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Optimal structures for a model of crop irrigation with constraints

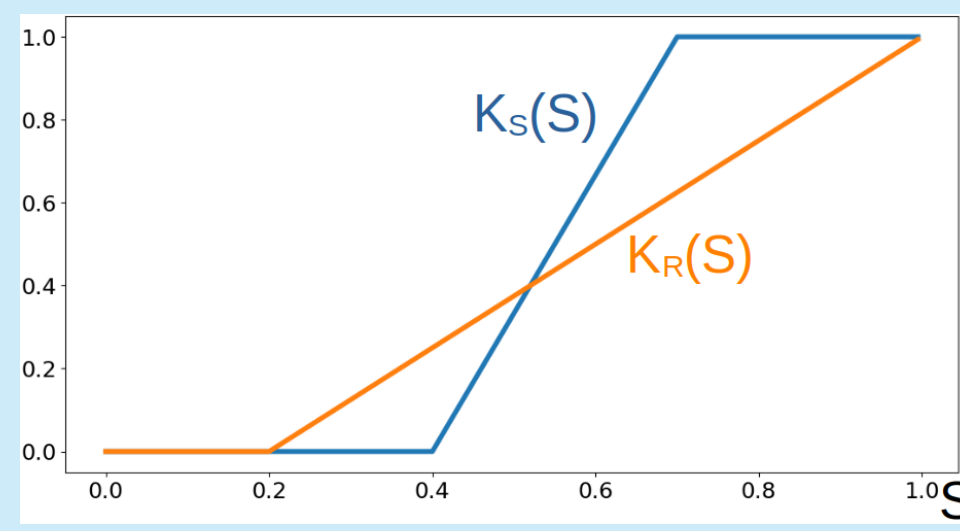
Ruben Chenevat, Bruno Cheviron, Alain Rapaport, Sébastien Roux



Irrigation model

$$(S) \begin{cases} \dot{S}(t) = k_1(-\varphi(t)K_S(S(t)) - (1-\varphi(t))K_R(S(t)) + k_2u(t)) \\ \dot{B}(t) = \varphi(t)K_S(S(t)) \\ \dot{V}(t) = u(t) \end{cases}$$

S : soil moisture
 B : biomass
 V : water consumption



Initial conditions : $S(0) = S_0 \in (S^*, 1]$, $B(0) = V(0) = 0$.

Constraints : $\begin{cases} S_{tol} \leq S(t) \leq 1 \text{ (state constraints)} \\ V_{tol} < V_{init} < V^* \text{ (deficit irrigation)} \end{cases}$

Objectives and targets

Maximizing biomass, limited water quota

$$(B1) : \max_{u(\cdot)} B(T) \quad , \text{st} : V(T) \leq V_{init}$$

Minimizing biomass, wasting water quota

$$(W1) : \min_{u(\cdot)} B(T) \quad , \text{st} : V(T) \geq V_{init}$$

Structures of optimal solutions

Time-varying feedback controls, parametrized "best" and "worst" strategy families :

$$\Psi_{t_d, V_c, t_c}^+(t, \mathbf{X}) =$$

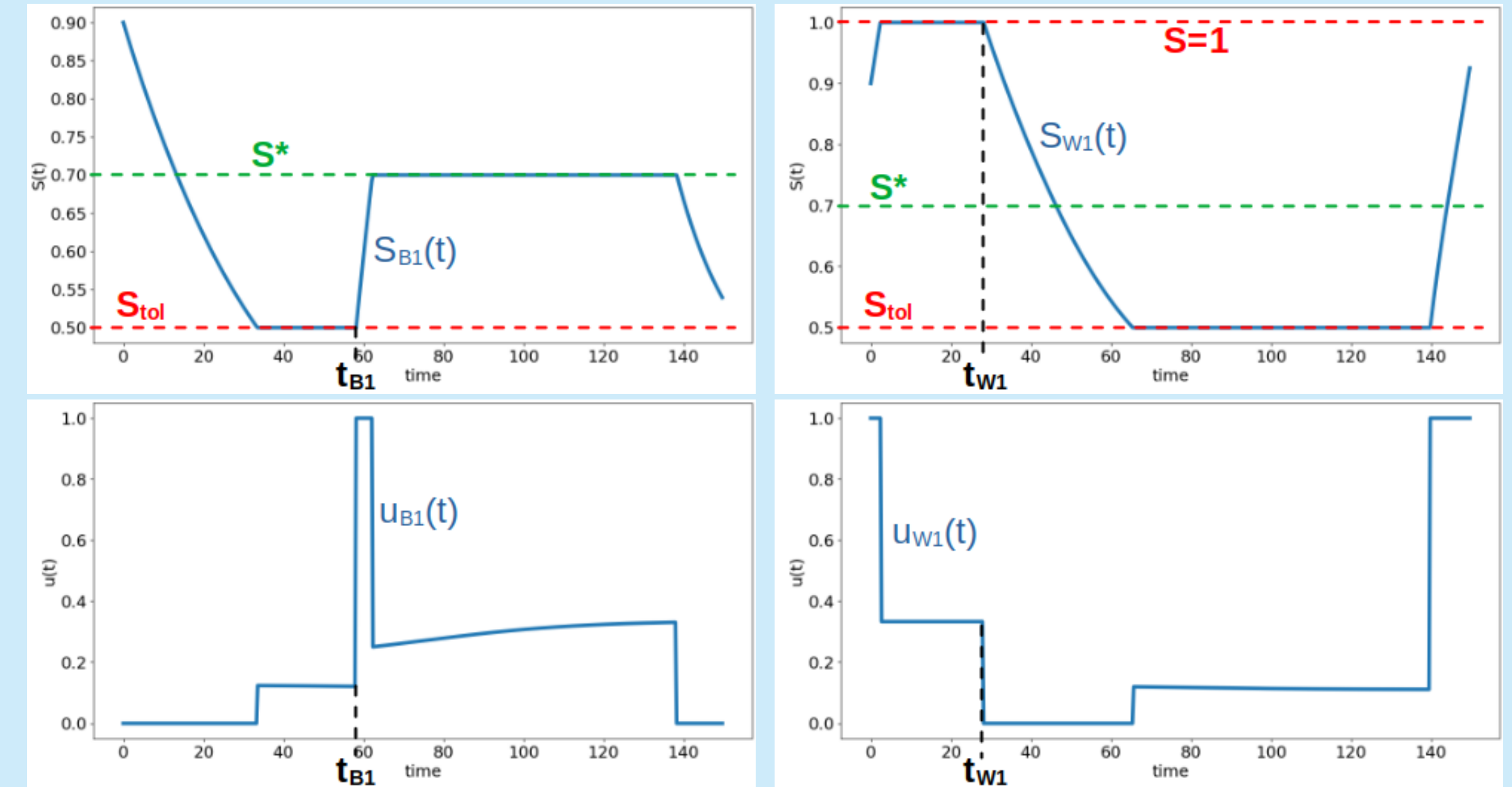
$$\begin{cases} 0 & \text{if } (S \geq S_{tol} \wedge t \leq t_d) \vee V = V_c \vee t \geq t_c \\ \tilde{u}(t, S_{tol}) & \text{if } S = S_{tol} \wedge t \leq t_d \\ \tilde{u}(t, S^*) & \text{if } S = S^* \wedge V < V_c \wedge t < t_c \\ 1 & \text{otherwise} \end{cases}$$

$(t_d : \text{trigger time})$

$$\Psi_{V_d, V_c, t_c}^-(t, \mathbf{X}) =$$

$$\begin{cases} 1 & \text{if } (V < V_d \wedge S < 1) \vee V_c - V = T - t \\ \tilde{u}(t, 1) & \text{if } V < V_d \wedge S = 1 \\ \tilde{u}(t, S_{tol}) & \text{if } S = S^* \wedge V < V_c \wedge t < t_c \\ 0 & \text{otherwise} \end{cases}$$

$(V_d : \text{trigger volume})$

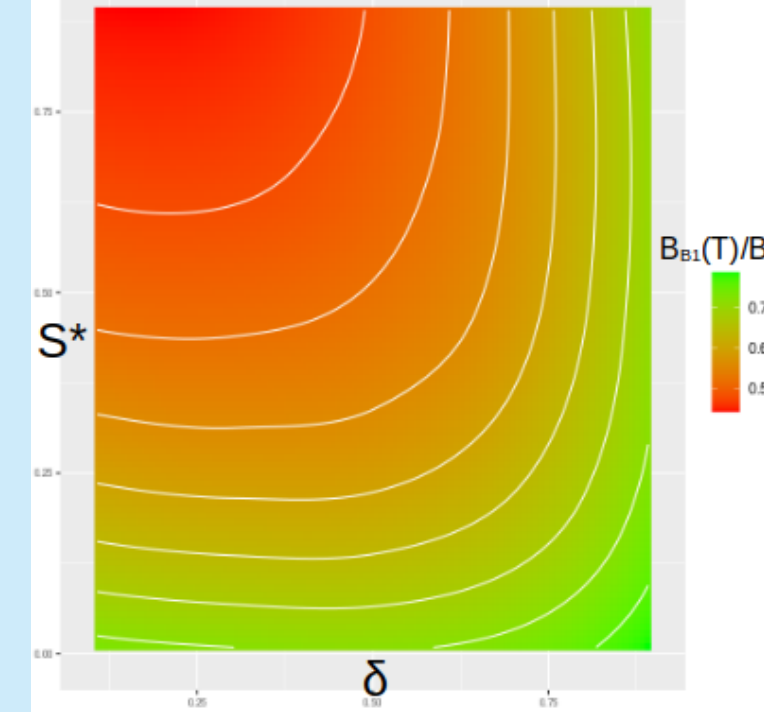
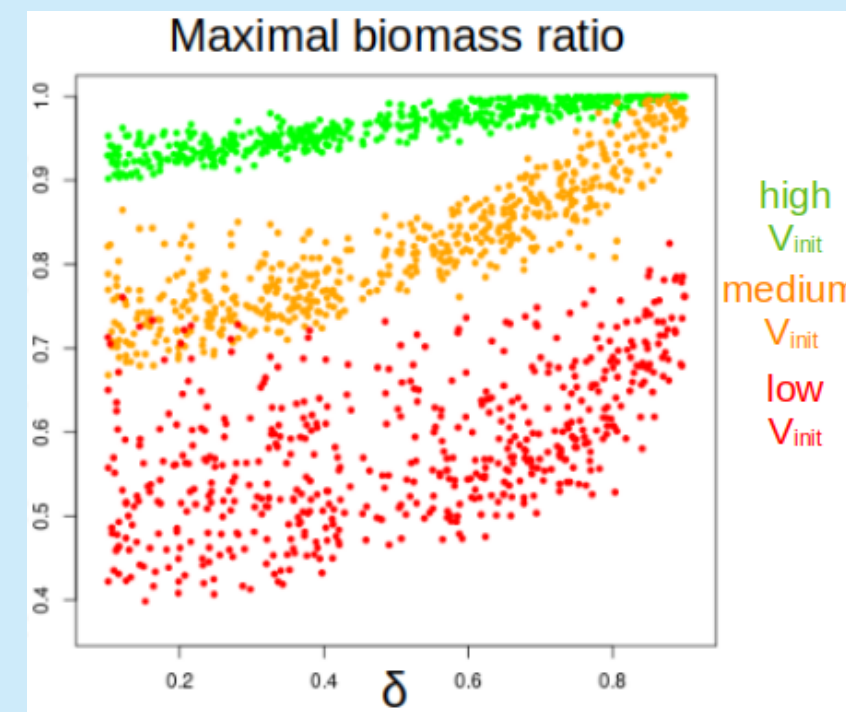
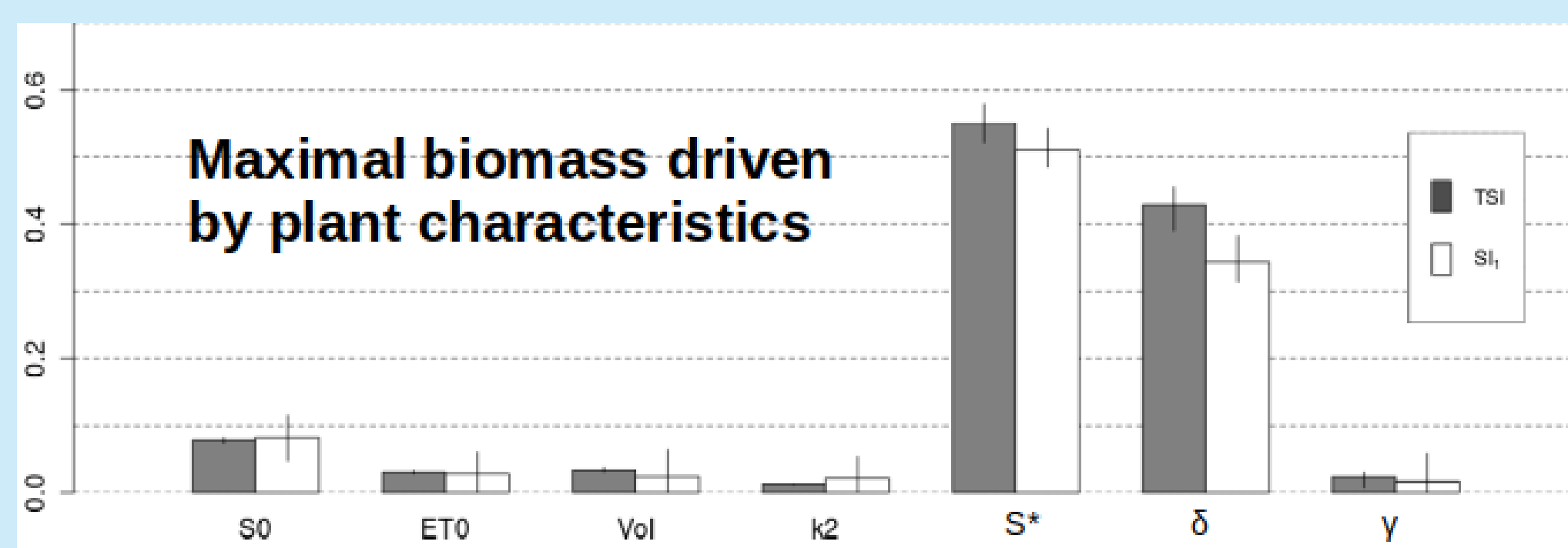


Sketch of proof for optimality

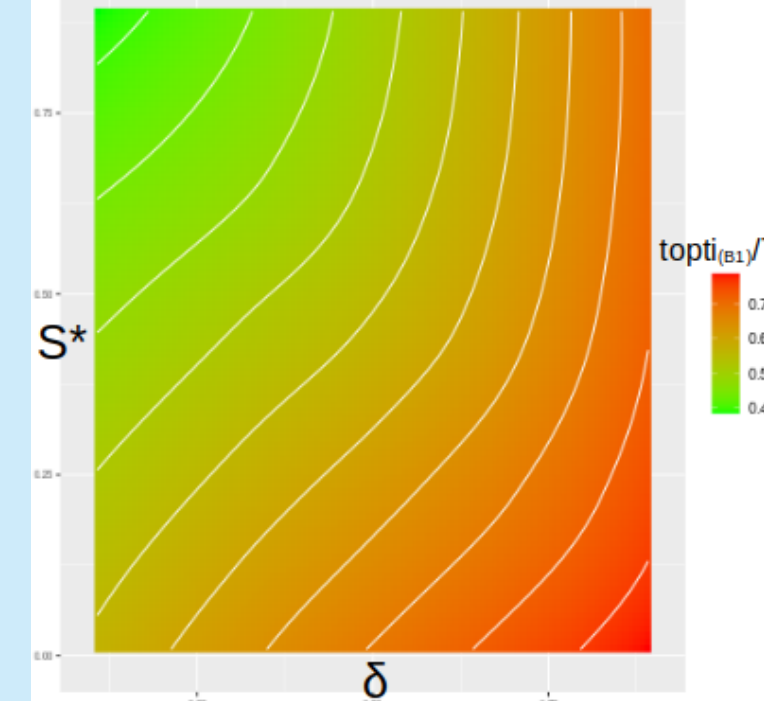
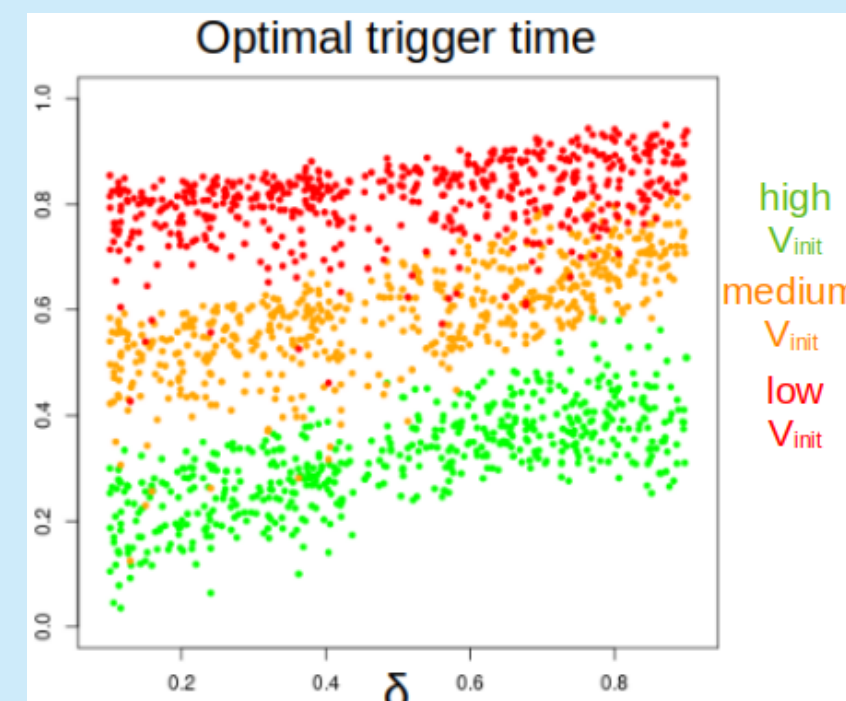
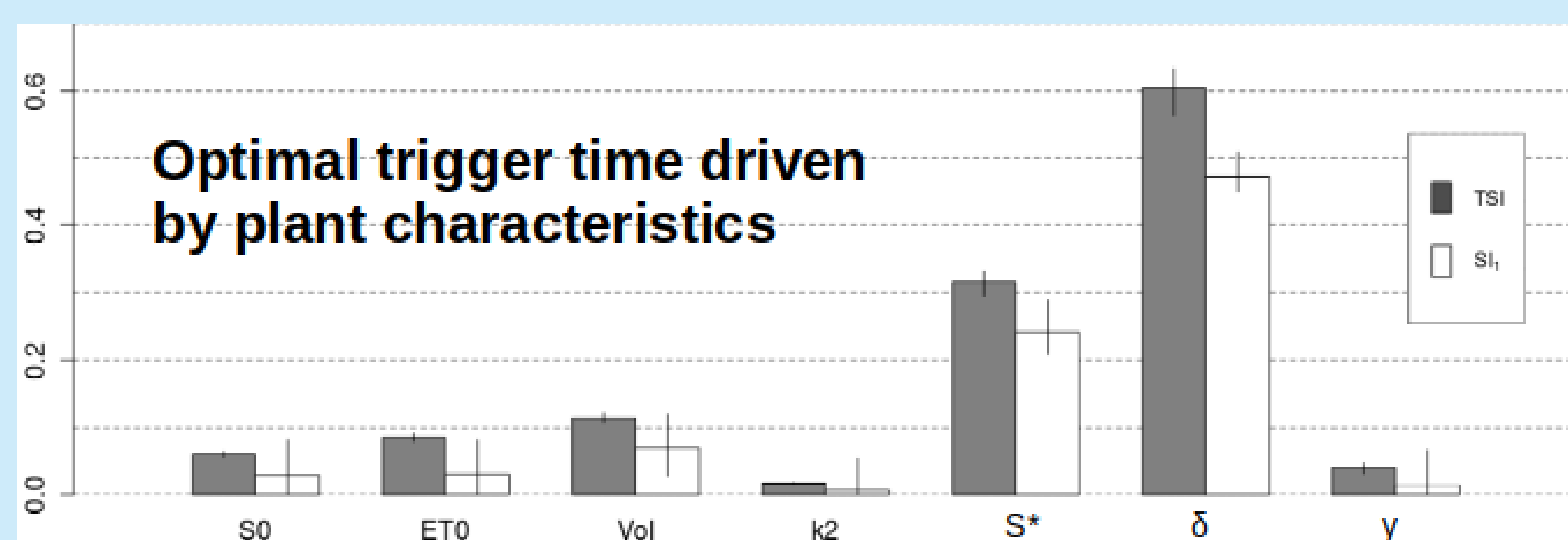
- To apply the non-smooth PMP, **additional state variables** and targets : $\dot{Y}(t) = \min(0, S(t) - S_{tol})$, $\dot{Z}(t) = \max(0, S(t) - 1)$; $Y(0) = Z(0) = Y(T) = Z(T) = 0$
- Denoting u_P the optimal control for problem (P), and $(S_P, B_P, V_P, Y_P, Z_P)$ the corresponding solution, then **the target is saturated** : $V_{B1}(T) = V_{W1}(T) = V_{init}$
- Common Hamiltonian** for the problems : $H_P = \lambda_P^S k_1 (k_2 u - (\varphi(t)K_S(S(t)) + (1-\varphi(t))K_R(S(t)))) + \lambda_P^B \varphi(t)K_S(S(t)) + \lambda_P^V u + \lambda_P^Y f_Y(S(t)) + \lambda_P^Z f_Z(S(t))$
- Adjoint equations**, with the Clarke generalized gradients : $\begin{cases} \dot{\lambda}_P^S(t) \in \varphi(t)(\lambda_P^S(t)k_1 - \lambda_P^B) \partial_C K_S(S(t)) + (1-\varphi(t))\lambda_P^S(t)k_1 \partial_C K_R(S(t)) - \lambda_P^Y \partial_C f_Y(S(t)) - \lambda_P^Z \partial_C f_Z(S(t)) \\ \dot{\lambda}_P^B = \dot{\lambda}_P^V = \dot{\lambda}_P^Y = \dot{\lambda}_P^Z = 0 \end{cases}$
- Switching function** : $\phi_P(t) = \lambda_P^S(t)k_1k_2 + \lambda_P^V$. Signs of adjoints : $\lambda_{B1}^S(T) = 0 \quad \lambda_{B1}^S \geq 0 \quad \lambda_{B1}^B = 1 \quad \lambda_{B1}^V < 0 \quad \lambda_{W1}^S(T) = 0 \quad \lambda_{W1}^S \leq 0 \quad \lambda_{W1}^B = -1 \quad \lambda_{W1}^V > 0$
- Singular arcs** : $S_P(t) = \tilde{S}$ where \tilde{S} is a **non-differentiable point** of K_S, K_R, f_Y or f_Z . **Singular control** : $\tilde{u}(t, \tilde{S}) = \frac{\varphi(t)K_S(\tilde{S}) + (1-\varphi(t))K_R(\tilde{S})}{k_2}$
- The sets $\chi_{B1} = \{t \in [t^*, T], \phi_{B1}(t) \geq 0\}$ and $\chi_{W1} = \{t \in [0, T], \phi_{W1}(t) \leq 0\}$ are **connected**.

Numerical exploration

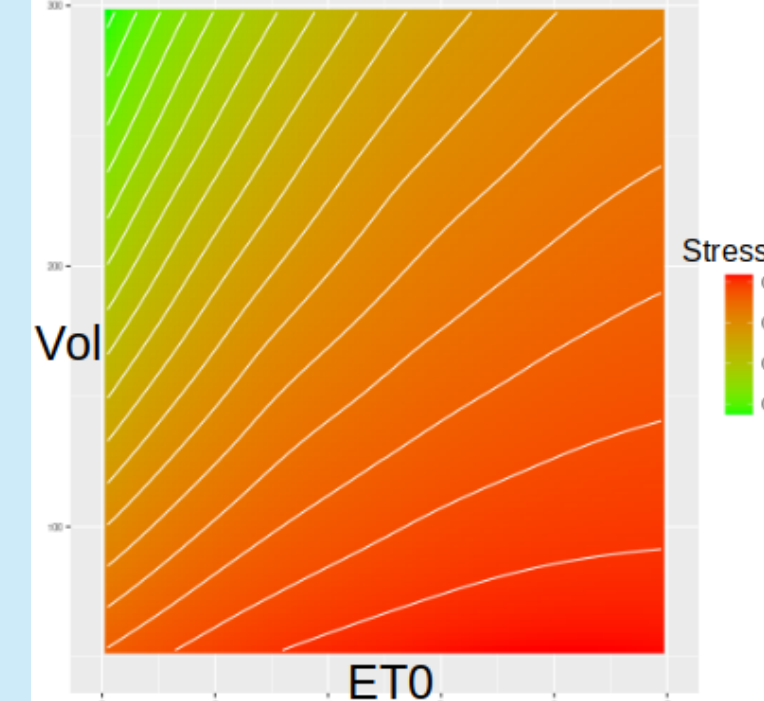
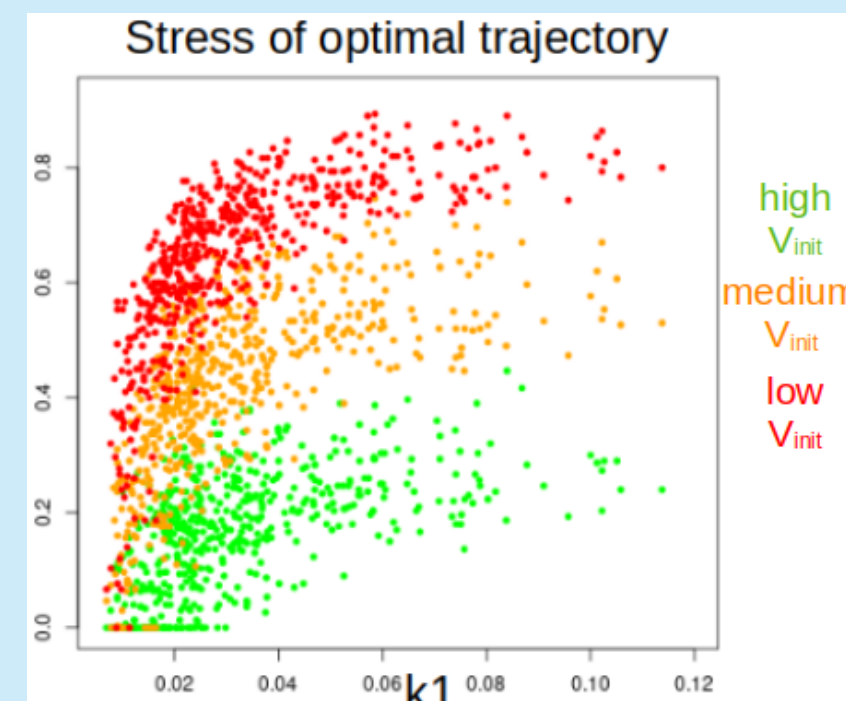
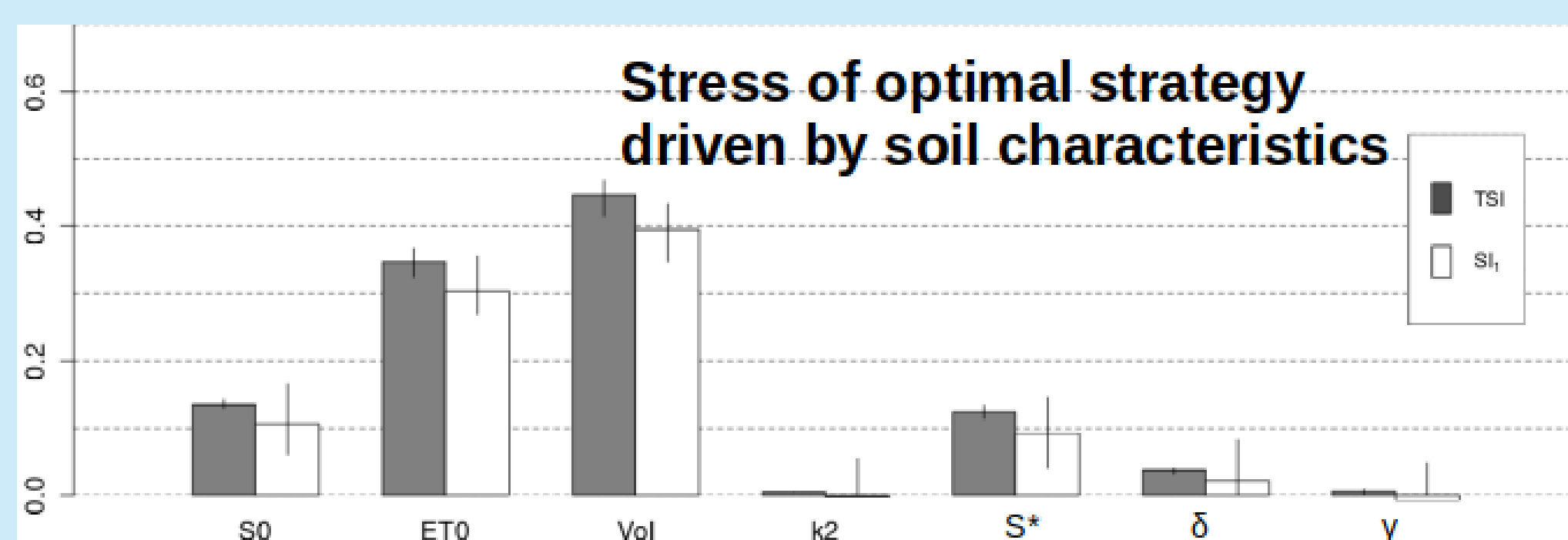
Sensitivity analysis for the maximal biomass :



Sensitivity analysis for the optimal trigger time :



Sensitivity analysis for the stress of optimal strategy :

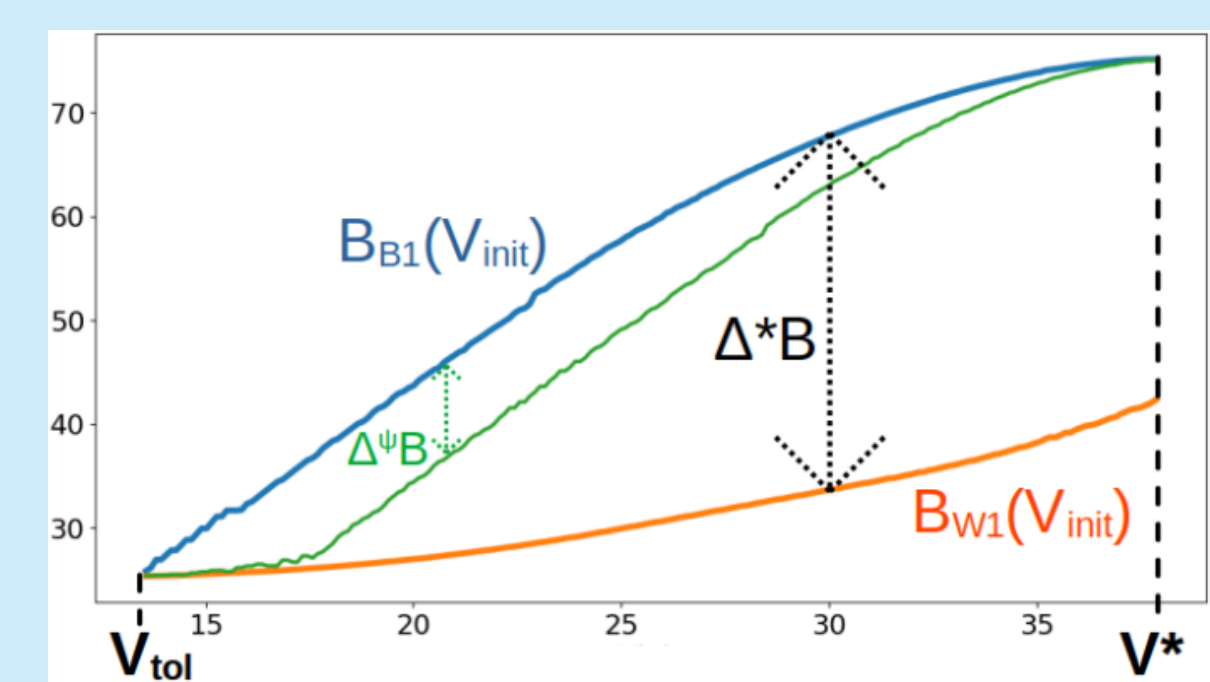


Exploration settings

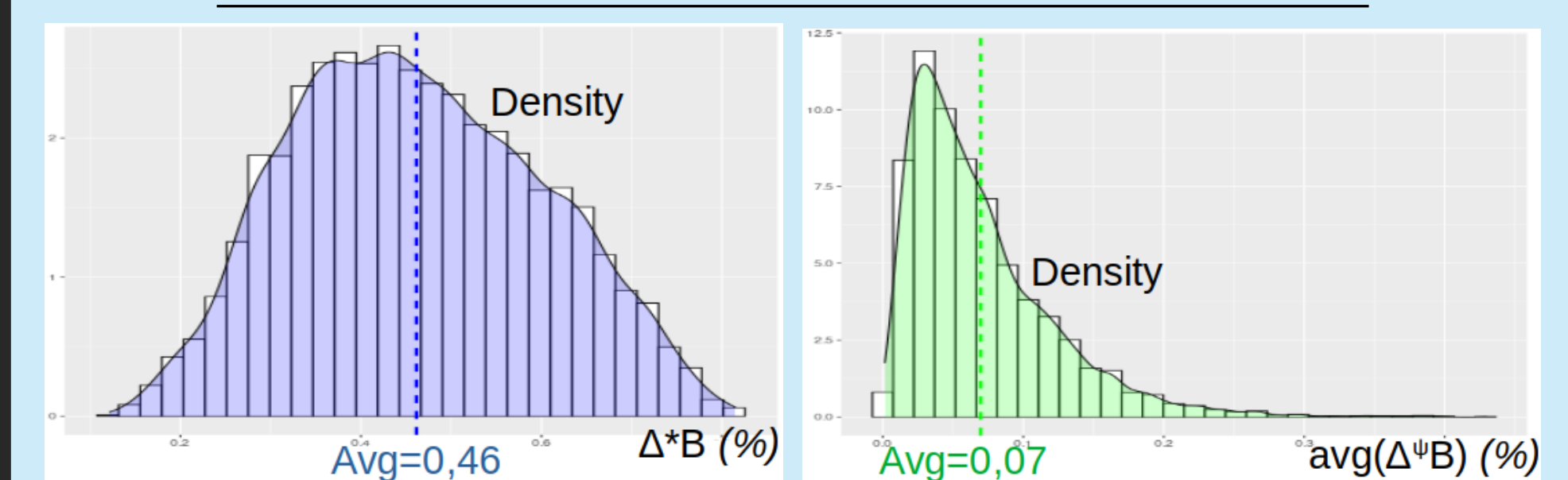
- $S_0 \in [0.7, 1]$
 - $ET_0 \in [2, 7]$
 - $Vol \in [50, 300]$
 - $S^* \in [0.55, S_0]$
 - $k_2 \in [2, 6]$
 - $\delta \in [0.1, 0.9]$
 - $\gamma \in [2, 8]$
- Parametrization of φ : $(Rk : k_1 = \frac{ET_0}{Vol})$
-

Relative gain best/worst

Gap between best Ψ^+ and worst Ψ^-
Gap between best and worst Ψ^+



Distributions and average of relative gain :



Reference

[1] R. Chenevat et al., <https://hal.science/hal-04506794>, Working paper, March 2024.

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