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Damping assignment of boundary controlled port-Hamiltonian systems with unknown open-loop damping

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Abstract: A damping assignment control law for infinite-dimensional port-Hamiltonian systems in one-dimensional space with actuators and sensors located at the spatial boundaries is proposed with the novelty that the boundary damping is unknown. This allows us to fix a desired decay of energy for the cases in which the system is over-damped, poorly damped, and even with negative damping. We propose an observer composed of an infinite-dimensional model and a finite-dimensional one for the state and parameter estimation. The asymptotic convergence of the observer is shown using LaSalle's invariance principle assuming that the trajectories are precompact. Finally, an observer-based adaptive output feedback controller is proposed for the damping assignment in the closed loop. The passivity of the closed-loop system is guaranteed with respect to the initial Hamiltonian of the system under the assumption that the observer is initialized identically to the current state and close enough to the parameter value. The transmission line is used to exemplify this approach.

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Keywords: Boundary control systems, distributed parameter systems, port-Hamiltonian systems, damping assignment, observer design, adaptive control.

1. INTRODUCTION

The port-Hamiltonian approach, initially introduced for finite-dimensional systems in [Maschke and van der Schaft, 1992], is a framework that takes energy considerations for the modelling and control of dynamical systems. One of the main advantages of this approach is that it can be seen as a *lingua franca* between different physical domains [van der Schaft and Jeltsema, 2014, Márquez et al., 2020]. The port-Hamiltonian representation has been extended to infinite-dimensional systems in [van der Schaft and Maschke, 2002] and the class of Boundary Controlled Port-Hamiltonian Systems (BC-PHS) has been characterized in [Le Gorrec et al., 2005]. Using this approach one can model beams, waves, transmission lines, transport phenomena, among others.

Since the Hamiltonian can be, in general, used as Lyapunov function, the stability of port-Hamiltonian systems can be characterized by the internal energy dissipation. Although in most physical systems, energy dissipation is natural due to intrinsic dissipative elements like resistors and dampers, their influence on the system can be different to the desired one. Then, control laws are important to modify the decay of energy into a desired one. For BC-PHS, this has been investigated using static and dynamic output feedback controllers [Villegas, 2007, Jacob and Zwart, 2012]. Asymptotic stability has been reported in [Macchelli, 2013, Macchelli et al., 2017], exponential stability in [Villegas et al., 2009, Ramirez et al.,

2014, Trostorff and Waurick, 2022], and more recently, the damping assignment technique has been proposed for the vibrating string model with distributed damping [Redaud et al., 2022]. In order to precisely assign a desired decay of energy, the values of the intrinsic damping (or the one added by design) are required by the controller.

One way to estimate the current parameters of the system is using dynamic observers and Kalman filters [Dochain, 2003, Demetriou and Ito, 2003]. For port-Hamiltonian systems, observers have been mainly used for state estimation. See for instance [Venkatraman and van der Schaft, 2010, Vincent et al., 2016, Yaghmaei and Yazdanpanah, 2018, Biedermann and Meurer, 2021, Pfeifer et al., 2021] for the finite-dimensional case and [Kotyczka et al., 2019, Toledo et al., 2020, Malzer et al., 2021b,a] for the infinite-dimensional case. To the best of the authors' knowledge, the estimation of parameters using observers has not been investigated for BC-PHS. In particular, in this paper we propose an observer composed of an infinite-dimensional model and a finite-dimensional one for the state and parameter estimation, being the observed parameter the damping coefficients at the spatial boundaries.

By estimating the damping terms at the spatial boundaries, in this paper, we present a preliminary step towards the damping assignment of the closed-loop of BC-PHS under unknown damping in open-loop. The conditions to guarantee asymptotic stability and the passivity of the closed-loop system with respect to the initial Hamiltonian are limited to a proper initialization of the observer. The main contributions of this paper are therefore twofold: (i)

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a dynamic observer for the estimation of the damping at the spatial boundaries of BC-PHS, and (ii) an adaptive output feedback control law for the assignment of damping in closed-loop.

The paper is organized as follows. In Section 2, we recall the class of system of interest and we introduce the main problem. In Section 3, we propose the observer and we show its convergence. In Section 4, we present the conditions to guarantee asymptotic stability using the adaptive output feedback control law. In Section 5, we illustrate the approach using numerical simulations on the transmission line. Finally, in Section 6, we give some conclusions and perspectives.

2. PROBLEM FORMULATION

We consider the following boundary controlled port-Hamiltonian system:

$$\Sigma : \begin{cases} \frac{\partial z}{\partial t}(\zeta, t) = P_1 \frac{\partial}{\partial \zeta} (\mathcal{H}(\zeta)z(\zeta, t)) + P_0 \mathcal{H}(\zeta)z(\zeta, t), \\ z(\zeta, 0) = z_0(\zeta), \\ y(t) := W_C \begin{pmatrix} f_{\partial}(t) \\ e_{\partial}(t) \end{pmatrix}, \\ W_B \begin{pmatrix} f_{\partial}(t) \\ e_{\partial}(t) \end{pmatrix} = -Ry(t) + v(t) := u(t), \end{cases} \quad (1)$$

where $\zeta \in [a, b]$ is the one-dimensional space, $t \geq 0$ is the time, $z(\zeta, t) \in \mathbb{R}^n$ is the state variable with initial condition $z_0(\zeta)$, $P_1 = P_1^T \in \mathbb{R}^{n \times n}$ is a non-singular matrix, $P_0 = -P_0^T \in \mathbb{R}^{n \times n}$, $\mathcal{H}(\zeta)$ is a bounded and continuously differentiable matrix-valued function satisfying for all $\zeta \in [a, b]$, $\mathcal{H}(\zeta) = \mathcal{H}^T(\zeta)$ and $mI < \mathcal{H}(\zeta) < MI$, with $0 < m < M$ being scalars independent of ζ , $\begin{pmatrix} f_{\partial}(t) \\ e_{\partial}(t) \end{pmatrix}$ are the *boundary port variables* defined as

$$\begin{pmatrix} f_{\partial}(t) \\ e_{\partial}(t) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} P_1 & -P_1 \\ I & I \end{pmatrix} \begin{pmatrix} \mathcal{H}(b)z(b, t) \\ \mathcal{H}(a)z(a, t) \end{pmatrix}, \quad (2)$$

and W_B and $W_C \in \mathbb{R}^{n \times 2n}$ are two matrices defining the input $u(t) \in \mathbb{R}^n$ and output $y(t) \in \mathbb{R}^n$, respectively.

The Hamiltonian associated to (1) is given by

$$H(t) = \frac{1}{2} \int_a^b z(\zeta, t)^\top \mathcal{H}(\zeta)z(\zeta, t) d\zeta, \quad (3)$$

and in this paper, we are interested to the cases in which the boundary output and boundary input in (1) are conjugated in the sense that $\dot{H}(t) = y(t)^\top u(t)$, *i.e.*, the cases in which the input and output matrices satisfy $W_B \Sigma W_B^\top = W_C \Sigma W_C^\top = 0$ and $W_C \Sigma W_B^\top = I$, with $\Sigma = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$. The matrix $R \in \mathbb{R}^{n \times n}$ is related to dissipative elements placed at the spatial boundaries of the domain and for simplicity, in this paper, is considered that is diagonal. Finally, $v(t) \in \mathbb{R}^n$ represents the control action. The reader is refer to [Le Gorrec et al., 2005, Jacob and Zwart, 2012] for more details on the BC-PHS (1).

Since $\dot{H}(t) = y(t)^\top u(t)$, we obtain the following energy balance through (1):

$$\dot{H}(t) = -y(t)^\top Ry(t) + y(t)^\top v(t). \quad (4)$$

Using the control action $v(t) = -(R_d - R)y(t) + r(t)$, with $R_d \in \mathbb{R}^{n \times n}$ and $r(t) \in \mathbb{R}^n$, one can fix some desired decay of energy by properly choosing R_d . Thus, the following desired energy balance is achieved:

$$\dot{H}(t) = -y(t)^\top R_d y(t) + y(t)^\top r(t). \quad (5)$$

The objective of this paper is to approach the desired energy balance in (5) **without the knowledge of the matrix R** . To this end, we propose an observer composed of an infinite-dimensional model (a copy of (1)) and a finite-dimensional one containing as state the estimation of the parameters. This observer allows us to estimate the damping matrix R , which can be then used in the following adaptive output feedback control law:

$$v(t) = -\hat{K}(t)y(t) + r(t), \quad \hat{K}(t) = R_d - \hat{R}(t), \quad (6)$$

where $\hat{R}(t)$ is the estimated value of the matrix R . As soon as $\hat{R}(t)$ approaches R , the control law approaches the desired one.

3. ESTIMATION OF THE DAMPING

We assume that R is a diagonal matrix and we define the unknown vector θ as follows:

$$\theta := \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix}, \quad R = \begin{pmatrix} \theta_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \theta_n \end{pmatrix}. \quad (7)$$

We assume that $\hat{\theta}(t) = 0$, and we propose the following observer for the system (1):

$$\hat{\Sigma} : \begin{cases} \frac{\partial \hat{z}}{\partial t}(\zeta, t) = P_1 \frac{\partial}{\partial \zeta} (\mathcal{H}(\zeta)\hat{z}(\zeta, t)) + P_0 \mathcal{H}(\zeta)\hat{z}(\zeta, t), \\ \hat{z}(\zeta, 0) = \hat{z}_0(\zeta), \\ \dot{\hat{\theta}}(t) = -L_2(y)(\hat{y}(t) - y(t)), \quad \hat{\theta}(0) = \hat{\theta}_0, \\ \hat{y}(t) := W_C \begin{pmatrix} \hat{f}_{\partial}(t) \\ \hat{e}_{\partial}(t) \end{pmatrix}, \\ W_B \begin{pmatrix} \hat{f}_{\partial}(t) \\ \hat{e}_{\partial}(t) \end{pmatrix} = -M(y)\hat{\theta}(t) + v(t) - L_1(\hat{y}(t) - y(t)), \end{cases} \quad (8)$$

where $\hat{z}(\zeta, t) \in \mathbb{R}^n$ is the estimation of $z(\zeta, t)$ with initial condition $\hat{z}_0(\zeta)$, $\hat{\theta}(t) \in \mathbb{R}^n$ is the damping estimation with initial condition $\hat{\theta}_0$, $L_1 \in \mathbb{R}^{n \times n}$ and $L_2(y) \in \mathbb{R}^{n \times n}$ are the observer matrices defined here after, $M(y) \in \mathbb{R}^{n \times n}$ is a diagonal matrix with the i -th diagonal element equal to the i -th element of the vector $y(t)$, and the *observed boundary port variables* $\begin{pmatrix} \hat{f}_{\partial}(t) \\ \hat{e}_{\partial}(t) \end{pmatrix}$ are defined as in (2) using $\hat{z}(b, t)$ and $\hat{z}(a, t)$ instead of $z(b, t)$ and $z(a, t)$, respectively.

Let the observer errors be defined as $\tilde{z}(\zeta, t) = \hat{z}(\zeta, t) - z(\zeta, t)$ and $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$. Then, computing estimation error dynamics as follows:

$$\tilde{\Sigma} : \begin{cases} \frac{\partial \tilde{z}}{\partial t}(\zeta, t) = P_1 \frac{\partial}{\partial \zeta} (\mathcal{H}(\zeta)\tilde{z}(\zeta, t)) + P_0 \mathcal{H}(\zeta)\tilde{z}(\zeta, t), \\ \tilde{\theta}(t) = -L_2(y)\tilde{y}(t), \\ \tilde{z}(\zeta, 0) = \tilde{z}_0(\zeta) = \hat{z}_0(\zeta) - z_0(\zeta), \\ \tilde{\theta}(0) = \tilde{\theta}_0 = \hat{\theta}_0 - \theta, \\ \tilde{y}(t) = W_C \begin{pmatrix} \tilde{f}_{\partial}(t) \\ \tilde{e}_{\partial}(t) \end{pmatrix}, \\ W_B \begin{pmatrix} \tilde{f}_{\partial}(t) \\ \tilde{e}_{\partial}(t) \end{pmatrix} = -M(y)\tilde{\theta}(t) - L_1\tilde{y}(t), \end{cases} \quad (9)$$

where $\tilde{z}(\zeta, t) \in \mathbb{R}^n$ is the state estimation error with initial condition $\tilde{z}_0(\zeta)$ and $\tilde{\theta} \in \mathbb{R}^n$ is the damping estimation error with initial condition $\tilde{\theta}_0$. The following proposition is instrumental for the forthcoming convergence proof of the observer.

Proposition 1. If the gain matrices L_1 and $L_2(y)$ are such that $L_1 = L_1^\top > 0$, and $L_2(y) = -Q_o^{-1}M(y)^\top$, then the

error system in (9) can be equivalently written as the passive interconnection between an infinite-dimensional port-Hamiltonian system (represented by $\tilde{\mathcal{P}}$) and a finite-dimensional one (represented by $\tilde{\mathcal{O}}$) as shown in Fig. 1.

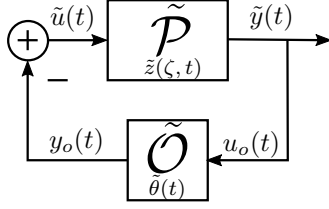


Fig. 1. Equivalent representation of the error dynamic.

Proof. If we define $\tilde{u}(t) := W_{\mathcal{B}} \begin{pmatrix} \tilde{f}_{\partial}(t) \\ \tilde{e}_{\partial}(t) \end{pmatrix}$, and the Hamiltonian error as $\tilde{H}(t) = \frac{1}{2} \int_a^b \tilde{z}(\zeta, t)^{\top} \mathcal{H} \tilde{z}(\zeta, t) d\zeta$, one can verify $\dot{\tilde{H}}(t) = \tilde{y}(t)^{\top} \tilde{u}(t)$ (This can be verified according to the assumptions on the matrices $W_{\mathcal{B}}$ and $W_{\mathcal{C}}$). Then, we can define the following BC-PHS associated to $\tilde{H}(t)$:

$$\tilde{\mathcal{P}} : \begin{cases} \frac{\partial \tilde{z}}{\partial t}(\zeta, t) = P_1 \frac{\partial}{\partial \zeta} (\mathcal{H}(\zeta) \tilde{z}(\zeta, t)) + P_0 \mathcal{H}(\zeta) \tilde{z}(\zeta, t), \\ \tilde{z}(\zeta, 0) = \tilde{z}_0(\zeta), \\ \tilde{y}(t) = W_{\mathcal{C}} \begin{pmatrix} \tilde{f}_{\partial}(t) \\ \tilde{e}_{\partial}(t) \end{pmatrix}, \\ W_{\mathcal{B}} \begin{pmatrix} \tilde{f}_{\partial}(t) \\ \tilde{e}_{\partial}(t) \end{pmatrix} = \tilde{u}(t). \end{cases} \quad (10)$$

Then, we define the following finite-dimensional port-Hamiltonian system $\tilde{\mathcal{O}}$ with associated energy $H_o(t) = \frac{1}{2} \tilde{\theta}(t)^{\top} Q_o \tilde{\theta}(t)$:

$$\tilde{\mathcal{O}} : \begin{cases} \dot{\tilde{\theta}}(t) = (J_o - R_o) Q_o \tilde{\theta}(t) + B_o(y) u_o(t), \\ y_o(t) = B_o(y)^{\top} Q_o \tilde{\theta}(t) + D_o u_o(t), \\ \tilde{\theta}(0) = \tilde{\theta}_0 = \hat{\theta}_0 - \theta, \end{cases} \quad (11)$$

with $J_o = 0$, $R_o = 0$, $0 < Q_o \in \mathbb{R}^{n \times n}$, $B_o(y) = -L_2(y)$, and $D_o = L_1$. Finally, using the passive interconnection as shown in Fig. 1

$$\tilde{\mathcal{I}} : \begin{cases} u_o(t) = \tilde{y}(t), \\ \tilde{u}(t) = -y_o(t), \end{cases} \quad (12)$$

and the fact that $L_2(y) = -Q_o^{-1} M(y)^{\top}$, one can write down the system (9) using (10), (11) and (12). This concludes the proof.

Since the error dynamic can be described by the passive interconnection between two port-Hamiltonian systems, its convergence can be shown using the total energy as Lyapunov function. In the next proposition, we present the main contribution of this paper. Using the fact that the BC-PHS in (10) is exactly observable (see [Jacob and Kaiser, 2019]) and assuming that the solution trajectories of the error system (9) are precompact, one can show the asymptotic convergence of the observer using LaSalle’s invariance principle applied to infinite-dimensional systems. This is summarized in the following proposition.

Proposition 2. Consider the error dynamic in (9) with $L_1 = L_1^{\top} > 0$, $L_2(y) = -Q_o^{-1} M(y)^{\top}$, and $Q_o = Q_o^{\top} > 0$. If $\dot{y}(t)$ is different to zero, the states $\tilde{z}(\zeta, t)$ and $\tilde{\theta}(t)$ converge

to zero asymptotically for any $\tilde{z}_0(\zeta) \in L_2([a, b], \mathbb{R}^n)$ and $\tilde{\theta}_0 \in \mathbb{R}^n$.

Proof. Using the fact that the error system (9) can be written as the interconnection between two port-Hamiltonian systems (see Proposition 1), a natural candidate Lyapunov function is the sum of the energies of both systems. That is

$$V(t) = \tilde{H}(t) + H_o(t), \quad (13)$$

with the functions $\tilde{H}(t) = \frac{1}{2} \int_a^b \tilde{z}(\zeta, t)^{\top} \mathcal{H} \tilde{z}(\zeta, t) d\zeta$ and $H_o(t) = \frac{1}{2} \tilde{\theta}(t)^{\top} Q_o \tilde{\theta}(t)$. The time derivative of (13) is:

$$\begin{aligned} \dot{V}(t) &= \tilde{y}(t)^{\top} \tilde{u}(t) + y_o(t)^{\top} u_o(t) - u_o(t)^{\top} D_o u_o(t), \\ &= -\tilde{y}(t)^{\top} y_o(t) + y_o(t)^{\top} \tilde{y}(t) - \tilde{y}(t)^{\top} L_1 \tilde{y}(t), \\ &= -\tilde{y}^{\top} L_1 \tilde{y}(t), \end{aligned}$$

where we have used (10), (11) and (12). Now, provided that the solution trajectories of the error system are precompact, we use LaSalle’s invariance principle applied to infinite-dimensional systems. To this end, we have to show that the maximal invariant set of the following set:

$$\mathcal{I} = \{ \tilde{z}(\zeta, t) \in L_2([a, b], \mathbb{R}^n), \tilde{\theta}(t) \in \mathbb{R}^n \mid \dot{V}(t) = 0 \},$$

contains the origin only, *i.e.*, $\tilde{z}(\zeta, t) = 0$ and $\tilde{\theta}(t) = 0$. Since $L_1 > 0$, the condition $\dot{V}(t) = 0$ implies $\tilde{y}(t) = 0$. Since the BC-PHS in (10) is exactly observable (see [Jacob and Kaiser, 2019, Theorem 3]), (10) is approximately observable (weaker condition). This implies that if for some period of time $t \in [0, \tau]$, $\tilde{y}(t) = 0$, then $\tilde{z}(\zeta, t) = 0$ (see [Curtain and Zwart, 2012, Corollary 4.1.14 b]). The latter implies $\tilde{u}(t) = 0$ (since $\tilde{z}(\zeta, t) = 0$ implies $\tilde{f}_{\partial}(t) = 0$ and $\tilde{e}_{\partial}(t) = 0$ (see (2) replacing f_{∂} by \tilde{f}_{∂} , e_{∂} by \tilde{e}_{∂} , and z by \tilde{z})). $\tilde{u}(t) = 0$ and $\tilde{y}(t) = 0$ imply $y_o(t) = 0$ and $u_o(t) = 0$, respectively (see (12)). The previous implies $\dot{\tilde{\theta}}(t) = 0$ and $B_o(y)^{\top} Q_o \tilde{\theta}(t) = 0$ (See (11)). Replacing $B_o(y) = -L_2(y) = Q_o^{-1} M(y)^{\top}$, we obtain $M(y) \tilde{\theta}(t) = 0$. Differentiating the latter one obtain $M(\dot{y}) \tilde{\theta}(t) + M(y) \dot{\tilde{\theta}}(t) = 0$. Since by definition $\dot{y}(t) \neq 0$ and since $\dot{\tilde{\theta}}(t) = 0$ (since $u_o(t) = 0$), the only possible value is $\tilde{\theta}(t) = 0$. This concludes the proof.

Remark 3. Note that, the main condition to use LaSalle’s invariance principle, in this case, is the approximate observability of the BC-PHS in (10). This allows applying Proposition 2 to cases where there are fewer than n sensors. This is exemplified in Section 5 with the transmission line example.

In the following section, we use the observed value $\hat{\theta}(t)$ to build an adaptive output feedback control law allowing the assignment of the desired damping in the closed loop.

4. OBSERVER-BASED DAMPING ASSIGNMENT

Now, we apply the adaptive output feedback control law (6) with $\tilde{R}(t)$ as a diagonal matrix containing the estimated values $\hat{\theta}_1(t), \dots, \hat{\theta}_n(t)$ in its diagonal (similarly to (7)). Then, the Hamiltonian of (1) becomes:

$$\dot{H}(t) = -y(t)^{\top} \left[R_d - \tilde{R}(t) \right] y(t) + y(t)^{\top} r(t), \quad (14)$$

where $\tilde{R}(t)$ being a diagonal matrix containing the elements of $\hat{\theta}(t)$.

The control objective is to guarantee that $\tilde{R}(t)$ converges faster than the state $z(\zeta, t)$ and that $\dot{H}(t) \leq y(t)^\top r(t)$ for all $t \geq 0$. In this way, the original system (1) is stable with a desired dissipation matrix R_d and the passive structure is preserved with respect to the new input $r(t)$ and the initial Hamiltonian (3). One way to guarantee this is to initialize the observer such that $\tilde{z}_0(\zeta) = 0$ and $R_d > \tilde{R}(0)$. This is a conservative condition and represents a trade-off between the choice of the desired dissipation matrix R_d in the closed loop and the initial estimation of the state and parameters. In the following proposition, we summarize this condition.

Proposition 4. Consider the system (1), the observer (8) with gains L_1 and $L_2(y)$ as in Proposition 2, and the adaptive output feedback control law (6) with $r(t) = 0$, $R_d > 0$, and $\hat{R}(t)$ as defined in (7) with $\hat{\theta}(t)$ instead. For any $z_0(\zeta) \in L_2([a, b], \mathbb{R}^n)$, the trajectory $z(\zeta, t)$ converges to zero asymptotically if the observer is initialized such that $\hat{z}_0(\zeta) = z_0(\zeta)$ and $\hat{R}(0) - R < R_d$. Moreover, if $r(t) \neq 0$, the Hamiltonian of (1) is such that $\dot{H}(t) \leq y(t)^\top r(t)$ for all t .

Proof. Let the Hamiltonian (3) of the system (1) as a candidate Lyapunov function. By replacing $u(t)$ by the observer-based control law (6) in (4) and we obtain the following balance equation

$$\dot{H}(t) = -y(t)^\top \left[R_d - \hat{R}(t) + R \right] y(t) + y(t)^\top r(t)$$

We use the fact that the estimation error is asymptotically stable as shown in Proposition 2. Then, using $V(t) = \tilde{H}(t) + H_o(t)$ from (13), we can write $\dot{V}(t) \leq 0$, implying that $V(t) \leq V(0)$. Since $\hat{z}_0(\zeta) = z_0(\zeta)$, $\tilde{z}_0(\zeta) = 0$, and $V(0) = H_o(0) = \frac{1}{2} \tilde{\theta}_0^\top Q_o \tilde{\theta}_0$. Then, since $V(t) \leq V(0)$ we can conclude that for all $t \geq 0$, the following is satisfied $\frac{1}{2} \tilde{\theta}(t)^\top Q_o \tilde{\theta}(t) \leq \frac{1}{2} \tilde{\theta}_0^\top Q_o \tilde{\theta}_0$. The last implies that $\|\tilde{\theta}(t)\|_2^2 \leq \|\tilde{\theta}_0\|_2^2$, implying as well that the diagonal elements of $\tilde{R}(t) = \hat{R}(t) - R$ can not be bigger than $\tilde{R}(0) = \hat{R}(0) - R$, *i.e.*, $\tilde{R}(t) \leq \tilde{R}(0)$ or $\hat{R}(t) \leq \hat{R}(0)$. Then, since $\hat{R}(0) < R_d + R$ (by definition) and $\hat{R}(t) \leq \hat{R}(0)$, $0 < R_d - \hat{R}(t) + R$ for all t . This allows to conclude that for all t , $y(t) \neq 0$ and $r(t) = 0$, the Hamiltonian is strictly decreasing, *i.e.*, $\dot{H}(t) < 0$, implying that $z(\zeta, t)$ converges to zero asymptotically. Finally, since $R_d - \hat{R}(t) + R > 0$, $\dot{H}(t) \leq y(t)^\top r(t)$ for all t and $r(t) \neq 0$. This concludes the proof.

Remark 5. The condition on the initialization of the observer in Proposition 4 is restrictive in the sense that this is not always possible due to the fact that the state is not always measured. However, this is only needed to preserve the passivity of the system with respect to the original Hamiltonian (3). Since by Proposition 2, the convergence of the observer is guaranteed for any initial condition on the L_2 space, the matrix $R_d - \hat{R}(t) + R = R_d - \tilde{R}(t)$ is guaranteed to converge to R_d (since $\tilde{R}(t)$ converges to zero). However, the free initialization of the observer may cause $\dot{H}(t) > 0$ before the convergence of $\hat{R}(t)$.

Remark 6. As for lumped linear time-invariant systems there is a trade-off on the design of the observer and the controller. In this case, the design parameter of the controller is the desired damping R_d and the design

parameters of the observer are the matrices L_1 and $L_2(y)$. The gains L_1 and $L_2(y)$ have to be designed such that the convergence of the observer is faster than the convergence of the state $z(\zeta, t)$. This is important for the case in which $r(t) = 0$, where due to the asymptotic stability of the closed-loop system, the state $z(\zeta, t)$ converges to zero, implying $\dot{y}(t) = 0$ and thus, loosing the asymptotic behaviour of the estimation error.

5. THE TRANSMISSION LINE EXAMPLE

Consider infinite-dimensional model of a transmission line with zero voltage at one end and an unknown resistor with a voltage source at the other end. The voltage source is considered as the actuator and we assume that the supply current to the transmission line is measured. In Fig. 2, we show a diagram of the circuit where $t \geq 0$ is the time, $\zeta \in [a, b]$ is the spatial variable, $v(t)$ is the voltage of the source, $\theta \in \mathbb{R}$ is the unknown resistor, and $i(a, t)$, $u(a, t)$, and $u(b, t) = 0$ are respectively, the input current, the input voltage, and the voltage at the end of the transmission line.

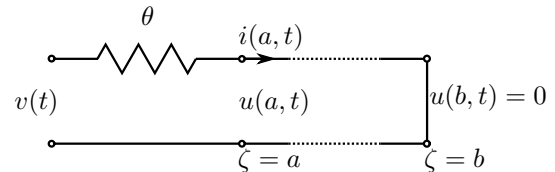


Fig. 2. Transmission line diagram.

The dynamic can be described by the following equations

$$\Sigma : \begin{cases} \frac{\partial Q}{\partial t}(\zeta, t) = -\frac{\partial}{\partial \zeta} \left(\frac{\phi(\zeta, t)}{L(\zeta)} \right), & Q(\zeta, 0) = Q_0(\zeta), \\ \frac{\partial \phi}{\partial t}(\zeta, t) = -\frac{\partial}{\partial \zeta} \left(\frac{Q(\zeta, t)}{C(\zeta)} \right), & \phi(\zeta, 0) = \phi_0(\zeta), \\ \frac{Q(b, t)}{C(b)} = 0, \\ \frac{Q(a, t)}{C(a)} = -\theta \frac{\phi(a, t)}{L(a)} + v_s(t), \\ y_1(t) = \frac{\phi(a, t)}{L(a)}, \end{cases}$$

where $Q(\zeta, t) \in \mathbb{R}$ is the electric charge with initial condition $Q_0(\zeta)$, $\phi(\zeta, t) \in \mathbb{R}$ is the magnetic flux with initial condition $\phi_0(\zeta)$, $C(\zeta) > 0$ for all $\zeta \in [a, b]$ is the capacity density and $L(\zeta) > 0$ for all $\zeta \in [a, b]$ is the inductance density. The voltage of the transmission line is given by $u(\zeta, t) = \frac{Q(\zeta, t)}{C(\zeta)}$ and the current by $i(\zeta, t) = \frac{\phi(\zeta, t)}{L(\zeta)}$

The previous system can be represented as the BC-PHS in (1) with

$$z(\zeta, t) = \begin{pmatrix} Q(\zeta, t) \\ \phi(\zeta, t) \end{pmatrix}, \quad z_0(\zeta) = \begin{pmatrix} Q_0(\zeta) \\ \phi_0(\zeta) \end{pmatrix},$$

$$P_1 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad P_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathcal{H} = \begin{pmatrix} \frac{1}{C(\zeta)} & 0 \\ 0 & \frac{1}{L(\zeta)} \end{pmatrix},$$

$$W_B = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix}, \quad W_C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix},$$

$$u(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}, \quad y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix},$$

$$\begin{aligned}
 u_1(t) &= -\theta y_1(t) + v_s(t), & y_1(t) &= \frac{\phi(a,t)}{L(a)}, \\
 u_2(t) &= 0, & y_2(t) &= -\frac{\phi(b,t)}{L(b)}, \\
 R &= \begin{pmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{pmatrix}, & \theta_1 &= \theta, & v(t) &= \begin{pmatrix} 0 \\ v_s(t) \end{pmatrix}.
 \end{aligned}$$

Now, we use the fact that the system is approximate observable with respect to $y_1(t)$ (since $\|\mathcal{H}(a)z(a,t)\| \leq \|u_1(t)\|^2 + \|y_1(t)\|^2$ (see [Mattioni, 2021, Theorem 3.2.1])). This makes also applicable Proposition (2) (see Remark 3) using $u_1(t)$, $u_2(t)$ and $y_1(t)$, only. Then, the observer (8) is designed with the following boundary conditions:

$$W_B \begin{pmatrix} \hat{f}_\theta(t) \\ \hat{e}_\theta(t) \end{pmatrix} = \begin{pmatrix} -M(y)\hat{\theta}(t) + v_s(t) - L_1(\hat{y}_1(t) - y_1(t)) \\ 0 \end{pmatrix},$$

and the finite-dimensional part:

$$\dot{\hat{\theta}}(t) = -L_2(y_1)(\hat{y}_1(t) - y_1(t)),$$

where

$$L_1 = \alpha, \quad L_2(y) = -\beta y_1, \quad M(y_1) = y_1,$$

with $\alpha > 0$ and $\beta > 0$ represent two degrees of freedom of design. Finally, by applying the following control law (equivalent to (6), but reduced to one actuator only):

$$v_s(t) = -\hat{k}_1(t)y_1(t) + r_1(t), \quad \hat{k}_1(t) = \theta_d - \hat{\theta}_1(t),$$

where θ_d is the degree of freedom related to the desired resistor, $\hat{\theta}_1(t)$ is the estimation of $\theta_1 = \theta$, and $r_1(t)$ is an external input.

Simulation The system and the observer are simulated in *Matlab* using the solver *ode23t*. The infinite-dimensional transmission line and the observer are spatially discretized using the finite differences method described in [Trenchant et al., 2018] with 200 states per state variable. In total there are 400 states for the system and 401 states for the observer (400 states for the infinite-dimensional part and 1 state for the finite-dimensional one).

Scenario 1 The transmission line is commissioned from the zero equilibrium through sinusoidal voltages and currents. To this end, we use the external input $r_1(t) = \sin(t)$. The resistance θ is unknown by the controller. In this case, we consider a poorly damped system with $\theta_1 = \theta = 0.1$, and such a low damping generates long peaks in the storage energy of the transmission line, as shown in Fig. 3. The objective is to assign a desired damping $\theta_d = 1$. To do so, we use $\alpha = 2$ and $\beta = 2$ for the design of the observer. The system and the observer are initialized with zero initial conditions. In Fig. 3, we show the Hamiltonian obtained using the observer-based damping assignment and the desired one. Similar behaviors can be observed, since the estimation $\hat{\theta}$ is always close to the real value of θ . The damping estimation is illustrated in Fig. 4, taking around 40 second to estimate the true value in this case.

Scenario 2 Now, consider the same design and initial conditions as Scenario 1, but in two different scenarios: (i) the open-loop system is unstable with a negative damping, and (ii) the system is overdamped ($\theta \gg 1$). In both cases, the observer-based approach achieves the desired behavior, as show in Fig. 5. For the unstable case, one can see a big peak on the Hamiltonian during the first seconds. This is due to the poor estimation of the initial condition $\hat{\theta}(0) = 0$ and the unstable nature of the transmission line. For the

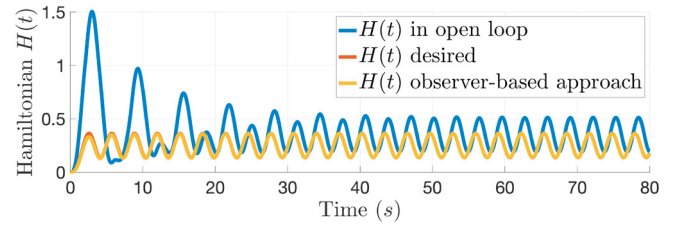


Fig. 3. Hamiltonian in open loop (blue), desired one (red) and in closed-loop with the observer-based approach.

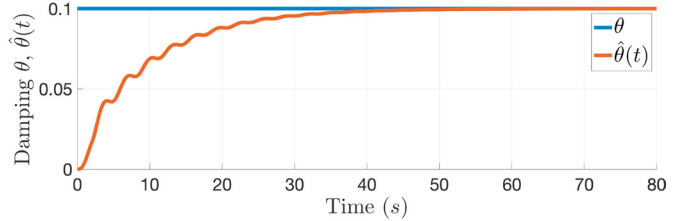


Fig. 4. Estimation of the parameter $\theta = 0.1$.

overdamped case, the Hamiltonian takes more time to achieve the desired behavior. This is also due to the poor initialization of $\hat{\theta}(0)$ and the design of the observer. The time response can be improved by modifying the observer design parameters α and β .

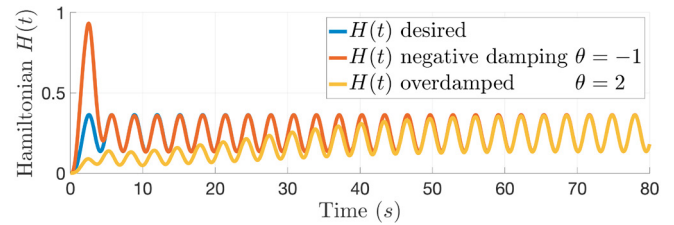


Fig. 5. Hamiltonian with desired damping (blue), negative damping (red), and overdamped (yellow).

6. CONCLUSIONS AND PERSPECTIVES

In this paper, a novel method for the assignment of damping in closed-loop for boundary-controlled port-Hamiltonian systems under the assumption that the damping at the spatial boundaries is unknown. To do so, we have proposed a dynamic observer composed of an infinite-dimensional model and a finite-dimensional one for the state and damping estimation. The convergence of the observer is guaranteed to be globally asymptotically stable if the conjugated output is not static. Then, an adaptive output feedback control law has been proposed to assign a desired damping to the closed-loop system. The conditions that guarantee closed-loop stability and passivity preservation are presented in terms of the initial conditions of the observer, being restrictive in the sense that the observer initialization should be identically to the state of the system and close enough to the parameters. The transmission line has been used to exemplify the proposed approach. The incorporation of the energy shaping technique is part of the future work, as well as the estimation of the damping inside the spatial domain. The application to the detection and suppression of (possibly unstable) mechanical oscillations (*e.g.*, flutter) are currently under investigation.

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