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## Separation of vibratory components in complex systems for condition monitoring

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### ABSTRACT

Mechanical signals are often a mixture of multiple components produced by different sources within multiple subsystems. The separation of these components is beneficial for differential diagnosis, fault severity assessment and prognosis. The synchronous average (SA) is a powerful and widely used technique for this purpose, providing optimal estimation performance and simplicity. However, the complexity of such systems often implies the existence of several spectral interferences, which jeopardize the straightforward application of the SA. This paper addresses this issue by proposing a general strategy based on the synchronous average and the set theory, to isolate each vibratory contribution related to a source of interest, while accounting for spectral interferences. This approach enables a comprehensive understanding of the system's health status and the ability to locate specific issues within a subsystem. The proposed methodology is demonstrated through an example of an accessory gearbox vibration signal acquired during a ground test campaign on a CFM56 turbojet engine.

*Keywords:* Synchronous average; Mechanical contribution separation; Condition monitoring; Vibrations analysis.

*Article history:* .

### 1. Introduction

The study of mechanical systems has long been a crucial area of research in various industries [1]. Effective monitoring and diagnosis of mechanical systems is essential for maintaining smooth operations. Vibration and acoustic analysis is a widely used method for monitoring the health of mechanical systems. This approach is non-intrusive and provides valuable information about the internal forces in the system, offering a reliable tool for fault detection and diagnosis [2]. However, these systems are often complex, involving the conversion of multiple speed and torque ratios to achieve different tasks. Therefore, this often results in a large number of rotating parts, each of which contributes differently to the overall vibration signal. This makes it difficult to identify and diagnose specific components, thus the importance of signal separation in [3]. Furthermore, the presence of multiple subsystems and rotating parts can lead to some interferences between different sources. This problem is often addressed through frequency analysis, which assumes that the sources do not share their frequency content. However, in many real-world applications, frequency content overlap is unavoidable, which results in a spectral interference due to having the information of a certain line contribution masking the other. This superimposition can compromise the accuracy of diagnosis in terms of fault detection and localization by tracking the faulty component over the healthy one and vice versa, therefore, a false health state assumption. Consequently, to diagnose each mechanical component independently, it is essential to extract the exclusive contribution of each one by

eliminating the overlapped frequencies caused by other components.

Some generic signal processing techniques were proposed for signal decomposition and mechanical component contribution extraction using vibration analysis. Starting with a basic Synchronous Average (SA); this method involves averaging the signal over one or several periods to extract the sinusoidal component that refers to a certain frequency and its multiples. The advantage of this method is its simplicity making it possible to implement without any estimation. However, when considering a set of lines, the sum of all contributions would result in a replication of the interference mentioned before. This concept was addressed in [4] by exploiting the corresponding operator of the synchronous average and this replication problem when extracting the whole subsystem. However, the remaining spectral overlap for each contribution still exists which triggers again the problem stated before. Next, [5] involves the use of a band pass filter to extract the frequency content of the components of interest. This method is simple to implement and can provide good results for simple mechanical systems. However, it can be sensitive to the choice of filter parameters and may include some interferences within the chosen bandwidth around the centered frequency. In addition, the problem of frequency overlap is still considered having several sources sharing the same frequency. Otherwise, the Optimal Subspace-Kurtosis and Subspace-Extended Kalman Filter involves combining the advantages of both approaches to achieve optimal source separation [6]. It uses the subspace-kurtosis method to

extract periodic signals of interest, while the extended Kalman filter is used to estimate the corresponding contributions. The advantage of this method is that it provides a comprehensive solution to the problem of interferences and source separation, taking into account both the presence of periodic signals and the correction of interferences. Other works use Kalman filters to improve frequency analysis and order tracking methods [7, 8, 9, 10, 11] and recent techniques aim to improve the Vold Kalman Filtering for resampling techniques, high frequency resolution in Fourier analysis and crossing orders detection [12, 13, 14, 15, 16]. However, when many harmonics are considered in typical complex mechanical systems, it can be challenging for these approaches to accurately track all of them, which can lead to an increased computation time and cost.

In this paper, the proposed method addresses the challenge of interferences and contribution separation in complex mechanical systems. In Section 2, the problem statement defines the periodicity identification in such systems and its replication when extracting several sources. Section 3 presents the methodology of the proposed method systematically to solve the defined problem. Section 4 demonstrates the application of the method on an aircraft accessory gearbox. Finally, in Section 5, a conclusion wraps this paper up and highlights the most important details.

## 2. Problem Statement

Vibratory signals generated by rotating machines are periodic in nature. This periodicity is directly linked to the angle of rotation of the machine. In fact, when machines operate under a perfectly constant speed, this periodicity remains consistent over time. In practice, machines are subject to speed fluctuations, which can introduce errors when analyzing time domain signals. One widely used solution in mechanical signal processing is angular description [17, 18]. This can be achieved by resampling the time domain signal in the angular domain using an encoder signal indicating the position of the angle of rotation.

### 2.1. The synchronous average (SA)

In this context, the synchronous average (SA) is a powerful and widely used tool in vibration analysis, which particularly deals with the extraction of periodic waveforms (such as produced by imbalances, misalignments, anisotropic rotors, flexible coupling, gear meshing and other phenomena) from other competing but random sources, assuming that its period is known a priori [19, 20, 21]. The basic idea of the SA is to average the signal over a number of samples called a cycle. This is done by dividing the signal into segments, each corresponding to one cycle of the signal, and then averaging the values within each segment. Mathematically, let  $x[\theta]$  be an angular signal of cycle  $\Theta$  (for instance  $\Theta = 2\pi$ ); its SA is defined as follows:

$$SA_{\alpha}(\bar{\theta}) = \left\langle x\left(\bar{\theta} + m \frac{2\pi}{\alpha}\right) \right\rangle_m, \quad m \in \mathbb{N} \quad (1)$$

where the operator  $\langle f(m) \rangle_m$  denotes the empirical average of the function  $f(m)$ ,  $\alpha$  being the fundamental cyclic order (so that

the averaging period equals  $2\pi/\alpha$ ) and  $\bar{\theta} \in \left[0, \frac{2\pi}{\alpha}\right]$  for the

angular location in the cycle (i.e.  $\bar{\theta} = \theta - k \frac{2\pi}{\alpha}$ , with  $k$  the greatest integer smaller than  $\frac{\theta}{2\pi/\alpha}$ ).

The obtained SA signal is defined over a corresponding period with respect to a cyclic order  $\alpha$  and then replicated to match with initial signal length as follows:

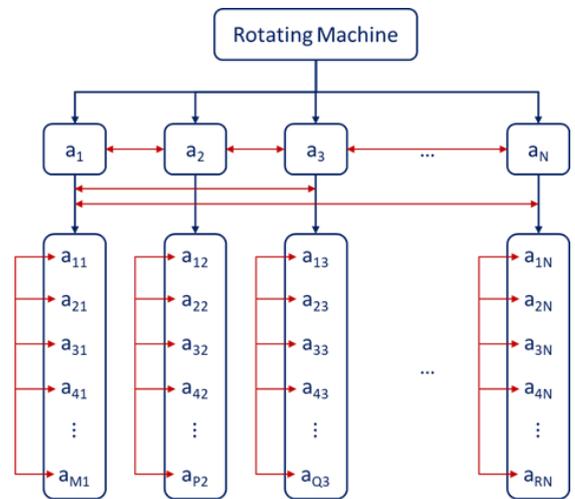
$$m_{\alpha}(\theta) \leftrightarrow SA_{\alpha}(\bar{\theta}) \quad (2)$$

The SA is typically used to extract the vibratory component associated with a rotating shaft, resulting in a Dirac comb in the frequency domain at the line order and its multiple harmonics.

The order of each shaft is defined as the ratio between the frequency of the shaft and the reference frequency, with the reference shaft having an order of 1.

### 2.2. Frequency Interferences

When approaching the problem of a complex mechanical system decomposition, given a general model illustrated in Figure 1 with the corresponding interferences in red, it is essential to firstly isolate the subsystem of interest ( $a_j$ , such that  $j = 1, \dots, N$ ). Some examples of subsystems in mechanical systems are: an accessory gearbox (AGB), a gas generator, a combustion chamber, etc...



**Figure 1.** General model of a rotating machine considering different interferences (red arrows) between different lines ( $a_{11}, a_{12}, \dots$ ) in each subsystem ( $a_1, a_2, \dots$ ).

Once the isolation of the potentially defective subsystem has been carried out, it is important in a second phase to proceed with the separation of the various components linked to the rotating parts of this subsystem (i.e.  $a_{i1}$ , such that  $i = 1, \dots, M$ , from the subsystem  $a_i$ ). This separation considers the exclusive contribution of each component (without interferences) to accurately locate the fault in the isolated subsystem.

In the following, and without loss of generality, the problem addressed in this paper is explained through a simplistic example. Considering a gearbox with an input shaft rotating at order 1 (reference shaft) and an output shaft rotating at 1.2, the associated frequency set with the SA of each shaft would lead to the following order set:

Input Shaft:

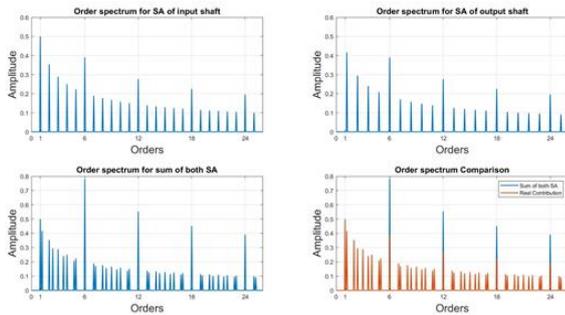
$$S_i : \left\{ \alpha_i = 1; 2; 3; 4; 5; 6; \dots, \text{ such that } \alpha_i \leq \frac{N_S}{2} \in \mathbb{N} \right\}$$

Output Shaft:

$$S_o : \left\{ 1.2 \times k = 1.2; 2.4; 3.6; 4.8; 6; \dots, \text{ such that } 1.2 \times k \leq \frac{N_S}{2} / k \in \mathbb{N} \right\}$$

with  $N_S$  being the number of samples per reference cycle assumed  $2\pi$  in this paper.

The vibratory contribution of this gearbox (the subsystem) cannot be directly obtained by summing up the SA of both shafts. In fact, as seen in the Figures 2a – 2b, there are several orders in common between the shafts (i.e. 6 and its multiples 12, 18, 24, etc.). These common orders represent the frequency interferences of the subsystem components. The sum of both SAs leads to incorrect estimation, which manifests through higher energy at the shared frequencies in the spectrum (see in Figures 2c – 2d). To be noted that this interference is not meant to be completely removed for the subsystem but rather its replication because it can provide important information about its global health state. In fact, these shared harmonics, representing the meshing harmonics and their energies, are widely used as efficient diagnostic indicators in gearboxes.



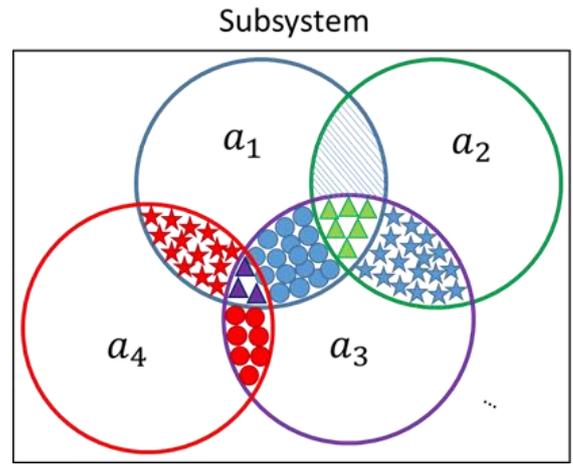
**Figure 2.** Order Spectra for: a- (Top left) SA of input shaft, b- (Top right) SA of output shaft, c- (Bottom left) Sum of both SAs, d- (Bottom right) Comparison between the sum of both SAs and the real subsystem SA.

Another interesting issue addressed in this paper is the localization of a fault. This needs to be achieved by considering the exclusive contribution of each shaft so the faulty one does not affect the others. As seen in Figures 2a – 2b, the SAs of the two shafts present an interference at the sixth order and its multiples, which could mislead the identification of the faulty shaft. Therefore, it is essential to eliminate the remaining interferences between each source to evaluate the health indicator applied on the exclusive contribution of each one. This paper goes beyond this simple example to the case of more complex systems, where multiple shafts with different rotating frequencies having multiple common frequencies in play.

### 3. Proposed methodology

The proposed methodology in this paper is designed to isolate the desired vibratory contribution of a given subsystem in a complex mechanical system using the SA, as well as the exclusive vibration response of each of the subset component. On the one hand, this is achieved by using the set theory to identify the common frequencies that represent the interferences between each line and eliminate its replication as seen in [4]. On the other hand, it can be implemented by the computation of the least common multiple (LCM) of each pair of orders.

Figure 3 illustrates this theory, where multiple sets are considered, having the set “ $a_1$ ” representing the reference shaft. Each set corresponds to the orders ensemble of each shaft’s contribution resulted from its SA. By comparing the orders of shafts  $a_1$  and  $a_2$ , it can be seen that some interferences exist (hatched part) between these two sets and others that are jointly common with shaft  $a_3$  (green triangles).



**Figure 3.** Illustration of the sets of SA orders with the different interferences.

#### 3.1. Subsystems Isolation

Generally, one can consider  $N$  shafts with the corresponding set of fundamental orders  $A = \{\alpha_i \in \mathbb{R} / i = 1, 2, \dots, N\}$  constituting a certain subsystem. The goal is to find the signal associated with this specific subsystem, which encompasses the total set of constituent lines excluding the replications of the interference between each considered line. This leads to the following equation:

$$\sum_{\alpha_i \in A} m_{\alpha_i}(\theta) = s(\theta) + \Delta(\theta) \quad (2)$$

With  $m_{\alpha_i}(\theta)$  being the SA of the angular signal  $x(\theta)$  with respect to the order  $\alpha_i$ ,  $s(\theta)$  being the subsystem deterministic signal and  $\Delta(\theta)$  being the replication contribution caused by the interfering components.

Taking the replication to be the unique interference between each two sets (i.e.  $a_1 \cap a_2$  without  $a_2 \cap a_1$ ; the interference is not totally eliminated between these two sets, but rather the replication of the interference).

The replication set requires a more generic definition because common orders between physically unlinked shafts can still exist. In addition, this interference will repeat for integer multiples of the common order, leading to two major problems that prompted the proposal of two solutions:

- Calculate the LCM between each pair of shafts to find the fundamental common order.
- Compute the synchronous average relative to the LCM order found to take into consideration all its multiples.

In order to calculate the corresponding LCM of all common orders for each shaft, a matrix of common orders (MCO) is proposed. To be able to free the replication of the interference between each two shafts without the elimination of the latter, this matrix must be an upper triangular matrix whose diagonal corresponds to the common order between two consecutive shafts. By taking the LCM between each pair of shafts, only the lowest common order is considered, eliminating the redundant frequencies when isolating the subsystem. Thus, the computation of the following matrix of common orders (MCO):

$$\mathbf{M}(i, j) = \begin{cases} LCM(\alpha_i, \alpha_{j+1}), & \text{if } i \leq j \\ 0, & \text{if } i > j \end{cases} \quad (3)$$

with  $LCM(a,b) = \frac{a \times b}{GCD(a,b)}$ ,  $i$  and  $j$  iterating from  $1 \rightarrow N-1$ , therefore  $\mathbf{M}$  would be of size  $(N-1, N-1)$ .

The greatest common divisor (GCD) can be computed using several methods, i.e. the Euclidean algorithm that considers a  $k^{th}$  step that performs a division with remainder to find the quotient  $q_k$  and remainder  $r_k$  so that:  $r_{k-2} = q_k r_{k-1} + r_k$  with  $r_{k-1} > r_k \geq 0$ . In the initial step  $k=0$ , the remainders are set to  $r_{-2} = a$  and  $r_{-1} = b$ , thus the algorithm must eventually produce a zero remainder  $r_N = 0$ . Therefore, the final nonzero remainder is the greatest common divisor of  $a$  and  $b$ :  $GCD(a,b) = r_{N-1}$ .

Once the MCO is calculated, it is used to find the SAs with respect to each found order and subtract it from the total sum of SA of all considered shafts. This ensures that the final result contains a single interference between each shaft, without modifying the structure of the total set. In conclusion, the formula (3) becomes:

$$s(\theta) = \sum_{\alpha_i \in A} m_{\alpha_i}(\theta) - \sum_i \sum_j m_{\mathbf{M}(i,j)}(\theta) \quad (5)$$

with  $m_{\mathbf{M}(i,j)}(\theta)$  being the SA of the angular signal  $x(\theta)$  with respect to the order found at element  $\mathbf{M}(i,j)$ .

### 3.2. Exclusive contributions extraction

Furthermore, to extract the exclusive contributions of each shaft from the isolated subsystem, the same set theory and the matrix MCO are used to find the following:

$$s(\theta) = \sum_{\alpha_i \in A} (c_{\alpha_i}(\theta) + \varepsilon_{\alpha_i}(\theta)) \quad (6)$$

with  $c_{\alpha_i}(\theta)$  being the exclusive contribution of shaft of order  $\alpha_i$ . The remaining interference  $\varepsilon_{\alpha_i}(\theta)$  represents all common contributions between the shaft of order  $\alpha_i$  and all remaining ones as seen in the following:

$$\varepsilon_{\alpha_i}(\theta) = \sum_j s_{\mathbf{M}(i,j)}(\theta) \quad (7)$$

with  $s_{\mathbf{M}(i,j)}(\theta)$  being the SA of the subsystem deterministic signal  $s(\theta)$  with respect to the order found at element  $\mathbf{M}(i,j)$  and  $i$  being the index of  $\alpha_i$  in the set  $A$ . One can identify the following:

$$c_{\alpha_i}(\theta) = s_{\alpha_i}(\theta) - \varepsilon_{\alpha_i}(\theta) \quad (8)$$

with  $s_{\alpha_i}(\theta)$  being the SA of the subsystem deterministic signal  $s(\theta)$  with respect to order  $\alpha_i$ .

In conclusion, each shaft's exclusive contribution is extracted as follows:

$$c_{\alpha_i}(\theta) = s_{\alpha_i}(\theta) - \sum_j s_{\mathbf{M}(i,j)}(\theta) \quad (9)$$

In summary, the proposed methodology in this paper uses the set theory and the calculation of MCO to isolate each subsystem within a complex mechanical system. By eliminating the redundant frequencies present in the synchronous averages of the shafts' orders, one can finally obtain the exclusive contribution of each component.

## 4. Application

### 4.1. Data description

In this section, an experimental example is considered to understand how to apply such method on aircraft subsystems. It concerns a vibration signal acquired during a ground test campaign on the CFM56 turbojet engine (Figure 4), which is a civil aircraft engine with two main shafts low and high pressure, and an accessory gearbox linked to the high-pressure shaft. The acquired signal is obtained from the accelerometer called Acc2 with a sampling frequency of 50,000 Hertz and a time duration of 200 seconds. The acquisition was performed during a steady state at idle followed by a slow acceleration up to full power. For this application, the desired subsystem to isolate is the accessory gearbox.

The kinematics of the gearbox and the rotating speeds of its shafts are described in Figure 5. The corresponding signal provided by the Acc2 accelerometer (located near the shafts L4 and L5 of the AGB) and the rotating frequency of shaft L1 is depicted on Figure 6. This signal has been initially provided for a contest during the International Conference Surveillance 8, October 20 and 21, 2015, at the Roanne Institute of technology, France. More details about the signal can be found in [22].

Accordingly, the orders are structured in a well-designed table to clearly distinguish the order of each shaft. A table was provided during the contest for the same specific reason by listing all characteristic frequencies of the gearbox components. In this embodiment, 12 frequencies are given and associated with the rotation of the 12 shaft lines of the CFM56 AGB. Table 1 summarizes the dynamics of the AGB with the 12 frequencies of the set  $f_{reference}$  of shaft frequencies referenced by  $N_2$ . It is specified that these frequencies are obtained with a rotational speed of shaft  $N_2$  of 10,000 rpm. The orders considered in this kinematic correspond to the Tooth line  $F_1(N_2)$ . Therefore, the goal is to find all the interferences between the HP shaft and all the others, then RDS and all the others, etc. In addition, it is necessary to analyze the different sources that interfere jointly (an interference might exist for several sources even if they are not physically linked).

Consequently, the next step would be to calculate the MCO with the corresponding orders. Figure 7 shows the resulting matrix after including all orders provided by the Table 1. It is essential to note that a non-integral number is mostly anticipated for each order (aside the reference one). However, the synchronous average takes an integer number of samples as the period to average the signal. As a result, it is possible to find a common order between each one. It can be identified in the matrix above that the diagonal presents the meshes of the gears located on the consecutive shafts or a divisor of this mesh in case a LCM smaller than the mesh was found.

Afterwards, using the given instantaneous angular speed, the signal undergoes resampling into its angular domain having the corresponding trajectory with its order spectrum in Figure 8a. The first phase of the proposed method extracts the contribution of the AGB by utilizing MCO to eliminate the replication of interferences among the subsystem lines. As a result, Figures 8b and 8c depict the signals of the AGB (deterministic component) and the residual, along with each corresponding order spectra. This deterministic signal corresponds to the periodic part of all the shafts of the gearbox including the common parts (the interferences) between the different lines without any replication.

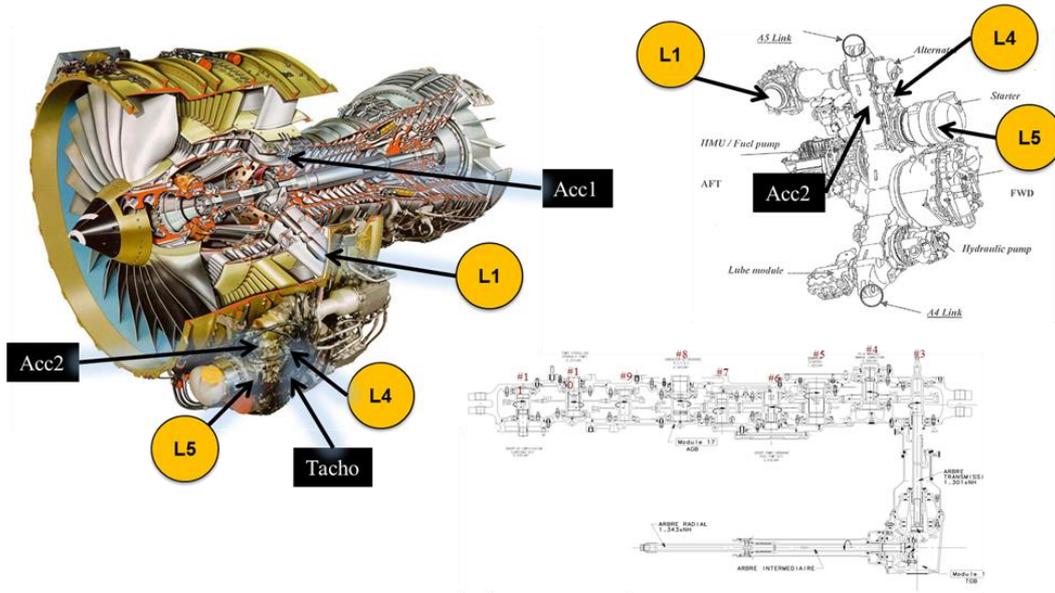


Figure 4. General overview of the engine and the accessory gearbox.

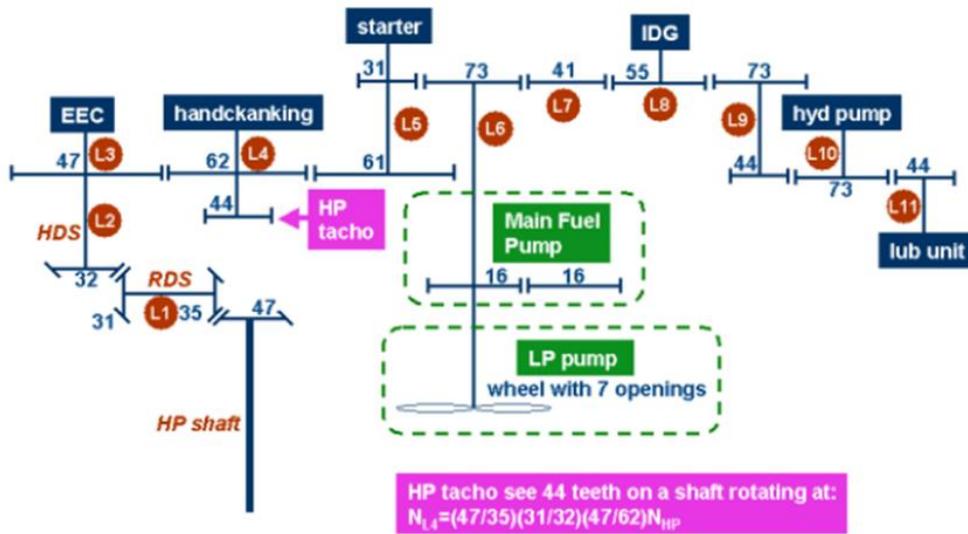


Figure 5. Kinematics of the gearbox.

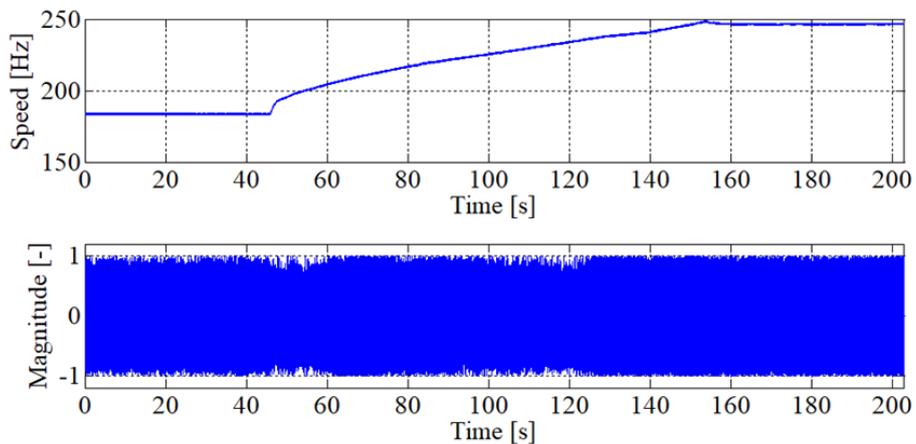


Figure 6. (Top) Evolution of the gas generator shaft speed N1 in Hertz and (bottom) the raw vibration signal measured on the gas generator casing.

**Table 1.** Characteristic frequencies of the gearbox.

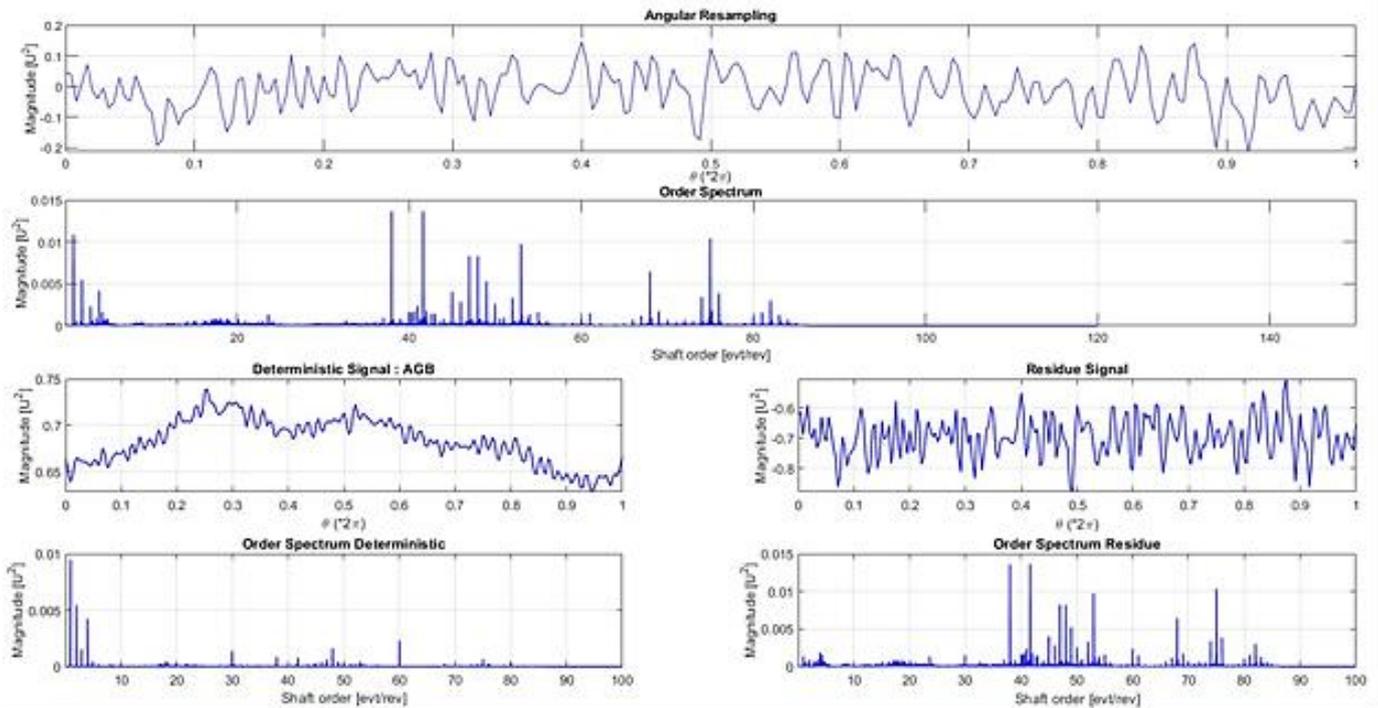
N2 speed (rpm)	N2																					
	Line 1		Line 2		Line 3		Line 4		Line 5		Line 6		Line 7		Line 8		Line 9		Line 10		Line 11	
10000	HP Shaft	RDS	Horizontal Drive Shaft		EEC Alternator		Handcranking		Starter		Fuel Pump		Intermediate		IDG		Intermediate		Hydro Pump		Lubrication Unit	
	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont
Number of teeth	47	35	31	32	47	47	47	62	62	61	31	73	73	41	41	55	55	73	44	73	73	44
Rotating Speed (rpm)	10000	13429	13009	13009	13009	13009	13009	9862	9862	10023	10023	4256	4256	7579	7579	5649	5649	4256	4256	2566	2566	4256
F1 tooth (Hz)	167	224	224	217	217	217	217	164	164	167	167	71	71	126	126	94	94	71	71	43	43	71
F1 tooth (N2)	1.0000	1.3429	1.3429	1.3009	1.3009	1.3009	1.3009	0.9862	0.9862	1.0023	1.0023	0.4256	0.4256	0.7579	0.7579	0.5649	0.5649	0.4256	0.4256	0.2566	0.2566	0.4256
Gear Mesh Frequency (Hz)	7833	6938	6938	10190	10190	10190	10190	10190	10190	5179	5179	5179	5179	5179	5179	5179	5179	3121	3121	3121	3121	3121
Gear Mesh (mult N2)	47.0000	41.6286	41.6286	61.1420	61.1420	61.1420	61.1420	61.1420	61.1420	31.0721	31.0721	31.0721	31.0721	31.0721	31.0721	31.0721	31.0721	18.7284	18.7284	18.7284	18.7284	18.7284
F coincidence (Hz)	4.8	7.0	7.0	216.8	216.8	216.8	216.8	3.5	3.5	2.7	2.7	2.3	2.3	1.7	1.7	2.3	2.3	1.3	1.3	1.0	1.0	1.0

$\alpha_i \backslash \alpha_{j+1}$	1.3429	1.3009	1.3009	0.9862	1.0023	0.4256	0.7579	0.5649	0.4256	0.2566	0.4256
1.0000	47.0000	147.0000	147.0000	285.0000	1.0000	63.0000	72.0000	274.0000	63.0000	137.0000	63.0000
1.3429	0	41.6286	41.6286	63.1143	209.4857	77.8857	76.5429	256.4857	77.8857	22.8286	77.8857
1.3009	0	0	1.3009	61.1420	61.1420	300.5062	165.2134	98.8679	300.5062	92.3634	300.5062
1.3009	0	0	0	61.1420	61.1420	300.5062	165.2134	98.8679	300.5062	92.3634	300.5062
0.9862	0	0	0	0	61.1420	80.8652	553.2362	54.2388	80.8652	233.7201	80.8652
1.0023	0	0	0	0	0	31.0721	31.0721	31.0721	31.0721	11.0256	31.0721
0.4256	0	0	0	0	0	0	31.0721	31.0721	0.4256	18.7284	0.4256
0.7579	0	0	0	0	0	0	0	31.0721	31.0721	65.9336	31.0721
0.5649	0	0	0	0	0	0	0	0	31.0721	109.0350	31.0721
0.4256	0	0	0	0	0	0	0	0	0	18.7284	0.4256
0.2566	0	0	0	0	0	0	0	0	0	0	18.7284

**Figure 7.** resulting MCO values (in black) with respect to each order (in blue).

Hence obtaining the contribution of each shaft without any remaining interference using the second phase of the proposed approach with the same MCO computed before. Regarding the

12 lines constituting the AGB, the different SAs are visualized in Figure 9 and their order spectra in Figure 10.



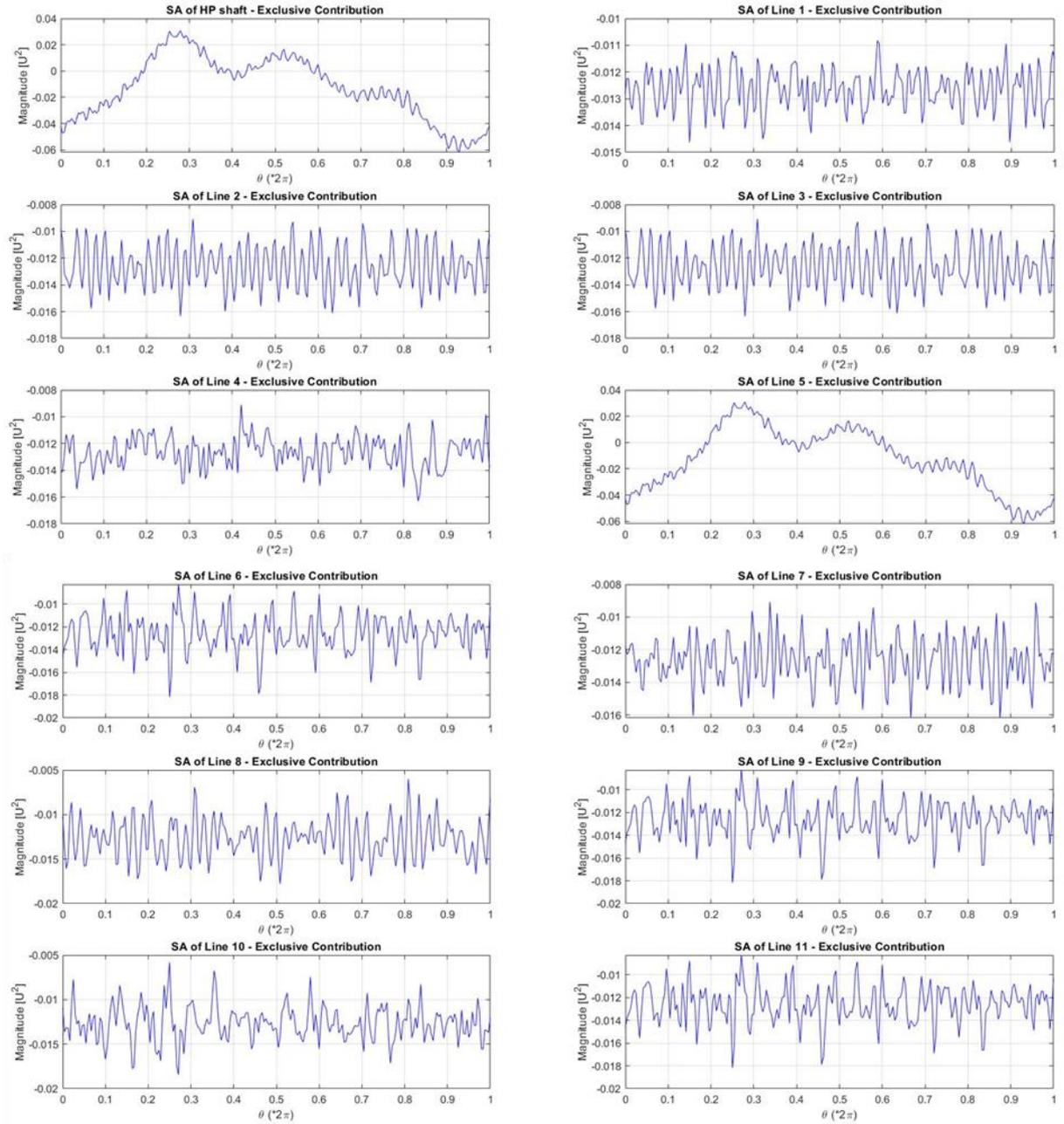
**Figure 8.**

- a - (Top Center) CFM56 angular signal and its order spectra
- b - (Bottom Left) Exclusive CFM56 AGB deterministic Signal and its order spectra
- c - (Bottom Right) CFM56 signal without AGB and its order spectra

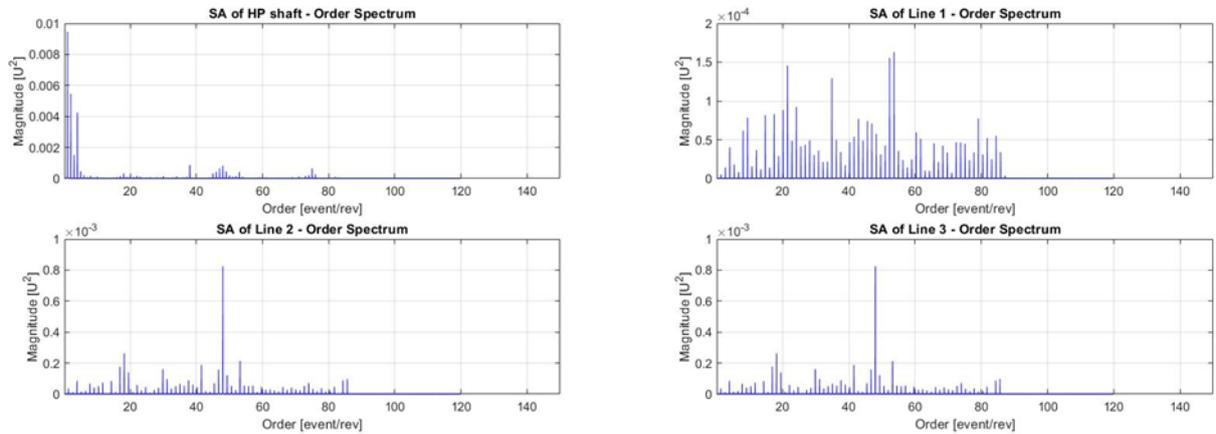
**4.2. Results and discussion**

The proposed methodology successfully extracts the AGB's deterministic signal and its constituent lines while eliminating the interferences and its replication for subsystem contribution. However, as it is seen in the MCO (Figure 7) and in the extracted

lines with their spectra (Figures 9-10), it is possible for some shafts to share the same order. To overcome this issue, multiple input sensors can be employed to identify the exclusive contribution of each shaft while using the same proposed methodology.



**Figure 9.** The signals of the 12 sources extracted from the AGB of the CFM56.



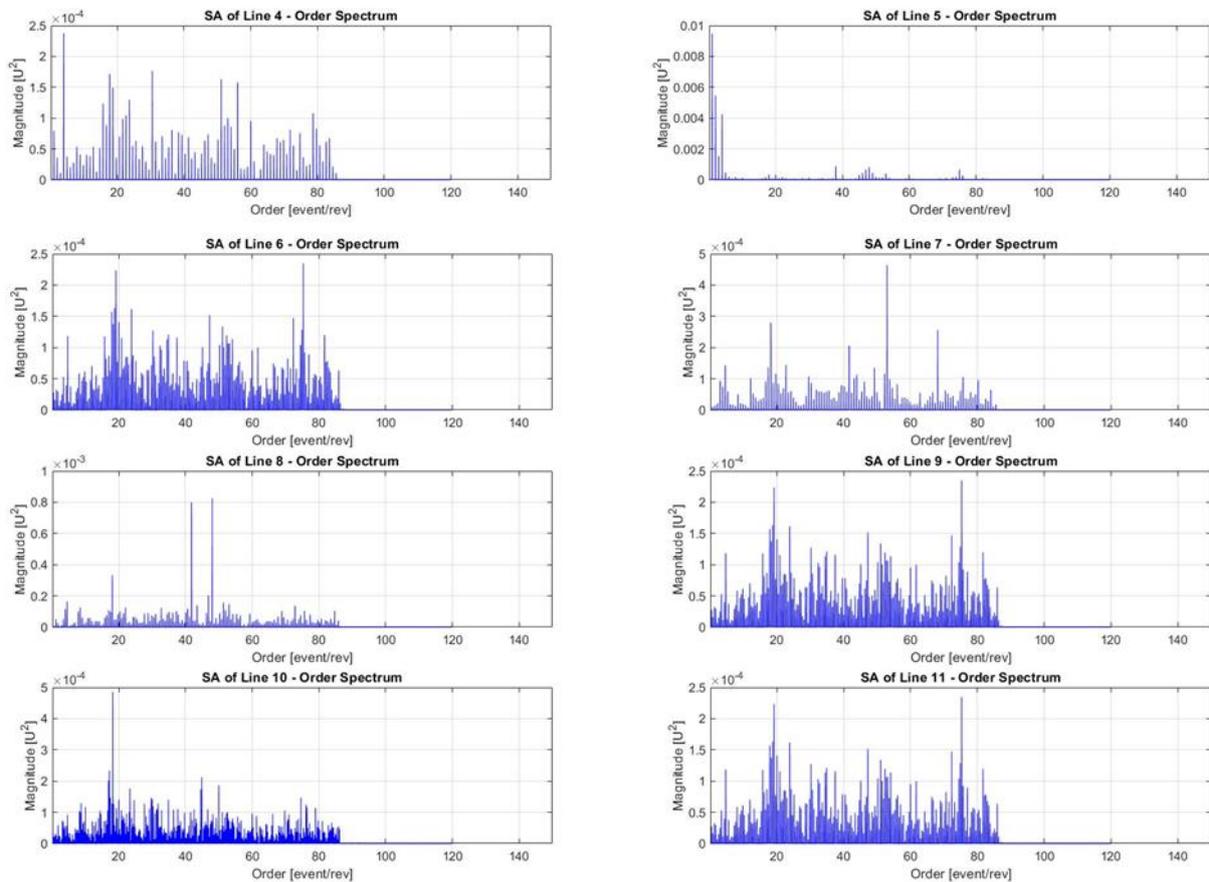


Figure 10. The order spectra of the 12 lines.

## 5. Conclusion

This paper presents a new approach for isolating the subsystems in a complex mechanical system, specifically an aircraft accessory gearbox. The method leverages synchronous averaging, set theory, and the matrix of common orders to eliminate redundant frequencies and uncover the unique contributions of each subsystem. This allows for a cleaner analysis for having each contribution well isolated from interferences, as well as the ability to pinpoint specific issues within a subsystem. Overall, the proposed method can improve the maintenance and global performance of mechanical systems while providing a good understanding of the behavior of complex mechanical systems.

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