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► **To cite this version:**

Edouard Challe. Uninsured Unemployment Risk and Optimal Monetary Policy in a Zero-Liquidity Economy. *American Economic Journal: Macroeconomics*, 2020, 12 (2), pp.241-283. 10.1257/mac.20180207 . hal-04645688

HAL Id: hal-04645688

<https://hal.science/hal-04645688>

Submitted on 11 Jul 2024

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Uninsured Unemployment Risk and Optimal Monetary Policy in a Zero-Liquidity Economy*

EDOUARD CHALLE[†]

April 2019[‡]

ABSTRACT. I study optimal monetary policy in a sticky-price economy wherein households precautionary-save against uninsured, endogenous unemployment risk. In this economy greater unemployment risk raises desired savings, causing aggregate demand to fall and feed back to greater unemployment risk. This deflationary spiral is constrained-inefficient and calls for an accommodative monetary policy response: after a contractionary aggregate shock the policy rate should be kept significantly lower and for longer than in the perfect-insurance benchmark. For example, the usual prescription obtained under perfect insurance of a hike in the policy rate in the face of a bad supply (i.e., productivity or cost-push) shock is easily overturned. The optimal policy breaks the deflationary spiral and takes the dynamics of the imperfect-insurance economy close to that of the perfect-insurance benchmark. These results are derived in an economy with zero asset supply (“zero liquidity”) and are thus independent of any redistributive effect of monetary policy on household wealth.

Keywords: Unemployment risk; imperfect insurance; optimal monetary policy.

JEL codes: E21; E32; E52.

*I thank Daniel Carrol, Wouter den Haan, Brigitte Hochmuth, Roozbeh Hosseini, Monika Merz, Xavier Ragot and three referees for their detailed comments on earlier versions of this paper. I also received helpful feedback from the participants to the SED meeting, ASSA meetings, the Oxford-New York Fed Monetary Economics Conference, the Konstanz Seminar in Monetary Theory and Policy, the Vienna Macro Workshop, the T2M Annual Meeting, the Macro Week in Marrakech, as well as from seminar participants at various places. I acknowledge financial support from the French National Research Agency (Labex Ecodec/ANR-11-LABX-0047). I have no relevant material or financial interests that relate to the research described in this paper.

[†]Ecole Polytechnique and CREST; Address: 5 av. Le Chatelier, 91120 Palaiseau, France; Email: edouard.challe@gmail.com.

[‡]First draft: February 2017.

1. INTRODUCTION

Households' precautionary-saving response to uninsured unemployment risk may generate substantial aggregate volatility, relative to a hypothetical situation of perfect insurance. The reason for this is that greater unemployment risk strengthens the precautionary motive for saving, causing aggregate demand, output and employment to fall, which ultimately feeds back to greater unemployment risk.¹ In this paper I ask how *should* the central bank respond to aggregate shocks when faced with this feedback loop, *by how much* does this response differ from that under perfect insurance, and *how effective* is it at stabilising welfare-relevant aggregates.

To this purpose, I construct a New Keynesian model with imperfect unemployment insurance and a frictional labour market and then derive the optimal monetary policy response to two prominent aggregate shocks, namely transitory (but persistent) productivity and “cost-push” shocks.² The optimal policy is that which best tracks a well-defined constrained-efficient allocation derived from a social welfare function exactly aggregating the intertemporal utilities of heterogeneous households and capturing all the frictions they are facing. I show the feedback loop between unemployment risk and aggregate demand to be constrained-inefficient and to affect the optimal path of the policy interest rate in important ways. To summarise, while the ultimate goals of monetary policy are the same as under perfect insurance –namely, stabilising prices and aligning output to its efficient level–, the implementation of this outcome *may require (much) more policy accommodation during recessions, so as to counter the inefficient rise in desired precautionary savings and associated fall in aggregate demand*; and conversely, it may require significantly more policy tightening in expansion, as consumption demand is boosted by the fall in desired precautionary savings. Put differently, the combination of endogenous unemployment risk and imperfect insurance leads to specific variations in desired savings that must be adequately

¹See, e.g., Auclert and Rognlie (2018), Beaudry et al. (2018), Challe et al. (2017), Chamley (2014), Den Haan et al. (2018), Heathcote and Perri (2018), Kekre (2018), McKay and Reis (2017), Ravn and Sterk (2017, 2018) and Werning (2015) for alternative formulations of this feedback loop. Challe et al. (2017), Den Haan et al. (2018) and Ravn and Sterk (2017) provide quantifications of this feedback loop for the U.S. and the euro area.

²In Representative-Agent New Keynesian models, persistent productivity shocks move the IS curve that determines the dynamics of the output gap, while cost-push shocks move the Phillips curve that determines the dynamics of inflation (see Clarida et al., 1999; Gali, 2008; Woodford, 2003).

stabilised by the central bank in order to avoid inefficient fluctuations in aggregate demand, output, employment and prices. The paper characterises the path of the policy rate that best achieves this stabilisation.

To understand why and how the precautionary motive affects the optimal path of the policy rate, consider first the response to a contractionary cost-push shock, that is, an exogenous increase in unit production costs that is passed through to final goods prices. With time-variations in desired precautionary savings the optimal response of the policy rate is in general ambiguous. On the one hand, the central bank should act to mitigate the direct inflationary impact of the shock, which typically commands a persistent increase in the policy rate. On the other hand, the shock harms job creation and sets in motion a *deflationary* feedback loop between unemployment risk and aggregate demand; this calls for a muted, or even reverted, response of the policy rate. Under a parametric restriction that gives the optimal response of the policy rate in closed form, these two effects can be additively decomposed into a *perfect-insurance response* and an *imperfect-insurance correction*. The perfect-insurance response calls for an increase in the policy rate, but the imperfect-insurance correction pushes the policy rate in the opposite direction and is greater the larger workers' mean consumption drop upon unemployment (a summary measure of the lack of consumption insurance). Away from this parametric restriction the contribution of imperfect insurance can be recovered numerically by comparing the optimal responses of the policy rate in the imperfect-insurance economy and in the perfect-insurance benchmark. In the calibrated imperfect-insurance model the central bank adopts a much more accommodative stance after a contractionary cost-push shock in order to offset its inefficient impact on aggregate demand; in most specifications the policy rate should be persistently *lowered*, not raised, after the shock. Moreover, implementation of the optimal path of the policy rate is effective, in that it breaks the deflationary spiral and takes the aggregate dynamics of the imperfect-insurance economy close to that of the perfect-insurance benchmark.

Uninsured unemployment risk also affects the optimal response of the policy rate to productivity shocks in important ways. Indeed, a persistent productivity-driven contraction (say) generates an increase in unemployment risk and elicits a precautionary response on

the part of the households. The resulting fall in aggregate demand exerts an inefficient downward pressure on inflation and employment that the central bank must stabilise. I show that under imperfect insurance the required degree of policy accommodation after a contractionary productivity shock depends on the two forces that ultimately determine workers' consumption response, namely the *precautionary motive* (against unemployment risk) and *aversion to intertemporal substitution* (as determined by the expected path of the real wage). The optimal policy is to cut the policy rate whenever the precautionary motive dominates aversion to intertemporal substitution. This happens to be the case under my baseline calibration, but even away from it, any plausible alternative calibration implies that substantially more accommodation against contractionary shocks is needed under imperfect insurance than under perfect insurance. Finally, just as in the case of cost-push shocks, implementation of the optimal policy after a productivity shock successfully undoes much of the propagating effect of imperfect insurance on aggregate dynamics.

I reach these conclusions by first focusing on a baseline specification of the model, and then exploring several departures from this baseline. For example, in my baseline imperfect-insurance model the real wage that splits the match surplus between a firm and a worker is determined via Nash bargaining, suitably augmented to allow for a plausible degree of wage inertia. This decentralised wage is generically inefficient, but I check that my results continue to hold when the real wage is the efficient one –which ensures that my baseline results are not driven by the inefficiency of the decentralised wage. Another feature of the baseline specification is that there is a set of (constant) taxes and subsidies that align the steady state of the decentralised equilibrium to its constrained-efficient counterpart. This ensures that the optimal policy I obtain is not unduly driven by steady state distortions, but this requires some subsidies that one does not observe in practice. I therefore check that my results continue to hold without these subsidies. Finally, I systematically compare my baseline results not only to the perfect-insurance benchmark –wherein the precautionary motive for saving is shut down– but also to a model specification with a constant wage –wherein the precautionary motive is maintained but it is aversion to intertemporal substitution that is shut down instead.

Broadly speaking, there are two reasons why imperfect insurance may affect positive or normative conclusions one may reach about monetary policy, relative to the perfect-insurance benchmark. One reason is that monetary policy may have redistributive effects on heterogeneous households' earnings or wealth. Another reason is that imperfect insurance brings about countercyclical variations in desired savings that may destabilise aggregate demand. Much of the recent literature on Heterogenous-Agent New Keynesian ("HANK") models has focused on the first issue, largely abstracting from the second (Auclert, 2018; Bhandari et al., 2018; Kaplan et al., 2018; Gornemann et al., 2016; Nuño and Thomas, 2017.) I do the opposite: I almost entirely bypass distributional issues (by using a model with minimal household heterogeneity) and I exploit the afforded tractability to study the optimal stabilisation of aggregate demand when unemployment risk drives a countercyclical precautionary motive.³

In so doing, my paper also complements two other strands of the literature. One is the positive analysis of business cycles and monetary policy in Heterogenous-Agent New Keynesian models with Search and Matching ("HANK & SaM"), wherein monetary policy is not optimized but obeys an *ad hoc* interest-rate rule (Challe et al., 2017; Den Haan et al., 2017; Gornemann et al., 2016; Ravn and Sterk, 2017, 2018).⁴ The model I construct is a variant of Ravn and Sterk (2018), wherein a time-varying precautionary motive is operative while a zero debt limit prevents the emergence of a cross-sectional wealth distribution. Relative to all these papers, the present one shows that it is both desirable and possible to neutralise the feedback loop between aggregate demand and unemployment risk via a suitable adjustment of the path of the policy rate. The other strand of the literature that my paper complements is the analysis of optimal monetary policy in New Keynesian models *without* a time-varying precautionary motive. This includes the basic RANK model (Clarida et al., 1999; Woodford, 2003; Galí, 2008), as well as extensions with SaM and *perfect* insurance (Thomas, 2008; Faia,

³In particular, I construct the model in such a way that uninsured unemployment risk is the *only* source of cyclicity in the income risk of precautionary savers. Alternative (but potentially controversial) sources of cyclicity, such as that coming from the distribution of firm rents, fiscal transfers, or the unemployment insurance scheme, are deliberately shut down.

⁴Acharya and Dogra (2018), Auclert and Rognlie (2018), Bilbiie (2019) and Werning (2015, Section 3.4) examine the sensitivity of aggregate demand to monetary policy in models wherein the cyclicity of individual income risk is potentially time-varying but parameterised –rather than derived from an underlying SaM structure.

2009; Blanchard and Galí, 2010; Ravenna and Walsh, 2011) or model variants with partial asset-market participation (Bilbiie, 2008; Bilbiie, 2018; Bilbiie and Ragot, 2017; Debortoli and Galí, 2018).⁵ Inasmuch as I consider a cashless economy with Calvo pricing, my model shares with those the efficiency of price stability –possibly implemented gradually in case of short-run policy trade-off (e.g., after to a cost-push shock). However, I stress that the way the policy rate should be adjusted in response to aggregate shocks in order to achieve this goal differs considerably when desired precautionary savings are time-varying.

Last but not least, an early paper and a precursor to the present one is Braun and Nakajima (2012), wherein the optimal monetary-policy response to productivity shocks is studied within a zero-liquidity HANK economy. My analysis extends theirs in several directions, notably by considering transitory idiosyncratic (i.e., unemployment) shocks, an endogenous cyclical risk of income risk, as well as inefficient cost-push shocks.

Section 2 presents the model and characterises the equilibrium. Section 3 derives the constrained-efficient allocation and associated steady state. Section 4 calibrates the model and studies two positive-policy benchmarks, namely one with flexible prices and the other with a simple interest rate rule. Section 5 formulates and solves the optimal monetary policy problem.

2. THE MODEL

2.1. Households. Time is discrete: $t \in \{0, 1, \dots\}$. Households are of two types: there is a unit measure of “workers”, who can be employed or unemployed, and a measure $\nu > 0$ of “firm owners” who manage the firms and collect dividends. All households are infinitely-lived and discount the future at the factor $\beta \in [0, 1)$, and none of them can borrow against future income.

⁵Two papers examine optimal *unemployment insurance* (UI) policies under the same frictions as those I consider: McKay and Reis (2017), who show that they raise the optimal *ex ante* level of UI (due to its role as an automatic stabiliser), and Kekre (2018), who show that they rationalise state-contingent UI duration. One advantage of monetary policy over state-contingent UI is that a change in the policy rate can be implemented readily and at virtually no cost to the public authority (aside from potential losses in seigniorage revenue). But UI policies can usefully complement monetary policy in situations where the policy rate is constrained (e.g., by an effective lower bound).

Workers. A worker $i \in [0, 1]$, who can be employed or unemployed, chooses the consumption sequence $\{c_{i,t+k}\}_{k=0}^{\infty}$ that maximises $V_t^i = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k u(c_{i,t+k})$, where $c_{i,t} \geq 0$ is consumption and $e_{i,t} \in \{0, 1\}$ worker's i status in the labour market –with $e_{i,t} = 1$ if the worker is employed and 0 otherwise. \mathbb{E}_t is the rational-expectations operator and $u(\cdot)$ is a period utility function such that $u' > 0$ and $u'' < 0$ for all $c \geq 0$. Employed workers earn the real wage $w_t > 0$, while unemployed workers earn the exogenous home production income $\delta_t \in (0, w_t)$. Workers transit randomly between labour market statuses and the associated income risk is uninsured. The budget and borrowing constraints of worker $i \in [0, 1]$ at date t are given by, respectively:

$$a_{i,t} + c_{i,t} = e_{i,t}w_t + (1 - e_{i,t})\delta_t + R_t a_{i,t-1} \quad \text{and} \quad a_{i,t} \geq 0, \quad (1)$$

where $a_{i,t}$ is the real value of worker's bond wealth at the end of date t and R_t the gross real return on assets. Workers hold no wealth initially, i.e., $a_{i,t-1} = 0$ for all $i \in [0, 1]$. Workers' optimal consumption-saving choices must satisfy the Euler condition $\beta \mathbb{E}_t[u'(c_{i,t+1}) R_{t+1}/u'(c_{i,t})] \leq 1$, with an equality if the borrowing constraint is slack and a strict inequality if it is binding.

Firm owners. Firm owners share the period utility function $\tilde{u}(c)$, with $\tilde{u}' > 0$ and $\tilde{u}'' \leq 0$, which may differ from $u(c)$.⁶ They do not face any idiosyncratic income risk, and they all hold the same asset wealth a_{-1}^F at the beginning of time; they thus stay symmetric at all times and I denote their common individual consumption and end-of-period asset wealth by c_t^F and a_t^F , respectively. In every period they get an equal share of the aggregate dividend D_t that results from firms' rents (see below), as well as a home production income, of amount $\varpi \geq 0$ in the aggregate, and a lump sum fiscal transfer, of amount τ_t in the aggregate. A firm owner thus maximises $V_t^F = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \tilde{u}(c_{t+k}^F)$, subject to:

$$a_t^F + c_t^F = (D_t + \varpi + \tau_t)/\nu + R_t a_{t-1}^F \quad \text{and} \quad a_t^F \geq 0. \quad (2)$$

⁶As shown in Section 3, the preferences of workers and firm owners will affect the efficient sharing of aggregate risk between the two groups.

Firm owners hold no wealth initially, i.e., $a_{t-1}^F = 0$. Given their preferences and constraints, the optimal consumption plan of a firm owner must satisfy $\mathbb{E}_t[M_{t+1}^F R_{t+1}] \leq 1$, where M_{t+1}^F denotes firm owners' common marginal rate of intertemporal substitution (“MRIS” henceforth):

$$M_{t+1}^F = \beta \tilde{u}(c_{t+1}^F) / \tilde{u}(c_t^F). \quad (3)$$

This market structure with two household types is consistent with the fact that in practice equity holding (whether public or private) is limited, while the MRIS of households who own firms –and consequently decide for them– differs from that of households enjoying labour earnings only. Moreover, Werning (2015), Bilbiie (2018) and others have stressed that the cyclicality and distribution of household transfers (including firm rents) in HANK models may affect, in a somewhat artificial –and potentially controversial– way the cyclicality of income risk and implied savings response of precautionary savers. This issue does not arise here because, as explained below, in equilibrium only the workers have a precautionary motive for saving while only the firm owners collect firm rents and fiscal transfers.⁷

2.2. Firms. The production structure has three layers: intermediate goods firms produce out of workers' labour units, which they hire in a frictional labour market with search costs. Those goods are sold to wholesale firms, each of whom turn them into a differentiated good. Finally, wholesale goods are purchased and reassembled by final goods firms, the output of which is used for consumption and search costs.

Final goods sector. There is a representative, competitive firm that produces the final good by combining wholesale inputs according to the function:

$$y_t = \left(\int_0^1 y_{h,t}^{\frac{\theta-1}{\theta}} dh \right)^{\frac{\theta}{\theta-1}}, \quad (4)$$

where $y_{h,t}$ is the quantity of wholesale good h used in production and $\theta > 1$ the cross-partial elasticity of substitution between wholesale inputs. Denoting $p_{h,t}$ as the price of wholesale good h in terms of the final good, the optimal combination of inputs gives the following

⁷More generally, in the present paper I deliberately neutralise any source of cyclicality in the income risk of precautionary savers, other than that arising from time-variations in unemployment risk.

demands:

$$y_{h,t} = y_t p_{h,t}^{-\theta}, \quad h \in [0, 1], \quad (5)$$

while the zero-profit condition in the final goods sector implies that $\int_0^1 p_{h,t}^{1-\theta} dj = 1$.

Wholesale sector. Wholesale firm $h \in [0, 1]$ turns every intermediate good into a specialised good that is monopolistically supplied to the final goods sector. The profit of wholesale firm h is

$$\Pi_{h,t}^W = y_{h,t} [p_{h,t} - \varphi_t (1 - \tau^W)], \quad (6)$$

where φ_t is the price of intermediate goods in terms of the final goods and τ^W a production subsidy to the wholesale sector, financed through a lump sum tax on firm owners.⁸

Wholesale firms face nominal pricing frictions a la Calvo: in every period a fraction $1 - \omega \in [0, 1]$ of the firms are able to reset their price optimally, while the other firms keep it unchanged. The resulting time-varying distribution of wholesale prices can be summarised by three moments, namely the optimal reset price common to all price-resetting firms \tilde{p}_t , final good inflation π_t , and the price dispersion index $\Delta_t \equiv \int_0^1 p_{h,t}^{-\theta} dh \geq 1$ (see Woodford, 2003, for details). These moments evolve as follows. First, the optimal reset price is given by:

$$\tilde{p}_t = \frac{\theta(1 - \tau^W)\Xi_t}{(\theta - 1)\Sigma_t}, \quad (7)$$

where Ξ_t and Σ_t obey the following forward recursions:

$$\Xi_t = \varphi_t y_t + \omega (1 + \pi_{t+1})^\theta \mathbb{E}_t[M_{t+1}^F \Xi_{t+1}] \quad \text{and} \quad \Sigma_t = y_t + \omega (1 + \pi_{t+1})^{\theta-1} \mathbb{E}_t[M_{t+1}^F \Sigma_{t+1}],$$

and M_{t+1}^F is given by equation (3).

Second, current inflation depends on the optimal reset price according to:

$$\pi_t = [\omega^{-1} - (\omega^{-1} - 1) (\tilde{p}_t)^{1-\theta}]^{\frac{1}{\theta-1}} - 1. \quad (8)$$

⁸This subsidy will serve in Section 3 to correct the steady-state distortion due to monopolistic competition.

Third, the dynamics of the price dispersion index as a function of (\tilde{p}_t, π_t) is given by:

$$\Delta_t = (1 - \omega) (\tilde{p}_t)^{-\theta} + \omega (1 + \pi_t)^\theta \Delta_{t-1}, \quad (9)$$

and I assume that prices are symmetric at the beginning of time (i.e., $\Delta_{-1} = 1$).

From equations (5)–(6) and the definition of Δ_t , the total rent generated by the wholesale sector, which will contribute to the aggregate dividend paid out to firm owners, is given by:

$$\Pi_t^W = \int_0^1 \Pi_{h,t}^W dh = y_t [1 - \varphi_t (1 - \tau^W) \Delta_t]. \quad (10)$$

Intermediate goods sector and labour market flows. Intermediate goods firms produce z_t units of goods out of one unit of labour, and labour productivity evolves as follows:

$$z_t = 1 + \mu_z (z_{t-1} - 1) + \epsilon_{z,t}, \quad (11)$$

where $\mu_z \in [0, 1)$, $\epsilon_{z,t}$ is a white noise process with mean zero, and $\epsilon_{z,t}$ and z_t have small bounded support.

These firms hire labour in a frictional market with search costs. At the beginning of date t a constant fraction $\rho \in (0, 1]$ of existing employment relationships are destroyed, at which point the size of the unemployment pool goes from $1 - n_{t-1}$ to $1 - (1 - \rho) n_{t-1}$. At that time intermediate goods firms post v_t vacancies, at a unit cost $\eta > 0$, a random matching market opens and $m (1 - (1 - \rho) n_{t-1})^\gamma v_t^{1-\gamma}$ (with $m > 0$ and $\gamma \in (0, 1)$) new employment relationships are formed.⁹ It follows that the job-finding and vacancy-filling rates are, respectively:

$$f_t = m \left[\frac{v_t}{1 - (1 - \rho) n_{t-1}} \right]^{1-\gamma} \quad \text{and} \quad \lambda_t = m \left[\frac{v_t}{1 - (1 - \rho) n_{t-1}} \right]^{-\gamma}. \quad (12)$$

⁹This timing assumption implies that firms may fill vacancies within the period in which they are opened, while workers may change jobs without going through a period of unemployment. This is the standard timing assumption for business-cycle models calibrated at the quarterly frequency, since it accommodates labor market flows taking place within the period (see, e.g., Den Haan et al., 2000; Blanchard and Gali, 2010; Ravenna and Walsh, 2011; Gornemann et al., 2016; Ravn and Sterk, 2017, 2018; Challe et al., 2017; and many others).

The value to firm owners of an employment relationship, denoted J_t , is the sum of a flow payoff—the after-tax rent generated by the match—and a continuation value that depends on the survival rate of the match and firm owners' MRIS:

$$J_t = (1 - \tau^I)(z_t \varphi_t - w_t + T - \zeta_t) + (1 - \rho) \mathbb{E}_t[M_{t+1}^F J_{t+1}], \quad (13)$$

where $\tau^I \in [0, 1]$ is the corporate tax rate and T a wage subsidy. ζ_t is a random wage tax evolving as follows:

$$\zeta_t = \mu_\zeta \zeta_{t-1} + \epsilon_{\zeta,t}, \quad (14)$$

where $\mu_\zeta \in [0, 1)$ and $\epsilon_{\zeta,t}$ is a white noise process with mean zero, and $\epsilon_{\zeta,t}$ and ζ_t have small bounded support.

The taxes and subsidy τ^I and T will serve the same purpose as the production subsidy τ^W in the wholesale sector: they will be set in such a way that the steady state of the decentralised equilibrium be constrained-efficient. Unlike in the basic RANK model the production subsidy τ^W does not suffice for this here because the economy has two additional distortions, namely congestion externalities in the intermediate-good sector (due to labour-market frictions) and imperfect insurance against unemployment risk. We will see in Section 3 below how τ^W , τ^I and T should be set to ensure constrained-efficiency of the steady state. Finally, the random tax ζ_t perturbs the real marginal cost of intermediate goods firms and is partly passed through to wholesale goods prices, and ultimately to final good prices. It will manifest itself as a *cost-push shock* and make the decentralised equilibrium of the stochastic economy generically constrained-inefficient.¹⁰ The net proceeds of all taxes and subsidies to the intermediate goods sector are rebated lump-sum to firm owners (they enter the transfer τ_t in equation (2)).

Under free entry, the cost of a vacant job (η) must equate its expected payoff ($\lambda_t J_t$, since vacancies can be filled immediately). Then, using equations (12)–(13) and the fact that

¹⁰When the aggregate supply block of the model takes the form of a simple New Keynesian Phillips curve (which is the case under the parametric restrictions of Section 5.1), then a change in the random distortionary tax ζ_t is isomorphic to a shock to the cross-partial elasticity of substitution between wholesale goods θ (i.e., it shows up a residual in the New Keynesian Phillips curve).

$\lambda_t^{-1} = f_t^{\frac{\gamma}{1-\gamma}}/m^{\frac{1}{1-\gamma}}$, I get the following forward recursion for the job-finding rate:

$$f_t^{\frac{\gamma}{1-\gamma}} = (1 - \tau^I)m^{\frac{1}{1-\gamma}}\eta^{-1}(z_t\varphi_t - w_t + T - \zeta_t) + (1 - \rho)\mathbb{E}_t[M_{t+1}^F f_{t+1}^{\frac{\gamma}{1-\gamma}}]. \quad (15)$$

Since employed workers are separated from their firm with probability ρ at the very beginning of the period, but can immediately find a job with probability f_t , the period-to-period transition rate from employment to unemployment is given by:

$$s_t = \rho(1 - f_t). \quad (16)$$

Note that it is the transition probability from employment to unemployment s_t , and not the beginning-of-period match destruction rate ρ per se, that measures the extent of unemployment risk faced by employed workers; consequently, it is this variable that will determine their desired precautionary savings. The transition rates s_t and f_t are perfectly correlated here because ρ is constant by assumption, but it would be straightforward to introduce shocks to ρ to relax this tight connection.¹¹

From (f_t, s_t) in equations (15)–(16), we obtain the law of motion for total employment:

$$n_t = f_t(1 - n_{t-1}) + (1 - s_t)n_{t-1}. \quad (17)$$

Finally, from the flow payoff in equation (13), the aggregate rent generated by intermediate goods firms at time t is:

$$\Pi_t^I = n_t(1 - \tau^I)(z_t\varphi_t - w_t + T - \zeta_t) - \eta v_t, \quad (18)$$

¹¹Shimer (2005) showed that high-frequency variations in U.S. unemployment are dominated by fluctuations in the exit flow from unemployment. This implies that if one time-aggregates labor market flows to compute lower-frequency (e.g., quarterly) transition rates between employment and unemployment, then both rates have a common underlying driver – namely, the sequence of higher-frequency job-finding rates – and are thus highly correlated. It follows that a specification like equation (16) captures well the dynamics of quarterly labor market flows in the US. For example, Challe et al. (2017) treat ρ as a residual after time-aggregating f_t and s_t and find that it moves very little over the business cycle. In Section 5.2 below the model is calibrated on the basis of quarterly US data, so the specification of s_t in equation (16) is warranted (but again, it can be relaxed at no cost).

so the aggregate dividend D_t paid out to firm owners in equation (2) is $D_t = \Pi_t^W + \Pi_t^I$.

Ultimately, firms' vacancy-posting decisions –and implied employment dynamics– depend on the real wage w_t . In the remainder of this section I fully characterise the equilibrium under the minimal requirement that all matches be bilaterally efficient –that is, w_t is sufficiently low for firms to find it worthwhile to post vacancies, and at the same time sufficiently large for matched workers to accept job offers. In Section 4 below I uniquely pin down the wage through generalised Nash bargaining, suitably augmented to generate a plausible degree of wage inertia.

2.3. Policymakers. There are two policymakers, namely the government and the central bank. The government sets the (constant) taxes and subsidies τ^W , τ^I and T and rebates the (possibly negative) net revenue to firm owners in a lump sum manner. From equations (10) and (18), the net transfer to firm owners is:

$$\tau_t = \underbrace{\tau^I n_t (z_t \varphi_t - w_t)}_{\text{corporate taxes}} - \underbrace{\tau^W \varphi_t \Delta_t y_t}_{\text{production subsidies}} - \underbrace{n_t (1 - \tau^I) (T - \zeta_t)}_{\text{wage subsidies}}. \quad (19)$$

In most of my analysis I assume that the taxes and transfers are set in a way that decentralises the constrained-efficient allocation in the absence of aggregate shocks. However, in Section 5.2 I also explore a model variant wherein the government has a more restricted set of instruments, which results in a distorted steady state.

The central bank controls the nominal interest rate on bonds i_t (the “policy rate”). The gross real ex post return that results from the policy rate and the dynamics of inflation is:

$$R_t = (1 + i_{t-1}) / (1 + \pi_t). \quad (20)$$

2.4. Market clearing. Given the measures of workers and firm owners (1 and ν , respectively) and the market and home production of final goods, the market-clearing conditions for bonds and final goods are given by $\int_{[0,1]} a_{i,t} di + \nu a_t^F = 0$ and $\int_{[0,1]} c_{i,t} di + \nu c_t^F + \eta v_t = y_t + (1 - n_t) \delta_t + \varpi$, respectively. The supply of intermediate goods is $z_t n_t$, while from (5) the demand for intermediate goods is $\int_{[0,1]} y_{h,t} dh = \Delta_t y_t$. Hence, clearing of the market for

intermediate goods requires:

$$\Delta_t y_t = z_t n_t \quad (21)$$

2.5. Equilibrium: definition and characterisation. An equilibrium is a set of sequences of (i) optimal households' decisions ($\{c_t^F, a_t^F, c_t^i, a_t^i, a_t^i\}_{t=0}^\infty, i \in [0, 1]$), firms' decisions ($\{y_t, y_{h,t}, p_t^*\}_{t=0}^\infty, h \in [0, 1]$) and central bank's decisions ($\{i_t\}_{t=0}^\infty$) given prices; and (ii) aggregate variables $\{v_t, J_t, \lambda_t, f_t, s_t, n_t, \Delta_t, \varphi_t, \pi_t, \Pi_t^W, \Pi_t^I, R_t\}_{t=0}^\infty$ that solve equations (8) to (21), together with the free entry condition $\eta = \lambda_t J_t$.

Under the assumptions made so far, the model does not generate a distribution of wealth across workers, despite imperfect unemployment insurance. The reason for this is that with a zero debt limit no one is issuing the assets that the precautionary savers would be willing to purchase for self-insurance. The structure of the equilibrium here is the same as in Ravn and Sterk (2017, 2018). Intuitively, employed workers' precautionary-saving behaviour pushes down the real interest rate below households' common rate of time preference. And at that interest rate, both unemployed workers (who face a rising expected income profile) and firm owners (who face no idiosyncratic risk) would like to borrow against future income, but they cannot due to a binding debt limit. Hence the supply of assets is zero in equilibrium and all households turn out to consume their current income. This feature of the equilibrium allows the precautionary motive to be operative –as shows up in the fact that the interest rate fluctuates *below* households' rate of time preference– without the need of tracking a time-varying wealth distribution.

Two remarks about the quantitative implications of this no-trade property are in order here. First, since in equilibrium all households consume their current income, δ_t/w_t should not be interpreted as the unemployment insurance replacement ratio but, rather, as the ratio of consumption levels across employment statuses. In my calibration below I set $\delta/w = 0.9$, implying a 10% consumption loss upon unemployment, in the lower range of available estimates for the US and the euro area.¹² Therefore the zero-liquidity property will *not* translate into an unrealistically low level of consumption insurance that could overestimate the precautionary motive. Second, one may argue that it is *liquid wealth*, rather

¹²See Den Haan et al. (2018, Appendix A) for a discussion of the evidence on this parameter.

than the entire net worth, that households can use to insulate nondurables consumption from income fluctuations, and liquid wealth is very low for many households in the US (see, e.g., Challe et al., 2017). For both reasons, focusing on an equilibrium with zero liquidity may not hugely distort the response of desired savings to aggregate shocks.

The existence of the no-trade equilibrium can be established formally by spelling out the corresponding equilibrium conditions and showing that they hold in steady state. Provided that aggregate shocks have small bounded support (my maintained assumption), then these conditions will also hold in stochastic equilibrium. The first property of the equilibrium is that employed workers do not face a binding debt limit (because they wish to precautionary-save). Hence their Euler condition holds with equality:

$$\mathbb{E}_t[M_{t+1}^e R_{t+1}] = 1, \quad (22)$$

where their MRIS, incorporating both aggregate and idiosyncratic risk, and taking account of the fact that all workers consume their current income (δ_t or w_t), is given by:

$$M_{t+1}^e = \beta \frac{(1 - s_{t+1}) u'(w_{t+1}) + s_{t+1} u'(\delta_{t+1})}{u'(w_t)}. \quad (23)$$

The MRIS in equation (23) summarises an employed workers' desire to save and it is driven by two forces here: *aversion to intertemporal substitution* and the *precautionary motive*. Aversion to intertemporal substitution shows up in the fact that transitory wage fluctuations affect M_{t+1}^e : employed workers wish to save more for future consumption when the current wage is unusually high, but less when the wage is unusually low. The precautionary motive shows up in the fact that changes in unemployment risk also affect M_{t+1}^e : the greater this risk (as measured by s_{t+1}), the stronger the desire to save (since by assumption $\delta_t < w_t \forall t$, hence $u'(w_{t+1}) > u'(\delta_{t+1})$). Hence, by equation (22), a declining wage profile or an increase in unemployment risk both exert a downward pressure on the equilibrium real interest rate R_{t+1} . Holding the policy rate i_t constant, a fall in R_{t+1} is brought about by deflationary pressures in the current period associated with a rise in expected inflation.

The second feature of the equilibrium is that unemployed workers face a binding debt

limit, i.e., their Euler condition holds with strict inequality:

$$\mathbb{E}_t[M_{t+1}^u R_{t+1}] < 1, \quad (24)$$

where

$$M_{t+1}^u = \beta \frac{(1 - f_{t+1}) u'(\delta_{t+1}) + f_{t+1} u'(w_{t+1})}{u'(\delta_t)}.$$

Conditions (22) and (24) can jointly hold because employed workers face a decreasing expected consumption profile –due to the risk of losing one’s job– while unemployed workers face a rising expected consumption profile –due to the possibility of finding one. Hence current marginal utility is low relative to expected marginal utility for the former, while the opposite is true for the latter.

The third feature of the equilibrium is that firm owners also face a binding debt limit, i.e.,

$$\mathbb{E}_t[M_{t+1}^F R_{t+1}] < 1. \quad (25)$$

Conditions (22) and (25) are mutually consistent because employed workers’ precautionary motive take the gross real interest rate below $1/\beta$, while firm owners face no idiosyncratic income shocks and hence have no reason to precautionary-save. Thus, instead of accepting a low return on their savings, they turn (frustrated) borrowers and consume their current income in every period. From equations (10), (18), (19) and (21), the consumption of a firm owner, after all taxes and subsidies have been rebated lump-sum, is given by:

$$c_t^F = \nu^{-1}(\Pi_t^W + \Pi_t^I + \tau_t) = \nu^{-1}(n_t(z_t/\Delta_t - w_t) - \eta v_t + \varpi). \quad (26)$$

Equation (26) shows that, holding labour market conditions (n_t, v_t, w_t) (hence workers’ welfare) fixed, price dispersion Δ_t creates a productive inefficiency that is directly borne by firm owners. By how much this inefficiency is passed through to workers ultimately depends on the cyclicity of the real wage (discussed below).

It is easy to verify that equations (22), (24) and (25) hold in steady state. From (22)–(23),

in steady state R is given by:

$$R = 1 + i = \beta^{-1} [1 - s + su'(\delta)/u'(w)]^{-1} < \beta^{-1}. \quad (27)$$

For $f \in (0, 1)$ we have $s = \rho(1 - f) > 0$ and hence (since $\delta < w$), $M^u < M^e$ and $M^F = \beta < M^e$. Thus, with $M^e R = 1$ –i.e., employed workers are not borrowing-constrained– we have $M^u, M^F < M^e$ –so that both unemployed workers and firm owners are. The same is true in stochastic equilibrium provided that aggregate shocks are sufficiently small.¹³

3. CONSTRAINED EFFICIENCY

The economy is potentially plagued by four distortions: monopolistic competition in the wholesale sector, asymmetric wholesale prices due to nominal rigidities, congestion externalities in the labour market, and imperfect insurance against unemployment risk. In what follows I characterise the constrained-efficient allocation of the economy and derive the values of steady-state inflation (π) and the tax instruments (τ^W, T, τ^I) that decentralise this allocation in steady state.

3.1. Social welfare function. Since in equilibrium all households consume their current income in every period, the ex ante intertemporal utilities of employed workers, unemployed workers and firm owners are given by, respectively:

$$V_t^e = u(w_t) + \beta \mathbb{E}_t[(1 - s_{t+1}) V_{t+1}^e + s_{t+1} V_{t+1}^u], \quad (28)$$

$$V_t^u = u(\delta_t) + \beta \mathbb{E}_t[f_{t+1} V_{t+1}^e + (1 - f_{t+1}) V_{t+1}^u], \quad (29)$$

and

$$V_t^F = \tilde{u}(c_t^F) + \beta \mathbb{E}_t[V_{t+1}^F]. \quad (30)$$

The social welfare function W_t aggregates the intertemporal utilities of all the households, potentially assigning different welfare weights to households that are ex ante heterogenous

¹³I am focusing here on the unique equilibrium that is robust to the introduction of an arbitrarily small amount of liquidity (e.g., through a marginal relaxation of the zero debt limit). See Werning (2015, Section 3.1) for further discussion of equilibrium uniqueness and selection in zero-liquidity, incomplete-market economies.

(i.e., workers versus firm owners). Normalising the welfare weight of workers to 1 and letting $\Lambda \geq 0$ denote the relative welfare weight of firm owners, the social welfare function is:

$$W_t = n_t V_t^e + (1 - n_t) V_t^u + \Lambda \nu V_t^F.$$

Using (17), (26) and (28)–(30) and rearranging, W_t can be written recursively as follows:

$$W_t = U_t + \beta \mathbb{E}_t[W_{t+1}], \quad (31)$$

where the flow payoff U_t is given by:

$$U_t = \underbrace{n_t u(w_t) + (1 - n_t) u(\delta_t)}_{\text{workers}} + \underbrace{\Lambda \nu \tilde{u}([\varpi + n_t(z_t/\Delta_t - w_t) - \eta v_t]/\nu)}_{\text{firm owners}}. \quad (32)$$

3.2. Constrained-efficient allocation. The constrained-efficient allocation is the sequence $\{\Delta_t, w_t, n_t, v_t\}_{t=0}^{\infty}$ that maximises W_t in (31)–(32), given (i) the initial conditions (n_{-1}, Δ_{-1}) , (ii) the law of motion for Δ_t (equation (9)) and (iii) the economy-wide relationship between employment and vacancies $n_t = (1 - \rho)n_{t-1} + (1 - (1 - \rho)n_{t-1})^\gamma v_t^{1-\gamma}$, which can equivalently be written as follows:

$$v_t = \left[\frac{n_t - (1 - \rho)n_{t-1}}{(1 - (1 - \rho)n_{t-1})^\gamma} \right]^{\frac{1}{1-\gamma}}, \quad (33)$$

and then substituted into (32). Equation (33) makes clear that, at any level of employment inherited from the previous period (i.e., $(1 - \rho)n_{t-1}$), raising current employment n_t can only be achieved by raising vacancies and hence the total hiring cost borne by firm owners. On the other hand, inherited employment $(1 - \rho)n_{t-1}$ is a state variable, which affects the amount of vacancies needed to reach a particular value of n_t in two ways. First, high past employment reduces the need for new vacancies (the numerator); and second, it reduces the size of the unemployment pool, which makes hiring more difficult and raises the need for new vacancies.

Formally, the constrained-efficient allocation is the solution to

$$W_t(n_{t-1}, \Delta_{t-1}, z_t) = \max_{\tilde{p}_t, w_t, n_t \geq 0} \{U_t + \beta \mathbb{E}_t[W_{t+1}(n_t, \Delta_t, z_{t+1})]\}, \quad (34)$$

subject to (8), (9) and (33). From equations (8)–(9), it is clear that $\tilde{p}_t = 1$ for all t is optimal: starting from $\Delta_{-1} = 1$, this sequence ensures that $(\pi_t, \Delta_t) = (0, 1)$ for all t , which maximises U_t in (32) in every period. Hence the constrained-efficient allocation has zero inflation and symmetric wholesale prices at all times. Given this and (32), the value of w_t that maximises W_t satisfies:

$$u'(w_t^*) = \Lambda \tilde{u}'(\nu^{-1}[n_t^*(z_t - w_t^*) - \eta v_t^* + \varpi]), \quad (35)$$

where starred variables denote their values in the constrained-efficient allocation.

The latter condition states that the efficient wage is that which equates the (weighted) marginal utilities of employed workers and firm owners. This condition determines how the burden of aggregate shocks is efficiently shared between workers and firm owners over the business cycle. In the extreme case where firm owners are risk neutral, the condition results in the constant wage $w_t^* = u'^{-1}(\Lambda)$ because firm owners are happy to fully insure risk-averse workers against wage fluctuations. Away from this limiting case efficiency requires employed workers to bear some of the burden of aggregate fluctuations through time-variations in their wage income.

Finally, the first-order and envelope conditions with respect to n_t give, respectively:

$$u(w_t^*) - u(\delta_t) + \Lambda \tilde{u}'(c_t^{F*}) \left[z_t - w_t^* - \frac{\eta}{(1-\gamma)\lambda_t^*} \right] + \beta \mathbb{E}_t \left[\frac{\partial W_{t+1}(n_t, 1, z_{t+1})}{\partial n_t} \right] = 0$$

and

$$\frac{\partial W_t(n_{t-1}, 1, z_t)}{\partial n_{t-1}} = \Lambda \tilde{u}'(c_t^{F*}) \eta \frac{\partial v_t}{\partial n_{t-1}} = \frac{\Lambda \tilde{u}'(c_t^{F*}) \eta (1-\rho)(1-\gamma f_t^*)}{\lambda_t^* (1-\gamma)}.$$

Combining those two expressions, and using equations (16)–(17) and the fact that $\lambda_t^{*-1} = f_t^{*\frac{\gamma}{1-\gamma}}/m^{\frac{1}{1-\gamma}}$, gives the following forward recursion for the constrained-efficient job-finding

rate:

$$f_t^{*\frac{\gamma}{1-\gamma}} = (1 - \gamma) \frac{m^{\frac{1}{1-\gamma}}}{\eta} \left[z_t - w_t^* + \frac{u(w_t^*) - u(\delta_t)}{u'(w_t^*)} \right] + (1 - \rho) \mathbb{E}_t[M_{t+1}^{F*} f_{t+1}^{*\frac{\gamma}{1-\gamma}} (1 - \gamma f_{t+1}^*)], \quad (36)$$

from which I recover the constrained-efficient employment level n_t^* using equations (16)–(17).

It is instructive to compare the dynamics of employment in the constrained-efficient allocation with that in decentralised equilibrium. Since equations (16)–(17) apply to both dynamics, this amounts to comparing the two recursions for the job-finding rate, namely (15) and (36). First, in decentralised equilibrium the flow payoff to intermediate goods firms, and hence the job-finding rate, are affected by variations in intermediate goods prices φ_t , while they are not in the constrained-efficient allocation (where the corresponding price is equal to 1 at all times). Second, even without relative price distortions the decentralised equilibrium is generically not constrained-efficient in the absence of appropriate taxes and transfers. On the one hand, imperfect insurance tends to make the decentralised job-finding rate excessively low, since firm owners do not internalise the impact of their hiring intensity on workers' idiosyncratic income risk. Formally, this shows up in the fact that $[u(w_t^*) - u(\delta_t)]/u'(w_t^*) > 0$ in equation (36), which calls for a positive wage subsidy T in equation (15). On the other hand, congestion externalities cause intermediate goods firms to crowd out each other in the labour market, which tends to generate excessive hiring. There are two sides to this crowding out: a static one operating in the current period, which shows up in the fact that $1 - \gamma < 1$ in equation (36); and an intertemporal one coming from the fact that current hiring persists over time (whenever $\rho < 1$) and hence crowds out hiring *in the next period*—which shows up in the term $1 - \gamma f_{t+1}^*$ in equation (36). Both types of crowding out call for setting $\tau^I > 0$ in equation (15).

3.3. Constrained-efficient steady state. The restriction that taxes and subsidies (τ^W, τ^I, T) are constant implies that they cannot, in general, decentralise the constrained-efficient allocation in the presence of aggregate shocks.¹⁴ However, the government can at least set the tax instruments, and the central bank trend inflation, in such a way that

¹⁴For example, equation (15) makes it clear that a suitably time-varying wage subsidy T_t would undo the impact of inefficient cost-push shocks, while a constant subsidy cannot.

(π, τ^W, τ^I, T) decentralise the constrained-efficient allocation *in steady state*. First, as shown above the constrained-efficient allocation has $(\tilde{p}_t, \pi_t, \Delta_t) = (1, 0, 1) \forall t$, while from equation (7) we have $\varphi_t = (\theta - 1) / \theta(1 - \tau^W) \forall t$ in any zero-inflation steady state. Then, comparing equations (15) and (36), we get that the steady state of the decentralised equilibrium is constrained-efficient provided that:

$$\pi = 0, \quad \tau^W = \frac{1}{\theta}, \quad T = \frac{u(w^*) - u(\delta)}{u'(w^*)} \quad \text{and} \quad \tau^I = 1 - \frac{(1 - \gamma)[1 - \beta(1 - \rho)]}{1 - \beta(1 - \rho)(1 - \gamma f^*)}, \quad (37)$$

where f^* satisfies

$$f^{*\frac{\gamma}{1-\gamma}} = \frac{(1 - \tau^I)m^{\frac{1}{1-\gamma}}}{\eta[1 - \beta(1 - \rho)]} \left[1 - w^* + \frac{u(w^*) - u(\delta)}{u'(w^*)} \right], \quad (38)$$

and w^* solves the steady state counterpart of equation (35). Intuitively, inflation creates relative price dispersion in wholesale prices and having $\pi = 0$ eliminates this distortion; the production subsidy τ^W corrects for monopolistic competition and is greater when wholesale goods are less substitutable (i.e., when wholesale firms have more market power); the hiring subsidy T corrects for the lack of insurance and is greater when the utility cost of falling into unemployment ($u(w^*) - u(\delta)$) is high; and the corporate tax rate τ^I corrects for congestion externalities in the labour market and is greater when the elasticity of total matches with respect to vacancies $(1 - \gamma)$ is low. In what follows I assume that (37) always holds, except in Section 5.2 where I investigate the robustness of my results to the introduction of steady-state distortions.

4. POSITIVE POLICY ANALYSIS

In this section I study two positive benchmarks that will shed light on the normative results of the following sections: one in which prices are flexible, and another in which the nominal interest rate is determined by a simple policy rule. These benchmarks will help understand the extent to which the traditional conclusions derived in the RANK literature about the transmission and ultimate goals of monetary policy carry over to my imperfect-insurance model.

4.1. The flexible-price model and the policy-rule model. The first benchmark I use is a *flexible-price model*, i.e., one in which the share of un-optimised prices ω is set to zero. The reason for examining this benchmark is that in the RANK model the optimal monetary policy typically consists in replicating the flexible-price allocation whenever the latter is constrained-efficient, which is implemented by tracking the natural interest rate.¹⁵ Deriving the natural interest rate implied by my model will allow me to show that this principle does extend to my imperfect-insurance model –although the natural interest rate, and implied optimal policy rate, may behave quite differently than in perfect-insurance economies.

The flexible-price equilibrium is determined as follows. First, given the paths of the exogenous state (z_t, ζ_t) and the real wage w_t , equations (3), (15), (16) and (26) (with $\Delta_t = 1$) determine the dynamics of the labour-market transition rates (f_t, s_t) . Then, equations (22)–(23) can be used to recover the corresponding (gross) natural interest rate. Assuming that a one-off aggregate shock occurs at time 0, the perfect-foresight path of the natural rate is given by:

$$R_{t+1}^n = \frac{1}{M_{t+1}^e} = \frac{\beta^{-1} u'(w_t)}{(1 - s_{t+1}) u'(w_{t+1}) + s_{t+1} u'(\delta_{t+1})}. \quad (39)$$

The second benchmark I examine in this section is a version of the baseline model wherein monetary policy is not optimized, but instead determined by an ad hoc *policy rule* for the nominal interest rate. This benchmark is informative for two reasons. First, it clarifies the monetary transmission mechanism at work in my imperfect-insurance model –an issue that is distinct from that of the *optimality* of monetary policy. Second, it will serve to illustrate the deflationary feedback loop that may arise when monetary policy is not set optimally. The policy rule I consider has the following simple form:

$$1 + i_t = R^{1-\mu_i} (1 + i_{t-1})^{\mu_i} (1 + \pi_t)^{\frac{3}{2}(1-\mu_i)} \xi_t, \quad (40)$$

where μ_i parameterises the degree of interest-rate inertia, $2/3$ is the elasticity of the policy

¹⁵See Clarida et al. (1999), Woodford (2003) and Gali (2008) for the derivation of this principle in the context of the basic RANK model, and Thomas (2008) and Blanchard and Gali (2010) for its extension to New Keynesian models with labor-market frictions. When the flexible-price equilibrium is not constrained-efficient the optimal policy rate in general departs from the natural rate at least in the short run, and this will also be true in my model.

rate to inflation and ξ_t is an i.i.d. monetary policy shock with mean 1 and small bounded support.¹⁶

4.2. Wage setting. In Section 2 I characterised the decentralised equilibrium under the minimal requirement that all worker-firm matches be bilaterally efficient. I now further specify the wage-setting process, using the results of Section 3 to make sure that the decentralised wage is efficient in steady state.

I assume that the real wage is determined by (generalised) Nash bargaining, possibly augmented with wage inertia.¹⁷ Under this wage-setting mechanism, the (notional) Nash wage is given by $w_t^N = \arg \max (S_t^W)^{1-\alpha} J_t^\alpha$, $\alpha \in (0, 1) \forall t$, where S_t^W and J_t are the values of the match to the worker and the firm. J_t is given by equation (13), while $S_t^W = V_t^e - V_t^u$, where V_t^e and V_t^u are given by equations (28)–(29). Note that S_t^W can be written recursively as:

$$S_t^W = u(w_t) - u(\delta_t) + \beta \mathbb{E}_t[(1 - f_{t+1} - s_{t+1}) S_{t+1}^W], \quad (41)$$

The first-order condition associated with the bargaining problem gives:

$$(1 - \alpha) J_t = \alpha S_t^W / u'(w_t). \quad (42)$$

The requirement that the steady state be undistorted (my maintained assumption) uniquely pins down the bargaining weight α . First, observe that the steady state values of J_t and S_t^W are given by

$$J = \left[\frac{1 - \tau^I}{1 - (1 - \rho)\beta} \right] \left[1 - w + \frac{u(w) - u(\delta)}{u'(w)} \right] \quad \text{and} \quad S^W = \frac{u(w) - u(\delta)}{1 - \beta(1 - s - f)}.$$

Then, setting $w = w^*$ in those expressions gives the values of J and S^W in the constrained-efficient allocation without aggregate shocks. It must thus be that $\alpha = [1 + S^W / Ju'(w^*)]^{-1}$ in equation (42) for the decentralised wage to be efficient in steady state.

¹⁶For the reasons discussed by Acharya and Dogra (2018), Bilbiie (2019) and Ravn and Sterk (2018), under countercyclical idiosyncratic risk a sufficient condition tighter than the Taylor principle is required for the policy rule to ensure the local determinacy of the equilibrium. This condition is satisfied under my baseline calibration.

¹⁷See, e.g., Krause and Lubik (2007), or Leduc and Liu (2016), for similar specifications.

As is well known at least since Shimer (2005), a typical feature of calibrated SaM models with Nash bargaining is to generate too strong a response of the real wage –and thus too small a response of unemployment– to labour productivity shocks. Moreover, in the present model the joint cyclicalities of wages and unemployment determines the relative strengths of the two forces that ultimately drive fluctuations in desired savings: the cyclicalities of wages determines changes in desired savings due to aversion to intertemporal substitution, while the implied cyclicalities of unemployment determines changes in desired savings due to the precautionary motive. To generate a plausible joint cyclicalities of wages and employment I assume that the notional Nash wage (w_t^N) is weighted against the long-run wage (w^*) in determining the actual wage w_t :

$$w_t = (w_t^N)^{1-\phi} (w^*)^\phi, \quad (43)$$

where $\phi \in [0, 1]$ is the degree of wage inertia.¹⁸

Discussion. Note that while the Nash-bargained wage w_t in equation (43) and the constrained-efficient wage w_t^* in equation (35) are the same in steady state (by construction), they generically differ over the business cycle. The constrained-efficient wage w_t^* cannot be decentralised through Nash bargaining here (whatever the values of α and ϕ) because the real wage has a redistributive role –between employed workers and firm owners–, in addition to determining firms’ incentives to hire. As a consequence, the Hosios (1990) criterion that allows decentralising the constrained-efficient wage in Representative-Agent economies does not apply here. However, I verify in Section 5.2 below that the Nash wage with inertia in equation (43) and the constrained-efficient wage in equation (35) are almost indistinguishable in a calibrated version of the model where both wages are parameterised to generate the

¹⁸Formally, the assumption that aggregate shocks have small bounded support implies that the real wage stays inside the bargaining set for any degree of wage inertia. Since this may not hold for large shocks, I check this condition numerically when feeding the simulated model with actual aggregate shocks. For my baseline specification (imperfect insurance and wage inertia), I find that it would take implausibly large aggregate shocks –causing output to fall at least 25% below steady state after a productivity shock, and at least 12% below steady state after a cost-push shock– for the real wage to exit the bargaining set. Relatedly, simulated aggregate shocks may in principle cause inequality (25) to be temporarily violated. However, under the baseline calibration this only occurs for productivity shock causing output to *rise* by at least 17% on impact and for cost-push shocks causing output to rise by at least 8% on impact.

same degree of flexibility.¹⁹ As a consequence, the inefficiency induced by the Nash wage with inertia is small and has almost no impact either on the optimal policy or on the implied aggregate dynamics.

4.3. Calibration. I solve the two benchmarks described above numerically and calibrate the model as follows (see Table 2 for a summary). A time period is a quarter. The cross-partial elasticity of substitution θ is set to 6, which generates a mean markup rate of 20% for wholesale firms. The fraction of unchanged wholesale goods prices ω is set to 0.75, so that the mean duration of wholesale prices is a year. Regarding labour market variables, I first set γ to $2/3$, very close to the values estimated by Shimer (2005) and Monacelli et al. (2015). I then have four parameters (η , w , m and ρ) for four targets (f , s , λ and η/w). Quarterly series for f_t and s_t were computed in Challe et al. (2017) by time-aggregating monthly series constructed as in Shimer (2005); their averages are very close to 80% and 5%, respectively. The targets for λ and η/w are, respectively, 70% (see, e.g., Den Haan et al., 2000; Walsh, 2005; Monacelli et al. 2015) and 4.5% (Hagedorn and Manovskii, 2008).

A key parameter in the model is workers' home production δ_t , which determines the extent of consumption insurance and hence the strength of the precautionary motive. There are two important dimensions to take into account in the calibration of δ_t , namely its *mean* (steady-state) level and its *cyclical*ity. Regarding the first dimension, one possibility would be to parameterise δ/w to match the UI replacement ratio. However, this would underestimate the amount of consumption insurance that households effectively enjoy, notably by ignoring self-insurance as well as other forms of direct, but unobserved or mismeasured, insurance. Following this concern I broadly interpret $(w - \delta)/w$ as the average proportional consumption loss upon unemployment and give it the (conservative) value of 10% (see den Haan et al., 2018, Appendix A, for an extensive discussion of this parameter).

Aside from its mean, the cyclical

ity of δ_t is also important, for the reason stressed by Werning (2015) that under incomplete markets the shape of the relevant Euler condition –and thus the impact of monetary policy– depends on the cyclicality of idiosyncratic income

¹⁹Under the Nash wage with inertia, wage flexibility is parameterised by ϕ , while under the constrained-efficient wage it is determined by the relative curvature of workers' and firm owners' utility functions (see equation (35)).

risk. In my model the relevant Euler condition is that of employed workers, and the income risk they are facing may vary for two reasons, namely (i) the risk of falling into unemployment (s_{t+1}) and (ii) the proportional income loss upon unemployment (determined by δ_t/w_t). As stressed earlier, I entirely focus on the first source of cyclicity in this paper; I thus assume that δ_t varies in such a way that δ_t/w_t stays constant over time, at the value $\delta/w = 0.9$ (the results are almost unchanged if I instead let δ_t/w_t vary such that δ_t stay constant). Finally, I interpret $vc_t^F = D_t - \tau_t + \varpi$ as aggregate capital income and accordingly set ϖ to 1/2; this generates a labour share of 65%.

Preferences are as follows. I restrict my attention to the following utility functions:

$$u(c) = \ln c \quad \text{and} \quad \tilde{u}(c) = (c^{1-\tilde{\sigma}} - 1)/(1 - \tilde{\sigma}), \quad \text{with } \tilde{\sigma} \geq 0. \quad (44)$$

As discussed above, I calibrate the model in such a way that the decentralised real wage of the flexible-price model (given by equation (43) and parameterised by ϕ) and the constrained-efficient real wage (given by equation (35) and parameterised by $\tilde{\sigma}$) not only share the same steady-state value (w^*) but also the same *cyclicity*.²⁰ More specifically, I parameterise ϕ and $\tilde{\sigma}$ so that both wages match the elasticity of the real wage to labour productivity, i.e. $\text{dln } w_t/\text{dln } z_t$. Setting $\phi = 0.948$ generates an elasticity of 1/3 for the decentralised wage of the flexible-price model, in the ballpark of available estimates –see e.g. Blanchard and Galí (2010); Hagedorn and Manovskii (2008); Den Haan et al. (2017). I reach the same elasticity of the constrained-efficient wage to productivity shocks by setting $\tilde{\sigma} = 0.283$.

Given household preferences, the replacement ratio δ/w and the transition rates in the labour market (f, s), equation (23) determines the value of the subjective discount factor β consistent with a given interest rate. Following McKay et al. (2016), β is set such that the annualised real interest rate $(1+i)^4 - 1 \simeq 4i$ be equal to 2%.

The steady-state real wage w is determined as follows. Given (44) and targets for $f, \delta/w$

²⁰I explore several departures from this baseline specification in Section 5.2.

and η/w , the value of w is recovered as the unique solution to equation (38):

$$w = \{1 + \ln(\delta/w) + (1 - \gamma)^{-1} m^{-\frac{1}{1-\gamma}} (\eta/w) [1 - \beta(1 - \rho)(1 - \gamma f)] f^{\frac{\gamma}{1-\gamma}}\}^{-1}. \quad (45)$$

Then, given w , the steady-state value of n and v are computed using equations (17) and (33)).

Table 1. Calibration.

<i>Parameters</i>				<i>Targets</i>		
Symb	Description	Value	P.I.L.	Symb	Description	Value
β	Discount factor	0.989	0.995	$4i$	Annual interest rate	2%
θ	Elasticity of subst.	6.000		$\frac{1}{\theta-1}$	Markup rate	20%
ω	Share of constant prices	0.750		$\frac{1}{1-\omega}$	Mean price duration	1 y.
η	Vacancy cost	0.044	0.040	η/w	Vacancy cost (% of wage)	4.5%
w	Real wage (Eq. (45))	0.979	0.893	f	Job-finding rate	80%
m	matching efficiency	0.765		λ	Vacancy-filling rate	70%
ρ	Job-destruction rate	0.250		s	Job-loss rate	5%
δ	Workers' home prod.	0.882	0.888	δ/w	Cons. loss upon employment	90%
ϖ	Firm owners' home prod.	1/2		$\frac{nw}{vc^F + nw}$	Labour share	65%
ϕ	Wage inertia	0.948		$\frac{d \ln w}{d \ln z}$	Wage cyclicalilty	1/3
$\tilde{\sigma}$	Firm owners' risk aversion	0.283		(same)	(same)	(same)

In what follows I often compare the baseline imperfect-insurance model to a counterfactual *perfect-insurance limit*, wherein δ/w is close to 1. In constructing this benchmark I adjust the deep parameters of the model to keep matching all the steady-state targets in Table 1 (that is, I interpret the same observed steady-state moments in the right-hand side of Table 1 as having been generated by the perfect-insurance limit of the model, rather than by the imperfect-insurance baseline). This requires adjusting β (to keep matching the interest rate, given no precautionary motive), w (since δ/w in equation (45) has changed), and thereby η (to keep matching η/w , given the new value of w). Those changes are indicated

in the fourth column of Table 1 (“P.I.L.”). In theory, given small aggregate shocks (my maintained assumption), any value of $\delta/w < 1$ is consistent with inequality (25) holding at all times. However, this inequality may not always hold in the simulated model, which relies on “large” shocks. It does hold, however, for all variants of the perfect-insurance limit when $\delta/w = 0.993$, which is the value I set (the implied dynamics are almost the same as with values of δ/w much closer to 1.)

4.4. Impulse responses.

Flexible-price model. Figure 1 shows the responses of the natural rate, the real wage and employment to aggregate shocks under flexible prices. The productivity shock is a 1% proportional fall in z_t while the cost push shock a level increase in ζ_t of size equal to 1% of the baseline steady-state wage w , and both shocks have persistence 0.95. My baseline specification is the imperfect-insurance model with an inertial Nash wage (see Table 1). This baseline is compared to three benchmarks, namely (i) the perfect-insurance limit of the imperfect-insurance model, (ii) the imperfect-insurance model with the basic (non-inertial) Nash wage, and (iii) the imperfect-insurance model with a constant wage (i.e., $\phi = 1$).

To understand those responses, recall the two determinants of employed workers’ consumption and saving decisions discussed in Section 2.5. The first determinant is *aversion to intertemporal substitution*: an employed worker contemplating a rising wage profile (conditional on remaining employed) is willing to save less in order to consume more in the present –and the other way around for a worker contemplating a falling wage profile. The second determinant of employed workers’ consumption is the *precautionary motive*: an employed worker who expects to lose his or her job with greater probability in the near future tends to consume less in the present. The competition between the two motives for saving is reflected in the natural interest rate. Linearising equation (23) (with $u(c) = \ln c$ and $\delta_t/w_t = \delta/w$) gives

$$\hat{R}_{t+1}^n \simeq \underbrace{\hat{w}_{t+1} - \hat{w}_t}_{\text{aversion to intertemporal substitution}} - \underbrace{\Psi \hat{s}_{t+1}}_{\text{precautionary motive}}, \quad (46)$$

where \hat{s}_t is the level deviation of the job-loss rate from steady state, \hat{w}_t is the proportional

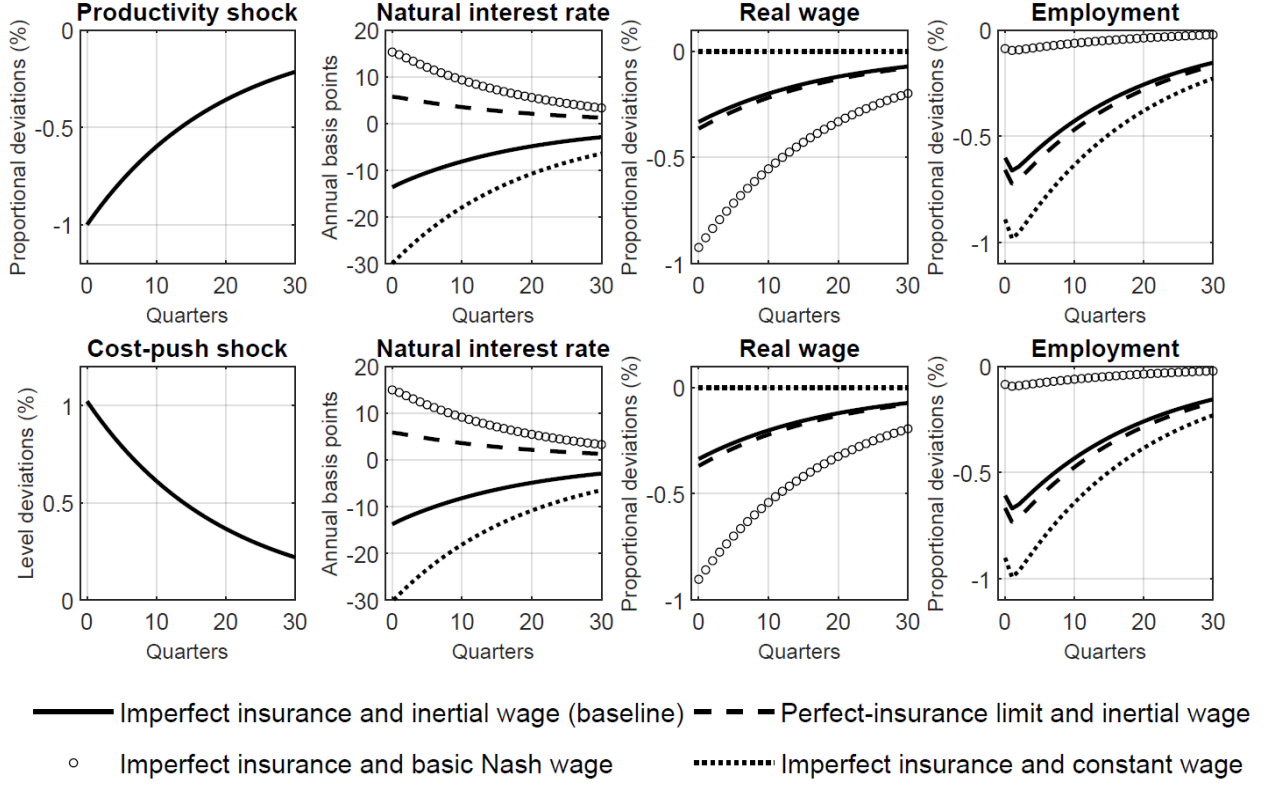


Figure 1: Responses to aggregate shocks under flexible prices.

deviations of the real wage from steady state, and

$$\Psi = [s + ((\delta/w)^{-1} - 1)^{-1}]^{-1} \in (0, s).$$

We observe that $\partial\Psi/\partial(\delta/w) < 0$ while $\lim_{\delta/w \rightarrow 1} \Psi = 0$: as the mean level of consumption insurance rises, desired precautionary savings become less and less responsive to changes in unemployment risk, up to the point of becoming fully unresponsive in the perfect-insurance limit. In the latter case the natural rate is entirely driven by aversion to intertemporal substitution; as a consequence, the natural rate necessarily *rises* after any aggregate shock that triggers a transitory recession, since the latter is associated with an expected wage recovery. In contrast, in the constant-wage specification it is $\hat{w}_{t+1} - \hat{w}_t$ that vanishes (since $w_t = w$ at all times). The natural rate is then entirely determined by the precautionary motive and thus it necessarily *falls* after a contractionary aggregate supply shock. The baseline imperfect-insurance model lies between these two extreme cases, because the two

savings motive are operative and pull the natural rate in opposite directions. This rate falls under my calibration, but less than in the constant-wage specification.

Figure 1 also shows the responses of the flexible-price economy to aggregate shocks under imperfect insurance when there is no wage inertia (i.e., $\phi = 0$). Unsurprisingly, in this scenario the wages fall considerably (about three times more than targeted), and employment barely falls as a result. It follows that $\hat{w}_{t+1} - \hat{w}_t$ is large and \hat{s}_{t+1} is small: aversion to intertemporal substitution largely dominates the precautionary motive in the determination of desired savings, leading the natural interest rate to rise even more than in the perfect-insurance limit with an inertial wage.

Last, one observes from Figure 1 that under flexible prices the dynamic responses to contractionary productivity and cost-push shocks are similar. The reason for this is as follows. First, under flexible prices job creation is governed by equation (13) with $\varphi_t = 1$ at all times; this implies that the *direct* effects (i.e., holding w_t constant) of the two shocks on job creation are the same (up to their magnitudes). Second, from equations (13) (with $\varphi_t = 1$) and (41)–(42), the difference $z_t - \zeta_t$ is also the only determinant of w_t ; this implies that the *indirect* effect (i.e., taking into account the endogenous response of w_t) of the two aggregate shocks on job creation are also the same. By way of consequence, their impact on employment and unemployment risk, and hence on employed workers' desired savings and the equilibrium (natural) interest rate, are the same. This exact symmetry is specific to the flexible-price model; it is broken under sticky prices because φ_t is no longer constant, at least after a cost-push shock.²¹

Policy-rule model. Figure 2 shows the responses of the nominal interest rate, inflation and employment in the sticky-price economy and under the interest-rate rule (40). The interest-rate inertia parameter is set to $\mu_i = 0.85$ (see, e.g., Challe and Giannitsarou, 2014, for a discussion of this parameter), and the size of the shock is set so that the equilibrium response of the policy rate be of 25 annualised basis points in the imperfect-insurance model. In this economy, only employed workers respond to changes in the real interest rate (since the other households face a binding borrowing limit), and the policy shock induces them

²¹The IRFs are different but close to each other in the policy-rule model, because the policy rule itself induces correlated changes in φ_t across the two shocks.

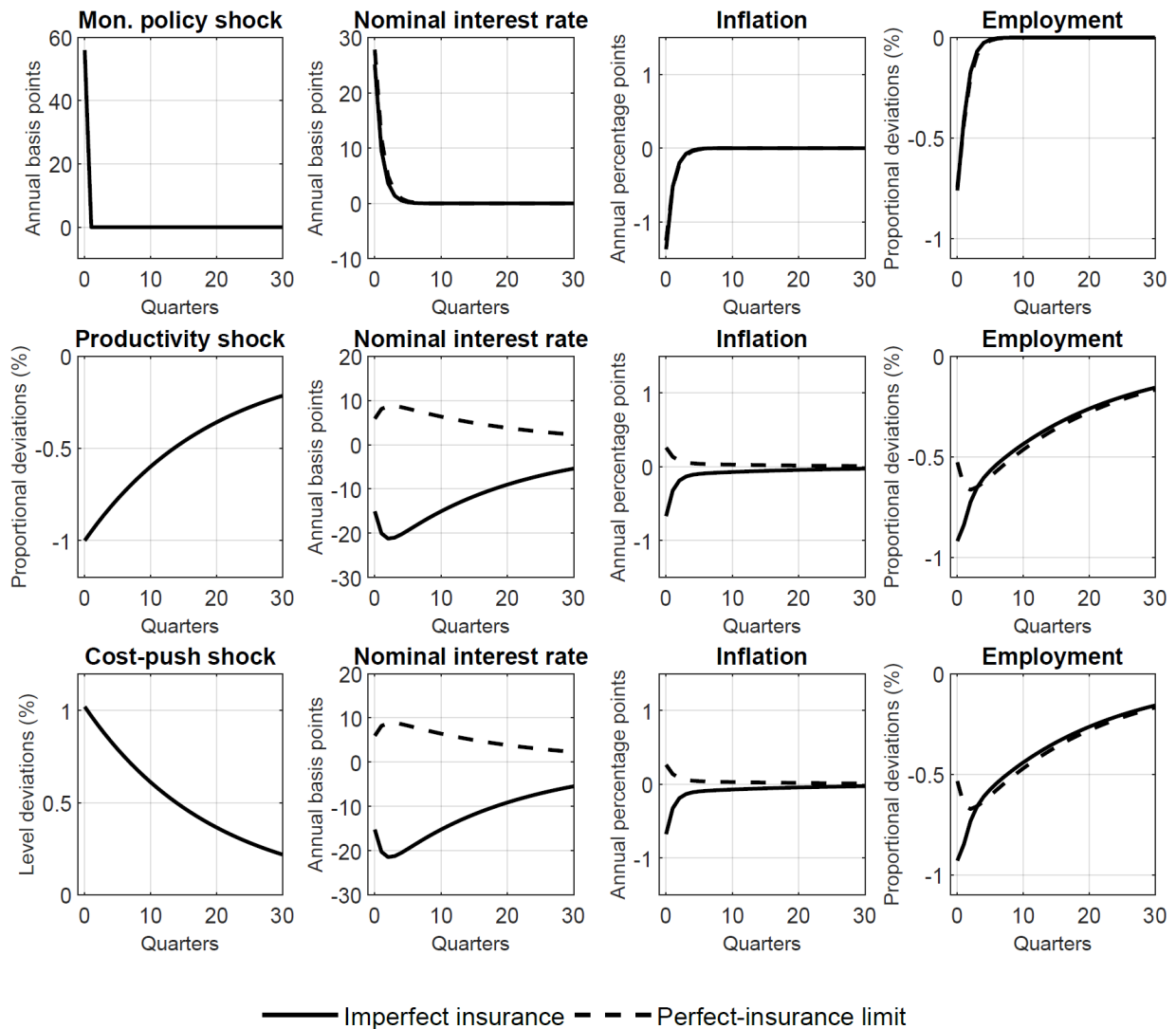


Figure 2: Responses to aggregate shocks under the simple policy rule.

to defer consumption. Interest-rate inertia implies that a contractionary monetary policy shock ($\xi_t > 0$) persistently lowers their consumption demand, which ultimately pulls down output, employment and inflation. Under imperfect insurance, the resulting increase in unemployment risk also fosters employed workers’ precautionary response, an effect that tends to magnify the direct impact of the policy shock. The reason why the *equilibrium* responses of inflation and employment in the imperfect- and perfect-insurance economies are close to each other is that the policy rate contains both a direct, exogenous component (the policy shock) and an indirect, endogenous component (the response to inflation). While

imperfect insurance tends to magnify the direct effect of the policy shock on aggregates, the endogenous component leans against the resulting deflationary pressures by providing more accommodation. As a result, the policy rate rises less under imperfect insurance than under perfect insurance, and this extra accommodation aligns the equilibrium responses of inflation and employment under perfect and imperfect insurance.

The responses of the policy rate, inflation and employment to contractionary productivity and cost-push shocks under the simple policy rule (bottom eight panels of Figure 2) also inform us about the consequences of an inefficient monetary policy. Under perfect insurance, we recover the usual result that after a contractionary aggregate supply shock of either type, inflation transitorily rises while employment falls. Under imperfect insurance, however, the deflationary spiral between uninsured unemployment risk and aggregate demand is set in motion. As a result inflation *falls* instead of rising, while employment contracts substantially more than in the flexible-price equilibrium –as can be verified by comparing the employment responses to productivity and cost-push shocks in Figures 1 and 2.²²

5. OPTIMAL POLICY ANALYSIS

I now derive my main results about optimal monetary policy under uninsured unemployment risk. In so doing, I proceed in two steps. I first solve in Section 5.1 the optimal policy problem under a set of parametric restrictions that delivers (i) a simple linear-quadratic approximation to the optimal policy problem and (ii) a closed-form expression for the optimal policy rate. Both features help develop intuition about the role of imperfect insurance in affecting optimal policy. In Section 5.2 I solve the full nonlinear Ramsey problem numerically, away from the parametric restrictions of Section 5.1.

5.1. Optimal policy with full worker reallocation. When aggregate shocks have small magnitude and the steady state is undistorted, the true optimal policy problem can be approximated by a simple (and easily interpreted) linear-quadratic (LQ) problem (Woodford, 2010). To arrive at an analytical formula for the optimal policy rate I also impose two additional restrictions here. The first assumption is that ρ is equal to 1, so that all employed

²²In contrast, under perfect insurance employment moves less under sticky prices than under flexible prices. This is because, under the policy rule, the real interest rate responds less to aggregate shocks than does the natural rate, leading to an (inefficient) buffering of aggregate supply shocks.

workers are reallocated (either towards other firms or towards unemployment) in every period; by way of consequence employment ceases to be a state variable, both in the social welfare function and in the value function for intermediate goods firms. In Section 5.2 the parameter ρ is instead calibrated to match the size and cyclicity of empirical worker flows in the U.S. economy.

The second assumption made here is that the real wage is constant: the inertia parameter ϕ in equation (43) is set to 1, and firm owners are assumed to be risk neutral (i.e., $\tilde{u}(c) = c$) for this (a-)cyclicity of the wage to be efficient (by equation (35)). A constant wage implies that aversion to intertemporal substitution plays no role in driving employed workers' desired savings: only the precautionary motive is at work (see equation (46)). In Section 5.2 I instead calibrate ϕ and $\tilde{u}(c)$ as in Table 1 and I examine how aversion to intertemporal substitution and the precautionary motive jointly determine workers' savings. For expositional clarity, but with no loss of generality, in this section I also normalise the matching efficiency parameter m to 1 and I set $u(c) = \ln c$.

Constrained-efficient, natural, and actual employment levels. With $\tilde{u}(c) = c$ and $\phi = \rho = m = 1$, equations (12), (17), (35) and (37) imply that $w_t = w^* = u'^{-1}(\Lambda)$ (i.e., the real wage is at its constrained-efficient, constant level w^* at all times), $\tau^I = \gamma$ (i.e., the corporate tax rate equals the elasticity of matches w.r.t. to the size of the unemployment pool) and

$$f_t = n_t = \lambda_t v_t = v_t^{1-\gamma}. \quad (47)$$

Equation (36) then gives the following expression for the *constrained-efficient* level of employment n_t^* :

$$n_t^* = \left[(1 - \gamma) \eta^{-1} (z_t - w^* (1 + \ln(\delta/w))) \right]^{\frac{1-\gamma}{\gamma}}, \quad (48)$$

where as in Section 4 the replacement ratio δ/w is treated as a exogenous composite parameter. On the other hand, from equations (15) and (37) the *actual* level of employment n_t is given by:

$$n_t = \left[(1 - \gamma) \eta^{-1} (\varphi_t z_t - \zeta_t - w^* (1 + \ln(\delta/w))) \right]^{\frac{1-\gamma}{\gamma}}. \quad (49)$$

Finally, the *natural* level of employment –i.e., that which would prevail under flexible

prices— is the same as n_t in equation (49) except $\varphi_t = 1 \forall t$ —reflecting the fact that in this scenario wholesale firms’ real marginal cost is constant across firms and across time.

In the remainder of this section I will use the linearised versions of equations (48) and (49). Using hatted variables to denote first-order level-deviations from the steady state, we have:

$$\hat{n}_t^* = \Phi \hat{z}_t \quad \text{and} \quad \hat{n}_t = \hat{n}_t^* + \Phi(\hat{\varphi}_t - \hat{\zeta}_t), \quad (50)$$

where $\Phi = (1 - \gamma)^2 n^* \frac{1-2\gamma}{1-\gamma} / \gamma \eta$. Looking at (50) makes it clear that the central bank cannot replicate the constrained-efficient allocation after a cost-push shock $\hat{\zeta}_t$, because it cannot simultaneously close the employment gap $\hat{n}_t - \hat{n}_t^* = \Phi(\hat{\varphi}_t - \hat{\zeta}_t)$ and stabilise intermediate goods prices $\hat{\varphi}_t$.

Linear-quadratic problem. One may now derive the linear-quadratic approximation to the optimal policy problem. Appendix A shows that, to second order, maximising W_t in equation (31) is equivalent to minimising

$$L_t = \frac{1}{2} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k (\tilde{n}_{t+k}^2 + \Omega \pi_{t+k}^2), \quad (51)$$

where $\tilde{n}_t \equiv \hat{n}_t - \hat{n}_t^*$ denotes the employment gap and

$$\Omega = \theta n \Phi / \kappa \quad \text{and} \quad \kappa = (1 - \omega)(1 - \beta \omega) / \omega.$$

The constraints faced by the central bank are the bond Euler equation for employed workers (equations (22)–(23)) and the optimality conditions for firms in the wholesale (equations (7)–(9)) and intermediate goods (equation (15)) sectors. Linearising equation (16) with $\rho = 1$ gives $\hat{s}_t = -\hat{f}_t = -\hat{n}_t$. Linearising equations (22)–(23) around the zero-inflation steady state gives:

$$\Psi \mathbb{E}_t[\hat{n}_{t+1}] = \hat{i}_t - \mathbb{E}_t[\pi_{t+1}], \quad (52)$$

where we recall from equation (46) that $\Psi \in (0, s)$ determines the strength of the precautionary-saving response to changes in unemployment risk. In the perfect-insurance limit ($\delta/w \rightarrow 1$) we have $\Psi \rightarrow 0$, so the precautionary motive vanishes and labour-market risk no longer

affects the equilibrium real interest rate. As δ/w falls and Ψ increases, the precautionary motive gains strength and has a larger impact on the real interest rate.

Under the assumptions of this section, linearising equations (7)–(8) and rearranging gives the following New Keynesian Phillips curve:

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa \hat{\varphi}_t. \quad (53)$$

Note that in the present framework there are two potential sources of procyclical variations in the real marginal cost faced by intermediate goods firms, which are then passed through to φ_t (by intermediate goods firms) and ultimately to π_t (by wholesale and final goods firms): variations in the *real wage* w_t as well as in the *real marginal search cost* η/λ_t (this cost is procyclical because aggregate hiring costs are convex, given the shape of the matching function). In the present section the real wage is constant, so the cyclicity of φ_t is entirely driven by the marginal search costs, but in Section 5.2 both sources of cyclicity will be playing out.

One may now use equations (11) and (50) to express (52) and (53) in terms of the employment gap \tilde{n}_t that enters the loss function (51). This gives the two constraints, imposed by households' and firms' optimal behaviour, that the central bank faces when attempting to minimise its loss:

$$\Psi \mathbb{E}_t[\tilde{n}_{t+1}] = \hat{i}_t - \mathbb{E}_t[\pi_{t+1}] - r_t^*, \quad (54)$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + (\kappa/\Phi) \tilde{n}_t + \kappa \hat{\zeta}_t, \quad (55)$$

where r_t^* is the *efficient* real interest rate (in terms of level-deviation from steady state) that would equate actual employment \hat{n}_t with its efficient level \hat{n}_t^* :

$$r_t^* = \Psi \Phi \mu_z \hat{z}_t. \quad (56)$$

The efficient interest rate covaries with productivity because of the precautionary motive: a persistent productivity slump worsens future labour market conditions and urges workers

to save more (and all the more so that Ψ is large). To close the employment gap the central bank should close the interest-rate gap, i.e., the difference between the actual and efficient interest rates (the right hand-side of (54)). However, because the inefficiency of the employment level due to cost-push shocks persists even under flexible prices, the efficient interest rate differs from the *natural* interest rate, which (from equations (14), (50) and (52)) is given by:

$$r_t^n = r_t^* - \Psi\Phi\mu_\zeta\hat{\zeta}_t. \quad (57)$$

Just like negative productivity shocks, persistent cost-push shocks reduce future hiring, which raises unemployment risk and employed workers' precautionary response; thus the impact of the cost-push shock ($\hat{\zeta}_t$) on the natural interest rate (r_t^n) adds up to the effect of labour productivity (\hat{z}_t) working through the efficient interest rate (r_t^*).

Optimal Ramsey policy. The optimal Ramsey policy is the sequence of policy rates $\{i_{t+k}\}_{k=0}^\infty$ that minimises L_t in equation (51) subject to (54)–(55). Formally, I first minimise (51) subject to (55) to solve for the optimal target sequences $\{\tilde{n}_t, \pi_t\}_{t=0}^\infty$ after one-off productivity and cost push innovations \hat{z}_0 and $\hat{\zeta}_0$ occurring at $t = 0$. Under full worker reallocation this problem turns out to be *isomorphic* to the optimal policy problem in the textbook RANK model (see, e.g., Galí, 2008, Chap. 5): one minimises a quadratic loss function featuring inflation and the employment gap (or equivalently the output gap, since productivity is independent of policy), subject to a linear New Keynesian Phillips curve. There are some differences in the underlying mechanisms –here the loss function (51) aggregates the welfare of heterogenous households, rather than of symmetric households, and the procyclicality of the real marginal cost in (53) or (55) comes from the convexity of aggregate search costs, rather than of the disutility of labour supply–, but these differences do not change the form of the optimal policy problem. Therefore, they do not change the tradeoffs that the central bank faces.²³

Table 2, derived in Appendix B, shows the optimal targeted paths of inflation and the employment gap. Because of the isomorphy stressed above, they are the same as in the

²³This isomorphy echoes that in Bilbiie (2008) between the RANK model and the Two-Agent New Keynesian (“TANK”) model with hand-to-mouth agents.

textbook RANK model. Following a cost-push shock the central bank promises, and then implements, a durable recession so as to mitigate the impact of the shock on current inflation; for example, when $\alpha + \mu_\zeta > 1$ the responses of inflation and the employment gap to the shock are both U-shaped, hence the response of the output gap also is. In contrast, productivity shocks do not generate a policy trade-off, thereby making it possible for the central bank to simultaneously close both gaps; this implies that under the optimal policy neither inflation nor the employment gap respond to \hat{z}_0 .²⁴

Table 2. Optimal targets for inflation and the employment gap (see Appendix B).

	\tilde{n}_t	π_t
$t = 0$	$-\Upsilon\theta n\hat{\zeta}_0$	$\Upsilon\hat{\zeta}_0 > 0$
$t = 1$	$-\Upsilon\theta n(\alpha + \mu_\zeta)\hat{\zeta}_0$	$\Upsilon(\alpha + \mu_\zeta - 1)\hat{\zeta}_0$
$t \geq 2$	$-\Upsilon\theta n(\sum_{k=0}^t \alpha^k \mu_\zeta^{t-k})\zeta_0$	$\Upsilon[\mu_\zeta^t - (1 - \alpha) \sum_{k=0}^{t-1} \alpha^k \mu_\zeta^{t-k}]\zeta_0$

Note: $\Upsilon = \frac{\alpha\kappa}{1-\alpha\beta\mu_\zeta} > 0$ and $\alpha = \frac{1+\beta+\kappa\theta n/\Phi}{2\beta} [1 - (1 - 4\beta(1 + \beta + \frac{\kappa\theta n}{\Phi})^{-2})^{1/2}] \in (0, 1)$.

Next, one may use equation (54) to infer the sequence of policy rates $\{i_t\}_{t=0}^\infty$ that implements the target sequences $\{\tilde{n}_t, \pi_t\}_{t=0}^\infty$. First, from equations (54) and (56) we can write the policy rate as follows:

$$\hat{i}_t = \Psi\Phi\mu_z\hat{z}_t + \Psi\tilde{n}_{t+1} + \pi_{t+1}. \quad (58)$$

Substituting the values of \tilde{n}_{t+1} and π_{t+1} in Table 2 into (58) and rearranging gives the following path of policy rates:

$$\text{For } t = 0: \quad \hat{i}_0(\hat{z}_0, \hat{\zeta}_0) = \underbrace{\Upsilon(\alpha + \mu_\zeta - 1)\hat{\zeta}_0}_{\text{perfect-insurance response}} - \underbrace{\Psi\Upsilon\theta n(\alpha + \mu_\zeta)\hat{\zeta}_0 + \Psi\Phi\mu_z\hat{z}_0}_{\text{imperfect-insurance correction}},$$

²⁴The exact isomorphy between the two optimal policy problems is lost under *partial* worker reallocation (studied in the next section), notably because employment becomes a state variable and hence the optimal policy problem is no longer purely forward-looking. Nevertheless, the same logic applies and the paths of inflation and employment gaps are very close to their full-reallocation counterparts.

and, for $t \geq 1$:

$$i_t(\hat{z}_0, \hat{\zeta}_0) = \underbrace{\Upsilon[\mu_\zeta^t - (1 - \alpha) \sum_{k=0}^t \alpha^k \mu_\zeta^{t-k}] \hat{\zeta}_0}_{\text{perfect-insurance response}} - \underbrace{\Psi \Upsilon \theta n [\sum_{k=0}^t \alpha^k \mu_\zeta^{t-k}] \hat{\zeta}_0 + \Psi \Phi \mu_z^{t+1} \hat{z}_0}_{\text{imperfect-insurance correction}}.$$

Discussion. The optimal policy responses to productivity and cost-push shocks can be explained as follows. First, after a *productivity shocks* the policy rate i_t should perfectly track movements in the efficient interest rate r_t^* , which is itself equal (under this shock) to the natural interest rate r_t^n (see equations (56)–(57)). Because a persistent productivity-driven contraction ($\hat{z}_0 < 0$) causes the natural interest rate to fall –due to the precautionary-saving response to the rise in unemployment risk–, this leads to a persistent cut in the optimal nominal interest rate and hence an equal fall in the real interest rate –since optimal inflation stays at zero after a productivity shock, see Table 2. Without this policy accommodation employment and inflation would deviate from target *downwards*, while a suitably sized cut in the policy rate can simultaneously close the employment and inflation gaps. Crucially, the size of the cut depends on the extent of imperfect insurance (as encoded in Ψ), because the latter determines the strength of the precautionary motive and hence the size of the fall in aggregate demand that would occur without the offsetting action of the central bank.

How does this optimal response to productivity shocks under imperfect insurance compare to the optimal response under perfect insurance? Again, the optimal policy here is to track the natural rate r_t^n and this remains true *whatever the degree of insurance* (i.e., whatever the value of Ψ). In other words, the general principle, traditionally derived under perfect insurance, that the central bank should track the natural interest rate in order to replicate the flexible-price allocation whenever that allocation is constrained-efficient *directly extends to my imperfect-insurance economy*. What does differ between the baseline imperfect-insurance economy and its perfect-insurance limit is the response of the natural interest rate. We will see in Section 5.2 below that the same logic applies to the economy with partial worker reallocation and a time-varying real wage, wherein both the precautionary motive and aversion to intertemporal substitution compete in determining the response of the natural interest rate to productivity shocks.

Second, the strength of the precautionary motive affects both the size and sign of the optimal interest-rate response to *cost-push shocks*. Just as in the textbook RANK model, the optimal policy response is such that both inflation and the employment gap persistently deviate from target (inflation upwards and the employment gap downwards). However, the fall in employment triggered by the cost-push shock strengthens the precautionary motive and generates deflationary pressures in the current period; this mutes down the optimal response of the policy rate and even reverts it if the precautionary motive is sufficiently strong (i.e., if insurance is sufficiently poor).

Importantly, unlike productivity shocks, cost-push shocks generate a persistent gap between the constrained-efficient allocation and the flexible-price equilibrium –and hence between the efficient interest rate and the natural interest rate. Therefore, merely tracking the natural rate is not the optimal response to cost-push shocks; instead, the current value of the policy rate as well as promises about its future values must be set in a way that best solves the tradeoff between stabilising the two gaps in the loss function (51). Here again, one notices that the same principle that governs optimal monetary policy under *perfect* insurance directly extends to the imperfect-insurance economy. In fact, the problem of minimising (51) subject to (55), which yields the optimal “targeting rule” for $\{\tilde{n}_t, \pi_t\}_{t=0}^{\infty}$ in Table 2, is *independent of the degree of insurance* (Ψ). The extent of unemployment insurance only appears in the second step, when it comes to policy implementation via the nominal interest rate.

Finally, the fact that the optimal policy implements a targeting rule $\{\tilde{n}_t, \pi_t\}_{t=0}^{\infty}$ that is independent of the degree of insurance (Ψ) means that the optimal policy *effectively undoes the effect of imperfect insurance on the propagation of aggregate shocks*.

5.2. Optimal policy with partial worker reallocation. Having analytically identified how the precautionary motive affects optimal policy in the special case of full worker reallocation –and a constant wage–, I now study the optimal interest-rate response to aggregate shocks under partial worker reallocation (i.e., $\rho < 1$) and a time-varying wage (i.e., $\phi < 1$). The first feature makes hiring decisions intertemporal: in decentralised equilibrium firms take into account the future rents they will earn on newly hired employees (in addi-

tion to the current rent), while the constrained-efficient allocation incorporates the impact of current employment on future aggregate hiring costs (in addition to the current aggregate hiring costs). The second feature implies that employed workers' desired savings will not only be governed by the precautionary motive but also by aversion to intertemporal substitution.

Since those generalisations preclude the derivation of an analytical formula for the policy rate, I solve numerically the Ramsey problem of finding the sequence $\{i_t\}_{t=0}^{\infty}$ that maximises W_t subject to (7)–(9), (15)–(17), (21), (22)–(23), and (37), after one-off productivity and cost-push innovations occurring at $t = 0$. To be more specific, the numerical algorithm involves three steps:

- First, the optimal policy is derived by computing the first-order conditions of the Lagrangian function involving the social welfare function $W_0 = \sum_{t=0}^{\infty} \beta^t U_t$ (see equation (31)) as well as two sets of constraints, namely (i) the laws of motion of the endogenous state variables (equations (9) and (17)) and (ii) the optimality conditions of the private sector (equations (7), (15) and (22)–(23)). Under the functional forms in (44), the flow payoff U_t in the social welfare function is given by:

$$U_t = \ln w_t + (1 - n_t) \ln (\delta/w) + \frac{\tilde{\Lambda}}{1 - \tilde{\sigma}} ([\varpi + n_t (z_t/\Delta_t - w_t) - \eta v_t]^{1 - \tilde{\sigma}}),$$

where $\tilde{\Lambda} \equiv \Lambda \nu^{\tilde{\sigma}}$ is recovered using the steady-state counterpart of the efficiency condition (35) (which gives $\tilde{\Lambda} = (n(1 - w) - \eta v + \varpi)^{\tilde{\sigma}} / w$);

- Second, the resulting nonlinear dynamic system, made of the optimal-policy conditions and the constraints faced by the central bank, and having as unknowns the sequences of endogenous variables including the policy instrument (i_t) and the Lagrange multipliers on the constraints, is (i) linearised around the steady state (characterised by equations (27), (37) and (45)), and (ii) stacked in state-space form;
- Third, the VAR representation of the model's dynamics is recovered by generalised Schur decomposition (Klein, 2000).²⁵

²⁵One advantage of solving the full nonlinear Ramsey problem is that the solution handles potential steady-

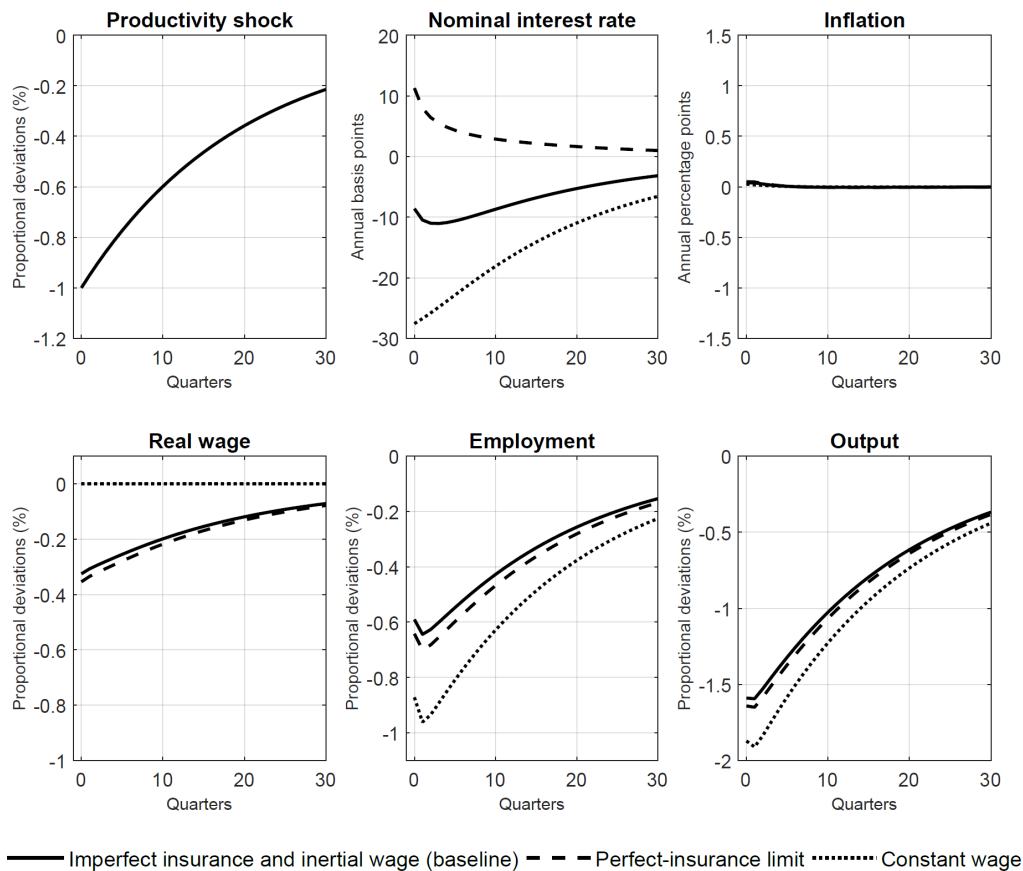


Figure 3: Responses to a contractionary productivity shock (imperfect versus perfect insurance).

The calibration of the baseline model is in Table 1. While I mostly focus on optimal monetary policy analysis around the undistorted steady state, later on I also evaluate the robustness of my baseline results with respect to steady state distortions.

Baseline results. Figures 3 and 4 show the optimal responses of the policy rate, and implied macroeconomic aggregates, after contractionary productivity and cost-push shocks. To understand those responses, recall the two determinants of employed workers' consumption demand, namely *aversion to intertemporal substitution* and the *precautionary motive*. The competition between these two effects explains the optimal policy response to productivity shocks, displayed in Figure 1. The response of the policy rate and macroeconomic aggregates in the perfect-insurance limit is essentially the same as that of the typical RANK state distortions, unlike the (naive) LQ approach (see Woodford, 2010, for a discussion). The algorithm is implemented using the DYNARE toolbox (www.dynare.org).

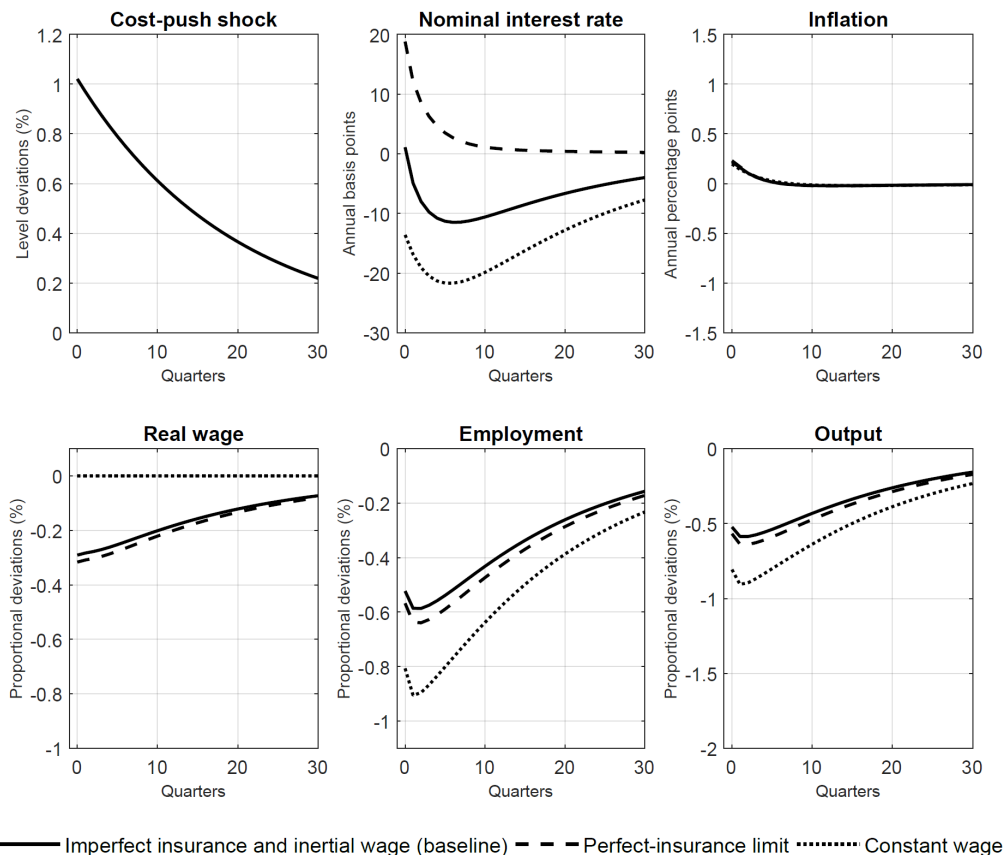


Figure 4: Responses to a contractionary cost-push shock (imperfect versus perfect insurance).

model: after a transitory fall in productivity the central bank should contract demand to align it to supply, otherwise there would be excess (i.e., inflationary) consumption demand (based on workers' high *future wages* relative to the current wage). At the other extreme, the constant-wage response is obtained by setting $\phi = 1$ (and also $\tilde{\sigma} = 0$ to make this constant wage efficient, by equation (35)). In this case the central bank should aggressively support aggregate demand, else workers' consumption would *fall* too much after a contractionary productivity shock (due to their fear of unemployment), and this would be deflationary. The optimal policy in the baseline imperfect-insurance model lies between these two extremes (the perfect-insurance limit and the constant-wage model), and under my baseline calibration it implies a persistent cut in the policy rate. Put differently, without a suitable monetary policy response the precautionary motive would dominate aversion to intertemporal substitution and aggregate demand would be too *low*, not too high. Quantitatively,

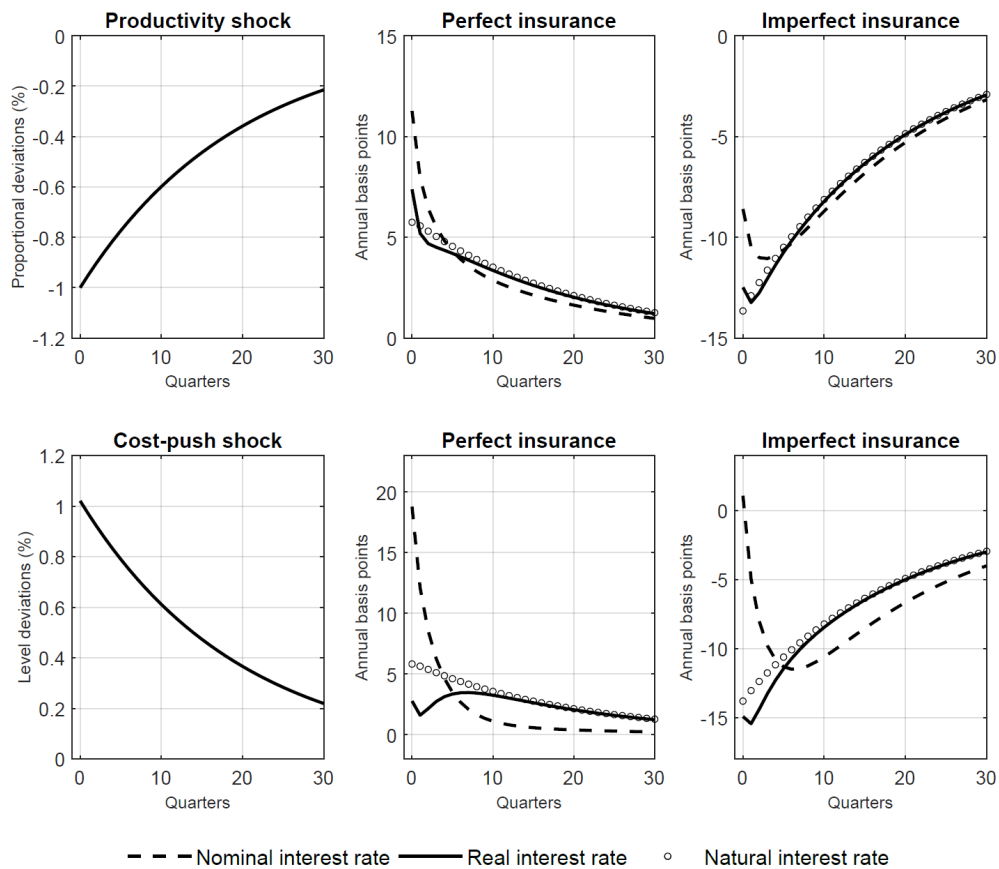


Figure 5: Nominal, real and natural interest rates compared.

beyond the sign reversal, imperfect insurance causes the impact response of the policy rate to be 20 annual basis point lower than under perfect insurance (for a normalised 1% fall in productivity), and this adjustment almost aligns the dynamics of inflation, employment and output under imperfect insurance to those under perfect insurance. Of course, one should keep in mind that the model is highly stylised and abstracts from many features that are likely to matter in practice.²⁶ Thus, while its qualitative predictions (i.e., the need for more policy accommodation, and the relative effectiveness of the optimal policy) are likely robust, its quantitative predictions (e.g., on the exact size of the required interest-rate cut) should not be taken too literally.

Finally, we can see from Figure 5 that after a productivity shock the optimal (real) inter-

²⁶For example, it abstracts from the redistributive effect of inflation on wealth, since all households have zero wealth in equilibrium (see Bhandari et al., 2018, on how redistributive concerns affect the sign and size of the optimal response of the policy rate, relative to the complete-market response)

est rate closely tracks the natural interest rate, both under perfect and imperfect insurance. Unlike in the case of full worker reallocation, optimal monetary policy cannot exactly replicate the natural rate: it implies a very small departure of inflation from zero (Figure 3) and a very small departure of the real rate from the natural rate (Figure 5). This is because, when $\rho < 1$, constant taxes and subsidies no longer exactly decentralise the constrained-efficient outcome. As a consequence, the equilibrium under sticky prices and the optimal policy does not exactly replicate the constrained-efficient outcome. However, the gap between the two (which was inexistent under full worker reallocation) remains very small here.²⁷

A related pattern emerges from the optimal response to cost-push shocks, as can be seen from Figure 4: more accommodation than under perfect insurance is required, and the optimal policy almost aligns the dynamics of the imperfect-insurance baseline with that of the perfect-insurance benchmark. Achieving this stabilisation under imperfect insurance requires an impact response of the policy rate that is about 18 basis points lower under imperfect insurance than under perfect insurance. This is again significant –though, as already stressed, one should not overinterpret the quantitative predictions of the model, given its stylised nature.

As already explained in Section 5.1, cost-push shocks take the natural interest rate away from the efficient interest rate, which in turn implies that the real rate interest rate under the optimal policy departs from the natural interest rate (most notably in the first few quarters, here). This is no different from what happens under perfect insurance –whether one considers the perfect-insurance limit of my mode (see Figure 5) or more generally the basic RANK model (see Galí, 2008, Chap. 5).

Efficient and inefficient wage-setting mechanisms. In my baseline specification, I assumed that the wage was generated by equation (43) and I calibrated the parameter ϕ to match the targeted elasticity of the wage with respect to productivity. To minimise the aggregate distortions generated by this decentralised wage, I also calibrated the relative risk aversion of firm owners ($\tilde{\sigma}$) so that the constrained-efficient wage matched the same

²⁷See Correia et al. (2008) for a general discussion of the conditions under which monetary policy can replicate the constrained-efficient allocation. In general, time-varying taxes rate are required, and when they are not available optimal monetary policy attempts to make up for them –though in a second-best manner.

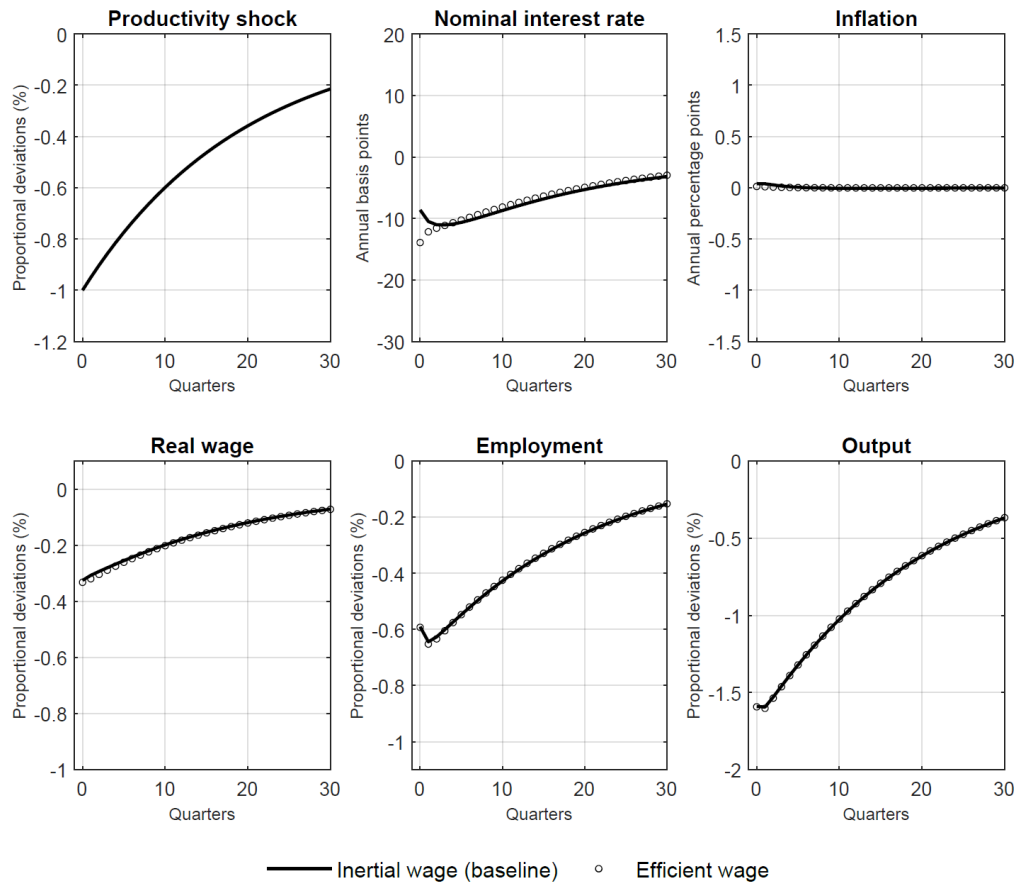


Figure 6: Responses to a contractionary productivity shock (inertial versus efficient wage).

elasticity (see Section 4.3 for details).

I now consider two departures from the baseline wage-setting mechanism. First, I study optimal policy when the real wage *is* the efficient wage characterised in Section 3, rather than the inertial Nash wage with the same elasticity with respect to productivity. By construction the former is not a source of inefficiency that the central bank might want to lean against, so any feature of optimal policy derived above that survives this scenario cannot be due an inefficient wage-setting mechanism. Figure 6 shows that the response of the policy rate to productivity shocks under the efficient wage tracks very closely that under the inertial wage, resulting in paths of inflation, the real wage, employment and output that are indistinguishable from one another. The main difference that arises between the inertial and the efficient wage pertains to the policy and aggregate responses to cost-push shocks, as shown in Figure 7. This is because the efficient wage does not respond to such shocks

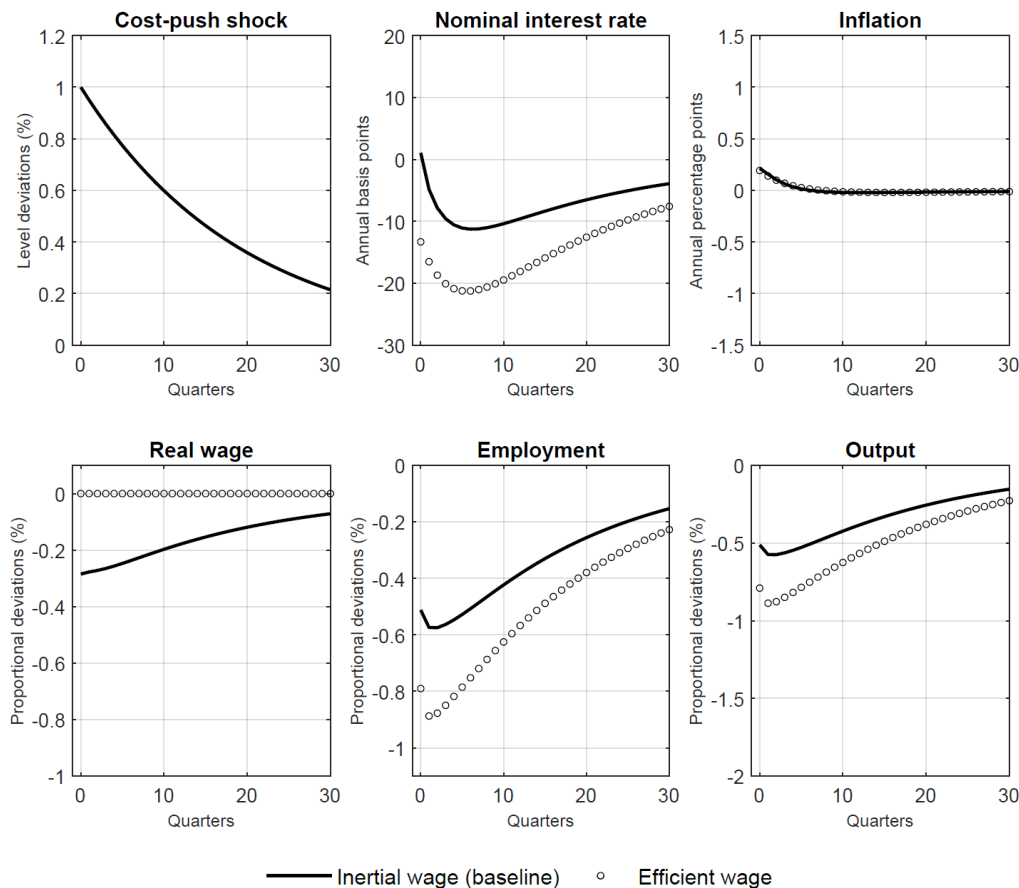


Figure 7: Responses to a contractionary cost-push shock (inertial versus efficient wage).

(see Section 3) while the Nash wage with inertia always does (provided that $\phi < 1$). But the constancy of the efficient wage can only reinforce the need for policy accommodation, since employment and output fall more than under a time-varying wage. To summarise: my main results about the need for more policy accommodation under imperfect insurance are not driven by my baseline (inefficient) wage-setting mechanism.

The second departure from the baseline wage-setting mechanism that I consider is when the actual cyclicalty of the wage is *farther away*, rather than closer to, that of the efficient wage. If the actual cyclicalty does not satisfy condition (35), then the gains and losses associated with productivity shocks are inefficiently shared between employed workers and firm owners, and monetary policy may want to lean against this. Under the maintained assumption that ϕ match the target $\ln w_0 / \ln z_0 = 1/3$ (see Table 1), one may evaluate the effect of an inefficient wage cyclicalty on optimal policy by varying firm owners' risk aversion

$\tilde{\sigma}$. For example, if $\tilde{\sigma}$ is below its baseline value (0.283), then the actual wage is excessively volatile, since firm owners would be willing to bear a larger fraction of aggregate fluctuations than they actually do. On the contrary, values of $\tilde{\sigma}$ above the baseline imply that the wage is too smooth –i.e., firm owners bear an excessive share of aggregate wage fluctuations. As it turns out, simulations of the model (not represented here) show that the baseline results are only marginally changed when $\tilde{\sigma}$ is moved within reasonable bounds. For example, lowering $\tilde{\sigma}$ to 0 (so that a constant wage would be efficient) changes the impact response of the policy rate by less than 3 annual basis points relative to the baseline scenario, and has no discernible effect on aggregates. Symmetrically, doubling $\tilde{\sigma}$ relative to its baseline value changes the response of the policy rate by the same magnitude, with again almost no effect on aggregates. To summarise: optimal policy is mostly governed by the *actual* cyclicality of the real wage, and very little by the *inefficiency* in this cyclicality.

Steady state distortions. I have so far been working under the assumption that taxes and subsidies were set in a way that aligned the steady state of the decentralised equilibrium with that of the constrained-efficient allocation. This is a natural assumption to start with, for it ensures that observed differences in optimal policies according to the degree of consumption insurance are not unduly driven by differences in steady-state distortions. However, this assumption is unrealistic in the sense that one does not observe, in practice, significant wage or production subsidies (i.e., T and τ^W) of the type that I considered. I therefore explore the optimal response of the policy rate to aggregate shocks under alternative assumptions about those subsidies. In so doing I am still careful to adjust the deep parameters of the model so as to keep matching all the steady state targets in Table 1 –so that the same observed steady state as in the baseline scenario is now considered as distorted rather than undistorted. To operate the required adjustments in the model’s deep parameters, write the steady state counterpart of equation (15) as follows:

$$f^{\frac{\gamma}{1-\gamma}} [1 - \beta (1 - \rho)] \eta = (1 - \tau^I) m^{\frac{1}{1-\gamma}} [\varphi - w + T].$$

Recalling that η/w is among the targets while $\varphi = (\theta - 1) / \theta(1 - \tau^W)$, one can write the

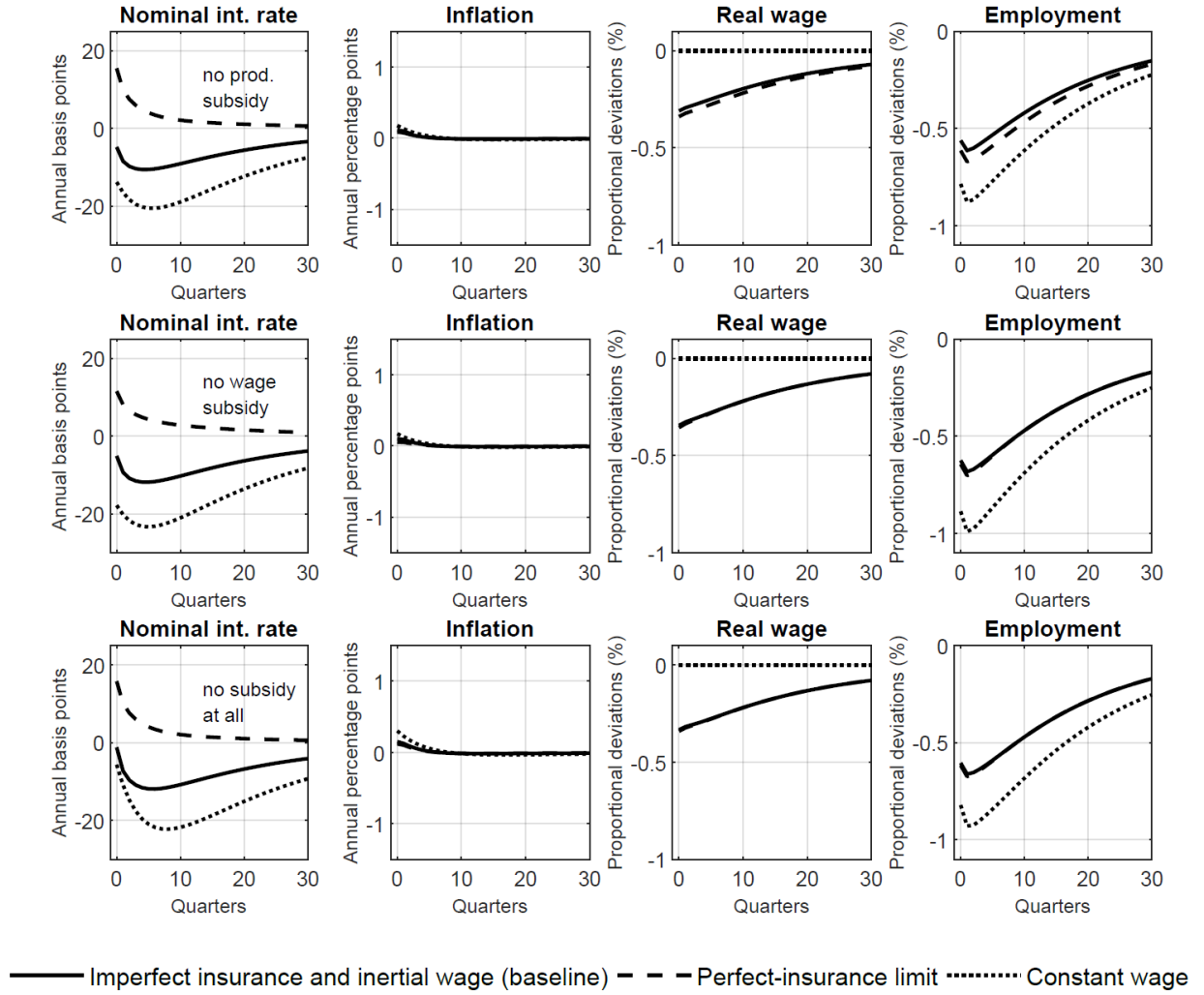


Figure 8: Responses to a contractionary productivity shock (distorted steady state)

steady-state real wage as follows:

$$w = \left[\frac{\theta - 1}{\theta(1 - \tau^W)} + T \right] \left\{ 1 + \frac{(\eta/w) \times f^{*\frac{\gamma}{1-\gamma}} [1 - \beta(1 - \rho)]}{(1 - \tau^I)m^{\frac{1}{1-\gamma}}} \right\}^{-1}. \quad (59)$$

In short, removing the wage subsidy T requires lowering w , else job creation would fall and the model would fail to match the labour-market targets. Similarly, the removal of the production subsidy τ^W should be offset by an appropriate reduction in w .

Figures 8 and 9 show the optimal-policy responses to aggregate shocks when the subsidies τ^W and T are alternatively, and then jointly, set to zero. The optimal responses to the shocks

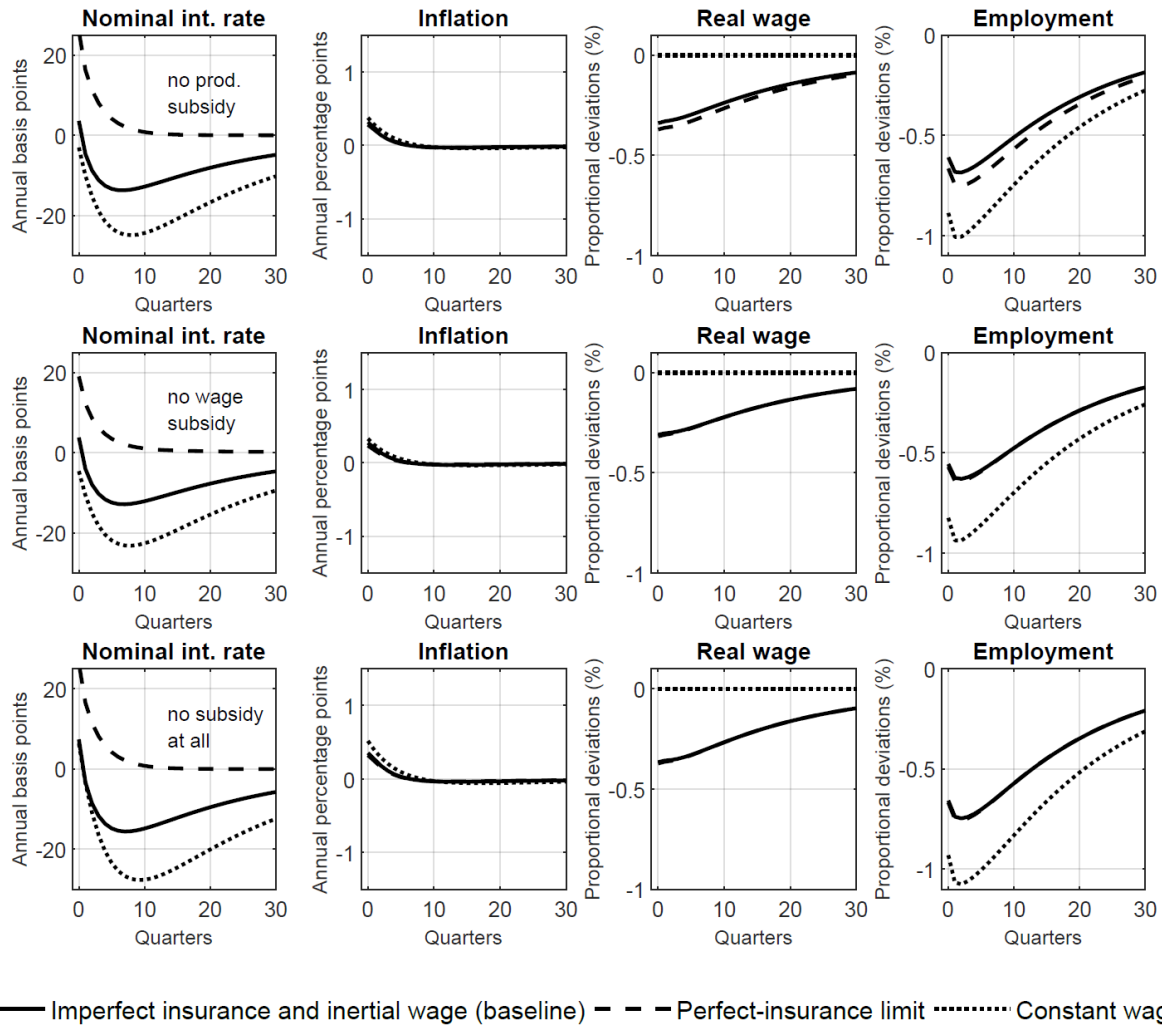


Figure 9: Responses to a contractionary cost-push shock (distorted steady state).

are affected by steady state distortions, a reflection of the fact that these distortions are nonnegligible (see Woodford, 2010). However, the general lessons that imperfect insurance calls for more policy accommodation following contractionary aggregate shocks, and that such an accommodation almost eliminates the destabilising impact of imperfect insurance, unambiguously survive.

6. CONCLUDING REMARKS

In this paper, I have computed the optimal interest-rate response to aggregate shocks in a model economy wherein workers have a precautionary motive against uninsured, endogenous unemployment risk. In this economy aggregate “supply” shocks such as productivity or cost-

push shocks may have powerful aggregate demand effects, due to the feedback loop between unemployment risk, desired savings and aggregate demand, and this loop calls for a response of the policy rate to aggregate shocks that is substantially more accommodative than under perfect insurance. However, provided that the appropriate correction to the policy rate is implemented, then the deflationary spiral is almost fully –if not completely– neutralised. In this sense, conditional on the appropriate correction to the policy rate, “incomplete markets do not matter” for the aggregate dynamics.

Of course, my results have been derived under a fairly restrictive set of assumptions. I only considered unemployment risk and ignored other potentially important sources of idiosyncratic income risk, such as wage risk. Crucially, my zero-liquidity analysis abstracts from distributional considerations, which play an important role in the recent Heterogenous-Agent New Keynesian literature reviewed in the introduction. My analysis of optimal monetary policy is thus best interpreted as providing a simple benchmark against which richer and more realistic imperfect-insurance models can be compared in the future.

APPENDIX TO SECTION 5.1

A. Derivation of the quadratic loss function. With $\rho = m = 1$ and $w_t = w_t^*$ we have:

$$U_t = u(\delta) + n_t [u(w_t^*) - u(\delta)] + \Lambda[\varpi + n_t(z_t/\Delta_t - w_t^*) - \eta n_t^{\frac{1}{1-\gamma}}].$$

I will use the facts that $\hat{n}_t^* = \Phi \hat{z}_t$ and that

$$\partial U_t / \partial n_t = u(w^*) - u(\delta) + \Lambda[z_t/\Delta_t - w^* - \eta n_t^{\frac{\gamma}{1-\gamma}} / (1-\gamma)] = 0.$$

This gives the following quadratic flow utility:

$$\begin{aligned} U_t &= \{u(w^*) - u(\delta) + \Lambda[1 - w^* - \eta n^{\frac{\gamma}{1-\gamma}} / (1-\gamma)]\} \hat{n}_t - \frac{\Lambda \gamma \eta n^{\frac{2\gamma-1}{1-\gamma}}}{2(1-\gamma)^2} \hat{n}_t^2 \\ &\quad + \Lambda \hat{z}_t \hat{n}_t - \Lambda n(\Delta_t - 1) + \text{terms independent of policy (t.i.p.)} + \mathcal{O}(\|\zeta\|^3) \\ &\simeq -(\Lambda/2\Phi) \hat{n}_t^2 + (\Lambda/\Phi) \hat{n}_t^* \hat{n}_t - \Lambda n(\Delta_t - 1) + \text{t.i.p.} \\ &= -(\Lambda/2\Phi) \tilde{n}_t^2 - \Lambda n(\Delta_t - 1) + \text{t.i.p.}, \text{ where } \tilde{n}_t = \hat{n}_t - \hat{n}_t^*. \end{aligned}$$

Now use the facts (derived in Woodford, 2003, chapter 6) that $\Delta_t \simeq 1 + (\theta/2) \text{Var}(p_t(i))$ and $\sum_{t=0}^{\infty} \beta^t \text{Var}(p_t(i)) = \kappa^{-1} \sum_{t=0}^{\infty} \beta^t \pi_t^2$. This allows writing the social welfare function as follows:

$$\begin{aligned} W_t &= \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k U_{t+k} \simeq \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[-\frac{\Lambda}{2\Phi} \tilde{n}_{t+k}^2 - \frac{\Lambda n \theta}{2} \text{Var}(p_{t+k}(i)) \right] + \text{t.i.p.} \\ &= -\frac{\Lambda}{2\Phi} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \tilde{n}_{t+k}^2 - \frac{\Lambda n \theta}{2} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \text{Var}(p_{t+k}(i)) + \text{t.i.p.} \\ &= -\frac{\Lambda}{2\Phi} \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k (\tilde{n}_{t+k}^2 + \Omega \pi_{t+k}^2) + \text{t.i.p.}, \quad \text{with } \Omega = \frac{\theta \Phi n}{\kappa}. \end{aligned}$$

Maximising W_t is thus equivalent to minimising L_t in equation (51).

B. Optimal Ramsey policy. This adapts Galí (2008, Chap. 5) to the present model. The Lagrangian function associated with the central bank's problem is:

$$\mathcal{L}_t = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[(\tilde{n}_{t+k}^2 + \Omega \pi_{t+k}^2) / 2 + \Gamma_{t+k} (\pi_{t+k} - \beta \pi_{t+1+k} - (\kappa/\Phi) \tilde{n}_{t+k}) \right].$$

The first-order conditions with respect to the \tilde{n}_{t+k} s and π_{t+k} s are:

$$\begin{aligned} \mathbb{E}_t[\tilde{n}_{t+k}] - (\kappa/\Phi) \mathbb{E}_t[\Gamma_{t+k}] &= 0 \text{ for all } k \geq 0, \quad \Omega \pi_t + \Gamma_t = 0, \\ \text{and } -\mathbb{E}_t[\Gamma_{t+k}] + \Omega \mathbb{E}_t[\pi_{t+1+k}] + \mathbb{E}_t[\Gamma_{t+k+1}] &= 0 \text{ for all } k \geq 1. \end{aligned}$$

Dropping the \mathbb{E}_t -operator (since I am looking at the response to a one-time shock) and using the expressions for Ω and κ in the loss function, I find that $\{\tilde{n}_{t+k}, \pi_{t+k}\}_{k=0}^{\infty}$ must satisfy:

$$\text{for } k = 0 : \tilde{n}_t + (\theta n) \pi_t = 0; \tag{60}$$

$$\text{for } k \geq 1 : \tilde{n}_{t+k} - \tilde{n}_{t+k-1} + (\theta n) \pi_{t+k} = 0. \tag{61}$$

Equations (60) and (61) can be more compactly written as, for all $k \geq 0$:

$$\tilde{n}_{t+k} = -(\theta n) \hat{p}_{t+k}, \quad \text{with } \hat{p}_{t+k} \equiv p_{t+k} - p_{t-1}, \tag{62}$$

and where p_{t-1} was the price level before the shock hit. Substituting this expression into (55) and rearranging, we obtain the following difference equation for \hat{p}_t :

$$(1 + \beta + \kappa\theta n/\Phi)\hat{p}_{t+k} = \hat{p}_{t+k-1} + \beta\hat{p}_{t+k+1} + \kappa\zeta_{t+k}.$$

The stationary solution to this equation is $\hat{p}_{t+k} = \alpha\hat{p}_{t+k-1} + \Upsilon\zeta_{t+k}$, where Υ and α are defined in Table 2. This solution can be used to recover $\{\tilde{n}_{t+k}, \pi_{t+k}\}_{k=0}^{\infty}$ using (60)–(62). For $k = 0$ we get $\tilde{n}_t = -(\theta n)\hat{p}_t = -\Upsilon\theta n\zeta_t$, where I have used the fact that $\Omega = \theta\Phi n/\kappa$. For $k \geq 1$ we have:

$$\tilde{n}_{t+k} = \alpha\tilde{n}_{t+k-1} + \Upsilon\theta n\zeta_{t+k} = -\Upsilon\theta n(\sum_{\iota=0}^k \alpha^{\iota}\mu_{\zeta}^{k-\iota})\zeta_t.$$

Then, we recover the path of inflation using (60)–(61). We obtain:

$$\begin{aligned} \text{for } k = 0 : \pi_t &= -\frac{\tilde{n}_t}{\theta n} = \Upsilon\zeta_t; \text{ for } k = 1 : \pi_{t+1} = \frac{\tilde{n}_t - \tilde{n}_{t+1}}{\theta n} = \Upsilon(\alpha + \mu_{\zeta} - 1)\zeta_t; \\ \text{and for } k \geq 2 : \pi_{t+k} &= \frac{\tilde{n}_{t+k-1} - \tilde{n}_{t+k}}{\theta n} = \Upsilon[\mu_{\zeta}^k - (1 - \alpha)\sum_{\iota=0}^{k-1} \alpha^{\iota}\mu_{\zeta}^{k-\iota}]\zeta_t. \end{aligned}$$

Table 1 summarises the effect of a shock occurring at $t = 0$.

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