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Radiative transfer: scattering processes

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► **To cite this version:**

| Benjamin Godard. Radiative transfer: scattering processes. Master. France. 2023. hal-04645954

HAL Id: hal-04645954

<https://hal.science/hal-04645954v1>

Submitted on 12 Jul 2024

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Scattering processes

International Research Track

2023

Outline

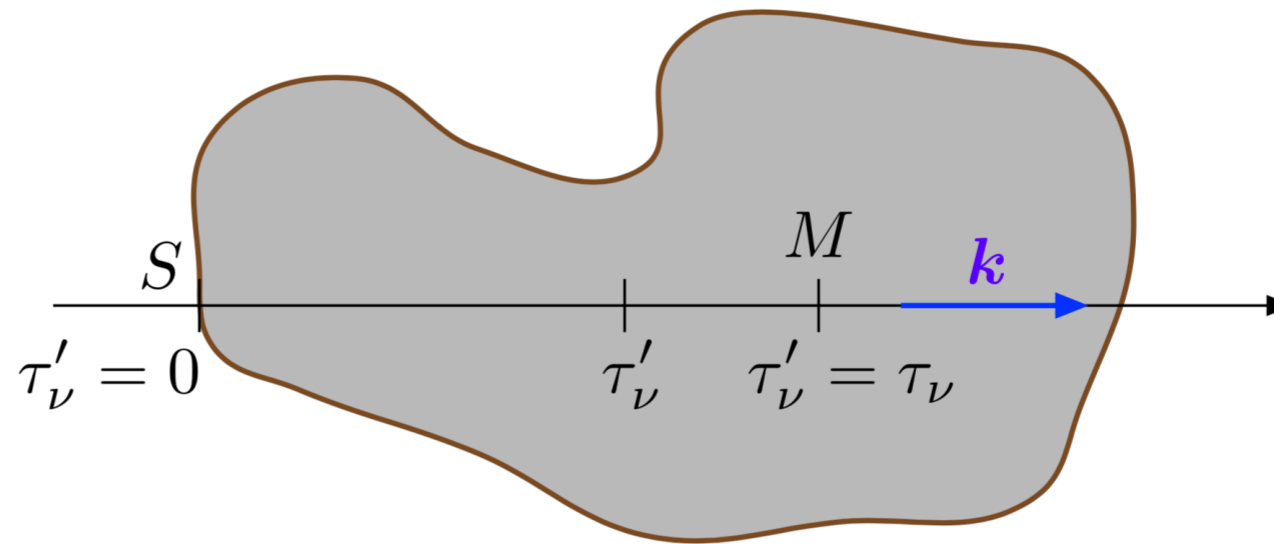
- introduction
- Thomson scattering
- Rayleigh scattering
- Mie scattering
- Compton scattering
- Monte-Carlo num method
- other num method
- conclusions

1. Introduction
2. Thomson scattering
3. Rayleigh scattering
4. Mie scattering
5. Compton scattering
6. Monte-Carlo method
7. Other methods
8. Conclusions

Radiative transfer equation

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F. Levrier - radiative transfer lecture

$$\frac{dI_\nu}{dl} = -[\alpha_\nu + \sigma_\nu] I_\nu + j_\nu + \frac{1}{4\pi} \int_{\Omega'} \tilde{\sigma}_\nu(\mathbf{k}' \rightarrow \mathbf{k}) I_\nu(\mathbf{k}') d\Omega'$$

- | | | |
|--------------------------|--|--|
| • absorption coefficient | $\alpha_\nu = \sum_a n_a s_{\nu,a}$ | cm^{-1} |
| • diffusion coefficient | $\sigma_\nu = \sum_d n_d s_{\nu,d}$ | cm^{-1} |
| • emissivity | j_ν | $\text{erg s}^{-1} \text{cm}^{-3} \text{Hz}^{-1} \text{sr}^{-1}$ |
| • opacity | $d\tau_\nu = [\alpha_\nu + \sigma_\nu] dl$ | |
| • source function | S_ν | $\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$ |

Questions related to the scattering

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what systems are involved in the scattering ?

- electrons, ions
- bound systems (atoms or molecules)
- macroscopic systems (dust, droplets, ...)

what are the properties of the microphysical processes ?

- elastic or inelastic scattering
- interaction cross section, angular dependence
- quantum and relativistic effects

how to solve problems with scattering

- analytical considerations
- numerical methods

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Scattering processes

Field induced by a single charge in motion

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consider a charge q in motion at a velocity $\mathbf{u}(t)$

→ Lienard-Wiechart retarded potentials $t' = t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}$

$$\phi(\mathbf{r}, t) = \frac{q}{R(t') \left[1 - \frac{\mathbf{n}(t') \cdot \mathbf{u}(t')}{c} \right]}$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{q\mathbf{u}}{cR(t') \left[1 - \frac{\mathbf{n}(t') \cdot \mathbf{u}(t')}{c} \right]}$$

→ for non-relativistic particles

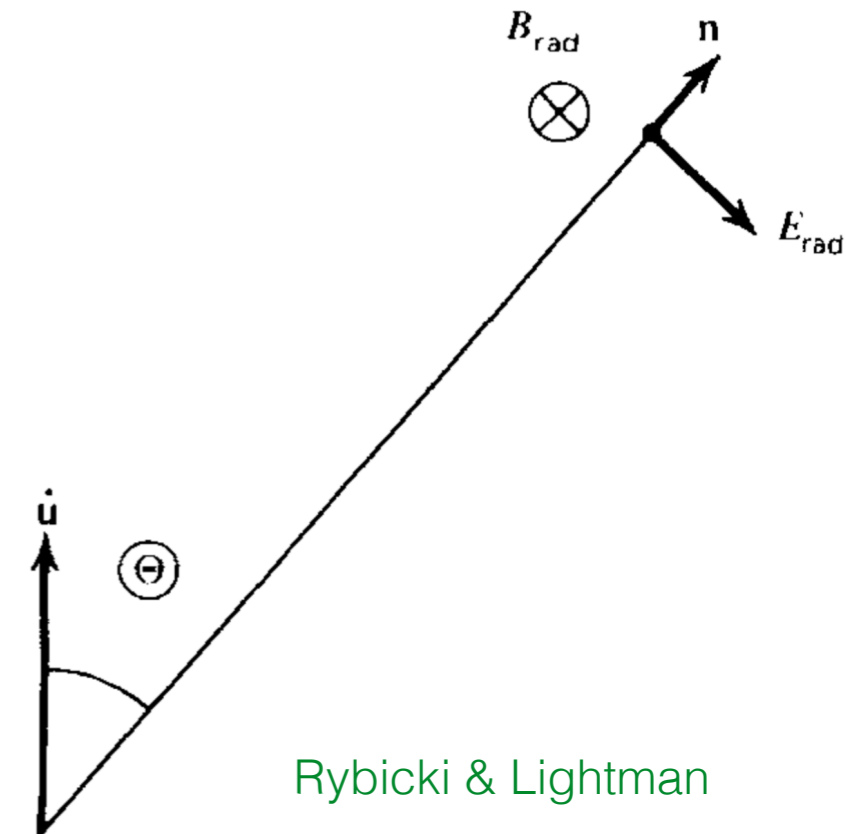
$$\mathbf{E}_{\text{rad}} = \frac{q}{c^2 R(t')} \mathbf{n}(t') \times [\mathbf{n}(t') \times \dot{\mathbf{u}}]$$

$$\mathbf{B}_{\text{rad}} = \mathbf{n}(t') \times \mathbf{E}_{\text{rad}}$$

→ Larmor formula (total power emitted)

$$\frac{dP_{\text{rad}}}{d\Omega} = \frac{q^2 \dot{\mathbf{u}}^2}{4\pi c^3} \sin^2(\Theta)$$

$$P_{\text{rad}} = \frac{2q^2 \dot{\mathbf{u}}^2}{3c^3}$$



Rybicki & Lightman

Dipole approximation

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consider a system of charged particles

→ dipole moment

$$\mathbf{d}(t) = \sum_i q_i \mathbf{r}_i(t)$$

→ if all points emit in phase (i.e. $\lambda \gg L$)

$$\frac{dP_{\text{rad}}}{d\Omega} = \frac{\ddot{\mathbf{d}}^2}{4\pi c^3} \sin^2(\Theta)$$

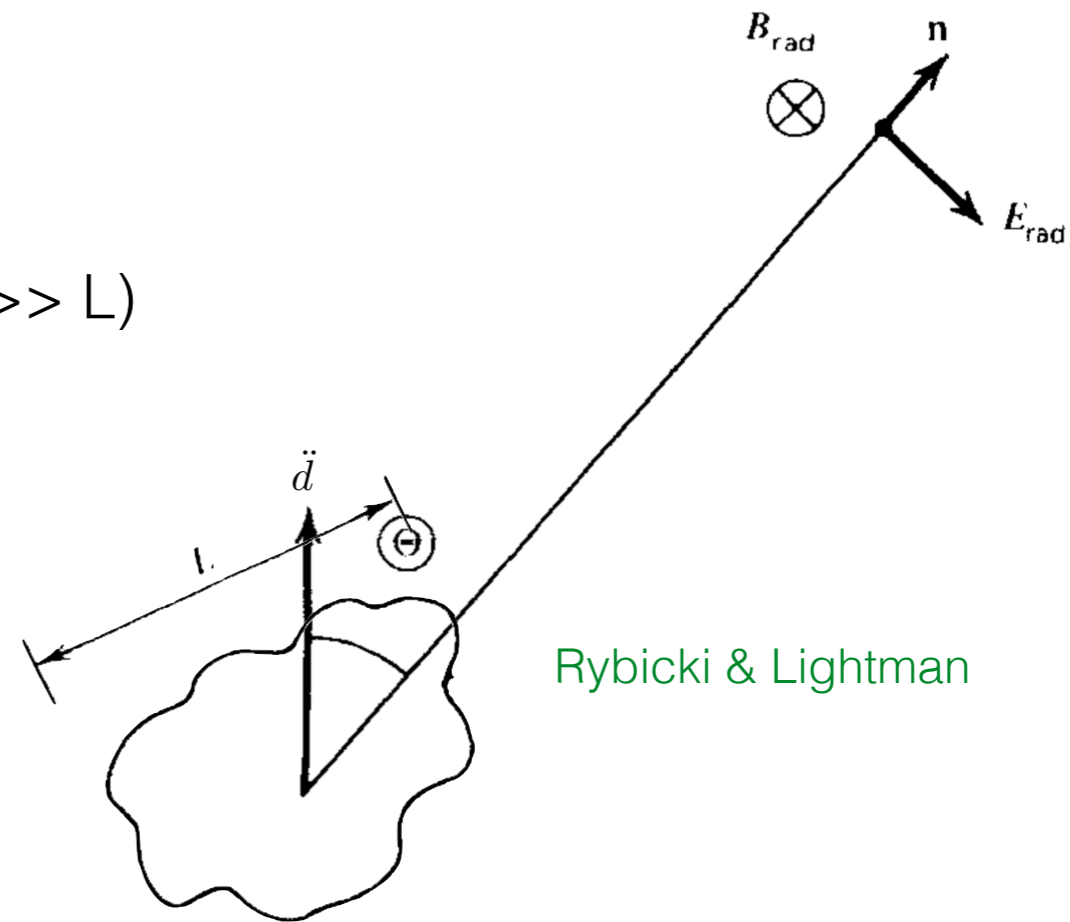
$$P_{\text{rad}} = \frac{2\ddot{\mathbf{d}}^2}{3c^3}$$

→ emitted wavelength

$$\mathbf{d}(t) = \int_{-\infty}^{\infty} e^{-i\omega t} \hat{\mathbf{d}}(\omega) d\omega$$

$$\frac{dP_{\text{rad}}}{d\omega d\Omega} = \frac{\omega^4}{c^3} |\hat{\mathbf{d}}(\omega)|^2 \sin^2(\Theta)$$

→ spectrum depends on oscillation frequencies of the dipole



Scattering by free electron - classical treatment

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free electron at rest in interaction with radiation of pulsation ω

→ incident energy flux $\langle S \rangle = \frac{c}{8\pi} E_0^2$

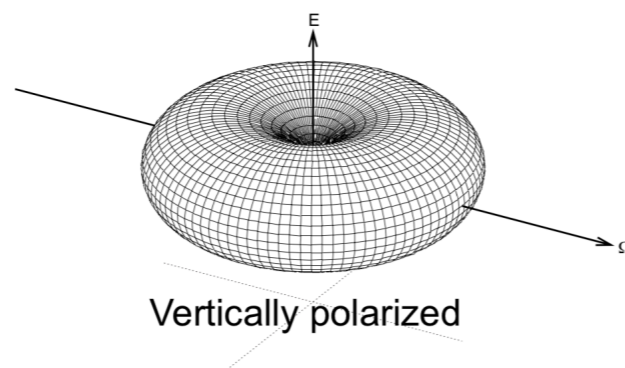
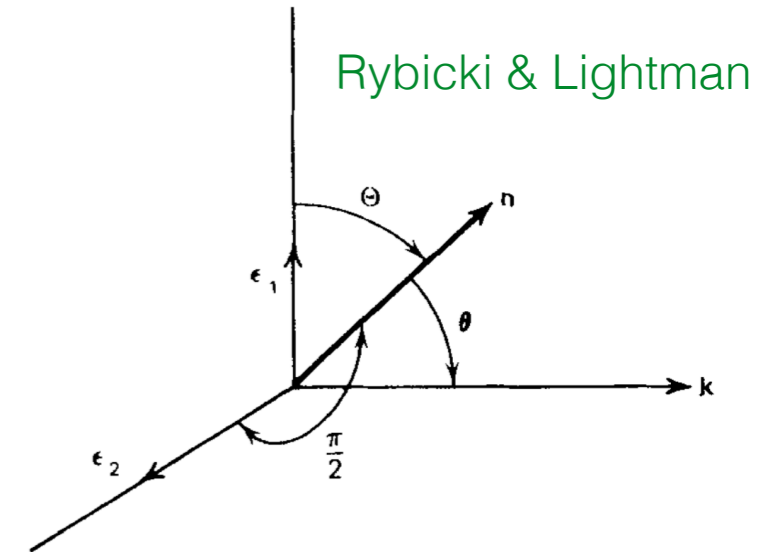
→ classical treatment

dipole of amplitude $d_0 = \frac{e^2 E_0}{m_e \omega^2}$

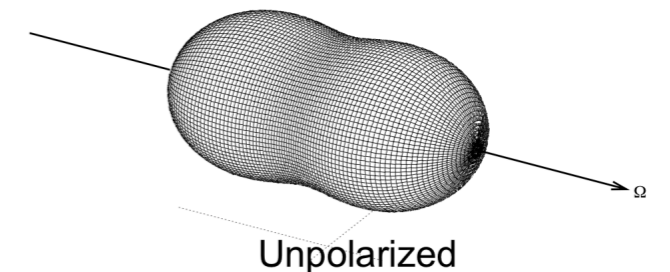
energy conservation $\frac{dP_{\text{rad}}}{d\Omega} = \langle S \rangle \frac{d\sigma}{d\Omega}$

→ total cross section $\sigma_T = \frac{8\pi}{3} r_0^2 \sim 6.65 \times 10^{-25} \text{ cm}^2$

$r_0 = \frac{e^2}{mc^2} \sim 2.8 \times 10^{-13} \text{ cm}$



$$\frac{d\sigma}{d\Omega} = r_0^2 \sin^2(\Theta)$$



$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_0^2 [1 + \cos^2(\theta)]$$

Scattering by bound electrons

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atom / molecule of polarizability η in interaction with radiation of pulsation ω

→ incident energy flux $\langle S \rangle = \frac{c}{8\pi} E_0^2$

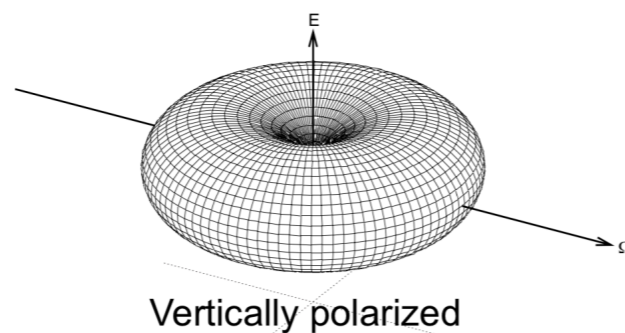
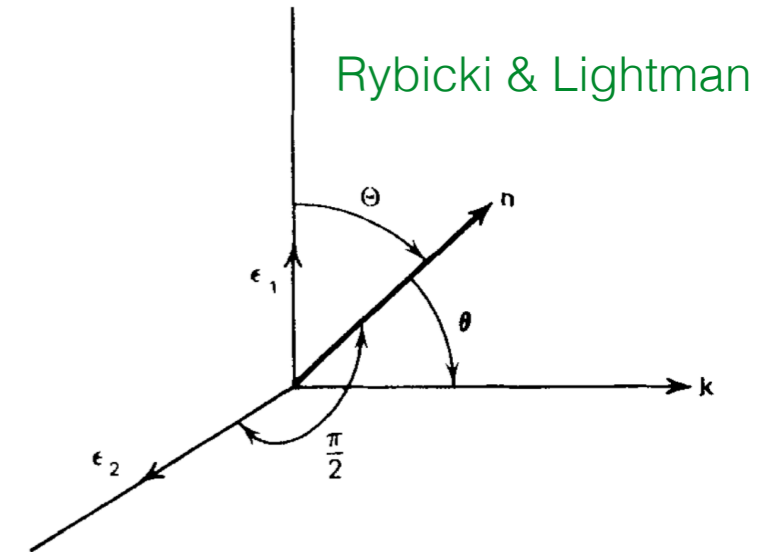
→ classical treatment

dipole of amplitude $d_0 = \eta E_0$

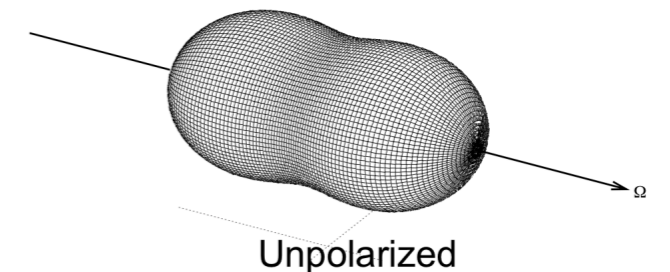
energy conservation $\frac{dP_{\text{rad}}}{d\Omega} = \langle S \rangle \frac{d\sigma}{d\Omega}$

→ total cross section $\sigma = \frac{8\pi}{3} r_0^2$

$$r_0 = \frac{\omega^2 \eta}{c^2}$$



$$\frac{d\sigma}{d\Omega} = r_0^2 \sin^2(\Theta)$$



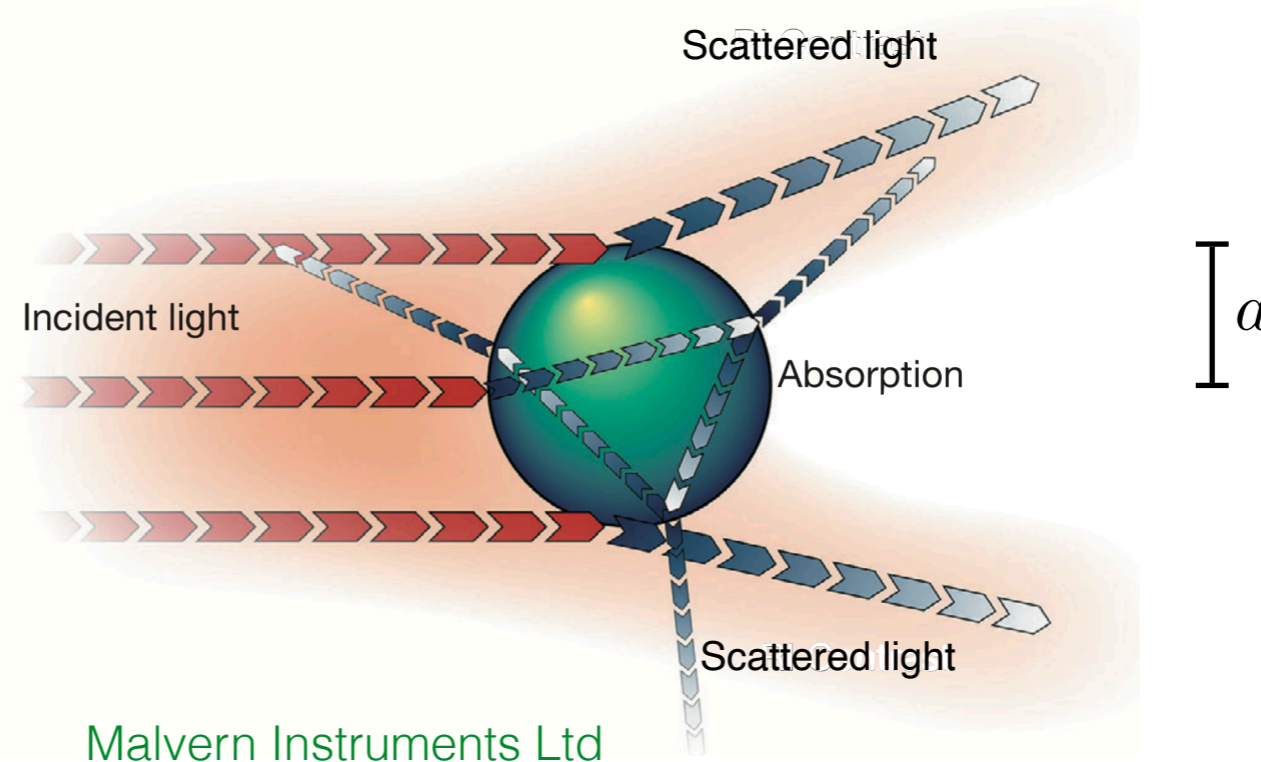
$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_0^2 [1 + \cos^2(\theta)]$$

From microscopic to macroscopic systems

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diffusion by macroscopic systems (dust, water droplets, objects)



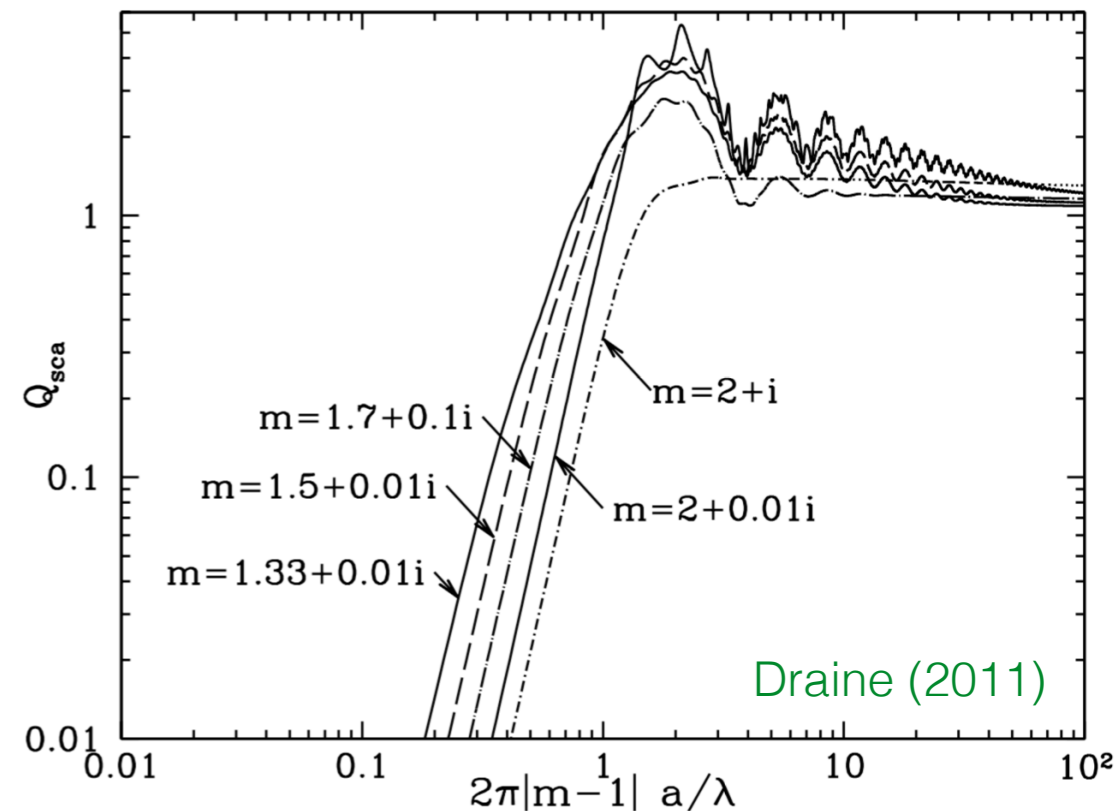
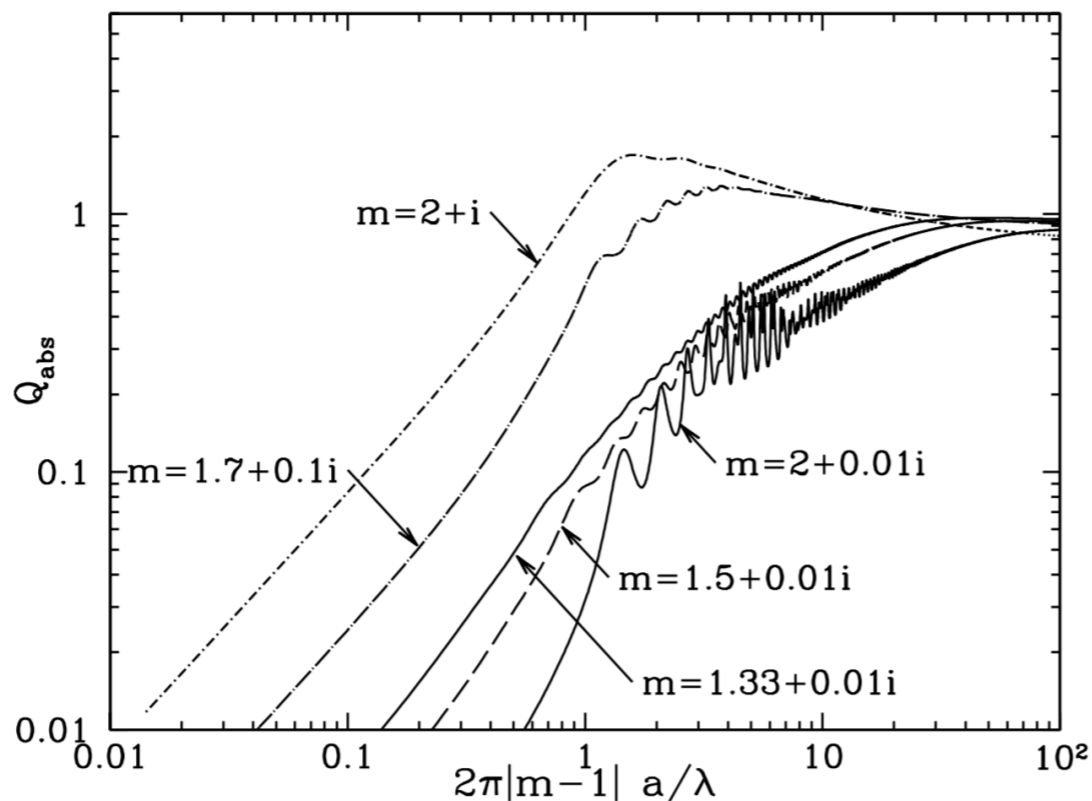
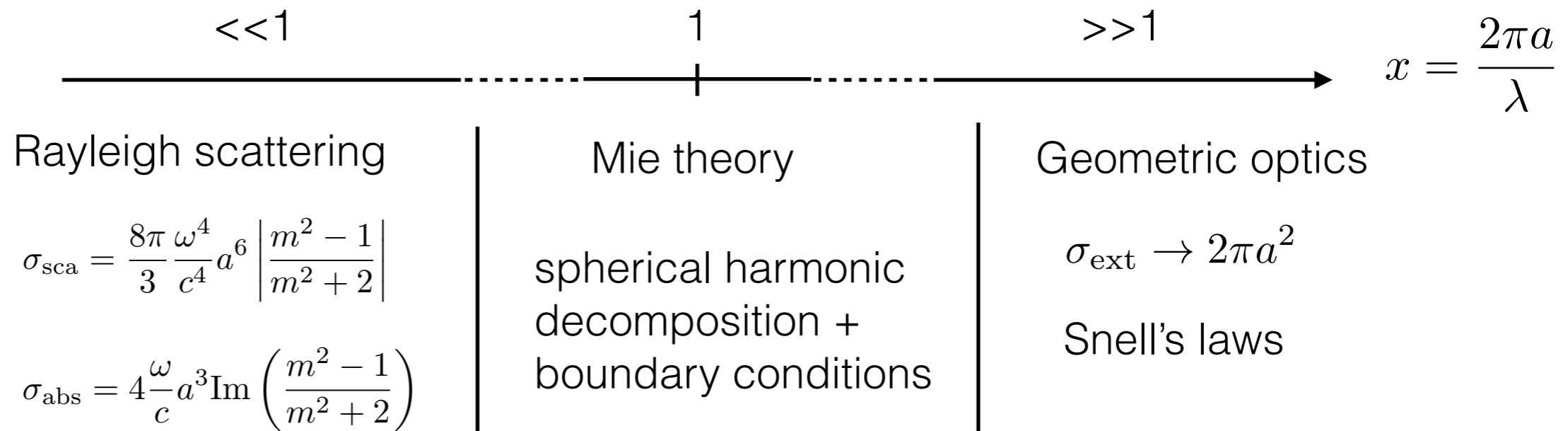
- size parameter $\longrightarrow x = \frac{2\pi a}{\lambda}$
- abs and scatt efficiencies $\longrightarrow Q_{\text{abs}} = \frac{\sigma_{\text{abs}}}{\pi a^2}, Q_{\text{sca}} = \frac{\sigma_{\text{sca}}}{\pi a^2}$
- index of refraction $\longrightarrow m(\lambda) = n(\lambda) + ik(\lambda)$
 $\downarrow \qquad \qquad \downarrow$
scattering absorption

From microscopic to macroscopic systems

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spherical particles → Mie theory (1908)

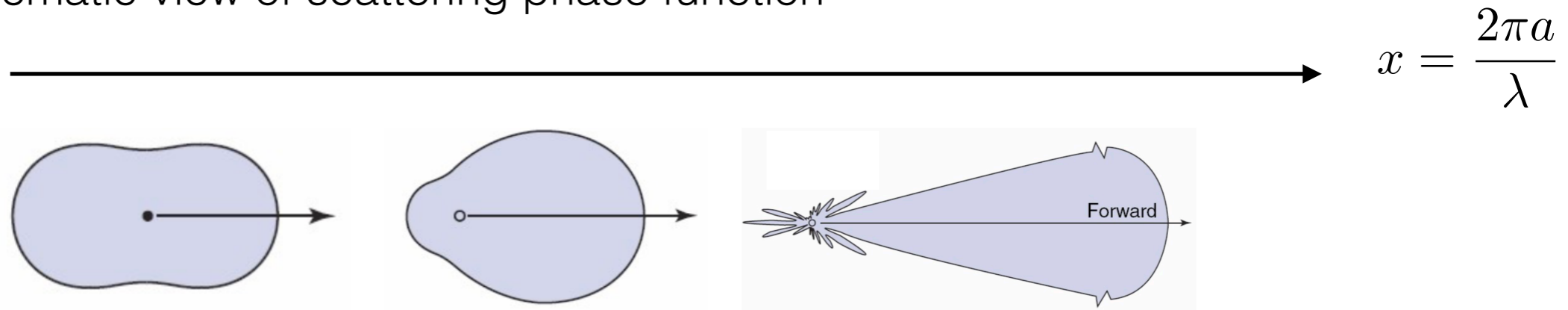


From microscopic to macroscopic systems

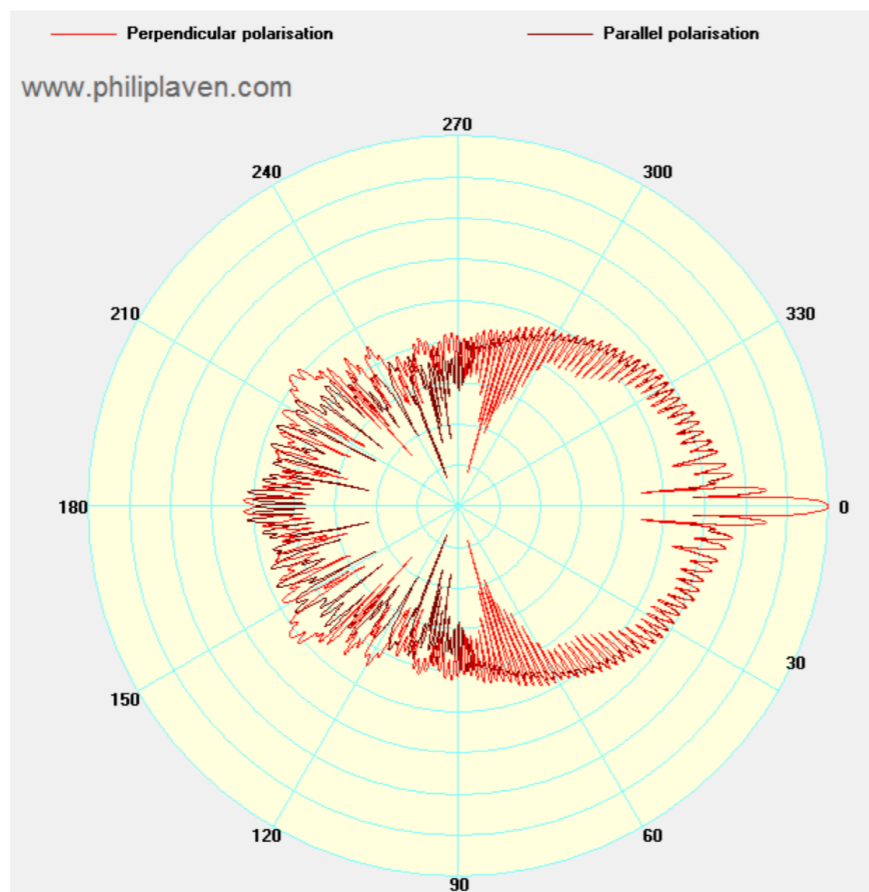
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schematic view of scattering phase function

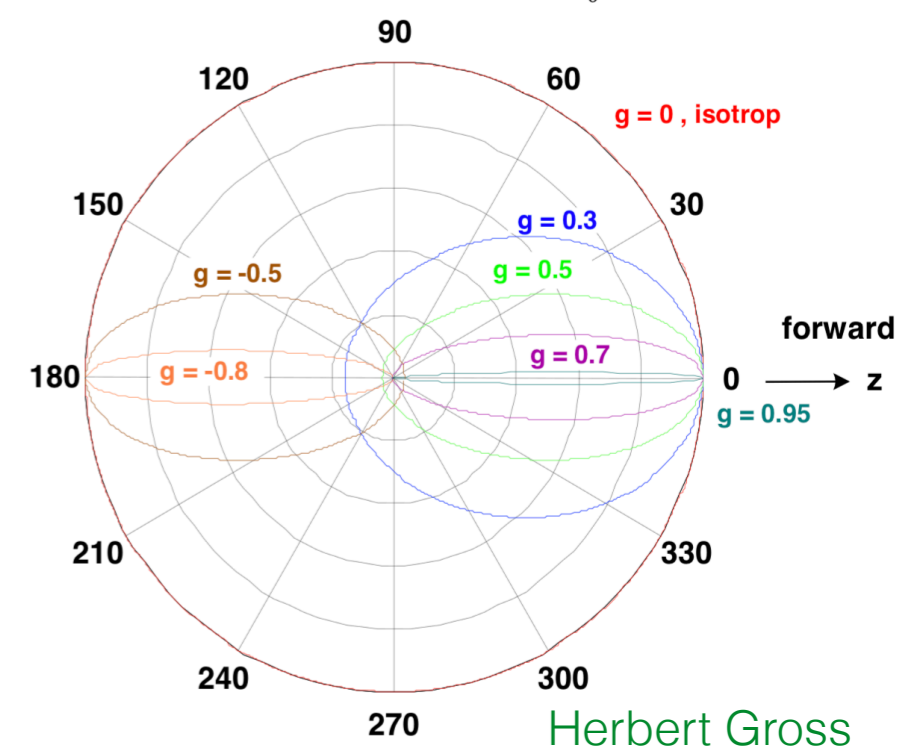


example of Mie scatt phase function



simplified model: Henyey-Greenstein

$$p_g(\mu) = \frac{1}{2} \frac{1 - g^2}{(1 + g^2 - 2g\mu)^{3/2}}$$



Examples of classical scattering effects

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Stellar Coronae



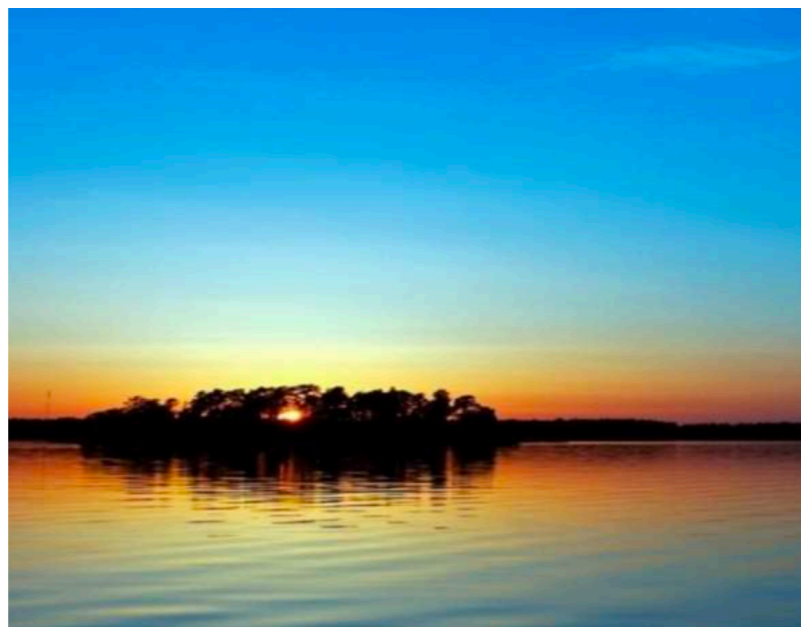
NASA / Aubrey Gemignani

Reflexion nebulae

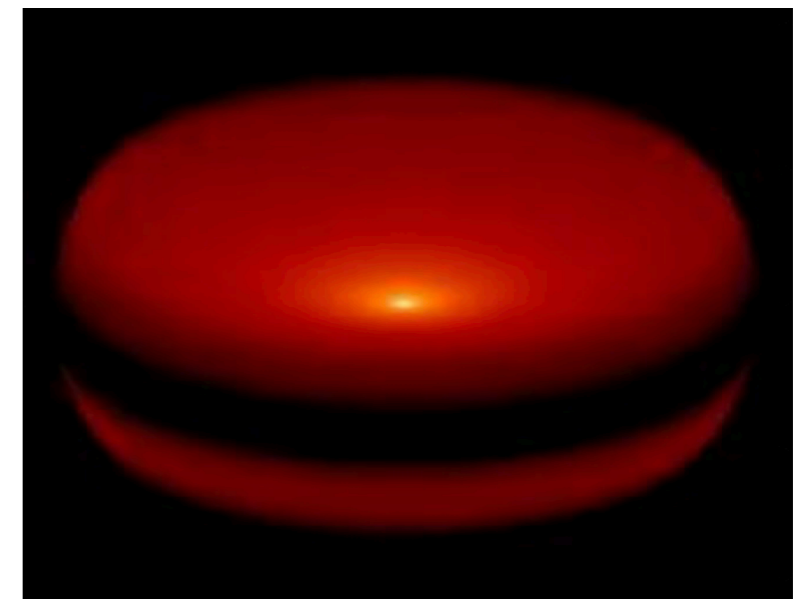


NASA / Gary Stevens

Sky and cloud colors



circumstellar envelopes & disks



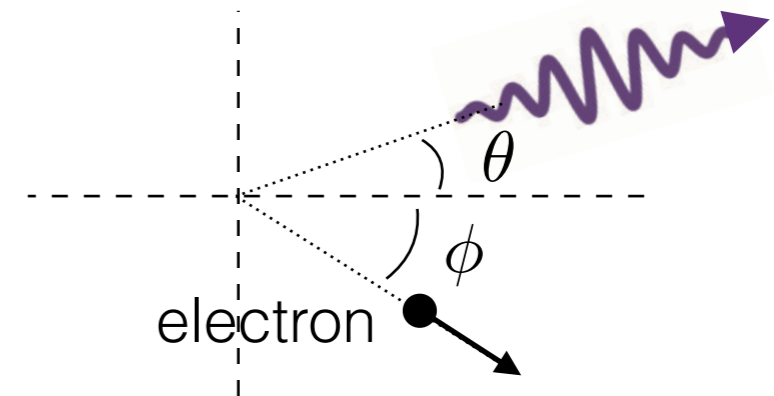
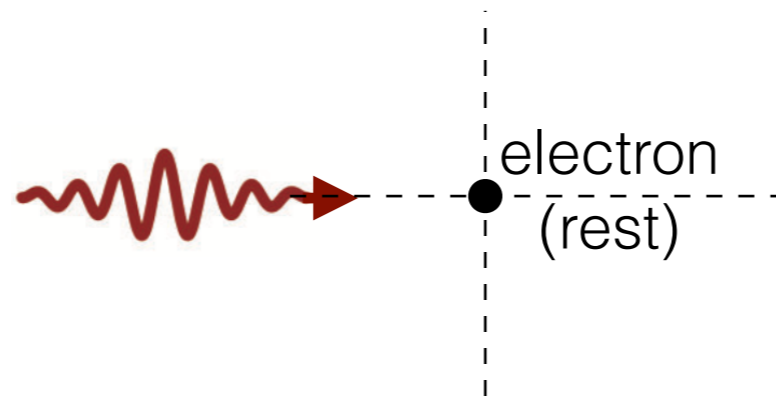
model from RADMC3D - Dullemond

Scattering by free electrons - quantum treatment

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free electron at rest in collision with a photon



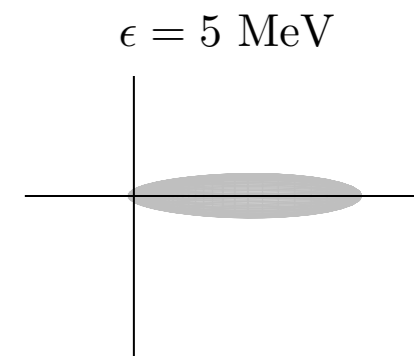
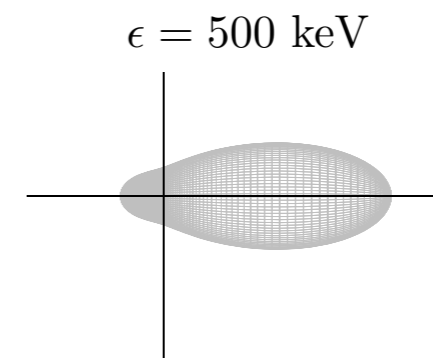
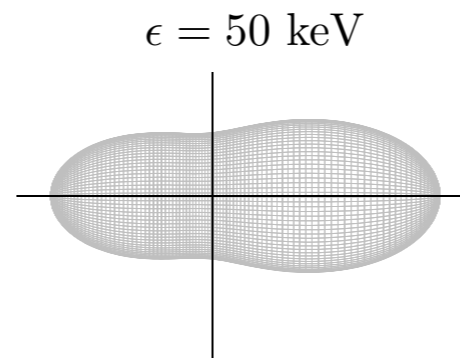
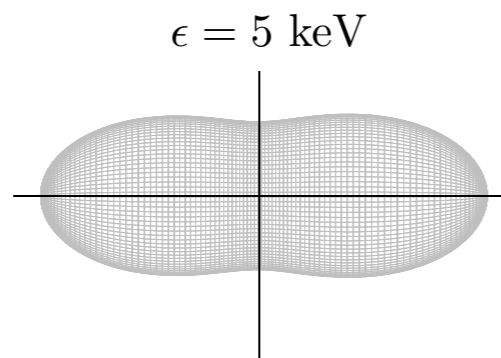
→ quantum mechanics treatment

- recoil of the electron (inelastic)
- differential cross section (QED)
Klein-Nishina (unpolarized)

$$\lambda_1 - \lambda = \frac{h}{m_e c} (1 - \cos\theta)$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_0^2 \left(\frac{\lambda}{\lambda_1} \right)^2 \left(\frac{\lambda_1}{\lambda} + \frac{\lambda}{\lambda_1} - \sin^2\theta \right)$$

→ simplifies to Thomson scattering for $\epsilon \ll m_e c^2$



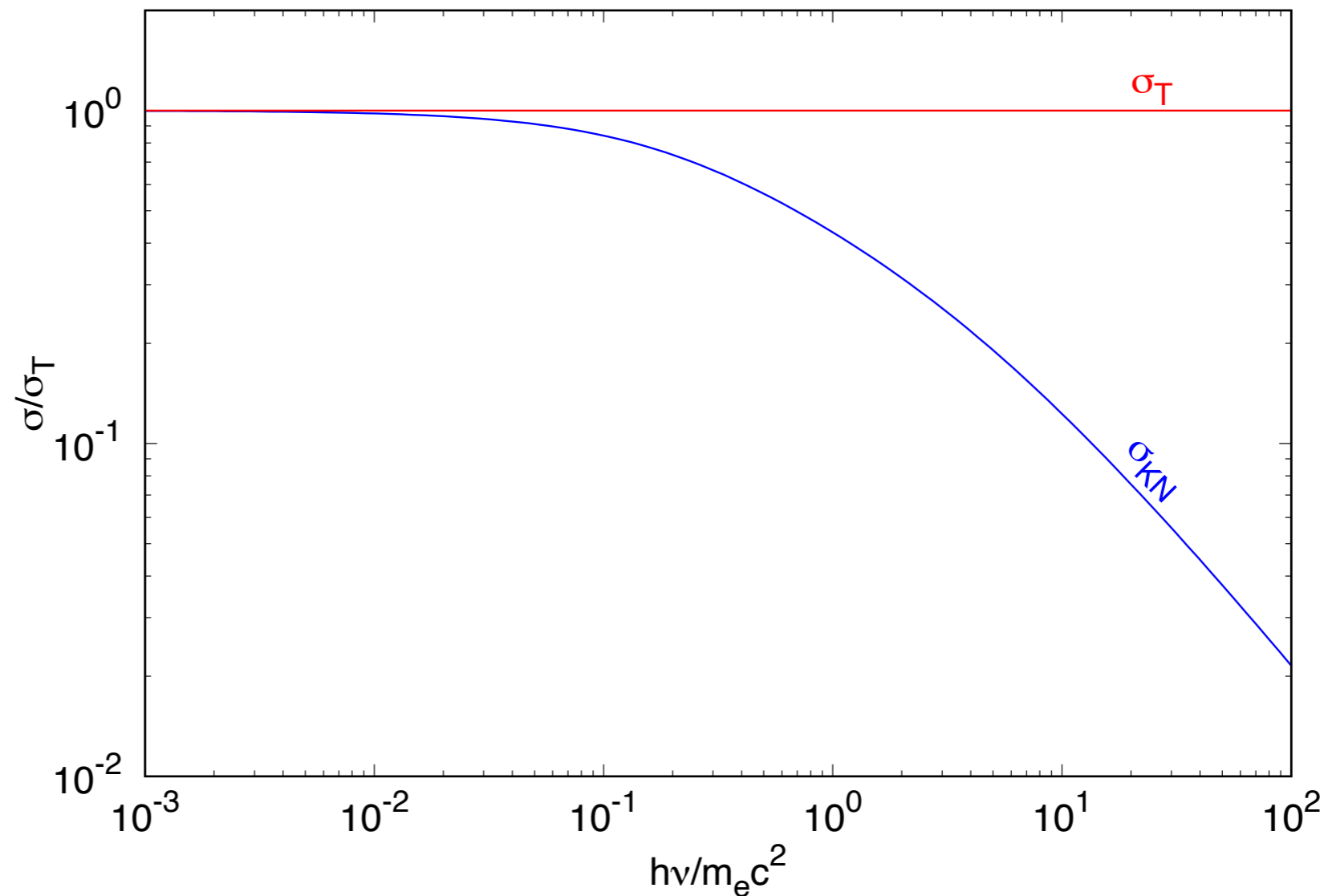
Scattering by free electrons - quantum treatment

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quantum effects on electron - photon scattering

- energy shift of the photon - loss of energy
- photon preferentially scattered forward
- reduction of the total cross section

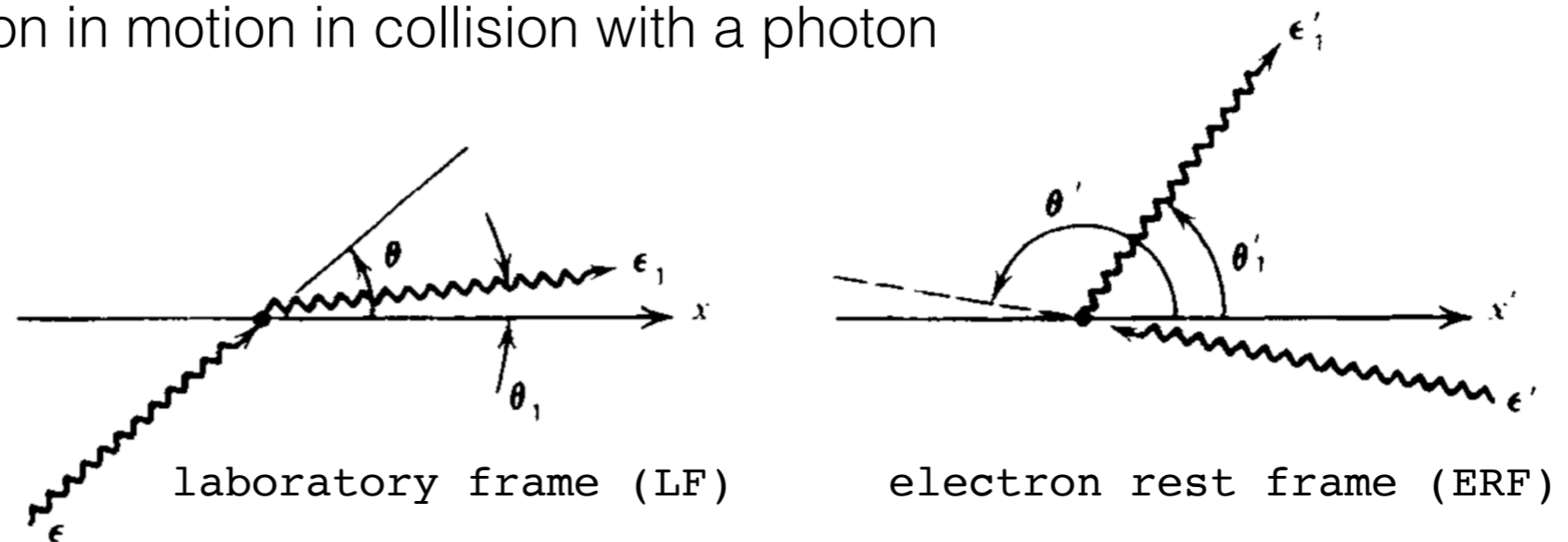


Inverse Compton scattering

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free electron in motion in collision with a photon



- step 1: from LF to ERF

$$\epsilon' = \epsilon \gamma (1 - \beta \cos \theta)$$

- step 2: scattering in ERF

$$\epsilon'_1 = \frac{\epsilon'}{1 + \frac{\epsilon'}{m_e c^2} (1 - \cos \theta')}$$

- step 3: from ERF to LF

$$\epsilon_1 = \epsilon'_1 \gamma (1 + \beta \cos \theta'_1)$$

if electron has a sufficient kinetic energy

→ transfer of energy to the photon : inverse Compton scattering

Inverse Compton scattering

Outline

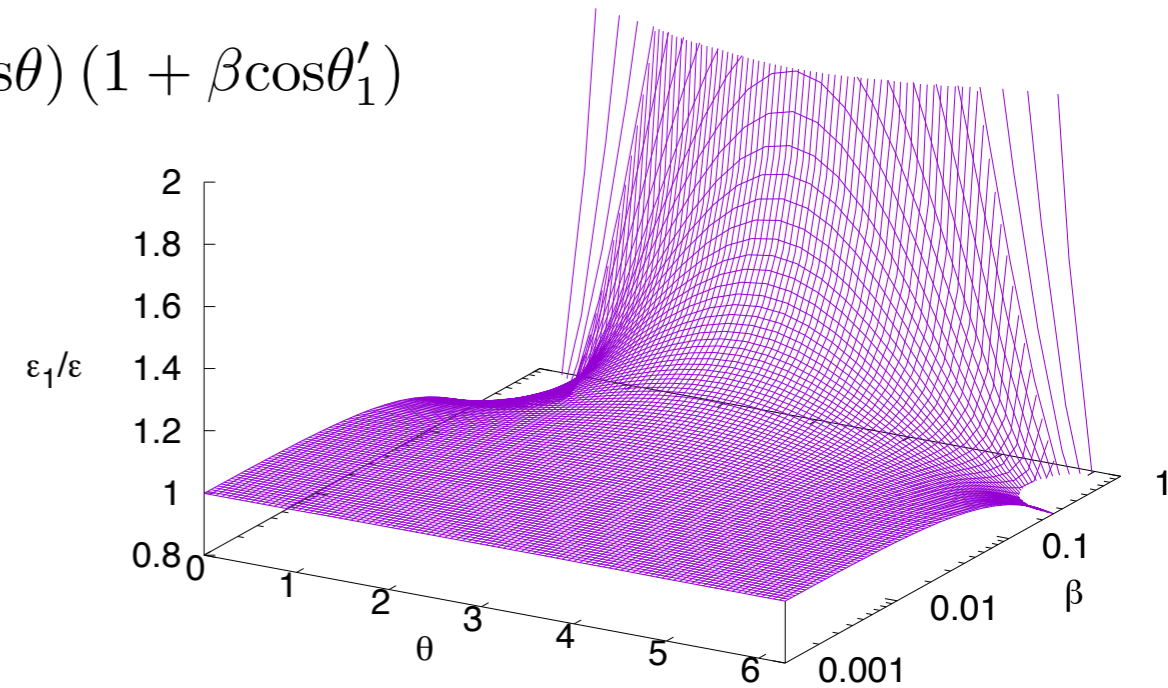
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in the Thomson regime $\gamma\epsilon \ll m_e c^2$

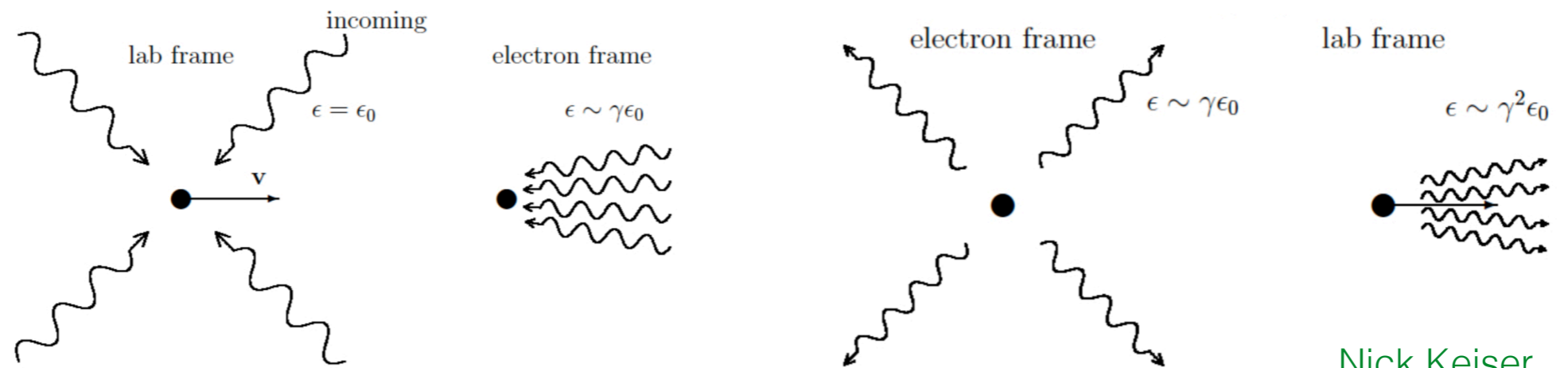
→ energy gain $\frac{\epsilon_1}{\epsilon} = \gamma^2 (1 - \beta \cos\theta) (1 + \beta \cos\theta'_1)$

depends on angle but
> 1 on average

strong gain for relativistic e^-



→ schematic view of energy transfer for relativistic e^-



Nick Keiser

Inverse Compton scattering

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in the Thomson regime $\gamma\epsilon \ll m_e c^2$ and for isotropic radiation

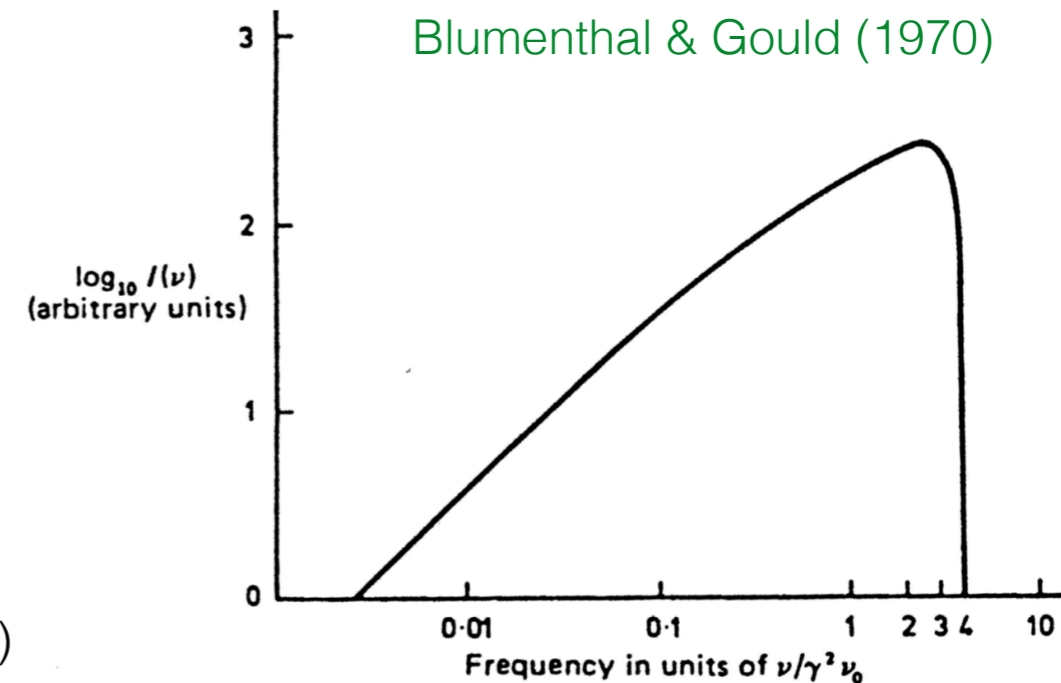
→ single scattering radiated power

$$P_{\text{Comp}} = 4/3 \sigma_T c \gamma^2 \beta^2 U_{ph} \quad (\text{erg s}^{-1})$$

→ incident field at ν_0 only spectrum

→ thermal dist of non-relativistic e^-

$$P_{\text{tot}} = \left(\frac{4kT}{m_e c^2} \right) \sigma_T c n_e U_{ph} \quad (\text{erg cm}^{-3} \text{ s}^{-1})$$



energy transfer for repeated scattering in finite thermal medium

→ non-relativistic e^- : per scattering

$$\frac{\Delta\epsilon}{\epsilon} \sim \frac{(4kT - \epsilon)}{m_e c^2}$$

→ ultra-relativistic e^- : per scattering

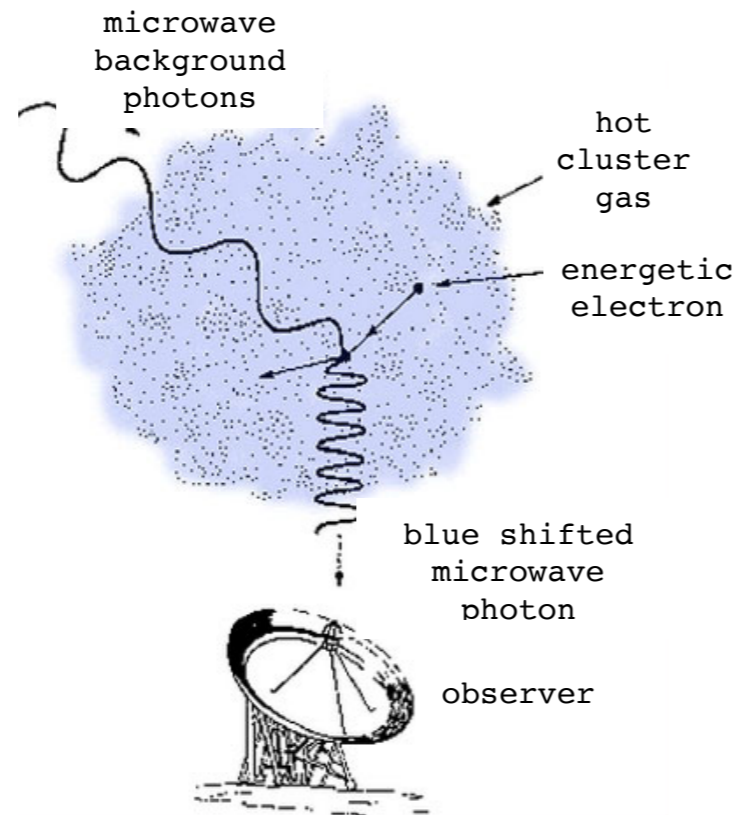
$$\frac{\Delta\epsilon}{\epsilon} \sim \frac{4}{3} \gamma^2 = 16 \left(\frac{kT}{m_e c^2} \right)^2$$

Example of inverse Compton scattering

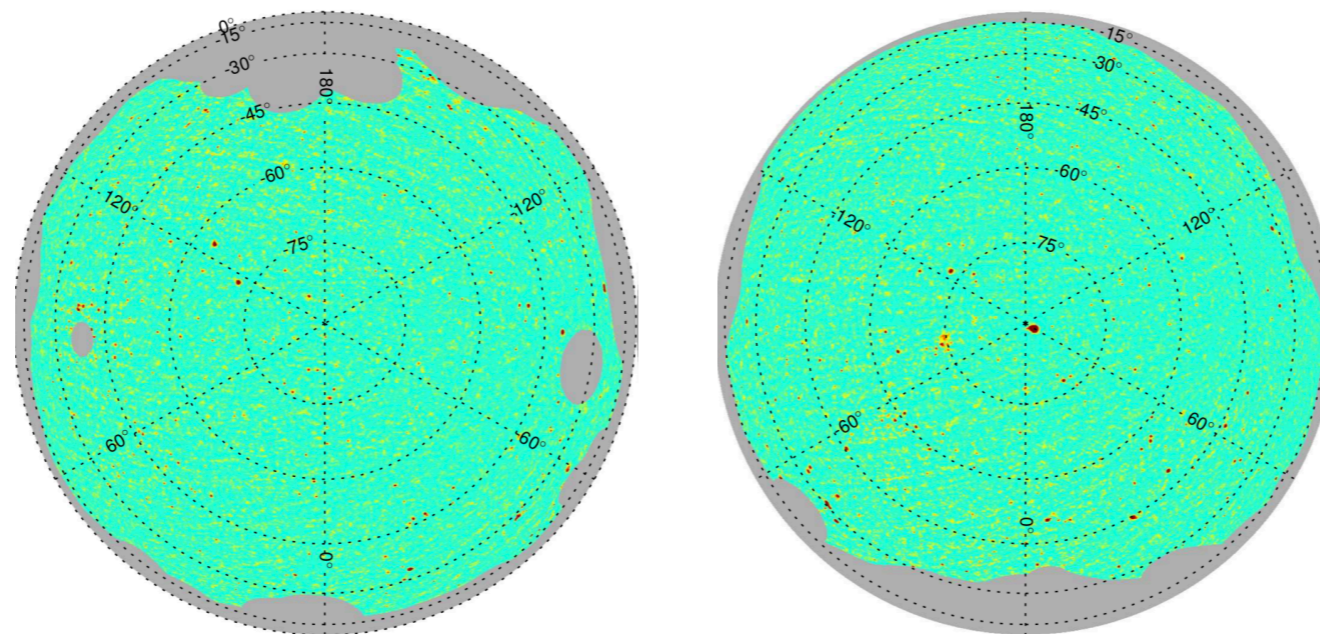
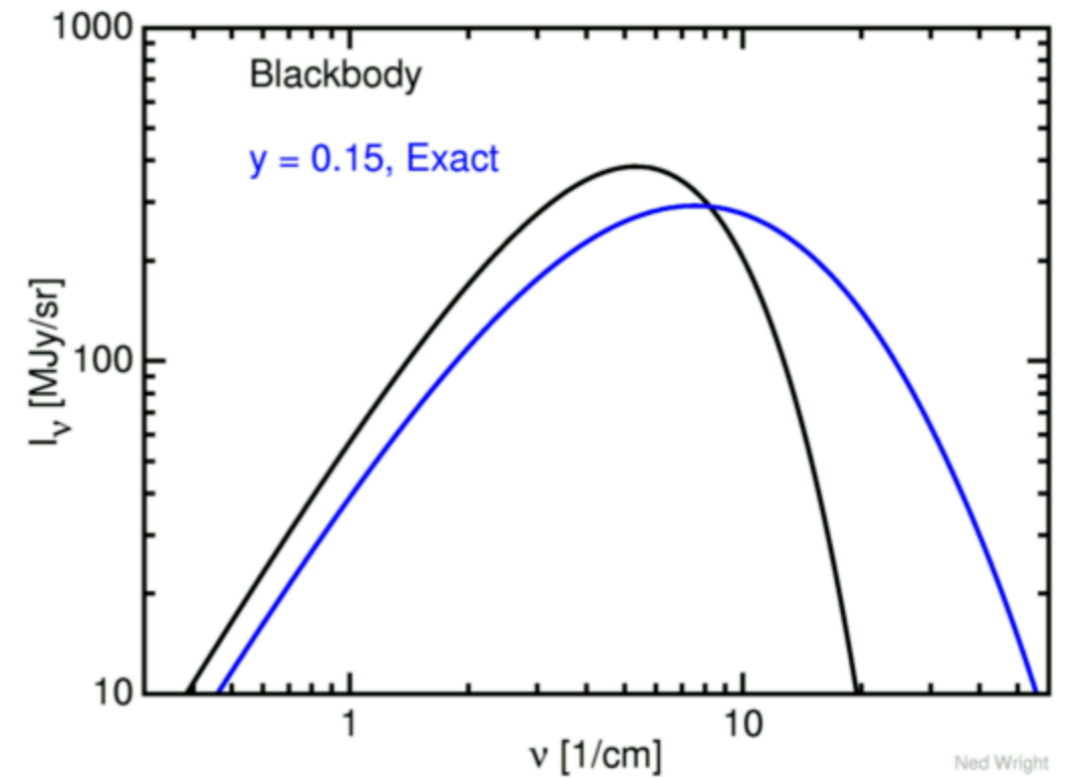
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Sunyaev-Zeldovich effect



Edward L. Wright



map of galaxy clusters
e- density & temperature
cluster size

Planck Collaboration (2015)

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methods to solve transfer
with scattering

Complexity of radiative transfer

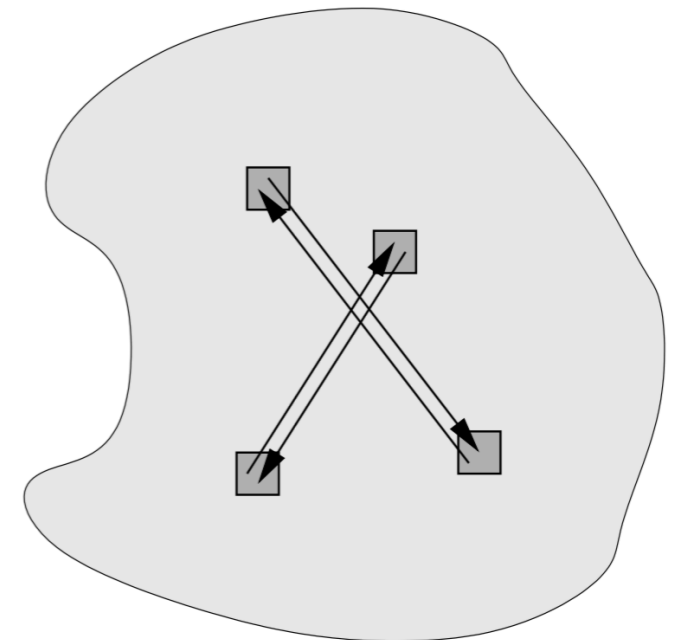
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$$\frac{dI_\nu}{dl} = -[\alpha_\nu + \sigma_\nu] I_\nu + j_\nu + \frac{1}{4\pi} \int_{\Omega'} \tilde{\sigma}_\nu(\mathbf{k}' \rightarrow \mathbf{k}) I_\nu(\mathbf{k}') d\Omega'$$

- goal: find the source function or the mean intensity at each point
- complexity: scattering & thermochemistry couple all rays & cells
- simpler (yet complicated) question:
how to solve the full radiative transfer (including absorption & scattering) knowing the thermal and chemical state of the medium ?

Radiative cell coupling



Dullemond (2012)

Scattering and random walk

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the fate of a photon expressed in terms of probabilities

$$l_\nu = \frac{1}{\alpha_\nu + \sigma_\nu} \longrightarrow \text{mean free path}$$

$$\varepsilon_\nu = \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu} \longrightarrow \text{proba that a free path ends with abs}$$

$$\omega_\nu = 1 - \varepsilon_\nu = \frac{\sigma_\nu}{\alpha_\nu + \sigma_\nu} \longrightarrow \text{proba that a free path ends with scatt}$$

$$l_\star = \sqrt{n} l_\nu \longrightarrow \text{rms net displacement for } n \text{ free paths}$$

for a medium of size L

$$\tau_\nu^a = \alpha_\nu L, \quad \tau_\nu^s = \sigma_\nu L \longrightarrow \text{absorption and scattering optical depth}$$

$$N_\nu = \frac{1 - \varepsilon_\nu}{\varepsilon_\nu} \longrightarrow \langle \text{number} \rangle \text{ scatt before absorption}$$

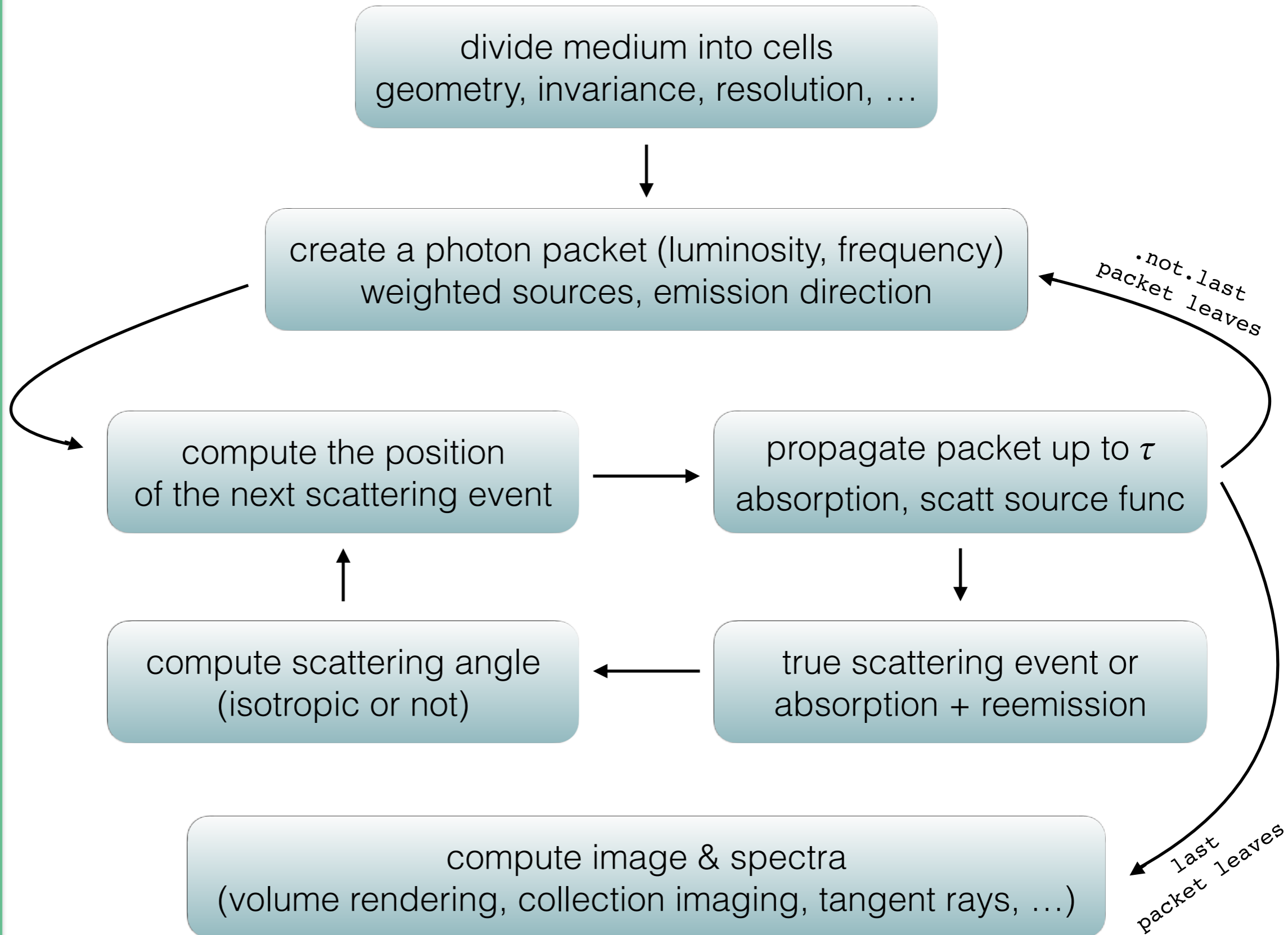
$$N_\nu \approx \max(\tau_\nu^s, \tau_\nu^{s2}) \longrightarrow \langle \text{number} \rangle \text{ scatt before escape}$$

$$l_\star = \min \left(L, \frac{1}{\sqrt{\alpha_\nu (\alpha_\nu + \sigma_\nu)}} \right) \longrightarrow \text{rms displacement in the cloud}$$

Monte Carlo basic algorithm

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Examples of probabilities and further readings

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assign probability distributions to each step

→ frequency of the photon packet $p(\nu) = \frac{L_\nu^*}{\int_0^\infty L_\nu^* d\nu}$

→ emission direction (Lambertian source ? else ?)

→ position of next scattering event $p(\tau) = \exp(-\tau)$

→ scattering phase function $p(\mu), p(\phi) = \frac{1}{2}, \frac{1}{2\pi}$

isotropic or more complex $p_g(\mu), p(\phi) = \frac{1}{2} \frac{1 - g^2}{(1 + g^2 - 2g\mu)^{3/2}}, \frac{1}{2\pi}$

further readings on the subject

→ amazing textbook by Dullemond (radiative transfer in astrophysics, 2012)

→ Lucy (1999), Bjorkman & Wood (2001)

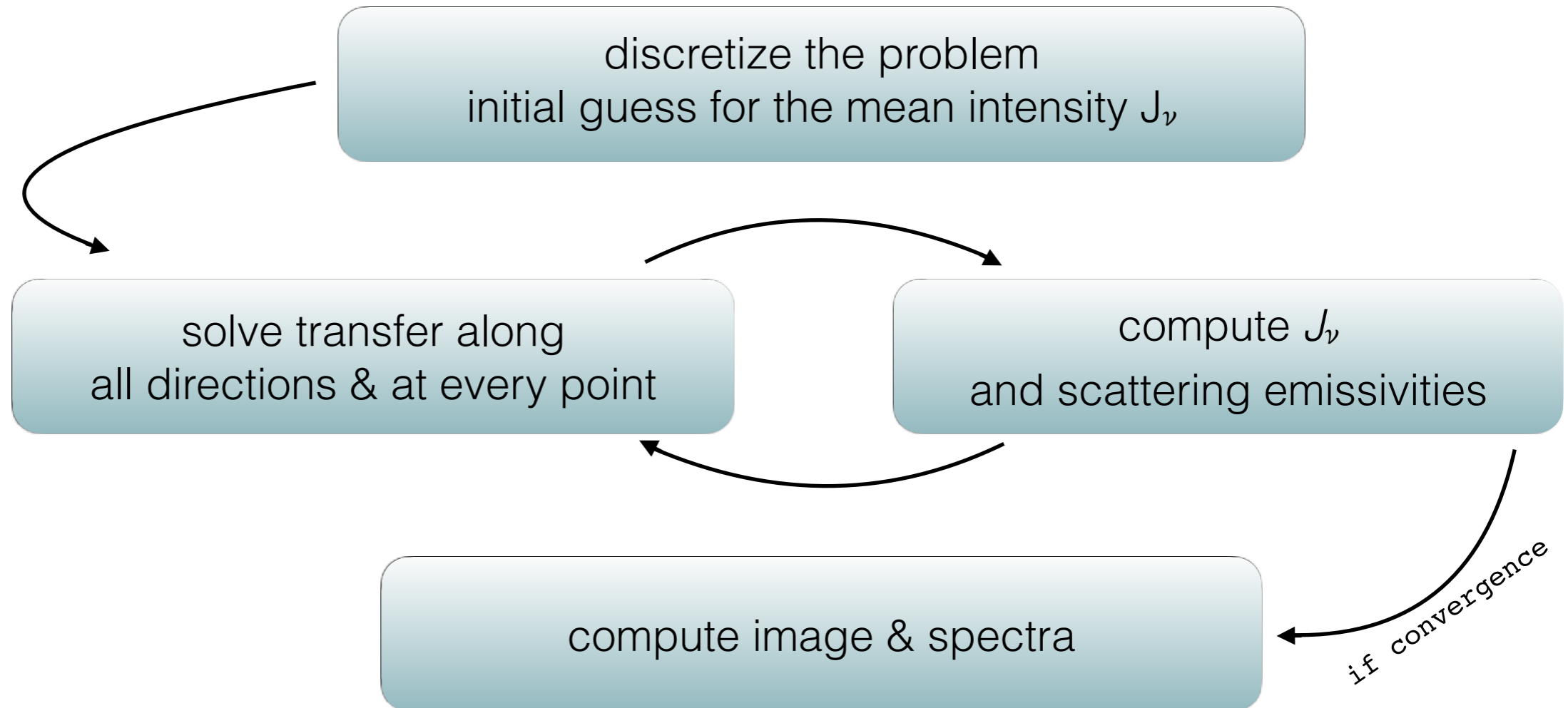
→ Min et al. (2009), Robitaille et al. (2010)

Discrete ordinate method

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class of method where all coordinates $(x, y, z, \nu, \theta, \phi)$ are discretized



→ examples of discrete ordinate methods

- lambda iteration (LI) or accelerated lambda iteration (ALI)
Cannon (1973), Olson et al. (1986)
- decomposition of in Legendre polynomials + spectral method
Roberge (1983), Kopp (1996), Goicoechea & Le Boulot (2007)

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reduce angular resolution by computing only the moments of I_ν

→ moments of I_ν

transfer in 1D

$$J_\nu(x) = \frac{1}{4\pi} \int I_\nu(x, \mathbf{k}) d\Omega$$

$$\mu \frac{dI}{d\tau} = I - S$$

$$\mathbf{H}_\nu(x) = \frac{1}{4\pi} \int I_\nu(x, \mathbf{k}) \mathbf{k} d\Omega$$

$$\frac{dF}{d\tau} = 4\pi(J - S)$$

$$\mathbf{K}_\nu(x) = \frac{1}{4\pi} \int I_\nu(x, \mathbf{k}) \mathbf{k} \otimes \mathbf{k} d\Omega$$

$$c \frac{dp}{d\tau} = F$$

...

→ closure equation (e.g. Eddington approximation)

$$K_{i,j,\nu} = \frac{1}{3} \delta_{ij} J_\nu$$

$$p = \frac{4\pi}{3c} J$$

→ other closure equation: variable Eddington tensor

Outline

- introduction
- Thomson scattering
- Rayleigh scattering
- Mie scattering
- Compton scattering
- Monte-Carlo num method
- other num method
- conclusions

Scattering processes

- at the heart of radiative transfer (impact all astrophysics environments)
- microprocesses explained by multipolar decomposition
- bound / free charges, small / large λ , micro / macroscopic systems
- change the direction and the energy of the photon (Compton)
- makes radiative transfer complicated

Methods to solve the transfer with scattering require several hours lecture

- amazing textbook by Dullemond (radiative transfer in astrophysics)
- algorithm & optimization are active fields of current research
- intuition comes from practice