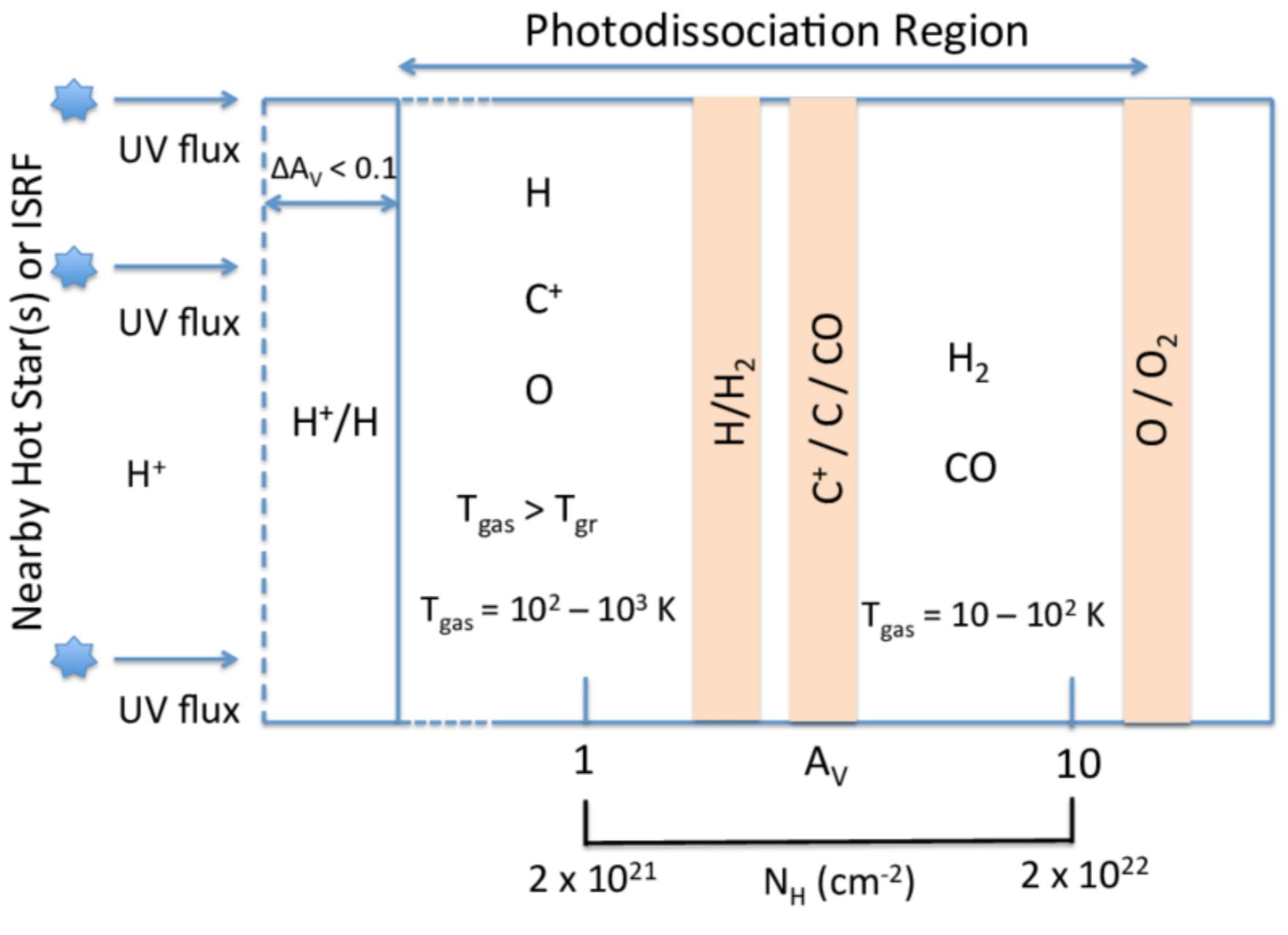


Example : H₂

Self-shielding process

Outline

- introduction
- energy structure
- molecular lines
- photo-processes
- conclusion



Molecular structure and spectroscopy

Outline

- introduction

- simplifications gives strong insight on molecular structure

Born-Oppenheimer → elec orbitals & quantum numbers

separation vibration / rotation → vib oscillator & rotational orbitals

additional couplings → fine & hyperfine structure

- electric dipole transitions obey to stringent transition rules

molecular spectra highly structured (band, branches, fine / hyperfine)

- homonuclear diatomic molecules radiate through quadrupole transitions

Interactions with radiation field

- conclusion

- ionization / dissociation / fluorescence originate from mol transitions

- non linear coupling between transfer and composition (e.g. self-shielding)