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Victory Elias, Emmanuel Dubois, Nikolay Smagin, Frédéric Rivart, Alex  
Vercampt, Georges Nassar

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# Enhanced Fluid Characterization with a Cantilever Microbeam System excited by an Embedded Electroactive Source

Victory Elias

UPHF- INSA Hauts-de-France, IEMN-  
UMR CNRS 8520  
Valenciennes, France  
victory.elias@uphf.fr

Frédéric Rivart

UPHF- INSA Hauts-de-France, IEMN-  
UMR CNRS 8520  
Valenciennes, France  
frederic.rivart@uphf.fr

Emmanuel Dubois

Université de Lille, IEMN-UMR CNRS  
8520  
Lille, France  
emmanuel.dubois@univ-lille.fr

Alex Vercampt

Polytech Lille, IEMN-UMR CNRS 8520  
Lille, France  
alex.vercampt.etu@univ-lille.fr

Nikolay Smagin

UPHF- INSA Hauts-de-France, IEMN-  
UMR CNRS 8520  
Valenciennes, France  
nikolay.smagin@uphf.fr

Georges Nassar

UPHF- INSA Hauts-de-France, IEMN-  
UMR CNRS 8520  
Valenciennes, France  
Georges.Nassar@uphf.fr

**Abstract**— *When there is a variation in fluid concentration at the free end of a microcantilever, this phenomenon can be utilized in applications such as chemical analysis, biological testing, pharmaceutical screenings, and environmental monitoring. This paper introduces a novel sensor based on a silicon microcantilever, designed for fluid characterizing. Our approach employs an embedded electromechanical source that excites the microbeam cantilever array, achieving superior sensitivity compared to existing models. An Ultra High-Frequency Vibrometer is utilized to measure the shifts in peak frequencies and the displacement motion of the vibrating cantilever. Experimentally, we evaluate changes in viscosity and density as indicated by the frequency shift, with a particular focus on the desired harmonic mode.*

**Keywords**— **Cantilever MicroBeam, Dynamic Mode, Shifting Frequency, Fluid Characterization.**

## INTRODUCTION

Historically, there have been extensive studies on microbeam's vibrational behavior when immersed in a fluid (1–3). Comprehensive theoretical models have been developed to understand the interactions between the microbeam and the fluid environment. These models were later substantiated through rigorous experimental measurements (4–6). More recently, advanced research by G. Brunetti et al. has broadened the scope of microbeam applications to include the detection of multiple target molecules (7).

Today, these microbeams are prevalent for detecting biological and chemical microparticles (9, 10, 28, 29). J. Duffy et al. observed the effective application of miRNA as a genosensor for cancer detection in serum samples (8). Similarly, Wang et al. (27) introduced a highly sensitive nanomechanical approach using a microcantilever array combined with PNA probes to identify the SARS-CoV-2 virus. Moreover, they serve as effective tools for probing the viscosity and density of gases and liquids, thereby replacing traditional rheometers (11-16). They are also used to detect changes in mass, stress, or temperature (17-19).

A notable advancement was the integration of a microcantilever array and a piezoelectric driver,

resulting in a comprehensive "lab-on-a-chip" system. Several researchers have pioneered innovations in this domain: In 2010, Ricciardi et al. (30) integrated a microcantilever array and piezoelectric driver into a chip. Huang et al. (31–33), in 2011, successfully combined a microcantilever, a self-calibrating readout circuit, a programmable microcontroller, a voltage regulator, and a wireless transceiver, among other modules, onto a singular chip. Khemthongcharoen et al. (34), in 2015, designed an integrated microfluidic detection chip. This chip married PCR technology with a pressure-sensitive microcantilever at its heart. In 2014, Wang and his team contributed extensively to the field, detailing works on microcantilever integration. Their research spanned areas such as surface antibody modification (35,36) and the intricate design and creation of a microcantilever that featured a cavity, piezoelectric propulsion, and a frequency-monitoring circuit (36–38). Wang's team, in their 2021 study (39-40), developed a microcantilever based on an SOI substrate using standard CMOS techniques like RIE and PECVD.

In our study, we examine how modifications in design and operational approach can enhance the performance of microcantilever-based sensors, particularly focusing on the strain and stress continuity characteristics of the overall structure. Specifically, we investigate the benefits of employing vibratory microbeams as diagnostic sensors. We favor the dynamic mode (21) over the static mode (22) for measurements and utilize an integrated electromechanical source for excitation.

## I. Theoretical & Numerical Framework

Our focus is on movements that occur above the surface to which the beams are attached, specifically in out-of-plane vibrations (Flexion and Torsion). This approach, by comparison to in-plane motion, makes the corresponding shifts in resonant frequency more pronounced and easier to detect with high precision.

### A. Theory

In the analysis of cantilever microbeam dynamics, two fundamental modes of deformation are considered:

flexion and torsion. The flexural mode is described by the natural frequency of vibration in a vacuum corresponding to a mode number  $n$  can be expressed as:

$$\omega_{vac,n}^f = \frac{C_n^2}{L^2} \sqrt{\frac{EI}{\mu}} \quad (2)$$

Where  $n$  is the mode order of flexural vibration and  $C_n$  is the  $n$ th positive root of  $1 + \cos C_n \cosh C_n = 0$ . For  $\omega_{vac,n}^f$  is derived from the relation incorporating the characteristic length  $L$  and  $C_n$ , which depends on the mode shape and boundary conditions.

Conversely, the torsional mode is depicted by the natural frequency for the  $n$ -th mode of torsional vibration in vacuum:

$$\omega_{vac,n}^t = \frac{D_n}{L} \sqrt{\frac{GK}{\rho c I_p}} \quad (4)$$

Where  $D_n = \frac{\pi}{2}(2n - 1)$ , and  $n = 1, 2, 3, \dots$ .  $\omega_{vac,n}^t$  is similarly determined by the torsional wave speed  $D_n$  and the characteristic length  $L$ . These equations serve as the foundation for exploring the dynamic response of cantilever microbeams, enabling the prediction and analysis of their behavior under various loading conditions and contributing to the design of microelectromechanical systems (MEMS).

Building upon the analysis of cantilever microbeams, we introduce two critical parameters: the quality factor ( $Q$ ) and the sensitivity ( $S$ ). The quality factor  $Q$  is a dimensionless parameter that characterizes the damping of the system and is inversely proportional to the energy loss per cycle.

$$Q = \frac{2\Delta w}{w_0} \quad (5)$$

It is defined as the ratio of the resonant frequency  $w_0$  to the bandwidth  $2\Delta w$  of the resonance peak. A higher  $Q$  indicates a lower rate of energy loss relative to the stored energy, which is desirable in applications requiring high precision and minimal energy dissipation.

The sensitivity  $S$ , on the other hand, is a measure of the system's responsiveness to changes in mass.

$$S = \frac{\Delta m}{\Delta \omega} \quad (6)$$

It is given by the ratio of the mass change  $\Delta m$  to the shift in resonant frequency  $\Delta \omega$ . This parameter is paramount in applications such as mass sensing, where the detection of minute mass changes is essential. The sensitivity is influenced by the physical properties of the beam and the mode of vibration. A higher sensitivity means that even small mass changes can be detected through significant shifts in the resonant frequency, enabling the cantilever to act as an effective sensor in MEMS applications. These parameters are crucial for the design and optimization of micro beam-based sensors, where high-quality factors and sensitivities are sought to achieve the best performance.

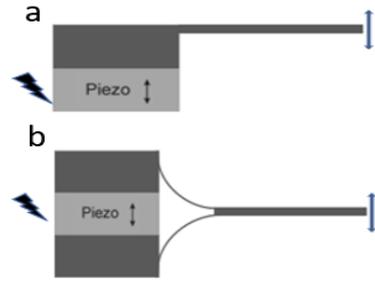


Fig. 1. Vibrating design: (a) an electromechanical source (PZT) deposited by bonding with a free side (44); (b) Excitation of the concept with an electromechanical source (PZT) stressed on both sides in the structure (Our Design).

The figure 1 illustrates two different setups of a piezoelectric element within a sensor system that includes a clamped microbeam. In the first setup, configuration (a), the piezoelectric element is placed either above or below the sensor's housing box, following a traditional design. Common in current applications, this configuration typically positions an actuator beneath a counterweight. This setup creates unilateral stress on the electromechanical element and facilitates energy transfer in a single direction when the piezoelectric material deforms.

In contrast, our innovative approach, depicted in configuration (b), integrates the source element within the housing box, aligning it with the clamped beams. Inspired by studies on the resonance behavior of conical structures, which aim to improve energy transfer, this design incorporates the piezoelectric material into the counterweight section. This arrangement imposes bilateral constraints on the electromechanical element. It is coupled with a hinge designed based on the same exponential principle, thereby optimizing energy transfer.

Our approach, emphasizing bidirectional stress and energy transfer, stands in stark contrast to the conventional unidirectional stress model. By evenly distributing stress across the beam, we expect a more dynamic interaction between the piezoelectric element and the beam, thus potentially increasing the sensor's responsiveness. This could lead to a more consistent excitation of multiple beams, enhancing the system's overall efficiency and precision.

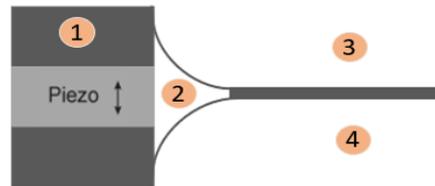


Fig. 2. Main elements of vibrating micro-system: (1) Rigid chip body. (2)Hinge. (3) Upper Surface (4) Bottom Surface.

The unique positioning of the piezoelectric material in our design as shown in Figure 2, sandwiched between the rigid chip body (1) and the solid cantilever-support structure, diverges from traditional practices where the material is placed on one side of the structure. This bidirectional stress application could significantly improve the sensor's sensitivity and accuracy by ensuring uniform stress distribution and energy transfer

to the cantilever (3,4) through the hinge (2). This design represents a paradigm shift in MEMS sensor functionality, leveraging the dynamic interaction between piezoelectric actuation and cantilever response to enhance sensor performance.

### B. Numerical Approach

We will translate this chosen theory into the design depicted in Figure 2, which illustrates our new proposal for a structure, and test it numerically.

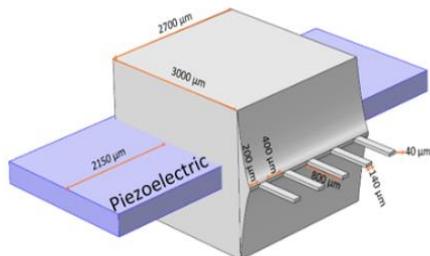


Fig. 3. 3D Illustration of a Composite Microcantilever Beam Array Featuring Integrated Piezoelectric Elements.

Referring to Figure 3, the beam is spaced at intervals of  $400 \mu\text{m}$ . The beam's dimensions are  $(800 \times 120 \times 40) \mu\text{m}^3$ . The piezoelectric component's housing is notably large, measuring  $3000 \mu\text{m}$  in width and  $2700 \mu\text{m}$  in length. Its exponential shape is a distinct feature, designed to support and carry the cantilever beam, ensuring optimal device performance for precision and stability.

We conducted several numerical tests to optimize the detection accuracy of our device. The first simulation focused on the beam's dynamic behavior in a fluid medium and the pressure's influence on the bending modes (Figure 4a) and torsion modes (Figure 4b), and its effects on the beam [our article].

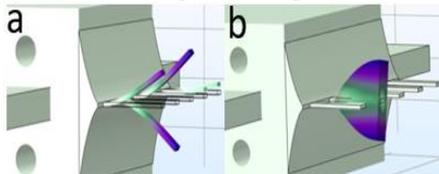


Fig. 4. Representation of Out-of-Plane Modes for a Microcantilever: (a) Flexural Mode and (b) Torsional Mode.

In flexion, the beam displaces the fluid symmetrically, while in torsion, the fluid's asymmetric movement can create additional forces resisting torsion.

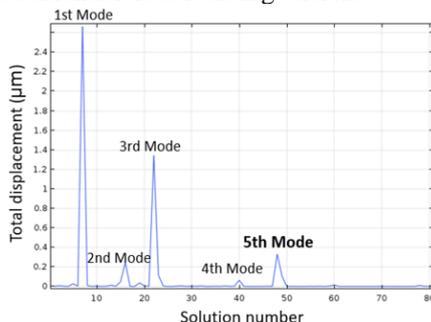


Fig. 5. Total Displacement (in micrometers) for a Microcantilever Beam Across Different Modes: The 1st, 2nd, 3rd, 4th, and 5th Modes.

We chose to focus on all bending modes of the beam that exhibit a natural frequency in the fluid, requiring little energy to induce movement, particularly evident in an aquatic environment. Mode 5, the last mode with significant beam displacement, was used and can be detected as shown in Figure 5.

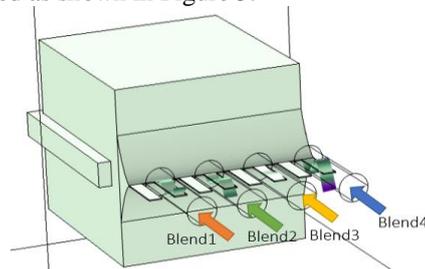


Fig. 6. Array of Cantilevers with Integrated Microcapillaries, Each Encasing Distinct Blends, Excited by Fifth Harmonic Mode Oscillation.

The third test shown in Figure 6 involved exciting the eigenfrequency without damping on mode 5 for all beams, which is the 3rd mode of flexion. Tests were conducted using microcapillaries on certain beams to observe each beam's behavior in different environments. This clearly showed the different environments and the different behaviors of each beam with a rheological change in the environment.

## II. SENSOR

In this section, we delve into a critical phase of our innovative project, where theoretical concepts in metrology are brought to life through practical application. We outline our comprehensive protocol, bridging the gap between digital metrology and experimental implementation. This approach not only emphasizes the meticulous process of design selection and realization but also underscores the importance of precise and methodical fabrication in achieving our project's objectives.

### A. Design & Fabrication

Our design process began with the utilization of digital metrology to ensure precise dimensioning, which is depicted in Figure 7. The subsequent engraving process is critical to the physical realization of the device, requiring constant metrological oversight. After engraving, we perform a meticulous cleaning of the sample—an essential step that is often underestimated. For the development of the cantilever microbeam array sensor, we selected silicon wafers due to their consistent properties. We then etched these wafers into individual tiles and assembled them with precision, using spacers to enhance structural integrity. The piezoelectric element was cut to precise dimensions and symmetrically integrated into the rigid body slot, a key factor for maintaining stability and establishing the electrical connections necessary for device functionality, as can be seen in the upper portion of Figure 7. The final assembly step involved securing the sample in the PCB holder, and preparing it for connection to the power source, as shown in the bottom views of Figure 7. This connection is vital for the device's operation. Our meticulous fine-tuning of the

equipment guarantees the high quality of the final product.

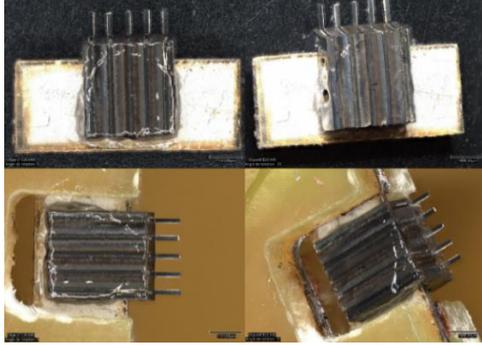


Fig. 7. Top Views: Microcantilever Device Structure with an Integrated Piezoelectric Element in the Rigid Body Slot. Bottom Views: The Integrated Device Attached to the PCB Holder.

### B. Experimental Setup

The experimental setup, provides a comprehensive framework for analyzing the vibrational characteristics of beams using advanced technology. The integration of the LDV-UHF-120, MATLAB software, and other critical components has allowed for precise, non-contact measurements of beam vibrations across a wide frequency spectrum. This setup not only facilitates a deeper understanding of the vibrational properties of beams but also sets a new standard in precision measurement techniques.

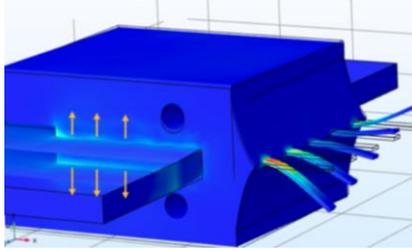


Fig. 8. Electric Excitation of Piezoelectric at Fundamental Frequency (88kHz).

In the context of defining safety margins, it is crucial to determine the frequency and applied voltage to the piezoelectric element that may risk beam breakage. According to the RDM simulation shown in Figure 8, it is essential not to exceed a voltage of 50 volts to avoid any risk of breakage, as the maximum displacement of the beam in the fundamental mode is 7.43 micrometers. This calculation aims to establish the nominal operating range of the sensor in terms of excitation voltage and beam resistance. The Vibrometer was employed to gauge cantilever vibrations, which were stimulated by a 7.5 Volt peak-to-peak chirp signal. For actuation, piezoelectric integration ensured efficient energy transfer to the cantilevers, facilitating vibration readouts using the Ultra High-Frequency Vibrometer.

Cantilever vibration shifts were detected in real-time using the Doppler Effect principle with the vibrometer, described by :

$$f_D = 2 \cdot \frac{v}{\lambda} \quad (7)$$

where  $v$  represents speed and  $\lambda$  wavelength. The device was subsequently immersed in a fluid medium,

aligned with the laser's measurement cycle, and scanned for roughly 10 minutes per beam.

### III. RESULTS & DISCUSSION

Our experimental analysis began by immersing the beams in various fluids to observe their frequency response behavior. Scanning five beams simultaneously across a broad frequency range (10 kHz–4 MHz), we identified eight distinct mode shapes. Given the inherent variations in fabrication and material properties, each beam naturally resonates at its unique frequency. Consequently, it is essential to select the most appropriate beam for performance in higher harmonic modes, aiming for the one with the optimal harmonic mode that possesses the highest quality factor (Q-factor) for energy efficiency and sensitivity. In air, our experiments determined that the fourth harmonic torsion mode, which is the first torsion mode, demonstrated superior sensitivity. Notably, Beam 3 exhibited the highest Q-factor in this mode based on Equation 5. In contrast, when submerged in water, the fifth harmonic vertical mode—equivalent to the third flexion mode—proved to be more sensitive despite the damping effects, as evidenced by the impressive Q-factor of Beam 3. These findings underscore the effectiveness of employing the 4th harmonic torsional mode for air sensitivity tests and the 5th harmonic mode for underwater applications. Subsequently, the ideal beam and harmonic mode selection was based on the magnitude of the frequency shift observed, indicative of heightened sensitivity in water, particularly for the 5th harmonic mode used in rheological measurements. This is depicted in Figure 9, which illustrates the frequency shifts with water as the reference medium, comparing the effects of different solutions, including ethanol and a 10% water solution. In this setup, density variations were introduced while maintaining a constant viscosity.

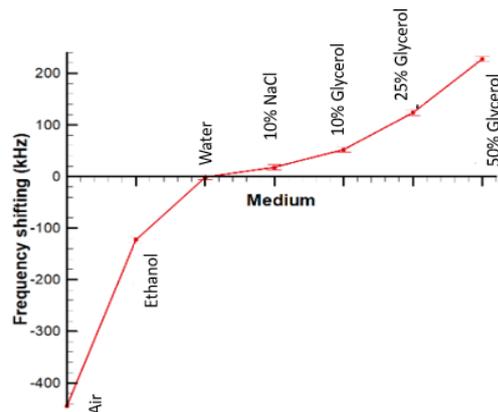


Fig. 9. Frequency Shift ( $\Delta f$ ) in kHz for the Third Beam of the Device, Excited at the Fifth Harmonic Mode Shape Oscillation, Across Various Solutions of Different Concentrations.

Conversely, in experiments with three different glycerol concentrations, we observed changes in viscosity while keeping density constant. Reference table 1 is vital for calibrating the sensor's sensitivity to these changes in medium viscosity and density, as shown by the frequency shift in Mode 5, based on

Equation 6. Table 1 demonstrates the sensitivity of the frequency shift (kHz) to changes in density and viscosity for various mediums, with water as the baseline and presents the comparative shifts in frequency for mediums ranging from air to a 50% glycerol solution, reflecting the sensor's calibrated response to these differing conditions.

TABLE 1. SENSITIVITY OF FREQUENCY SHIFTING (KHZ) RELATIVE TO CHANGES IN DENSITY AND VISCOSITY FOR VARIOUS MEDIUMS, WITH WATER SERVING AS THE REFERENCE POINT.

Medium	$\Delta$ Density (g/cm <sup>3</sup> )	$\Delta$ Viscosity (Pa.s)	$\Delta f$ in Mode 5 (kHz)
Air	---	---	-445.2
Ethanol	$\approx -0.211$	---	-21.0
Water (W)	0	0	0
W+10% Salt	$> 0.165$	---	16.2
W+10% Glycerol	---	0.148	39.7
W+25% Glycerol	---	0.371	113.5
W+50% Glycerol	---	0.7435	227.5

#### IV. CONCLUSION & PERSPECTIVES

Our research marks a significant advancement in micro-cantilever sensor technology, enhancing sensitivity and reliability for a range of industrial applications. A crucial aspect of these developments lies in optimizing the placement of the source within the structure. This optimization specifically enhances the sensitivity of certain modes, particularly the third mode of bending, as demonstrated in our study. The study investigates the relationship between beam configuration and sensor performance, addressing fabrication challenges like precision in laser engraving. These advancements have implications for various applications including chemical analysis, biological testing, and environmental monitoring, marking progress in scientific and medical fields. Future work includes refining beam structures and integrating microsystems for accurate fluid analysis, moving towards lab-on-a-chip systems. This progress brings us closer to fully harnessing microfluidic technology in numerous scientific and technological domains.

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