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# Complex behaviour in day-to-day dynamics of Transportation systems

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**Abstract.** The dynamics of large scale transportation systems result from: i) traffic flow in the system, a strongly nonlinear process, ii) traffic assignment, i.e. the route/departure time choice of travellers. Route and departure time choice occur on different time scales, real time and day-to-day. Travellers make their choices based on their assessment of their travel costs. Traffic assignment impacts the congestion patterns, thus retroacts with the travellers' choices. Available information and learning play a crucial role in this process. The aim of the paper is to analyse the day-to-day dynamics of the transportation system on a medium time scale. The questions of interest are: existence and unicity of equilibria, periodic orbits, possibly complex dynamics.

**Keywords.** *Nonlinear dynamical systems; infinite dimensional system; complex system; fixed point; equilibrium; dynamic traffic assignment; traffic flow model; GSOM model*

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## 1 Introduction, setting the problem, aims.

In this contribution we consider the transportation system at a regional level, and focus on vehicular traffic, which contributes to the bulk of passenger transportation in most regions. The question addressed is: what is the evolution of the transportation system over a time range of a month to a year, and specifically does the system reach an equilibrium, and if yes, how is the equilibrium reached, is it unique, is it stable, do other dynamics eventually occur? The dynamics of networks are the result of the process of Dynamic Traffic Assignment (DTA), that is to say the choice by travellers of their route and their departure time. DTA modelling constitutes an essential tool for analysis, planning and management of the transportation system at the regional level. The reader is referred to [5] and [24]

Following Wardrop ([25] the general behavioral assumption for DTA is that travellers make their choices by minimizing their travel cost in order to achieve the object of their trip. In the case of DTA the travel cost includes mainly travel time and penalty for late/early arrival with respect to the desired arrival time, and possible financial costs (tolls). Travellers' choices have an obvious impact on the supply side of the transportation system. If many travellers chose a route because of its attraction (low travel cost) then this route becomes congested and its attraction diminishes. A similar process applies to departure time choice. Hence the fundamental question: does the system reach an equilibrium, and if the answer is positive, is

this equilibrium unique, is it stable, what are the day-to-day dynamics. All these questions have a direct impact on managing the system and on its planification.

In the case of static single mode traffic and monotonous diagonal travel costs (i.e. costs increasing with demand) the problem is well understood and the path flows can be obtained as the solution of a fixed point (or variational inequality) problem, which guarantees the existence of an equilibrium. The reader is referred for instance to [21, 22, 23]. Nevertheless it can be shown in some simple configurations that chaotic behaviour is liable to occur, depending on the assumptions made on the learning behaviour of travellers [6, 16, 4].

Analysis of the dynamic case is much more difficult. One reason is that models of very large transportation networks need to be both very fast and precise. Another reason lies with the innate complexity of the problem, the setting of which is a graph (the network) times a time interval (containing the possible departure times). Thus the natural functional setting is an infinite dimensional space. Many models have been considered for DTA: point queue models [12], simple and efficient, cellular automata (with a large-scale application in the NordRhein-Westphalen region [18]), hydrodynamic models [17], MFD (macroscopic fundamental diagram) based models [1], 2D (bidimensional) models [20, 10] which are emerging for addressing very large scale DTA problems, microsimulation/multiagent models [8, 2, 3] which constitute the core of simulation-based applications (MATSIM [7], commercial softwares such as AIMSUN, PTV, CALLIPER).

The present contribution uses a model of the GSOM family [15, 13, 14]. These models rely on the hydrodynamic paradigm for traffic modelling: they approximate the flow of traffic as a flow of liquid in a network and are well-suited for traffic on large networks. The approach outlined in the contribution also takes into account the evolution of technology which impacts directly DTA. Indeed, crowd sourcing, internet services and V2V (vehicle to vehicle) or V2I (vehicle to infrastructure) communication provide an increasing fraction of travellers with real-time information on the totality of the network. We will use the term ITT (instantaneous travel time) to designate the result of this real-time information. One important aspect of our approach is that travellers in DTA make choices at two different levels and time-scales. They base their route choice in real time on ITT, but base their departure time choice on a day-to-day basis on PTT (predictive travel times). Predictive travel times result from past experienced travel times (ETT) by a learning process. The learning process can also affect the route choice.

Three factors add to the complexity of the system. i) ITT is not a good motive for route choice, as traffic conditions change while the traveller moves along his route, which leads to suboptimal route choice and network dynamics [9, 11]. ii) The two different time-scales necessarily interact, especially since learning (a day-to-day process) can also impact route choice. iii) The learning behaviour of travellers is likely to retroact adversely on the dynamics of the system and may induce chaotic behaviour as shown in the static/quasi-static case [16, 4]. In [19] suggestion of possible chaotic behaviour has been shown with a simplified point queue model.

The contribution aims to extend these ideas with the more precise GSOM model, by considering various learning strategies in the iterative process of day-to-day dynamic assignment. We will analyze in this context the questions of multiplicity of equilibria, of the convergence and stability of the equilibrium reaching process, as well as possible bifurcation phenomena occurring in this process.

## 2 Outline of the traffic assignment process

The main data of the problem is  $D_w^{t_a}$ : the travel demand (number of trips per unit of time) of OD (origin-destination) couple  $w \in \mathcal{W}$  of travellers with desired arrival time  $t_a \in \mathcal{T}_a$ . The main unknowns are:

i) the distribution of departure times  $\varphi_w^{t_a}(t)dt \quad \forall w \in \mathcal{W}, t_a \in \mathcal{T}_a$  with respect to departure time  $t \in \mathcal{T}_d$ . Thus

$$(\mathcal{K}) \left| \begin{array}{l} \int_{t_d \in \mathcal{T}_d} \varphi_w^{t_a}(t) dt = 1 \quad \forall w \in \mathcal{W}, \forall t_a \in \mathcal{T}_a \\ \varphi_w^{t_a} \geq 0 \quad \forall w \in \mathcal{W}, \forall t_a \in \mathcal{T}_a \end{array} \right. \quad (1)$$

ii) the fraction  $\varpi_p^{t_a}(t)$  of travellers departing at time  $t \in \mathcal{T}_d$ , with desired arrival time  $t_a$ , using path  $p \in \mathcal{P}_w$  to complete their  $w$  OD trip.  $\mathcal{P}_w$  denotes the set of plausible sets joining the OD  $w \in \mathcal{W}$ . Thus the fraction  $\varpi_p^{t_a}$  are positive and satisfy:

$$\sum_{q \in \mathcal{P}_w} \varpi_q^{t_a}(t) = 1 \quad \forall t_a \in \mathcal{T}_a, t \in \mathcal{T}_d$$

Let us note by  $f_p^{t_a}(t)$  the flow of travellers departing at time  $t \in \mathcal{T}_d$ , with desired arrival time  $t_a \in \mathcal{T}_a$ , using path  $p \in \mathcal{P}_w$  to complete their trip joining the OD couple  $w \in \mathcal{W}$ . By the above definitions:

$$f_p^{t_a}(t) = \varphi_w^{t_a}(t) \varpi_p^{t_a}(t) D_w^{t_a} \quad \forall w \in \mathcal{W}, p \in \mathcal{P}_w, t_a \in \mathcal{T}_a, t \in \mathcal{T}_d$$

The  $f_p^{t_a}(t)$  constitute the input of the GSOM traffic flow model. The output of this model includes: the instantaneous and experienced travel times and costs, and the predictive travel costs, after resolution of the Wardrop optimality conditions.

The late/early arrival time penalty takes the form  $L(t_a - TA)$  where TA denotes the arrival time and  $L$  denotes a convex function which admits a minimum at  $L(0) = 0$ . Thus if the instantaneous travel time of path  $p$  at time  $t$  is  $ITT_p(t)$  then the corresponding instantaneous travel cost  $ITC_p^{t_a}(t)$  is obtained by  $ITC_p^{t_a}(t) = ITT_p(t) + L(t_a - t - ITT_p(t))$ . Note that the instantaneous travel times are additive and express  $ITT_p(t) \approx \int_p d\xi/V(\xi, t)$  where  $V$  denotes the velocity (an output of the GSOM model). The Wardrop principle applied to route choice, i.e. to the calculation of the fractions  $\varpi_p^{t_a}(t)$  can be expressed as

$$\varpi_p^{t_a}(t) \cdot \left( ITC_p^{t_a}(t) - \min_{q \in \mathcal{P}_w} ITC_q^{t_a}(t) \right) = 0 \quad \forall p \in \mathcal{P}_w, t_a \in \mathcal{T}_a \quad (2)$$

which must be solved at any departure time  $t \in \mathcal{T}_d$  and for all OD couples  $w \in \mathcal{W}$ .

The GSOM model also yields experienced travel times  $ETT_p(t)$  for all paths  $p \in \mathcal{P}_w, w \in \mathcal{W}$  and arrival times  $t$ . Note that the experienced travel times are not additive but satisfy a semi-group property. They are estimated on each path  $p \in \mathcal{P}_w$  by keeping track of the departure time of travellers. Departure time is a traveller attribute which is advected by the traffic flow and thus is easily calculated in the GSOM model. The first step consists in inverting the experienced travel times in order to obtain the predictive travel times  $PTT_p(t)$  with  $t$  being the departure time:  $PTT_p(t) = ETT_p(t + PTT_p(t))$ . Then the predictive path travel cost is obtained by  $PTC_p^{t_a}(t) = PTT_p(t) + L(t_a - t - PTT_p(t))$ . Finally we obtain the predictive OD travel costs as the expectation of the predictive path travel costs:

$$PTC_w^{t_a}(t) = \sum_{q \in \mathcal{P}_w} \varpi_q^{t_a}(t) \cdot PTC_q^{t_a}(t) \quad \forall t_a \in \mathcal{T}_a, t \in \mathcal{T}_d$$

The Wardrop principle applied to the departure time choice can be expressed as

$$\int_{t \in \mathcal{T}_d} dt \left[ \varphi_w^{t_a}(t) \cdot \left( PTC_w^{t_a}(t) - \min_{s \in \mathcal{T}_d} PTC_w^{t_a}(s) \right) \right] = 0 \quad \forall w \in \mathcal{W}, t_a \in \mathcal{T}_a, \varphi \in (\mathcal{K}) \quad (3)$$

where  $\varphi$  must be constrained by  $(\mathcal{K})$ .

### 3 Implementation, concluding remarks.

Let us consider the resolution of (3) in a day-to-day process. We denote by  $\tau$  the day index. (3) can be viewed as a fixed point problem (a natural functional setting would be a  $L^2$  space of square integrable functions with respect to  $t \in \mathcal{T}_d$ ):

$$\varphi = P_{\mathcal{K}}[\varphi - PTC(\varphi)] \tag{4}$$

Here  $\varphi$  denotes the vector of OD costs,  $P_{\mathcal{K}}$  the projector on  $(\mathcal{K})$  and  $PTC$  the vector of OD costs, which can be considered as a function of  $\varphi$  because (2) must be solved with respect to  $\varpi$  given  $\varphi$ . Various schemes are conceivable in order to solve (4), for instance

$$\varphi^{\tau+1} = (1 - \beta^{\tau}) \varphi^{\tau} + \beta^{\tau} P_{\mathcal{K}}[\varphi^{\tau} - \alpha^{\tau} PTC(\varphi^{\tau})] \tag{5}$$

Here the coefficients  $\alpha^{\tau}$  and  $\beta^{\tau}$  express learning behaviours of travellers.  $\alpha^{\tau}$  expresses the sensitivity to expected OD travel costs whereas  $\beta^{\tau}$  would designate the fraction of travellers who actually react to OD travel costs. These coefficients can also depend on the day.

Preliminary results show various patterns: simple convergence, periodic orbits, lack of convergence suggestive of possible chaotic behaviour, depending on the learning strategies and on the demand level which appears also as a critical parameter. These results are consistent with previous results reported in the literature for the static/quasi-static case.

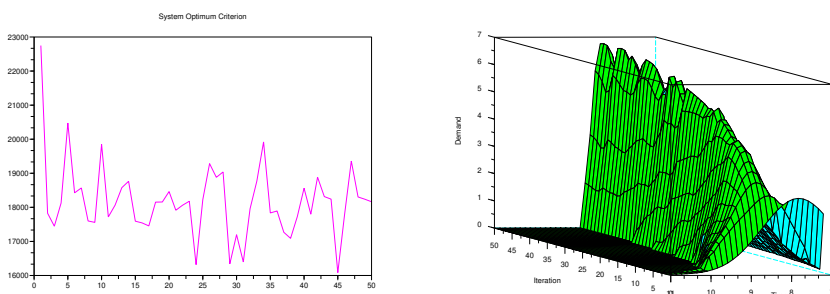


Figure 1: Non convergence. Left: Total network travel cost as a function of time, right: demand  $f$  for one path and desired arrival time, as a function of  $\tau$  (iteration) and departure time

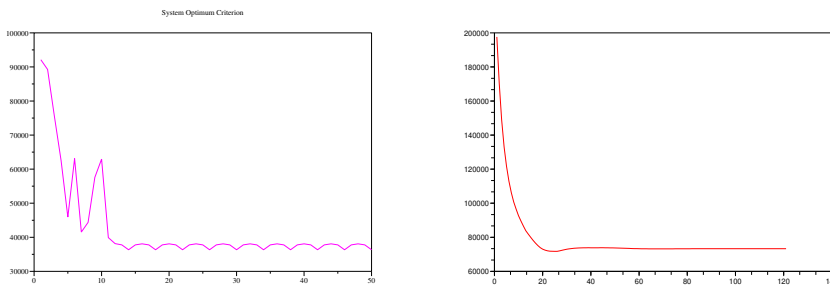


Figure 2: Left: the system converges towards an orbit of period 4, right the system converges to an equilibrium (index: total network travel cost as a function of iteration  $\tau$ )

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