Teaching mathematics to non-specialists at university: a praxeological approach

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Abstract: In this paper, we focus on the practices of teachers who teach mathematics to non-specialist students. Referring to the anthropological theory of the didactic, we investigate university teachers' personal didactical praxeologies. Drawing on the literature on the specific issues met by non-specialist students, we identified five didactical types of tasks situated at the level of the discipline belonging to 'the praxeological problem of the teacher'. We conducted interviews with three experienced teachers teaching mathematics to first year non-specialist students about their practices and collected their resources. The data analyses indicated that the three teachers developed their didactical praxeologies for 'Teach basic mathematics' and 'Link mathematics and other disciplines'. The analysis at the lower levels (domain/sector/theme/question) and the consideration of the mathematical praxeologies they proposed to their students allowed us to question the relevance of the actual implementation of the techniques they use.

Keywords: Anthropological theory of the didactic, Teacher and student practices at university level, Teaching and learning of mathematics in other fields, Teaching mathematics to non-specialist students, Learning of mathematics by non-specialist students

1. Introduction

Research has recently been conducted on mathematics for non-specialist students at the university level (Biza et al., 2016; Biehler et al., 2022). The expression 'non-specialist students' encompasses a variety of situations but broadly describes students who are not studying pure mathematics. They usually specialise in other disciplines, such as biology, health or computer science; here, we focus only on STEM subjects. The students could be enrolled in highly selective courses (e.g. engineering schools) or in courses accessible to all students with a secondary school diploma (e.g. calculus for first year students in a given course). Nevertheless, in all these situations, studies of mathematics education have shown that the learning of mathematics raises specific issues for these students.

Teaching at university level is another theme that has recently attracted considerable research interest (Biza et al., 2016; Biehler et al., 2022). Since non-specialist students face specific issues in their learning of mathematics, it can be hypothesised that their teachers develop specific teaching practices to support them and address these issues. However to date, research in university mathematics education has never addressed the question: Do teachers who teach mathematics to non-specialist students develop specific practices for these students, and if so, what are the features of these practices? Our study intends to fill this gap in the literature concerning first-year non-specialist students.

In a previous study (Gueudet et al., 2022), analysis of teachers' interviews enabled us to identify teaching practices likely to be specific for non-specialist students. In the work presented here, we revisit the same data to deepen our analysis of specific practices. In the next section, we present background literature concerning university teachers' practices and the theoretical framework which guided our study: the anthropological theory of the didactic (ATD; Chevallard, 1999, 2002). In section 3, we draw on the

research literature concerning the difficulties met by non-specialist students to identify potential specific aspects in the practices used by their teachers; we also present our research questions. In section 4, we introduce our methods, in section 5, we present our results and discuss them in section 6.

2. Related work and theory

2.1 Studies concerning teaching non-specialist students

Studies concerning teachers' practices for non-specialist students are rare. While focusing on mathematics in engineering education, Pepin et al. (2021) observed that, while several studies addressed the issues of instructors' expectations and their views on the mathematics courses that should be taught to future engineers, only a few have investigated the ordinary practices of teachers in their mathematics courses for future engineers. Focusing on the connections made by teachers in their courses for non-specialists, Sabra (2019) analysed interviews with three teachers teaching future engineers. Their study provided evidence that the teachers' different backgrounds influenced the connections they made in their courses, such as those between different mathematical concepts, for a mathematician; those between mathematics and other disciplines, for a physicist; and those with engineering courses, for an engineer. Considering that courses for future engineers are a specific institution, González-Martín and Hernandes-Gomes (2020) used ATD (Chevallard, 1999) to study teachers' practices in this institution. As we use the same approach in our work, we present our theoretical framework before returning to González-Martín and Hernandes-Gomes' (2020) study.

2.2 ATD and didactical praxeologies

ATD (Chevallard, 1999) considers that the knowledge taught is shaped by institutions, here defined as any socially legitimate group. While secondary schools and universities

are institutions, a mathematics or physics course for first year students is also considered as an institution. In ATD, how knowledge is shaped is described through the concept of praxeology. A praxeology comprises four elements: a type of tasks T; a technique τ to perform this type of tasks; a technology θ, which is a discourse explaining the technique; and a theory Θ, grounding the technology. According to Chaachoua (2020), a technique is itself composed of a set of types of tasks, each of which is called a 'ingredient of technique'.

In mathematical praxeologies, the type of tasks concerns mathematics (e.g. T_{cf}: 'Calculating with fractions'). This type of tasks can be present in mathematics and other disciplines. In other courses, students probably meet this type of task rather as an ingredient of technique; for example, in chemistry, as an ingredient of technique for addressing the type of tasks 'Compute a concentration'. Chevallard (2002) also considered that the teacher faces a *praxeological problem*; this means that the teacher has to identify didactical types of tasks (e.g. 'Teach the mathematical praxeology associated with T_{cf}') and develop didactical praxeologies associated with them. Didactical praxeologies comprise a didactical type of tasks, a didactical technique and technology; the didactical theory usually remains implicit. The didactical and mathematical praxeologies mutually influence each other (Bosch & Gascón, 2002).

2.3 Studying teachers' practices at universities with ATD

González-Martín and Hernandes-Gomes (2020) studied teachers' practices in engineering education. In particular, they investigated how different relationships with mathematics, which resulted from teachers' different backgrounds, influence teachers' didactical praxeologies. They focused on two teachers with different academic and professional backgrounds who supervised capstone projects for future engineers (one of

them had significant professional experience as an engineer, while the other had limited experience in the field. The students worked on these projects during the last three semesters of their training (years 4 and 5 at university)—using previously acquired theoretical and practical knowledge, particularly mathematical knowledge. González-Martín and Hernandes-Gomes (2020) analysed the interviews conducted with these two teachers and provide evidence that the two teachers had different personal relationships with mathematics, for example, concerning the rigour needed when using mathematics in engineering tasks. Both teachers declared that their background influenced this relationship. The two teachers developed different didactical praxeologies; the projects they proposed were different (e.g. the mathematical component was very important for one of them but not for the other). The teacher who worked as an engineer incorporated workplace practices in their work with the students.

The present study has points of convergence but also differs from the study by González-Martín and Hernandes-Gomes (2020). Like these authors, we tried to identify didactical praxeologies developed by university teachers using interviews with these teachers. However, we do not only focus on courses for future engineers as we are interested in all types of non-specialist students. What is more, our aim is not to investigate how different teachers' backgrounds influence the personal didactical praxeologies they developed but to analyse didactical praxeologies that are potentially specific to non-specialist students. Do teachers identify teaching challenges linked with mathematics for non-specialists? Do they tackle these challenges, and if so, how?

Florensa et al. (2018) considered the *praxeological problem* of teachers (Chevallard, 2002). In the context of professional development courses at the university, they collected professional questions proposed by the participants (lecturers), and then analysed the questions to study the teaching challenges that lecturers formulate (i.e. the

didactical types of tasks they identify). Florensa et al. (2018) referred to the concept of levels of didactic co-determinacy (Chevallard, 2002). The conditions and constraints underpinning any teaching or learning process can be located and analysed at different levels, classified on a scale extending from the more general to the more precise viewpoint. This scale can be separated between an 'upper scale' (Humanity <> Civilisation <> Society <> School <> Pedagogy) and a 'lower scale' (Discipline <> Domain <> Sector <> Theme <> Question). Florensa et al. (2018) classified the questions proposed by the lecturers according to their level of didactic co-determinacy. For example, 'How to deal with student diversity?' was situ ated at the pedagogy level, while 'How to relate the subject with the real world?' was situated at the discipline level. Analysing the collected questions, the authors identified a phenomenon they termed 'pedagogical generalism': a separation made by the lecturers between the instructional processes and their precise mathematical content.

In our study, we focus on didactical praxeologies situated on the lower scale, i.e. concerning the following five levels: discipline, domain, sector, theme and question. We chose this focus because we hypothesise that these levels will allow us to investigate the links between didactical and mathematical praxeologies.

In a previous study (Gueudet et al., 2022), we interviewed teachers and by analysing their interviews, we identified the different didactical types of tasks they addressed, Some of these types of tasks were likely to be specific for non-specialist students. For the present study, we went back to the literature concerning the difficulties met by non-specialist students to identify didactical types of tasks that could belong to the praxeological problem of their teachers.

3. Potential challenges in teaching non-specialists and didactical types of tasks

Difficulties with mathematics are an important cause of failure in courses for non-specialist students (González-Martín et al., 2021). Some of these difficulties can also concern mathematics majors; here we are interested in difficulties that are likely to be specific, or especially severe, for first year non-specialist students and potentially create challenges for their teachers. Our aim is not to draw up an exhaustive list but to identify some difficulties that are likely to be specific for non-specialist students, and therefore, part of the praxeological problem of their teachers. We associate each of the difficulties identified with a didactical type of tasks. The aim of our study is to investigate the personal praxeologies developed by teachers for these types of tasks.

One of the issues demonstrated by the literature concerns the disconnection between the mathematics taught in mathematics courses and that present in the courses of other disciplines (Faulkner et al., 2020; González-Martín et al., 2021; Harris et al., 2015). The vocabulary, the notations and, more generally, the discourse used in these courses can differ (Flegg et al., 2011; Viirman & Nardi, 2019); moreover, the mathematics courses can emphasise theoretical aspects that are not present when mathematics is part of other courses (González-Martín & Hernandes-Gomes, 2017). This disconnection can result in the students perceiving the mathematics courses as useless (Faulkner et al., 2020) or perceiving the usefulness of mathematics only as 'foundational for methods used in the other disciplines' (Jablonka & Bergsten, 2022). Thus, for our study, we retained the didactical type of tasks 'Link mathematical content to other course content (T_{lmo})'.

One possible direction for improving this situation by making connections between mathematics and the other discipline courses is related to the practice of modelling in mathematics lessons. Several authors (e.g. Härterich et al., 2012; Quéré, 2022; Viirman

& Nardi, 2019) have demonstrated that modelling can contribute to building connections between mathematics and other disciplines. Moreover, modelling competencies are needed in, for example, physics courses (Redish, 2017). However, modelling can be difficult for first year students, and some of the difficulties encountered by non-specialist students are linked to mathematical modelling (Doukhan, 2020). Accordingly, we are also interested in the type of task 'Teach mathematical modelling (T_{mm})'. In ATD, modelling is defined as 'a process of reconstruction of mathematical praxeologies that become progressively broader and more complex' (Barquero et al., 2013, p. 312). In the present study, we use a weaker meaning for modelling, encompassing incomplete mathematical modelling activities. In fact, we consider it unlikely that teachers teach their students the full process of reconstructing mathematical praxeologies; nevertheless they may try to teach some aspects of modelling, using contextualised exercises. In the example described in part 5.2.1, we consider that it is a modelling activity first because the teacher uses this term in the course and second because the technique he proposed consists of identifying the random experiment, the universe, the probability measure and the different events. According to Doukhan (2020), these elements constitute a first key step in the probabilistic modelling activity at the start of a university course.

Another possible source of difficulties at the beginning of university that has been increasingly investigated recently concerns prior knowledge (i.e. secondary school knowledge needed for the university courses) (Rach & Ufer, 2020). Mastering mathematical prerequisites is an important factor for success in scientific studies, for mathematics courses as well as for courses in other disciplines (Culpepper et al., 2010; Greefrath et al., 2017). The lack of prior knowledge can be more severe for non-specialist students, since some of them may have had difficulties in mathematics in secondary school; moreover, depending on the country, they may have taken fewer mathematics

courses in secondary school. Linked with this potential challenge, we retained the didactical type of tasks 'Teach basic mathematics' (T_{bm}).

The consequences for non-specialist students' of difficulties they experienced with mathematics at secondary school can also concern affective dimensions. Some non-specialist students enter university with low self-efficacy expectations in mathematics (i.e. the students' confidence in their own mathematical abilities is low), which can further decline during their first year at university (Kürten, 2017). This is a major problem since mathematical self-efficacy is a central factor for retention and success in STEM studies (Lin et al., 2018; Tossavainen et al., 2021). This led us to consider the didactical type of tasks 'Restore students' confidence in their mathematical abilities (T_{scm})'.

Eichler and Gradwohl (2021) demonstrated that motivational factors (self-efficacy, self-concept, interest and values) have a significant influence of on first year engineering students' achievement in mathematics. We hypothesise that non-specialist students are especially subject to a low motivation for mathematics (compared to mathematics majors) and thus retained the didactical type of tasks 'Foster students' interest and engagement in mathematics (T_{iem})'.

To sum up, we identified five didactical types of tasks that belong to the 'praxeological problem' of the teacher for non-specialist students: (1) Link mathematical content to other course content (T_{lmo}), (2) Teach mathematical modelling (T_{mm}), (3) Teach basic mathematics (T_{bm}), (4) Restore students' self-confidence in their mathematical abilities (T_{scm}) and (5) Foster students' interest and engagement in mathematics (T_{iem}).

These five types of tasks are situated at the discipline level. This is straightforward for T_{bm}, T_{mm} and T_{lmo} but may be more questionable for T_{scm} and T_{iem}. The students' confidence in their mathematics abilities and their interest in mathematics are linked with the features of the discipline. For example, lack of interest may be due to difficulty in

understanding the structure and the role of mathematical proofs. Demonstrating the need for proofs is a technique that can be used by mathematics teachers to arouse their students' interest, and it clearly belongs to the discipline level. These five types of tasks at the discipline level also entail subtypes of tasks at the domain/sector/theme/question levels. For example, 'Teach calculation with fractions (to first year science students)' is a subtype of T_{bm}. Investigating the didactical praxeologies corresponding to such subtypes of tasks can help refine our understanding of teachers' practices.

We do not claim that these are the only didactical types of tasks encountered by mathematics teachers with non-specialist students at universities. However, these types of tasks are important in our context, since they correspond to the difficulties faced by non-specialist students provided as evidence by previous research. In addition, we do not claim that all teachers develop personal praxeologies associated with these types of tasks. In fact, an important aspect of the praxeological problem of teachers is that they have to identify the didactical types of tasks they need to tackle (Chevallard, 2002). Some teachers might consider that the difficulties met by non-specialist students are the result of insufficient preparation at secondary school (what Pinto & Koichu, 2022, call 'the deficit discourse' of teachers) and will not develop praxeologies associated with the five types of tasks cited above. Of course, when dealing with personal praxeologies, one must also be aware of possible biases in the analysis; in particular, teachers' descriptions of their practices are not necessarily objective. We return to this issue in the Methods section.

The research questions we address in this paper are the following:

RQ1: Do teachers who teach non-specialist students develop didactical praxeologies associated with the types of tasks (1) to (5)?

RQ2: For teachers who develop such praxeologies (level of the discipline), which are the didactical techniques they declare they are using, do they justify their choice of techniques and if so, how (didactical technologies)?

RQ3: Which didactical praxeologies do these teachers develop at the levels of the domain/sector/theme/question for subtypes of tasks of types of tasks (1) to (5), and how are they linked with mathematical praxeologies?

4. Methods

In the study reported here, we re-used data collected in the context of a project concerning the practices of teachers who teach mathematics to non-specialist STEM students. Here, we present the method we used for data collection in this context, the data selected for the present study, and how we analyse these data to answer our research questions.

4.1 Data collection

We designed an interview protocol (see Appendix) for semi-structured interviews to be conducted in the context of a project concerning teaching practices for non-specialist students in a French research network called Didactics and Epistemology of Mathematics, Computer science and Physics at Tertiary level (French acronym DEMIPS).

The interview was divided into 4 parts. In part 1, after general questions about their background and teaching experience, we asked the teachers to specify the courses they would be teaching during the academic year. In part 2, they were asked to focus on a particular course, which they were free to choose under the explicit condition of being able to share the material they used with us. We asked them to present the course, how it was organised and their role and freedom in its management. In part 3, they were asked how the course was prepared, what resources they generally used and the resources the

institution possibly offers to students (e.g. a digital platform and the texts of previous exams). We collected resources such as slides, exercises sheets and sometimes sheets with some corrected exercises. We also asked them about their views on the needs and potential difficulties of students in this particular course. Finally, in part 4, we asked the teachers about the practices they adopted to prepare for this course and its assessment (the resources they designed, how they designed them and their collective work with other teachers) and for implementing it with the students. Ten teachers were interviewed in the DEMIPS network.

We selected three of these teachers for the present study according to the following criteria. The teachers teach a course entirely focused on mathematics to first year non-specialist students. We selected experienced teachers (i.e. with more than 10 years of teaching experience), assuming that the didactical praxeologies they had developed were likely to be stabilised. That is, these teachers had had the opportunity to test their techniques and adjust their praxeologies if they did not enable them to achieve their objectives. We hypothesise they have developed more complete didactical praxeologies than novice teachers, in particular in that they incorporate technological elements. The profiles of the teachers and the chosen courses are presented in Table 1 below.

Teacher	Rob	Pete	Gene
PhD in	Theoretical Physics	Mathematics	Mathematics
Personal information	Male, 16 yrs teaching experience	Male, 28 yrs teaching experience	Male, 18 yrs teaching experience
Chosen course	Remedial mathematics course for students who gave up in a STEM course	Mathematics for future chemistry engineers	Probabilities for biology students

Table 1. Profiles of the three teachers selected and the courses they taught.

The background elements and the courses taught listed in the table are varied. They are mentioned only to inform the reader about the profiles of the three teachers and their teaching context. Our aim was not to investigate factors likely to influence the teaching practices. We wanted to know whether, when describing their practices, the teachers would mention the five didactical types of tasks that interest us here, and if so, which techniques and technologies they associate with them.

4.2 Data analysis

All the interviews were recorded and transcribed. For the praxeological analysis of these interviews focused on types of tasks (1) to (5), we used parts 3 and 4 of the interviews.

Since the parts of the interview we used here concern students' needs and teachers' practices, the three teachers did not use exactly the same expressions we chose here to characterise the types of tasks. Consequently we searched the interviews for terms associated with these types of tasks, and summarise them in table 2 below.

$\mathbf{T_{lmo}}$	Connections (with other disciplines); Links (with other disciplines); Context (of other disciplines)
$\mathbf{T_{mm}}$	Model; Modelling
$\mathbf{T_{bm}}$	Mathematical content learned before grade10, e.g.: Fractions; First-degree equations
$\mathbf{T_{scm}}$	Confidence (lack of); worried; anxious
$\mathbf{T_{iem}}$	Interest; motivation; engagement

Table 2. Terms used to identify the types of tasks in the teachers' answers.

When we identified a type of tasks mentioned by a teacher, we searched that teacher's discourse for the didactical techniques evoked and the associated technologies. We systematically compared each teacher's declaration with the resources he provided: Does the teacher actually propose, e.g., exercises contextualised in another discipline? During his course, does he recall methods concerning, e.g., computation with fractions? Comparing the discourse and the resources designed by the teacher for his students could

confirm or lead him to amend a didactical technique or technology. We also analysed the mathematical content of the resources collected in terms of praxeologies and investigated the links between didactical praxeologies at the domain/sector/theme/question levels and mathematical praxeologies.

5. Results

5.1. Didactical praxeologies at the discipline level, specific for non-specialist students

In Table 3 below, we note whether the teachers evoked a didactical praxeology for types of tasks (1) to (5) and then discuss these praxeologies.

	Rob	**Pete**	**Gene**
(1) Link mathematical content to other course content (T_{lmo})	Yes	Yes	Yes
(2) Teach mathematical modelling (T_{mm})	No	No	Yes
(3) Teach basic mathematics (T_{bm})	Yes	Yes	Yes
(4) Restore students' self-confidence in their mathematical abilities (T_{scm})	Yes	No	No
(5) Foster students' interest and engagement in mathematics (T_{iem})	Yes	Yes	No

Table 3. Praxeologies evoked by the teachers.

5.1.1 Link mathematical content to other courses content (T_{lmo})

The didactical type of tasks T_{lmo} was mentioned by the three interviewees with different techniques. To connect the mathematical content and the content of other courses, Rob gave his students contextualised exercises that he created by researching resources from other disciplines. He also introduced the students to the mathematical concepts used in other scientific domains (e.g. in physics). He justified this by explaining, 'I feel like it

works because it shows students that maths is connected to a lot of things, that it's not just maths for maths' sake, but it's also tools for reasoning and whatever course they're going to take they're going to have to reason'. Like Rob, Gene also presented his students with contextualised examples. In Gene's case, the mathematical examples were presented in a biological context and thus were linked with biology courses. For instance, he used chemistry situations (solutions to be diluted) in his examples related to analysis. Gene build his examples following discussions with his biology colleagues. He explained, 'Through these discussions we can come up with exercises that create a link with biology'. Gene looked for examples of how biologists use statistical tools for data analysis, or how they collect their data, etc. Finally, Pete declared that, for some chapters, he gave end-of-lesson exercises contextualised in other disciplines. We interpreted this to mean that linking mathematics and other disciplines was not a central objective for him.

5.1.4 Teach mathematical modelling (T_{mm})

The didactical type of tasks T_{mm} was only mentioned in the interview with Gene. Since it concerns a probability course, Gene did not explain how to model in mathematics in general but did explain how to model a discrete random experiment. In part 5.2.1 we investigate the didactical praxeologies developed by Gene associated with the type of tasks T_{pm} 'Teach probabilistic modelling in the discrete case' (sector level).

5.1.3 Teach basic mathematics (T_{bm})

Pete considered teaching basic mathematics tools to be necessary but did not systematically teach them himself, since his engineering school dedicated one week at the beginning of the school year to autonomous work on these basic mathematical tools for students who had just obtained their Baccalaureate (end of secondary school exam in

France, equivalent of A-Levels in the British system, of a High School Diploma in the USA). The technique used by Rob for T_{bm} involved making students do many practice exercises to develop their procedural fluency with basic contents: e.g. developing, factoring or solving first-degree equations. He gave the students long lists of exercises. Some of these were collectively solved in the tutorials, while for the others, corrections were provided and the students were invited to attempt the exercises as homework. Rob said in his interview, 'The most important is to practise, practise, practise to improve their fluency [...]. It is important that they have the solutions, so that they can try, try, repeat exercises and check if their solution is correct'. Gene also tackled this type of tasks. It was a specific query posed by the biology teachers that caused huge difficulties for students with these basic competencies. He tried to improve the students' basic computational skills by providing exercises on discrete probabilities that only require basic mathematical notions. This is an indirect way to practise percentages or fractions: "Discrete probabilities allow us to cover these notions, whereas if we give students exercises about fractions, they might be upset."

5.1.4 Restore students' self-confidence in their mathematical abilities (T_{scm})

Rob aimed to restore his students' self-confidence in terms of their mathematical reasoning. He urged them to check each step of their reasoning to be sure it's correct: 'The aim is [...] to give them confidence and make them feel at ease with the basic tools of calculation, and also to teach them to reason step-by-step'. Neither Pete nor Gene mentioned the need to restore their students' self-confidence in mathematics.

5.1.5 Foster students' interest and engagement in mathematics (T_{iem})

To foster students' interest and engagement in mathematics (T_{iem}), Rob chose

contextualised exercises (that referred to physics but also to daily life). He justified this choice by explaining that it is likely to foster students' engagement but also that students are used to contextualised exercises at secondary school. Another technique Rob used for the didactical type tasks T_{iem} was to explain to the students that they will need these mathematical tools in their future professional activities or in their training. He also tried to show the fun aspect of mathematics and their 'calming abstraction': 'It's almost like therapy, this slightly abstract side of mathematics. You're cut off from emotional things, problems and so on. It's a bit of an abstract world and it also has a bit of a calming effect.'

Pete stated, 'I would think that we are not really here to foster students' interest... We are here to make them learn things'. Nevertheless, he provided mathematical videos that he found interesting, because they were visually attractive and called up historical aspects, the origin of the mathematical concepts, to foster the students' engagement. We consider this to be a technique for T_{iem}. Gene did not use the term 'interest' or 'motivation' in the interview. For this reason, we consider he did not refer to a personal praxeology for T_{iem}.

5.2. Examples of didactical praxeologies at the domain/sector/theme/question levels, links with mathematical praxeologies

In this section, we use two examples to highlight links between teachers' didactical praxeologies and the mathematical praxeologies taught to students.

5.2.1 Teach probabilistic modelling in the discrete case, Gene

In the first example, we focus on the probability course taught by Gene, and recall that we interpreted what Gene declared in his interview as the description of a didactical praxeology for the type of tasks 'Teach probabilistic modelling', since he presented it in this way himself. Nevertheless, we claim that understanding what he describes in this way

requires considering the lower levels (domain, sector, theme and/or question) and analysing the resources he designed and used with the students. We chose the following example from his probability course to highlight this important fact.

> A lake contains roach and trout. 30% of the roach are of breeding age, 60% of the trout are of breeding age. We know that roach represent 80% of the fish in the lake. We catch a fish (with equiprobability). What is the probability of catching a fish of breeding age?

This example presents the computation of a simple probability by using the total probability formula; it is a task encountered at secondary school in France. It does not correspond to an authentic situation and can even be considered a model of an authentic situation. Equiprobability is assumed for the sake of simplicity, and some percentages are already given. Nevertheless, the statement is in natural language, and the universe is not explicitly described. Figure 1 presents the correction included in Gene's slide show.

R = catch a fish of breeding age ⊂ Ω = all fish capable of reproduction

G = catch a roach ⊂ Ω

T = catch a trout ⊂ Ω

$$P(G) = \frac{|G|}{|\Omega|} = \frac{80}{100}$$

$$P(T) = \frac{20}{100}$$

$$P(R|G) = \frac{P(R \cap G)}{P(G)} = \frac{\frac{|R \cap G|}{|\Omega|}}{\frac{|G|}{|\Omega|}} = \frac{30}{100}$$

So $P(R) = P(R|G)P(G) + P(R|T)P(T) = \frac{30}{100} \times \frac{80}{100} + \frac{60}{100} \times \frac{20}{100} = \frac{36}{100}$

Figure 1. Slide showing the correction of the "Roach and trout" example.

Gene aimed to teach probabilistic modelling by presenting several examples in the slide show of his course. There is no national curriculum at university but referring to the national French curriculum for grade 12, where this mathematical content is also taught

(BOEN, 2011), the example above belongs to the domain “probability and statistics”, sector “discrete probabilities”, theme “conditional probabilities and independence”. Furthermore, the type of tasks can be considered as belonging to the question: “How to calculate the probability of an event knowing its conditional probabilities related to a partition of the universe?”. For the sake of brevity, here we do not address the mathematical and didactical praxeologies at each of the four levels. We focus on the level of the sector: ‘Teach probabilistic modelling in the discrete case’. Indeed, in Gene's course, modelling a random experiment in the continuous case is done in a completely different way.

Using this example (and others), Gene presented his students with a mathematical technique for the type of tasks: ‘Modelling a random experiment.’ The technique consisted in identifying the random experiment, the universe, the probability measure, and the different events. For the “Roach and trout” example, he told the students that the outcome of the experiment is a particular fish and the set of possible outcomes is the set of all fish, so the universe is all the fish. Regarding the choice of the probability measure **P**, in the correction of this exercise Gene explained the link between uniform distribution and equiprobability to his students. Since there is no reason to choose one fish over another, they have to choose a uniform distribution, i.e. all fish can be caught with the same probability, called equiprobability. Gene continued to model the random experiment by defining specific events and interpreting the statement to obtain the probability values of these events. The final probabilities were calculated as the ratio of the number of favourable cases to the number of total cases.

For the didactical type of task ‘Teach probabilistic modelling in the discrete case’, Gene developed a didactical technique. He presented several examples, and explained to the students that, when doing such exercises, they should start as follows: either Ω (the

universe) and **P** (the probability measure) are given in the text, or they have to be determined. He emphasised the differences between secondary school and university: what is new at university is that students are asked to model the situation using a set **Ω** and a probability measure **P**, before doing these calculations.

According to his interview, this technique seems to be linked to technologies like: "at secondary school, the students did not learn to introduce the universe and the probability measure, since they were present in the text of the exercise" and "at secondary school, students systematically use probability trees for modelling conditional probabilities, and this mathematical technique is inadequate." According to Gene, using a probability tree is not appropriate because it reinforces the chronological conceptualisation of conditional probability in students.

We note that Gene is aware of the secondary school curriculum, and of the differences between this curriculum and what is expected at university. His didactical technologies are evidence of an epistemological reflection about probability trees and the model they convey. Nevertheless, concerning probabilistic modelling, through the analysis presented above, we can highlight that, even if in his interview, Gene says he teaches it, the associated didactical praxeology remains incomplete compared to a complete modelling activity (Barquero et al., 2011). The conditions and constraints of a first year course of lectures whose objective is rather to revisit the theoretical foundations of probability hinder the proposal of a task which is not already a model of an authentic situation or which involves a complete modelling activity.

5.2.2 Teach calculation with fractional algebraic expressions, Rob

The second example comes from Rob. We chose this example because it is evidence that considering the lower levels: domain, sector, theme and/or question is essential when

questioning the relevance of the didactical praxeologies developed by the teachers.

The reader will recall that Rob taught low-achieving students from different STEM courses. These students encountered serious difficulties during the first two months at university and were at risk of dropout. For this reason, they were taught a special course. Here we consider an exercise that is representative of exercises chosen by Rob to start a course.

Exercise 6 – session 2

Concerning a manipulation involving pouring an specified quantity of a liquid solvent (distilled water) from bottle *A* to bottle *B* thereby diluting hydrochloric acid molecules, as described in the left-hand part of the diagram below, what is the concentration by volume of the hydrochloric acid molecules in the solution in bottles *A* and *B* after the dilution (compared to that in bottle *A* before the operation)? Answer the same question for the diagram on the right.

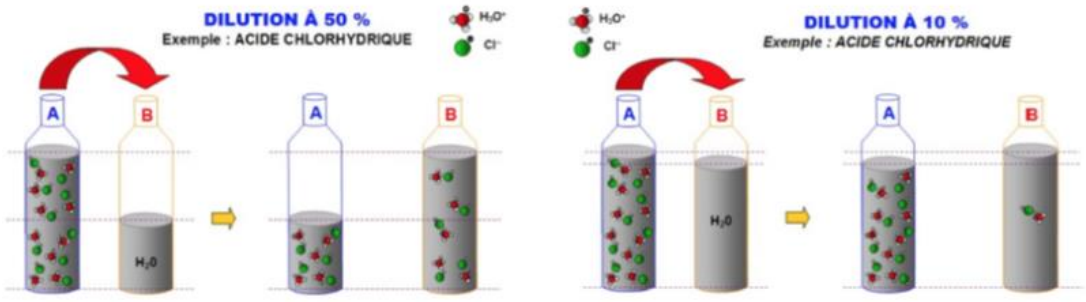

Figure 2. An exercise on fractions designed by Rob.

The solution to the first question (on the left of the diagram) the students are expected to produce is:

> Let *n* be the number of *HCl* molecules in bottle *A* before pouring, *V* its volume and $C_{HCl}(A)=n/V$ the corresponding concentration of *HCl*. In the diagram on the left this gives:

$C_{HCl}^{Left}(A)$= (n/2)/(V/2)=(n/2)*(2/V)=n/V= C_{HCl}(A), and

C_{HCl}^{Left} (B) = (n/2)/V=(1/2)*(n/V)=(1/2)* C_{HCl}(A).

As mentioned above, at the level of the discipline, for Rob, two important didactical types of tasks with these students were T_{iem} and T_{bm}. The didactical technique he used here for T_{iem} was to create and propose an exercise in the chemistry context to provide evidence for the usefulness of mathematical tools. It is associated with the didactical technology: "students often think that mathematics are useless, so proving mathematics are useful is likely to arouse the students' interest." This issue of usefulness is linked to the didactical technique used here for T_{bm}: propose an exercise where basic mathematical contents are used as tools. Rob's associated personal didactical technology is: "proposing an exercise using mathematics taught before grade 10 can lead students to recall this old knowledge, and justifies the need for a reminder if they do not succeed."

The mathematics involved in this task belong to French national curriculum for grade 10. Based on this curriculum (BOEN, 2019), this mathematical content belongs to the domain "Numbers and calculation", sector "Use algebraic calculation", theme "calculation with fractional algebraic expressions." We consider that the question here can be presented as "How to use fractional algebraic expressions to compute a concentration in a chemistry context?" For the sake of brevity, here we do not discuss the mathematical and didactical praxeologies at each of the four levels. We only focus on the level of the theme, "calculation with fractional algebraic expressions."

In terms of praxeologies in chemistry and mathematics, the task proposed here belongs to the type of tasks 'Compute a concentration after a dilution'. The technique expected by Rob involved introducing unknown quantities to express the initial concentration,

using information included in the text of the exercise to express how these quantities change during the dilution, writing an algebraic formula based on this information and finding a relationship between the initial and final concentrations. Solving an equation then provides the final concentration. The definition of concentration is the only technological element justifying this technique.

At the level of the theme, for the didactical type of tasks ‘Teach computation with fractional algebraic expressions’, Rob uses a didactical technique: propose an exercise where fractional algebraic expressions are used as tools. This didactical technique is justified by the didactical technology: “proposing an exercise using fractional algebraic expressions (taught at grade 10) can lead students to recall this old knowledge, and justifies the need for a reminder if they don’t succeed.”

However, the relevance of the choice of this particular exercise for implementing this didactical praxeology can be questioned. Observing that the concentration in A did not change and that in B was half of A could be quite straightforward: nothing has been poured into A, while B contains the same volume of water as A (before pouring) and half the molecules of HCl. Thus students could solve the exercise without using fractional algebraic expressions. Moreover, even in grade 10, students do not compute fractional algebraic equations involving different letters (BOEN, 2019). Thus this exercise, considered by Rob as dealing with ‘old’ knowledge, in fact proposes something with which the students are not familiar. Hence they are unlikely to “recall” anything; and the “reminder” presented by Rob after the exercise introduces new mathematical knowledge. Finally, the expected solution of this exercise involves a modelling phase: introducing unknown quantities, writing the relations, which is difficult for students and for which Rob does not appear to have developed a didactic praxeology.

Rob developed didactical praxeologies linked to the difficulties of non-specialist students; however, this example and the analysis at the level of the theme demonstrate that the implementation of this didactical praxeology was not always relevant.

5. Discussion

In this section, we first return to our research questions and present the elements of answers to the questions obtained from the analyses presented above. We then discuss the contributions of our work to research and the training of university teachers more generally and suggest avenues for further research.

In this study, we interviewed only three teachers, which is certainly a major limitation. Nevertheless, analysis of the data we collected produced partial answers to our research questions.

Our first research question was 'RQ1: Do teachers teaching non-specialist students develop didactical praxeologies associated with the types of tasks (1) to (5)?'

Two didactical types of tasks in our list were cited by each of the three teachers: T_{bm} and T_{lmo}. The other three types of tasks in our list of five were cited by two teachers: T_{iem} (mentioned by Rob and Pete); or only one teacher: T_{mm} (only mentioned by Gene); and T_{scm} (only mentioned by Rob). The number of respondents was too small to draw general conclusions from these observations. We hypothesise that T_{bm} can be tackled by many teachers for non-specialist students. The first year students may have different backgrounds and may therefore have lacked even basic mathematical practice in the years preceding their arrival at university. Teachers may have the same opinion concerning first year non-specialist students' lack of mastery of basic mathematics. Concerning T_{lmo}, the fact that all three teachers cited it is surprising, since the literature reports that non-specialist students complain about the absence of links

between courses in mathematics courses and courses in other disciplines (e.g. Faulkner et al., 2020). The three teachers we interviewed were experienced, dedicated teachers; however, this indicates the need to investigate the associated didactical praxeologies they developed and question their relevance.

It should be noted that, in the interviews, we did not ask the teachers direct questions to discover if they addressed those types of tasks. Instead, we asked them about the students' needs and their own practices and listened to see whether they spontaneously mentioned these didactical types of tasks in their interviews. The method we used did not allow us to identify whether the teachers who do not mention a type of tasks are not aware of it or deliberately choose not to address it (e.g. because they consider that it is not their responsibility to do so). In future research, we plan to present the types of tasks to the teachers and ask if they tackle them, and if they answer no, to ask the reasons for their choice.

Our second research question was 'RQ2: For teachers who develop such praxeologies (level of the discipline), what didactical techniques and associated didactical technologies do they use?'

These experienced teachers developed their own techniques to address the didactical types of tasks they tackled. Mostly, they also provided a rationale for their choice of technique (we consider it a personal didactical technology). The techniques varied from one teacher to another. For 'Teach basic mathematics', which they all addressed, Rob provided lists of basic exercises with their corrections because he wanted his students to practise. Gene 'concealed' work on basic mathematics in probability exercises because he did not want to upset his students with basic exercises. One technique used by all three teachers was contextualised exercises. This technique was used for both T_{iem} (by Rob) and T_{lmo} (by all three teachers, although Pete did not

systematically use such exercises). Rob's technological discourse suggests that T_{lmo} could be considered as a part of the technique for accomplishing T_{iem}. Indeed, linking mathematical content to other course content was used by Rob as a lever to foster students' interest and engagement in mathematics.

Are contextualised exercises likely to convince students of the usefulness of mathematics and to increase their interest in mathematics? Answering this question requires an analysis at the lower levels (domain/sector/theme/question), considering mathematical praxeologies (see the answers to RQ3 below).

Investigating the techniques and technologies informs us about the levels of didactic co-determinacy. Above, we explained why we considered T_{scm} to be at the discipline level. Our analyses confirmed this fact. Rob wanted to support his students' confidence in their own reasoning abilities. He urged them to check each step of their reasoning, which is clearly a mathematical practice. For T_{iem}, Rob proposed contextualised exercises because they provide evidence for the usefulness of mathematics, which is also linked with the discipline. Pete showed his students mathematical videos that presented historical elements and the origins of the concepts learned. This reflects not only a change in the media used but also a choice linked with the discipline.

Concerning RQ3: 'Which didactical praxeologies do these teachers develop at the levels of the domain/sector/theme/question, for subtypes of tasks of types of tasks (1) to (5), and how are they linked with mathematical praxeologies?', above, we presented some examples of didactical praxeologies applied by Gene at the level of the sector for the didactical type of tasks 'Teach probabilistic modelling in the discrete case' and by Rob at the level of the theme for the didactical type of tasks 'Teach calculation with fractional algebraic expressions.'

We chose these two examples to demonstrate that an analysis at the lower levels (domain/sector/theme/question) is required to understand what the didactical techniques used by the teachers really are and to question their relevance.

Gene was aware of the secondary school curriculum and mathematical praxeologies. He wanted his students to develop new mathematical praxeologies, in particular for modelling discrete probabilities. Gene did not want his students to use the representation by a probability tree. In secondary schools in France, on the contrary, probability trees play a central role in mathematical praxeologies (Doukhan, 2022). Gene knew why he wanted to ban probability trees (they can convey a wrong meaning). Nevertheless, it does not seem that the didactical praxeology he has developed can provide students with an alternative effective tool, and convince them to abandon the use of probability trees. Moreover, in his interview, Gene said he taught mathematical modelling to his biology students after discussing it with his biology colleagues. However, the analysis conducted above provides evidence that he only taught certain aspects of modelling. This is in line with the results of previous research which provided evidence that 'applicationism' is the dominant epistemology at the university level (Barquero et al., 2011). The mathematical contents are taught in mathematical courses and should then be applied to other disciplines, with no specific work on modelling. Accordingly, we hypothesise that the didactical praxeology for T_{mm} is rare in the institution.

Rob develop a didactical praxeology for 'Teach calculation with fractional algebraic expressions.' However, the mathematical praxeologies in the exercises he proposed were much more complex than those actually used in secondary schools. Moreover, the exercise he considered as evidence for the usefulness of computing with fractions can be solved without using such calculation; its potential for increasing the

interest of students is thus limited. Proposing exercises originating from another discipline, and using basic mathematical content may appear to be a relevant didactical technique for T_{bm} and T_{iem} at the level of the discipline, but at the level of the theme, other techniques are required for the choice of relevant exercises; it seems that Rob did not use such techniques.

Our analysis of the mathematical praxeologies highlights the fact that at the lower levels (domain/sector/theme/question), Gene and Rob, and possibly other teachers, seem to be lacking some didactical praxeologies.

In addition to providing these answers to our research questions, this study contributes to the field of research on teaching non-specialist students at universities. It provides evidence that, despite the different types of institutions involved (e.g. university level engineering schools and first year of biology at university), teachers try to perform the same didactical types of tasks, particularly to teach basic mathematics and to link mathematics with other disciplines. In terms of didactical praxeologies, the approach can provide evidence for personal praxeologies developed at the level of the discipline, but also, through the analysis of the mathematical praxeologies proposed by the teachers, identify didactical praxeologies that are missing at the lower levels (domain/sector/theme/question). Our study can also contribute to the training of university teachers by presenting didactical types of tasks they could address and organising collective work to develop relevant didactical praxeologies.

This study opens up several avenues for further research. We will complement our interviews by classroom observations and triangulate the teachers' declarations, the resources they designed and their actual practices. This qualitative study could also be completed by an online survey, which would allow us to reach a significant number of teachers.

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Appendix

Interview protocol

Teaching mathematics to first year non-specialist students

Ahead of the interview, the teachers were informed that the interview concerned their teaching of mathematics to first year students, in general and for a specific course they were invited to choose amongst the courses they teach non-specialist students. For this specific course, we asked them to bring the resources they use with them to the interview.

Part 1. Presentation Personal details?

- Family name, given name, age, experience (number of years teaching at tertiary level)
- Current position, teaching component, etc.
- What initial training did you receive? Have you done any specific teacher training?
- Which course(s) did you teach at the University in 2021/2022, and to which students?

Part 2. Presentation of the chosen course

- Please explain the general organisation of the course:
    - What methods of instruction did you use: lectures, tutorials, projects? (please state duration of each in hours)
    - Are any hybrid or distance learning courses planned as a result of the experience gathered during the COVID crisis? (length of such a course in hours + methods).
- What method(s) of instruction do you use in this course or module?
- What was your degree of freedom and your role in the choice of the course contents and the method of instruction ?

Part 3. Student needs, resources, preparation (for the chosen course)

Resources

- What resources (course handouts, exercise booklet, videos, platform etc.) are provided by the institution for this course?
    - by the university in general
    - by the course team
    - others?

Preparation

- How did you prepare this course?
- Did you discuss the course with colleagues from the same or other disciplines?
- What resources did you actually use?

    - among those offered by the institution
    - from those available elsewhere (thematic digital university; platforms, online exercises, etc.)
- What resources did you create yourself?
    - for your own use
    - to make available to students

Needs

- What do you identify as specific needs for the students who take this course, in terms of mathematical content or skills?
- What mathematical difficulties and lacks do you usually observe among these students?

At the end of this part of the questionnaire, the teacher presented the resources they brought to the interview, and we discussed the mathematical content of the resource.

Part 4. The uses of the resources and interactions with students (for the chosen course)

Implementation in the classroom

- How do you organise the sessions with the students (individual or group work, correction of exercises during the sessions, by whom? students and/or teachers?)
- What type(s) of resources do you disseminate, communicate and authorise during the sessions? Example: course slides, computer? files with dynamic representations, calculators, etc.
- How are these resources used and why?
- Do you make links with lectures and tutorials in the same course or in other courses? If so, how?
- What do you expect from the students during the tutorial, in terms of attitude, engagement, and how do you foster these expected attitudes and engagement?
- What personal assignments do you give your students?
- Do students return their assignments to you? If so, in what form?

At the end of this part of the questionnaire, we discuss how the specific resource the teacher brought to the interview was set up in class, and the interactions with the students around this resource.

Assessment

- What are the assessment procedures for this course?
- What are your expectations in terms of learning (technical or conceptual)?
- What form(s) of assessment do you use in the case of your own teaching in this course?
- Can you give us an example?