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1 Optimized Stokes imaging for highly resolved 2 optical speckle fields, part III: Topological 3 analysis of polarimetric states distributions with 4 optimized data representations

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9 **Abstract:** In this last article of a three-paper series focusing on Stokes polarimetry of optical
10 speckle fields resolved at the individual speckle grain scale, experimental results are provided on
11 test samples of varying nature and polarization properties, and are analyzed extensively. For
12 this purpose, a review of the classical ways of displaying Stokes polarimetric information is
13 provided. Then, some original alternative graphical representations are introduced that ensure
14 optimal readability and interpretability of the Stokes imaging data in the context of speckle field
15 polarimetry, and it is shown how they can be adapted to various observation scales. Finally, these
16 tools are implemented in order to provide a topological analysis of the distribution of the states of
17 polarization across a speckle pattern, and in the vicinity of polarimetric singularities of the field.

18 1. Introduction

19 This article is the third of a paper series titled “Optimized Stokes imaging for highly resolved
20 optical speckle fields”. After having detailed an optimized experimental Stokes polarimetric
21 imaging setup resolved at the speckle grain scale in the first article of the series [1], we have
22 analyzed in the second article [2] optimum acquisition and processing strategies which can be
23 implemented to estimate the polarimetric state with the best accuracy, precision, and robustness
24 to experimental imperfections. In this last paper, we present a set of experimental results that have
25 been acquired with this setup and these optimized acquisition/processing modalities. In addition,
26 we propose original graphical representations of the polarimetric information measured, and we
27 discuss their relevance according to the various scales of observation of the speckle patterns that
28 we analyze. This discussion about the choice of a relevant representation as a function of the
29 scale of analysis is finally illustrated through several analyses of the “topological” behaviour of
30 the state of polarization (SOP) across a speckle field, at the frontier of two speckle grains, or in
31 the vicinity of polarization singularities of the speckle field.

32 This paper is organized as follows: in Section 2, we report the experiments conducted and
33 results gathered that will be the basis of discussion of all the remainder of the article, and we
34 provide a first global polarimetric analysis of these results. Then, we propose and compare in
35 Section 3 various modalities of graphical representation of polarimetric information. We discuss
36 their respective relevance for better visual understanding of the physical situation at hand, since
37 producing a legible two-dimensional graphic representation that displays as well as possible such
38 a 6-dimensional polarimetric information is not trivial. In Section 4, we provide a thorough
39 analysis of the “topological” behaviour of the distribution of the SoP across a speckle pattern,
40 at various spatial scales (population of grains, neighbouring grains, single grain), using the
41 graphical representations proposed in Section 3 that can be refined and adapted depending on
42 the scale at which the speckle pattern is observed. A similar topological analysis is provided
43 in Section 5 at the vicinity of polarization singularities in a speckle pattern. Finally, a global
44 conclusion and some perspectives of this article series are given in Section 6.

45 **2. Overview of experiments and global polarimetric analysis**

46 This article relies on experimental Stokes imaging acquisitions that have been performed with
 47 the imaging setup detailed in the first article of this series [1]. In this first article, we also
 48 reported the principle of Stokes polarimetric imaging, as well as the exact final experimental
 49 configuration retained after optimization of the acquisition setup. We therefore refer the reader
 50 to this first article in order to get more details about these points, and to fix the notations used
 51 in the present article. Moreover, the data gathered and discussed here have been obtained and
 52 processed according to the optimal acquisition/processing strategy that has been theoretically
 53 studied and characterized in the second article of this series [2], namely, the optimized so-called
 54 SOPAFP approach detailed and advocated in [2]. For data reproducibility, we invite the reader to
 55 refer to these two papers in order to get full details about the acquisition and processing pipelines
 56 that have been used.

57 The experimental results reported and analyzed here were acquired from the polarimetric
 58 analysis of speckle pattern obtained upon interaction of a highly coherent single-mode green
 59 laser with four samples of varying polarimetric nature, ranging from non-depolarizing to highly
 60 depolarizing (see also [1] for more details), namely:

- 61 • a metal plate (brushed aluminum), whose polarimetric behaviour is (quasi perfectly)
 62 non-depolarizing [1, 3, 4];
- 63 • a block of marble painted in green whose partially depolarizing nature is due to a volume
 64 scattering regime of the incident green light source (monochromatic at $\lambda = 532 \text{ nm}$) [3].
- 65 • a Spectralon sample exhibiting a highly depolarizing polarimetric behaviour [5–8], coupled
 66 with high reflectance ($\approx 99\%$), and whose strongly Lambertian behaviour is generally
 67 used for the calibration of optical instruments [9, 10], particularly in Spectralon-coated
 68 integrating spheres.
- 69 • a piece of tissue phantom (Polydimethylsiloxane (PDMS) polymer with scattering
 70 nanoparticles of titanium dioxide (TiO_2) and absorbing India ink) whose optical behaviour
 71 approximates the optical properties of biological tissue [11, 12].

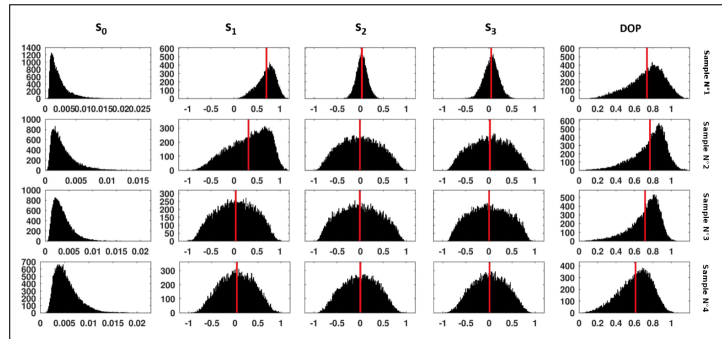


Fig. 1. Histogram of the various Stokes parameters and the degree of polarization for the following samples: metal plate ($n^\circ 1$), brick covered with green paint ($n^\circ 2$), Spectralon ($n^\circ 3$) and PDMS phantom ($n^\circ 4$).

72 In order to check the polarimetric nature of each sample used in this experimental work, a
 73 statistical analysis of the polarization properties of the speckle fields produced on each of these
 74 samples is first provided below, at a macroscopic scale, over the whole resolved speckle field.
 75 For that purpose, the histogram of the different Stokes parameters for a region of interest (ROI)

76 of 150×150 superpixels (each superpixel resulting from a 4×4 binning of the pixels of the
77 detector [1]) for each sample is shown in Figure 1. We recall here that the illumination source
78 is vertically polarized, and thus with conventions of [1, 2] the theoretical incident polarization
79 state corresponds to a Stokes vector $\mathbf{S}^{in} = S_{in} [1 \ 1 \ 0 \ 0]^T$ with S_{in} the input illumination
80 intensity.

81 As expected, it can be seen in Fig. 1 that polarization state of the speckle pattern obtained with
82 the metallic sample (sample n° 1) basically retains the SOP of the incident laser source, as shown
83 by the mean values of the reduced Stokes parameters s_1 , s_2 and s_3 , which are respectively equal
84 to 0.704 ± 0.002 , 0.035 ± 0.002 and 0.054 ± 0.002 , while retaining a high degree of polarization
85 (DOP) when evaluated locally across the speckle field, with an average spatial DOP, denoted
86 \overline{DOP} , of 0.736 ± 0.002 (The given precision correspond to the standard deviation of estimation
87 over sets of 150×150 superpixels). In this article, the reduced Stokes parameters s_i ($i = 1, \dots, 3$)
88 refer to the elements of the Stokes vector normalized by the first Stokes parameter S_0 which
89 represents the total intensity of the field at the observed location (see [1] for more details on
90 notations). The mean values of the s_i and of the DOP indicated above are displayed in the
91 histograms of Fig. 1 with a solid vertical red line.

92 As noted in previous similar works [3, 13, 14], the SOP of light across a speckle field (when
93 a static sample is illuminated with coherent polarized light) is well defined at each location of
94 the field, i.e., the DOP should be equal to one at each location of the speckle pattern. It can
95 be observed in the top right histogram of Fig. 1 that the distribution of the DOP values differs
96 from the theoretical expected value of 1, as the statistical (spatial) average comprises also the
97 “dark” pixels (corresponding to destructive interferences in the speckle field, i.e., pixels with
98 low values of S_0) for which the estimated SOP cannot be trusted. When the speckle pattern is
99 preliminary thresholded in order to retain only the pixels that have a significantly high level of S_0 ,
100 the averaged DOP value then tends to one on such a metallic non-depolarizing sample [3].

101 For the marble brick covered with green paint (sample n° 2), the standard deviation of the
102 parameters s_1 , s_2 and s_3 are wider than for the previous sample and the respective mean of each
103 parameter is 0.315 ± 0.003 , -0.006 ± 0.003 and 0.025 ± 0.003 . The decrease of the mean value
104 of s_1 , coupled with an increase in the standard deviation of the s_i parameters corresponds to a
105 “dispersion” of the incident polarimetric state, suggesting that a green diffusive sample acts as a
106 depolarizing material where volume scattering occurs when illuminated with green laser light.
107 Meanwhile, it can be observed by looking at the histogram of the DOP that these experiments
108 confirm again the fact that the SOP is pretty well defined at each location of the field, with a local
109 DOP value remaining close to one, as the average DOP value is 0.767 ± 0.002 .

110 Finally, the Spectralon and tissue phantom samples (samples 3 and 4) share a similar
111 polarimetric behaviour, with the histograms of the three reduced Stokes parameters s_1 , s_2 and
112 s_3 showing similar distributions, corresponding to a modification of the incident polarization
113 state resulting in an almost complete loss of the incident polarimetric state used to illuminate the
114 sample. The analysis of the histograms of the DOP for these two samples reveals that on a static
115 depolarizing sample such as Spectralon, the local DOP values remain close to one (0.715 ± 0.002),
116 whereas the distribution tends to show slightly lower values on the PDMS sample 0.612 ± 0.002 .
117 We hypothetically attribute such observation to a stronger influence of mechanical and thermal
118 drifts during the acquisition of the intensity images as the PDMS phantom is more deformable
119 than the Spectralon, even though it was thermalized for about twenty minutes before the images
120 were acquired. As a consequence, and since they share comparable behaviour, we rather focus in
121 the remainder of this study on the spectralon sample.

122 To complete this macroscopic analysis, it is interesting to analyze jointly the histograms of the
123 local DOP (as displayed in the right column of Fig. 1) and their respective mean values \overline{DOP}
124 (red vertical lines), along with the DOP of the average SOP of the light analyzed in the ROI,
125 i.e., the DOP that would have been obtained in a standard polarimetric imaging situation if the

126 speckle grains of the ROI had been averaged on a single detector (single pixel). In that case, the
127 DOP of the average resulting SOP is denoted \overline{DOP} and computed as $\overline{DOP} = (\overline{s_1^2} + \overline{s_2^2} + \overline{s_3^2})^{1/2}$,
128 where $\overline{s_i}$ indicates the spatially averaged mean value of the i^{th} reduced Stokes component. From
129 the experimental results obtained, we get a value of \overline{DOP} equal to 0.707 ± 0.002 for the metallic
130 sample, showing its imperfect non-depolarizing nature even when experimental imperfections
131 are accounted for and compensated (see part 1 of this article series [1]). As for the other samples,
132 the value of \overline{DOP} is 0.316 ± 0.003 for the green-painted marble and 0.041 ± 0.003 for the
133 Spectralon and 0.046 ± 0.003 for the phantom sample, which is in clear agreement with the
134 expected respectively non-depolarizing/depolarizing nature of the samples studied, and with
135 the results of previous works on similar samples [1, 3, 5]. This again confirms the fact that the
136 depolarization in such samples and with such active coherent polarimetric imaging experiments
137 is due to a spatial average of well-defined SOPs but which are more or less scattered out across
138 the Poincaré's sphere [3].

139 Such a statistical analysis can provide an interesting insight on the polarimetric nature of the
140 samples, as discussed above and in several anterior works, however, it does not allow any spatial
141 analysis of the distribution of the SOPs, spatially across the speckle field, or across the Poincaré's
142 sphere. Classically, such Stokes imaging results are analyzed by providing the spatial maps
143 of the various Stokes parameters S_i for $i = 0, \dots, 3$ and of the DOP, as displayed for instance
144 in Fig. 13 of Section 6 of the first article of this series [1]. In the next section, we recall this
145 classical modality of displaying Stokes polarimetric information, but we propose alternative
146 representations that can be of interest depending on the physical situation analyzed and on the
147 spatial scale at which it is observed.

148 3. Graphical representations of Stokes polarimetric data

149 A speckle field is a pattern of light intensity varying spatially in two dimensions in the (x, y)
150 plane. At any point in this plane, the polarimetric information is defined and is characterized by
151 a four-dimensional vector, usually the four Stokes parameters, or the set of intensity, polarization
152 azimuth, polarization ellipticity and DOP of the light. Even in a monochromatic situation (using
153 a graylevel camera) such as the one studied in this article series, representing a 4-dimensional
154 information across a plane is a mere challenge that exceeds the capacities of our human senses
155 and mind. As a result, a compromise is necessarily made when representing such data, and the
156 "dimensions" that one must privilege clearly depend on the problem at hand and on the purpose
157 of the imaging experiment.

158 The most widely used representation consists in showing each polarimetric parameter as a
159 2D image, thus preserving the spatial information. This is clearly the solution that has to be
160 advocated for most polarization imaging situations, when one wants to increase contrasts, such
161 as in the biomedical or defense domains for instance. We shall refer in the following to such
162 representations as "spatial-structure-preserving representations", which prioritize the readability
163 of the spatial (imaging) information to the detriment of the polarimetric information.

164 From another standpoint, some specific situations of polarimetric imaging can afford to relegate
165 the spatial distribution of the pixels in the image to the secondary level. In that case, the main
166 information lies in the way the SOP is distributed among the pixels along the "polarization"
167 dimensions. This is clearly the case in our present study of the polarimetric properties of a
168 speckle field, whose spatial structure is relatively poor in terms of interpretable information, as it
169 consists of a complex interference pattern of speckle grains. In such situations, the analysis of the
170 distribution of SOPs is more easily done by plotting them across a Poincaré's sphere for instance.
171 We shall refer in the following to such representations as "spatial-structure-non-preserving
172 representations", which prioritize the readability of the polarimetric information to the detriment
173 of the spatial information.

174 In the following subsections, we will introduce and describe several alternative graphical

175 representations, based on the above definitions, which will allow us to propose optimal readability
 176 of the Stokes imaging data, depending on the observation scale (grain population, interface
 177 between neighbouring grains, single grain). All these representations detailed below are
 178 summarized in Table 1.

Table 1. **Graphical representations developed and used according to the desired scale of observation. (PQ: Peirce-Quincuncial projection)**

| Observation scale | “Dimension” of representation | |
|----------------------|---------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------|
| | Spatial | Polarimetric |
| Population of grains | - S_0 , s_1 , s_2 , s_3 and DOP -Stokes vector RGB color coding - S_0 + polarization ellipses | -Poincaré’s sphere mapping -PQ mapping -ROI + thresholding + colour coding of grain membership/boundaries |
| Neighbouring grains | - S_0 , s_1 , s_2 , s_3 and DOP -Stokes vector RGB color coding - S_0 + polarization ellipses | -Poincaré’s sphere mapping -PQ mapping -Triang. ROI + RGB coding of spat. coord. + boundaries color coding |
| Single grain | - S_0 , s_1 , s_2 , s_3 and DOP -Stokes vector RGB color coding - S_0 + polarization ellipses | -Poincaré’s sphere mapping -PQ mapping - ROI/thresholding (+ meshing) + colour coding of polygonal contour |

179 3.1. Polarimetric representations with preserved spatial structure

180 To understand polarimetric information on top of a spatial information in a 2D-image, we need to
 181 encode it visually in the ‘dimensions’ accessible to our senses (intensity, colour, 3D, etc.) or by
 182 using representation devices superimposed on the image (arrows, bars, ellipses, etc.). The answer
 183 to this problem is far from being simple and unique, given the variety of polarimetric parameters
 184 and measurement techniques, as well as the applications that require specific information. A
 185 great deal of work has been done on encoding polarimetric information while preserving certain
 186 information that is visible to the human eye [15–17]. We list below some of the available
 187 spatial-structure-preserving representations:

- 188 • **Set of 2D maps:** This representation enables spatial information to be perfectly preserved,
 189 by providing one spatial 2-D map for each polarimetric channel. In our work, these
 190 parameters are represented as follows:
 - 191 – S_0 : A grayscale colormap is commonly used, corresponding to the standard image
 192 obtained with a non-polarimetric grayscale camera.
 - 193 – s_1 & s_2 : As those parameters vary between -1 and $+1$, the representation we propose
 194 consists in using a scale whose extreme values show green and fuchsia colours
 195 corresponding respectively for the s_1 parameter to a vertical ($s_1=1$) and horizontal
 196 ($s_1=-1$) polarization and for the s_2 parameter to a polarization of $+45^\circ$ ($s_2=1$) and
 197 -45° ($s_2=-1$). The saturation of these colours is directly linked to the values of these
 198 parameters. Zero values are represented by a white hue.
 - 199 – s_3 : Similarly to the previous case, the representation of this parameter uses a scale
 200 whose extreme values are represented by the colours blue and red, corresponding

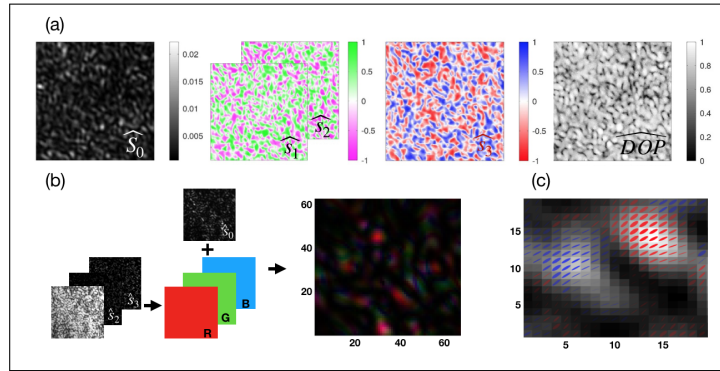


Fig. 2. a) 2D representation of the Stokes parameters: S_0 , s_1 , s_2 and s_3 as well as the DOP obtained on a Spectralon sample. (b) False colors representation of the polarimetric information (RGB coding with s_1 , s_2 and s_3 , S_0 encodes the level of luminosity). (c) 2D image of the intensity in graylevels with the polarimetric information implemented as a polarization ellipse in each "superpixel".

201 respectively to a state of left-circular and right-circular polarization. As with the
 202 previous colour scale, intermediate values are represented by a white hue. The
 203 different choice of colors (red vs blue) is justified by the fact that these colors will
 204 directly indicate the sign of ellipticity of the local SOP in the s_3 map.
 205 – (Optionally) DOP : Although introducing some redundancy with the above
 206 parameters, the DOP map is very commonly used in polarimetric imaging. As it is a
 207 scalar bounded between 0 and 1, a grayscale colormap is used, with higher values of
 208 the DOP represented by clearer pixels.

209 The main drawback of this classical representation, which is schematically illustrated in
 210 Fig. 2.a on real data, lies in the lack of global interpretability from a single glance, as it
 211 requires the joint analysis of 4 to 5 spatial maps (Stokes parameters + DOP).

- 212 • **Stokes vector RGB color coding:** This approach, illustrated in Fig. 2.b, is an attempt
 213 to summarize all the polarimetric information into a unique map, more suited to visual
 214 interpretation. This approach amounts to encoding each SOP into a corresponding colour
 215 using an RGB colorimetric encoding (HSV encoding could also be used), obtained from the
 216 value of the reduced Stokes parameters s_i , $i = 1, \dots, 3$. In order to retain the information
 217 related to intensity, the brightness of each pixel is linked to the values of the S_0 parameter,
 218 i.e., an area of low luminosity is represented by dark colours. Although this representation
 219 makes it easier to visualize speckle grains and the spatial evolution of polarization states, it
 220 is still inefficient to ensure a clear visual interpretation of the SOP evolution, and then to
 221 carry out a careful topological study of the polarization state distribution.
- 222 • **Optimized representation: S_0 map + polarization ellipses:** The last representation that
 223 we propose in this section consists in visually encoding the SOP on top of the intensity
 224 image (S_0 parameter in graylevel) using visual indications that are provided at a lower
 225 spatial resolution. This representation, illustrated in 2.c, is inspired from [13] but refined
 226 as detailed below to ensure best readability. For that purpose, a layer containing the
 227 polarimetric information, consisting of polarimetric ellipses, is superimposed on the
 228 intensity image, thereby providing a direct information about the polarization azimuth and
 229 ellipticity. A blue (resp. red) ellipse represents a left (resp. right) circular polarization
 230 state, with positive (resp. negative) ellipticity. We also propose to encode the polarization

231 degree in this polarimetric information layer, by linking the “normalized” size of the ellipse
232 to the value of the DOP. For a purely polarized state (DOP=1), the length of the major axis
233 of the ellipse is made equal to the width of the *superpixel*, whereas for a depolarized light,
234 the superimposed ellipse exhibits a vanishing size. A satisfactory magnification scale used
235 on the actual resolution of the S_0 image was shown to be of 16 pixels (i.e., one pixel of the
236 original S_0 is represented with a uniform square of 16×16 pixels, on top of which the
237 corresponding local ellipse is plotted).

238 This refined representation proves very efficient at quickly visualizing the evolution of
239 the polarization state inside a speckle grain, or across a population of speckle grains. By
240 zooming out on this image, it is also possible to recover a global view of the intensity
241 image without being too much perturbed by the polarization ellipses. For that purpose,
242 we implemented a variable transparency parameter in the display of each ellipse, directly
243 given by the S_0 intensity level. In that way, bright areas (speckle grains) show opaque and
244 clearly visible ellipses, while in dark regions, the ellipses are almost transparent and do
245 not perturb the dark background of the intensity image. Furthermore, this transparency
246 tuning is also directly related to the level of confidence one can expect, as low intensity
247 regions will lead to estimated valued of the Stokes parameters with poor accuracy and
248 precision [2]. This representation, illustrated Fig.2.c, appears to be a good compromise in
249 order to preserve spatial information in an image, while at the same time offering quite
250 clear readability of the polarimetric information on intense pixels. In the present context
251 of Stokes imaging at the speckle grain scale, it can be used at all scales of observation to
252 study the spatial evolution of the SOP and of the DOP.

253 3.2. *Polarimetric representations with non-conserved spatial structure*

254 Polarimetric representations which do not preserve the 2D-spatial structure of the data may be
255 inconceivable in many imaging applications (medical imaging, defence) without impoverishing
256 the interpretability of the results. However, this type of representation can be well indicated
257 to analyze the distribution of polarimetric SOPs in a readable way, making understanding
258 almost immediate once you are familiar with the polarization formalism. In the context of our
259 study, the spatial information is not very informative as noted above, and it becomes acceptable
260 for us to "omit" the spatial structure of the 2D images in favour of a better analysis of the
261 distribution/evolution of the polarimetric states. Thus, in this subsection, we will list some
262 alternative approaches for displaying polarimetric information while improving the readability of
263 the distribution/evolution of the polarimetric states of a speckle pattern at different observation
264 scales.

- 265 • **Poincaré’s sphere plot:** The Poincaré’s sphere representation is a well-known tool
266 allowing all possible polarimetric states to be plotted in a three-dimensional frame of
267 reference which corresponds to the normalized Stokes parameters (s_1 , s_2 and s_3). Fully
268 polarized states correspond to the sphere of unit radius, whereas a partially polarized SOP
269 is represented by a point that lie inside this unit radius sphere, its DOP corresponding to
270 the distance of this point to the center of the frame. Using such representation, it is thus
271 possible to map each pixel in a Stokes imaging experiment with its corresponding location
272 in the Poincaré’s sphere [3, 14]. An example of such mapping is provided in Fig. 3.a, which
273 shows that in the context of our experiments, the well-defined and polarized SOPs observed
274 at each location of a speckle pattern with sufficiently high intensity correspond to a group
275 of points located at the surface of the Poincaré’s sphere. However, the main drawback
276 of such representation lies in its 3-dimensional nature (sphere), preventing the eye from
277 encompassing all possible SOPs with a single glance, which can be the case, as will be
278 seen below with very depolarizing samples, or at the vicinity of polarization singularities.

279 Furthermore, antipodic SOPs may be superimposed, leading to confusion or occlusion,
280 which can be very annoying on a static representation, such as on a printed 2D figure.

281 • **Planispheric projections:** An obvious way to circumvent such difficulties is to resort
282 to classical planispheric projection methods, which allow the surface of a sphere to be
283 mapped on a 2D plane. There are tens of planispheric projections that have been proposed
284 with very different properties and characteristics. We can cite here the stereographic
285 projection that has already been proposed to analyze polarization states, and which maps
286 the sphere to a 2D plane [18]. Although such projection has the interesting property of
287 being conformal, it is however barely usable as it maps the sphere to an infinite plane,
288 which also prevents the observer to encompass all SOPs at a time.

289 • **Preferred representation: Peirce's quincuncial projection:** Among all the alternative
290 planispheric projections at hand, we propose to use the Peirce's quincuncial (PQ) projection
291 [19], which is illustrated in Fig. 3.b. Although we do not detail here the formulation of
292 such projection for the sake of concision, this PQ projection allows all the polarimetric
293 states lying at the surface of the sphere ($DOP=1$) to be displayed instantaneously on a two-
294 dimensional square map. This conformal map representation is obtained by concatenating
295 eight partial stereographic transformations of the eight octants of the sphere, leading to
296 eight isosceles right triangles forming a square, as shown in Fig. 3.b. The six specific
297 points of coordinates $(1, 0, 0)$, $(-1, 0, 0)$, $(0, 1, 0)$, $(0, -1, 0)$, $(0, 0, 1)$ and $(1, 0, -1)$
298 on the sphere are respectively indicated by letters V, H, P, M, L and R in Fig. 3.b.

299 Classically, and as shown in Fig.3.b, the centre of the PS projection corresponds to the
300 north pole of the globe corresponding to the left-hand circular polarization state L on the
301 Poincaré's sphere. In this configuration, the south pole is represented simultaneously in all
302 four corners of the map. The equator of the sphere, corresponding to all the linear states, is
303 shown as a dotted red line on the map (Fig.3.b), forming a diamond shape. For the sake of
304 readability, it can be advantageous to modify the center of the PQ projection, so that it is
305 centered on the average polarization state of the ROI studied, thus improving the readability
306 of the information for the reader. This corresponds to modifying the reference state of the
307 Poincaré's sphere from which the stereographic projection is performed. An example is
308 shown in Fig.3.c, where the center of the map represents the vertical polarization state
309 corresponding to the polarimetric state of the laser source used in the experiment [1]. As
310 can be observed, it also allows to avoid the distortions caused by the PQ projection when
311 the observed SOPs are located close to the corners of the equator in red (Fig.3.b).

312 This PQ projection offers the interesting property of encompassing at a single glance
313 all SOPs, without occlusion or possible confusion between them. Furthermore, it is a
314 conformal projection (except at four singular points on the equator, located at the middle
315 of each vertex of the square), which tends to avoid strong deformations on the planispheric
316 projection, except at the vicinity of the four singular points on the equator. As a result, in
317 terms of practical use of such projection, we always tended to select a centering point
318 for the projection that minimizes the quantity of SOPs located at the vicinity of these four
319 singular points.

320 It the remainder of this article, we will analyze the distribution of SOPs across a speckle field
321 using the Poincaré's sphere representation, and with the corresponding PQ projection, which, as
322 will be shown below greatly improves the readability and interpretability of the results. It can
323 be noted here that contrary to the Poincaré's sphere representation, the PQ projection does not
324 directly enable the DOP to be "encoded" in the projected map. This will not be an issue in the
325 context of this study, as the SOP is well defined with a DOP very close to one at any location

326 of the speckle field under polarized illumination [3, 13, 14]. In a more general context, such
 327 additional information could be displayed onto the PQ map using false color coding for instance.

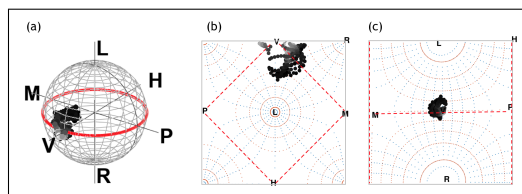


Fig. 3. Representation of the Poincaré's sphere with a distribution of SOPs (a) and its corresponding Peirce's quincuncial projection (b) where the left circular state noted L (+s3 on the Poincaré's sphere) is at the centre of the map. (c) Same as (b) but with a Peirce's projection centered on the linear vertical SOP (corresponding to the illumination state of the laser source in the experiment).

328 4. Multiscale polarimetric topology analysis of a speckle pattern

329 In this section, the various representations described above will be used, adapted and refined in
 330 order to offer the best interpretability of experimental Stokes imaging results obtained on speckle
 331 patterns at distinct scales of observation: grain populations, neighbouring grains and single grain
 332 scale. Such multiscale representation tools can indeed prove interesting for upcoming works
 333 aiming at analyzing the origin of spatial depolarization of a speckle pattern. We will illustrate
 334 below how such experimental data along with optimized polarimetric data representation can lead
 335 to interesting observations of the topology of the spatial distribution of SOPs across a speckle
 336 field.

337 4.1. Population of grains

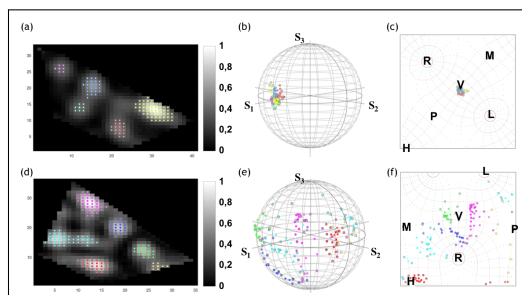


Fig. 4. First column: image of the S_0 parameter with colored markers for the selected pixels. The second column represents the SOPs plotted on the Poincaré's sphere. The last column is the Peirce's projection of the Poincaré's sphere. The first line corresponds to the metal sample and the second to a Spectralon sample.

338 In this subsection, the aim is to analyze the polarimetric states distribution within speckle grains
 339 for a population of grains located in a same selected ROI. As an additional objective, we aim at
 340 refining the above data representation strategies, in order to ensure best interpretability of the
 341 SOPs distribution, while preserving as much spatial information as possible, although we will
 342 resort to "spatial-structure-non-preserving representations" such as Poincaré's sphere and PQ
 343 projection.

344 From the Stokes images acquisition, a ROI is selected using a polygonal contour, as represented
345 in the intensity (S_0) maps in Fig. 4.a and d for two examples acquired respectively on a metallic
346 blade and on a Spectralon sample. A thresholding operation on the values of S_0 enables the
347 brightest pixels in the intensity map to be selected, thereby isolating distinct speckle grains. In
348 order to analyze the spatial distribution of the SOPs across the speckle field, a colour is assigned
349 to each grain detected, and a coloured marker is displayed on each pixel of the grain on the
350 displayed S_0 map, using a magnification scale similar to the one used in Section 4. The colour
351 saturation of each marker is linked to the level of intensity, so that high intensity is represented by
352 a bright colour for readability purpose. All these markers can then be plotted on the Poincaré's
353 sphere (Fig. 4.b and e) and on the PQ projection (Fig. 4.c and f). As a result, using such colored
354 markers makes it possible to distinguish the SOPs of different speckle grains by referring to the
355 reference intensity image (Fig. 4.a and b).

356 In addition, we wanted to identify in the Poincaré's sphere or in the PQ map the pixels lying
357 on the edge of the speckle grains selected by the thresholding operation. For that purpose, we
358 implemented a black edge on the colored markers corresponding to the pixels belonging to the
359 grain boundary. This makes it possible to visualize the polarimetric state evolution within a grain
360 when it is represented on the Poincaré's sphere and PQ projection.

361 The two ROIs selected and plotted in Fig. 4 represent a population of grains (<10 grains) of
362 the speckle pattern obtained at the surface of a non-depolarizing sample (polished metal plate)
363 and a highly depolarizing sample (Spectralon), shown respectively in Fig. 4.a and d. It can be
364 seen that all the speckle grains observed on the surface of the non-depolarizing sample have
365 polarization states close to that of the illumination source linearly polarized along the vertical
366 axis (corresponding to reduced Stokes vector $s = [1 \ 0 \ 0]$) as shown in Figs. 4.b and c. As
367 detailed above, PQ map has been centered on the average polarimetric state, which is close to a
368 state with reduced Stokes vector $(1, 0, 0)$. This observation is clearly in agreement with previous
369 studies [3, 13, 14]

370 For the Spectralon sample, as expected for a highly depolarizing sample, the SOPs are scattered
371 around the surface of the Poincaré's sphere and the PQ projection (Fig.4.e and f). The polarimetric
372 states observed are distributed over almost all eight octants of the sphere for the six grains
373 analyzed in this ROI. This representation clearly shows that spatial depolarization occurs by the
374 averaging of distinct polarimetric states, but also that each pixel of a speckle grain has a specific
375 distribution of SOP, centered around an average SOP which differs from one grain to the other.

376 Interestingly, the clear readability of the PQ projection seems to indicate a noticeable property.
377 Indeed, in some situations, a contour which encloses a spatial region in the vicinity of the center
378 of a bright speckle grain seems to be mapped, on the SOP distribution, to a contour on the
379 Poincaré's sphere (or the PQ projection) that encloses the distribution of SOPs of the pixels
380 inside the initial spatial contour. However, as will be discussed later, this property does not seem
381 to be universal for all speckle grains.

382 4.2. *Interface between neighbouring grains*

383 We now propose to focus on the evolution of the SOP in regions corresponding to the interface
384 between neighbouring grains. Two kinds of situations have been analyzed: in the first one we
385 analyze the SOP in a triangular region defined by the centers of the three neighbouring grains.
386 The second situation corresponds to analyze the transition of SOP along a linear trajectory
387 between two grains.

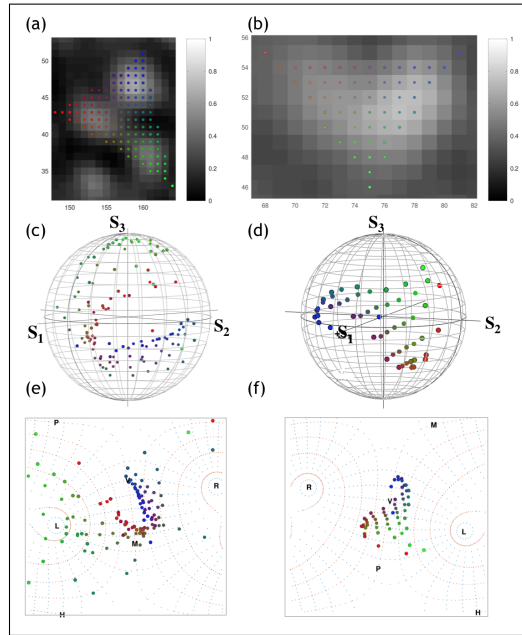


Fig. 5. First column: Representation of the selected ROI on the S_0 image with colored spatial encoding of the pixels locations. Second column: Representation of the Stokes vector at each pixel on the Poincaré's sphere and on the PQ projection, represented in the last column.

388 **Triangular area of interest** In order to represent the evolution of the SOP across a triangular
 389 area while preserving as much spatial information as possible, we proposed to use colored
 390 markers, as introduced in the previous subsection, but with a distinct color encoding. In that
 391 case indeed, we assigned the markers colors in such a way that the three triangle summits are
 392 respectively assigned a red, green and blue colour, with respective RGB encoding: $[1\ 0\ 0]$, $[0\ 1\ 0]$
 393 and $[0\ 0\ 1]$. The pixels corresponding to all the other points of the triangle are then assigned a
 394 colored marker whose RGB encoding is directly given by the spatial triangular coordinates of
 395 the point (while ensuring normalization to 1 of the sum of the RGB values). As a result, the
 396 pixel located at the barycentre of the triangular ROI is therefore assigned the RGB color triplet
 397 $[1/3\ 1/3\ 1/3]$ corresponding to a gray level. By referring to the reference image intensity image,
 398 as shown in Fig. 5.a, this representation allows us to better visualize the spatial evolution of the
 399 polarization state on the Poincaré's sphere (Fig. 5.c) or on the PQ projection (Fig. 5.e) as a
 400 function of the pixel location at the interface between three neighbouring grains. In addition, and
 401 as described previously, the boundary of the region of interest is distinguished by markers with a
 402 black edge.

403 Thus the first ROI shown in Fig.5.a is selected from a speckle pattern obtained on the partially
 404 depolarizing marble sample painted in green and is composed of three distinct grains. It can be
 405 seen on the Poincaré's sphere, and more particularly on the PQ projection, that these three grains
 406 have a circular polarimetric state (green markers), a vertical polarimetric state (blue markers)
 407 and a -45° polarimetric state, represented by the letter M (red markers). In this case, we are unable to
 408 visualize an organisation of polarization states in a form "enclosed" by the polarization states of
 409 the boundary. The field adopts a high diversity of states, with very rapid spatial evolution of the
 410 polarisation.

411 The second ROI, Fig.5.b, is obtained from a speckle pattern formed on the surface of the
 412 spectralon sample and is composed of a single V-shaped grain that we were able to fit into

413 a triangular ROI. We note in this case that, firstly, the distribution of polarization states on
 414 the surface of the Poincaré's sphere (Fig.5 d) and on the PQ map (Fig.5 f) is "denser" than
 415 in the previous case and, secondly, that the distribution of polarimetric states preserves the
 416 spatial distribution of the pixels. The polarimetric states appear to remain "enclosed" within the
 417 spatial distribution and remain within the polarization states located at the boundary of the ROI
 418 (represented by black border markers).

419 The two discussed examples show very distinct properties for the distribution of states across
 420 the Poincaré's sphere, the first one being very scattered over the entire sphere, while the second
 421 seems much more localized. We conjecture that the difference is due to the fact that the ROI of
 422 the first example encompasses a dark region in the intensity image. As a result, firstly, the level
 423 of confidence on the estimated SOPs is lower in such dark region, and secondly, it may happen
 424 that a field singularity lying in the triangular ROI leads to a very large spreading of the SOPs in
 425 the vicinity of the singularity, as will be observed and discussed in the next section.

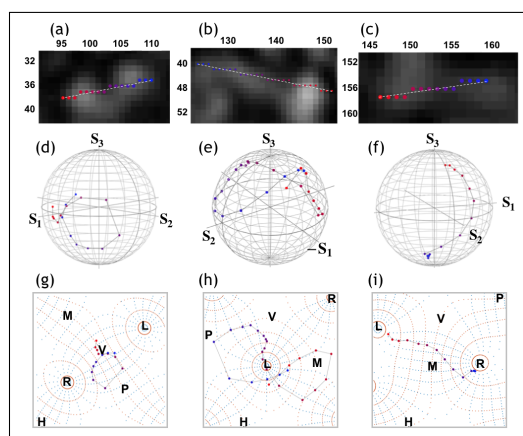


Fig. 6. First line: image of parameter S_0 with colored marking (from red to blue) of the pixels selected along a linear trajectory (dashed white line). Second line: representation of the selected pixels on the Poincaré's sphere, retaining the color coding of the markers. Third line: similar representation on the PQ projection.

426 **Linear ROI** In a previous work [3], a linear "trajectory of interest" on the images of the Stokes
 427 parameters, and in particular of S_0 , was used to study the evolution of the polarization state
 428 between two grains or even within a single grain. In this seminal article, it was observed that
 429 the spatial evolution of the polarization state was continuous along such trajectory. In order to
 430 reproduce such results, we implemented a similar ROI by selecting the trajectory on the reference
 431 image S_0 (Fig.6.a, b and c), while assigning a colored marker (from red to blue, in RGB color
 432 encoding) to all the pixels located on this trajectory. Thus the pixels at each end are respectively
 433 assigned the RGB color code red [1 0 0] and blue [0 0 1] and the marker color code for the
 434 central pixel is purple [1/2 0 1/2]. As before, the sum of the RGB values is constrained to 1 and
 435 as this analysis is performed on a straight trajectory, the green colour is not used and its value
 436 remains zero. In addition, when we represent the corresponding SOPs on the Poincaré's sphere
 437 and on the PQ mapping, the color encoding of the SOPs links the points together by a continuous
 438 line corresponding to the spatial trajectory, thereby clearly guiding the reader through complex
 439 polarimetric evolution.

440 To visualize interesting evolutions of the SOP, we chose to study speckle patterns obtained from
 441 depolarizing samples such as Spectralon and green-painted brick. The three selected trajectories

442 shown in Fig.6.a-c were chosen to spatially link two adjacent speckle grains by a straight line.
 443 The results show that the evolution of the polarization state along a linear spatial trajectory
 444 (dashed white line) on the S_0 parameter images can represent a wide variety of trajectory shapes
 445 at the surface of the Poincaré's sphere and on the PQ projection map, as shown in Fig. 6 d-i.
 446 These trajectories represent a rather linear path from circular left (red markers) to circular right
 447 (blue markers) such as in Fig. 6.d and g; an " ∞ -shaped" trajectory such as in Fig. 6 e and h; or
 448 finally a loop-shaped trajectory such as in Fig. 6.f and i.

449 As observed in [3], the spatial evolution of the SOP is continuous, as these trajectories do
 450 not cross field singularities, but without a clearly repeatable trajectory structure, contrary to
 451 what could have been conjectured in [3]. It can be noted that, in order to check the validity of
 452 the experimental results presented here, it was verified that the DOP of the detected light along
 453 the selected trajectories is well preserved, as shown in Fig.7. The DOP along the trajectories
 454 is preserved and remains high (DOP > 0.75) for all ROIs (a), (b) and (c) which respectively
 455 correspond to the trajectories selected and displayed in Fig.6.a, b and c. Such verification confirms
 456 the validity of the experimental results presented in these figures, but these measurements do not
 457 reveal a clear trend in the behavior of the polarimetric trajectory at the transition between two
 458 neighboring speckle grains.

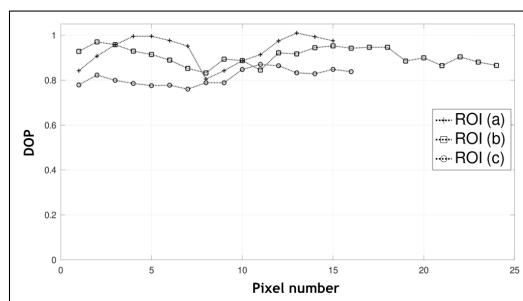


Fig. 7. DOP evolution along the selected linear path of the ROI displayed in Fig. 6.a-c

459 4.3. Single grain scale

460 The objective of this last representation for speckle polarimetric topology analysis is to provide a
 461 simple way to represent, as well as possible, the connexity between adjacent pixels on a Poincaré's
 462 sphere or on the corresponding PQ map, two representations that belong to the above-defined
 463 category of "spatial-structure-non-preserving representations". For that purpose, we proposed to
 464 introduce a spatial mesh on the pixels of the ROI (rectangular, polygonal or circular in shape)
 465 selected on the reference image S_0 , as shown for instance in Fig. 8.a, b. This same mesh, referred
 466 to in the following as connexity mesh is then applied and plotted on the Poincaré's sphere (Fig.
 467 8.c, d) or on the PQ map (Fig. 8.e, f) in order to "link" the SOPs represented depending on their
 468 spatial vicinity in the S_0 image.

469 To enhance the reader's understanding, the different edges of the selected polygonal ROI are
 470 assigned a distinct color, and vertical and horizontal lines of the mesh are represented in the
 471 projections by blue and red lines respectively. Finally, the color of the markers within the ROI is
 472 linked to the S_0 value, where high intensity is represented by a dark marker to enhance contrast
 473 with the white background of the Poincaré's sphere and PQ map. The results shown in Fig. 8
 474 were obtained from speckle patterns produced on the Spectralon sample.

475 This representation, especially when projected on the PQ map, enables us to further analyze
 476 the above question, namely whether the SOPs within a chosen ROI delimiting a speckle grain
 477 remain (on the Poincaré's sphere and its PQ projection) within a region enclosed by the SOPs
 478 of the pixels of the edge of the spatial ROI, as was suggested in [3] and in Section 4.2. This

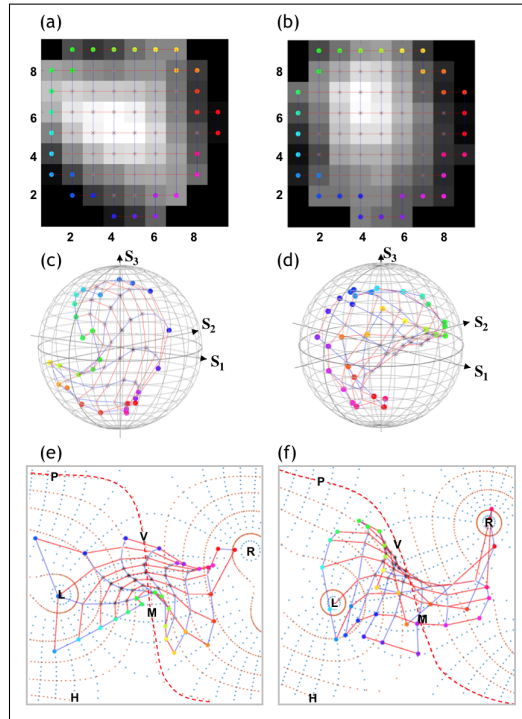


Fig. 8. First line: image of parameter S_0 for a Spectralon sample with a spatial mesh of selected pixels, where the markers corresponding to the boundary are colored. Second line: representation of the connectivity mesh on the Poincaré's sphere of polarization states at each pixel. Third line: same representation on the PQ projection, where the equator of the Poincaré's sphere is highlighted with the dashed red line.

479 is sometimes the case, since we can see that the spatial mesh produced in Fig. 8.a retains its
 480 spatial structure when projected onto the Poincaré's sphere (Fig. 8.c and d) and the PQ map
 481 (Fig. 8.e and f), with a one-to-one correspondance between the pixel location and the SOP. The
 482 SOPs corresponding to the pixels inside the ROI indeed remain within the perimeter delimited
 483 by the SOPs of the pixels forming the ROI boundary. However, we observed on other grains
 484 that this situation is not a generality. Indeed, when projected onto the Poincaré's sphere and PQ
 485 representation, this spatial connectivity mesh can "fold", allowing polarization states located inside
 486 the mesh to lie outside the perimeter formed by the polarization states corresponding to the ROI
 487 boundary, and there is no one-to-one correspondance between spatial location and SOP, as shown
 488 in the right column of Fig.8. This is an interesting observation for which, to our best knowledge,
 489 we do not have physical interpretation, but which could be the subject of further investigation in
 490 upcoming works.

491 Whether for the grain population, grain neighbourhood, or single grain analysis, for the sake of
 492 concision, only a small set of experimental "topological" analyses results have been presented
 493 here in this Section 4 among those that have been observed, focusing on a small number of cases.
 494 However, this study will need to be completed in subsequent work involving a large population
 495 of different grains and cases, in order to conduct a statistical analysis of the different situations
 496 observed. The automated experimental set-up developed and the data processing/analysis
 497 algorithms detailed in [1, 2] now offer this possibility with a high degree of confidence in the
 498 experimental results. In the last section of this paper, we will extend the analysis and the use
 499 of these proposed graphical tools to study the polarimetric behaviour of light at the vicinity of

500 polarization singularities.

501 5. Preferred representation for analyzing polarimetric singularities

502 Experimental and numerical studies of polarization singularities present in a speckle field have
503 been the subject of many research studies [5, 20–23], finding more and more applications in very
504 distinct domains such as super-resolved microscopy [24], or optical communications [25]. The
505 kind of polarization singularities we will address below represent specific locations in the speckle
506 field, characterized by non-zero intensity (i.e., not a field singularity) and a polarization state
507 whose azimuthal angle of the polarization ellipse is undefined, and in the vicinity of which the
508 field exhibits extremely fast azimuthal variations of the polarization ellipse (polarization state
509 with a linear component). In this case three main configurations of polarization singularities
510 are commonly identified [22] and referred to as “Lemon”, “Star” and “Monstar”, and can be
511 described as follows (We refer the interested reader to [22] for further details):

- 512 • A Lemon-type singularity corresponds to lines of slow azimuth angle variations around a
513 polarization ellipse line of constant azimuth
- 514 • A Star-type singularity corresponds to the intersection of three lines of constant azimuth,
515 where the azimuth angle is zero at the point of singularity,
- 516 • A Monstar-type singularity is a hybrid situation between Lemon and Star singularities.

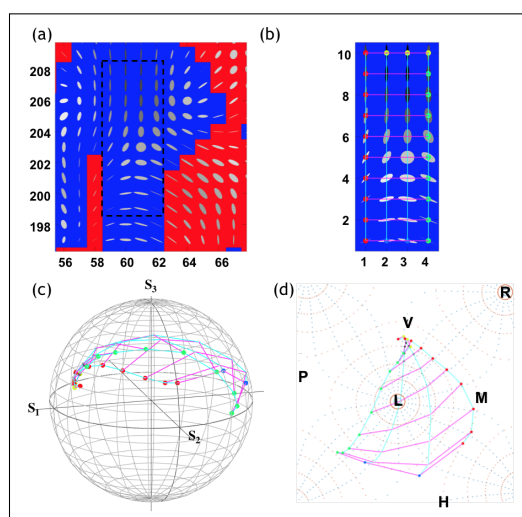


Fig. 9. (a) Star singularity observed in a speckle field produced with a Spectralon sample. Black dashed rectangular area: selected ROI. (b) Connexity mesh of pixels in the ROI. (c) Distribution of SOPs across the Poincaré's sphere displayed with connexity mesh. (d) Same as (c) for PQ projection.

517 To improve our understanding of the polarimetric evolution in the vicinity of a singularity, we
518 rely on the previous representations and in particular the use of the connexity mesh as introduced
519 in Section 4.3. Unlike previous representations, the ROI is represented on a reference image
520 where the blue (red) colored background corresponds to a left (right) circular polarimetric state
521 instead of the intensity S_0 image. As discussed in previous sections, a polarization ellipse
522 is represented at each pixel location: the intensity (S_0) information is encoded by the brightness
523 of the ellipse, where a white ellipse corresponds to a very bright pixel. As before, the DOP is

524 encoded by the size of the ellipse: hence, a state with a high DOP will be represented by a large
 525 ellipse size. This representation is shown for instance in Fig. 9.a, where a star singularity can be
 526 clearly identified in the center of the ROI.

527 The connexity mesh generated in this ROI and displayed in Fig. 9.b is then plotted on the
 528 Poincaré's sphere and on the PQ mapping (Fig. 9.c and d). Here, each side of the ROI has been
 529 associated with markers of the same color, and the vertical and horizontal lines of the mesh are
 530 identified by cyan/magenta colored lines. The Star singularity shown in Fig. 9 corresponds to the
 531 intersection of three trajectories whose azimuth angle varies slowly, and whose polarization state
 532 at the intersection is of indefinite azimuth (circular SOP), on which the ROI was centered. We
 533 note that the vertical linearly polarized states located on the top edge of the ROI (yellow markers)
 534 are all located close to the polarization state corresponding to the vertical state on the Poincaré's
 535 sphere (1, 0, 0) (Fig. 9.c), and close to the V symbol on the PQ projection map (Fig. 9.d). The
 536 SOPs of pixels located along the left edge of the ROI spatially evolve towards a linear state close
 537 to -45° (corresponding to point M on the (Fig. 9) and are represented with red markers. The
 538 pixels on the right edge of the ROI correspond to the green markers, and their SOPs evolve
 539 towards a linear state with an azimuth close to $+22.5^\circ$ (located between the horizontal H state
 540 and the $+45^\circ$ state marked P). Finally, the SOPs located along the ROI's central vertical axis
 541 vary from a vertical state to a near-horizontal state (H), transiting via a left-hand circular state
 542 (L). The mesh of this singularity is very insightful, particularly due to the fact that each of the
 543 three branches follows an evolution of the SOPs between linear polarization states: vertical state
 544 (0° azimuth) (yellow markers), -45° azimuth (intersection between blue and red markers) and
 545 $+22.5^\circ$ azimuth (intersection between blue and green markers). Interestingly, this corresponds
 546 on the PQ projection to a rather triangular shape. The great advantage of Peirce's projection over
 547 the classical Poincaré's sphere mapping is obvious in this example: there is no ambiguity on
 548 Fig. 9.d to distinguish clearly the location of the SOPs, unlike in Fig. 9.c where it is difficult to
 549 distinguish whether the points are organized on the hemisphere facing the observer or on the
 550 opposite one. Next, we will focus on situations where two singularities are located close to each
 551 other, leading to interesting configurations of the SOP distribution.

552 5.1. Lemon-Monstar configuration

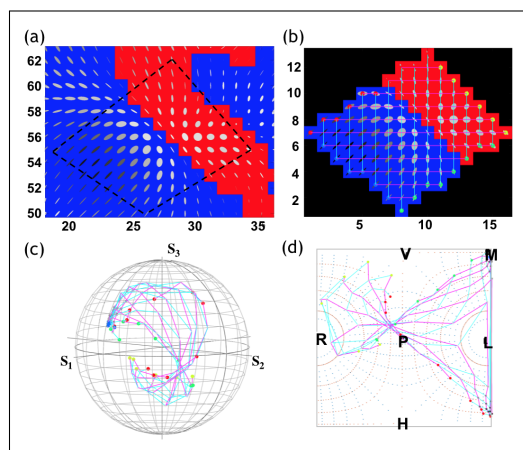


Fig. 10. (a) Lemon-Monstar configuration observed in a speckle field produced with a Spectralon sample. Black dashed polygonal area: selected ROI. (b) Mesh of selected pixels. (c) Distribution of SOPs across the Poincaré's sphere displayed with connexity mesh. (d) Same as (c) for PQ projection.

553 In this subsection, we first focus on an experimentally observed configuration of two singularities
 554 located in a close neighbourhood, namely a Lemon and a Monstar singularity (Fig. 10.a). As in
 555 the above subsection, a spatial mesh of the chosen polygonal ROI is produced and displayed in
 556 Fig. 10.b. The SOP at each pixel and the connexity mesh are plotted on the Poincaré's sphere
 557 and its PQ projection (Fig. 10.c and d).

558 It can be seen that in this configuration, all the horizontal lines of the mesh intersect, on the
 559 Poincaré's sphere and on the PQ projection, close to the linear polarimetric state of azimuth
 560 $+45^\circ$. This intersection corresponds to the SOP of the pixels located at the frontier between
 561 positive (left-handed) ellipticity states (blue background in Fig. 10.b) and negative (right-handed)
 562 ellipticity states (red background). Once again, the proposed PQ projection associated to the
 563 connexity mesh displayed offers a very good readability of the spatial evolution to the SOP across
 564 the speckle field, and in the vicinity of polarization singularities.

565 5.2. Star-Star configuration

566 To end up this analysis of the topological distribution of SOPs around polarization singularities,
 567 let us focus on the experimentally observed Star-Star configuration, composed of the two Star
 568 singularities described above. This configuration shows a particularly interesting distribution of
 569 the SOPs, especially in the ROI displayed in Fig. 11.a.

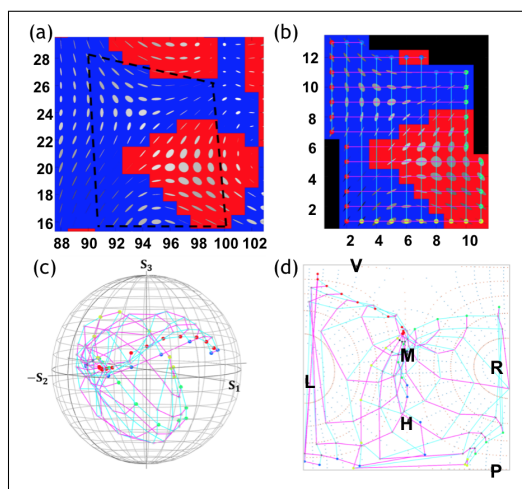


Fig. 11. Same as Fig. 10 for a Star-Star configuration of singularities.

570 This ROI can be basically split into two regions: a first upper triangle with positive (left-handed)
 571 ellipticity SOPs, and a second lower triangle with negative ellipticity SOPs. In such configuration
 572 of two Star polarization singularities in the vicinity of each other, the mapping of the SOPs
 573 across the Poincaré's sphere and on the PQ projection shows that such configuration tends to
 574 cover all possible SOPs. Indeed, the spatial mesh seems to spread over almost all the surface
 575 of the Poincaré's sphere (although with radius lower than unity, indicating partial polarization).
 576 The projection of this incomplete sphere onto the PQ is even clearer to demonstrate the fact that
 577 the observed SOPs almost cover all the surface of the PQ map. This interestingly shows that
 578 such Star-Star singularity configuration exhibits almost all possible polarimetric states in a very
 579 limited spatial extent on the imaged scene.

580 **6. General conclusion and perspectives**

581 This third part of the article series about “Optimized Stokes imaging for highly resolved optical
582 speckle fields” has permitted to illustrate the wide variety of results and analyses that can be
583 conducted when performing accurate Stokes imaging experiments on a speckle pattern, at a
584 spatial scale such that the SOP of light can be mapped inside a single speckle grain. As discussed
585 in this series, this was made possible by optimizing an experimental Stokes imaging setup in
586 order to compensate carefully any source of bias or non-linearity in the measurement process,
587 as detailed in the first article [1]. Furthermore, as we have established in the second article [2],
588 the performance of such polarimetric imaging experiment has been optimized, in terms of
589 accuracy, precision and robustness to experimental imperfections, by resorting to the so-called
590 SOPAFP approach, using appropriate polarization analysis states, which enabled us to provide
591 the experimental results presented in the present article.

592 In order to complete this study and provide the reader with useful tools in the particular context
593 of this study, we have also introduced and discussed various ways of representing and analyzing
594 the polarimetric data. In the context of imaging the SOPs on a speckle pattern, we have shown
595 that using a particular planispheric projection of the SOPs on the Poincaré’s sphere, namely the
596 Peirce’s quicuncial projection, appears to be a very interesting approach to enhance the readability
597 of such results and to help in the analysis of the spatial behaviour of the distribution of SOPs.
598 In particular, we have demonstrated the benefit of the PQ projection over the Poincaré’s sphere
599 representation in order to avoid confusion or occultation of SOPs when the SOPs distribution
600 tends to occupy the entire sphere. Lastly, based on these general modalities, we have proposed
601 several refined graphical representations, adapted to various scales of observation (“macroscopic”
602 analysis, population of a few grains, interface between neighbouring grains, and finally single
603 grain), and to different “topological” properties that can be investigated on the spatial distribution
604 of SOPs across a speckle field. In all cases, the objective is to preserve at best the spatial
605 information (even though it is naturally lost in representations such as Poincaré’s sphere or
606 PQ projection), but resorting to colored markers, or connexity meshes... In addition, we were
607 also able to observe polarization singularities of various kinds, and to analyze their spatial
608 distributions with optimized graphical tools.

609 This study thus provides a complete review of the optimal methodology to perform polarimetric
610 state metrology at the speckle grain scale, from the experimental device to the processing and
611 estimation strategies, as well as original representations of the polarimetric information. With
612 such automated and optimized experiment, we are now able to conduct extensive investigations
613 of the polarization properties of speckle fields. This work opens many perspectives, mainly
614 experimental, but which could provide interesting insights to open theoretical questions that are
615 still pending in the domain. In particular, it will be very interesting to analyze the evolution of
616 the SOPs distribution with modification of the physical properties of the illumination laser, such
617 as tuning the wavelength, the coherence properties of the source, and/or modifying the DOP of
618 the illumination beam ranging from fully polarized to totally unpolarized.

619 These experiments and this optimized setup also paves the way to experimental measurement
620 of original depolarization metrics proposed in the field of statistical optics that could be very
621 interesting to investigate on a speckle pattern, which could lead to identify and measure more
622 subtle statistical descriptors of light depolarization, such as non-isotropic and/or non-gaussian
623 depolarization processes, already proposed theoretically [26] but to be observed experimentally.
624 Another class of interesting parameters could be the second-order (two-point) polarimetric
625 statistics (such as two-point DOP, or two-point Stokes parameters [27, 28]) which could enable
626 the definition of a “spatial polarization coherence”, and somehow be linked to less standard
627 spatial coherence properties [29, 30]. Identifying specific samples or materials that could lead to
628 particular spatial depolarization patterns across the speckle field (such as an anisotropic spatial
629 depolarization) is another perspective to this work. Combined with the non-standard descriptors
630 of light properties mentioned above, original polarization metrology approaches such as the

631 one described in this work could also exhibit an interest for applications: they could indeed
632 give relevant clues to discriminate materials sharing identical “macroscopic” characteristics
633 (spectrum, degree of polarization;. . .) but which could differ in terms of the internal structural
634 properties of their SOPs distribution.

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636 as well as F. Théry and F. Murie for insightful debates on the mental representation of polarimetric
637 information.

638 **Disclosures.** The authors declare no conflicts of interest.

639 **Data availability.** Data underlying the results presented in this paper are not publicly available at this time
640 but may be obtained from the authors upon reasonable request.

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