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IDEALIZED NONLINEAR DYNAMICS OF THE FOUR-WAVE MIXING IN OPTICAL FIBERS

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RÉSUMÉ

We design a fiber-based experimental setup to demonstrate an ideal four-wave mixing process. After revealing the complete phase-space topology, we discuss the impact of the second-order harmonics on the Hamiltoninan dymanics. Using a perturbative approach, we discuss how invariants of the ideal motion experience changes in a real system; and develop a modified four-wave mixing model.

MOTS-CLEFS : *Four-wave mixing; nonlinear fiber optics; Hamiltoninan dyamics*

1. INTRODUCTION

Interaction between waves in an optical fiber is governed by the nonlinear Schrodinger equation (NLSE) which describes the impact of nonlinearity and dispersion on phases and amplitudes of the involved waves. If the system is limited to only three harmonics $\sum_{n=-1}^{1}\psi_n\exp(-in\omega_m t+i\varphi_n)$ where ω_m is the modulation angular frequency, ψ_n and φ_n are spectral amplitude and phase of n-wave, the process can be referred to as a four-wave mixing (FWM) in its degenerate case. The dynamics can be then described in terms of reduced variables $\eta=|\psi_0|^2/\sum_{n=-1}^{1}|\psi_n|^2$ and $\phi=\varphi_{-1}+\varphi_1-2\varphi_0$ by a set of analytical equations [1]. In this contribution we discuss experimental demonstration of this ideal FWM and its extension which takes into account impact of the second-order harmonics.

2. EXPERIMENTAL SETUP AND RESULTS

Experimental demonstration of the ideal FWM has been recently done developing a special type of the setup (Figure 1 (a)) relying on iteration of initial conditions [2], where a phase-modulated continuous wave laser is shaped according to initial conditions η_j,ϕ_j and then propagates in a segment of fiber ($L=500$ m, $\gamma=1.7$ km^{-1} W^{-1}, $\beta_2=-8$ ps^{2}km^{-1}). Fiber parameters are carefully selected, so that generation of higher-order harmonics is limited. The output parameters η_{j+1},ϕ_{j+1} are then measured and used as inputs for the next iteration of the input so one can reconstruct waves dynamics over tens of kilometers while limiting the process to three-waves.

The experimental results of the phase-space plane $(\eta\cos\phi,\eta\sin\phi)$ investigation are depicted in Figure 1 (b1) where several initial conditions have sequentially propagated over $L=50$ km (or $\xi=L(\gamma P_0)^{-1}=12$ at $\kappa=\omega_m\sqrt{\frac{|\beta_2|}{\gamma P_0}}=-2$). We note a good agreement with the ideal FWM model as well as an appearance of other typical characteristics of the process such as position of the separatrix and existence of stationary solutions [2].

In addition, a feed-forward neural Network (NN) has been successfully trained [3] so that it can reproduce the idealized dynamics, including experimental deviations from the model (Figure 1 (b2)). We also develop a method to control the FWM dynamics by introducing an abrupt power change, so we can force a transition between two arbitrary states [4].

Finally, we discuss the impact of higher order harmonics on the dynamics. Indeed, in our experimental configuration, generation of the second-order (and further) sidebands is limited but not excluded. Thus, even if their level stays below 1% of the total spectral amplitude, the very existence of additional nonlinear mixing processes impacts the system, especially at longer propagation distances. This leads to

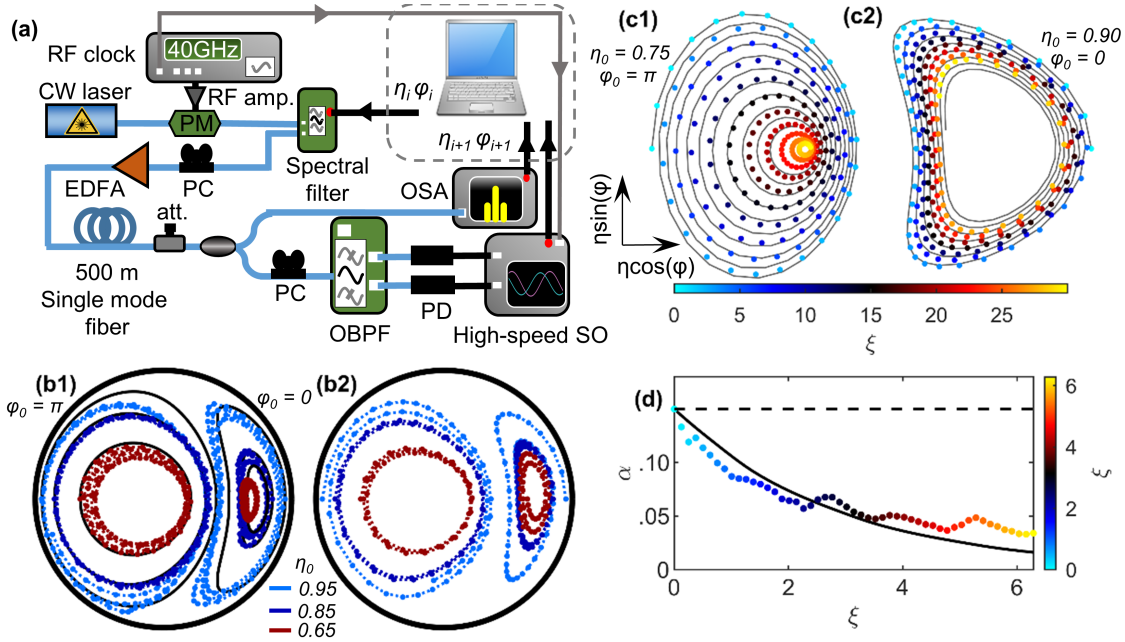

FIGURE 1 : (a) Experimental setup : CW - continuous wave, RF amp. - radiofrequency amplifier, PM - phase modulator, EDFA - erbium-doped fiber amplifier, PC - polarization controller, OSA - optical spectrum analyzer, PD - photodiode, SO - sampling oscilloscope, OBPF - optical band-pass filter, att. - attenuator. (b1) Experimental FWM dynamics displayed on the phase-space plane $(\eta\cos\phi, \eta\sin\phi)$. Black lines show the respective theoretical dynamics. (b2) Dynamics reconstructed by a NN trained on the experimental data. (c) Showcase of the spiraling dynamics over long propagation distance ($\xi = 30$) and $\kappa = -2$: numerical simulation - black, updated model - colored dots. (d) Experimental recording of the asymmetry variation (colored dots) is compared to numerical data (black solid line) and ideal four-wave mixing (black dashed line). The parameters are : $\kappa = -1.2, \alpha_0 = 0.15, \eta_0 = 0.75, \phi_0 = 0$.

deviation of the dynamics from the ideal closed orbits as depicted in Figure 1 (c) (black lines - numerical simulations). To take into account the existence of other photon nonlinear scattering processes of other routes of photons' exchange, we consider the development of the second-order sidebands in the perturbative approximation which allows us to develop a modified FWM model, which still retains the three photons (hence, three equations) framework (Figure 1 (c), dots - updated model). That allows to attribute the spiraling behavior to changes in the Hamiltonian.

Another invariant in the ideal model that our modified theory can describe is the asymmetry parameter $\alpha = |\psi_{-1}|^2 - |\psi_1|^2$. We show that in the experiment α is no longer conserved but decays towards zero (Figure 1 (d)). This behavior can be again attributed to impact of the higher-order sidebands on the wave-mixing dynamics [5].

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