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## ► To cite this version:

Nguyen Huynh Tan Tai, Anh Le Van. Numerical computation of the deployment of a folded inflatable tube. IV International Conference on Textile Composites and Inflatable Structures, 2009, Barcelone, Spain. ⟨hal-04621269⟩

**HAL Id: hal-04621269**

**<https://hal.science/hal-04621269v1>**

Submitted on 23 Jun 2024

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# NUMERICAL COMPUTATION OF THE DEPLOYMENT OF A FOLDED INFLATABLE TUBE

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**Key words:** Membrane, Inflatable structure, Contact, Finite element method.

**Summary.** This work deals with the numerical simulation of the deployment of a folded inflatable structure subjected to an internal pressure. The computation is performed by means of the *finite element method* as opposed to most of other works which used the control volume method. During the deployment process, *self contact* can occur at some portions of the boundary where contact force play a crucial role to deploy the structure. Contact problem will be treated by making use of a new weighted residual principle.

## 1 THEORETICAL FORMULATION

Consider two bodies – labeled 1 and 2 – undergoing motions  $\phi^{(1)}$  and  $\phi^{(2)}$  in the three-dimensional space during some time interval  $[O, T]$ . Assuming that the bodies may come into contact with each other, we formulate the contact problem using a Lagrangian description. The notations, rather standard, are summarized as follows. The reference configuration of the two bodies are represented by the regions  $\Omega_o^{(1)}$  and  $\Omega_o^{(2)}$ . The prescribed body force per unit mass in body  $i$  is denoted  $\mathbf{f}^{(i)}$ . The boundary  $S_o^{(i)}$  of body  $i$  is partitioned into three parts denoted  $S_{oU}^{(i)}$ ,  $S_{oT}^{(i)}$  and  $S_{oc}^{(i)}$ , where  $S_{oU}^{(i)}$ ,  $S_{oT}^{(i)}$  are Dirichlet and Neumann boundary parts and  $S_{oc}^{(i)}$  is the part where contact potentially takes place. The stress state in body  $i$  is defined by the first Piola-Kirchhoff stress tensor  $\mathbf{\Pi}^{(i)}$ . The nominal traction vector at any point in  $S_o^{(i)}$  with normal vector  $\mathbf{N}^{(i)}$  is denoted  $\mathbf{T}^{(i)} = \mathbf{\Pi}^{(i)} \cdot \mathbf{N}^{(i)}$ . The spatial counterparts of surface  $S_o^{(i)}$  is denoted  $S^{(i)}$ .

Given a point  $\mathbf{x} \in S_c^{(1)}$  one defines a contact point  $\mathbf{y} \in S_c^{(2)}$  as the closest point to  $\mathbf{x}$  via  $\mathbf{y} = \arg \min_{\mathbf{x}^{(2)} \in S_c^{(2)}} \|\mathbf{x} - \mathbf{x}^{(2)}\|$ , and the proximity as  $g = -\boldsymbol{\nu}(\mathbf{x} - \mathbf{y})$  where  $\boldsymbol{\nu}$  is the outward normal at point  $\mathbf{y}$ . One also defines the point  $\mathbf{X} \in S_{oc}^{(1)}$  related to point  $\mathbf{x}$  in question by  $\mathbf{x} = \phi^{(1)}(\mathbf{X})$  and  $\mathbf{Y}(\mathbf{X}) \in S_{oc}^{(2)}$  related to point  $\mathbf{y}$  by  $\mathbf{y} = \phi^{(2)}(\mathbf{Y})$ . The tangential kinematics is characterized by the slip velocity  $\mathbf{V}_T = \dot{\eta}^\alpha \mathbf{a}_\alpha$  which is resolved in terms of the local spatial basis  $(\mathbf{a}_1, \mathbf{a}_2)$  at point  $\mathbf{y} \in S_c^{(2)}$ . Likewise, the nominal traction vector at any point  $\mathbf{X} \in S_{oc}^{(1)}$  is resolved as  $\mathbf{T} = T_N \boldsymbol{\nu} - \mathbf{T}_T$ . The coefficient of friction is  $\mu$ .

The weak form presented here is stated as a mixed relationship which involves both the displacement fields  $\mathbf{U}^{(i)}$ ,  $i \in \{1, 2\}$ , defined in  $\Omega_o^{(i)}$  and the multiplier fields  $\lambda_N$  and  $\lambda_T$  defined on  $S_{oc}^{(1)}$ . Accordingly, the weighting functions are the virtual displacements  $\mathbf{U}^{(1)*}$ ,  $\mathbf{U}^{(2)*}$ , and the

virtual multipliers  $\lambda_N^*$ ,  $\lambda_T^*$ . Two positive constants  $\epsilon_N, \epsilon_T$  being chosen, the weighted residual relationship is given by<sup>1</sup>

$$\begin{aligned}
 & \forall t, \forall \mathbf{U}^{(1)*}, \forall \mathbf{U}^{(2)*}, \forall \lambda_N^*, \forall \lambda_T^* \\
 & \sum_{i=1}^2 \left\{ - \int_{\Omega_o^{(i)}} \boldsymbol{\Pi}^{(i)T} : \nabla_{\mathbf{X}^{(i)}} \mathbf{U}^{(i)*} d\Omega_o + \int_{\Omega_o^{(i)}} \rho_o^{(i)} \mathbf{f}^{(i)} \mathbf{U}^{(i)*} d\Omega_o + \int_{S_{oU}^{(i)} \cup S_{oT}^{(i)}} \mathbf{T}^{(i)} \mathbf{U}^{(i)*} dS_o \right\} \\
 & + \int_{S_{oc}^{(1)}} \left[ \langle \lambda_N + \epsilon_N g \rangle \boldsymbol{\nu} - \left( 1 - \left\langle 1 - \frac{\mu \langle \lambda_N + \epsilon_N g \rangle}{\|\lambda_T + \epsilon_T \mathbf{V}_T\|} \right\rangle \right) (\lambda_T + \epsilon_T \mathbf{V}_T) \right] \left( \mathbf{U}^{(1)*}(\mathbf{X}) - \mathbf{U}^{(2)*}(\mathbf{Y}(\mathbf{X})) \right) dS_o \\
 & + \int_{S_{oc}^{(1)}} \left\{ (\lambda_N - \langle \lambda_N + \epsilon_N g \rangle) \frac{\lambda_N^*}{\epsilon_N} + \left[ \lambda_T - \left( 1 - \left\langle 1 - \frac{\mu \langle \lambda_N + \epsilon_N g \rangle}{\|\lambda_T + \epsilon_T \mathbf{V}_T\|} \right\rangle \right) (\lambda_T + \epsilon_T \mathbf{V}_T) \right] \frac{\lambda_T^*}{\epsilon_T} \right\} dS_o \\
 & = \sum_{i=1}^2 \int_{\Omega_o^{(i)}} \rho_o^{(i)} \ddot{\mathbf{U}}^{(i)} \mathbf{U}^{(i)*} d\Omega_o
 \end{aligned} \tag{1}$$

## 2 SEMI-DISCRETE EQUATIONS AND TIME INTEGRATION

The mixed relationship (1) is discretized in space by the finite element method, considering that the displacements and multipliers are two independent fields. One eventually obtains the following coupled equation system with unknowns  $\mathbf{U}$  and  $\boldsymbol{\Lambda}$

$$\begin{cases} [\mathbf{M}] \{\ddot{\mathbf{U}}\} + \{\boldsymbol{\Psi}(\mathbf{U})\} = \{\boldsymbol{\Phi}(\mathbf{U})\} + \{\boldsymbol{\Phi}_{contact}(\mathbf{U}, \boldsymbol{\Lambda})\} \\ \{\mathbf{R}_{\boldsymbol{\Lambda}}(\mathbf{U}, \boldsymbol{\Lambda})\} = \{\mathbf{0}\} \end{cases} \tag{2}$$

where  $[\mathbf{M}]$  is the mass matrix,  $\{\boldsymbol{\Psi}\}$  the internal force vector,  $\{\boldsymbol{\Phi}\}$  the external force and  $\{\boldsymbol{\Phi}_{contact}\}$  the contact force. The second Equation results from the discretization of third line of Equation (1).

In this work, use is made of both the Newmark<sup>2</sup> and the Moreau<sup>3,4</sup> schemes for the time stepping. Whereas the standard Newmark scheme is able to give satisfactory results, the Moreau scheme proves to be more efficient to deal with the velocity jumps at impact time since it only involves first order time derivatives.

The Moreau scheme is summarized as follows. Let the time interval of interest  $[0, T]$  be divided into  $N$  equal intervals  $\cup_{n=1}^N [t_{n-1}, t_n]$ , i.e. equal time steps  $\Delta t = t_n - t_{n-1}$ . By integrating Equation (2)<sub>1</sub> in time from  $t_{n-1}$  to  $t_n$  and making use of the  $\theta$ -method to approximate the displacement,  $\{\mathbf{U}_n\} = \{\mathbf{U}_{n-1}\} + \Delta t (\theta \{\mathbf{V}_n\} + (1 - \theta) \{\mathbf{V}_{n-1}\})$ , one obtains

$$\begin{cases} \frac{1}{\Delta t} \{\mathbf{R}\} \equiv \frac{1}{\theta \Delta t^2} [\mathbf{M}] \left( \{\mathbf{U}_n\} - \{\mathbf{U}_{n-1}\} - \Delta t \{\mathbf{V}\}_{n-1} \right) + \left( \xi \{\boldsymbol{\Psi}_n\} + (1 - \xi) \{\boldsymbol{\Psi}_{n-1}\} \right) \\ - \left( \xi \{\boldsymbol{\Phi}_n\} + (1 - \xi) \{\boldsymbol{\Phi}_{n-1}\} \right) - \{\boldsymbol{\Phi}_{contact,n}\} = 0 \\ \{\mathbf{R}_{\boldsymbol{\Lambda},n}\} = \{\mathbf{0}\} \end{cases} \tag{3}$$

where  $\xi, \theta \in [0, 1]$  are chosen parameters. Equation (3) is a nonlinear algebraic system which can be solved by the Newton-Raphson scheme.

## 3 A FIRST NUMERICAL EXAMPLE: INFLATION OF A FLAT TUBE

The example considered in this section is a first step for validating the contact computation of a membrane structure. Consider a membrane tube made of a hyperelastic Saint-Venant

Kirchhoff material with  $E = 10^7 N/m^2$ ,  $\nu = 0.3$ . In the reference configuration, the tube is flat, rectangular of dimensions  $0.15m \times 0.5m \times 50\mu m$  and rests on a rigid foundation as shown in Figure 1<sub>1</sub>. It is free from all kinematic boundary condition except for the contact with the foundation and is submitted to an internal pressure  $p = \min\{50.t, 10\}Pa$ . The tube is modeled by  $2 \times 24$  eight-node quadrilateral membrane elements. Figures 1 and 2 show the initial and deformed shapes corresponding to the Newmark and Moreau schemes, respectively, as time evolves. As predicted, the tube takes off due to the contact reaction force with the foundation before falling down under the gravity effect.

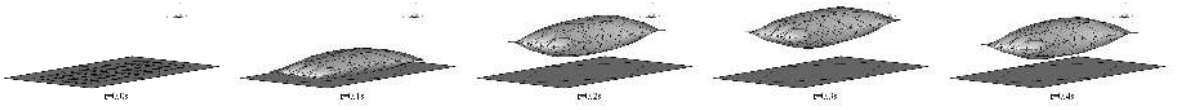


Figure 1: Initial and deformed shapes obtained with the Newmark scheme,  $\Delta t = 10^{-3}$ ,  $\gamma = 2\beta = \frac{1}{2}$

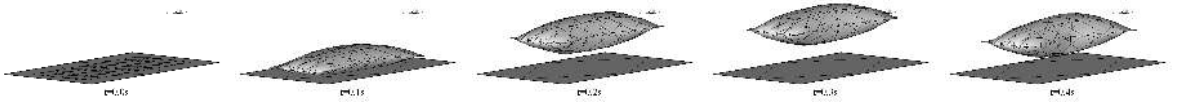


Figure 2: Initial and deformed shapes obtained with the Moreau scheme,  $\Delta t = 10^{-3}$ ,  $\xi = \theta = \frac{1}{2}$

#### 4 DEPLOYMENT OF A FOLDED TUBE

Consider again a flat tube made of an elastic membrane of dimension of  $0.3m \times 0.8m$  resting on a rigid foundation and made of the same material as in the previous section. Here, the tube is folded in two in the reference configuration; one end of the tube is clamped at the fixed plane and the middle point of the other end constrained to move along an axis parallel to  $\mathbf{z}$ , as shown in Figure 3. One assumes either gravity in the  $\mathbf{z}$  direction or no gravity. When submitted to an internal pressure  $p(t)$ , the tube is gradually unfolded and eventually reaches the straight shape. The deployment process is mainly governed in the initial stage by *the self-contact* of the tube as well as *the contact* with the rigid plane. The structure is modeled by  $2 \times 40$  8-node quadrilateral membrane elements.

Figure 3 shows the deployment sequence when a pressure  $p = 1000.tPa$  is applied while Figure 4 plots the internal pressure versus the tip displacement with the Newmark and Moreau schemes. An in-depth analysis reveals that the Newmark scheme may yield unlikely vibrations of the membrane and some inter-penetrations at the contact zones, while the Moreau scheme produces better numerical results. Under gravity  $g = 10m/s^2$ , the structure can not be deployed with the weak pressure rate  $p = 100.tPa$  as the inertial effect is not strong enough to win the gravity. Figure 5 shows that the greater the pressure rate is the faster the structure deploys, yet the deployment time is not a linear function of the pressure rate.

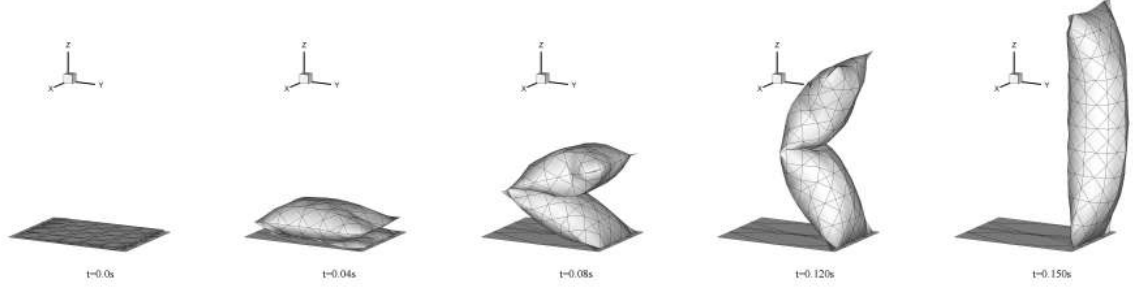


Figure 3: Deployment of the folded tube under internal pressure  $p(t) = 1000.t \text{ Pa}$

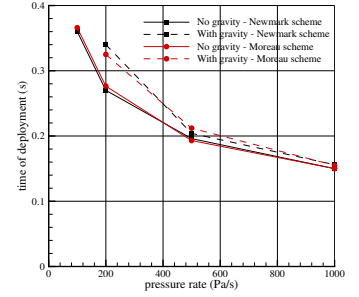
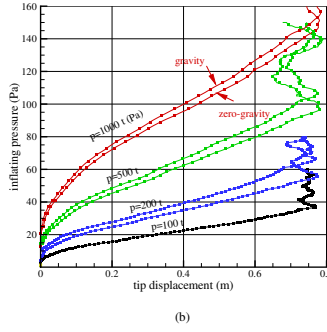
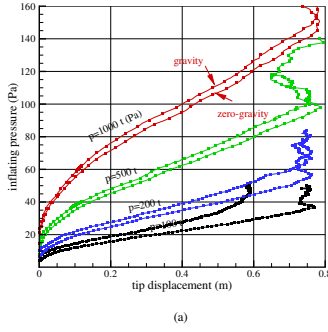


Figure 4: Inflating pressure versus the tip displacement. (a) Newmark scheme, (b) Moreau scheme.

Figure 5: Deployment time versus pressure rate.

## 5 CONCLUSION

The weighted residual principle (1) proposed for contact problems has been shown to be able to successfully compute the deployment process of an inflatable tube. The time discretization has been performed using the standard Newmark scheme and the first-order Moreau scheme, which is found to give more satisfactory results than the first one.

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