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1 3D Monte Carlo surface-atmosphere radiative transfer  
2 modelling with DART

3  
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18

19 **Research highlights:**

20 New 3D Monte Carlo surface-atmosphere RT modelling in DART model.

21 A virtual Earth-Atmosphere laboratory for simulating the adjacency effect.

22 Study of the impact of the surface 3D structure on the adjacency effect.

23 Study of the adjacency effect contribution on radiance and albedo at TOC and TOA.

24

25 **Abstract**

26 Significant errors can arise if the adjacency effect (*i.e.*, the contribution of neighbouring pixels  
27 to the radiance of a pixel) is neglected in the interpretation of remote sensing images. For  
28 example, adjacency radiances can account for more than 30% of the signal at the top of the  
29 atmosphere (TOA) of a white sand-lined coastline (Bulgarelli and Zibordi, 2018). This paper  
30 shows that 3D radiative transfer (RT) modelling can quantify this phenomenon. It presents a  
31 new 3D Monte Carlo surface-atmosphere RT modelling in the DART RT model, and a resulting  
32 virtual 3D Earth-Atmosphere laboratory for accurate simulation of atmospheric RT, including  
33 the adjacency effect. It was first validated with the atmosphere model SMART-G for 2D scenes:  
34 relative difference is 0.20% in TOA directional reflectance of infinite black surface for solar  
35 zenith equal to 60°, and 0.03% in TOA nadir reflectance of a black disc inside infinite white  
36 Lambertian surface, for solar zenith equal to 0° and 30°. Then, the adjacency effect on the TOA  
37 radiance and TOA albedo of a 3D scene was studied for a circular city (radius 2 km) surrounded  
38 by a forest (dimension 10 km), for four Sentinel-2A bands (blue, green, red, near infrared). Its  
39 contribution to TOA nadir radiance reaches ~20% in the near infrared band, and increases with  
40 viewing zenith angle (VZA): ~60% if VZA = 80°. The 3D scene TOA albedo was calculated  
41 after adapting the DART Bidirectional Reflectance Factor (BRF) camera to simulate TOA  
42 radiance for all upward directions at any angular resolution. For Sentinel-2A's four bands, the  
43 adjacency effect influences the TOA albedo of the circular city by up to 10% for the green band  
44 and up to 27% for the near infrared band, well above the maximum uncertainty 5% usually  
45 required in land surface applications. The adjacency effect of the city neighbourhood 3D  
46 structure was studied by replacing it by a Lambertian surface with its albedo. The city nadir  
47 TOA radiance changed by up to 1.3% with very small change in TOA albedo (< 0.5%). This  
48 new modelling greatly improves DART potential for accurate simulation of atmospheric effects.  
49 It is in the DART version freely available for research and education (<https://dart.omp.eu>).

50

51 **Key words**

52 DART, Monte Carlo, Atmosphere, Adjacency effect, Radiative transfer

53

54 **1 Introduction**

55 From a remote sensing point of view, the adjacency effect is the part of the measured signal  
56 due to land surface elements outside the instantaneous field of view (IFOV). This phenomenon  
57 is one of the most challenging sources of inaccuracy in the remote sensing study of land surfaces  
58 since the advent of high-resolution Earth observation satellites (Kiselev et al., 2015; Liang et  
59 al., 2001; Ouaidrari and Vermote, 1999; Pan et al., 2022; Reinersman and Carder, 1995; Sterckx  
60 et al., 2015). It depends on many factors: the spectral domain (Dave, 1980), the atmospheric  
61 conditions (Reinersman and Carder, 1995), the heterogeneity and anisotropic scattering of the  
62 target and its neighbouring land surfaces (Bulgarelli and Zibordi, 2018), and the light source  
63 and sensor configurations (Vermote et al., 1997), *etc.* In the study of (Bulgarelli and Zibordi,  
64 2018) the adjacency effect reached 30% of the top of atmosphere (TOA) radiance signal at 443  
65 nm at 1 km off the coast of northern Adriatic Sea and 15% at 5 km offshore. Therefore,  
66 neglecting it can lead to very inaccurate interpretations of remote sensing images.

67

68 Several methods to study and quantify the adjacency effect have been proposed since the late  
69 1970's. They broadly belong to three groups: (1) empirical methods, (2) point spread function  
70 (PSF) based methods and (3) 3D radiative transfer (RT) modelling. Empirical methods  
71 (Minomura et al., 2001; Ouaidrari and Vermote, 1999; Richter, 1990) first derive individually  
72 the Lambertian reflectance  $\rho^{(j)}$  of any pixel  $j$  from TOA reflectance assuming an infinite  
73 homogeneous plane ground, then approximate the adjacency effect on the TOA reflectance as  
74 the product of the averaged background reflectance  $\langle \rho \rangle$  and the atmospheric diffuse

75 transmittance.  $\langle \rho \rangle$  is the reflectance average over a  $N \times N$  pixel window around the pixel of  
76 interest using the previously computed reflectance  $\rho^{(j)}$ . Being very fast, empirical methods are  
77 often used in operational atmospheric corrections (*e.g.*, Landsat TM). They use windows whose  
78 size  $N$  needs prior experience, mainly depending on the pixel size, the atmospheric condition,  
79 and the surface heterogeneity (Richter, 1990).

80

81 In PSF-based methods, the adjacency effect is defined by the atmospheric PSF  $f(P, P_0, \Omega_i, \Omega_v)$   
82 that gives the scattering contribution of a land surface point  $P$  illuminated in direction  $\Omega_i$  to the  
83 observed point  $P_0$  in direction  $\Omega_v$ . The PSF can be computed analytically using the single  
84 scattering approximation (Kiselev et al., 2015; Santer and Schmechtig, 2000) or the two-stream  
85 approximation (Mekler and Kaufman, 1980). It can also be derived using a radiosity code  
86 (Borel and Gerstl, 1992) or using a Monte Carlo code (Adler-Golden et al., 2008; Miesch et al.,  
87 2005; Pearce, 1986; Reinersman and Carder, 1995). However, these methods tend to simplify  
88 the 3D RT mechanisms (*e.g.*, multiple scattering, 3D surface architecture, atmospheric RT) that  
89 give rise to the magnitude of the adjacency radiance.

90

91 A quantification of the adjacency radiance in remote sensing images needs a unified 3D RT  
92 modelling in a coupled Earth-Atmosphere system. This is increasingly possible as  
93 improvements in 3D RT modelling and computer technology reduce simulation times (Qi et al.,  
94 2019; Ramon et al., 2019; Wang et al., 2022). 3D RT modelling has recently been used to  
95 quantify the adjacency effect (Bulgarelli and Zibordi, 2018; Pan et al., 2022; Sun et al., 2021).

96

97 Most 3D RT models are either for the atmosphere and neglect the influence of the 3D structure  
98 and spatial heterogeneity of land surfaces (*e.g.*, 3DMCPOL, SPARTA) (Barlakas et al., 2016;  
99 Cornet et al., 2010) or for land surfaces and neglect the influence of the atmosphere (*e.g.*, librat,

100 raytran) (Govaerts et al., 1996; Lewis, 1999). Coupling a land surface RT model with an  
101 atmospheric RT model is often very approximate for simulating the adjacency effect, since it  
102 usually supposes a horizontal homogeneous land surface and atmospheric backscattering. This  
103 is why DART-FT (Flux-Tracking), the initial version of DART<sup>1</sup> (Gastellu-Etchegorry et al.,  
104 1996), included a radiative coupling mode, between the stratified atmosphere and the 3D land  
105 surface (Gascon et al., 2001; Grau and Gastellu-Etchegorry, 2013) instead of a coupling with  
106 an atmosphere model like MODTRAN (Berk and Hawes, 2017). DART-FT uses the discrete  
107 ordinates method (DOM) (Yin et al., 2013) to simulate the adjacency effect by convolving a  
108 pre-computed four-dimensional transfer function  $TF_{TOC \rightarrow Z}(\Delta i, \Delta j, \Omega, \Omega')$  that stores the  
109 incident power in direction  $\Omega'$  at three image planes  $Z$  (any altitude, TOA, and top of canopy:  
110 TOC) with  $\Delta i$  line and  $\Delta j$  column shifts relative to point on the TOC plane that gives rise to a  
111 unit power in direction  $\Omega$ . Pre-computing  $TF_{TOC \rightarrow Z}$  avoids repeated computations. However, it  
112 is only adapted to horizontally homogeneous atmosphere. Also, its efficiency greatly decreases  
113 with the increase in the numbers of atmospheric cells, and discrete directions  $\Omega$  and  $\Omega'$ , whereas  
114 these numbers must be large to accurately simulate the adjacency effect, especially for high-  
115 resolution remote sensing images of spatially heterogeneous landscapes.

116

117 Since 2018, a Monte Carlo mode called DART-Lux has been implemented in DART to improve  
118 the accuracy and efficiency of simulations of TOC remote sensing images of large-and complex  
119 landscapes (Wang et al., 2022). Its end-to-end bidirectional path tracing algorithm samples a  
120 group of stochastic paths connecting the light source and the sensor and unbiasedly estimates  
121 radiance measurement using the weighted contribution of these path samples. DART-Lux

---

<sup>1</sup> DART (<https://dart.omp.eu>) is an ever-evolving radiative transfer model. It simulates the 3D radiative budget, including sun induced chlorophyll fluorescence, and remote sensing (RS) satellite, airborne and in-situ signals (spectroradiometer image, LiDAR FWF, SPL, point cloud) of natural and urban landscapes, from visible to thermal infrared. It is a reference tool for a wide range of RS studies (sensitivity studies, inversion of RS images, design of new RS sensor, etc.). Licenses are free for research and education.

122 results are consistent with those of DART-FT (relative difference  $< 1\%$ ) with simulation times  
123 and computer memory reduced by factors over 100 for complex landscapes (without  
124 atmosphere). Recently, (Wang and Gastellu-Etchegorry, 2021) designed a hybrid method that  
125 couples the Monte Carlo land surface RT and the DOM atmospheric RT to simulate satellite  
126 and airborne images. Although fast and accurate for relatively homogeneous land surfaces (*e.g.*,  
127 dense forest), this method is inefficient to simulate the adjacency effect, because its DOM  
128 atmospheric RT needs small atmospheric voxels. Although usually more efficient than Monte  
129 Carlo method for 1D surface-atmosphere coupling (*i.e.*, spatially constant TOC upwelling and  
130 downwelling fluxes), the DOM is much less efficient for 3D radiative coupling. Therefore,  
131 there was a need to design a Monte Carlo atmospheric RT modelling in DART, in addition to  
132 the DOM atmospheric RT.

133

134 This paper presents two improvements of DART-Lux to simulate remote sensing images with  
135 accurate account of atmospheric effects (*e.g.*, adjacency effect). 1) A new representation of the  
136 Earth-Atmosphere system. 2) A generalization of our initial formulation of RT modelling  
137 (Wang et al., 2022) to scenes with surfaces and volumes. In addition, the DART tool (*i.e.*, BRF  
138 camera) that simulates the scene TOC radiance at  $1^\circ \times 1^\circ$  angular resolution is adapted to any  
139 altitude level and angular resolution in order to quantify the adjacency effect on the scene  
140 radiance and albedo at TOA and TOC. Section 2 summarizes the Monte Carlo modelling of the  
141 initial version of DART-Lux for land surfaces. Section 3 presents the new representation of the  
142 Earth-Atmosphere system and the new formulation of atmospheric RT modelling. Section 4  
143 presents the accuracy assessment of the new modelling and the study of the impact of the  
144 adjacency effect on TOA radiance and albedo. Finally, section 5 presents concluding remarks.

145

146 **2 Principles of DART-Lux**

147 DART-Lux is an unbiased Monte Carlo method for accurate and efficient simulation of  
 148 remotely sensed images of large and complex landscapes (Wang et al., 2022). Its features useful  
 149 for understanding the current work are summarized here. In DART-Lux, the 3D RT problem is  
 150 described by a multi-dimensional integral and is solved with Monte Carlo integration methods.  
 151 The radiance  $L^{(j)}$  measured at a pixel  $j$  of the image is an integral over all the possible paths  $\bar{r}$   
 152 from the light source to pixel  $j$ :

$$L^{(j)} = \int_{\mathcal{D}} f^{(j)}(\bar{r}) \cdot d\mu(\bar{r}) \quad (1)$$

153 with  $\mathcal{D} = \bigcup_{n=1}^{\infty} \mathcal{D}_n$  the path space,  $\mathcal{D}_n$  the set of paths  $\bar{r}_n$  of length  $n$ , and  $\bar{r}_n = r_0 r_1 \dots r_n$  a  
 154 series of vertices  $r_{k=0\dots n}$  on surfaces  $dA(r_k)$  of the scene with  $r_0$  on the sensor and  $r_n$  on the  
 155 light source.  $\mu(\mathcal{D}_n) = \int_{\mathcal{D}_n} d\mu(\bar{r}_n) = \int_{\mathcal{D}_n} dA(r_n) \cdot dA(r_{n-1}) \cdots dA(r_0)$  is the area-product of  $\mathcal{D}_n$ .

156 The term  $f^{(j)}(\bar{r})$  represents the measurement contribution of a path  $\bar{r}$ . For a path  $\bar{r}_n$ :

$$f^{(j)}(\bar{r}_n) = L_e(r_n \rightarrow r_{n-1}) \cdot G(r_0 \leftrightarrow r_1) \cdot W_e^{(j)}(r_0 \rightarrow r_1) \prod_{k=2}^n f(r_k \rightarrow r_{k-1} \rightarrow r_{k-2}) \cdot G(r_{k-1} \leftrightarrow r_k) \quad (2)$$

157 with  $f(r_k \rightarrow r_{k-1} \rightarrow r_{k-2})$  the bidirectional scattering distribution function (BSDF) at vertex  $r_{k-1}$ .

158 It is often written as  $f(r_k \leftrightarrow r_{k-1} \leftrightarrow r_{k-2})$  if there is Helmholtz reciprocity of scattering.  $L_e$  is  
 159 the radiance emitted by the light source and  $W_e$  is the sensor importance function (*cf.*  
 160 Appendix A in (Wang et al., 2022)) that transforms incident radiation into radiance.

161  $G(r_{k-1} \leftrightarrow r_k) = V_t(r_{k-1} \leftrightarrow r_k) \cdot \frac{\cos \theta_i^{r_{k-1}} \cdot \cos \theta_o^{r_k}}{\|r_{k-1} - r_k\|^2}$  is a geometric term, with  $V_t(r_{k-1} \leftrightarrow r_k)$  the

162 visibility function:  $V_t(r_{k-1} \leftrightarrow r_k)$  is 1 if  $r_{k-1}$  and  $r_k$  are mutually visible, and 0 otherwise.

163  $\theta_i^{r_{k-1}}$  is the incident angle at vertex  $r_{k-1}$  and  $\cos \theta_o^{r_k}$  is the exit angle at vertex  $r_k$ .

164

165 The importance sampling method, a major variance reduction technique in Monte Carlo  
 166 integration (Rubinstein and Kroese, 2016), can unbiasedly estimate  $L^{(j)}$  (Eq.) using:



$$F^{(j)} = \frac{f^{(j)}(\bar{r}_n)}{p(\bar{r}_n)} \quad (3)$$

167 with  $F^{(j)}$  an estimate of  $L^{(j)}$  and  $p(\bar{r}_n)$  the probability of a path sample  $\bar{r}_n$ .

168

169 Eq. (3) indicates that  $L^{(j)}$  can be efficiently derived if all paths  $\bar{r}_n$  are fast sampled and the  
 170 estimate  $F^{(j)}$  converges with less repetitive scholastic processes. DART-Lux estimates  $L^{(j)}$   
 171 with a robust and rapid bidirectional path tracing method. First, all vertices of two independent  
 172 paths, one starting from the light source ( $\bar{p}_N = p_0, p_1, \dots, p_s, \dots, p_{N-1}$ ) and one starting from the  
 173 sensor ( $\bar{q}_N = q_0, q_1, \dots, q_t, \dots, q_{N-1}$ ), are connected to get a group of random paths  
 174  $\bar{r}_{s,t} = p_0, p_1, \dots, p_{s-1}, q_{t-1}, q_{t-2}, \dots, q_0$  (cf. Figure 5 in (Wang et al., 2022)). Then, the multiple  
 175 importance sampling (MIS) estimator, proposed by (Veach and Guibas, 1997), computes  $L^{(j)}$   
 176 using a weight  $w_{s,t}(\bar{r}_{s,t})$  per path sample  $\bar{r}_{s,t}$  (Eq. (4)). In general, a larger weight corresponds  
 177 to a path sample with large probability density function (Wang et al., 2022).

$$F_{\text{MIS}}^{(j)} = \sum_{s \geq 0} \sum_{t \geq 0} w_{s,t}(\bar{r}_{s,t}) \cdot \frac{f^{(j)}(\bar{r}_{s,t})}{p(\bar{r}_{s,t})} \quad (4)$$

178

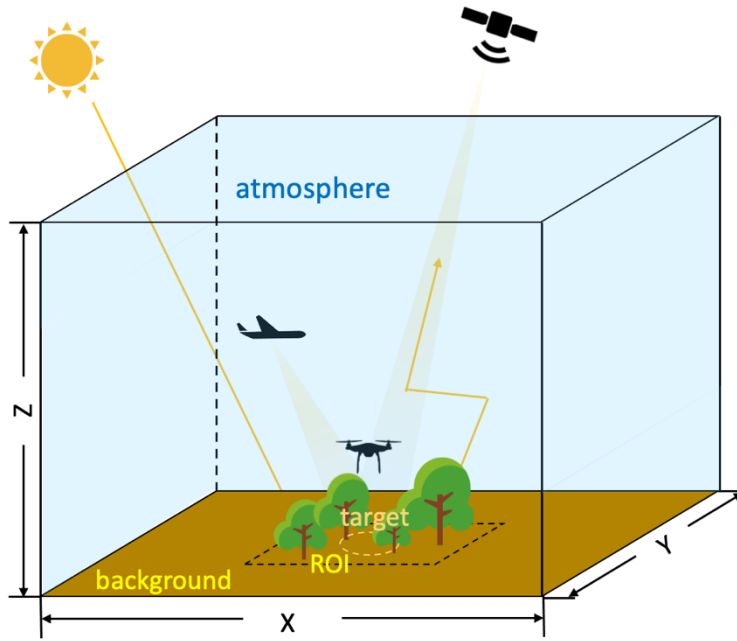
### 179 **3 Atmospheric radiative transfer modelling**

180 The initial version of DART-Lux (Wang et al., 2022) simulated RT with a Monte Carlo method  
 181 for Earth scenes (*i.e.*, landscapes) represented by surfaces (*i.e.*, facets), and with the DOM mode  
 182 of DART-FT for the atmosphere. As it was not suitable for simulating the adjacency effect, we  
 183 designed a new representation of the Earth-Atmosphere system and generalized the theoretical  
 184 formulation of DART-Lux to Earth scenes that contain both surfaces and volumes.

#### 185 **3.1 New Earth-Atmosphere system**

186 The new Earth-Atmosphere scene (Figure 1) is a square cuboid that contains the atmosphere  
 187 and a region of interest (ROI) at its bottom centre, surrounded by a horizontal background with

188 user-defined bi-directional reflectance (*e.g.*, Lambertian, Rahman-Pinty-Verstraete (RPV),  
189 specular). The ROI is a 3D landscape that includes the observed target. Its topography is  
190 smoothly connected to the background by the Bézier triangle (Farin, 2014; Wang et al., 2020).  
191 The optical properties of foliar and soil surfaces can be defined using the DART database of  
192 optical properties, or computed by the PROSPECT / Fluspect / MARMIT modules (Bablot et  
193 al., 2018; Dupiau et al., 2022; Féret et al., 2017; Jacquemoud and Baret, 1990; Vilfan et al.,  
194 2016). The atmosphere (*i.e.*, gases and, aerosols) is made of horizontally homogeneous and  
195 vertically continuous layers with any depths, at any altitudes; default depths are 1 km from 0 to  
196 15 km altitude and 5 km from 15 to 50 km altitude. Polynomials from the spline interpolation  
197 ensure vertical continuity of optical properties (*e.g.*, extinction coefficient) and temperature  
198 within and between layers (Wang and Gastellu-Etchegorry, 2020). Clouds can be imported as  
199 3D volumes with defined shape and homogenous properties. The scene default size is 500 km  
200  $\times$  500 km  $\times$  50 km, which is a good approximation of infinite plane-parallel atmosphere with  
201 relative difference less than 0.1% (*cf.* Appendix A). The DART atmosphere database includes  
202 six standard gas models (*i.e.*, TROPICAL, MIDLATSUM, MIDLATWIN, SUBARCSUM,  
203 SUBARCWIN and USSTD76 (Anderson et al., 1986)) and five standard aerosol models (*i.e.*,  
204 Rural, Urban, Maritime, Tropospheric and Fog (Shettle and Fenn, 1979)). These models can be  
205 adapted and data (*e.g.*, gas and temperature profiles) can be imported to match actual  
206 atmospheric conditions. The methods to compute the atmospheric vertical profiles are detailed  
207 in (Wang and Gastellu-Etchegorry, 2020). In this new Earth-Atmosphere scene, the sensor can  
208 be anywhere with any orientation and field of view (FOV).



209

210 Figure 1. The DART-Lux Earth-Atmosphere system: square cuboid filled by the atmosphere and a 3D  
 211 landscape (region of interest: ROI) at its bottom centre, surrounded by a horizontal background.

212 The observed target is in the ROI. Default size:  $X \times Y \times Z = 500 \text{ km} \times 500 \text{ km} \times 50 \text{ km}$ .

213

214 The DART BRF camera gives the TOC radiance of the total scene at  $1^\circ \times 1^\circ$  angular resolution

215 (Wang et al., 2022). It was adapted in order to give the directional radiance at any altitude from

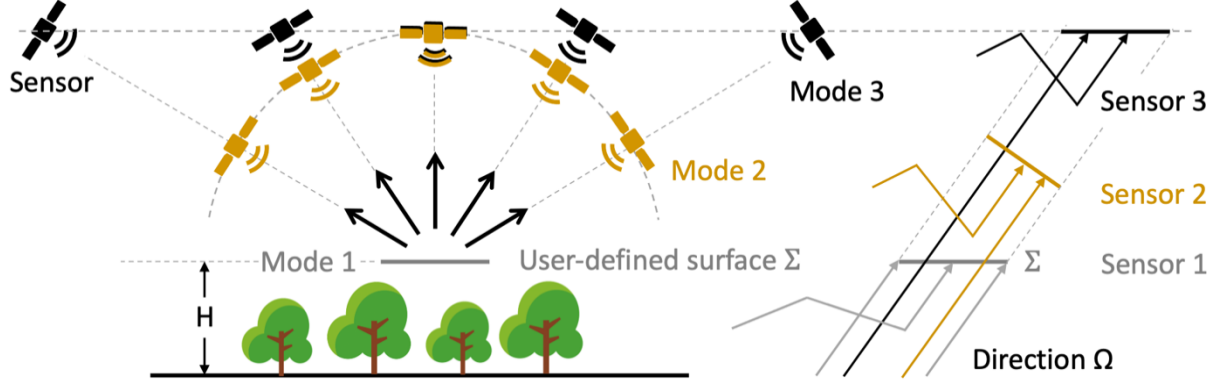
216 the ground to TOA, for any user-defined angular resolution and scene sub-zones. It provides

217 the directional radiance measured from a user-defined virtual surface  $\Sigma$ . Figure 2 shows its three

218 modes. In mode 1, the BRF camera provides the average directional radiance of  $\Sigma$  measured at

219  $\Sigma$ . In mode 2, it provides the average directional radiance at a given distance to the centre of  $\Sigma$ .

220 In mode 3, it provides the average directional radiance of  $\Sigma$  measured at a given altitude.



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Figure 2. The three modes of the improved BRF camera. They give the radiance from a user-defined surface  $\Sigma$  at altitude  $H$ , for any angular resolution and range. Mode 1: average directional radiance measured at  $\Sigma$ . Mode 2: average directional radiance measured at a given distance to the centre of  $\Sigma$ . Mode 3: average directional radiance measured at any user-defined altitude.

227

### 3.2 Light transport equation

228

#### 3.2.1 Radiative transfer equation and its formal solution

229

Eq. (1) is not adapted to simulate the propagation of radiation in fluids (*e.g.*, atmosphere) and

230

vegetation volumes treated as turbid media. Then, an integro-differential equation gives the

231

change of radiance per unit distance  $\frac{dL}{ds}$  in the considered medium at location  $r$  in direction  $\Omega$ .

232

For fluids, this equation is:

$$\frac{dL(r, \Omega)}{ds} = -\alpha_t(r) \cdot L(r, \Omega) + \frac{\alpha_s(r)}{4\pi} \cdot \int_{4\pi} L(r, \Omega') \cdot P(r, \Omega' \rightarrow \Omega) d\Omega' + \frac{dL_e(r, \Omega)}{ds} \quad (5)$$

233

with  $\alpha_t$  the extinction coefficient and  $\alpha_s$  the scattering extinction coefficient.  $\alpha_t$  is assumed

234

invariant to rotation:  $\alpha_t(r, \Omega) = \alpha_t(r)$ , as in a medium filled with microscopically isotropic

235

and mirror-symmetric particles (Mishchenko et al., 2006). The unit normalized scattering

236

phase function  $P(r, \Omega' \rightarrow \Omega)$  ( $\frac{1}{4\pi} \int_{4\pi} P(r, \Omega' \rightarrow \Omega) d\Omega = 1$ ) gives the angular distribution of the

237

scattered power to direction  $\Omega$  due to the incident power from direction  $\Omega'$ . The term  $\frac{dL_e}{ds}$  is

238

the variation of thermal radiance, *i.e.*, the Planck function, per unit distance. The notation of

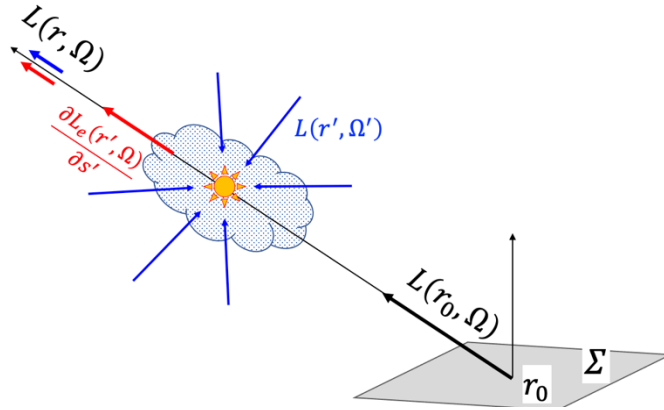
239 incident direction  $\Omega'$  differs for volume and surface scattering: it usually points towards the  
 240 interaction location in volume scattering and usually points away from the interaction  
 241 location in surface scattering.

242

243 Because it is an integro-differential equation, Eq. (5) is transformed to an integral form (Eq. (6))  
 244 that a Monte Carlo algorithm can solve. Eq. (6) shows that radiance  $L(r, \Omega)$  at a point  $r$  in a  
 245 medium is the sum of (1) radiance  $L(r_0, \Omega)$  at  $r_0$  transmitted to  $r$ , and (2) cumulated scattering  
 246 and thermal emission from  $r_0$  to  $r$  (Figure 3).

$$L(r, \Omega) = L(r_0, \Omega) \cdot \mathcal{T}(r \leftrightarrow r_0) + \int_0^s \left[ \frac{\alpha_s(r')}{4\pi} \int_{4\pi} L(r', \Omega') \cdot P(r', \Omega' \rightarrow \Omega) \cdot d\Omega' + \frac{\partial L_e(r', \Omega)}{\partial s'} \right] \cdot \mathcal{T}(r \leftrightarrow r') \cdot ds' \quad (6)$$

247 with  $s = \|r - r_0\|$  the distance between  $r$  and  $r_0$ ,  $r' = r - s' \Omega$  the location at the distance  $s'$  from  
 248  $r$  in direction  $\Omega$ , and  $\mathcal{T}(r \leftrightarrow r_0) = \exp(-\int_0^s \alpha_t(r') ds')$  the transmittance from  $r_0$  to  $r$ . The  
 249 left-right arrow emphasizes the Helmholtz reciprocity principle (i.e.,  $\mathcal{T}(r_0 \rightarrow r) = \mathcal{T}(r \rightarrow r_0)$ ).



250

251 Figure 3. Radiance  $L(r, \Omega)$  at  $r$ . It is the sum of radiance  $L(r_0, \Omega)$  at  $r_0$  of a surface  $\Sigma$  that is  
 252 transmitted to  $r$ , and radiance due to scattering and thermal emission along the path  $\overline{r_0 r}$ .

### 253 3.2.2 Unified light transport equation for the surface and the volume

254 In an infinite medium, Eq. (6) becomes  $L(r, \Omega) = \int_0^\infty \frac{\partial L(r', \Omega)}{\partial s'} \cdot \mathcal{T}(r \leftrightarrow r') \cdot ds'$  with:

$$\frac{\partial L(r', \Omega)}{\partial s'} = \int_{4\pi} L(r', \Omega') \cdot \frac{\alpha_s(r') \cdot P(r', \Omega' \rightarrow \Omega)}{4\pi} \cdot d\Omega' + \frac{\partial L_e(r', \Omega)}{\partial s'} \quad (7)$$

255 Eq. (7) becomes an integral over volume  $V$  by replacing directions with pairs of vertices (*e.g.*,

256  $\Omega \equiv (r' \rightarrow r)$ ), and solid angle  $d\Omega'$  by  $\frac{dA(r'')}{\|r'' - r'\|^2}$ , and using the visibility function  $V_t(r'' \leftrightarrow r')$ :

$$\frac{\partial L(r' \rightarrow r)}{\partial s'} = \int_V \frac{\partial L(r'' \rightarrow r')}{\partial s''} \cdot \frac{\alpha_s(r') \cdot P(r'' \rightarrow r' \rightarrow r)}{4\pi} \cdot \frac{V_t(r'' \leftrightarrow r') \cdot \mathcal{T}(r'' \leftrightarrow r')}{\|r'' - r'\|^2} \cdot dV(r'') + \frac{\partial L_e(r' \rightarrow r)}{\partial s'} \quad (8)$$

257 Since the derived volume light transport equation (Eq. (8)) and the surface light transport  
258 equation (Wang et al., 2022) have a similar formulation, a unified formulation can be used:

$$\tilde{L}(r' \rightarrow r) = \tilde{L}_e(r' \rightarrow r) + \int_{\mathcal{M}} \tilde{L}(r'' \rightarrow r') \cdot \tilde{f}_s(r'' \rightarrow r' \rightarrow r) \cdot \tilde{G}(r' \leftrightarrow r'') \cdot d\mathcal{M}(r'') \quad (9)$$

259 with  $\mathcal{M}$  a surface  $A$  or volume  $V$  depending on the material at the vertex. The radiance terms

260  $\tilde{L}$  and  $\tilde{L}_e$  correspond to radiance for a surface, and radiance per unit distance for a volume:

$$261 \quad \tilde{L}(r'' \rightarrow r') = \begin{cases} L(r'' \rightarrow r') & \forall r' \in A \\ \frac{\partial L(r'' \rightarrow r')}{\partial s''} & \forall r' \in V \end{cases}$$

262 The scattering distribution function  $\tilde{f}$  is either the BSDF for a surface, or the product of the  
263 scattering extinction coefficient and the scattering phase function for a volume:

$$264 \quad \tilde{f}_s(r'' \rightarrow r' \rightarrow r) = \begin{cases} f_s(r'' \rightarrow r' \rightarrow r) & \forall r' \in A \\ \alpha_s(r') \cdot \frac{P(r'' \rightarrow r' \rightarrow r)}{4\pi} & \forall r' \in V \end{cases}$$

265 The generalized geometric term  $\tilde{G}(r' \leftrightarrow r'') = V(r' \leftrightarrow r'') \cdot \mathcal{T}(r'' \leftrightarrow r') \cdot \frac{\tilde{\mu}_o^{r'} \cdot \tilde{\mu}_i^{r''}}{\|r'' - r'\|^2}$  is defined with:

$$266 \quad \tilde{\mu}_o^{r'} = \begin{cases} \cos \theta_o^{r'} & \forall r' \in A \\ 1, & \forall r' \in V \end{cases} \quad \text{and} \quad \tilde{\mu}_i^{r''} = \begin{cases} \cos \theta_i^{r''} & \forall r'' \in A \\ 1, & \forall r'' \in V \end{cases}$$

### 267 3.2.3 Path integral formulation

268 In an interaction, the incident radiance is the path attenuated exit radiance from a previous  
269 interaction. Therefore, Eq. (9) is iteratively expanded to an infinite sum of multidimensional  
270 integrals where each integral is the contribution of a specific scattering order to  $\tilde{L}(r_1 \rightarrow r_0)$ :

$$271 \quad \tilde{L}(r_1 \rightarrow r_0) = \tilde{L}_e(r_1 \rightarrow r_0) + \int_{\mathcal{M}} \tilde{L}_e(r_2 \rightarrow r_1) \cdot \tilde{f}_s(r_2 \rightarrow r_1 \rightarrow r_0) \cdot \tilde{G}(r_1 \leftrightarrow r_2) \cdot d\mathcal{M}(r_2)$$

$$272 \quad + \int_{\mathcal{M}} \int_{\mathcal{M}} \tilde{L}_e(r_3 \rightarrow r_2) \cdot \tilde{f}_s(r_3 \rightarrow r_2 \rightarrow r_1) \cdot \tilde{G}(r_2 \leftrightarrow r_3) \cdot \tilde{f}_s(r_2 \rightarrow r_1 \rightarrow r_0) \cdot \tilde{G}(r_1 \leftrightarrow r_2) \cdot d\mathcal{M}(r_3) \cdot d\mathcal{M}(r_2) + \dots$$

273 Since  $r_1 \rightarrow r_0$  is the direction to the sensor, the radiance measurement  $L^{(j)}$  at pixel  $j$  is computed

274 by integrating  $\tilde{L}(r_1 \rightarrow r_0)$  weighted by the importance function  $W_e^{(j)}(r_0 \rightarrow r_1)$  of the sensor:

$$\tilde{L}^{(j)} = \int_A \int_{\mathcal{M}} W_e^{(j)}(r_0 \rightarrow r_1) \cdot \tilde{L}(r_1 \rightarrow r_0) \cdot \tilde{G}(r_0 \leftrightarrow r_1) \cdot d\mathcal{M}(r_1) \cdot dA(r_0) \quad (10)$$

275  $L^{(j)}$  is a Lebesgue integral over the space  $\tilde{\mu}$  of surfaces and volumes of the set of paths  $\tilde{\mathcal{D}}$ :

$$\tilde{L}^{(j)} = \int_{\tilde{\mathcal{D}}} \tilde{f}^{(j)}(\tilde{r}) \cdot d\tilde{\mu}(\tilde{r}) \quad (11)$$

276  $\tilde{\mathcal{D}}_n$  and  $\tilde{\mu}(\tilde{\mathcal{D}}_n)$  are generalized paths that intersect surfaces and volumes, with  $\tilde{\mathcal{D}}_n$  the multiple

277 cartesian product of the space  $\{A, V\}$ , since a vertex is either in a volume  $V$  or on a surface  $A$ .

$$278 \quad \tilde{\mathcal{D}}_n = \bigcup_{c \in \{0,1\}^n} (A \times \mathcal{M}_1 \times \dots \times \mathcal{M}_n), \text{ with } \mathcal{M}_k = \begin{cases} A, & c_k = 0 \\ V, & c_k = 1 \end{cases}$$

$$279 \quad \tilde{\mu}(\tilde{\mathcal{D}}_n) = \int_{\mathcal{D}_n} d\mu(\tilde{r}_n) = \sum_{c \in \{0,1\}^n} \int_{(A \times \mathcal{M}_1 \times \dots \times \mathcal{M}_n)} dA(r_0) \cdot \prod_{k=1}^n \begin{cases} dA(r_k), & c_k = 0 \\ dV(r_k), & c_k = 1 \end{cases}$$

280 with Cartesian product such that  $\{0, 1\}^n = \{(c_1, c_2, \dots, c_k, \dots, c_n) \mid c_k \in \{0, 1\} \forall k \in \{1, 2, \dots, n\}\}$ .

281

282 For example,  $\tilde{\mathcal{D}}_1 = (A \times A) + (A \times V)$  describes a surface emission and a volume emission

283 event.  $\tilde{\mathcal{D}}_2 = (A \times A \times A) + (A \times A \times V) + (A \times V \times A) + (A \times V \times V)$  describes a surface

284 emission followed by a surface scatter event, a surface emission followed by a volume scatter

285 event, a volume emission followed by a surface scatter event and a volume emission followed

286 by a volume scatter event, and so on. Therefore, integration over  $\tilde{\mathcal{D}}_n$  is the sum of  $2^n (n+1)$ -

287 dimensional integrals.

288

289 The unified contribution function  $f^{(j)}(\tilde{r}_n)$  uses the unified quantities for both surfaces and

290 volumes in the formulation in Eq. (2):

$$\tilde{f}^{(j)}(\bar{r}_n) = \tilde{L}_e(r_n \rightarrow r_{n-1}) \cdot \tilde{G}(r_0 \leftrightarrow r_1) \cdot W_e^{(j)}(r_1 \rightarrow r_0) \cdot \prod_{k=2}^n \tilde{f}_s(r_k \rightarrow r_{k-1} \rightarrow r_{k-2}) \cdot \tilde{G}(r_{k-1} \leftrightarrow r_k) \quad (12)$$

291 Thus, a unified algorithm handles surface and volume interactions. Differences are only in  
 292 assessing the effective source emission, scattering distribution function, and geometric term at  
 293 each vertex  $r_k$  according to the material. (Regaieg, 2023) describes its adaptation to vegetation  
 294 treated as turbid medium.

295

### 296 3.3 Radiative transfer modelling

297 As the Earth-Atmosphere system (Figure 1) has a huge size (*i.e.*,  $X \times Y \times Z = 500 \text{ km} \times 500 \text{ km}$   
 298  $\times 50 \text{ km}$ ) compared to ROI, the forward random walk from the light source is inefficient. For  
 299 example, a ray from a source located 200 km away from the ROI has a very small probability  
 300 to be scattered into the sensor FOV. In addition, its intersection with a scene element (*e.g.*, a  
 301 leaf) can be very inaccurate due to computer decimal imprecision. Therefore, because it is time  
 302 consuming and inaccurate, the forward random walk is deactivated for simulating Monte Carlo  
 303 surface-atmosphere RT. In short, a random virtual ray is emitted from the sensor, and is  
 304 connected to the light source after each interaction with a volume or a surface, using a user  
 305 defined maximal scattering order  $M$  to limit the path length. Then, the estimator in Eq. (4) is:

$$F_{\text{IS}}^{(j)} = \sum_{n=1}^{\infty} \frac{\tilde{f}^{(j)}(\bar{r}_n)}{p(\bar{r}_n)} \quad (13)$$

#### 306 3.3.1 Light scattering

307 The sampling of a stochastic path  $\bar{r}_n$  and the evaluation of the path contribution  $\frac{\tilde{f}^{(j)}(\bar{r}_n)}{p(\bar{r}_n)}$  are  
 308 detailed here. The scattering direction is sampled with the angular distribution of the exit power  
 309 due to unit incident power. For surfaces, the probability density function (PDF) for incident  
 310 direction  $-\Omega_i$  (minus sign as  $\Omega_i$  is defined from the scattering point) and exit direction  $\Omega_o$  is:



$$p(\Omega_o|\Omega_i) = \begin{cases} P^* \cdot \frac{\tilde{f}(r, -\Omega_i \rightarrow \Omega_o) \cdot \cos \theta_o}{\int_{2\pi} \tilde{f}(r, -\Omega_i \rightarrow \Omega_o') \cdot \cos \theta_o' d\Omega_o'}, & \text{if } (\vec{n} \cdot \Omega_o) \cdot (\vec{n} \cdot \Omega_i) \geq 0 \\ (1 - P^*) \cdot \frac{\tilde{f}(r, -\Omega_i \rightarrow \Omega_o) \cdot \cos \theta_o}{\int_{2\pi} \tilde{f}(r, -\Omega_i \rightarrow \Omega_o') \cdot \cos \theta_o' d\Omega_o'}, & \text{otherwise} \end{cases} \quad (14)$$

311 with  $P^*$  the reflection probability and  $\vec{n}$  the surface normal.  $p(\Omega_i|\Omega_o)$  is similarly defined.

312 The conditional PDF of volume scattering (isotropic mirror-symmetric microscopic particles) is:

$$p(\Omega_o|\Omega_i) = \frac{\tilde{f}(r, \Omega_i \rightarrow \Omega_o)}{\int_{4\pi} \tilde{f}(r, \Omega_i \rightarrow \Omega_o') d\Omega_o'} = \frac{P(r, \Omega_i \rightarrow \Omega_o)}{4\pi} \quad (15)$$

313 with  $p(\Omega_i|\Omega_o)$  being similarly defined.

314 Two types of atmospheric scattering implemented in DART are presented here:

315 - **Rayleigh scattering** (particles with diameter  $d$  such that  $0.002 < \frac{\pi d}{\lambda} < 0.2$ ) (Petty, 2006)

316 The Rayleigh phase function uses the phase angle  $\gamma$  ( $\cos \gamma = |\Omega_i \cdot \Omega_o|$ ) and a depolarization  
 317 factor  $\delta$  to consider the influence of molecule anisotropy.  $\delta$  is a spectral quantity (Bates,  
 318 1984; Bucholtz, 1995) that decreases from 0.04545 at  $0.2 \mu\text{m}$  down to 0.02955 at  $0.4 \mu\text{m}$ ,  
 319 0.02730 at  $0.8 \mu\text{m}$ , and is nearly constant for longer wavelengths. Its relative influence on  
 320 Rayleigh scattering phase function of dry air is smaller than 0.1% for wavelengths larger  
 321 than  $0.4 \mu\text{m}$ . It is commonly assumed to be equal to 0.0279, as advised by (Young, 1980):

$$P(r, \Omega_i \rightarrow \Omega_o) = \frac{3}{2} \cdot \frac{1 - \delta}{2 + \delta} \cdot \left( \frac{1 + \delta}{1 - \delta} + \cos^2 \gamma \right) \quad (16)$$

322 - **Particle scattering** (particles with diameter  $d$  such that  $\frac{\pi d}{\lambda} > 0.2$ )

323 The double Henyey-Greenstein phase function simulates it, with asymmetry factors  $g_1 > 0$   
 324 (forward scattering) and  $g_2 < 0$  (backward scattering) and ratio  $a$  of forward scattering:

$$P(r, \Omega_i \rightarrow \Omega_o) = \frac{a \cdot (1 - g_1^2)}{(1 + g_1^2 - 2g_1 \cdot \cos \gamma)^{3/2}} + \frac{(1 - a) \cdot (1 - g_2^2)}{(1 + g_2^2 - 2g_2 \cdot \cos \gamma)^{3/2}} \quad (17)$$

### 3.3.2 Light transmission

Once an exit direction  $\Omega_o$  is sampled at  $r$ , an intersection test along  $\Omega_o$  gives the intersected vertex on the nearest surface (Figure 4). If there is no medium in the line of sight, the process continues by sampling the next scattering direction at the intercepted vertex. Otherwise, a free path  $S$  in the medium is sampled according to the path PDF  $p(s)$  and the cumulative distribution function  $P_S(s)$  (*i.e.*,  $\mathbb{P}(S \leq s)$ ):

$$P_S(s) = \mathbb{P}(S \leq s) = 1 - \mathcal{T}(s) \quad \text{and} \quad p(s) = \frac{dP_S(s)}{ds} = \alpha_t(s) \cdot \mathcal{T}(s) \quad (18)$$

with  $\mathcal{T}(s)$  the transmittance from  $r$  to  $r + s \cdot \Omega_o$  along  $\Omega_o$  and  $\alpha_t(s)$  the extinction coefficient at  $r + s \cdot \Omega_o$ . If the signal is modelled for a series of spectral bands,  $\alpha_t$  is an average extinction coefficient for all spectral bands.

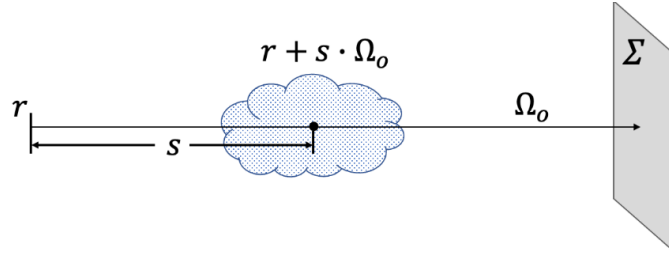


Figure 4. Distance sampling schema.  $s$  is the distance from the start location  $r$  to the current location  $r + s \cdot \Omega_o$  in direction  $\Omega_o$ .  $\Sigma$  is the nearest surface in the direction of propagation.

The probabilistic free path  $S$  is derived from a random variable  $\xi \in [0, 1)$  and an inversion function  $S = P_S^{-1}(\xi)$ . For a homogeneous medium (*i.e.*, constant  $\alpha_t$  in the line of sight):

$$S = P_S^{-1}(\xi) = -\frac{\ln(1 - \xi)}{\alpha_t} \quad (19)$$

Usually,  $\alpha_t$  varies in the line of sight, and  $S$  cannot be derived analytically. Then, the medium is represented by discrete volumes and the free path is sampled with the *Regular Tracking* method (Amanatides and Woo, 1987). For example, for a series of homogeneous segment medium along the line of sight, each one with extinction coefficient  $\alpha_i$  and segment length  $\Delta s_i$ :

$$\begin{aligned}
p(s) &= \alpha_i \cdot e^{-[\sum_{k=0}^{i-1} \alpha_k \cdot \Delta s_k + \alpha_i \cdot (s - \sum_{k=0}^{i-1} \Delta s_k)]} \\
P_S(s) &= 1 - e^{-[\sum_{k=0}^{i-1} \alpha_k \cdot \Delta s_k + \alpha_i \cdot (s - \sum_{k=0}^{i-1} \Delta s_k)]}
\end{aligned}
\quad \forall s \in [\sum_{k=0}^{i-1} \Delta s_k, \sum_{k=0}^i \Delta s_k] \quad (20)$$

343 The free path equation  $S = P_S^{-1}(\xi)$  is solved recursively until the path optical depth gives the  
344 randomly sampled value  $-\ln(1 - \xi)$ :

$$345 \quad -\ln(1 - \xi) = \sum_{k=0}^{i-1} \alpha_k \cdot \Delta s_k + \alpha_i \cdot (S - \sum_{k=0}^{i-1} \Delta s_k)$$

346 Once sampled, if the free path  $S$  is less than the distance  $s_\Sigma$  to the nearest surface (Figure 4), it  
347 defines the next scattering event in the medium. If  $S \geq s_\Sigma$ , the sampled vertex is on the nearest  
348 surface and the path probability density is multiplied by the probability  
349  $\mathbb{P}(S \geq s_\Sigma) = \int_{s_\Sigma}^{\infty} p(s) ds = \mathcal{T}(s_\Sigma)$  that the ray will be intercepted by the nearest surface.

### 350 3.3.3 Measurement evaluation

351 Starting from the sensor, a complete path is created by repeatedly sampling a vertex and free  
352 path. In detail, a vertex  $r_0$  on the lens and an emitted virtual ray direction  $\Omega_0$  are first sampled  
353 using sensor characteristics. Then, a vertex  $r_k$  is created by sampling a stochastic exit direction  
354  $\Omega_{k-1}$  at the vertex  $r_{k-1}$ . This sampling depends only on the local incident direction  $r_{k-2} \rightarrow r_{k-1}$ .  
355 The free path is always sampled after each bounce. Finally, the last vertex  $r_n$  on the light source  
356 is generated by uniformly sampling the solid angle subtended by the light source at the previous  
357 vertex  $r_{n-1}$ . Here the free path is not sampled. Instead, we force the radiation to connect the  
358 light source to increase the efficiency. The path PDF is evaluated by:

$$359 \quad p(\vec{r}_n) = p(r_0) \cdot p(r_1|r_0) \cdot \prod_{k=2}^{n-1} p(r_k|r_{k-1}, r_{k-2}) \cdot p(r_n|r_{n-1}) = \prod_{k=0}^n \tilde{p}(r_k) \quad (21)$$

360 Eq. (22) shows the expression of PDF  $p(r_k|r_{k-1}, r_{k-2}, \dots, r_0) \equiv \tilde{p}(r_k)$  for  $1 < k < n$  with a reverse  
361 arrow to indicate that it is for backward light transport (*cf.* Appendix C in (Wang et al., 2022)).  
362 PDFs  $p(r_0) \equiv \tilde{p}(r_0)$ ,  $p(r_1|r_0) \equiv \tilde{p}(r_1)$  and  $p(r_n|r_{n-1}) \equiv \tilde{p}(r_n)$  can be derived in a similar way.

$$\tilde{p}(r_k) = \begin{cases} p(\Omega_{k-1}) \cdot \frac{\mathcal{J}(r_{k-1} \leftrightarrow r_k) \cdot \cos \theta_o^{r_k}}{\|r_{k-1} - r_k\|^2} & \forall r_k \in A \\ p(\Omega_{k-1}) \cdot \frac{\alpha_e(r_k) \cdot \mathcal{J}(r_{k-1} \leftrightarrow r_k)}{\|r_{k-1} - r_k\|^2} & \forall r_k \in V \end{cases} \quad (22)$$

362 The contribution function  $\tilde{f}^{(j)}(\bar{r}_n)$  is computed as in Eq. (12):

$$F_{\text{IS}}^{(j)} = \sum_{n=1}^{\infty} \frac{\tilde{L}_e(r_n \rightarrow r_{n-1}) \cdot \tilde{G}(r_0 \leftrightarrow r_1) \cdot W_e^{(j)}(r_0 \rightarrow r_1)}{\tilde{p}(r_0) \cdot \tilde{p}(r_1)} \cdot \prod_{k=2}^n \frac{\tilde{f}(r_k \leftrightarrow r_{k-1} \leftrightarrow r_{k-2}) \cdot \tilde{G}(r_{k-1} \leftrightarrow r_k)}{\tilde{p}(r_k)} \quad (23)$$

363 Eq. (23) is analysed in Appendix B for a semi-infinite plane parallel atmosphere, illuminated  
 364 by parallel sunlight and observed by a TOA orthographic camera. Results are consistent with  
 365 the familiar Single Scattering Albedo (SSA) method (Spada et al., 2006).

366

## 367 **4 Model assessment and application**

368 DART-Lux already proved to be accurate (Wang et al., 2022) for land surfaces, as compared  
 369 with Monte Carlo models in the RAMI experiment (Widlowski et al., 2007), and remote sensing  
 370 and in-situ measurements (Malenovsky et al., 2023, Liu et al., 2023). For example, (Landier et  
 371 al., 2018) noted a urban TOC albedo (ratio of upward exitance and downward irradiance)  
 372 relative difference  $\approx 2.5\%$  between in-situ measurements and simulations using satellite-derived  
 373 optical properties of urban elements. This section presents the assessment of DART accuracy  
 374 for the adjacency effect modelling, including a quantitative study on this effect.

### 375 **4.1 Accuracy assessment**

376 DART accuracy in atmospheric RT modelling, in particular the adjacency effect, was assessed  
 377 with the Monte Carlo atmospheric model SMART-G (Ramon et al., 2019), already validated  
 378 with the reference MYSTIC model in the International Polarized Radiative Transfer (IPRT)  
 379 intercomparison study (<https://www.meteo.physik.uni-muenchen.de/~iprt/doku.php>). SMART-  
 380 G is developed by HYGEOS (<https://www.hygeos.com>) for the French space agency (CNES)

381 to simulate light propagation in the atmosphere and oceans. Our assessment consists of two  
 382 phases using atmospheric parameters of case B2 ([https://www.meteo.physik.uni-](https://www.meteo.physik.uni-muenchen.de/~iprt/doku.php?id=intercomparisons:b2_absorption)  
 383 [muenchen.de/~iprt/doku.php?id=intercomparisons:b2\\_absorption](https://www.meteo.physik.uni-muenchen.de/~iprt/doku.php?id=intercomparisons:b2_absorption)) of the IPRT study (Emde et  
 384 al., 2015). This atmosphere has 30 equal-thickness (1 km) layers, with given absorption and  
 385 scattering optical thickness per layer. Table 1 shows the DART and SMART-G configurations.

386

387 Table 1. Simulation configurations for Phase 1 and Phase 2 comparisons.

	Phase 1	Phase 2
<b>Atmosphere model</b>	IPRT case B2	IPRT case B2
<b>Land surface</b>	Infinite black surface	Black disc (R = 2 km) + Infinite white environment
<b>Solar direction</b>	SZA = 60° SAA = 0°	SZA = 0° and 30° SAA = 270°
<b>Sensor altitude</b>	30 km (TOA)	30 km (TOA)
<b>Output</b>	Directional reflectance for: VZA 0° – 80° with 5° step VAA 0° – 180° with 5° step	Nadir image of the disc: Spatial resolution: 10 m Image size: 400 × 400

388

389 In phase 1, DART and SMART-G agree (Figure 5): the relative root mean square difference

390  $\epsilon_{\text{DART}}^{\text{SMART-G}} = 0.20\%$  for 629 upward viewing directions (Table 1), by considering

391  $\epsilon_{\text{model}}^{\text{reference}} = \frac{\sqrt{\sum_{i=1}^N (\rho_{\text{model}}(i) - \rho_{\text{reference}}(i))^2}}{\sqrt{\sum_{i=1}^N \rho_{\text{reference}}(i)^2}}$  (Emde et al., 2015). This small value is mainly due to Monte

392 Carlo noise and rounding errors in calculations. The squared-average TOA reflectance

393  $\sqrt{\frac{\sum_{i=1}^N (\rho_{\text{SMART-G}}(i))^2}{N}}$  is 0.04866, with maximum standard deviation  $\sigma_{\text{Max}}^{\text{SMART-G}} = 1.1 \times 10^{-5}$  and

394  $\sigma_{\text{Max}}^{\text{DART}} = 3.8 \times 10^{-5}$ . It gives composed standard deviation  $\sigma_{\text{Max}}^{\text{tot}} = \sqrt{(\sigma_{\text{Max}}^{\text{SMART-G}})^2 + (\sigma_{\text{Max}}^{\text{DART}})^2} = 4.0 \times 10^{-5}$

395 and maximum tolerated relative difference  $\frac{3 \times \sigma_{\text{tot}}}{0.04866} = 0.25\%$ , which is a probability of 99.7% the

396 relative difference  $\epsilon_{\text{DART}}^{\text{SMART-G}}$  lies in 0.25%. Compared to the proposed reference model MYSTIC

397 in IPRT program, the relative differences are  $\epsilon_{\text{DART}}^{\text{MYSTIC}} = 0.21\%$  and  $\epsilon_{\text{SMART-G}}^{\text{MYSTIC}} = 0.013\%$ .

398

399 In phase 2, we determined the horizontal profiles of average TOA reflectance of the black disc<sup>2</sup>

400 from its centre to its edge, with a step of 0.1 km, for two solar directions<sup>3</sup> (SZA = 0° and SZA

401 = 30°) (Figure 6). The adjacency effect clearly increases from the centre to the edge of the disc.

402 The relative root mean square difference is 0.033% for SZA = 0° and 0.032% for SZA = 30°.

403 These small differences are due to Monte Carlo noise and rounding error in calculations. Indeed,

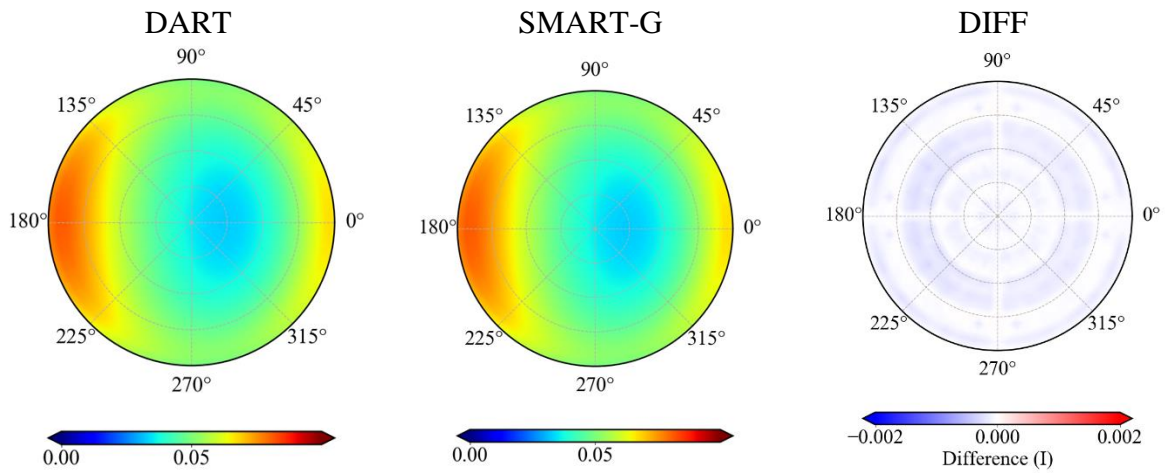
404 the average TOA nadir reflectance of the disc is 0.137 for SZA = 0° and 0.114 for SZA = 30°,

405 while DART and SMART-G maximum standard deviations are  $\sigma_{\text{MAX}}^{\text{SMART-G}} = 6 \times 10^{-6}$  and

406  $\sigma_{\text{MAX}}^{\text{DART}} = 1.8 \times 10^{-5}$ . Then, composed standard deviation  $\sigma_{\text{MAX}}^{\text{tot}} = \sqrt{(\sigma_{\text{MAX}}^{\text{SMART-G}})^2 + (\sigma_{\text{MAX}}^{\text{DART}})^2} = 1.9 \times$

407  $10^{-5}$  and maximum tolerated relative difference  $\frac{3 \times \sigma_{\text{tot}}}{0.114} = 0.05\%$ .

408

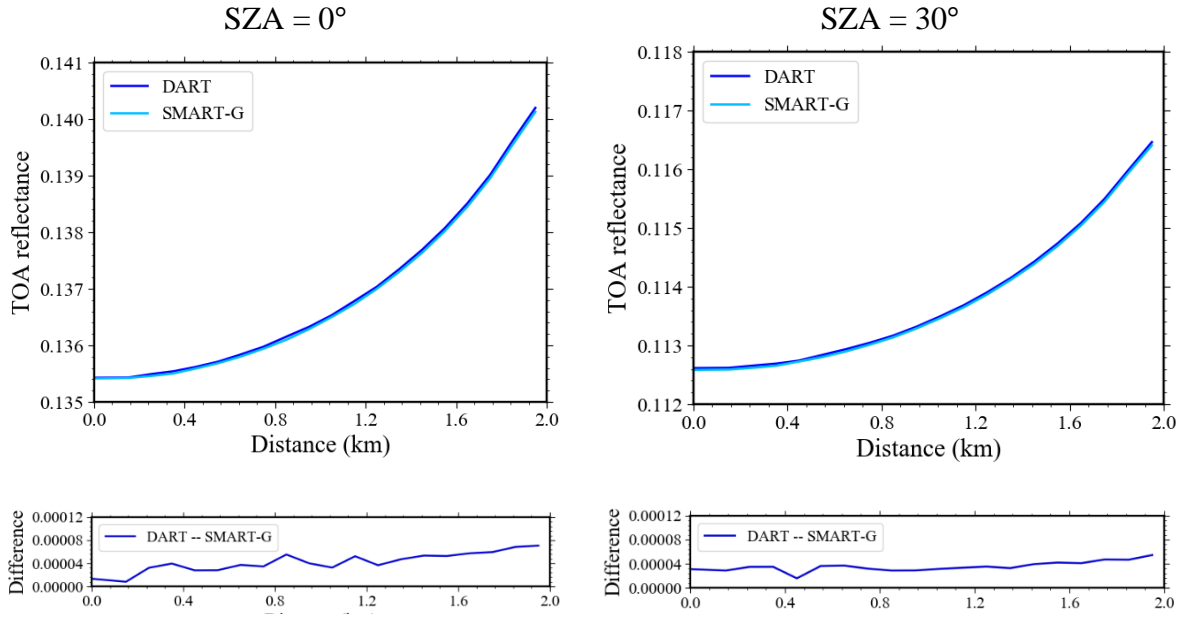


409 Figure 5. DART and SMART-G TOA directional reflectance and difference in phase 1, from left to

410 right. Simulation parameters are indicated in Table 1.

<sup>2</sup> In phase 2, DART simulates a nadir 400 x 400 image with spatial resolution of 10 m while SMART-G simulates  $1.6 \times 10^5$  sensors that are positioned at each pixel center of the image.

<sup>3</sup> The solar direction in DART is defined as a vector in local coordinates pointing towards the sun. Azimuth is measured from the north-south axis in an anticlockwise manner.



412 Figure 6. DART and SMART-G TOA reflectance value and difference from centre to edge of black  
 413 disc for sun zenith angle (SZA) = 0° and 30°. Simulation parameters are in Table 1.

414

## 415 4.2 3D modelling of the adjacency effect

### 416 4.2.1 The experiment

417 The influence of the adjacency effect on target radiance and albedo is shown for four Sentinel-  
 418 2A bands: blue (B:  $\lambda=0.4924 \mu\text{m}$ ,  $\Delta\lambda=0.066 \mu\text{m}$ ), green (G:  $\lambda=0.5598 \mu\text{m}$ ,  $\Delta\lambda=0.036 \mu\text{m}$ ,  
 419 red (R:  $\lambda=0.6646 \mu\text{m}$ ,  $\Delta\lambda=0.031 \mu\text{m}$ ) and near infrared (NIR:  $\lambda=0.8328 \mu\text{m}$ ,  $\Delta\lambda=0.106 \mu\text{m}$ ).  
 420 The atmosphere is defined by the USSTD76 gas model, and the Rural aerosol model. Table 2  
 421 gives its vertical scattering and absorption optical depth per band. Solar zenith and azimuth  
 422 angles are  $\theta_s=30^\circ$  and  $\varphi_s=270^\circ$ , respectively. The target is a circular city with a 2 km radius  
 423 in a 3D neighbourhood of 10 km sides inside a 500 km square Lambertian plane background  
 424 (Figure 7). The city has 55828 houses in regular rows (tiled roofs, concrete wall) and streets  
 425 (asphalt roads), with optical properties of urban materials (*e.g.*, asphalt, concrete, tile) from the

426 SLUM (<http://www.met.reading.ac.uk/micromet/LUMA/SLUM.html>) and ECOSTRESS  
 427 (<https://speclib.jpl.nasa.gov>) libraries. Two cases are considered (Figure 8):

428 - 3D case (Figure 8): the neighbourhood is a 10 km × 10 km 3D forest with 1553504 nearly-  
 429 randomly located *Tilia cordata* trees at five growing stages, with homogeneous understory.

430 The tree models and optical properties, including those of the understory, are from the  
 431 RAMI experiment ([https://rami-benchmark.jrc.ec.europa.eu/\\_www/index.php](https://rami-benchmark.jrc.ec.europa.eu/_www/index.php)).

432 - 1D case: the neighbourhood is a 10 km x 10 km Lambertian plane with the TOC albedo of  
 433 the forest. DART computes the TOC albedo of the forest and city for subzones (1 km × 1  
 434 km at city centre and 2 km × 2 km in top-left forest, in Figure 7).

435 Table 2. Atmosphere vertical scattering and absorption optical depth (OD), and city and forest TOC  
 436 albedos, for four Sentinel-2A bands.

Band	Wavelength	Scattering OD	Absorption OD	City TOC albedo	Forest TOC albedo
B	0.4924 $\mu\text{m}$	0.501	0.030	0.06	0.03
G	0.5598 $\mu\text{m}$	0.392	0.058	0.07	0.07
R	0.6646 $\mu\text{m}$	0.286	0.036	0.010	0.03
NIR	0.8328 $\mu\text{m}$	0.189	0.045	0.100	0.42

437



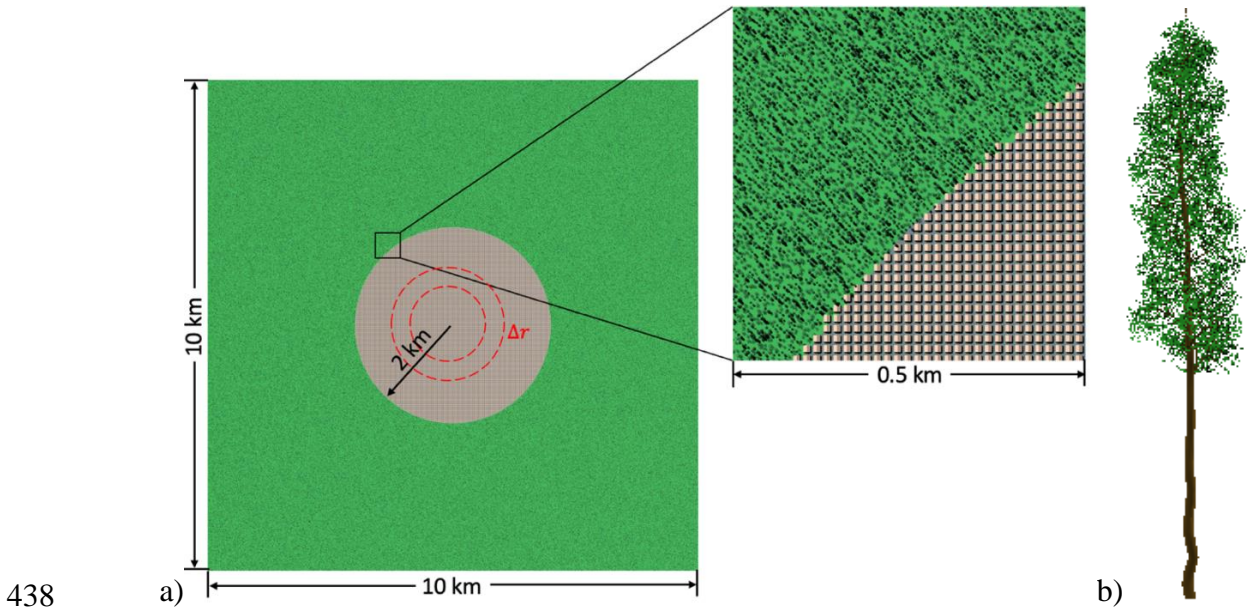


Figure 7. 3D case. a) Target: circular city of 2 km radius with 55828 houses spaced 15 m apart in a 10 × 10 km forest with 1553504 quasi-randomly distributed *Tilia cordata* trees at five growing stages, spaced ≈ 5 m apart). b) DART created house. b) Example of *Tilia cordata*: TICO 5.

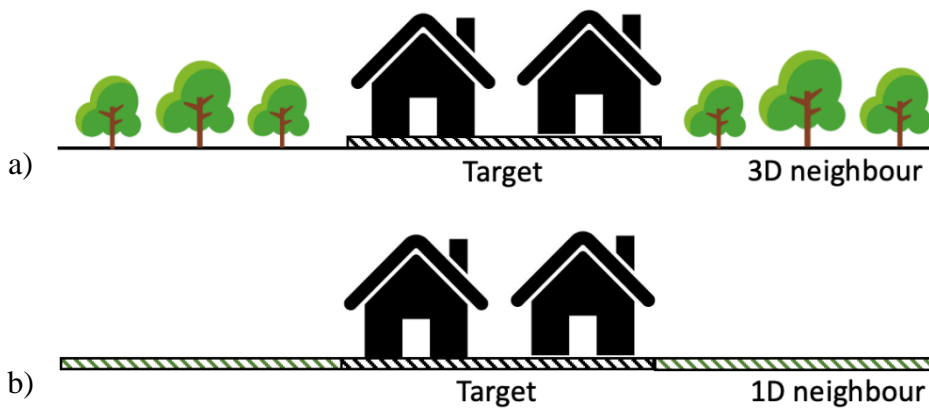
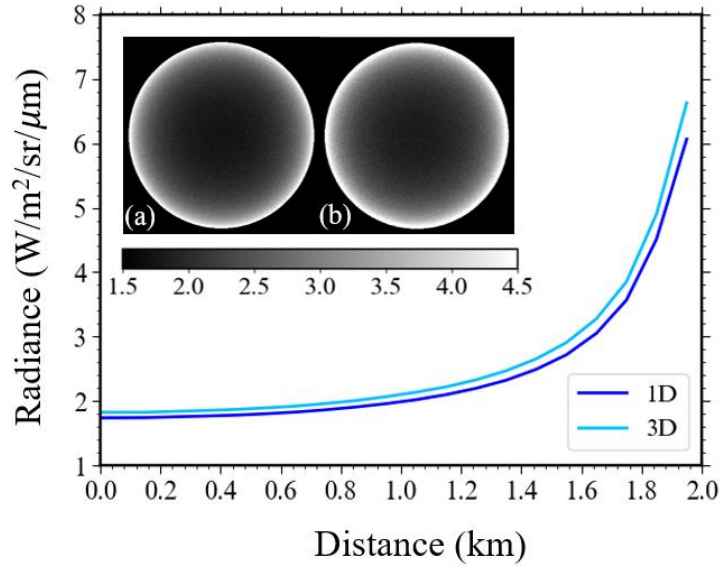


Figure 8. Two cases: city (*i.e.*, target) in a neighbourhood that is a forest (a: 3D case) or a Lambertian plane with the forest TOC albedo (b: 1D case). The two cases have the same background.

#### 444 4.2.2 Adjacency effect on TOA nadir radiance

445 The adjacency effect is evaluated as the difference of two simulations, one with the 1D/3D  
 446 neighbourhood and the background and the other one with the neighbourhood and background  
 447 as flat black surfaces. Figure 9 shows the TOA nadir adjacency NIR radiance  $L_{adj}(x, y)$  and  
 448  $L_{adj}(r)$  of the city with 1D (a) and 3D (b) neighbourhood, with  $r$  the distance from the city

449 centre to the edge ( $r = 2$  km).  $L_{adj}$  increases from the city centre. It is larger for the 3D  
 450 neighbourhood than for the 1D neighbourhoods, although both have the same TOC albedo. This  
 451 is mostly due to the forest anisotropic reflectance.



452  
 453 Figure 9. Adjacency radiance  $L_{adj}$  in the nadir TOA images for the 1D (a) and 3D (b) neighbourhoods  
 454 at NIR band.  $L_{adj}(r)$  greatly increases with distance  $r$  from the city centre to the edge.  $L_{adj}(r)$   
 455 is the average for all pixels  $L_{adj}(x, y)$  in the ring  $[r, r + \Delta r]$  with  $\Delta r = 0.1$  km (Table 3).

456 Table 3 shows nadir  $L_{adj}$  at city centre and edge, and its relative contribution  $\zeta_{L_{adj}} = L_{adj}/L_{TOA}$   
 457 to TOA radiance for four Sentinel-2A bands.  $L_{adj}$  is  $\approx 2 - 3$  times larger at the edge of the city  
 458 than at its centre, for all bands. It reaches  $\sim 20\%$  in the NIR band.  $L_{adj}$  and  $\zeta_{L_{adj}}$  increase with  
 459 the neighbour TOC albedo and the importance of atmospheric scattering. For example,  $\zeta_{L_{adj}}$  is  
 460 more than 5 times larger in NIR band than in the red band because the forest TOC albedo is  
 461 more than 12 times larger in the NIR band than in the red band. Also,  $\zeta_{L_{adj}}$  is more than 1.5  
 462 times larger in the blue band than in the red band, although the TOC albedo of the forest is  
 463 similar, because the atmosphere scatters more in the blue band than in the red band (*cf.*  
 464 atmospheric scattering optical depth in Table 2). The study of adjacency effect under different  
 465 aerosol loadings in the Appendix C confirms this point. We also note that the impact of 3D

466 structure on the adjacency radiance is maximal in the NIR band, the  $\zeta_{L_{adj}}$  values of the 1D and  
 467 3D cases differ by 0.3% at the city centre and by 1.3% at the city edge.

468 Table 3. Adjacency radiance  $L_{adj}$  ( $W/m^2/sr/\mu m$ ) and its relative contribution  $\zeta_{L_{adj}} = L_{adj}/L_{TOA}$  (%) to  
 469 total TOA nadir radiance at the city centre and edge, for the four Sentinel-2A bands.

	1D neighbour				3D neighbour			
	Centre		Edge		Centre		Edge	
	$L_{adj}$	$\zeta_{L_{adj}}$	$L_{adj}$	$\zeta_{L_{adj}}$	$L_{adj}$	$\zeta_{L_{adj}}$	$L_{adj}$	$\zeta_{L_{adj}}$
B	1.4	2.2%	2.5	4.2%	1.4	2.3%	2.8	4.7%
G	1.8	3.6%	3.9	8.0%	1.9	3.9%	4.4	9.0%
R	0.4	0.9%	1.1	2.5%	0.4	1.0%	1.2	2.8%
NIR	1.7	5.5%	6.1	17.5%	1.8	5.8%	6.6	18.8%

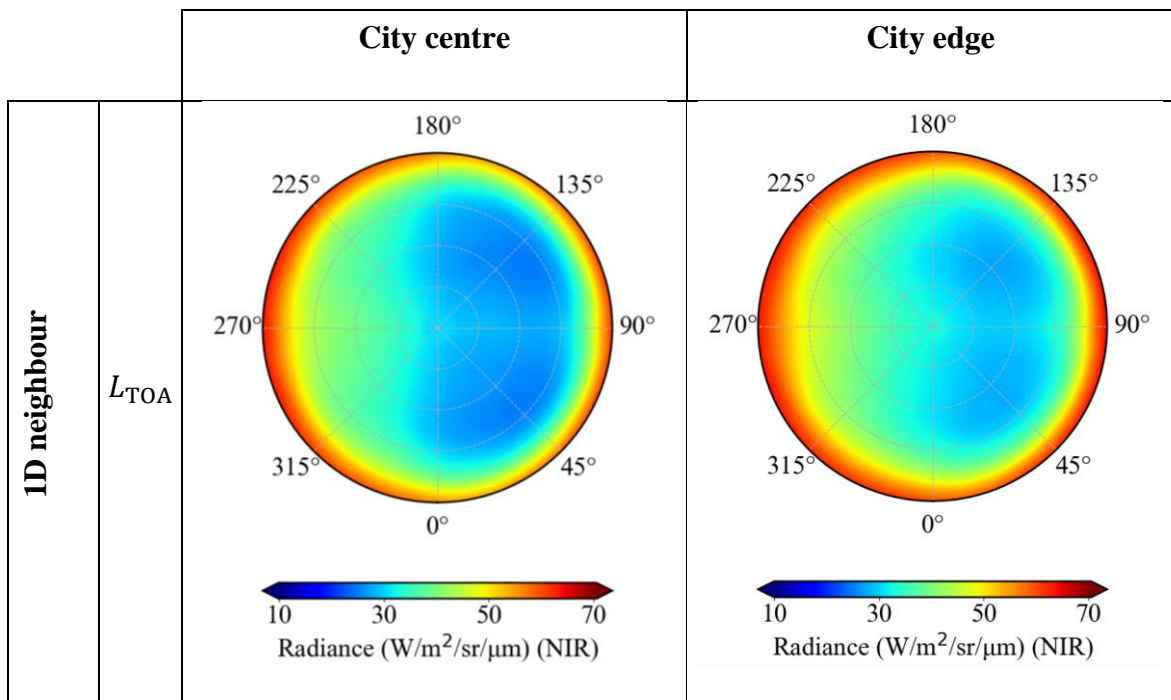
470

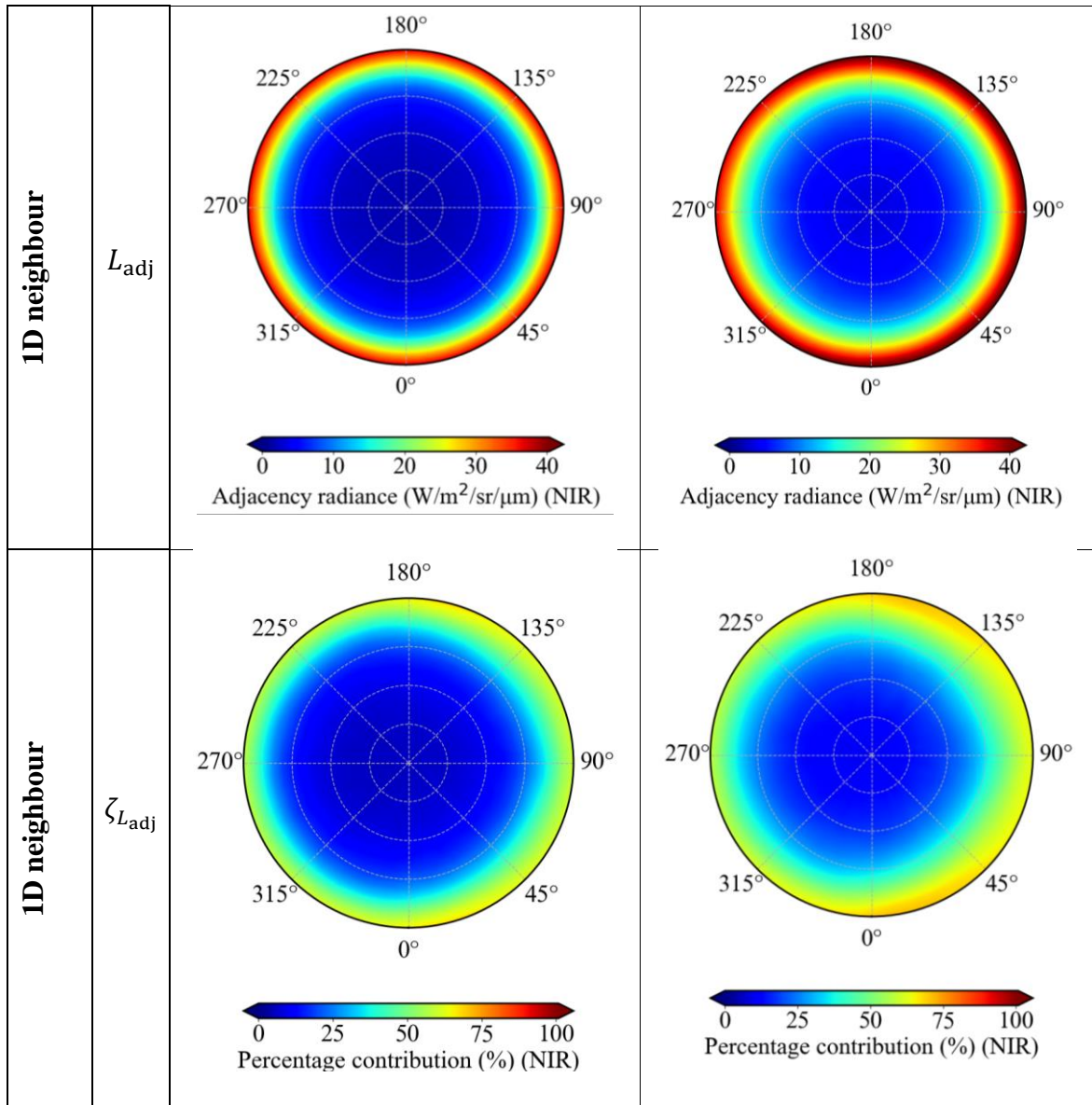
### 471 4.2.3 Adjacency effect on directional radiance and albedo

472 The TOA albedo of the city centre and the edge are simulated using the BRDF camera (Figure 2)  
 473 in mode 3, for a user-defined plane ( $\Delta x = \Delta y = 100$  m) with altitude  $H = 15$  m, mode 3 to obtain  
 474 the TOA signal and mode 1 to obtain the TOC signal. Figure 10 shows the directional  
 475 distributions of TOA radiance  $L_{TOA}$ , adjacency radiance  $L_{adj}$  and its relative contribution  $\zeta_{L_{adj}}$ .  
 476 at the city centre and edge, for the NIR band.  $L_{TOA}$  tends to be larger in the backward directions  
 477 due to the hot-spot effect of the 3D land surface and is smaller in the forward directions due to  
 478 the observation of the shadows of the city buildings.  $L_{adj}$  increases with the obliquity of the  
 479 observation directions. For example,  $\zeta_{L_{adj}}$  can be as high as 60% for directions with  $VZA = 80^\circ$ ,  
 480 because increasing the “sensor-city” path (*i.e.*, larger atmospheric optical depth for observation)  
 481 decreases the city signal and increases the city neighbourhood signal by atmospheric scattering.

482

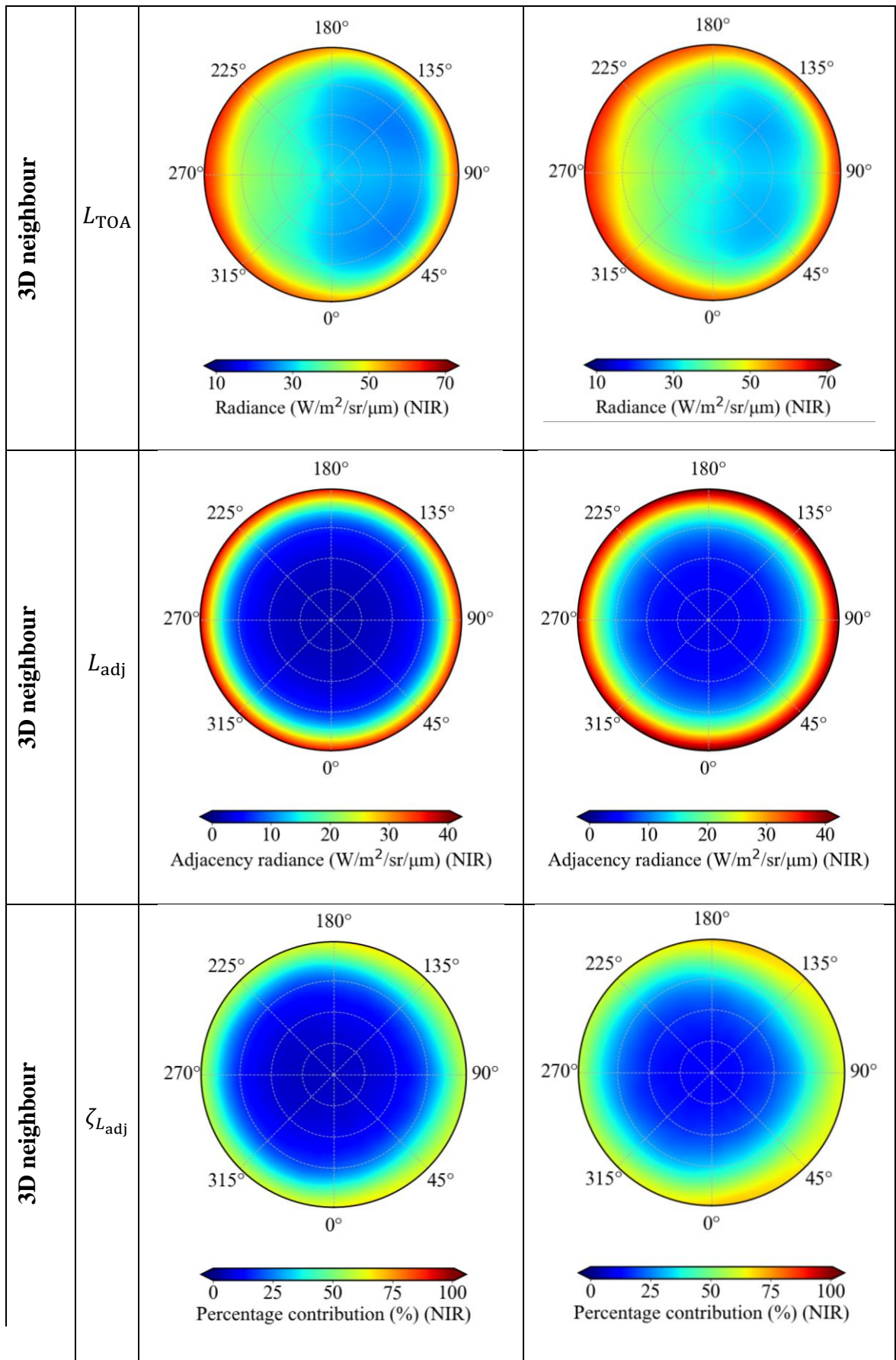
483 TOA exitance  $M_{TOA}$  is the hemispherical integral of the directional average upward TOA  
 484 radiance of a surface at TOC level. TOA albedo  $\mathcal{A}_{TOA}$  is the ratio of TOA exitance  $M_{TOA}$  to  
 485 TOA irradiance  $E_{TOA}$ . Table 4 shows the contribution  $\mathcal{A}_{adj}^Z$  (altitude level Z can be TOC or  
 486 TOA) due to the adjacency effect and its relative importance  $\zeta_{\mathcal{A}_{adj}^Z} = \mathcal{A}_{adj}^Z / \mathcal{A}_Z$  at the city centre  
 487 and edge, for the four Sentinel-2A bands.  $\zeta_{\mathcal{A}_{adj}^{TOA}}$  is usually larger than 5%, the maximum TOC  
 488 albedo uncertainty advised by the Global Climate Observing System (GCOS:  
 489 <https://gcos.wmo.int/en/essential-climate-variables/albedo/>). Therefore, the adjacency effect  
 490 can be a significant source of uncertainty for the interpretation of TOC albedo from RS images  
 491 if they are not corrected by the atmospheric correction algorithm. Here, the large TOC albedo  
 492 of the neighbour forest in the NIR can increase  $\mathcal{A}_{adj}^{TOA}$  up to 0.035 (*i.e.*, 28.1% of the total TOA  
 493 albedo). Although it impacts the directional radiance, the 3D structure of the neighbourhood  
 494 only slightly influences  $\mathcal{A}_{adj}^{TOA}$ : the difference of  $\mathcal{A}_{adj}^{TOA}$  between 1D and 3D neighbour case is  
 495 less than 0.8% for all four bands.





a)

<b>City centre</b>	<b>City edge</b>
--------------------	------------------



b)

497 Figure 10. Angular plots of TOA total radiance  $L_{\text{TOA}}$ , radiance  $L_{\text{adj}}$  due to the adjacency effect, and  
 498 relative contribution  $\zeta_{L_{\text{adj}}}$  at the city centre and edge. NIR band. a) 1D neighbour. b) 3D  
 499 neighbour.

500 Table 4. Adjacency TOA albedo  $\mathcal{A}_{\text{adj}}^{\text{TOA}}$  and its percentage contribution  $\zeta_{\mathcal{A}_{\text{adj}}^{\text{TOA}}} = \mathcal{A}_{\text{adj}}^{\text{TOA}} / \mathcal{A}_{\text{TOA}}$  (%) to  
 501 total TOA albedo at the city centre and edge, for four Sentinel-2A bands.

	1D neighbour				3D neighbour			
	Centre		Edge		Centre		Edge	
	$\mathcal{A}_{\text{adj}}^{\text{TOA}}$	$\zeta_{\mathcal{A}_{\text{adj}}^{\text{TOA}}}$	$\mathcal{A}_{\text{adj}}^{\text{TOA}}$	$\zeta_{\mathcal{A}_{\text{adj}}^{\text{TOA}}}$	$\mathcal{A}_{\text{adj}}^{\text{TOA}}$	$\zeta_{\mathcal{A}_{\text{adj}}^{\text{TOA}}}$	$\mathcal{A}_{\text{adj}}^{\text{TOA}}$	$\zeta_{\mathcal{A}_{\text{adj}}^{\text{TOA}}}$
B	0.004	3.8%	0.006	4.9%	0.005	3.9%	0.007	5.2%
G	0.008	7.4%	0.010	10.1%	0.008	7.6%	0.011	10.6%
R	0.003	2.6%	0.004	3.9%	0.003	2.6%	0.004	4.1%
NIR	0.021	18.6%	0.033	27.3%	0.022	19.0%	0.035	28.1%

502  
 503 The influence of the adjacency effect is smaller on the TOC irradiance and TOC exitance than  
 504 on the TOA exitance. Its relative contribution to local TOC albedo  $\mathcal{A}_{\text{TOC}}$  is even smaller  
 505 because it contributes in a similar way to TOC exitance  $M_{\text{TOC}}$  and irradiance  $E_{\text{TOC}}$ : it is only  
 506 about 0.13% at NIR band, 0.01% at the red band, 0.03% at the green band and 0.03% at the  
 507 blue band. Table 5 shows  $M_{\text{TOC}}$ ,  $E_{\text{TOC}}$  and  $\mathcal{A}_{\text{TOC}}$  at NIR band as well as their respective  
 508 increases  $\Delta_{\text{adj}}^{\text{TOC}}$  due to the adjacency effect, at the city centre and edge, for 1D and 3D neighbour  
 509 cases. The ratio of the components  $\Delta_{\text{adj}}^{\text{TOC}}$  to  $M_{\text{TOC}}$  and  $E_{\text{TOC}}$  is close to  $\mathcal{A}_{\text{TOC}}$  (*i.e.*,  $\Delta_{\text{adj}}^{\text{M}_{\text{TOC}}} /$   
 510  $\Delta_{\text{adj}}^{\text{E}_{\text{TOC}}} \approx \mathcal{A}_{\text{TOC}}$ ).

511 Table 5. TOC exitance  $M_{\text{TOC}}$  ( $\text{W}/\text{m}^2/\mu\text{m}$ ), irradiance  $E_{\text{TOC}}$  ( $\text{W}/\text{m}^2/\mu\text{m}$ ) and albedo  $\mathcal{A}_{\text{TOC}}$  at NIR band  
512 as well as their component  $\Delta_{\text{adj}}^{\text{TOC}}$  due to the adjacency effect.  $E_{\text{TOC}}$  is the integral of directional  
513 radiance captured by a fisheye camera looking upward and placed at the city centre and edge.

	1D neighbour				3D neighbour			
	Centre		Edge		Centre		Edge	
	Total	$\Delta_{\text{adj}}^{\text{TOC}}$	Total	$\Delta_{\text{adj}}^{\text{TOC}}$	Total	$\Delta_{\text{adj}}^{\text{TOC}}$	Total	$\Delta_{\text{adj}}^{\text{TOC}}$
$M_{\text{TOC}}$	87.67	0.92	87.87	1.06	87.67	0.91	87.88	1.07
$E_{\text{TOC}}$	857.99	8.90	858.24	9.26	858.03	8.94	858.33	9.35
$\mathcal{A}_{\text{TOC}}$	0.1022	0.0000	0.1024	0.0001	0.1022	0.0000	0.1024	0.0001

514

## 515 5 Concluding remarks

516 This paper presents the new 3D Monte Carlo modelling of surface-atmosphere RT in DART as  
517 an extension and adaptation of the initial light transport theory to scenes that comprise surfaces  
518 and volumes. It also presents a new Earth-Atmosphere system. It is designed to simulate  
519 accurate remote sensing observations and their atmospheric contributions, including those due  
520 to the adjacency effect. The default setting of this Earth-Atmosphere system (*i.e.*,  $X \times Y \times Z =$   
521  $500 \text{ km} \times 500 \text{ km} \times 50 \text{ km}$ ) is consistent (relative error  $< 0.1\%$ ) with a quasi-infinite parallel  
522 atmosphere. Its dimensions can be easily adapted to any experimental context. Its atmospheric  
523 RT modelling has been successfully evaluated with SMART-G with an atmosphere defined in  
524 the IPRT program. The relative root mean square difference of TOA reflectance is 0.20% for  
525 629 upward viewing directions from  $VZA = 0^\circ$  to  $80^\circ$  for infinite black surface, and 0.03% for  
526 the points on the radius of a black disc inside infinite white environment. These differences are  
527 mainly due to Monte Carlo noise and the roundoff error of computing.

528



529 This new DART modelling is very well adapted to study the influence of the adjacency effect  
530 on remotely sensed radiance of land surfaces at any altitude. We studied this influence for a  
531 scene made up of a target city (radius 2 km) surrounded by a forest (dimension 10 km), for four  
532 Sentinel-2A bands (blue, green, red, NIR). Results allow one to quantify the increase of the  
533 adjacency effect with larger TOC albedo of the neighbouring forest and atmospheric scattering.  
534 In our experiment, the adjacency effect contributes up to ~ 5.5% at the city centre and up to  
535 ~20% at the city edge to the TOA nadir NIR radiance, which is more than 5 times larger than  
536 that at red band (~1% at the city centre and ~3% at the city edge). It is explained by the fact  
537 that the forest TOC albedo at NIR band is more than 12 times larger than in the red band. The  
538 adjacency effect in the blue band is 1.5 times larger than that of the red band at the city centre  
539 and edge although forest TOC albedo is similar in the blue and red bands, because the  
540 atmosphere scatters more in the blue band than in the red band. The 3D structure of the  
541 neighbouring forest also plays a role because it drives the angular distribution of the radiation  
542 scattered by the neighbourhood of the city. Compared to the 1D Lambertian neighbour case, it  
543 can induce a difference of adjacency contribution up to 1.3% for the edge of the city for the  
544 NIR band.

545

546 The BRF camera of DART has been adapted to the new Earth-Atmosphere system in order to  
547 simulate the hemispherical distribution of the upward radiance, then to compute the  
548 corresponding albedo at a given altitude, a key parameter reflecting the radiative balance.  
549 Results show that the adjacency contribution increases if the observing direction is more  
550 inclined. In our virtual experiment, at observing zenith equal to  $80^\circ$ , the adjacency contribution  
551 in the NIR band can represent up to 60% of the TOA radiance. The increased TOA albedo due  
552 to adjacency effect for the four Sentinel-2A bands considered is generally larger than the  
553 required maximum TOC albedo uncertainty 5% in applications as reported in GCOS. For

554 example, the proportion of adjacency albedo in total TOA albedo can reach 10% in the green  
555 band and 27% in the NIR band. The anisotropic reflectance of the forest that surrounds the city  
556 influences the TOA radiance of the city, but only slightly influences the TOA albedo of the city.  
557 The adjacency effect changes TOC irradiance but hardly changes the TOC albedo since the  
558 adjacency effect increases both the TOC irradiance and exitance.

559

560 This work expands the potential of DART to study and correct atmospheric effects in remote  
561 sensing images through the consideration of realistic landscapes with their spatial heterogeneity,  
562 3D architecture, topography, *etc.* It also helps quantifying the error in the fast methods that  
563 estimate approximately the adjacency effect.

564

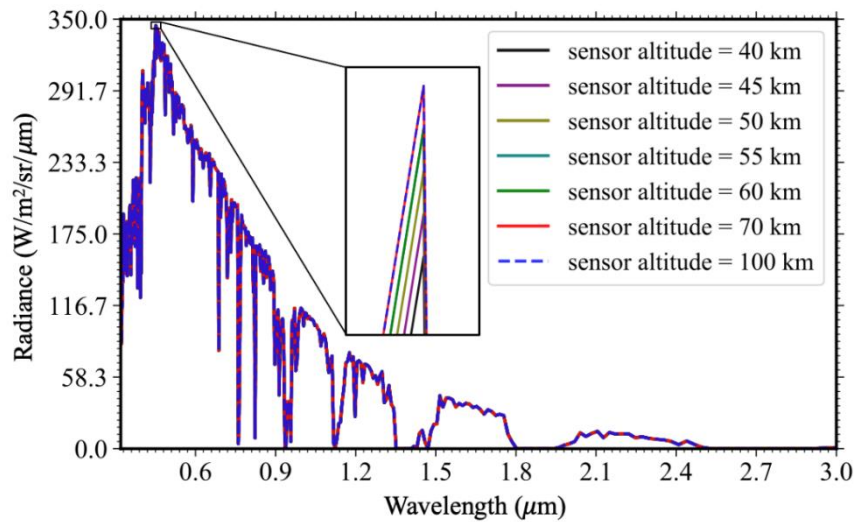
#### 565 **Acknowledgement**

566 This work is funded by the Région Occitanie, France, the Paul Sabatier University, and the  
567 TOSCA program of the French Space Agency (CNES).

568

569 **Appendix A. The default atmosphere dimension**

570 The default parameters (*e.g.*, 50 km altitude, 500 km × 500 km atmosphere horizontal dimension)  
 571 of the new Earth - Atmosphere system (*cf.* section 3.1) were derived from sensitivity studies in  
 572 order to be optimal in usual configurations. For example, larger dimensions improve accuracy  
 573 but increase simulation time. The optimal altitude was determined using MODTRAN as a  
 574 reference. Considered an atmosphere characterized by the USSTD76 gas model and the Rural  
 575 aerosol model, TOC albedo equal to 0.5, 2721 bands from 0.32 to 2.5 μm, nadir solar and  
 576 viewing directions, and sensors at 40 km, 45 km, 50 km, 55 km, 60 km and 70 km altitudes.  
 577 The relative differences  $\varepsilon$  between sensor and MODTRAN spectra are very small and decrease  
 578 with altitude (Figure A.1, Table A.1). For the selected 50 km altitude  $\varepsilon = 0.07\%$ .



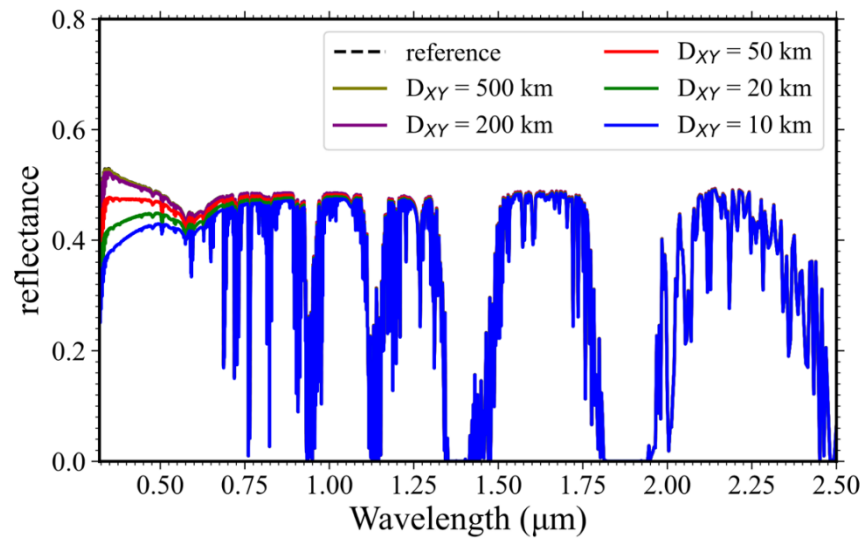
579  
 580 Figure A.1. MODTRAN TOA radiance spectra at seven sensor altitudes (40 km, 45 km, 50 km, 55 km,  
 581 60 km, 70 km, 100 km). TOC albedo is 0.5. USSTD76 atmosphere model with rural aerosols.

582  
 583 Table A.1. Mean relative error  $\varepsilon$  of radiance spectra at six sensor altitudes (Figure A.1) compared to  
 584 the reference radiance spectrum of a sensor at 100 km altitude.

Sensor altitude	40 km	45 km	50 km	55 km	60 km	70 km
$\varepsilon$	0.17%	0.07%	0.07%	0.06%	0.03%	0.01%

585

586 The optimal horizontal dimension  $D_{XY}$  of the atmosphere volume was studied using a DART  
 587 simulation with  $D_{XY} = 10000$  km as a reference, for a TOC albedo equal to 0.5, an atmosphere  
 588 characterized by the USSTD76 gas model and rural aerosol model, and nadir solar and viewing  
 589 directions.  $D_{XY}$  was set to 20 km, 50 km, 100 km, 200 km, 500 km, and 10000 km. Figure A.2  
 590 shows that a small  $D_{XY}$  value (*e.g.*, 20 km) leads to a large underestimate of TOA reflectance  
 591 in the visible domain. Table A.2 shows the average relative error  $\varepsilon$  for each  $D_{XY}$  value.  $D_{XY} =$   
 592 500 km is a good approximation for an infinite parallel atmosphere because  $\varepsilon=0.08\%$ .



593  
 594 Figure A.2. DART TOA nadir reflectance for five atmosphere horizontal dimensions  $D_{XY}$  (20 km, 50  
 595 km, 100 km, 200 km, 500 km). TOC albedo is 0.5. USSTD76 atmosphere model with rural  
 596 aerosols. The reference is for  $D_{XY}=10000$  km.

597  
 598 Table A.2. The mean relative error  $\varepsilon$  of all DART TOA nadir reflectance spectra (Figure A.2)  
 599 compared to the reference spectrum simulated with  $D_{XY} = 10000$  km.

$D_{XY}$	20 km	50 km	100 km	200 km	500 km
$\varepsilon$	7.74%	3.59%	0.55%	0.45%	0.08%

600

601 **Appendix B. Radiance estimate for a semi-infinite atmosphere**

602 For the semi-infinite plane-parallel atmosphere, illuminated by parallel sunlight and observed

603 by an orthographic camera, the radiance estimate  $F_{\text{IS}}^{(j)} = \sum_{n=1}^{\infty} \left[ \frac{\tilde{L}_e(r_n \rightarrow r_{n-1}) \cdot \tilde{G}(r_0 \leftrightarrow r_1) \cdot W_e^{(j)}(r_0 \rightarrow r_1)}{\tilde{p}(r_0) \cdot \tilde{p}(r_1)} \cdot \right.$

604  $\left. \prod_{k=2}^n \frac{\tilde{f}_s(r_k \rightarrow r_{k-1} \rightarrow r_{k-2}) \cdot \tilde{G}(r_{k-1} \leftrightarrow r_k)}{\tilde{p}(r_k)} \right]$  (Eq. (23)) can be simplified since all vertices, except those at

605 the light source and sensor, are in the medium, we have:

$$606 \quad \frac{\tilde{G}(r_0 \leftrightarrow r_1) \cdot W_e^{(j)}(r_0 \rightarrow r_1)}{\tilde{p}(r_0) \cdot \tilde{p}(r_1)} = \frac{\mathcal{T}(r_0 \leftrightarrow r_1) \cdot \frac{\cos \theta_i^{r_0}}{\|r_0 - r_1\|^2} \cdot \frac{\delta(\Omega_0 - \Omega_d)}{A_{\text{img}}}}{\frac{1}{A_{\text{img}}} \cdot \frac{\delta(\Omega_0 - \Omega_d)}{\|r_0 - r_1\|^2} \cdot \alpha_e(r_0 \rightarrow r_1) \cdot \mathcal{T}(r_0 \leftrightarrow r_1)} = \frac{1}{\alpha_e(r_0 \rightarrow r_1)}$$

$$607 \quad \frac{\tilde{f}_s(r_k \rightarrow r_{k-1} \rightarrow r_{k-2}) \cdot \tilde{G}(r_{k-1} \leftrightarrow r_k)}{\tilde{p}(r_k)} = \frac{\alpha_s(r_k \rightarrow r_{k-1}) \cdot \frac{P(r_k \rightarrow r_{k-1} \rightarrow r_{k-2})}{4\pi} \cdot \frac{\mathcal{T}(r_{k-1} \leftrightarrow r_k)}{\|r_{k-1} - r_k\|^2}}{\frac{P(r_k \rightarrow r_{k-1} \rightarrow r_{k-2})}{4\pi} \cdot \alpha_e(r_{k-1} \rightarrow r_k) \cdot \frac{\mathcal{T}(r_{k-1} \leftrightarrow r_k)}{\|r_{k-1} - r_k\|^2}} = \frac{\alpha_s(r_k \rightarrow r_{k-1})}{\alpha_e(r_{k-1} \rightarrow r_k)}$$

608 Besides, the direct connection of the light source indicates:

$$609 \quad \frac{\tilde{L}_e(r_n \rightarrow r_{n-1}) \cdot \tilde{f}_s(r_n \rightarrow r_{n-1} \rightarrow r_{n-2}) \cdot \tilde{G}(r_{n-1} \leftrightarrow r_n)}{\tilde{p}(r_n)}$$

$$610 \quad = \frac{E_s \cdot \delta(\Omega_{n-1} - \Omega_s) \cdot \alpha_s(r_n \rightarrow r_{n-1}) \cdot \frac{P(r_n \rightarrow r_{n-1} \rightarrow r_{n-2})}{4\pi} \cdot \frac{\mathcal{T}(r_{n-1} \leftrightarrow r_n)}{\|r_{n-1} - r_n\|^2}}{\delta(\Omega_{n-1} - \Omega_s) \cdot \frac{\cos \theta_o^{r_n}}{\|r_{n-1} - r_n\|^2}}$$

$$611 \quad = E_s \cdot \alpha_s(r_n \rightarrow r_{n-1}) \cdot \mathcal{T}(r_{n-1} \leftrightarrow r_n) \cdot \frac{P(r_n \rightarrow r_{n-1} \rightarrow r_{n-2})}{4\pi}$$

612 with  $E_s$  the solar constant at TOA, and  $\Omega_s$  the solar direction.

613

614 Assuming that atmospheric constituents are isotropic, extinction coefficients depend only on

615 location. After a few mathematical manipulations, we obtain a form consistent with the SSA

616 method in (Spada et al., 2006):

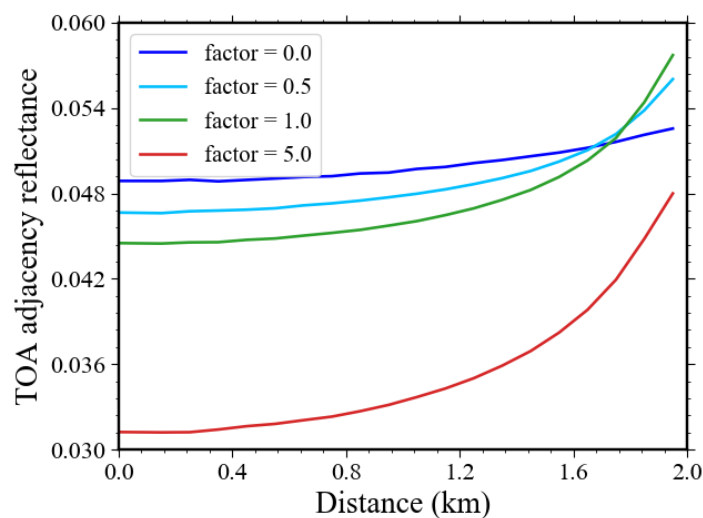
$$617 \quad F_{\text{IS}}^{(j)} = \sum_{n=1}^{\infty} \left[ E_s \cdot \mathcal{T}(r_n \leftrightarrow r_{n-1}) \cdot \frac{P(r_n \rightarrow r_{n-1} \rightarrow r_{n-2})}{4\pi} \cdot \prod_{k=1}^{n-1} \frac{\alpha_s(r_k)}{\alpha_e(r_k)} \right]$$

618

619 **Appendix C. Adjacency effect under different aerosol loadings**

620 The adjacency effect is mostly due to scene elements at short distance if the aerosol optical  
621 depth (AOD) is high, and at greater distances if the AOD is small (Houborg and McCabe, 2016).  
622 Here, we consider TOA reflectance for different AODs. The scene is a black disc with radius  
623 equal to 2 km surrounded by an infinite white environment. The atmosphere is characterized  
624 by a USSTD76 gas model and a Rural aerosol model. SZA = 30° and SAA = 270°. Figure C.1  
625 shows the TOA adjacency reflectance of the disc as a function of distance from the disc centre  
626 to edge for four different AODs. The AOD in DART was scaled by a so-called multiplicative  
627 factor applied to the aerosol vertical profile. For example, a factor equal to 0.5 means the  
628 resulted AOD is the AOD of Rural aerosol model multiplied by 0.5. Four factors were  
629 considered: 0, 0.5, 1 and 5. Figure C.1 shows that the adjacency effect decreases at the disc  
630 centre if AOD increases. The trend is more complex on the disc border: an AOD increase leads  
631 to an increase of the adjacency effect if AOD is low (i.e., factor less or equal to 1), and to a  
632 decrease of the adjacency effect if the AOD is very high (i.e., factor 5).

633



634

635 Figure C.1. TOA adjacency reflectance from the centre to border of the black disc, for four  
636 different aerosol loadings.

637

638 **Reference**

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640 Adler-Golden, S.M., Matthew, M.W., Berk, A., Fox, M.J., Lee, J., Ratkowski, A.J., 2008.

641 Improvements in aerosol retrieval for atmospheric correction, in: IGARSS 2008-2008

642 IEEE International Geoscience and Remote Sensing Symposium. IEEE, pp. III–130.

643 Amanatides, J., Woo, A., 1987. A fast voxel traversal algorithm for ray tracing., in:

644 Eurographics. pp. 3–10.

645 Anderson, G.P., Clough, S.A., Kneizys, F.X., Chetwynd, J.H., Shettle, E.P., 1986. AFGL

646 atmospheric constituent profiles (0.120 km). AIR FORCE GEOPHYSICS LAB

647 HANSCOM AFB MA.

648 Bablet, A., Vu, P.V.H., Jacquemoud, S., Viallefont-Robinet, F., Fabre, S., Briottet, X., Sadeghi,

649 M., Whiting, M.L., Baret, F., Tian, J., 2018. MARMIT: A multilayer radiative transfer

650 model of soil reflectance to estimate surface soil moisture content in the solar domain

651 (400–2500 nm). *Remote Sens Environ* 217, 1–17.

652 Barlakas, V., Macke, A., Wendisch, M., 2016. SPARTA–solver for polarized atmospheric

653 radiative transfer applications: introduction and application to Saharan dust fields. *J Quant*

654 *Spectrosc Radiat Transf* 178, 77–92.

655 Bates, D.R., 1984. Rayleigh scattering by air. *Planet Space Sci* 32, 785–790.

656 Berk, A., Hawes, F., 2017. Validation of MODTRAN® 6 and its line-by-line algorithm. *J Quant*

657 *Spectrosc Radiat Transf* 203, 542–556.

658 Borel, C.C., Gerstl, S.A.W., 1992. Adjacency-blurring effect of scenes modeled by the radiosity

659 method, in: *Atmospheric Propagation and Remote Sensing*. International Society for

660 *Optics and Photonics*, pp. 620–624.

661 Bucholtz, A., 1995. Rayleigh-scattering calculations for the terrestrial atmosphere. *Appl Opt*

662 34, 2765–2773.

663 Bulgarelli, B., Zibordi, G., 2018. On the detectability of adjacency effects in ocean color remote  
664 sensing of mid-latitude coastal environments by SeaWiFS, MODIS-A, MERIS, OLCI,  
665 OLI and MSI. *Remote Sens Environ* 209, 423–438.

666 Cornet, C., C-Labonnote, L., Szczap, F., 2010. Three-dimensional polarized Monte Carlo  
667 atmospheric radiative transfer model (3DMCPOL): 3D effects on polarized visible  
668 reflectances of a cirrus cloud. *J Quant Spectrosc Radiat Transf* 111, 174–186.

669 Dave, J. V., 1980. Effect of atmospheric conditions on remote sensing of a surface  
670 nonhomogeneity. *Photogramm Eng Remote Sensing* 46, 1173–1180.

671 Dupiau, A., Jacquemoud, S., Briottet, X., Fabre, S., Viallefont-Robinet, F., Philpot, W., Di  
672 Biagio, C., Hébert, M., Formenti, P., 2022. MARMIT-2: An improved version of the  
673 MARMIT model to predict soil reflectance as a function of surface water content in the  
674 solar domain. *Remote Sens Environ* 272, 112951.

675 Emde, C., Barlakas, V., Cornet, C., Evans, F., Korkin, S., Ota, Y., Labonnote, L.C., Lyapustin,  
676 A., Macke, A., Mayer, B., 2015. IPRT polarized radiative transfer model intercomparison  
677 project–Phase A. *J Quant Spectrosc Radiat Transf* 164, 8–36.

678 Farin, G., 2014. *Curves and surfaces for computer-aided geometric design: a practical guide*.  
679 Elsevier.

680 Féret, J.-B., Gitelson, A.A., Noble, S.D., Jacquemoud, S., 2017. PROSPECT-D: Towards  
681 modeling leaf optical properties through a complete lifecycle. *Remote Sens Environ* 193,  
682 204–215.

683 Gascon, F., Gastellu-Etchegorry, J.-P., Lefèvre, M.-J., 2001. Radiative transfer model for  
684 simulating high-resolution satellite images. *IEEE Transactions on Geoscience and Remote*  
685 *Sensing* 39, 1922–1926.

686 Gastellu-Etchegorry, J.-P., Demarez, V., Pinel, V., Zagolski, F., 1996. Modeling radiative  
687 transfer in heterogeneous 3-D vegetation canopies. *Remote Sens Environ* 58, 131–156.



688 Govaerts, Y.M., Jacquemoud, S., Verstraete, M.M., Ustin, S.L., 1996. Three-dimensional  
689 radiation transfer modeling in a dicotyledon leaf. *Appl Opt* 35, 6585–6598.

690 Grau, E., Gastellu-Etchegorry, J.-P., 2013. Radiative transfer modeling in the Earth–  
691 Atmosphere system with DART model. *Remote Sens Environ* 139, 149–170.

692 Houborg, R., McCabe, M.F., 2016. Adapting a regularized canopy reflectance model  
693 (REGFLEC) for the retrieval challenges of dryland agricultural systems. *Remote Sens*  
694 *Environ* 186, 105–120.

695 Jacquemoud, S., Baret, F., 1990. PROSPECT: A model of leaf optical properties spectra.  
696 *Remote Sens Environ* 34, 75–91.

697 Kiselev, V., Bulgarelli, B., Heege, T., 2015. Sensor independent adjacency correction algorithm  
698 for coastal and inland water systems. *Remote Sens Environ* 157, 85–95.

699 Landier, L., Gastellu-Etchegorry, J.P., Al Bitar, A., Chavanon, E., Lauret, N., Feigenwinter, C.,  
700 Mitraka, Z., Chrysoulakis, N., 2018. Calibration of urban canopies albedo and 3D  
701 shortwave radiative budget using remote-sensing data and the DART model. *Eur J Remote*  
702 *Sens* 51, 739–753. <https://doi.org/10.1080/22797254.2018.1462102>

703 Lewis, P., 1999. Three-dimensional plant modelling for remote sensing simulation studies  
704 using the Botanical Plant Modelling System. *Agronomie* 19, 185–210.

705 Liang, S., Fang, H., Chen, M., 2001. Atmospheric correction of Landsat ETM+ land surface  
706 imagery. I. Methods. *IEEE Transactions on geoscience and remote sensing* 39, 2490–2498.

707 Mekler, Y., Kaufman, Y.J., 1980. The effect of Earth’s atmosphere on contrast reduction for a  
708 nonuniform surface albedo and ‘two-halves’ field. *J Geophys Res Oceans* 85, 4067–4083.

709 Miesch, C., Poutier, L., Achard, V., Briottet, X., Lenot, X., Boucher, Y., 2005. Direct and  
710 inverse radiative transfer solutions for visible and near-infrared hyperspectral imagery.  
711 *IEEE Transactions on Geoscience and Remote Sensing* 43, 1552–1562.

712 Minomura, M., Kuze, H., Takeuchi, N., 2001. Atmospheric correction of visible and near-  
713 infrared satellite data using radiance components: an improved treatment of adjacency  
714 effect. *Journal of the Remote Sensing Society of Japan* 21, 260–271.

715 Mishchenko, M.I., Travis, L.D., Lacis, A.A., 2006. Multiple scattering of light by particles:  
716 radiative transfer and coherent backscattering. Cambridge University Press.

717 Ouaidrari, H., Vermote, E.F., 1999. Operational atmospheric correction of Landsat TM data.  
718 *Remote Sens Environ* 70, 4–15.

719 Pan, Y., Bélanger, S., Huot, Y., 2022. Evaluation of atmospheric correction algorithms over  
720 lakes for high-resolution multispectral imagery: Implications of adjacency effect. *Remote*  
721 *Sens (Basel)* 14, 2979.

722 Pearce, W.A., 1986. Monte Carlo study of the atmospheric spread function. *Appl Opt* 25, 438–  
723 447.

724 Petty, G.W., 2006. A first course in atmospheric radiation. Sundog Pub.

725 Qi, J., Xie, D., Yin, T., Yan, G., Gastellu-Etchegorry, J.-P., Li, L., Zhang, W., Mu, X., Norford,  
726 L.K., 2019. LESS: Large-Scale remote sensing data and image simulation framework over  
727 heterogeneous 3D scenes. *Remote Sens Environ* 221, 695–706.

728 Ramon, D., Steinmetz, F., Jolivet, D., Compiègne, M., Frouin, R., 2019. Modeling polarized  
729 radiative transfer in the ocean-atmosphere system with the GPU-accelerated SMART-G  
730 Monte Carlo code. *J Quant Spectrosc Radiat Transf* 222, 89–107.

731 Regaieg, O., 2023. Modélisation du transfert radiatif de la fluorescence induite par le soleil, de  
732 l'émission thermique et du bilan radiatif des couverts végétaux 3D: vers un modèle SIF  
733 complet.

734 Reinersman, P.N., Carder, K.L., 1995. Monte Carlo simulation of the atmospheric point-spread  
735 function with an application to correction for the adjacency effect. *Appl Opt* 34, 4453–  
736 4471.

737 Richter, R., 1990. A fast atmospheric correction algorithm applied to Landsat TM images.  
738 Title REMOTE SENSING 11, 159–166.

739 Rubinstein, R.Y., Kroese, D.P., 2016. Simulation and the Monte Carlo method. John Wiley &  
740 Sons.

741 Santer, R., Schmechtig, C., 2000. Adjacency effects on water surfaces: primary scattering  
742 approximation and sensitivity study. Appl Opt 39, 361–375.

743 Shettle, E.P., Fenn, R.W., 1979. Models for the aerosols of the lower atmosphere and the effects  
744 of humidity variations on their optical properties. Optical Physics Division, Air Force  
745 Geophysics Laboratory.

746 Spada, F., Krol, M.C., Stammes, P., 2006. McSCIA: application of the Equivalence Theorem  
747 in a Monte Carlo radiative transfer model for spherical shell atmospheres. Atmos Chem  
748 Phys 6, 4823–4842.

749 Sterckx, S., Knaeps, S., Kratzer, S., Ruddick, K., 2015. SIMilarity Environment Correction  
750 (SIMEC) applied to MERIS data over inland and coastal waters. Remote Sens Environ  
751 157, 96–110.

752 Sun, B., Schäfer, M., Ehrlich, A., Jäkel, E., Wendisch, M., 2021. Influence of atmospheric  
753 adjacency effect on top-of-atmosphere radiances and its correction in the retrieval of  
754 Lambertian surface reflectivity based on three-dimensional radiative transfer. Remote  
755 Sens Environ 263, 112543.

756 Veach, E., Guibas, L.J., 1997. Metropolis light transport, in: Proceedings of the 24th Annual  
757 Conference on Computer Graphics and Interactive Techniques. pp. 65–76.

758 Vermote, E.F., Tanré, D., Deuze, J.L., Herman, M., Morcette, J.-J., 1997. Second simulation of  
759 the satellite signal in the solar spectrum, 6S: An overview. IEEE transactions on  
760 geoscience and remote sensing 35, 675–686.

761 Vilfan, N., Van der Tol, C., Muller, O., Rascher, U., Verhoef, W., 2016. Fluspect-B: A model  
762 for leaf fluorescence, reflectance and transmittance spectra. *Remote Sens Environ* 186,  
763 596–615.

764 Wang, Y., Gastellu-Etchegorry, J.-P., 2021. Accurate and fast simulation of remote sensing  
765 images at top of atmosphere with DART-Lux. *Remote Sens Environ* 256.  
766 <https://doi.org/10.1016/j.rse.2021.112311>

767 Wang, Y., Gastellu-Etchegorry, J.-P., 2020. DART: Improvement of thermal infrared radiative  
768 transfer modelling for simulating top of atmosphere radiance. *Remote Sens Environ* 251,  
769 112082.

770 Wang, Y., Kallel, A., Yang, X., Regaieg, O., Lauret, N., Guilleux, J., Chavanon, E., Gastellu-  
771 Etchegorry, J.-P., 2022. DART-Lux: An unbiased and rapid Monte Carlo radiative transfer  
772 method for simulating remote sensing images. *Remote Sens Environ* 274, 112973.

773 Wang, Y., Lauret, N., Gastellu-Etchegorry, J.-P., 2020. DART radiative transfer modelling for  
774 sloping landscapes. *Remote Sens Environ*. <https://doi.org/10.1016/j.rse.2020.111902>

775 Widlowski, J., Taberner, M., Pinty, B., Bruniquel-Pinel, V., Disney, M., Fernandes, R.,  
776 Gastellu-Etchegorry, J., Gobron, N., Kuusk, A., Lavergne, T., 2007. Third Radiation  
777 Transfer Model Intercomparison (RAMI) exercise: Documenting progress in canopy  
778 reflectance models. *Journal of Geophysical Research: Atmospheres* 112.

779 Yin, T., Gastellu-Etchegorry, J.-P., Lauret, N., Grau, E., Rubio, J., 2013. A new approach of  
780 direction discretization and oversampling for 3D anisotropic radiative transfer modeling.  
781 *Remote Sens Environ* 135, 213–223.

782 Young, A.T., 1980. Revised depolarization corrections for atmospheric extinction. *Appl Opt*  
783 19, 3427–3428.

784