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A novel look at the double-slit experiment through the symmetries of the vacuum

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The renowned double-slit experiment encompasses some of the deepest mysteries of Quantum Mechanics: single particles behave as corpuscles, but their statistical behavior corresponds to delocalized wave packets. Here, we offer a novel, physically intuitive explanation for this dual behavior. The proposal is based on insight gained from a recently described statistical description of another renowned experiment: the CHSH-Bell experiment. According to the proposed explanation, while each particle follows a single path as it goes through the double-slit and into the detector, it also modifies the physical properties of the vacuum around it by spontaneously breaking its symmetries. The well-known pattern of interference fringes that appears in statistical analysis of long sequences of repetitions is then understood as the result of the self-interaction of the propagating particles with the polarized vacuum they create around themselves. An experimental setup aimed to test the offered explanation is suggested.

I. INTRODUCTION

The pattern of intensity fringes that emerges in the classical Young's double-slit experiment was widely regarded at the beginning of the nineteenth century when the experiment was first performed, as definitive experimental proof of the wave nature of light [1–3]. Maxwell's theoretical description of electromagnetic radiation developed a few decades later [4] and the subsequent experiments performed by Herz [5] solidly established this concept against the alternative corpuscular theory of light proposed two hundred years earlier by Newton [6]. However, the discovery at the turn of the twentieth century of the photoelectric effect [7] and then unexplained features in the spectrum of the black-body radiation [8] fully reopened the issue and laid the foundational stones for the development of Quantum Mechanics and its mysteriously precise description of the dual corpuscular/wave nature of both light and matter at the microscopic level [9].

The modern double-slit experiment [10, 11] is one of the most striking examples of this mysterious corpuscle/wave duality. The experiment is performed with a source of single microscopic particles (photons, electrons, neutrons, or even molecules [12]) - rather than the coherent pulses of light used in the classical version of the experiment, which are sent one by one towards a pair of nearby narrow slits opened in an opaque wall. The particles that successfully go through these slits are subsequently detected on a screen positioned behind the wall. While each one of the particles is detected at a single location on the screen, as would be expected from localized corpuscular objects, when a long sequence of such events is statistically analyzed, a characteristic pattern of intensity fringes appears, as it would be expected from self-interfering delocalized wave-packets [13, 14]. Despite the precise description of the experiment provided by Quantum Mechanics, a physically intuitive explanation

of Nature's workings in this phenomenon is still lacking [15].

A prevalent version of the double-slit experiment is performed with the help of a Mach-Zender interferometer (MZI) [10]. This device consists of two beam splitters connected one after the other through two imbalanced intermediate arms, as shown in Fig 1. Single microscopic particles enter the device, one at a time, through the input leg of the first beam splitter and are detected as localized objects as they leave the device through either one of the two exit legs of the second beam splitter. Similarly, when the particles are tested in separate experiments as they leave the first beam splitter, they are detected at either of its exit arms and never at both. Nonetheless, the probabilities of these single particles being detected at either one or the other of the two exit legs of the Mach-Zender interferometer, as measured in statistical analysis of long sequences of repetitions in which many single particles are recorded, show a very characteristic periodic dependence on the phase shift $\tilde{\Delta}$ introduced by the length imbalance between the two intermediate arms, see eq. (3) below, as it would correspond to self-interfering delocalized wave packets advancing at once through both intermediate arms.

Elaborated versions of this experiment, including nested Mach-Zender interferometers, have attracted a lot of attention in recent years, with several teams reporting their theoretical analysis and experimental results in a renewed search for novel clues about the physical mechanisms involved in this mysterious dual behavior [16–25]. Yet, while the quantum formalism describes these experiments as precisely as in the case of a single interferometer, no agreed physically intuitive understanding of the phenomenon has been reached.

This paper proposes a novel, physically intuitive explanation of the mechanisms involved in these experiments. The proposal is inspired by recent work that shows how to bypass the constraint imposed by Bell's theorem to build a statistical description of the CHSH-Bell experiment in terms of hidden configurations of the maximally entangled quantum states of two qubits [26–31], and ex-

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exploits the fact that the probabilities of single particles leaving the Mach-Zender interferometer through either one of its two exit legs are formally identical to the probabilities to obtain either equal or opposite outcomes in the two detectors of a CHSH-Bell experiment. Accordingly, the phase shift introduced by the length imbalance between the two intermediate arms of the Mach-Zender interferometer plays a role similar to the relative orientation between the two measuring devices in the CHSH-Bell experiment.

We follow this observation and build an explicit statistical model of hidden configurations for the double-slit experiment on a Mach-Zender interferometer. The proposed model suggests that while each single particle goes through the device along only one of the two available paths, it also modifies the physical properties of the idle path by spontaneously breaking the group of symmetries of its vacuum state. That is, according to the proposed explanation, the vacuum and its symmetries, which are known to play a crucial role in well-understood physical phenomena like the Casimir effect [32–34] and other electromagnetic effects [35–38], might also play a crucial role in unraveling the centuries-old mysteries about the double-slit experiment.

The paper is structured as follows. In Section II, we revisit the standard quantum mechanical description of the double-slit experiment. Section III summarizes our statistical model for the Bell-CHSH experiment and its application to the double-slit experiment. In Section IV, we draw insights from this model. Importantly, in Section V, we propose an experiment to test them. We conclude in Section VI with a summary of our findings.

II. THE DOUBLE-SLIT EXPERIMENT ON A MACH-ZENDER INTERFEROMETER: THE QUANTUM DESCRIPTION

A Mach-Zender interferometer consists of two beam splitters coupled one after the other through two imbalanced arms, as shown in Fig.1. In the experiment under consideration, microscopic particles (either photons, electrons, neutrons, or even molecules) enter the device through the input leg of the first beam splitter, BS_{in} , and leave it through either one of the two exit legs of the second beam splitter, BS_{out} , where they are recorded one by one by either one of the two detectors, D_A and D_B , located at their respective ends.

Thus, the operators describing the corresponding modes at the input and output legs of the device are related by the linear transformation

$$\begin{pmatrix} \hat{a}_{out,A} \\ \hat{a}_{out,B} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i(\Phi+\Delta)} \end{pmatrix} \times \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \hat{a}_{in,1} \\ \hat{a}_{in,2} \end{pmatrix}, \quad (1)$$

where Φ is a phase that needs to be experimentally calibrated and $\Delta = 2\pi L/\lambda$ is the phase shift introduced by

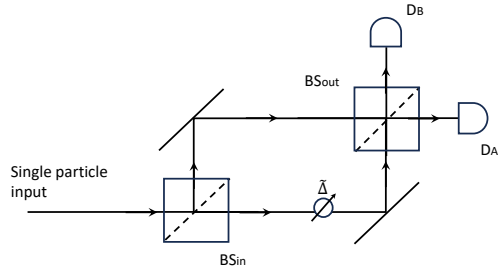


FIG. 1. Schematic set-up for a Mach-Zender interferometer, consisting of two splitters connected through two imbalanced intermediate arms and two detectors located at the two exit legs of the second splitter.

an additional length imbalance between the two intermediate arms relative to the calibrated setting. Hence, for a particle that enters the device through port #1 of the first splitter,

$$\langle \hat{a}_{in,1}^2 \rangle = 1 - \langle \hat{a}_{in,2}^2 \rangle = 1, \quad (2)$$

the probabilities that it exits through either one of the two output legs of the second splitter, #A or #B, are given, respectively, by $p_{A(B)} = \langle \hat{a}_{out,A(B)}^2 \rangle$. That is,

$$p_A = \frac{1}{2} (1 + \cos(\tilde{\Delta})), \quad p_B = \frac{1}{2} (1 - \cos(\tilde{\Delta})), \quad (3)$$

where $\tilde{\Delta} = \Phi + \Delta$.

These probabilities are formally identical to the probabilities predicted by quantum mechanics for either equal or opposite outcomes at the two detectors in the CHSH-Bell experiment with two maximally entangled qubits, with the phase shift $\tilde{\Delta}$ introduced by the length imbalance between the intermediate arms of the Mach-Zender interferometer playing a role analogous to the relative orientation between the two detectors in the CHSH-Bell experiment.

In a series of recent papers, we have described an explicit statistical model that reproduces the predictions of quantum mechanics for the CHSH-Bell experiment [26–30] and, hence, it also reproduces the predictions for the double-slit experiment as summarized by eq.(3). This paper focuses on the physical interpretation of this statistical model for the latter experiment.

III. A SIMPLE STATISTICAL MODEL FOR THE DOUBLE-SLIT EXPERIMENT ON A MACH-ZENDER INTERFEROMETER

Let us consider for any single particle going through the Mach-Zender interferometer a space of infinitely

many possible hidden configurations $\lambda \in \mathcal{S}$ distributed over the unit circle \mathcal{S} with a density of probability distribution

$$g(\lambda) = \frac{1}{4} |\sin(\lambda)|, \quad \lambda \in [-\pi, \pi]. \quad (4)$$

Let us now consider a binary response function given by

$$R(\lambda) = \text{sign}(\lambda) \cdot \text{sign}(\lambda'), \quad (5)$$

where

$$\lambda' = -L(\lambda; \tilde{\Delta}) \in [-\pi, \pi), \quad (6)$$

with

- If $\tilde{\Delta} \in [0, \pi)$,

$$L(\lambda; \tilde{\Delta}) = \begin{cases} q(\lambda) \cdot \text{arc-cos} \left(-\cos(\tilde{\Delta}) - \cos(\lambda) - 1 \right), & \text{if } -\pi \leq \lambda < \tilde{\Delta} - \pi, \\ q(\lambda) \cdot \text{arc-cos} \left(+\cos(\tilde{\Delta}) + \cos(\lambda) - 1 \right), & \text{if } \tilde{\Delta} - \pi \leq \lambda < 0, \\ q(\lambda) \cdot \text{arc-cos} \left(+\cos(\tilde{\Delta}) - \cos(\lambda) + 1 \right), & \text{if } 0 \leq \lambda < \tilde{\Delta}, \\ q(\lambda) \cdot \text{arc-cos} \left(-\cos(\tilde{\Delta}) + \cos(\lambda) + 1 \right), & \text{if } \tilde{\Delta} \leq \lambda < +\pi, \end{cases} \quad (7)$$

- If $\tilde{\Delta} \in [-\pi, 0)$,

$$L(\lambda; \tilde{\Delta}) = \begin{cases} q(\lambda) \cdot \text{arc-cos} \left(-\cos(\tilde{\Delta}) + \cos(\lambda) + 1 \right), & \text{if } -\pi \leq \lambda < \tilde{\Delta}, \\ q(\lambda) \cdot \text{arc-cos} \left(+\cos(\tilde{\Delta}) - \cos(\lambda) + 1 \right), & \text{if } \tilde{\Delta} \leq \lambda < 0, \\ q(\lambda) \cdot \text{arc-cos} \left(+\cos(\tilde{\Delta}) + \cos(\lambda) - 1 \right), & \text{if } 0 \leq \lambda < \tilde{\Delta} + \pi, \\ q(\lambda) \cdot \text{arc-cos} \left(-\cos(\tilde{\Delta}) - \cos(\lambda) - 1 \right), & \text{if } \tilde{\Delta} + \pi \leq \lambda < +\pi, \end{cases} \quad (8)$$

and

$$q(\lambda) = \text{sign}((\lambda - \tilde{\Delta}) \bmod([- \pi, \pi])).$$

Here, the function $y = \text{arc-cos}(x)$ is defined in its main branch, such that $y \in [0, \pi]$ while $x \in [-1, +1]$. This coordinates transformation implies, in particular, that

$$\lambda = -L(\lambda'; \tilde{\Delta}), \quad (9)$$

and, moreover,

$$d\lambda g(\lambda) = d\lambda' g(\lambda'), \quad (10)$$

so that both coordinates λ and λ' are distributed with the same density of probability over the unit circle. As

we will discuss in further detail in the next section, we interpret these two coordinates, λ and λ' , as describing two phases propagating, respectively, along each of the two intermediate arms of the Mach-Zender interferometer.

The response function (5) determines whether the particle shall leave the interferometer through exit leg $\#A$ or through exit leg $\#B$ according to the following rule,

$$\text{exit} = \begin{cases} \#A, & \text{if } R(\lambda) = +1, \\ \#B, & \text{if } R(\lambda) = -1. \end{cases} \quad (11)$$

It is then straightforward to obtain that the respective probabilities are given by

$$p_A = 1 - p_B = 1 - 2 \int_0^{\tilde{\Delta}} d\lambda g(\lambda) = \frac{1}{2} \left(1 + \cos(\tilde{\Delta}) \right), \quad (12)$$

which exactly reproduces the probabilities predicted by quantum mechanics, eq. (3) for this experiment. Nonetheless, there are obvious questions regarding the physical interpretation of this model that need to be addressed, and we do so in the coming section.

IV. DISCUSSION

One should wonder how can the phase shift $\tilde{\Delta}$ associated with the length imbalance between the two intermediate arms of the Mach-Zender interferometer affect the results of the described experiment, given that the collected evidence clearly shows that every single particle goes through either one or the other of the two intermediate arms, but never through both of them at once [10]. This question might be regarded as the ultimate mystery about the double-slit experiment.

In the statistical model presented in the previous section, the phase shift $\tilde{\Delta}$ enters the description of the experiment through the transformation (6),(9) that relates the pair of angular coordinates λ and λ' , which in turn determine the exit leg through which the particle leaves the interferometer according to the response function defined by (11) and (5). We then advanced that this pair of coordinates must be interpreted as describing two phases propagating, respectively, along the two intermediate arms of the interferometer. In other words, our model suggests that even though each single particle propagates along only one of the two available intermediate arms, it also polarizes the vacuum state of the second idle arm. Indeed, if quantum states accept a statistical description in terms of hidden configurations, the vacuum state must not be an exception. As noticed in [39], the vacuum must be understood as a "dielectric" medium that can get polarized.

As we noticed in previous papers, where this statistical model was already discussed in the context of the Bell-CHSH experiment, the relationship (10) may be associated with the spontaneous breaking of a phase rotational symmetry. Here, we interpret it as indicating that this

symmetry gets broken along both intermediate arms of the interferometer, the one carrying the particle and the idle one. In fact, the hidden configurations along the arm carrying the particle and along the other, idle arm must both be characterized by the exact 'conserved charges' (mass and momentum, electric charge, etc, ...) that identify the particle and the vacuum, respectively. In modern physics, conserved charges are determined by how physical objects transform under corresponding symmetry operations. Thus, the hidden configurations describing the polarization of the vacuum must not carry actual charges - that is, they must be invariant under the corresponding symmetry operations - unless the symmetry is spontaneously broken.

The role of the vacuum (and its symmetries) in explaining a wide range of physical phenomena has been recognized since long ago [32]. For example, breaking the translational symmetry due to boundary conditions is responsible for the measurable Casimir forces experienced between two conducting plates facing each other at very short distances [33, 34]. The effects of boundary conditions on the symmetries of the quantum vacuum are also responsible for shifts in the atomic energy levels, changes in the atomic spontaneous emission times, and modifications in the anomalous magnetic moment of the electron [35–38]. At a much larger scale, symmetry-breaking vacuum fluctuations are also widely believed to have seeded the development of the large-scale cosmological mass structures of the Universe [40, 41]. According to the discussion presented in this paper, symmetry-breaking vacuum fluctuations might also play a fundamental role in explaining the long-standing mysteries about the physical mechanisms involved in the double-slit experiment.

V. AN EXPERIMENTAL TEST

Let us start with the experimental setup described in Fig.2. A coherent pulse of light enters the device through the input beam splitter S_0 , where it is split into two coherent pulses of half intensity each. These two pulses are then headed toward beam splitters S_1 and S_2 , respectively, where they are evenly split again. One pulse emerging from splitter S_1 and one emerging from splitter S_2 are then fully recombined into a single pulse and directed to detector D . At the same time, the other two pulses are brought to interfere with each other at splitter S_f . The results of their interference are measured by detectors D_A and D_B as a function of the phase shift $\tilde{\Delta}$ introduced between the two pulses.

A modified version of the above setup is described in Fig.3. In this version, a source emits single pairs of coherent photons, γ_1 and γ_2 , and sends them toward the beam splitters S_1 and S_2 , respectively. After them, the experimental setup is identical to the one discussed in Fig.2, except for the detectors, which now can measure incoming single photons and time-correlate their respec-

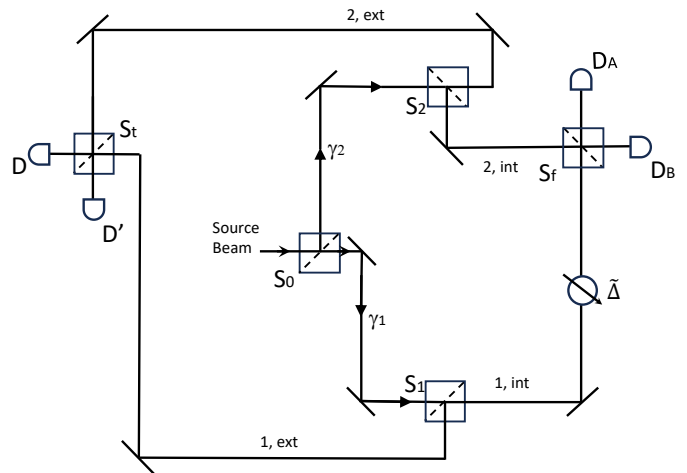


FIG. 2. Schematic description of an experiment setup with a modified Mach-Zender interferometer: a coherent input pulse is evenly split into two at splitter S_0 , as it enters the device; the two emerging pulses are directed to splitters S_1 and S_2 , respectively, where they are evenly split again; finally, two of the four pulses, one emerging from each secondary splitter, are brought together to interfere at splitter S_f , while the other two pulses are merged into a single one; the three final pulses are headed to detectors where they are measured. An interference pattern is expected at detectors D_A and D_B as a function of the phase shift $\tilde{\Delta}$ between the two interfering pulses if they have kept their coherence.

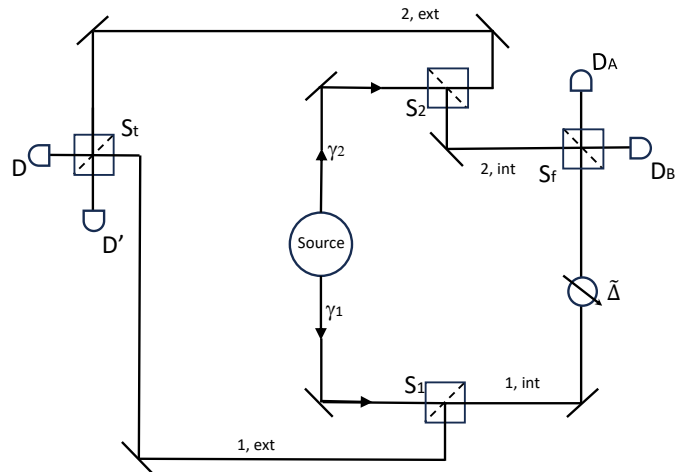


FIG. 3. Schematic description of the setup for the proposed experiment: instead of a coherent pulse split into two, as described in Fig.2, we consider a source of pairs of coherent photons, γ_1 and γ_2 , produced simultaneously. In this setup, the detectors can detect incoming single photons and time-correlate their arrivals.

tive measurements.

Let us say we perform a long sequence of repetitions of this last experiment, with many pairs of twin photons recorded, and post-select only those events in which one photon is detected at detector D_A while the other photon is detected at detector D . We shall call these events the post-selected collection of events.

We may now ask the following question regarding the post-selected collection of events: do we expect the number of counters at detectors D_A to show a pattern of interference as a function of a phase imbalance $\tilde{\Delta}$ introduced in one of the inner arms, similar to the pattern expected when coherent pulses were considered? Quantum Mechanics tells us that an interference pattern is indeed expected, see appendix A. If so, we would have experimental evidence of the interaction of one of the photons with the polarized vacuum created by the other photon.

VI. CONCLUSION

We have presented a statistical model that successfully reproduces the experimentally confirmed predictions of Quantum Mechanics for the double-slit experiment on a Mach-Zender interferometer. The proposed model suggests that even though every single particle goes through the device along only one of the two available paths, it also polarizes the vacuum along the second idle path. Our model thus offers a long-sought physically intuitive explanation to an experiment which, according to many, encloses the deepest of all the mysteries about the physical reality underlying the quantum formalism [9]. An experimental setup that might test this proposal was also suggested.

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Appendix A: Quantum mechanical description of the proposed experimental set-up

In this section, we discuss the quantum mechanical predictions for the experimental set-up discussed in Section V and shown in Fig.3.

The initial pair of photons γ_1, γ_2 is described by the

wavefunction

$$|\Psi\rangle = |\psi\rangle_1 \otimes |\psi\rangle_2. \quad (\text{A1})$$

The notation highlights that the two photons share the same frequency and polarization but are distinguishable. For example, they may be produced with orthogonal polarizations and then rotated to share the same polarization after being located in their respective arms of the optical device.

After passing through the beam splitters S_1 and S_2 at the entrance of the device, the pair of photons is described by the wavefunction

$$|\Psi'\rangle = \frac{1}{\sqrt{2}} \left(i|0\rangle_{1,int} \otimes |1\rangle_{1,ext} + |1\rangle_{1,int} \otimes |0\rangle_{1,ext} \right) \otimes \frac{1}{\sqrt{2}} \left(|0\rangle_{2,ext} \otimes |1\rangle_{2,int} + i|1\rangle_{2,ext} \otimes |0\rangle_{2,int} \right) \quad (\text{A2})$$

where the lower labels $|i\rangle_{i,int}$ and $|i\rangle_{i,ext}$, $i = 1, 2$, denote, respectively, the internal and external exit legs of each one of the two splitters according to the notation used in Fig.3, and $|1\rangle$ and $|0\rangle$ denote, respectively, quantum states for which the propagating mode along the corresponding leg is either filled or empty.

This wavefunction can be rewritten as

$$|\Psi'\rangle = \frac{1}{2} \left(i|0\rangle_{1,int} \otimes |1\rangle_{1,ext} \otimes |0\rangle_{2,ext} \otimes |1\rangle_{2,int} - |0\rangle_{1,int} \otimes |1\rangle_{1,ext} \otimes |1\rangle_{2,ext} \otimes |0\rangle_{2,int} + |1\rangle_{1,int} \otimes |0\rangle_{1,ext} \otimes |0\rangle_{2,ext} \otimes |1\rangle_{2,int} + i|1\rangle_{1,int} \otimes |0\rangle_{1,ext} \otimes |1\rangle_{2,ext} \otimes |0\rangle_{2,int} \right). \quad (\text{A3})$$

After introducing a phase shift $\tilde{\Delta}$ along the internal leg of one of the splitters with respect to its external leg, we bring the pair of photons into the state

$$|\Psi''\rangle = \frac{1}{2} \left(i|0\rangle_{1,int} \otimes |1\rangle_{1,ext} \otimes |0\rangle_{2,ext} \otimes |1\rangle_{2,int} - |0\rangle_{1,int} \otimes |1\rangle_{1,ext} \otimes |1\rangle_{2,ext} \otimes |0\rangle_{2,int} + e^{i\tilde{\Delta}} |1\rangle_{1,int} \otimes |0\rangle_{1,ext} \otimes |0\rangle_{2,ext} \otimes |1\rangle_{2,int} + i e^{i\tilde{\Delta}} |1\rangle_{1,int} \otimes |0\rangle_{1,ext} \otimes |1\rangle_{2,ext} \otimes |0\rangle_{2,int} \right). \quad (\text{A4})$$

This wavefunction can be written as a linear superposition of three terms:

$$|\Psi''\rangle = \frac{i}{\sqrt{2}} |\Psi''\rangle_{int,ext} - \frac{1}{2} |\Psi''\rangle_{ext,ext} + \frac{e^{i\tilde{\Delta}}}{2} |\Psi''\rangle_{int,int} \quad (\text{A5})$$

where the first term

$$|\Psi''\rangle_{int,ext} = \frac{1}{\sqrt{2}} \left(|0\rangle_{1,int} \otimes |1\rangle_{1,ext} \otimes |0\rangle_{2,ext} \otimes |1\rangle_{2,int} + e^{i\tilde{\Delta}} |1\rangle_{1,int} \otimes |0\rangle_{1,ext} \otimes |1\rangle_{2,ext} \otimes |0\rangle_{2,int} \right) \quad (\text{A6})$$

describes events in which one of the two photons, either γ_1 or γ_2 , exits its splitter through the internal arm while the other photon exits its splitter through the external arm; the second term

$$|\Psi''\rangle_{ext,ext} = |0\rangle_{1,int} \otimes |1\rangle_{1,ext} \otimes |1\rangle_{2,ext} \otimes |0\rangle_{2,int} \quad (\text{A7})$$

describes events in which both photons exit through the external arms of their respective splitters, so they both are detected either at detector D or D' ; and the third term

$$|\Psi''\rangle_{int,int} = |1\rangle_{1,int} \otimes |0\rangle_{1,ext} \otimes |0\rangle_{2,ext} \otimes |1\rangle_{2,int} \quad (\text{A8})$$

describes events in which both photons exit their respective splitters through their internal arms, so they both are detected either at detector D_A or D_B .

The first term (A6) is the focus of our interest for reasons that will become clear below. Let us rewrite this term as:

$$|\Psi''\rangle_{int,ext} = \frac{1}{\sqrt{2}} (|H_D\rangle_{int} \otimes |D\rangle_{ext} + |H_{D'}\rangle_{int} \otimes |D'\rangle_{ext}) \quad (\text{A9})$$

where

$$|D\rangle_{ext} = \frac{1}{\sqrt{2}} \left(|1\rangle_{1,ext} \otimes |0\rangle_{2,ext} + i|0\rangle_{1,ext} \otimes |1\rangle_{2,ext} \right) \quad (\text{A10})$$

and

$$|D'\rangle_{ext} = \frac{1}{\sqrt{2}} \left(|1\rangle_{1,ext} \otimes |0\rangle_{2,ext} - i|0\rangle_{1,ext} \otimes |1\rangle_{2,ext} \right) \quad (\text{A11})$$

are the orthogonal eigenstates corresponding, respectively, to the photon traveling through the external arms being detected either at detector D or D' after going through the splitter S_t , while

$$|H_D\rangle_{int} = \frac{1}{\sqrt{2}} \left(|0\rangle_{1,int} \otimes |1\rangle_{2,int} - i e^{i\tilde{\Delta}} |1\rangle_{1,int} \otimes |0\rangle_{2,int} \right) \quad (\text{A12})$$

and

$$|H_{D'}\rangle_{int} = \frac{1}{\sqrt{2}} \left(|0\rangle_{1,int} \otimes |1\rangle_{2,int} + i e^{i\tilde{\Delta}} |1\rangle_{1,int} \otimes |0\rangle_{2,int} \right) \quad (\text{A13})$$

are the states upon which the photon traveling through the internal arms gets projected when the external photon is detected, respectively, at detector D or D' .

The conditional probabilities for the photon traveling along the internal arms being detected either at detector D_A or detector D_B after going through splitter S_f , given

that the photon traveling through the external arms was detected at detector D are given, respectively, by

$$p_A = |{}_{int}\langle H_D | D_A \rangle_{int}|^2 = \frac{1}{2} (1 + \cos \tilde{\Delta}) \quad (\text{A14})$$

$$p_B = |{}_{int}\langle H_D | D_B \rangle_{int}|^2 = \frac{1}{2} (1 - \cos \tilde{\Delta}) \quad (\text{A15})$$

where

$$|D_A\rangle_{int} = \frac{1}{\sqrt{2}} \left(|0\rangle_{1,int} \otimes |1\rangle_{2,int} - i |1\rangle_{1,int} \otimes |0\rangle_{2,int} \right), \quad (\text{A16})$$

$$|D_B\rangle_{int} = \frac{1}{\sqrt{2}} \left(|0\rangle_{1,int} \otimes |1\rangle_{2,int} + i |1\rangle_{1,int} \otimes |0\rangle_{2,int} \right), \quad (\text{A17})$$

are the corresponding eigenstates for the internal photon being detected after going through the splitter S_f at detector D_A or detector D_B , respectively.

According to eq. (A14), in a long sequence of repetitions of the experiment in which many pairs of photons are tested, we should observe an interference pattern as a function of the phase shift $\tilde{\Delta}$ in the number of photons detected at device D_A when we post-select those events in which the other photon is detected at detector D .