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Ruben Chenevat, Bruno Cheviron, Alain Rapaport, Sébastien Roux

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# Optimal structures for a model of crop irrigation with constraints

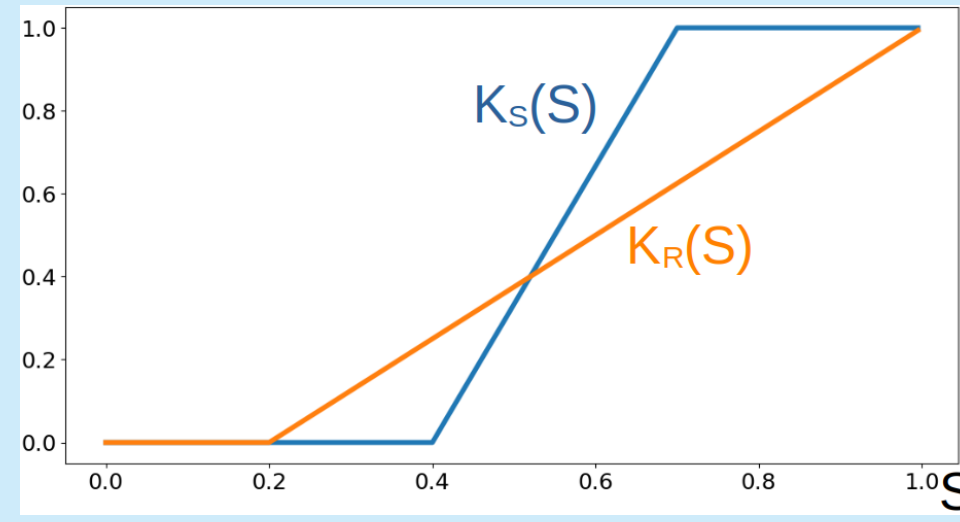
Ruben Chenevat, Bruno Cheviron, Alain Rapaport, Sébastien Roux



## Irrigation model

$$(S) \begin{cases} \dot{S}(t) = k_1(-\varphi(t)K_S(S(t)) - (1-\varphi(t))K_R(S(t)) + k_2u(t)) \\ \dot{B}(t) = \varphi(t)K_S(S(t)) \\ \dot{V}(t) = u(t) \end{cases}$$

$S$  : soil moisture  
 $B$  : biomass  
 $V$  : water consumption



**Initial conditions** :  $S(0) = S_0 \in (S^*, 1]$ ,  $B(0) = V(0) = 0$ .

**Constraints** :  $\begin{cases} S_{tol} \leq S(t) \leq 1 \text{ (state constraints)} \\ V_{tol} < V_{init} < V^* \text{ (deficit irrigation)} \end{cases}$

## Objectives and targets

**Maximizing biomass, limited water quota**

$$(B1) : \max_{u(\cdot)} B(T) \quad , \text{st} : V(T) \leq V_{init}$$

**Minimizing biomass, wasting water quota**

$$(W1) : \min_{u(\cdot)} B(T) \quad , \text{st} : V(T) \geq V_{init}$$

## Structures of optimal solutions

Time-varying feedback controls, parametrized "best" and "worst" strategy families :

$$\Psi_{t_d, V_c, t_c}^+(t, \mathbf{X}) =$$

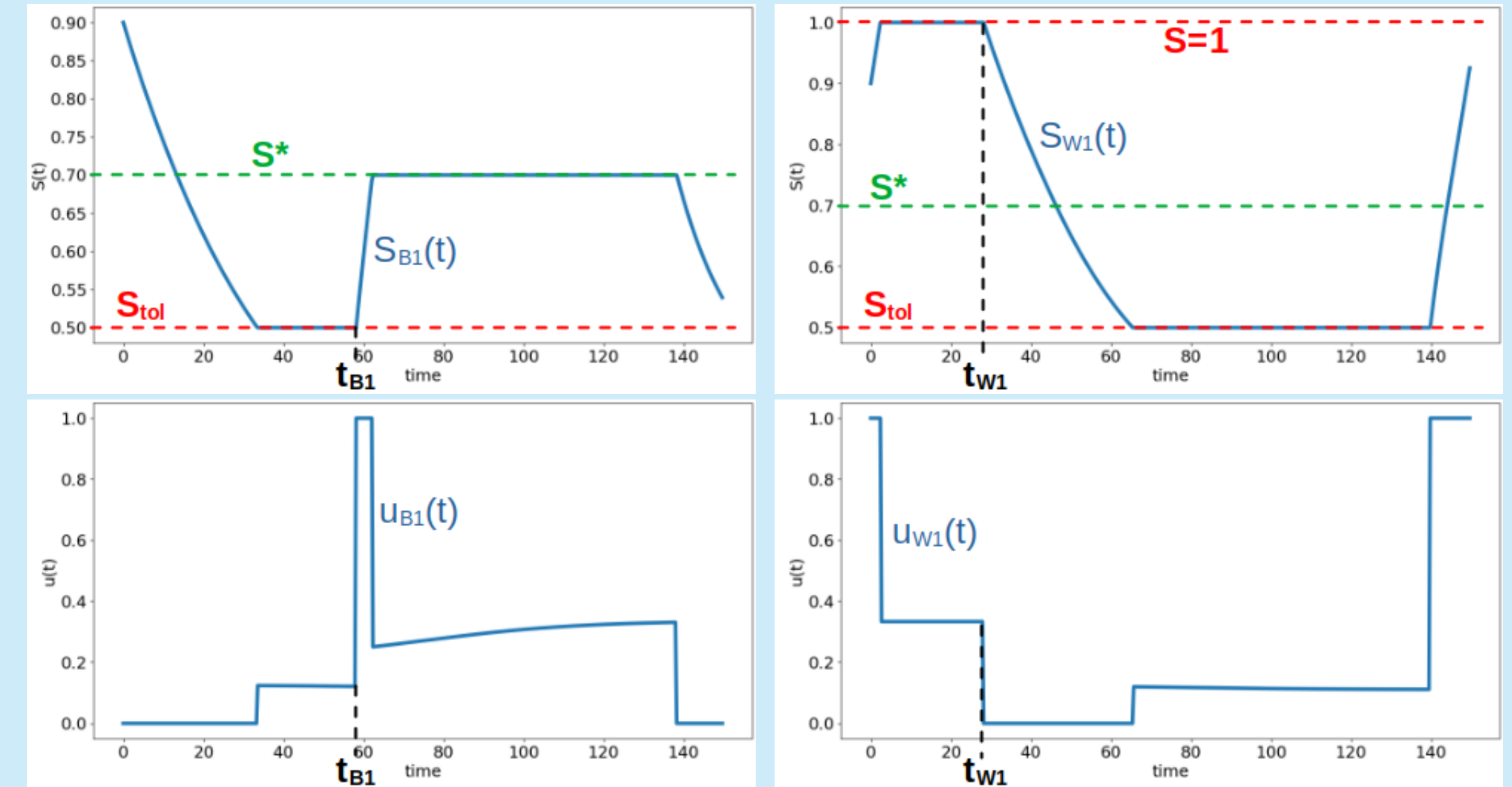
$$\begin{cases} 0 & \text{if } (S \geq S_{tol} \wedge t \leq t_d) \vee V = V_c \vee t \geq t_c \\ \tilde{u}(t, S_{tol}) & \text{if } S = S_{tol} \wedge t \leq t_d \\ \tilde{u}(t, S^*) & \text{if } S = S^* \wedge V < V_c \wedge t < t_c \\ 1 & \text{otherwise} \end{cases}$$

$(t_d : \text{trigger time})$

$$\Psi_{V_d, V_c, t_c}^-(t, \mathbf{X}) =$$

$$\begin{cases} 1 & \text{if } (V < V_d \wedge S < 1) \vee V_c - V = T - t \\ \tilde{u}(t, 1) & \text{if } V < V_d \wedge S = 1 \\ \tilde{u}(t, S_{tol}) & \text{if } S = S^* \wedge V < V_c \wedge t < t_c \\ 0 & \text{otherwise} \end{cases}$$

$(V_d : \text{trigger volume})$

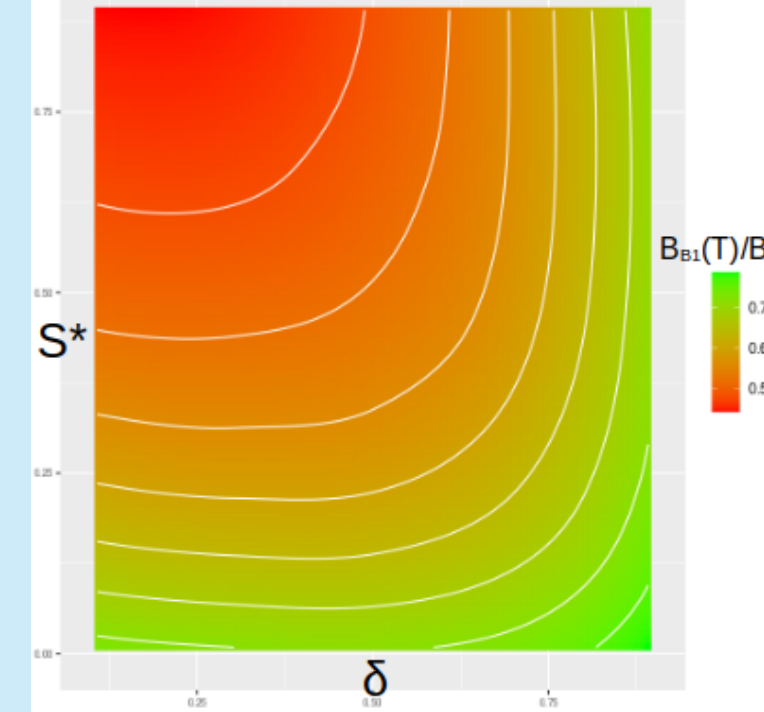
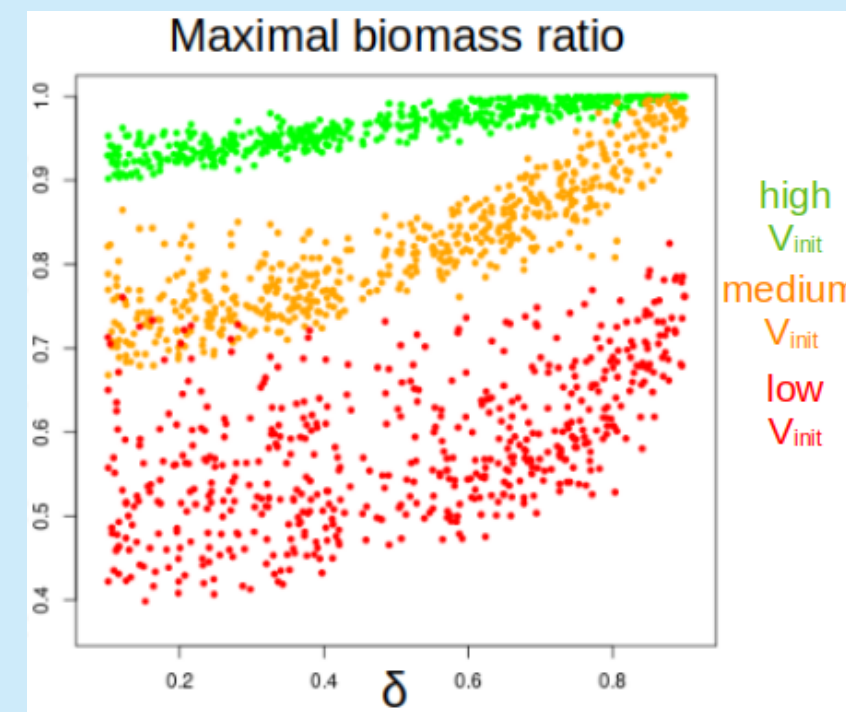
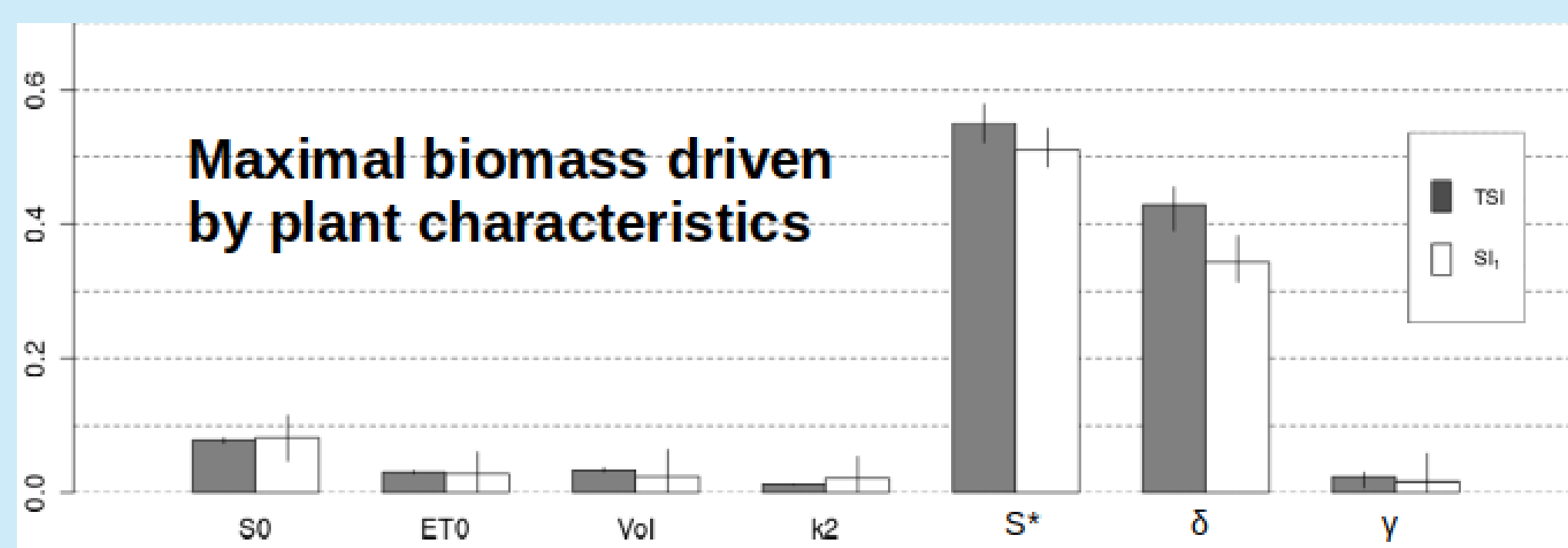


## Sketch of proof for optimality

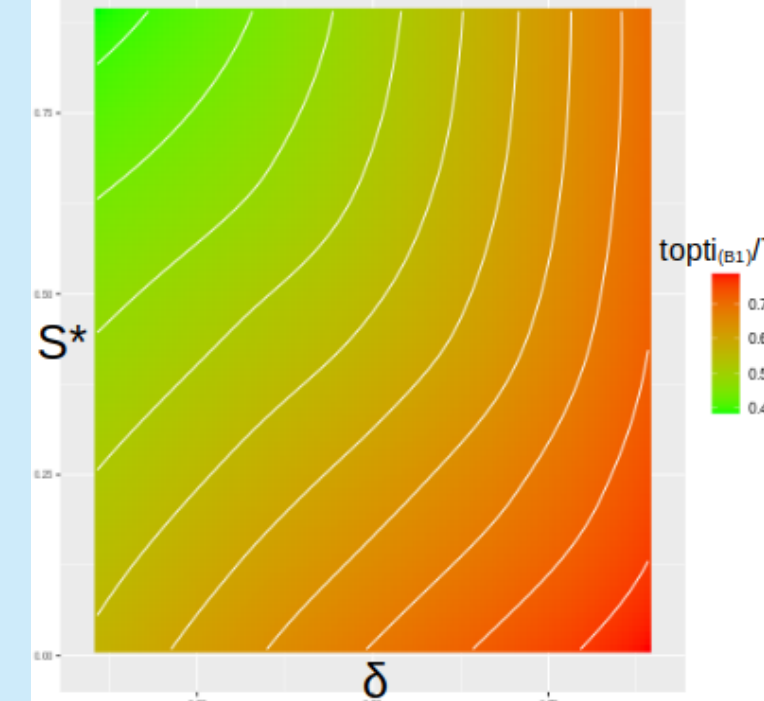
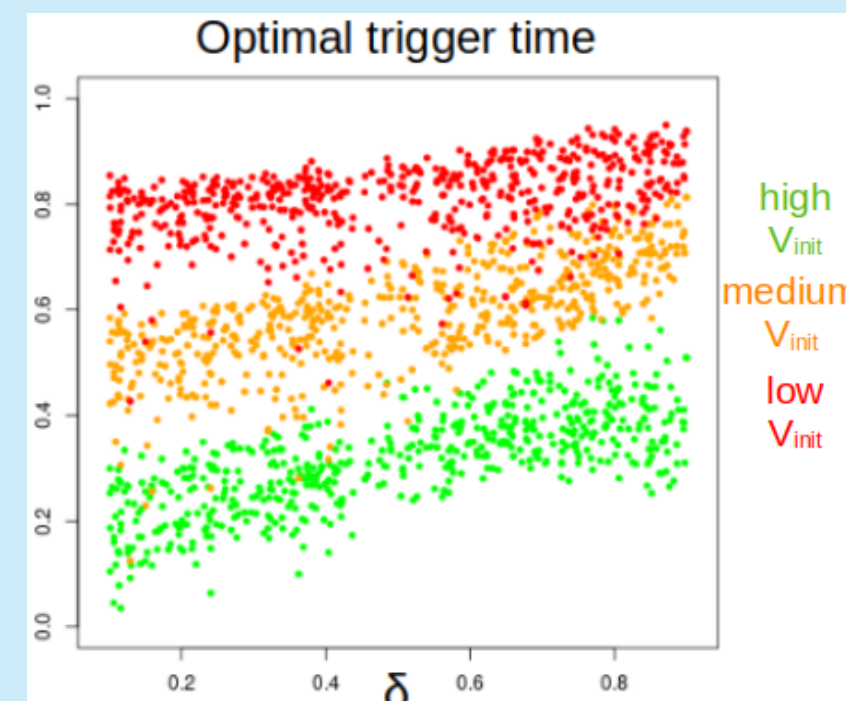
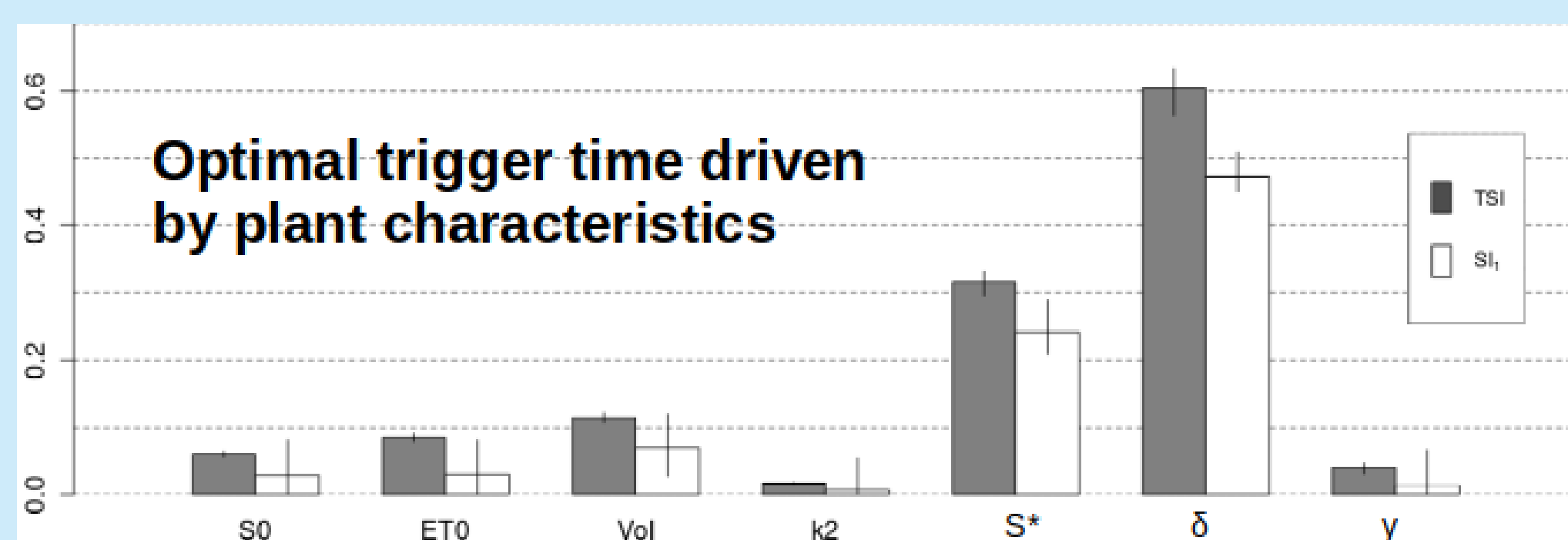
- To apply the non-smooth PMP, **additional state variables** and targets :  $\dot{Y}(t) = \min(0, S(t) - S_{tol})$  ,  $\dot{Z}(t) = \max(0, S(t) - 1)$  ;  $Y(0) = Z(0) = Y(T) = Z(T) = 0$
- Denoting  $u_P$  the optimal control for problem  $(P)$ , and  $(S_P, B_P, V_P, Y_P, Z_P)$  the corresponding solution, then **the target is saturated** :  $V_{B1}(T) = V_{W1}(T) = V_{init}$
- Common Hamiltonian** for the problems :  $H_P = \lambda_P^S k_1 (k_2 u - (\varphi(t)K_S(S(t)) + (1-\varphi(t))K_R(S(t)))) + \lambda_P^B \varphi(t)K_S(S(t)) + \lambda_P^V u + \lambda_P^Y f_Y(S(t)) + \lambda_P^Z f_Z(S(t))$
- Adjoint equations**, with the Clarke generalized gradients :  $\begin{cases} \dot{\lambda}_P^S(t) \in \varphi(t)(\lambda_P^S(t)k_1 - \lambda_P^B) \partial_C K_S(S(t)) + (1-\varphi(t))\lambda_P^S(t)k_1 \partial_C K_R(S(t)) - \lambda_P^Y \partial_C f_Y(S(t)) - \lambda_P^Z \partial_C f_Z(S(t)) \\ \dot{\lambda}_P^B = \dot{\lambda}_P^V = \dot{\lambda}_P^Y = \dot{\lambda}_P^Z = 0 \end{cases}$
- Switching function** :  $\phi_P(t) = \lambda_P^S(t)k_1k_2 + \lambda_P^V$ . Signs of adjoints :  $\lambda_{B1}^S(T) = 0 \quad \lambda_{B1}^S \geq 0 \quad \lambda_{B1}^B = 1 \quad \lambda_{B1}^V < 0 \quad \lambda_{W1}^S(T) = 0 \quad \lambda_{W1}^S \leq 0 \quad \lambda_{W1}^B = -1 \quad \lambda_{W1}^V > 0$
- Singular arcs** :  $S_P(t) = \tilde{S}$  where  $\tilde{S}$  is a **non-differentiable point** of  $K_S, K_R, f_Y$  or  $f_Z$ . **Singular control** :  $\tilde{u}(t, \tilde{S}) = \frac{\varphi(t)K_S(\tilde{S}) + (1-\varphi(t))K_R(\tilde{S})}{k_2}$
- The sets  $\chi_{B1} = \{t \in [t^*, T], \phi_{B1}(t) \geq 0\}$  and  $\chi_{W1} = \{t \in [0, T], \phi_{W1}(t) \leq 0\}$  are **connected**.

## Numerical exploration

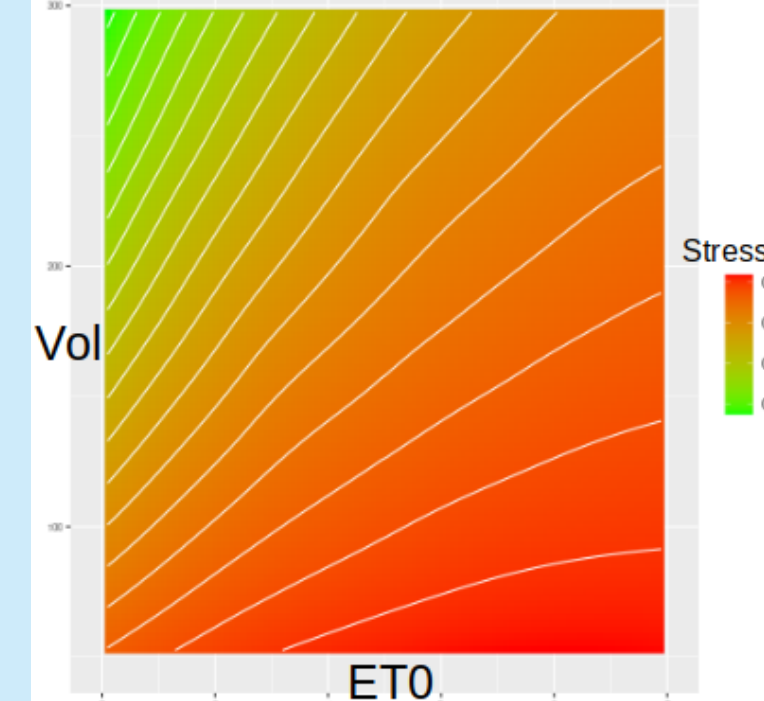
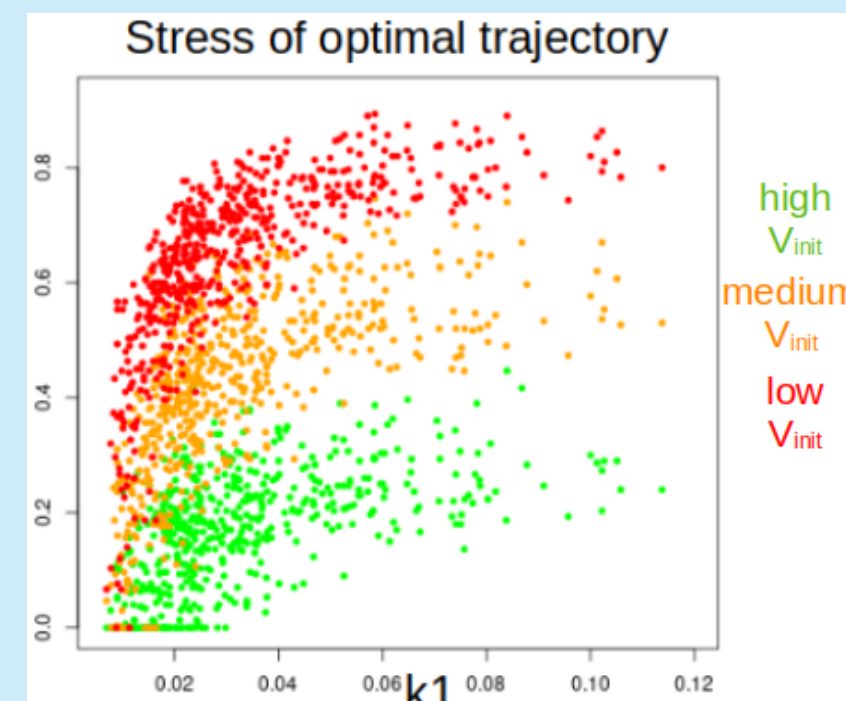
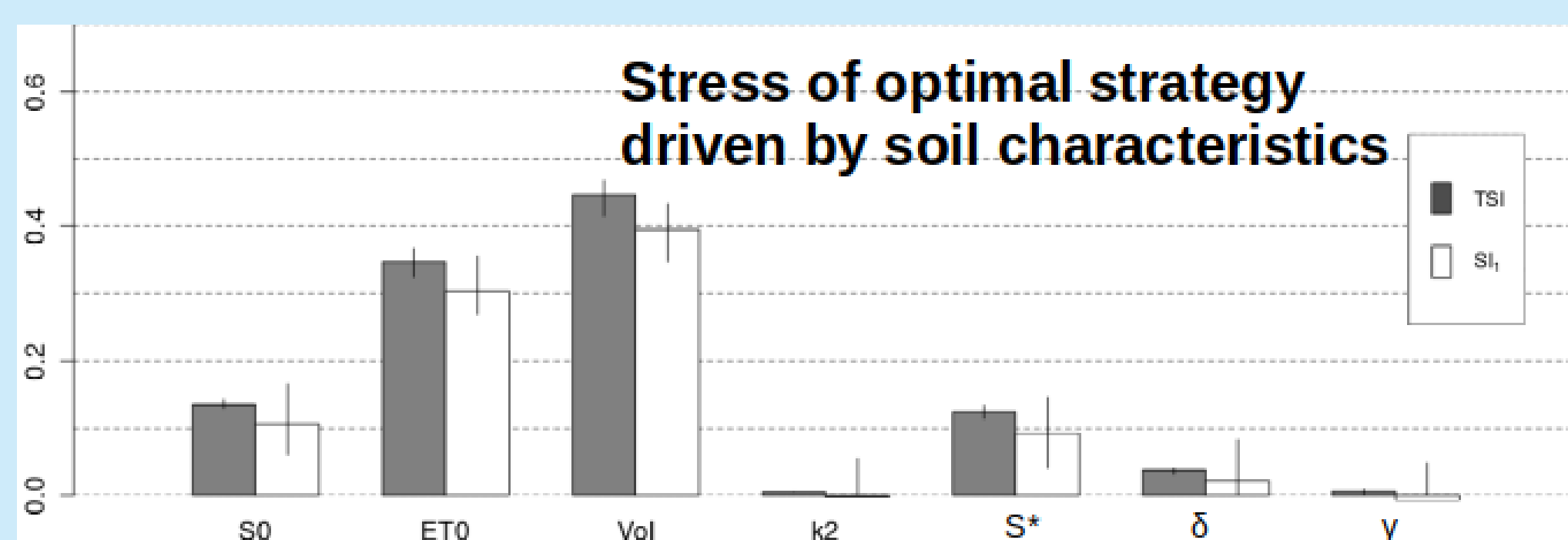
Sensitivity analysis for the maximal biomass :



Sensitivity analysis for the optimal trigger time :



Sensitivity analysis for the stress of optimal strategy :

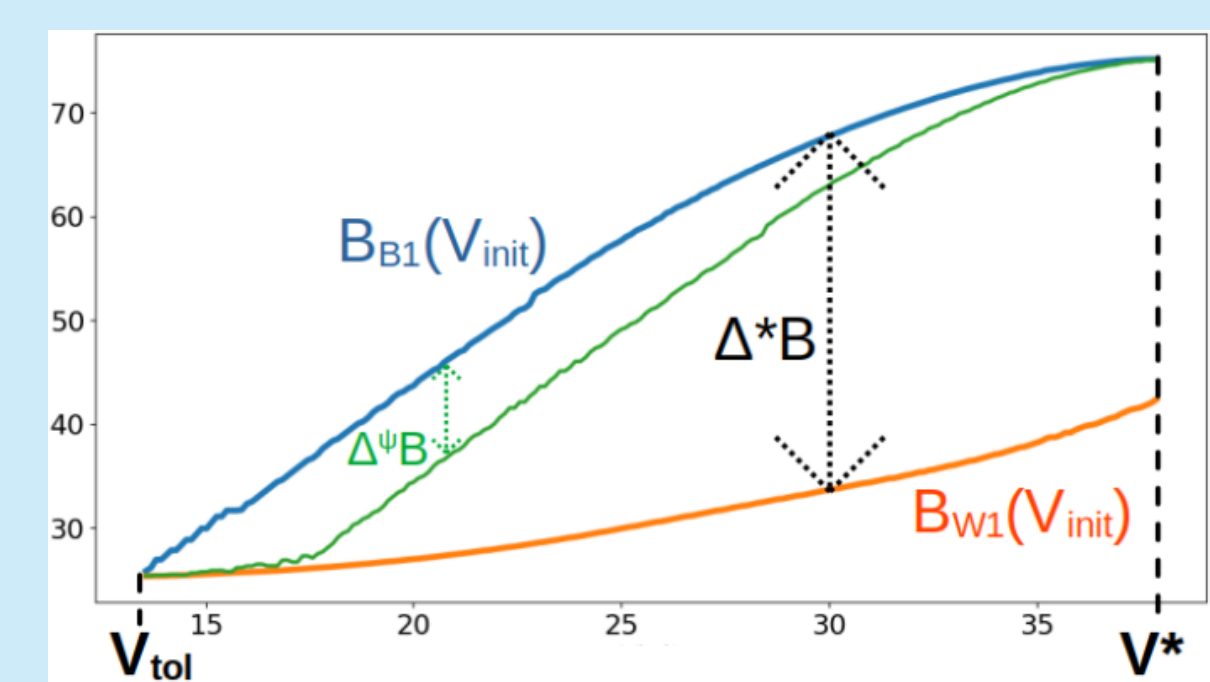


## Exploration settings

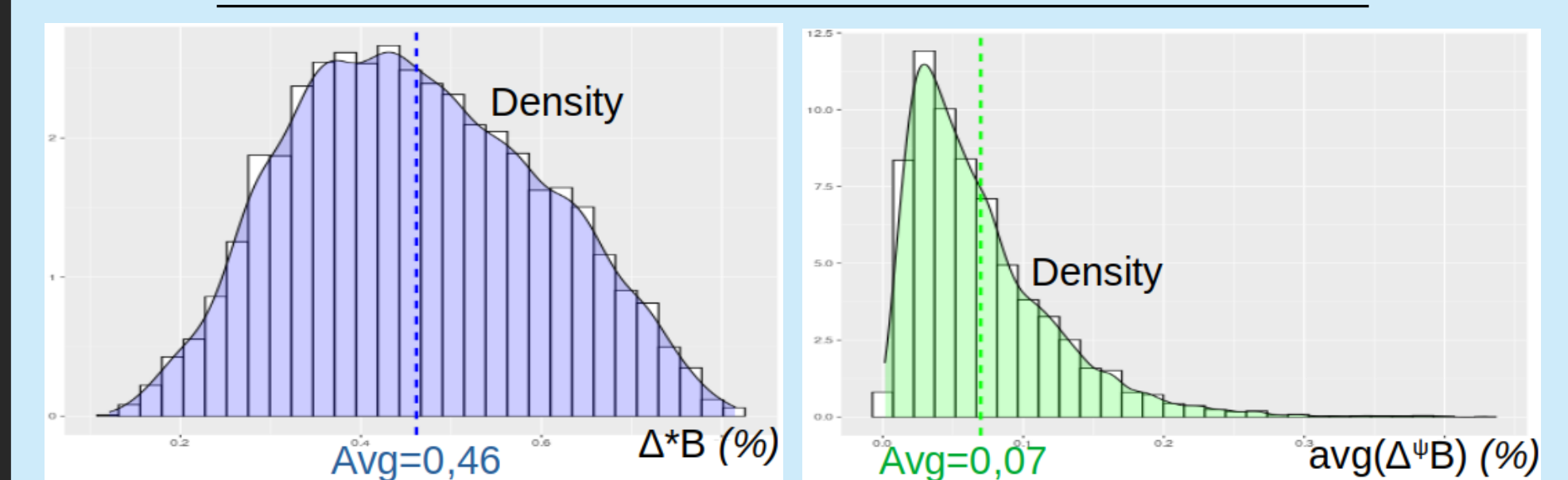
- $S_0 \in [0.7, 1]$
  - $ET_0 \in [2, 7]$
  - $Vol \in [50, 300]$
  - $S^* \in [0.55, S_0]$
  - $k_2 \in [2, 6]$
  - $\delta \in [0.1, 0.9]$
  - $\gamma \in [2, 8]$
- Parametrization of  $\varphi$  :  $(Rk : k_1 = \frac{ET_0}{Vol})$
- 

## Relative gain best/worst

Gap between best  $\Psi^+$  and worst  $\Psi^-$   
Gap between best and worst  $\Psi^+$



Distributions and average of relative gain :



## Reference

[1] R. Chenevat et al., <https://hal.science/hal-04506794>, Working paper, March 2024.

## Acknowledgements

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