Numerical Study of Fluid-Structure Interactions in Tube Bundles With Multiphase-POD Reduced-Order Approach

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NUMERICAL STUDY OF FLUID-STRUCTURE INTERACTIONS IN TUBE BUNDLES WITH MULTIPHASE-POD REDUCED-ORDER APPROACH

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ABSTRACT

Fluid-Structure Interactions are present in a large number of systems of nuclear power plants and nuclear on-board stoke-holds. Particularly in steam generators, where tube bundles are submitted to cross-flow which can lead to structure vibrations. We know that numerical studies of such a complex mechanism is very costly, that is why we propose the use of reduced-order methods in order to reduce calculation times and to make easier parametric studies for such problems.

We use the multiphase-POD approach, initially proposed by Liberge (*E. Liberge; POD-Galerkin Reduction Models for Fluid-Structure Interaction Problems, PhD Thesis, Université de La Rochelle, 2008*). This method is an adaptation of the classical POD approach to the case of a moving structure in a flow, considering the whole system (fluid and structure) as a multiphase domain. We are interested in the case of large displacements of a structure moving in a fluid, in order to observe the ability of the multiphase-POD technique to give a satisfying solution reconstruction. We obtain very interesting results for the case of a single circular cylinder in cross-flow (lock-in phenomenon). Then we present the application of the method to a case of confined cylinders in large displacements too. Here again, results are encouraging.

Finally, we propose to go further presenting a first step in parametric studies with POD-Galerkin approach. We only consider a flowing-fluid around a fixed structure and the Burgers' equation. A future work will consist in applications to fluid-structure interactions.

INTRODUCTION

Flow induced vibrations (FIV) mechanisms in tube bundle system [8][10][15][29] are still encountering a large interest in the community of researchers and industrialists, because there are found in a lot of energy systems components, like for example nuclear power plants [27][28] or nuclear on-board stokeholds (steam boilers), as well as various heat exchangers [6][21][33]. As the cost of numerical calculations to lead a complete study of FIV mechanisms in a tube bundle is well-known to be very high [18][34], the use of low-order methods represents an interesting alternative to achieve an efficient work at low cost. Model reduction have been used since years through various fields like automatism , fluid mechanics [1][13], structure mechanics [3],... And more recently to fluid-structure interactions (FSI) [4][11][22][24][30].

Here we propose to use the adaptation of the POD-Galerkin method to FSI which is the Multiphase-POD technique [22][23] to treat cases of a single circular cylinder and of a confined cylinder in tube-bundle, able to move under a cross-flow with large displacements. The main advantage of Multiphase-POD is that it gives a robust method to study large displacements of the structure, when other reduced-order techniques for FSI need, to the author's knowledge, the small displacements hypothesis to be set up.

The paper is organized as follows: first, a description of the multiphase-POD method is given as a recall. Then, we present two applications of this technique in a second part. The first model is the well-known case of large displacements (lock-in phenomenon) of a single circular cylinder under cross-flow at Re=100, the second case is a tube bundle configuration, simpler than real cases of heat exchangers but close to models that are used in complete calculation numerical studies. Finally, the third part is dedicated to two elementary cases of parametric studies for a fluid domain (without moving boundaries) and for the Burgers' equation.

1. MULTIPHASE-POD APPROACH

In the framework of this paper, we consider that POD-Galerkin method is well-known (see for example [17][20][35]) and we focus on its adaptation to FSI through Multiphase-POD. Main variables used in this article are depicted in Table 1:

Variable	Definition
ρ	Global density (kg.m^{-3})
μ	Global dynamic viscosity (kg.m^{-1}.s^{-1})
u	Global velocity field
p	Global pressure field
σ	Constraints tensor
g_f	A variable g in the fluid domain
g_s	A variable g in the solid domain

Tab. 1. Main variables of the system

Flow-Induced Vibrations problems that we want to numerically study, *i.e.* single circular cylinder and tube bundle vibrations under a cross flow are leaded with a classic ALE approach [14]. It is important to recall that the application of the POD-Galerkin method cannot be made directly in that case: as POD modes are exclusively spatial modes, and as mesh nodes take several positions during a complete ALE calculation, information of their position are lost while the POD base is constructed. It is thus necessary to work for example with a non-moving mesh technique in order to get round this problem. Liberge and Hamdouni [22] proposed an original method to treat the case of a fluid-structure interaction problem with the POD-Galerkin method, inspired by Glowinski *et al.* [16]. This technique is called "Multiphase-POD method".

The main idea is to consider a global domain containing the fluid and the solid, and to consider the latter as another fluid with its specific characteristics, in order to insure the non-deformable characteristic of the fluid. The domain is thus considered as a zone occupied by a two-phase flow. A unique and fixed mesh is created on this global domain. Then, data obtained from (for example) a classic ALE complete calculation are interpolated on this fixed mesh, so that a POD basis could be constructed. The description of the multiphase method is the following: lets consider a global domain Ω containing the fluid domain $\Omega_f(t)$ and the solid domain $\Omega_s(t)$ at each time step t, where the solid domain is considered as a particular fluid with its own physical characteristics (density, viscosity).

We have $\Omega = \Omega_f(t) \cup \Omega_s(t) \cup \Gamma_i(t)$ where $\Gamma_i(t)$ is the interface between fluid and structure domains. A global velocity field $u \in H(\Omega)$ (with H a Hilbert space) is considered:

$$u(x,t) = u_f(x,t)\chi_{\Omega_f}(x,t) + u_s(x,t)\chi_{\Omega_s}(x,t) \quad (1)$$

where $\chi_{\Omega_f}(x,t)$ and $\chi_{\Omega_f}(x,t)$ are respectively characteristics functions defining the considered point position:

$$\chi_{\Omega_s}(x,t) = \begin{cases} 1 \text{ if } x \in \Omega_s \\ 0 \text{ if } x \notin \Omega_s \end{cases} \text{ and } \chi_{\Omega_f}(x,t) = 1 - \chi_{\Omega_s}(x,t)$$

Taking into account these notations, a global weak form of Navier-Stokes equations on Ω is made possible to formulate:

$$\int_\Omega \rho \frac{\partial u(x,t)}{\partial t} u^* \mathrm{d}\Omega + \int_\Omega (u.\nabla)u.u^* \mathrm{d}\Omega = \int_\Omega (\nabla.\sigma) u^* \mathrm{d}\Omega \quad (2)$$

Where u^* is a test-function defined as $u^* \in H(\Omega)$ with the non-deformable solid constraint $D(u^*) = 0$ in $\Omega_s(t)$. Each field or variable is defined on the global domain Ω as:

$$\begin{cases} \rho(x,t) = \rho_s \chi_{\Omega_s}(x,t) + \rho_f \chi_{\Omega_f}(x,t) \\ \mu(x,t) = \mu_s \chi_{\Omega_s}(x,t) + \mu_f \chi_{\Omega_f}(x,t) \\ u(x,t) = u_s \chi_{\Omega_s}(x,t) + u_f \chi_{\Omega_f}(x,t) \\ \sigma(x,t) = \sigma_s \chi_{\Omega_s}(x,t) + \sigma_f \chi_{\Omega_f}(x,t) \end{cases} \quad (3)$$

Let's define both components of the constraints tensor σ.

$$\sigma_{f,ij}(x,t) = -p\delta_i^j + 2\mu_f D_{ij}(u_f) \qquad (4)$$

where δ_i^j is the Kroenecker symbol and D_{ij} is the deformation velocity tensor.

The definition of the structural component $\sigma_s(x,t)$ allows taking into account that the solid has its specific viscosity and the non-deformable structural condition. For the viscosity, a penalization term is used: in order to specify that the domain $\Omega_s(t)$ is solid, the viscosity is artificially increased. To insure the non-deformable condition, a Lagrange multiplier Λ is added. Thus, the structural component of the constraints tensor is:

$$\sigma_{s,ij}(x,t) = -p\delta_i^j + \Lambda + 2\mu_s D_{ij}(u_s) \qquad (5)$$

Developing the global weak form with these definitions and making the Proper Orthogonal Decomposition on the global velocity flow field lead to the construction of a dynamical system for the whole domain Ω which is fixed all over the studied time interval. Taking into account the space-time decomposition of the global velocity field as:

$$u(x,t) = \sum_{n=1}^{N} a_n(t)\Phi_n(x) \qquad (6)$$

Where $\{\Phi_n\}_{n=1,\dots,N^*}$ are elements of the POD basis and $a_n(t)$ are time coefficients, the final dynamical system is the following:

$$\begin{cases} \sum_{i=1}^{N} \frac{da_i}{dt} A_{in} = \sum_{i=1}^{N}\sum_{j=1}^{N} B_{ijn} a_i a_j + \sum_{i=1}^{N} C_{in} a_i + E_n & \text{(a)} \\ D(u) = 0 \text{ on } \Omega_s(t) & \text{(b)} \\ \frac{\partial \chi_{\Omega_s}}{\partial t} + u.\nabla \chi_{\Omega_s} = 0 & \text{(c)} \end{cases} \qquad (7)$$

for each $n = 1,\dots,N$ where N is the number of POD modes. Equation (7.b) is the non-deformability condition and (7.c) the characteristic function transfer.

Coefficients $\mathrm{A_{in}}, \mathrm{B_{ijn}}, \mathrm{C_{in}}, \mathrm{E_n}$ are not detailed here, but a very important point to notice is that they are not all exclusively spatial coefficients, because some of them contain the physical characteristics $\rho(x,t)$ and $\mu(x,t)$. Thus, they have to be re-calculated at each time step: the time calculation is increased in comparison with a classic POD model without moving structure. But this time calculation is still less than a complete calculation. Another approach consists in making the proper orthogonal decomposition of the characteristic function $\chi_{\Omega_f}(x,t)$ also, which allows avoiding the time dependence of all coefficients of the dynamical system. For more precisions, see [23].

Practical implementation of the Multiphase-POD technique is described below.

Multiphase method implementation:

1. Lead a complete ALE calculation of the fluid-structure interaction problem during a time interval [0,T]
2. Extract enough snapshots from this complete calculation
3. Create a unique Cartesian fixed mesh containing both fluid and solid domains
4. Interpolate each extracted snapshot onto the fixed reference mesh: creation of new fixed snapshots
5. Apply the classic POD approach for the new snapshots constructed on the reference mesh
6. Construct the dynamical system following (7) and resolve it with a classic method (Runge-Kutta for example).

2. FSI APPLICATIONS WITH MULTIPHASE-POD

2.1. Single circular cylinder under cross-flow

In order to test the multiphase method in the case of FIV, a simple case is considered before its application to the case of tube bundle. We test the configuration of a single circular cylinder under a cross-flow. Work on such a case is interesting to validate the method, as Liberge and Hamdouni proposed a similar configuration in the case of low structure displacement amplitude. The cylinder is allowed to move in the transverse direction; the fluid domain is considered as infinite, as boundaries are far enough from the structure.

Here we consider the case of large displacements of the cylinder (without small displacement hypothesis): the frequency lock-in phenomenon [19][31][36] is reproduced. The case of small displacements of a cylinder under cross-flow has been studied in [22]. The geometry of the studied case is visible on figure 1.

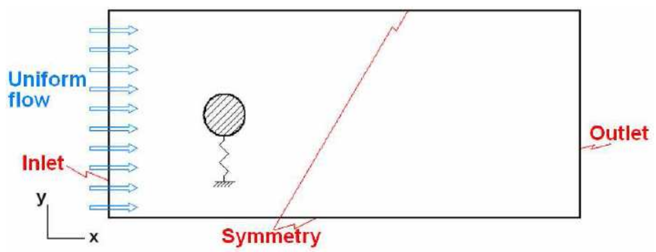


Fig. 1. Case of a single circular cylinder and boundary conditions

The cylinder is allowed to move in the y-direction (see figure 1) only. The effects that the flow exerts on the structure are modeled through a restoring force.

Reynolds number is Re=100, fluid is water. Cylinder displacement maximal amplitude is A*=0,58D, where D is the cylinder diameter: the frequency lock-in mechanism is reached.

Complete calculations are leaded with the CFD code *Code_Saturne* [5] and the cylinder is just considered as a mass point on its gravity center.

The reduced-order model is constructed with the following characteristics: 250 snapshots are extracted from the complete ALE calculation, 6 POD modes are constituting the POD basis. The fixed reference mesh contains 200 x 250 points. The dynamical system resolution in the present case is simplified: indeed, the penalization term is sufficient to guarantee the non-deformable condition. Time integration scheme is Runge-Kutta 4.

The two first time coefficients are represented on figure 2, they are well reconstructed by the reduced model. And, as they are containing the main part of the system energy, this good reproduction allows a good reconstruction of the velocity flow field and the cylinder displacement is also well reproduced (figure 3), which is confirming that 1) the Multiphase-POD method is able to reproduce a structure displacement and a fluid flow with its global formulation and 2) the Multiphase-POD method is able to reproduce large displacements of the structure. The latter point is interesting for the willingness of studying instability behaviors.

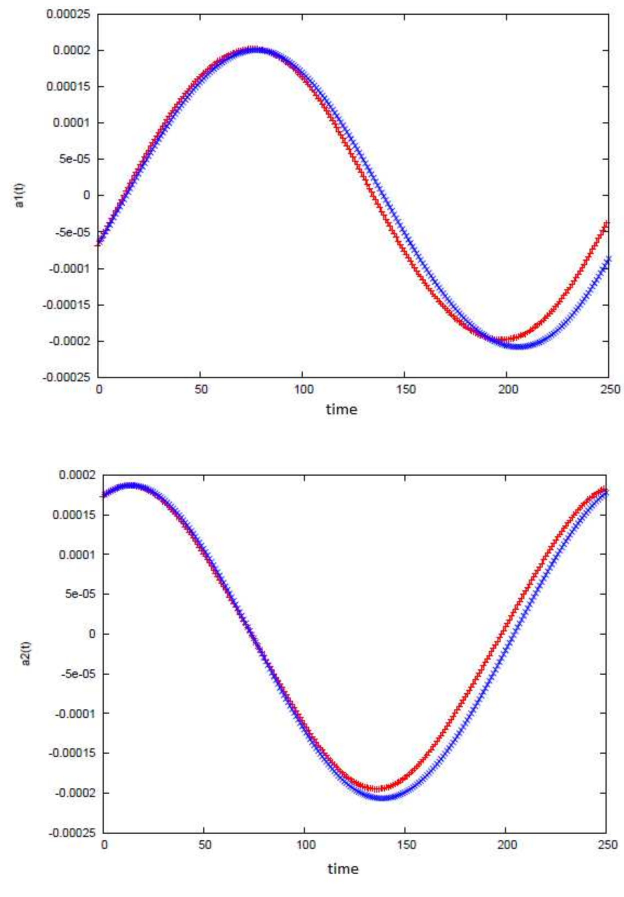


Fig. 2. Two first time coefficients of the velocity field in the single circular cylinder case with large displacements under cross-flow. +++ direct coefficient ; xxx multiphase-POD reconstruction

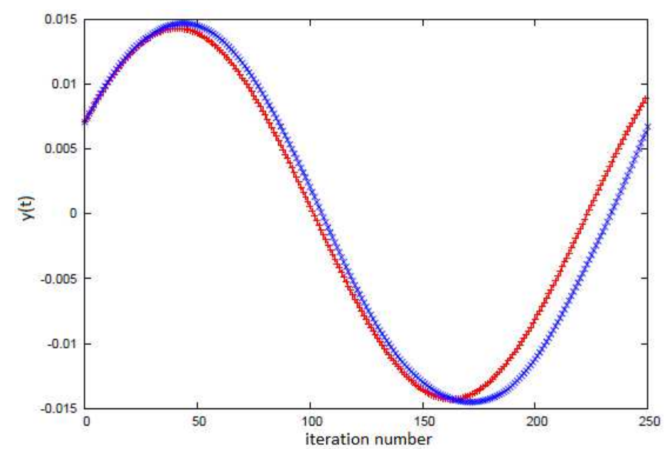


Fig. 3. Gravity center displacement reconstruction of the cylinder in the single circular cylinder case with large displacements under cross-flow. +++ direct calculation ; xxx multiphase-POD reconstruction

2.2. Tube bundle under cross-flow

In order to consider a configuration close to the case of a tube bundle of heat exchanger, we consider a circular cylinder in a confined configuration. Non-dimensional numbers are adapted to this configuration, here Reynolds number is defined as $R_e = \frac{\rho U_p D}{\mu}$. The step fluid velocity U_p takes into account the tube confinement. It is defined as: $U_p = U_\infty \frac{P}{P-D}$ where U_∞ is the equivalent mean flow velocity that would have been imposed in an infinite domain, D is the cylinder diameter and P is the pitch ratio (distance between two neighbouring cylinders centres). Geometry and boundary conditions are depicted on figure 4: a 2D domain and only one tube and its neighbors are considered, with periodic boundary conditions. Thus, the domain is representing an infinite regular tube bundle.

Reynolds number is fixed to $R_e = 2000$, complete calculation is also leaded with *Code_Saturne* which has been validated in various FSI studies in tube bundle systems [7][18][25]. Large displacements in the *y* direction (see figure 4) of the central cylinder are considered (A* = 0.35D when P/D = 0.44D). The reference fixed mesh contains 200 x 200 points.

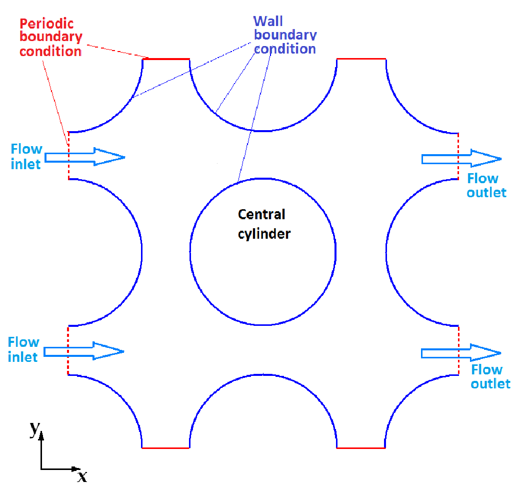


Fig. 4. Boundary conditions for the confined tube

Figure 5 represents the comparison between the global velocity flow field from the complete calculation and the interpolated velocity flow field. It allows to check the precision of the snapshots interpolation algorithm.

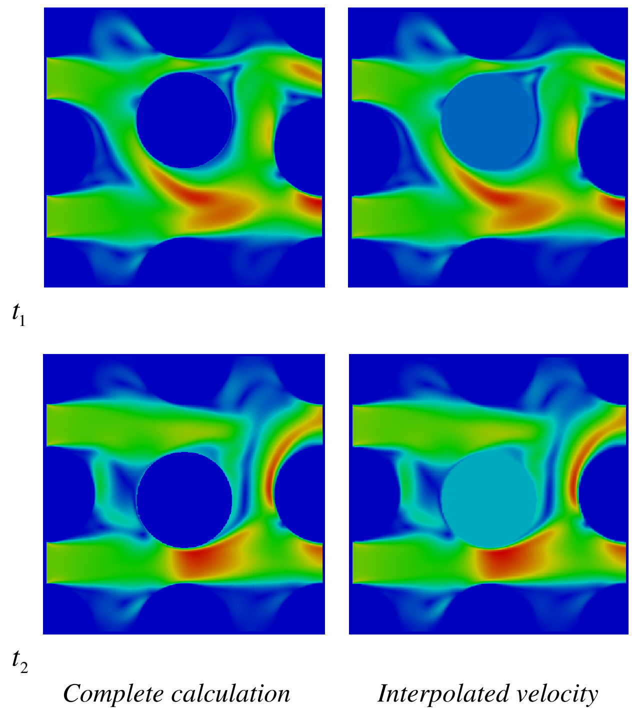


Fig. 5. Comparison between complete and interpolated fluid velocity field at two dates t_1 and t_2

Figure 6 shows the comparison between the central cylinder displacement calculated by complete calculation and by Multiphase-POD. The reconstruction gives very satisfying results, which is confirmed by the observation of the two first time coefficients of the global velocity flow field (figure 7).

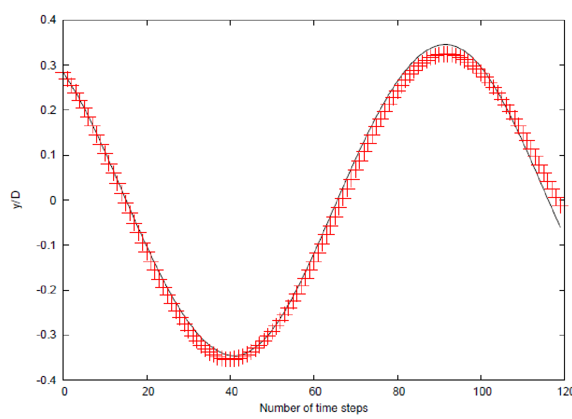


Fig. 6. Comparison between complete and Multiphase-POD reconstruction of the central cylinder displacement. — Complete calculation, +++ Multiphase-POD reconstruction

The reconstruction of large displacements with Multiphase-POD in the case of a confined tube bundle is very interesting. It allows to plan for its implementation to unstable fluid-structure interactions like fluid-elastic instability occurring in tube bundle systems [9][12][21][29][32].

3. REDUCED-ORDER MODEL AND PARAMETRIC STUDIES

3.1. Case of a single fixed cylinder

The main interest of reduced-order models consist in their ability to reconstruct various solutions of a system where one or several parameters have been changed. Indeed, the reconstruction of a solution for which we already have the complete calculation is not satisfying. Thus, we propose here a first step in this sense: for a given parameter, we use the POD basis obtained from a given value of the parameter and the system for other values of the parameter is projected onto this unique basis.

Here is an example: lets consider the case studied in the paragraph 2.1 with, here, a *fixed* single circular cylinder at Re=100. The classic POD-Galerkin method is applied, and a POD basis $\{\Phi_n^1\}_{n=1,\ldots,N}$ is constructed. Then, dynamical systems for $R_e = 110, 120, 130, 140, 150$ are projected onto $\{\Phi_n^1\}_{n=1,\ldots,N}$. Results are visible on figures 8 and 9 in terms of velocity flow field and vorticity.

The main characteristics of the flow field are well reproduced, but vortex shedding frequency observed on figure 9 is not exactly reproduced: it means that the unique POD basis $\{\Phi_n^1\}_{n=1,\ldots,N}$ obtained for Re=100 is not sufficient for the representation of the whole features of the flow in this panel of Reynolds number values.

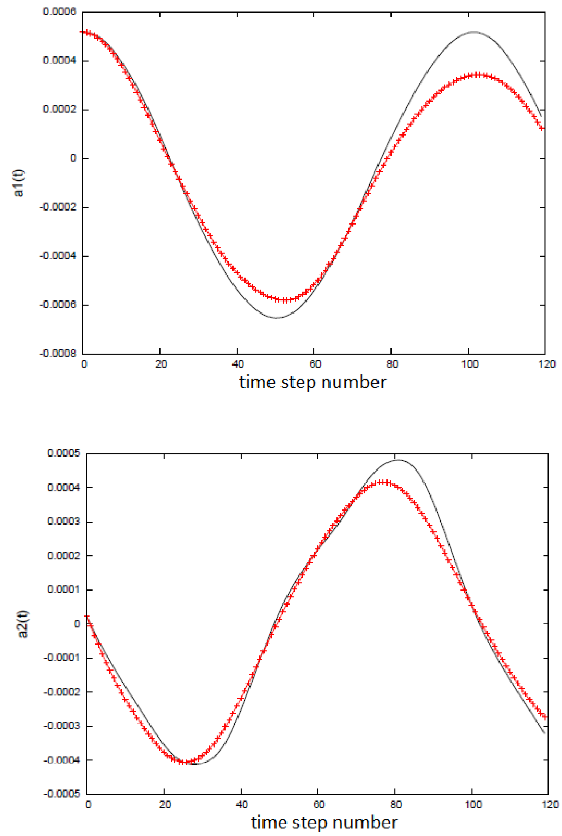


Fig. 7. Comparison between complete and Multiphase-POD reconstruction of two first time coefficients. —— Complete calculation, +++ Multiphase-POD reconstruction

3.2. Case of a tube bundle

The same method is applied to the case of a *fixed* tube bundle in a similar configuration than in part 2.2. Reynolds number for the reference case is Re=2600, a unique POD basis $\{\Phi_n^1\}_{n=1,\ldots,N}$ is constructed. Dynamical systems are calculated for $R_e = 2650$ and 2700. Results for the velocity field are presented on figure 10.

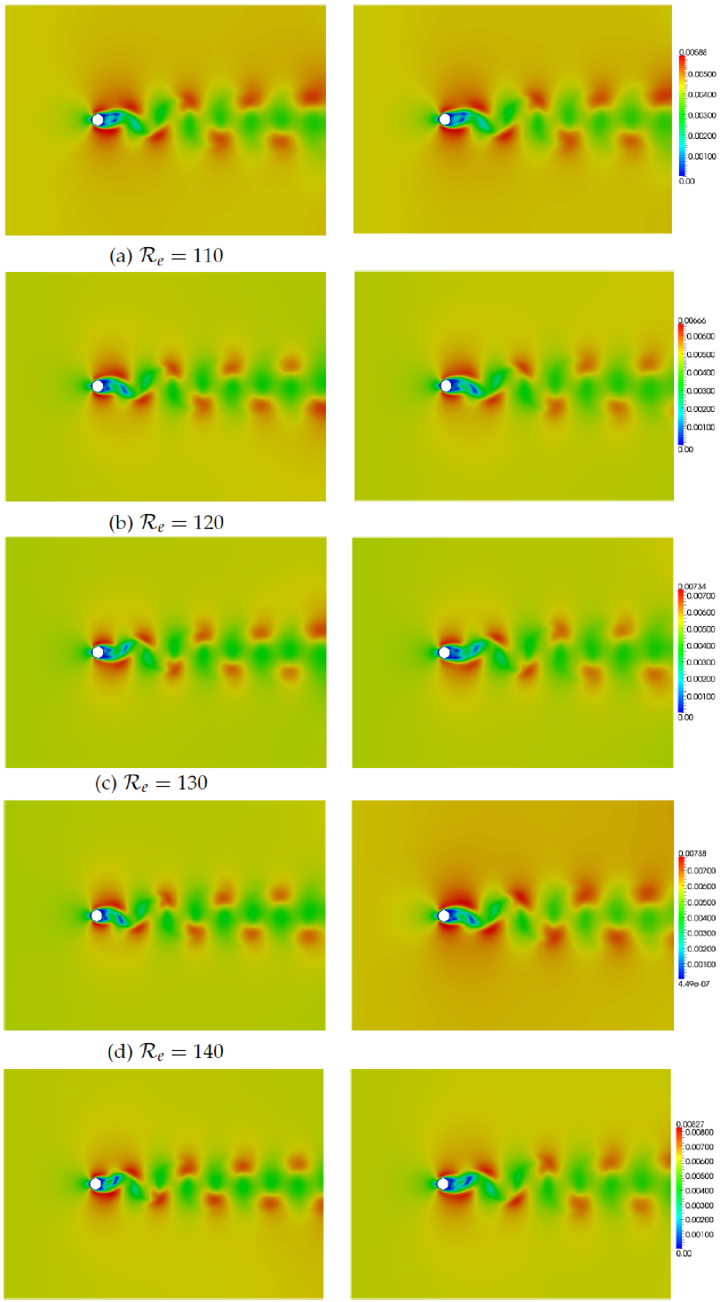


Fig. 8. Comparison between complete and POD-Galerkin reconstruction of velocity for various Reynolds number with a unique POD basis (Re=100)

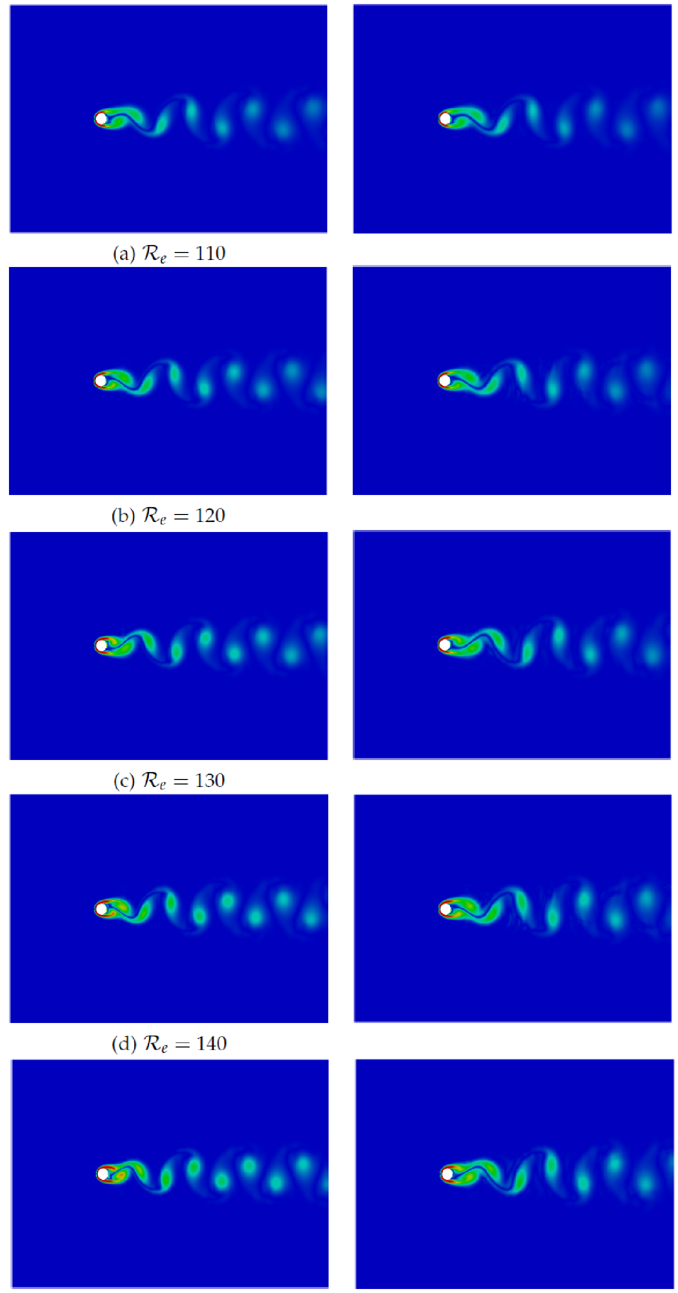


Fig. 9. Comparison between complete and POD-Galerkin reconstruction of vorticity for various Reynolds number with a unique POD basis (Re=100)

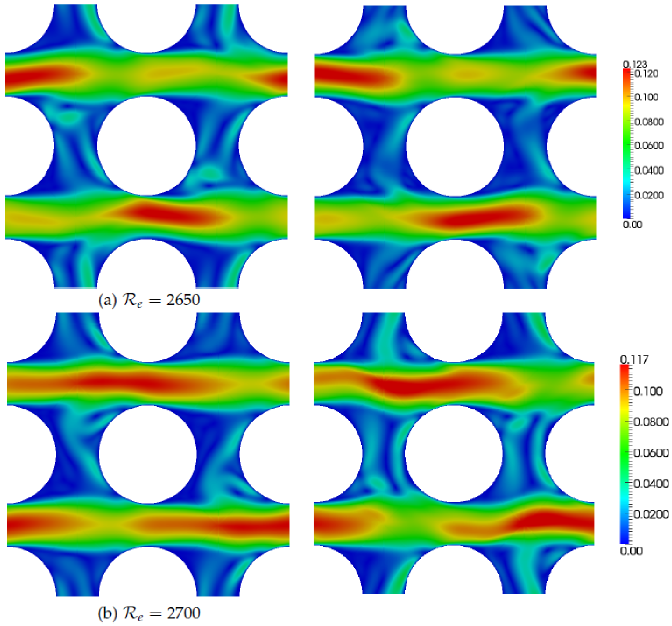


Fig. 10. Comparison between complete and POD-Galerkin reconstruction of velocity for various Reynolds number with a unique POD basis (Re=2600), case of a tube-bundle

The main reconstruction is good but not in all scales. We propose here an amelioration of this parametric method with the help of several basis interpolations.

3.3. Basis interpolation

Amsallem and Farhat [2] propose the introduction of basis interpolation in order to improve the reconstruction of solution for a new value of the considered parameter. Lets consider the following problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} + A(u,\lambda) = f \\ u(0) = u_0 \end{cases} \qquad (8)$$

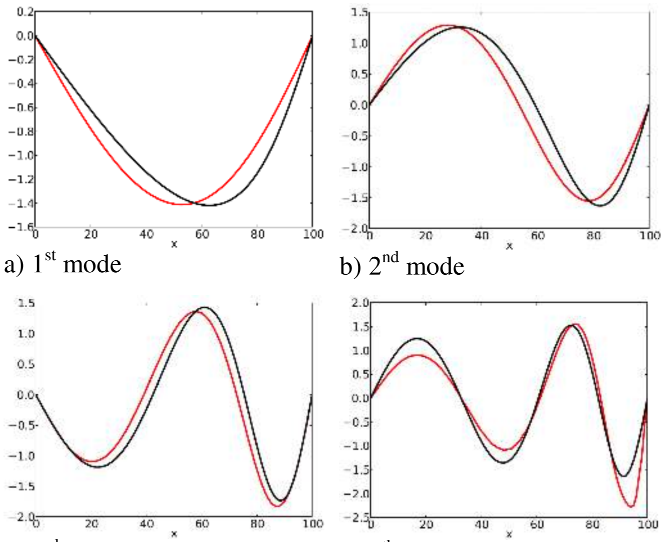
with the studied parameter λ. For example, if we know M POD basis $\{\Phi_n^1\}_{n=1,\dots,N}$, $\{\Phi_n^2\}_{n=1,\dots,N}$, …, $\{\Phi_n^M\}_{n=1,\dots,N}$ calculated to their corresponding parameter values λ_1, λ_2, …, λ_M, it is interesting to calculate the POD basis $\{\Phi_n^{\text{new}}\}_{n=1,\dots,N}$ associated with a new value λ_{new} of the parameter without leading the complete calculation. Thus, an interpolation of the known POD basis is proposed by Amsallem and Farhat thanks to the use of Grassmann manifolds, in order to keep the new basis orthonormal. The idea is the following: we consider the Grassmann manifold corresponding to the set of subspaces of dimension N in $\mathbb{R}^n$ (where n is the dimension of the complete system). Each known POD basis can be represented by a point on this manifold, and a tangent space can be considered at this point. The interpolation between basis will be done on this space which is a vector space. Then, a re-interpolation onto the Grassmann manifold is done in order to obtain the new POD basis $\{\Phi_n^{\text{new}}\}_{n=1,\dots,N}$. For a complete description of the method and algorithm, see [2].

Here we present an elementary example with the 1D Burgers equation on $H_0^1(\Omega)$ where $\Omega =]0,1[$ defined below:

$$\begin{cases} \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} - \nu\dfrac{\partial^2 u}{\partial x^2} = 0 \\ u(x,0) = u_0(x) = \sin(\pi x) \end{cases} \qquad (9)$$

where ν is the dynamical viscosity which is the considered parameter. We consider the case where $\nu_1 = 0.1\,m^2s^{-1}$. We construct the basis $\{\Phi_n\}_{n=1,\dots,N}^{1,\text{interp}}$ which is the interpolation of M=3 basis $\{\Phi_n\}_{n=1,\dots,N}^2$, $\{\Phi_n\}_{n=1,\dots,N}^3$ and $\{\Phi_n\}_{n=1,\dots,N}^4$ respectively associated with $\nu_2 = 0.01\,m^2s^{-1}$, $\nu_3 = 0.2\,m^2s^{-1}$ and $\nu_4 = 1\,m^2s^{-1}$. Then, we compare $\{\Phi_n\}_{n=1,\dots,N}^{1,\text{interp}}$ with the POD basis $\{\Phi_n\}_{n=1,\dots,N}^1$ that have been classically obtained with snapshots construction (see figure 11). We can observe that both are not exactly the same. But, when we compare the results obtained by the solution reconstruction with each POD basis $\{\Phi_n\}_{n=1,\dots,N}^1$ and $\{\Phi_n\}_{n=1,\dots,N}^{1,\text{interp}}$ (figures 12 and 13), we can conclude that the interpolation method gives a satisfying solution.

This method is interesting since with a small number M of POD basis relatively close with each other, it is possible to construct new basis that keep POD characteristics for new parameters with the assurance of a good reproduction of the unknown field.



a) 1st mode b) 2nd mode

c) 3rd mode d) 4th mode

Fig. 11. Comparison between both POD basis $\{\Phi_n\}^{1}_{n=1,...,N}$ and $\{\Phi_n\}^{1,interp}_{n=1,...,N}$ associated with $\nu_1 = 0.1\,m^2s^{-1}$ for 4 POD modes. —— $\{\Phi_n\}^{1}_{n=1,...,N}$ —— $\{\Phi_n\}^{1,interp}_{n=1,...,N}$

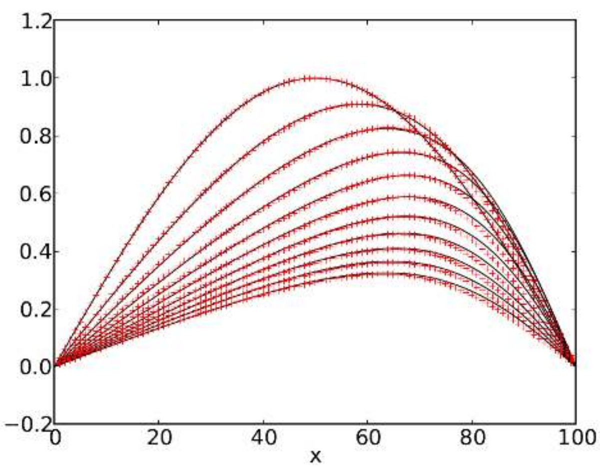


Fig. 12. Comparison between complete Burgers equation for $\nu = 0.1\,m^2s^{-1}$ and its reconstruction with the classical POD basis $\{\Phi_n\}^{1}_{n=1,...,N}$. —— Complete calculation, +++ POD reconstruction

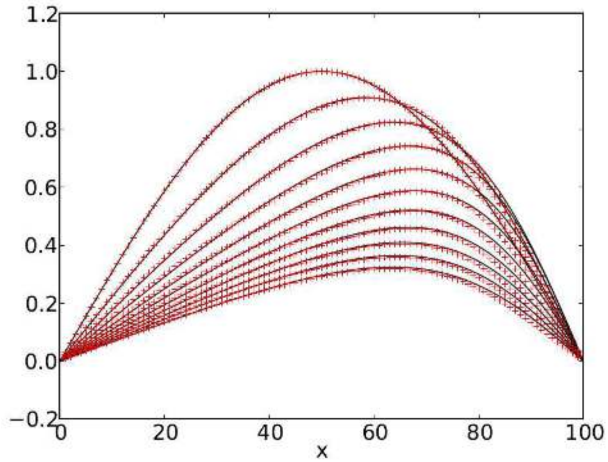


Fig. 13. Comparison between complete Burgers equation for $\nu = 0.1\,m^2s^{-1}$ and its reconstruction with $\{\Phi_n\}^{1,interp}_{n=1,...,N}$. —— Complete calculation, +++ POD reconstruction

CONCLUSION

In this paper, the Multiphase-POD method is presented and applied to the case of a single circular cylinder moving under cross-flow and a confined cylinder in tube bundle under cross-flow. The method was already shown to be efficient in the case of small displacements of a structure under flow solicitations and here, we show its efficiency in the case of large displacements of the structure. This is a very interesting point in order to treat instabilities that can appear in a large number of industrial systems. An on-going work presented here is the application of parametric studies with the help of reduced-order methods. The POD-Galerkin method already gives interesting results in fluid-flow studies, and an improvement of this technique is tested to the Burgers equation. Further work will consist in the application of these methods to the case of FSI.

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