

# What Model Is the “Ising Model?”

*When A Model Fulfills  
Different Epistemic Functions  
Across History*

Quentin Rodriguez  
& Vincent Ardourel

# Plan

I/ Introduction

II/ A foretaste of the Ising model

III/ The Ising model as an “approximate model”

IV/ The Ising model as a “toy model”

V/ The Ising model as a “minimal model”

# ▶ I / Introduction

# The “physicists’ drosophila” (1924- ?)

- ▶ (One of?) the most studied model of contemporary physics
- ▶ ~ 1’000 articles per year, vs. 50’000 to 100’000 for “*physics*”
- ▶ 100<sup>th</sup> anniversary!
- ▶ Not only in physics, but also in economics, biology, social studies...
  
- ▶ Much discussed about *emergence/reduction* and *mathematical or causal explanation debate* (Batterman, Morrison...), and *model transfers* (Knuuttila & Loettgers)

# An exemplary case study for the “Model as Mediators” approach

- ▶ Scientific practice --> mediators between theories, experience, data... for a specific epistemic purpose
- ▶ Hughes in Morgan & Morrison 1999
- ▶ But... which epistemic purpose does the Ising model serve?

# Disagreement among philosophers

- ▶ A “toy model?”
  - ▶ “Paradigmatic examples of toy models include the Ising model in physics” (Reutlinger, Hangleiter & Hartmann 2018)
  - ▶ “examples of models that are frequently classified as toy models are the Ising model in statistical mechanics” (Frigg 2022)
  - ▶ “toy models do not offer accurate representations, do not explain, and do not provide predictions; toy models serve another goal, namely providing understanding” (*id.*)

# Disagreement among philosophers

- ▶ If the Ising model possess a representational function, what is its target?
- ▶ “In some discussions [...] the Ising model comes to represent a large, and very disparate class of physical systems almost in the manner that a congressman represents a district, or member of parliament a riding. It represents a class of systems by being a representative of them.” (Hughes 1999)

# Disagreement among philosophers

- ▶ A “minimal” (Weisberg) or “minimalist model?”
  - ▶ “a representation that contains only the core causal factors that give rise to a phenomenon of interest. (...) Despite its simplicity, the Ising model explains phase changes and other critical phenomena by capturing the interactions that give rise to them.” (Elliott-Graves & Weisberg 2014)
- ▶ A “minimal model” that explains, but not by representing, esp. causal factors:
  - ▶ “a story that is focused on demonstrating why details do not matter. [...] [T]his story does not require minimally accurate mirroring of model and target system.” (Batterman & Rice 2014)



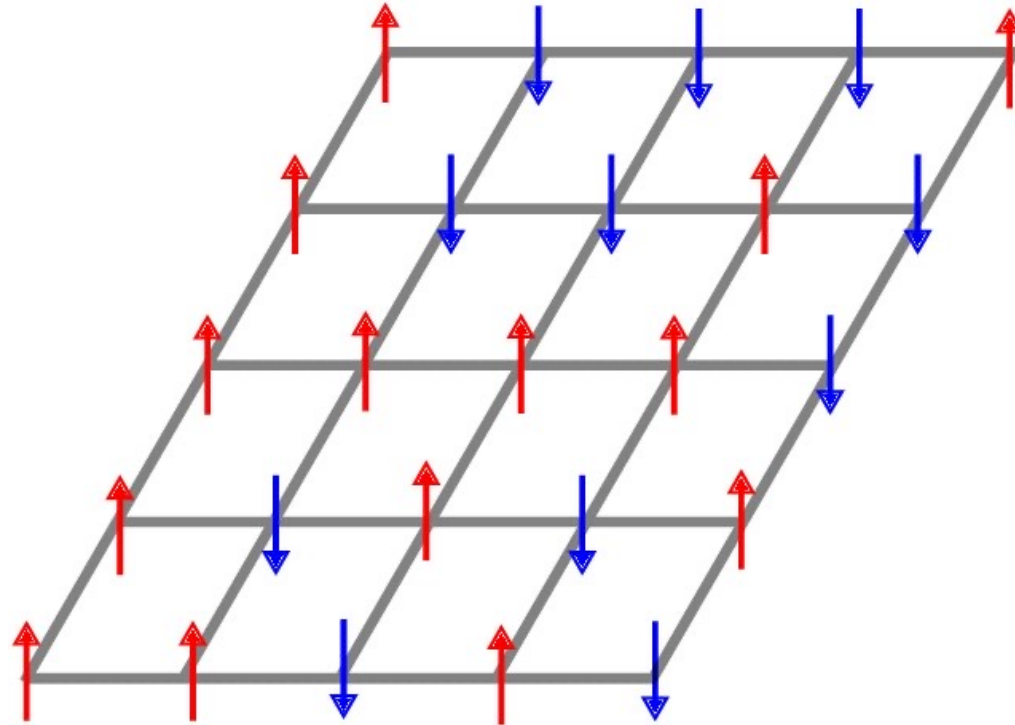
# Epistemic functions and context of use

- ▶ Our main claim:
  - ▶ The Ising model fulfilled three very distinct – sometimes contradictory – epistemic functions across its history, allowing indeed to label it under three different types of model.
- ▶ Build on Niss's historical work (2005, 2009, 2011)
- ▶ Three main views on the Ising model:
  1. An “approximate model” (1920's)
  2. A toy model (1930's–1960's)
  3. A minimal model (1970's)

# II / A Foretaste of ▶ the Ising Model

# A Model for Phase Transitions

- ▶ W. Lenz (1920) & E. Ising (1924)
- ▶ Ferromagnetism/paramagnetism
- ▶ Arrows that can flip “up” or “down” on a lattice → *Ising spins*
- ▶ First-neighbor only, interaction  $J$
- ▶  $H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j$ 
  - ▶  $\sigma_i = \pm 1$



# Statistical Mechanics

- ▶ Add temperature (chance):  
probability of a configuration given by the Boltzmann factor  $e^{-H/(kT)}$
- ▶ Trade-off between potential energy ( $H$ ) and thermal energy ( $kT$ )
  - ▶ at (sufficiently) low  $T$  : arrows align to decrease  $H$
  - ▶ at (sufficiently) high  $T$  : arrow directions make no difference

# A Caricature of Phase Transitions

- ▶ Physical question: how does the model go from one regime to another?
- ▶ Philosophical question: how what one can learn from this model relates to physical phenomena in the world?

# III / The Ising Model as an “Approximate ▶ Model”

# Atomistic Model of Magnetism

- ▶ 1920: no spin, no quantum mechanics. Kinetic theory of gases, Bohr atom model.
- ▶ Curie law:  $\chi_T \propto T^{-1}$
- ▶ Pierre Weiss (1907): iron is made of an assembly of “elementary magnets”
- ▶ Is the “turn over” of the arrows **realistic?**
- ▶ “Since free rotatability of the elementary magnets therefore has to be refuted, it can be concluded from the above that they have the ability to turn over. I want to show that this assumption is sufficient to explain the Curie law.” (Lenz 1920)  
--> *atomic crystal geometry*

# Atomistic Model of Magnetism

- ▶ To account for the transition to ferromagnetism, Weiss added a uniform “molecular field”, in which the elementary magnets are bathed -> mean magnetization.
- ▶ For Lenz, this hypothesis “offers only a purely phenomenological hint” into ferromagnetism (Lenz 1920)

--> replaced by the first-neighbor interaction by Ising:  
“we assume that they [the interactions] decay rapidly with distance, so that we, in general, to a **first approximation**, only have to take the influence on neighboring elements into account” (Ising 1924)

- ▶ “They perceived the model as quite realistic” (Niss 2005)



# Approximate Model

- ▶ Qualifies as a very common type of model, from which one can draw some knowledge about its target, to the extent that it is a **more or less close** realistic description of the target. The closer, the better the model is!
- ▶ “Modèle approché” --> versus Batterman’s “minimal model”
- ▶ Of course, the Ising model is highly simplified... “to a first approximation”

# IV/ The Ising

- ▶ Model as a “Toy Model”

# And the Quantum Mechanics Came

- ▶ 1925: quantum mechanics, electronic spin.
- ▶ 1928: Heisenberg's theory of ferromagnetism --> Heisenberg's model
- ▶ The basic mechanisms of the model are completely **discarded**:
  - ▶ **no magnetic interaction**, but “exchange interaction”, i.e. *electrostatic* interaction + Pauli exclusion principle
  - ▶ **not between atomic** magnetic moments, but between *electronic* spins
  - ▶ why should one limit electronic spins to up & down flips?

# A model of... what?

- ▶ “it clearly was not the right model, because the spin is a quantum object and not a classical object, so you couldn’t just say up spin and down spin, but have to permit them to change direction.” (Bethe, itw)
- ▶ “The Lenz-Ising model is therefore now only of mathematical interest.” (Peierls 1936)
- ▶ “a kind of interesting mathematical game, not to be taken seriously.” (Yang, itw)

# Does a Toy Model represent any target?

- ▶ Toy model:
  - ▶ not “approximate”
  - ▶ doesn’t support belief about real phenomena
  - ▶ intellectual handling ability
- ▶ A generic model of “cooperative phenomena,” or a purely mathematical tool?

# Mathematical results

- ▶ Onsager proof, 1944:
  1. In the thermodynamic limit, statistical mechanics *can* predict genuinely discontinuous phase transitions (singularity)
  2. Simple mathematical models *may* describe such a behavior in an exact way
- ▶ “The Ising model is a **crude attempt** to simulate the structure of a physical ferromagnetic substance. Its **main virtue** lies in the fact that a two-dimensional Ising model yields to an exact treatment in statistical mechanics. It is **the only nontrivial example** of a phase transition that can be worked out with mathematical rigor.” (Huang 1987)

# Back to an approximate model

- ▶ Pauli's interpretation of the arrows as “(classical) anisotropic, electronic spins”

Heisenberg  
quantum-mechanical  
hamiltonian



*uniaxial  
anisotropy*

Ising  
classical-mechanical  
hamiltonian

# V/ The Ising

- ▶ Model as a  
“Minimal Model”



# A Model for Critical Phenomena

- ▶ 1970's: theory of a class of phase transitions (“continuous” or “second order phase transitions”), depending on their “order parameter”
- ▶ Results: renormalization group (RG), universality classes, critical exponents, scaling laws... (Batterman, Morrison)

# Minimal Models

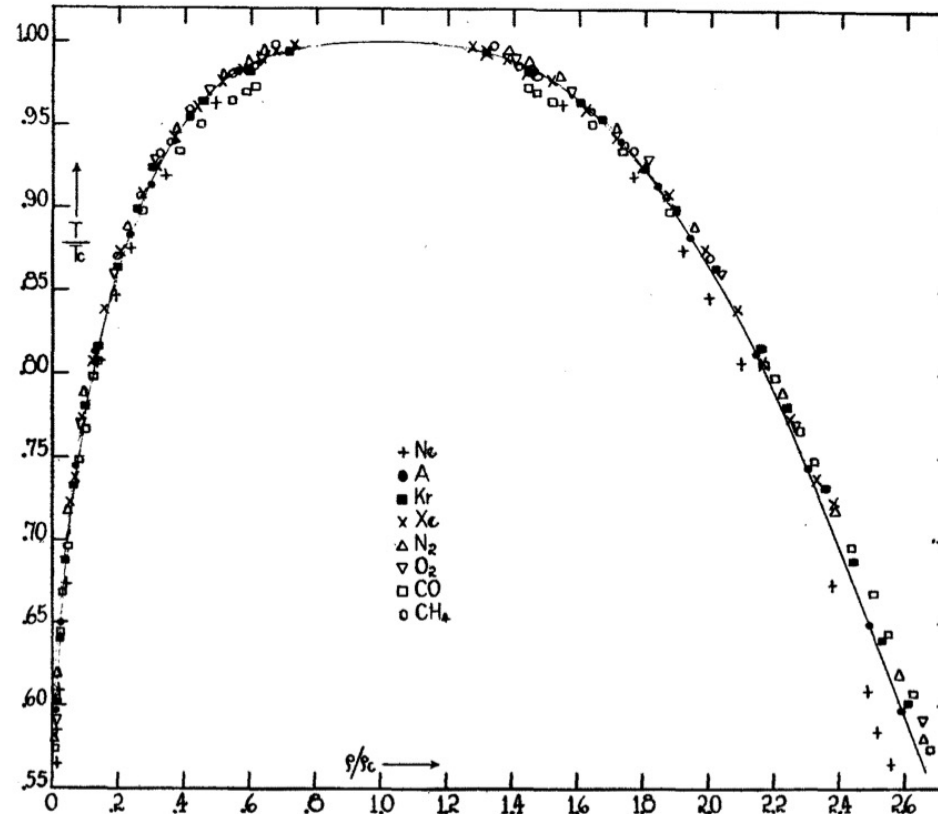
- ▶ Surprisingly, the Ising model became *explanatory* and even *predictive* for real phenomena

- ▶ Not iron, but for fluids, for instance!

- ▶  $\rho - \rho_c \sim \left| \frac{T - T_c}{T_c} \right|^\beta$  when  $T \rightarrow T_c$ ,  
with  $\beta \approx 0.325$

- ▶ The general idea:

*You don't need a model M of a phenomenon P to be realistic regarding P to explain something about P*



# What is essential?

- ▶ Physical insight:
  - ▶ Critical phenomena display *universal* properties, in the sense that these properties will not depend upon the microscopic details
  - ▶ RG embodies this idea into a mathematical framework that systematically wipe out the irrelevant details
  - ▶ If the only remaining relevant parameters are the same in the (simplistic) model and the real phenomenon, then they will exhibit the same properties, whatever the elements they are made of.
- ▶ “There are two **diametrically opposing views** about the way models are used. The “**traditional**” viewpoint has been to construct a faithful representation of the physical system (...). In this methodology, when theory is unable to explain the results of an experiment, the response is to fine-tune the parameters of the model, or to add new parameters if necessary. (...) **On the other hand**, such fine detail may actually not be needed to describe the particular phenomenon in which one is interested. Many of the parameters may be irrelevant (...). In such a case, it is only important to start with the correct **minimal model**, i.e. that model which most economically caricatures the essential physics.” (Goldenfeld 1992)

# Different definitions

- ▶ “which we will call a minimal model, is a representation that contains **only the core causal factors** that give rise to a phenomenon of interest. For example, consider the Ising model. (...) This model is extremely simple, building in almost no realistic detail about the substances being modeled. Despite its simplicity, the Ising model explains phase changes and other critical phenomena by **capturing the interactions that give rise to them.**” (Elliott-Graves & Weisberg 2014)
- ▶ Are minimal models just “(crudely) approximate models,” or does they explain without representing? (Batterman, Batterman & Rice)

# Thanks for your attention

Quentin Rodriguez, *UR Philosophies et Rationalités, Université Clermont Auvergne*

Vincent Ardourel, *IHPST, CNRS – Université Paris-I Panthéon-Sorbonne*

Funded by ASYMPTOPHYS project (ANR-22-CE54-0002).

► For details:

V. Ardourel & Q. Rodriguez, “Des usages pluriels d’un modèle en physique : Le cas du modèle d’Ising,” *Cahiers philosophiques* 176, 2024. HAL preprint: hal-04349962.

► [vincent.ardourel@univ-paris1.fr](mailto:vincent.ardourel@univ-paris1.fr) / [rodriguez.quentin@gmail.com](mailto:rodriguez.quentin@gmail.com)