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# Extension of the fundamental four wave-mixing model to large bandwidth systems

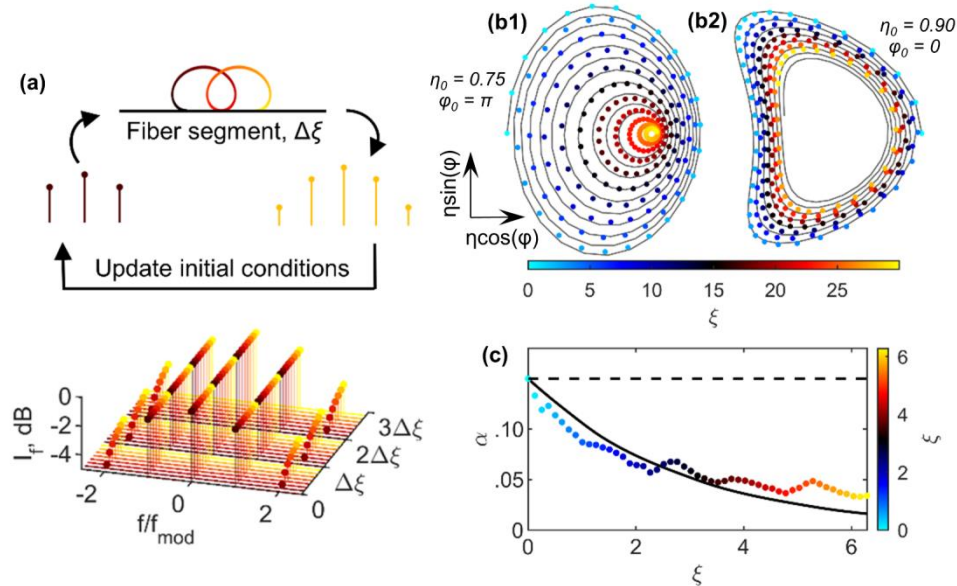
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**Abstract:** While retaining the nominal three-wave framework used for the fundamental four-wave mixing dynamics, we extend it using second order perturbation theory in order to include the impact of higher order sidebands. We discuss then the mechanisms at the origin of the divergence of intermediate systems from the ideal model.

Waves dynamics in optical fiber is governed by the nonlinear Schrödinger equation (NLSE) which describes the impact of dispersion and nonlinearity on phases and amplitudes of the involved waves. For the system containing only three waves  $\sum_{n=-1}^1 \psi_n e^{-in\omega_m t + i\phi_n}$ , where  $\omega_m$  is the modulation angular frequency,  $\psi_n$  and  $\phi_n$  are spectral amplitude and phase of  $n^{th}$  spectral line, the process can be referred to as the four-wave mixing (FWM) in its degenerate case. Then the complete dynamics can be described using a Hamiltonian framework by a set of coupled equations in terms of reduced variables  $\eta = \psi_0^2 / \sum_{n=-1}^1 \psi_n^2$  and  $\varphi = \phi_{-1} + \phi_1 - 2\phi_0$  [1].

However, this fundamental FWM process is valid only if the system is strictly limited to three spectral lines. Nevertheless, we have recently demonstrated it experimentally [2] by implementing an original setup relying on iterated initial conditions as depicted in Figure 1 (a). The idea is to limit light propagation to a small segment of fiber and filter-out the higher-order sidebands at each iteration while continuously updating the input conditions, hence preventing the growth of any extra-sidebands, as well as other detrimental mechanisms (e.g. Raman and Brillouin effect, propagation loss).



**Figure 1** (a) Simplified sketch of the experimental concept [2]. The bottom panel displays growth and filtering of the second-order sidebands over each segment length  $\Delta\xi$ . (b) FWM dynamics over long propagation distance  $\xi = \mathbf{z}\boldsymbol{\gamma}\mathbf{P}_0 = 30 = 236\Delta\xi$  and  $\boldsymbol{\kappa} = \boldsymbol{\beta}_2\omega_m^2/\boldsymbol{\gamma}\mathbf{P}_0 = -2$  (where  $\boldsymbol{\beta}_2$  is the second-order dispersion,  $\boldsymbol{\gamma}$  – nonlinear coefficient,  $\mathbf{P}_0$  – average power,  $\mathbf{z}$  – fiber length in m): numerical simulation of the segmented propagation - black, updated model - colored dots. (c) Experimental recording of the asymmetry parameter (colored dots) is compared to numerical data (black solid line)

and the fundamental FWM (black dashed line). The parameters are:  $\kappa = -1.2$ ,  $\alpha_0 = 0.15$ ,  $\eta_0 = 0.75$ ,  $\varphi_0 = 0$ .

In this case, the fundamental dynamics is reproduced over short propagation lengths ( $100\Delta\xi$ , about 50km in fiber); but gradually degrades at longer propagation distances exhibiting a gradual spiraling toward a fix point (of a limit cycle) visible when looking at the phase-portrait  $(\eta \cos \varphi, \eta \sin \varphi)$ . It happens because in the real fiber propagation, unlike in the perfect mathematical model, it's impossible to eliminate fundamental photons exchange processes between the main three lines and the higher-order harmonics. Even if these extra photonics states are not populated, their very existence opens up new interactions paths which eventually results into small (less than 1% of energy is transferred to the extra sidebands) but alas cumulative effect which drives the system off its idealized dynamics.

To improve the ideal FWM model we include the effect of the second-order sidebands. Because their resulting impact is small, it is actually possible to include them as an adiabatic perturbation. This leads to development of average values of  $\psi_{\pm 2}, \phi_{\pm 2}$  expressed solely in terms of the idealized three-wave parameters  $\eta, \varphi$  which are accumulated over  $\Delta\xi$ . After developing the reduced set of equations while taking into account extra photon exchange process paths, we obtain a modified model which retains the initial two-equation description:

$$\begin{cases} \frac{d\eta}{d\xi} = -2\eta(1 - \eta) \sin \varphi + \Delta\xi f_1(\eta, \varphi) \\ \frac{d\varphi}{d\xi} = (\kappa - 1) + 3\eta - 2(1 - 2\eta) \cos \varphi + \Delta\xi f_2(\eta, \varphi) \end{cases} \quad (1)$$

where  $f_{1,2}(\eta, \varphi)$  are modifications to the fundamental model which is depicted in the first part of the equations.

Solving numerically Eq. (1), we obtain a close agreement with numerical results imitating experimental iterated propagation (Figure 1 (b)). After analyzing the Eq. (1), we conclude that the non-conserved dynamics and the spiraling behavior are directly linked to the dissipations of the Hamiltonian whose energy no longer a constant for each orbit.

Another propagation invariant in the ideal FWM is the asymmetry parameter defined as  $\alpha = \psi_{-1}^2 - \psi_1^2$ . However, in our experimental configuration we observe its gradual decay towards zero (colored dots in Figure 1 (c)). This behavior is again attributed to the impact of the second-order harmonics, as confirmed by our improved model. We can quantify the decay rate of the asymmetry parameter depending on the initial conditions and parameters of the system [3].

In this work, we investigate changes in the FWM invariants that are observed during experimental and numerical simulation of the iterated propagation. These changes are attributed to the impact of the second-order sidebands. Even though their spectral amplitudes are kept almost negligible, even the very existence of extra open passages for photons interactions is enough to disturb the system. By taking the perturbative approach, we can quantify these impacts, and we demonstrate a detailed study and a derivation of a modified FWM model operating the same reduced variables. Hence, our approach can be generalized to other systems where the dynamics of a closed system becomes sensitive to the impact of its external environment.

## References

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