

Active learning reliability analysis for the design of continuous reinforced concrete beams

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Outline

- 1 Global Problematic
 - Uncertainties Consideration
 - Performance of reinforced concrete beams
- 2 Active Learning reliability of complex structures
 - Failure Probability Estimation (Reliability analysis)
 - Surrogate Modeling for reliability analysis
- 3 Active Learning reliability of complex structures
 - Active learning reliability analysis scheme
- 4 Case study: Reliability analysis for continuous reinforced concrete beam
 - Problem Description
 - Cast3M multi-fiber FEM model
- 5 Results
 - Comparison of different failure probabilities
 - Comparison of probability stability and probability boundaries
 - Convergence of PC-Kriging
- 6 Conclusions

Uncertainties Consideration

Bridge design in the context of uncertainties propagation for optimum performance.

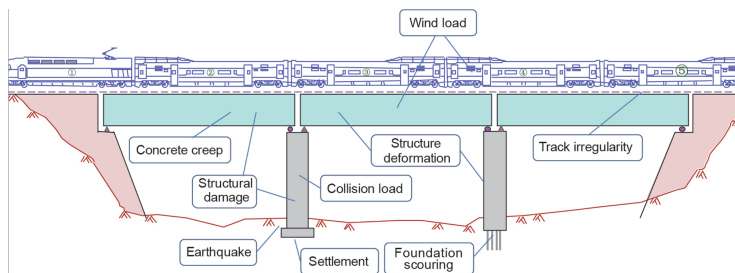


Figure: Sources of uncertainties in the operation of reinforced concrete rail bridges. Zhang, Tian, and Xia 2016

Performance of reinforced concrete beams

Safety (Adaptation to Climate change)
Ultimate Limit State ULS

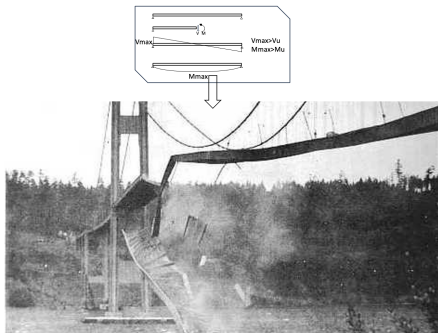


Figure: Tacoma bridge failure (ULS).

Serviceability (service limit state SLS)

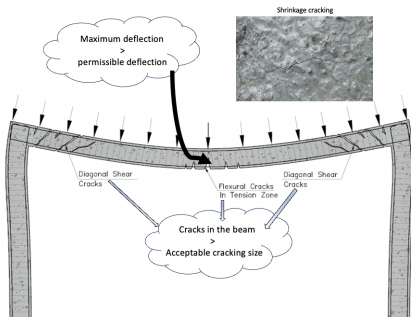


Figure: Cracking and deflection (SLS).

Failure Probability Estimation

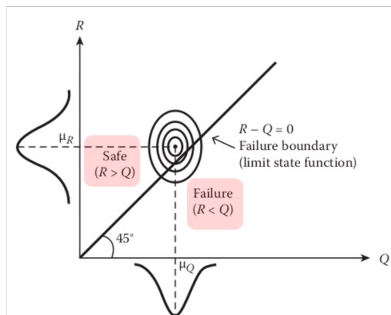


Figure: Reliability analysis: Function vs limit threshold

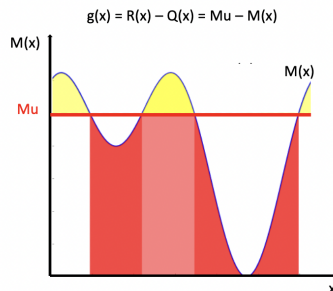


Figure: Reliability analysis: Failure vs safe zones

$$P_f = \mathbb{P}(g(X) \leq 0) = \int_{\mathcal{D}_f} f_X(x) dx.$$
$$P_f = \int_{\mathcal{D}_f} f_X(x) dx = \int_{\{x: g(x) \leq 0\}} f_X(x) dx$$

Surrogate Modeling for limit state estimation

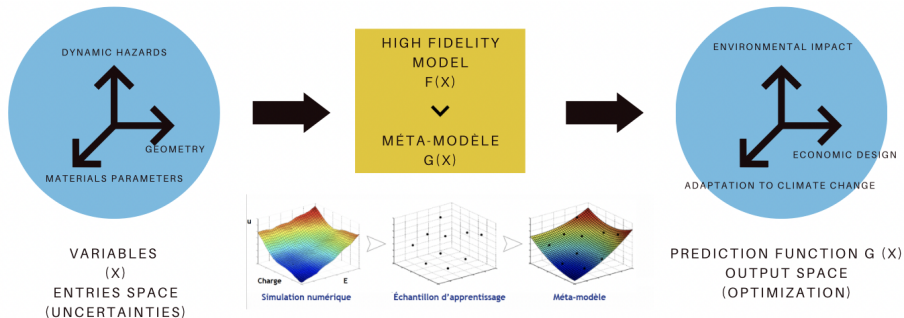
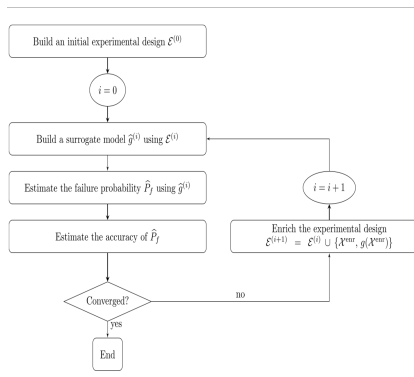


Figure: Surrogate modeling

- Kriging
- Radial Basis Functions
 - Neural Networks
- Support Vector Regression

Active learning reliability analysis scheme



- Sampling technique for data selection
- Finite Element model
- Surrogate model of the limit state
- Reliability analysis for failure probability estimation
- Active learning
- Stopping criterion

Figure: Active learning algorithm. Moustapha, Marelli, and Sudret 2022

Problem Description

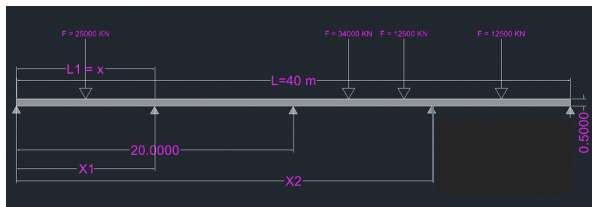


Figure: Longitudinal view of the reinforced concrete beam

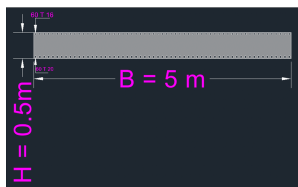
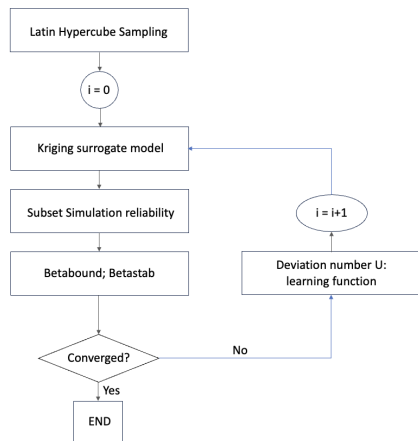


Figure: Transversal section of the beam

- Four-span continuous reinforced concrete beam
- Length = 40 m
- Height = 0.5 m
- 4 Concentrated loads: 5 m 24 m 28 m 35 m

Problem Description

- Considering the effect of 2 geometrical parameters:
 - position of second pier: $X1$
 - position of fourth pier: $X2$
- Quantity of interest: maximum deflection
- Limit state threshold:
 - $T=X1/1200$
 - $T=X1/600$
- Multi-Fiber Finite Element model
- 3 Reliability analysis problems performed using **UQLAB**:
 - Considering 2 failure thresholds
 - Choosing the most adequate stopping criterion
 - Kriging and PC-Kriging for reliability analysis



Cast3M multi-fiber FEM model

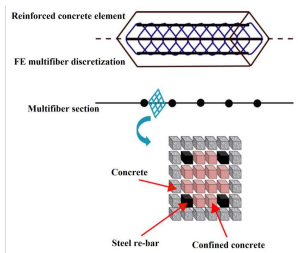


Figure: Representation of multifiber element. Omar, Grange, and Dufour 2017

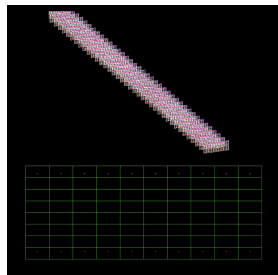


Figure: Geometric representation (Cast3M)

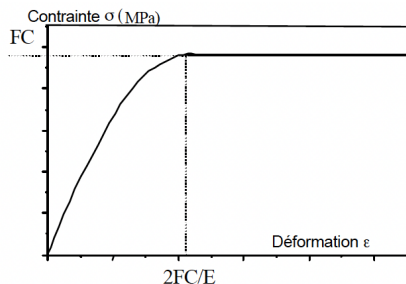
Timoshenko Fiber element

$$(\varepsilon_x)_i = \varepsilon_x - y_i \cdot \phi_z + z_i \cdot \phi_y$$

$$(\gamma_y)_i = \gamma_y - z_i \cdot \phi_x$$

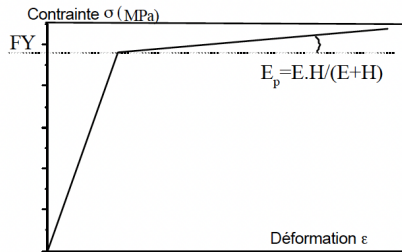
$$(\gamma_z)_i = \gamma_z + y_i \cdot \phi_x$$

Concrete and steel constitutive laws



Béton BAEL for concrete:

- $F_c = 40\text{MPa}$
- $E_c = 29.7\text{GPa}$
- Poisson ratio = 0.2



Parfait-Uni for steel (kinematic hardening):

- $E_p = \frac{E \cdot H}{E + H} F_y = 420\text{MPa}$
- $H = 10\text{GPa}$
- Poisson ratio = 0.3

Comparison of different failure probabilities($X1/600$)

- 10 initial simulations
- 27 added simulations through active learning
- Probability stability: 0.005

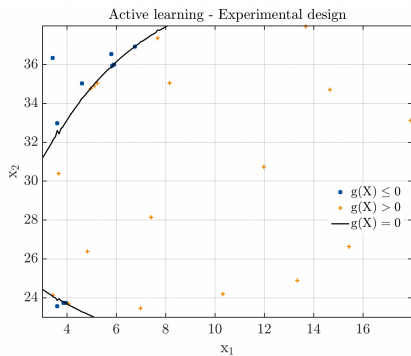


Figure: X1/600 Reliability analysis

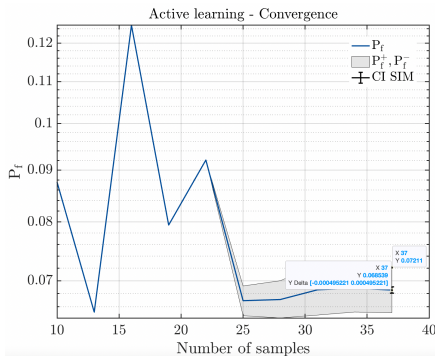


Figure: X1/600 Problem convergence

Comparison of different failure probabilities ($X1/600$)

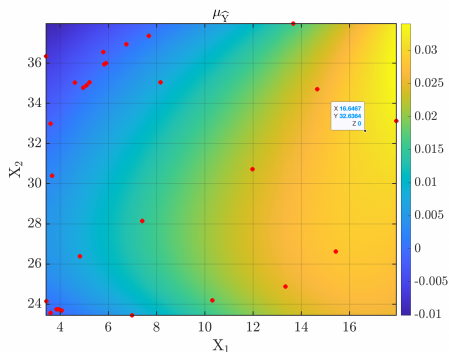


Figure: kriging model predictions

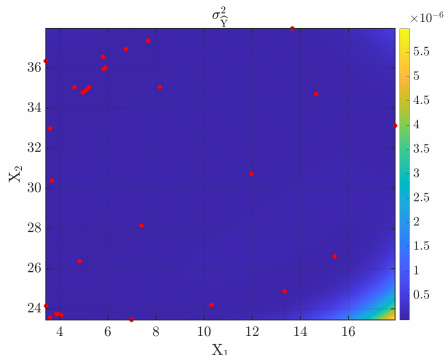


Figure: kriging model uncertainties

$$\mu_{\hat{\gamma}}(\mathbf{x}) = \mathbf{f}(\mathbf{x})^T \hat{\boldsymbol{\beta}} + \mathbf{r}(\mathbf{x})^T \mathbf{R}^{-1}(\mathcal{Y} - \mathbf{F}\hat{\boldsymbol{\beta}})$$
$$\sigma_{\hat{\gamma}}^2(\mathbf{x}) = \sigma^2 \left(1 - \mathbf{r}^T(\mathbf{x}) \mathbf{R}^{-1} \mathbf{r}(\mathbf{x}) + \mathbf{u}^T(\mathbf{x}) \left(\mathbf{F}^T \mathbf{R}^{-1} \mathbf{F} \right)^{-1} \mathbf{u}(\mathbf{x}) \right)$$

Comparison of different failure probabilities (X1/1200)

- 10 initial simulations
- 43 added simulations through active learning
- Probability stability: 0.005

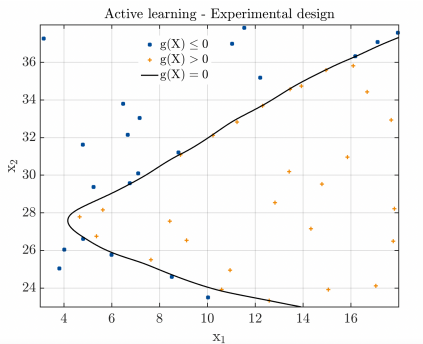


Figure: X1/1200 Reliability analysis

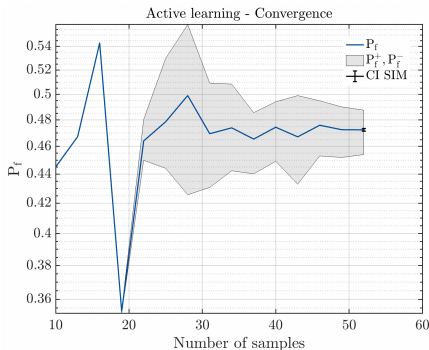


Figure: X1/1200 Problem convergence

Comparison of different failure probabilities ($X1/1200$)

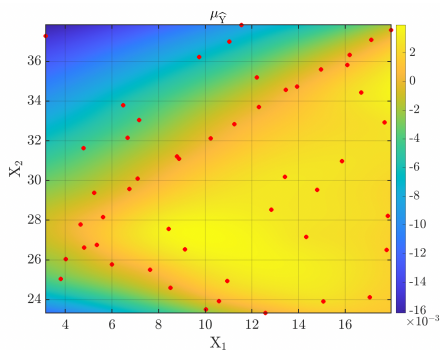


Figure: kriging model predictions

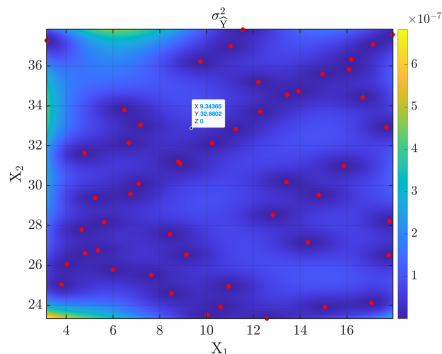
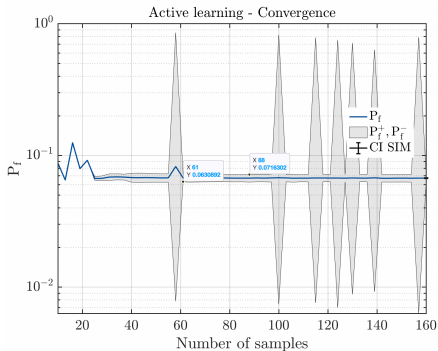
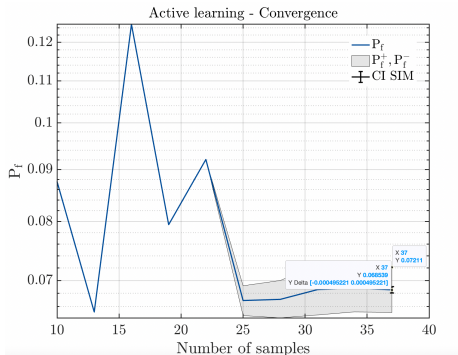


Figure: kriging model uncertainties

Comparison of probability stability and probability boundaries

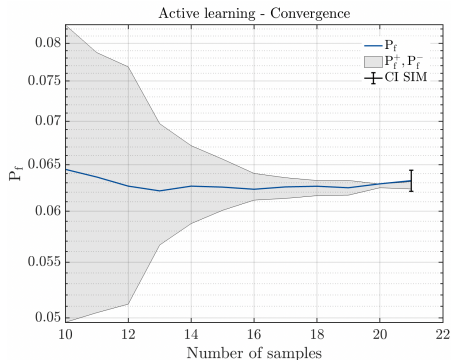
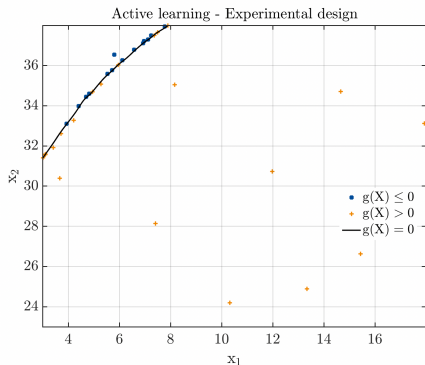
$$\frac{|\hat{P}_f^{(j)} - \hat{P}_f^{(j-1)}|}{\hat{P}_f^{(j)}} \leq \epsilon_{\hat{P}_f}^{\text{stab}} = 0.005$$

$$\frac{\hat{P}_f^+ - \hat{P}_f^-}{\hat{P}_f^0} \leq \epsilon_{\hat{P}_f}^{\text{bound}} = 0.01$$



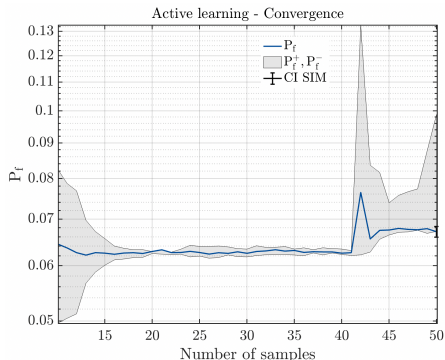
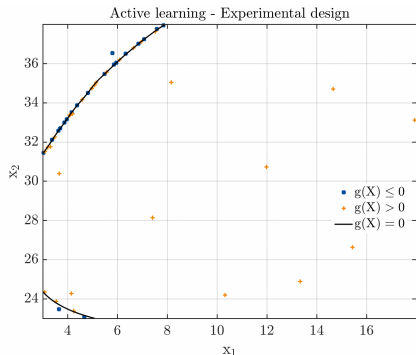
PC-Kriging surrogate for reliability analysis (Convergence of PC-Kriging)

$$\frac{\hat{P}_f^+ - \hat{P}_f^-}{\hat{P}_f^0} \leq \epsilon_{\hat{P}_f}^{\text{bound}} = 0.005$$



PC-Kriging surrogate for reliability analysis(Limit state estimation)

$$\frac{\hat{P}_f^+ - \hat{P}_f^-}{\hat{P}_f^0} \leq \epsilon_{\hat{P}_f}^{\text{bound}} = 0.001$$



Conclusions

- Active learning made it possible with few points to detect the influence of geometrical parameters on the performance of the beam.
- The problem with higher probability needed more FEM simulations to converge
- The convergence of the problem was possible in reference to the probability stability whereas the localised performance of the kriging model didn't permit the probability boundaries to converge
- PC-kriging model convergence (with both stopping criteria) is not enough to reproduce accurately the limit state
- For the beam to be considered as a system, multiple limit states with multi-dimensional input will be considered

References

references