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# Efficiency and Application Fees in School Choice\*

Cyril Rouault<sup>†</sup>

## Abstract

This paper examines the impact of application fees on student strategies within the deferred acceptance mechanism. We demonstrate that these fees can lead to Pareto-efficient allocations. However, they may also inhibit the existence of Nash equilibria that would result in assignments Pareto-dominating the student-optimal stable assignment. This issue always arises when application fees are positive for all students at a given school.

**JEL Classification:** C78, D47, D82.

**Keywords:** Matching; Application Fees; Efficiency; Stability; Nash Equilibrium

## 1 Introduction

Since Gale and Shapley's (1962) seminal paper, matching theory has influenced the design of college admission systems (Roth and Sotomayor 1990; Abdulkadiroğlu and Sönmez 2003). A commonly used mechanism is the student-proposing deferred acceptance mechanism (DA). DA leads to the stable assignment preferred by students and is strategy-proof for students (Roth, 1982). In DA, schools must rank students by reviewing applications, a process that incurs costs. To cover part of these costs or to limit the number of applicants, application fees are often introduced. This paper studies the impact of implementing these application fees on student strategies.

In this context, we consider lexicographic preferences on the outcomes of DA, where students prioritize assignments first and consider application fees second. This assumption is justified by the relatively low application fees, as students typically prefer being assigned to a better school even if it involves paying a fee. Empirical evidence from He and Magnac (2022) supports this.

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Implementing constraints on students impacts their strategies.<sup>1</sup> Although application fees are low, they constrain students in their choice of schools to which they apply. We examine application fee profiles that ensure the existence of a Nash equilibrium resulting in an assignment that Pareto-dominates the student-optimal stable assignment (Example 1).<sup>2</sup> Our main result identifies the conditions on application fee profiles that prevent a Nash equilibrium. Specifically, if a student faces a positive application fee for the school obtained in the student-optimal stable assignment, no Nash equilibrium results in a Pareto-dominating assignment that the student prefers.

To prove this result, we show that in a Nash equilibrium, students apply to a school with an application fee only if they are assigned to it. If a student does not apply to the school obtained in the student-optimal stable assignment, another student has a profitable deviation, contradicting the existence of a Nash equilibrium. Therefore, if all students face a positive application fee for a school, no Nash equilibrium under DA improves their assignment, implying that uniform application fees hinder assignment efficiency.

## 2 Model

A school choice problem with application fees is a tuple  $\pi = (I, S, P, \succ, q, C)$ .  $I$  and  $S$  denote the finite sets of *students* and *schools*, respectively. For each  $i \in I$ ,  $P_i$  is a *strict preference ordering* over  $S \cup \{i\}$ , where  $i$  denotes the *outside option*. Let  $P \equiv (P_i)_{i \in I}$ . For each  $s \in S$ ,  $\succ_s$  is a *strict priority ordering* over the set of students, and  $q_s$  is the *capacity* of school  $s$ .  $q \equiv (q_s)_{s \in S}$  denotes the *capacity vector* and  $\succ \equiv (\succ_s)_{s \in S}$ . The *application fees profile* is a  $|I| \times |S|$ -dimensional matrix  $C$ , where each element  $c_{i,s} \in \{0, 1\}$  represents the application fee of student  $i$  to school  $s$ :  $c_{i,s} = 0$  the application fee for student  $i$  to school  $s$  is zero,  $c_{i,s} = 1$  the fee is positive. Let  $C_0$  denote a *null-application fees profile* such that for each  $i \in I$  and  $s \in S$ ,  $c_{i,s} = 0$ . Let  $\Pi$  be the set of all problems. Throughout  $I, S, \succ$ , and  $\mathbf{q}$  are fixed, then we denote a problem by  $(P, C)$ .

An *assignment* is a mapping  $\mu : I \cup S \rightarrow I \cup S$  such that for each  $i \in I, \mu(i) \in S \cup \{i\}$ , for each  $s \in S, \mu(s) \in 2^I$  with  $|\mu(s)| \leq q_s$ , and for each  $i \in I, \mu(i) = s$  if and only if  $i \in \mu(s)$ . If  $\mu(i) = i$ , we say that  $i$  is *unassigned* at  $\mu$ .  $i$ 's preferences  $P_i$  over schools implicitly define a preference relation  $R_i$  over assignments as follows:  $\mu(i) R_i \mu'(i)$  if and only if  $\mu(i) P_i \mu'(i)$  or  $\mu(i) = \mu'(i)$ .

<sup>1</sup>See Haeringer and Klijn (2009) for capacity constraints and Chade et al. (2014) for application fees.

<sup>2</sup>Bando (2014), Dur and Morrill (2020), and Rouault (2023) study Nash equilibria achieving assignments that Pareto-dominate the student-optimal stable assignment without application fees.

An assignment  $\mu$  is *stable* if:

- $\mu$  is *individually rational*, i.e., for each  $i \in I$ ,  $\mu(i)R_i i$ ,
- $\mu$  is *non-wasteful*, i.e., for each  $i \in I$  and each  $s \in S$ ,  $sP_i \mu(i)$  implies  $|\mu(s)| = q_s$ ,
- there is *no justified envy*, i.e., for each  $i, j \in I$  with  $\mu(j) = s$ ,  $sP_i \mu(i)$  implies  $j \succ_s i$ .

Let  $S(P, C)$  denote the *set of stable assignments* in problem  $(P, C)$ . An assignment  $\mu$  *Pareto-dominates* an assignment  $\mu'$  if for each  $i \in I$ ,  $\mu(i)R_i \mu'(i)$  and there exists at least one  $i$  such that  $\mu(i)P_i \mu'(i)$ . An assignment is *efficient* if it is not Pareto-dominated by any other assignment. Let  $\mu_I$  be the *student-optimal stable assignment*.

A *mechanism*  $\varphi$  selects an outcome  $\varphi(P, C) = (\mu, c_P)$  for each problem  $(P, C)$  and  $\varphi(P, C)(i) = (\mu(i), c_{P_i})$  the outcome for  $i$ .  $\varphi$  defines a game in which students are the *players*. Let the *strategy*  $Q_i$  be an ordered list of schools corresponding to the reported preferences of student  $i$  over schools. We denote by  $A(Q_i)$  the *set of schools reported* by student  $i$  under strategy  $Q_i$ . Given  $C$ , let  $c_{Q_i} = \sum_{s \in A(Q_i)} c_{i,s}$  be the *number of school with a positive application fee* under strategy  $Q_i$ . Let  $Q \equiv (Q_i)_{i \in I}$  be a *strategy profile* of students and  $c_Q \equiv (c_{Q_i})_{i \in I}$ . We consider *lexicographic preference* over outcomes, such that  $(\mu(i), c_{Q_i}) \succ_i (\mu'(i), c'_{Q_i})$  if and only if  $\mu(i)P_i \mu'(i)$  or  $\mu(i) = \mu'(i)$  and  $c_{Q_i} < c'_{Q_i}$ .

A strategy profile  $Q$  is a *Nash equilibrium* under  $\varphi$  if for each  $i \in I$ , there is no strategy  $Q'_i$  such that  $Q'_i \neq Q_i$ , and  $\varphi((Q'_i, Q_{-i}), C)(i) \succ_i \varphi(Q, C)(i)$ . Let  $DA(P, C)$  denote the outcome of the *deferred acceptance mechanism* (DA) for problem  $(P, C)$  and  $NE(DA(P, C))$  the *set of strategy profiles  $Q$  that are Nash equilibria* under DA for problem  $(P, C)$ .

We now introduce an example to illustrate our model and analysis.

**Example 1.** Consider a problem  $\pi = (I, S, P, \succ, q, C_0)$  such that  $I = \{i_1, i_2, i_3\}$ ,  $S = \{s_1, s_2, s_3\}$ , for each  $s \in S$ ,  $q_s = 1$ . Preferences and priorities are given in the following tables, and  $(\cdot)$  indicates that priorities are irrelevant to the problem:

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$		$P_{i_1}$	$P_{i_2}$	$P_{i_3}$
$i_3$	$i_1$	$\cdot$		$s_1^*$	$s_1$	$s_2^*$
$i_1$	$i_2$	$\cdot$		<u><math>s_2</math></u>	$s_2$	<u><math>s_1</math></u>
$i_2$	$i_3$	$\cdot$		$s_3$	<u><math>s_3^*</math></u>	$s_3$

$\mu_I$  is underlined in students' preferences and  $\mu^*$  is denoted by a star (\*). It is clear that  $\mu^*$  is the only assignment that Pareto-dominates  $\mu_I$  and  $\mu^*$  is efficient. From the literature, we know that  $DA(P, C_0) \in NE(DA(P, C_0))$  (Gale and Shapley, 1962). Consider

now  $C$ , such that  $c_{i_2, s_2} = 1$  and 0 for other elements. There exists  $Q_{i_2} : s_1, s_3, i_2, s_2$  such that  $DA((Q_{i_2}, P_{-i_2}), C)(i_2) \succ_{i_2} DA(P, C)(i_2)$  and  $DA(P, C) \notin NE(DA(P, C))$ . By implementing the application fee profile  $C$ , there exists a profitable deviation for  $i_2$ , which is to not apply to  $s_2$ . Considering the strategy  $P_{-i_2}$ , it is straightforward that  $i_2$  is indifferent between applying to  $s_2$  or not. Following  $i_2$ 's deviation, the student assignment is Pareto improved. For each  $i \in I$ ,  $DA((Q_{i_2}, P_{-i_2}), C)(i) = (\mu^*(i), 0)$ .

### 3 Results

In this section, we introduce the reasoning to prove our main result with Proposition 1. Proposition 1 implies that for all Nash equilibria, students apply to at most one school with a positive application fee.

**Proposition 1.** For any problem  $(P, C)$  if  $Q \in NE(DA(P, C))$  with  $DA(Q, C) = (\mu, c_Q)$ , then for each  $i \in I$ ,  $c_{Q_i} \leq 1$ .

*Proof.* By contradiction, suppose there exists  $Q \in NE(DA(P, C))$  such that there exists  $i \in I$ , with  $DA(Q, C)(i) = (\mu(i), t)$  with  $t > 1$ . Consider  $Q'_i : \mu(i), i$ , such that  $DA((Q'_i, Q_{-i}), C)(i) = (\mu'(i), t')$ . We need to show that  $(\mu'(i), t') \succ (\mu(i), t)$ . Suppose  $(\mu(i), t) \succ (\mu'(i), t')$ . This implies either  $\mu(i) P_i \mu'(i)$  or  $\mu(i) = \mu'(i)$  and  $t' > t > 1$ . Since  $|A(Q'_i)| = 1$ , we have  $t > t'$ , meaning there exists  $s \in A(Q_i)$  such that  $s \neq \mu(i) \neq \mu'(i)$ . However,  $i$  has been rejected from  $s$ , and by the construction of  $DA$ , this leads to a contradiction. Hence,  $\mu(i) = \mu'(i)$  and  $Q$  is not a Nash equilibrium. ■

The intuition of Proposition 1 is that students do not pay application fees for schools to which they are not assigned. When students' strategies include more than one school with a positive application fee, and considering the strategies of other students as fixed, they have a profitable deviation by removing schools from their strategies.

Theorem 1 states that if for  $i$ , the application fee for  $\mu_I(i)$  is positive, then there is no Nash equilibrium with  $C$  such that the assignment obtained  $\mu$  Pareto-dominates  $\mu_I$ , and  $i$  prefers  $\mu(i)$  to  $\mu_I(i)$ .

**Theorem 1.** Consider a problem  $(P, C)$  and  $i \in I$  such that  $c_{i, \mu_I(i)} = 1$ . Then, there is no  $Q \in NE(DA(P, C))$  such that  $DA(Q, C) = (\mu, c_Q)$  and  $\mu$  Pareto-dominates  $\mu_I$  with  $\mu(i) P_i \mu_I(i)$ .

*Proof.* By contradiction, suppose there exists  $Q \in NE(DA(P, C))$  with  $DA(Q, C) = (\mu, c_Q)$  and  $\mu$  Pareto-dominates  $\mu_I$  with  $\mu(i) P_i \mu_I(i)$ . This implies that  $\mu_I$  is not efficient.

- If  $c_{i,\mu(i)} = 1$ , then  $c_{Q_i} > 1$ . By Proposition 1, we know that  $Q$  is not a Nash equilibrium.
- If  $c_{i,\mu(i)} = 0$ , following the argument of the proof of Proposition 1, we know that  $\mu_I(i) \notin A(Q_i)$ . We need to show that if  $\mu_I(i) \notin A(Q_i)$ , then  $Q$  is not a Nash equilibrium.

Since  $\mu_I$  is stable and  $\mu$  Pareto-dominates  $\mu_I$ , we have  $\mu(i) \neq i$  because  $\mu_I(i)$  is individually rational and  $\mu(i)P_i\mu_I(i)R_i i$ . Thus, there exists  $i' \in I, i \neq i', \mu_I(i') = \mu(i)$  such that  $i' \succ_{\mu_I(i')} i$ . By the construction of DA,  $i$  was rejected from  $\mu_I(i')$  in a chain of rejections initiated by some  $i^* \in I$ .

- **Case 1:** If  $i^* = i$ , then  $i$  generates a chain of rejections and is rejected from  $\mu_I(i')$ . According to Kesten (2010),  $i$  is an *interrupter*. Proposition 3 of Kesten (2010) states that  $i$  is indifferent between applying to  $\mu_I(i')$  or not, hence  $i$  cannot be assigned to  $\mu_I(i')$  in  $DA(Q, C)$ .
- **Case 2:** If  $i^* = i'$ , then according to cycle-solving methods (Dur et al., 2019), no improvement is possible, and  $\mu$  does not Pareto-dominate  $\mu_I$ , which contradicts our assumption.
- **Case 3:** If  $i^* \neq i$  and  $i^* \neq i'$ .  $i^*$  generates a chain of rejections and is rejected from  $\mu_I(i')$  at a later step of DA. Therefore,  $\mu_I(i')P_{i''}\mu_I(i'')$  and  $i' \succ_{\mu_I(i')} i'' \succ_{\mu_I(i')} i$ . After being rejected from  $\mu_I(i')$ ,  $i$  applies to  $\mu_I(i)$ , rejects a student, which in turn will lead to the rejection of  $i'$  who will apply to  $\mu_I(i')$ , and reject  $i^*$ . Thus, when  $\mu_I(i) \notin A(Q_i)$ , the chain of rejection cannot occur, and  $i^*$  can reject  $i$  without subsequently being rejected. Since  $I$  is finite the reasoning continues until  $i$  is rejected from  $\mu_I(i')$ .

Therefore, when  $\mu_I(i) \notin A(Q_i)$ , at least one student has a profitable deviation, and  $Q$  is not a Nash equilibrium. ■

Theorem 1 can be illustrated in Example 1. As mentioned, if an application fee profile is such that  $c_{i_2, s_2} = 1$  is introduced, then  $i_2$  does not apply to  $s_2$ . Suppose  $c_{i_3, s_1} = 1$ . Given  $Q_{i_1} = P_{i_1}$  and  $Q_{i_2}$ ,  $i_3$  has a profitable deviation:  $s_1 \notin A(Q'_{i_3})$ . Let  $Q'_{i_3} : s_2, s_3, i_3, s_1$  denote the new strategy of  $i_3$ . Then  $DA((Q'_{i_3}, Q_{-i_3}), C)(i_3) \succ_{i_3} DA((P_{i_3}, Q_{-i_3}), C)(i_3)$  with  $DA((Q'_{i_3}, Q_{-i_3}), C)(i_3) = (s_2, 0)$ . However,  $i_2$  has a profitable deviation. By applying to  $s_2$ ,  $i_2$  will reject  $i_3$ , who will be assigned to  $s_3$ .

In practice, application fees for a given school are uniform for all students. Following Theorem 1 if the application fee for a given school  $s$  is positive for all  $i$ , then there is no Nash equilibrium leading to  $\mu$  that Pareto-dominates  $\mu_I$  and students  $\mu_I(s)$  prefer  $\mu$  to  $\mu_I$ . Hence, application fees shrink the set of Nash equilibria and prevent Pareto

improvement for some students.

When all schools implement positive application fees for all students, Proposition 1 implies that students apply to at most one school in each Nash equilibrium. Proposition 2 completes Theorem 1 by stating that  $C$  shrinks the set of Nash equilibria and only stable assignments can be obtained.

**Proposition 2.** Consider a problem  $(P, C)$ . If for each  $i \in I, s \in S, c_{i,s} = 1$ , then for each  $Q \in NE(DA(P, C))$  with  $DA(Q, C) = (\mu, c_Q)$ , we have  $\mu \in S(P, C)$ .

*Proof.* From Proposition 1, we know that for each  $Q \in NE(DA(P, C))$ , for each  $i, |A(Q_i)| \leq 1$ . Then, by Theorem 2 of Gale and Sotomayor (1985) and Theorem 5.3. of Haeringer and Klijn (2009), it directly follows that only stable assignments can be obtained at Nash equilibrium when students apply to at most one school. ■

## 4 Conclusion

In this paper, we show that application fees can restrict the set of Nash equilibria under DA. Furthermore, Nash equilibria eliminated by these fees result in assignments that Pareto-dominate the student-optimal stable assignment.

This paper highlights the importance of the design of application fee profiles and college admission mechanisms. A natural follow-up research is to explore mechanisms that offer free applications to students. These could allow students to apply to schools to which they are assigned in the student-optimal stable assignment, thereby preserving the Nash equilibrium. Moreover, laboratory experiments could complement these theoretical studies.

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