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Towards physics-based inversions of deformation data

Valérie Cayol

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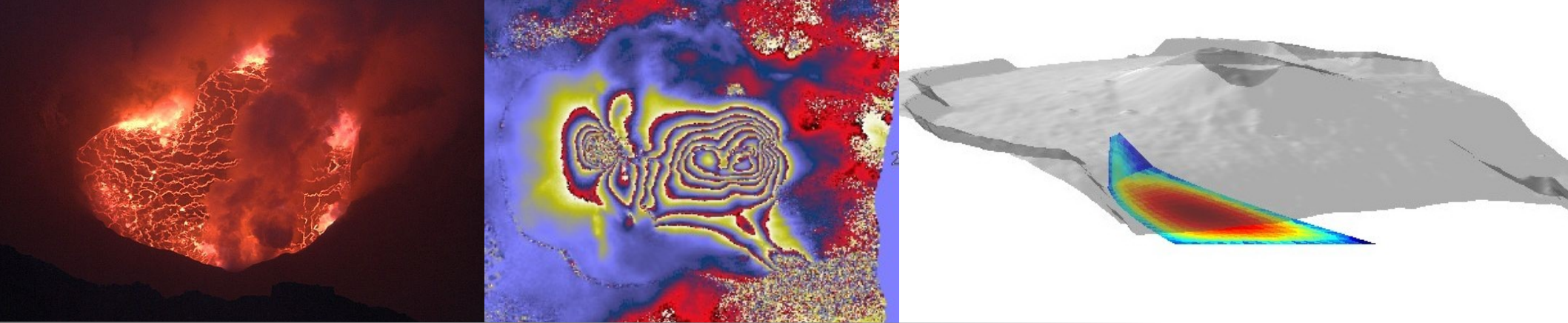
HAL Id: hal-04614798

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Submitted on 17 Jun 2024

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Towards physics-based inversions of deformation data

Valérie Cayol, valerie.cayol@uca.fr

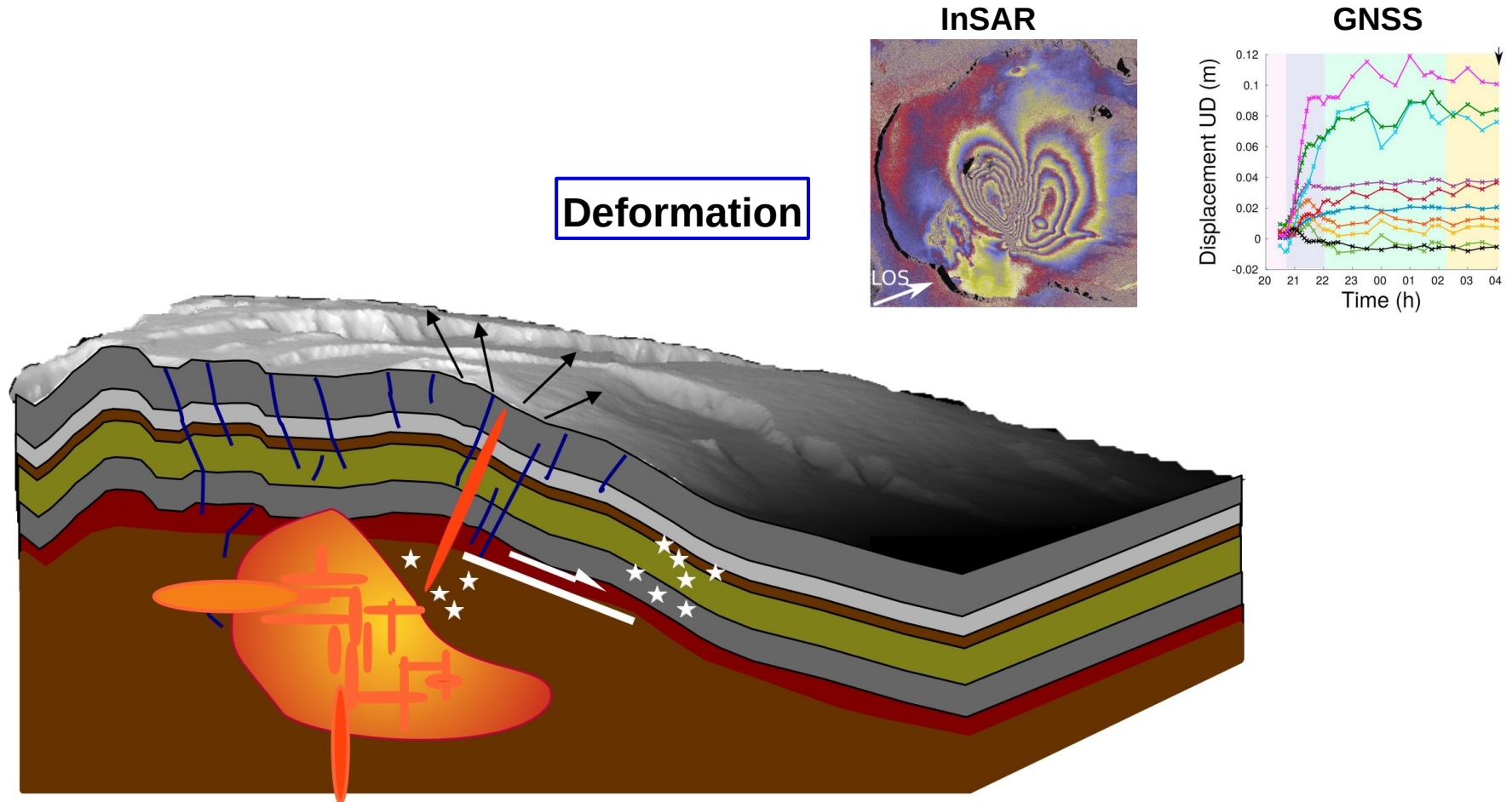
*Laboratoire Magmas et Volcans, CNRS, UCA, IRD, OPGC,
F-63000 Clermont-Ferrand, France*



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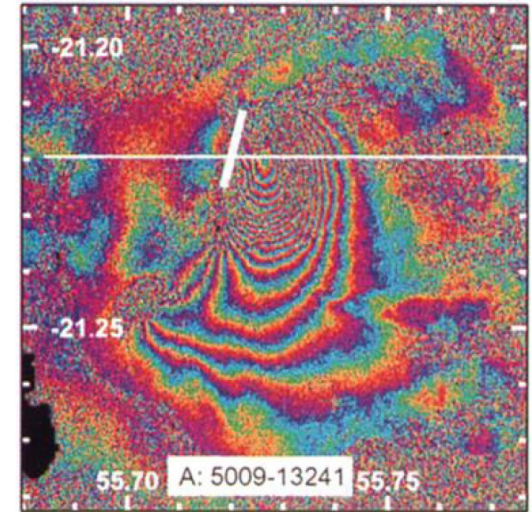
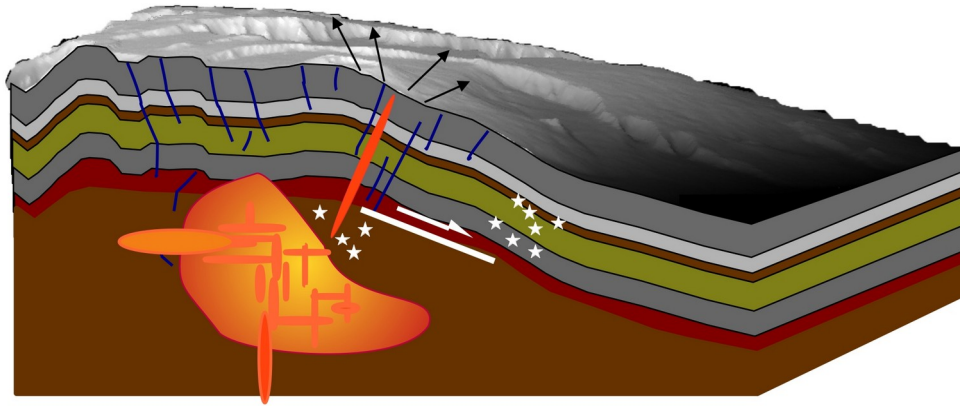
Why do we model deformation ?



- Where is magma stored ?
- What are the physical and mechanical parameters controlling magma transfer ?
- How do edifices grow and collapse ?

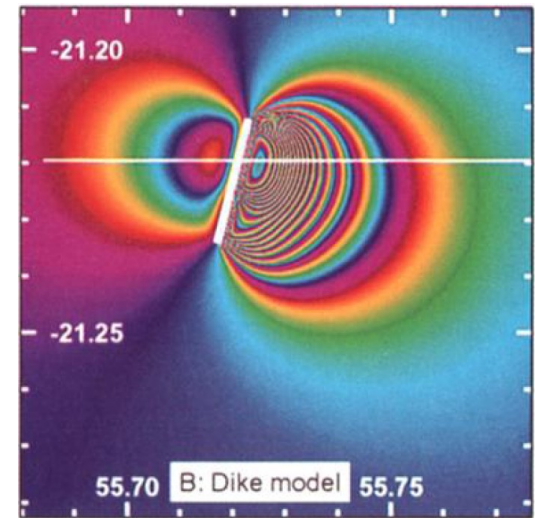
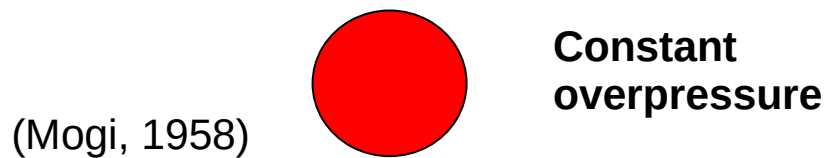
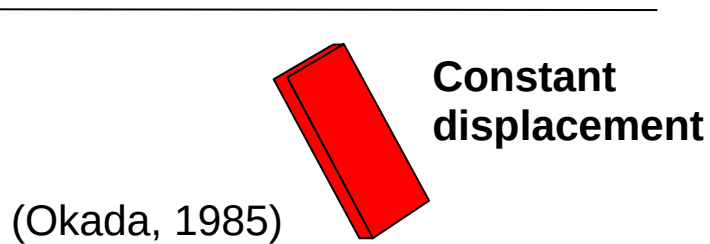
Classical elastostatic analytic models

Reality



Analytic Models

Models



(Sigmundsson et al., GRL, 1999)

➡ InSAR data call for more realistic models

Inversions: Definition of a cost-function

- The simplest cost-function :

$$\chi^2 = \sum_{i=1}^N (u_o^i - u_m^i)^2 = \|\mathbf{u}_o - \mathbf{u}_m\|^2$$

where u_o^i is the i^{th} observed displacement
 u_m^i is the i^{th} modelled displacements

- Normalized cost function:

$$\chi^2 = \sum_{i=1}^N \frac{(u_o^i - u_m^i)^2}{\sigma_i^2}$$

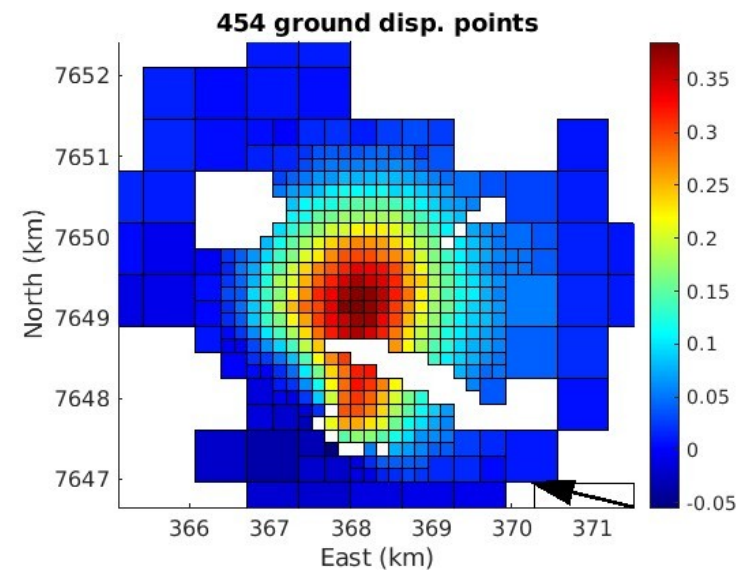
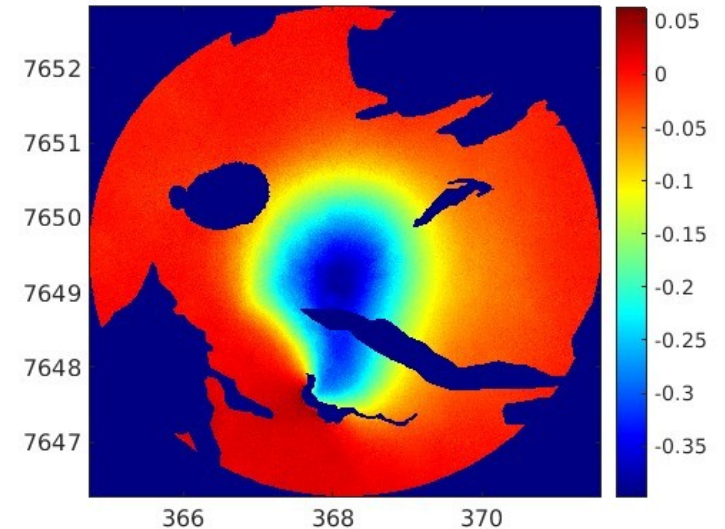
σ_i Standard deviation or error
on the i^{th} data

- Taking the data correlation into account:

$$\chi^2 = (\mathbf{u}_o - \mathbf{u}_m)^T \mathbf{C}_d^{-1} (\mathbf{u}_o - \mathbf{u}_m)$$

where \mathbf{C}_d is the full covariance matrix

Data point undersampling $\rightarrow \mathbf{u}_o$



Inversions : **linear** versus non-linear inversions

Linear inversions: there is a linear relation between the parameters \mathbf{m} and the observations, \mathbf{u}_m

$$\mathbf{u}_m = \mathbf{G} \mathbf{m}$$

Exemple: Okada's Model (1985, 1992); Mogi's solution (1958) are linear models

Typically, the location of a source is known, and the amplitude of the source is searched for.

To minimize the cost function: $\chi^2 = \|\mathbf{u}_o - \mathbf{u}_m\|^2 = \|\mathbf{u}_o - \mathbf{G} \mathbf{m}\|^2$

We seek \mathbf{m} such that : $\frac{\partial \chi^2}{\partial \mathbf{m}} = 0$

Which leads to solving a linear system of equations: $\mathbf{m} = (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \mathbf{u}_o$

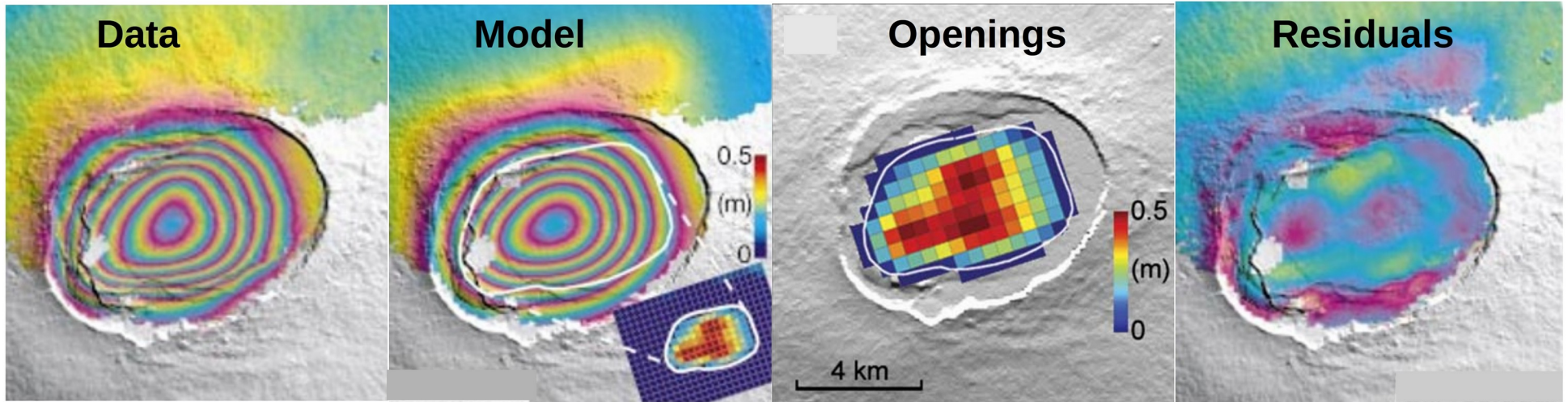
Pros: fast method

Cons: the source location has to be known

Example of a linear inversion: “kinematic” models

Uplift at Sierra Negra volcano in 1998-99 (Galapagos)

Minimization of $\chi^2 = \|\mathbf{u}_o - \mathbf{G} \mathbf{m}\|^2 + \beta^2 \|\nabla \mathbf{m}\|^2$, where \mathbf{m} is the opening vector



Amelung et al., Science, 2000

Still widely used

Science

Fracturing and tectonic stress drives ultra flow into dikes

Freysteinn Sigmundsson^{1*}, Michelle Parks², Halldór Geirsson¹, Andrew Hooper³, Vincent Drot G. Ófeigsson², Sonja H. M. Greiner^{1,4,5}, Yilin Yang¹, Chiara Lanzì¹, Gregory P. De Pascale¹, Kristí Valentyn Tolpekin⁷, Hildur María Friðriksdóttir², Páll Einarsson¹, Sara Barsotti²

JGR Solid Earth

RESEARCH ARTICLE
10.1029/2019JB019117

- Key Points:
- Imaging multidisciplinary continuous deformation data to improve dike ascent modeling
 - Detailed temporal model of the 2018 intrusion at Etna volcano

The 24 December 2018 Eruptive Intrusion at Etna Volcano as Revealed by Multidisciplinary Continuous Deformation Networks (CGPS, Borehole Strainmeters and Tiltmeters)

M. Aloisi¹, A. Bonaccorso¹, F. Cannavò¹, G. Currenti¹, and S. Gambino¹

RESEARCH ART



Cite as: Sigmundsson et al., 2020

AGU ADVANCING EARTH AND SPACE SCIENCE

Inversions : linear versus non-linear inversions

Non-linear inversions: there is a non-linear relation between the parameters m and the observations, u_m

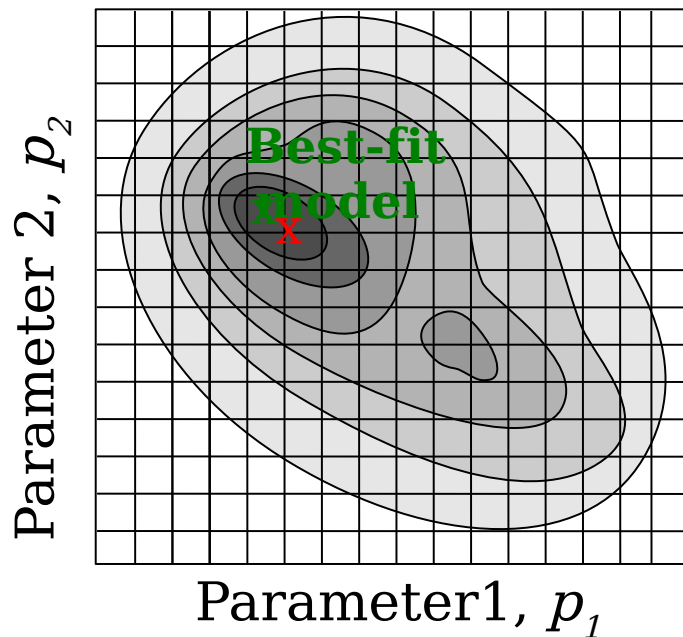
$$u_m = G(m)$$

The link between the source location, orientation parameters and the ground displacement is a non linear relation.

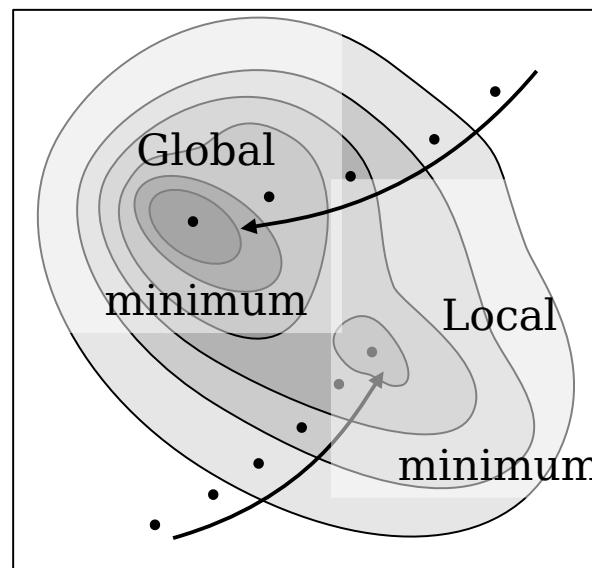
To minimize the cost function:
we can use different methods

$$\chi^2 = \|u_o - u_m\|^2 = \|u_o - G(m)\|^2,$$

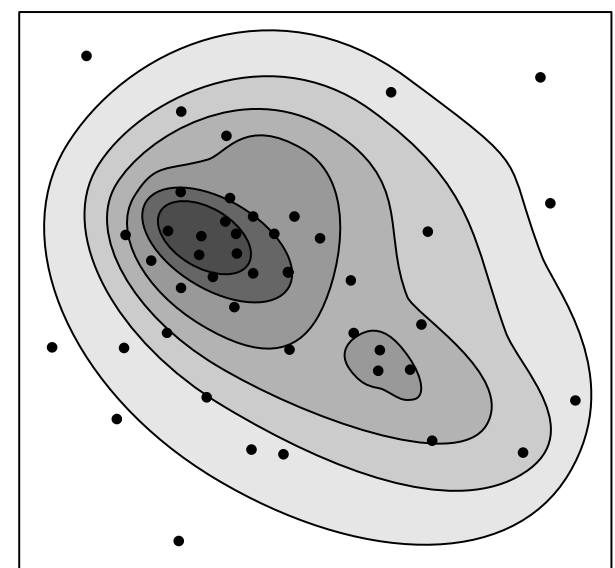
Grid search



Gradient-based method



Monte Carlo search

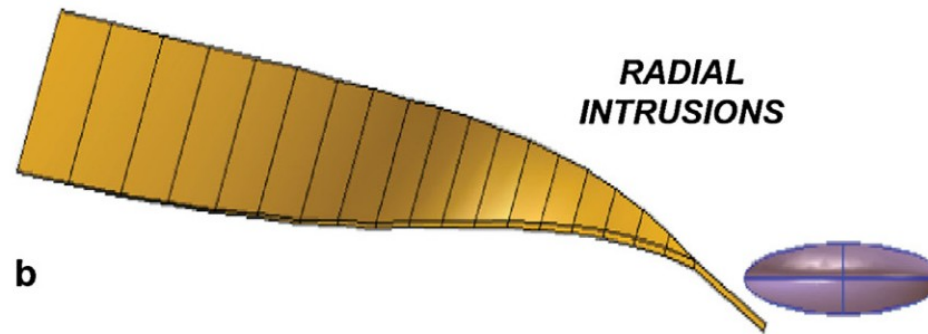
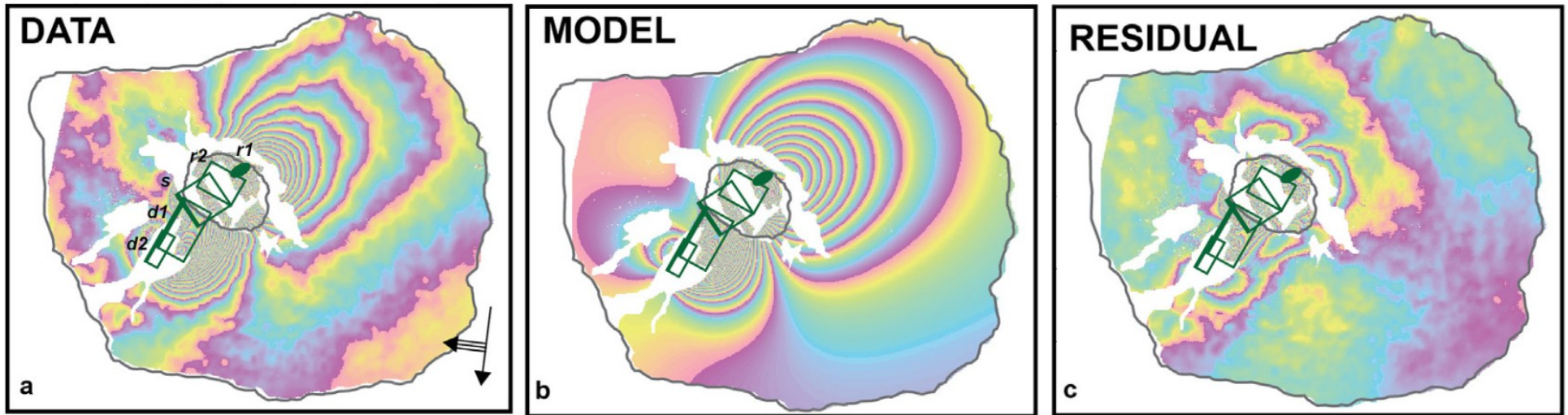


Pros: adapted to non-linear inversion

Cons: slow

Non-linear inversion to capture source geometries

Radial intrusion in 2009 at Fernandina volcano (Galapagos)

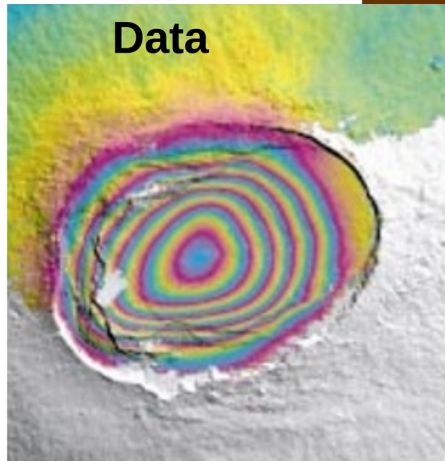
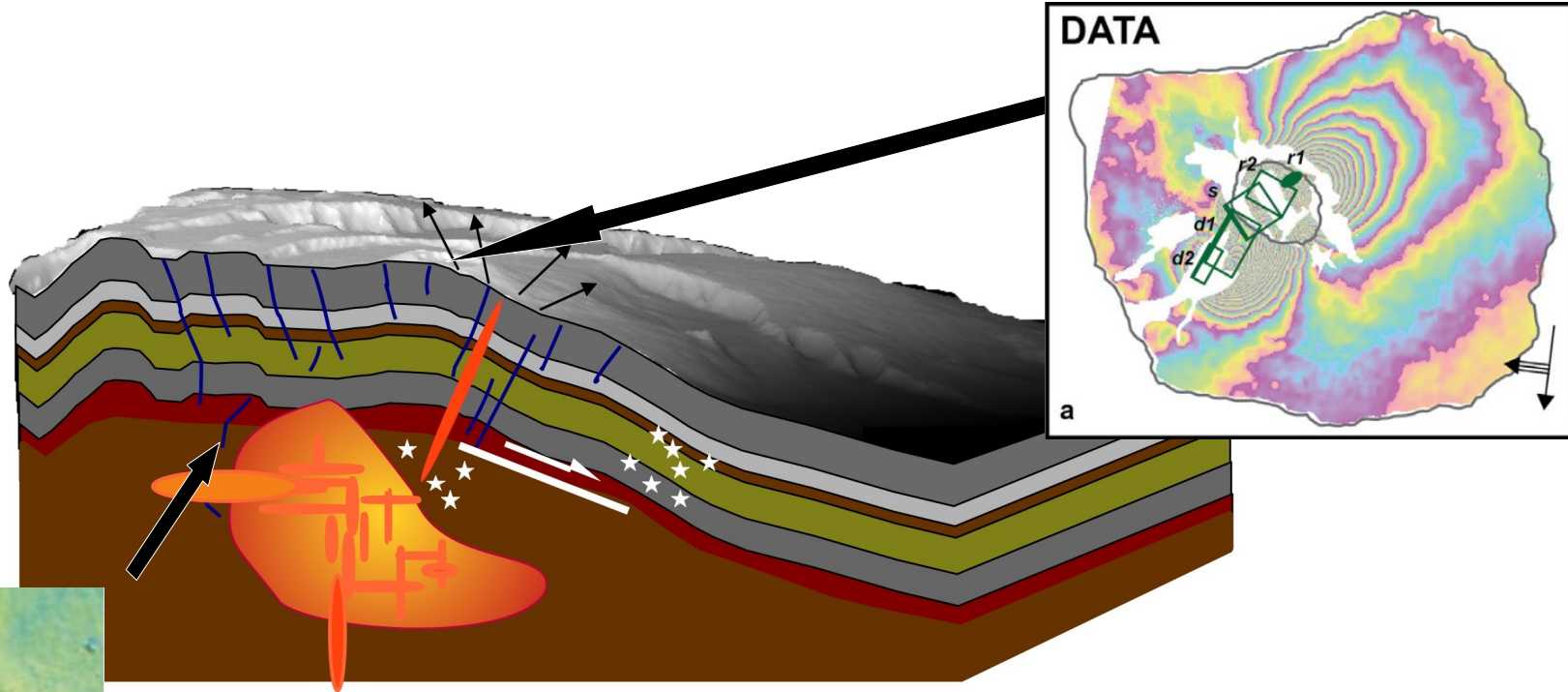


Bagnardi et al., EPSL, 2013

What can be learnt from analytic and kinematic models ?

Intrusion pathways

Bagnardi et al., EPSL, 2013



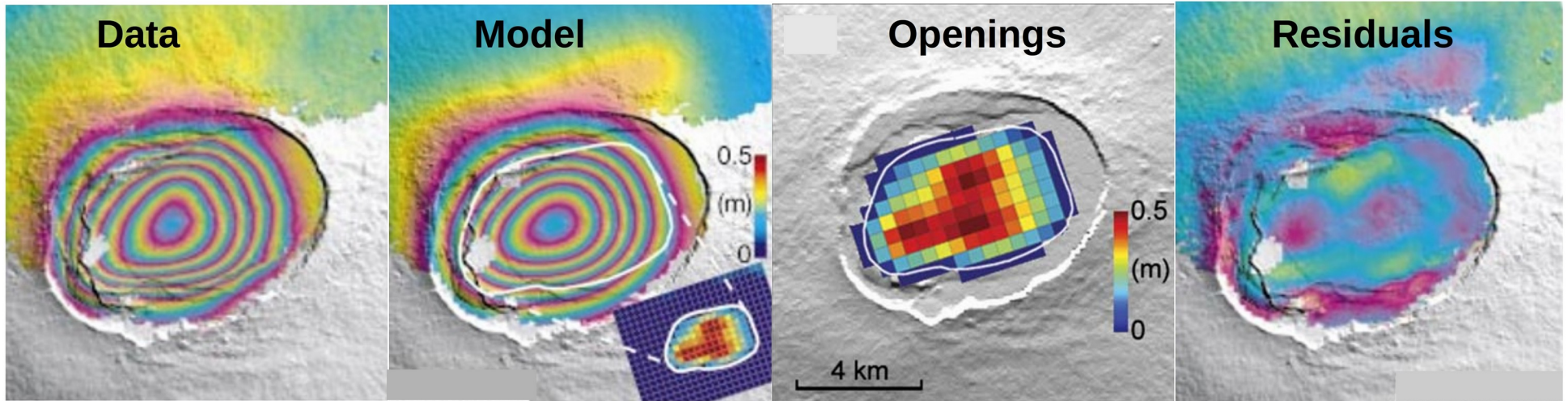
Characteristics of reservoirs

Amelung et al., Science, 2000

Analytic and kinematic models require many parameters

- To better capture openings: linear inversion of **77 parameters**

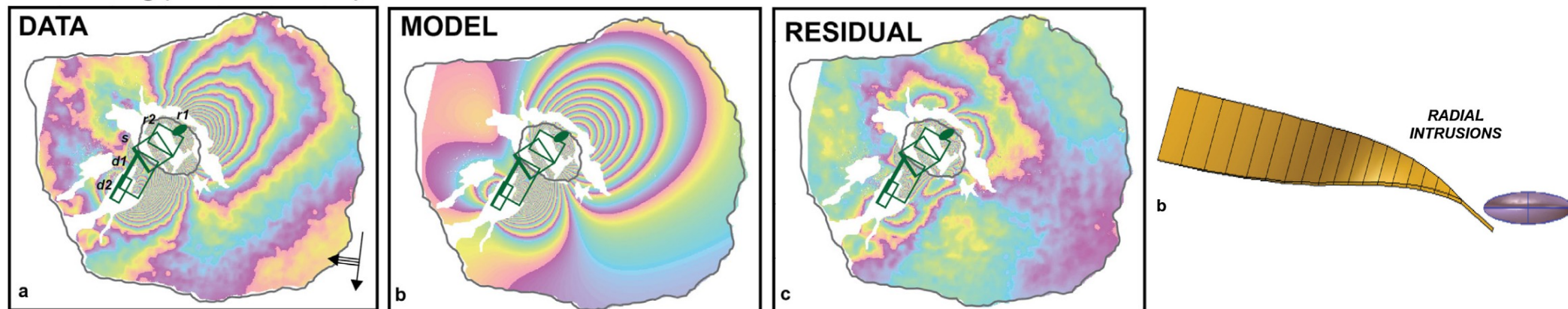
Uplift at Sierra Negra volcano in 1998-99 (Galapagos)



Amelung et al., 2000

- To better capture complex geometries: non linear inversion of **22 parameters**

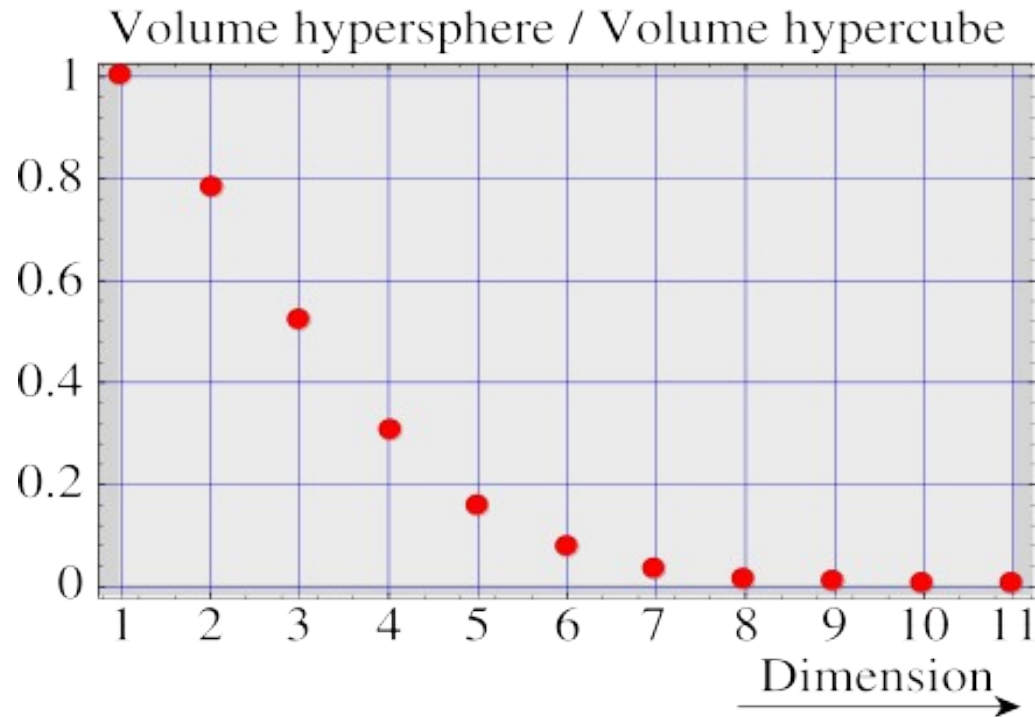
Radial intrusion in 2009 at Fernandina volcano (Galapagos)



Bagnardi et al., EPSL, 2013

Why should large numbers of parameters be avoided ?

- The probability of finding the best-fit solution decreases with the dimension of the search space ;



Tarantola, Inverse Problem Theory, 2005

- With non linear inversions, the search time increases exponentially with the number of parameters ;
- There is a risk of overfitting the data.

➡ Use of Akaike Information Criteria

$$\text{AIC} = 2 \cdot k + \chi^2 + \text{cst} \text{ with } k = \text{Nb parameters and } \chi^2 = \text{cost-function}$$

3D Mixed Boundary Elements

Cayol et Cornet, Int. J. Rock Mech. Min. Sc., 1997;
Cayol and Cornet, JGR, 1998; Cayol et al., JGR 2014

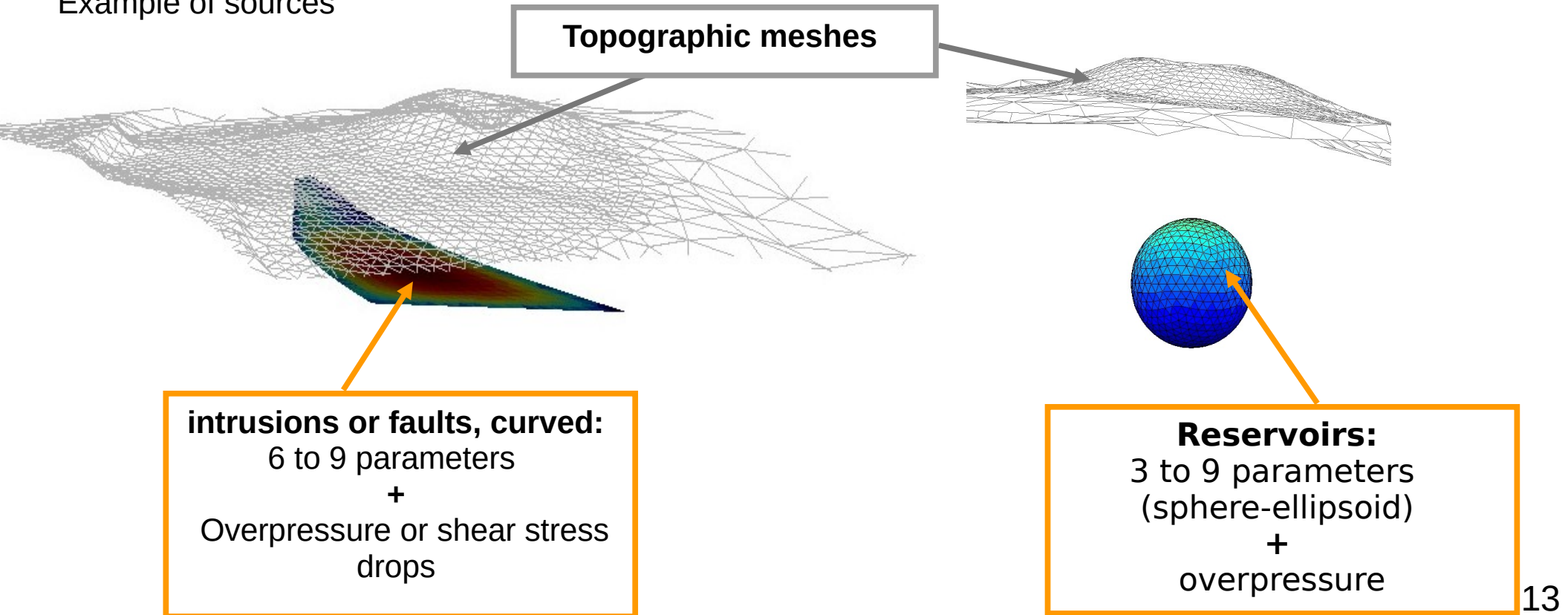
3D Numerical method:

- **Realistic topographies;**
- **Any number and geometry of fractures and pressure sources;**
- **Treats more than one source appropriately (interactions are taken into account);**

Assumptions:

- intrusions, faults, reservoirs are submitted to **constant stress** changes;
- Fractures may be **curved**.

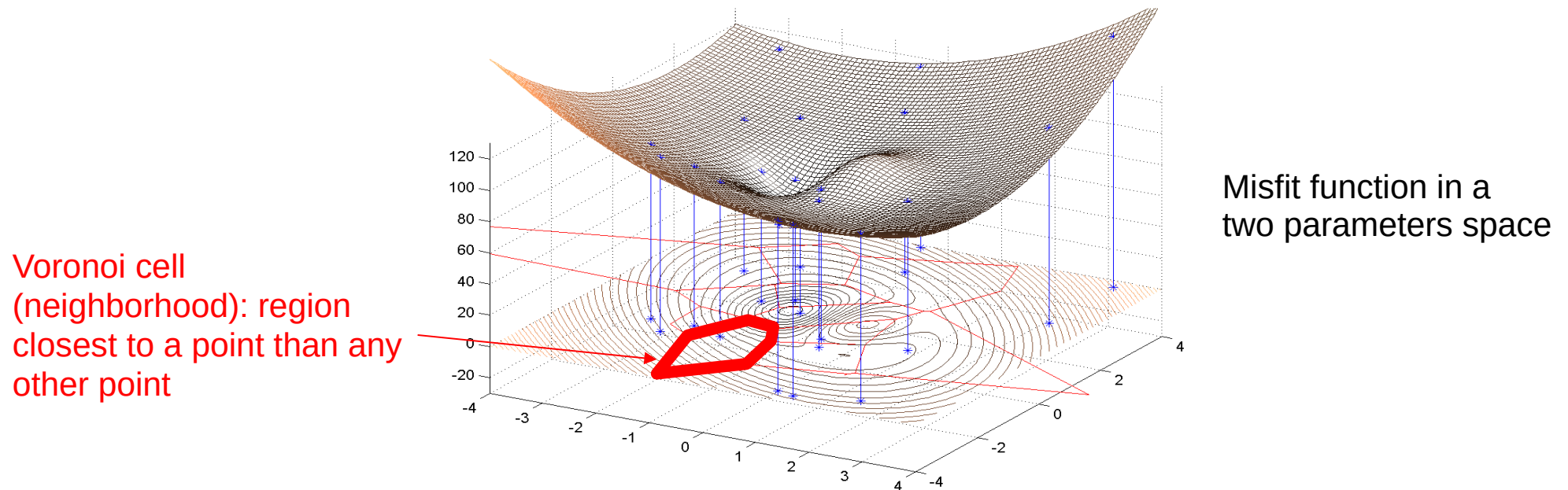
Example of sources



Combined with Non-linear inversions relying on a Neighborhood Algorithm

Fukushima et al., JGR, 2005

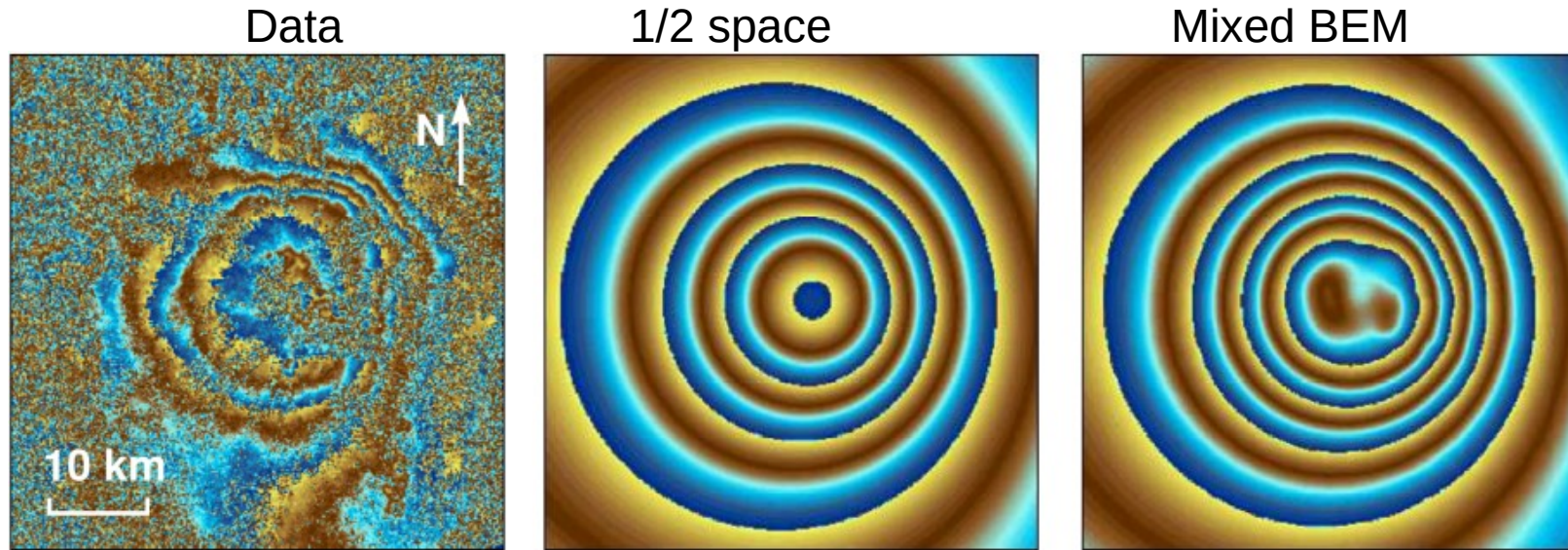
Near-neighborhood non-linear inversion (*Sambridge, JGI, 1999a*) to invert for geometrical parameters



- Linear inversions of pressure;
- Appraisal of model using Bayesian inference → confidence intervals and trade-offs between parameters (*Sambridge, JGI, 1999b*)
- Available to registered users at <http://www.opgc.fr/defvolc>

Topography is taken into account

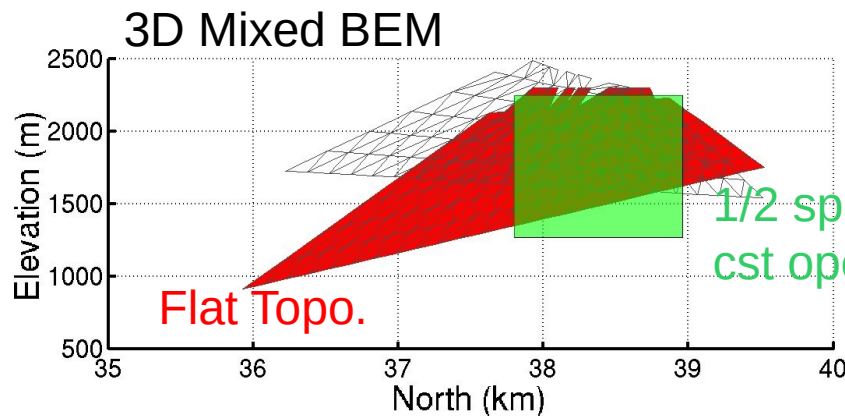
Topographies have an influence of computed displacements



Etna, 1992-1993 eruption,
Massonnet et al., Nature, 1995

Cayol and Cornet, GRL, 1998

Neglecting topographies bias results : volume errors, depths errors



Volume with Okada (1985) : 80 % overestimation

Max depth with Okada : 40 % overestimation

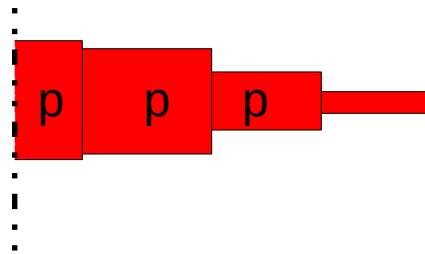
Fukushima et al., JGR, 2005

Crozier et al., Bull. Volc., 2023

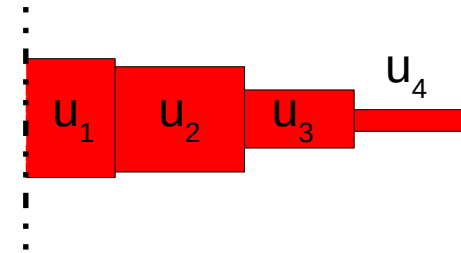
Boundary conditions are homogeneous stress



Field observation

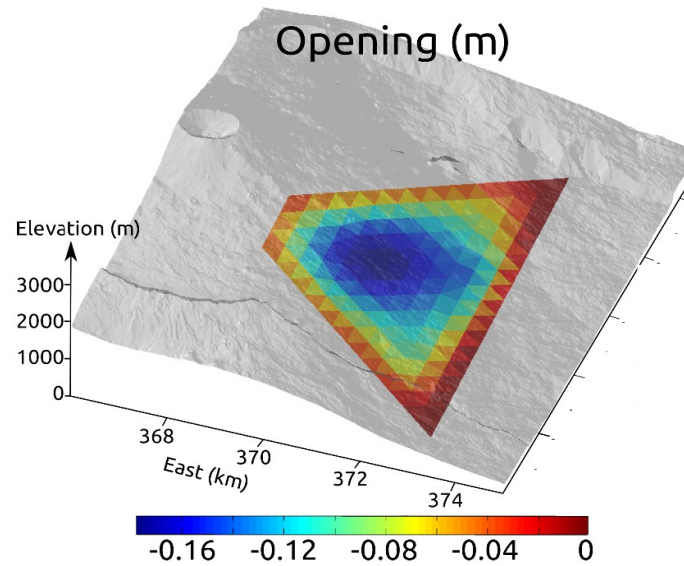


Pressure boundary condition

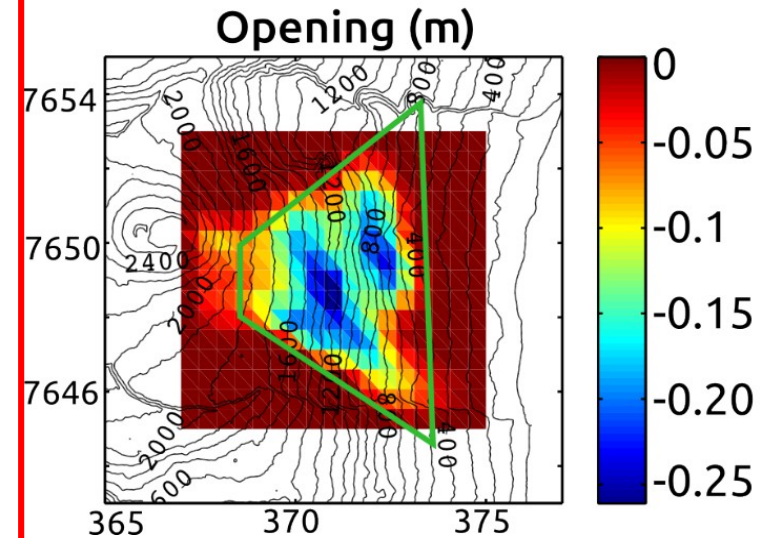


Displacement boundary condition : kinematic models

Inverted openings



One parameter
~ 5 % of inverse models



Tridon et al., JGR, 2016

500 parameters
~ 95 % of inverse models

These studies

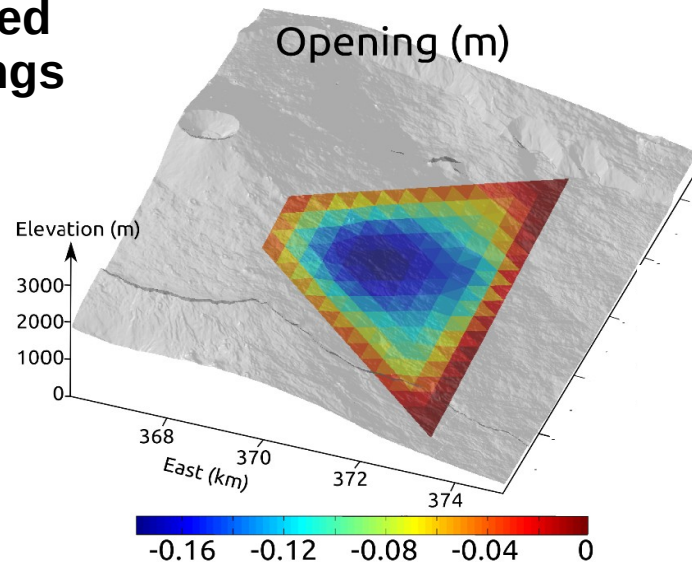
Stress boundary conditions leads to better models

$$\text{AIC} = 2 \cdot k + \chi^2 + \text{cst} \text{ with } k = \text{Nb parameters and } \chi^2 = (\mathbf{u}_o - \mathbf{u}_m)^T \mathbf{C}_d^{-1} (\mathbf{u}_o - \mathbf{u}_m)$$

Pressure boundary condition

Displacement boundary condition

Inverted openings

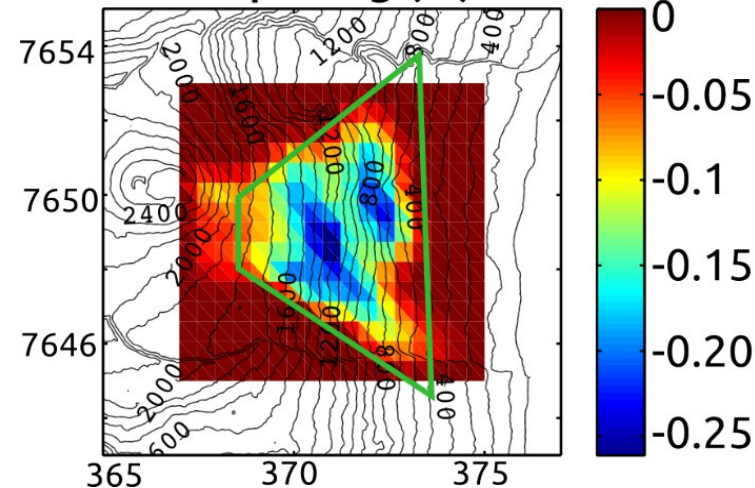


One parameter

$$\chi^2 \approx 2784$$

$$\text{AIC} \approx 2800$$

Opening (m)



500 parameters

$$\chi^2 \approx 2300$$

$$\text{AIC} \approx 3300$$

Tridon et al., JGR, 2016

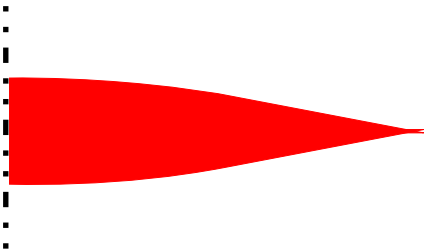
- ➡ Models with stress boundary conditions require less inversion parameters
- ➡ Inverting for stress leads to **better** models than inverting for dislocation amplitudes

Models with stress boundary conditions are closer to the physics

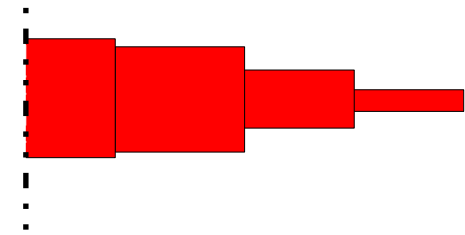
Field observation



Models



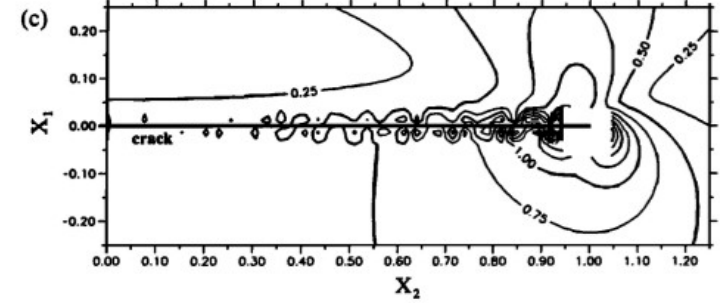
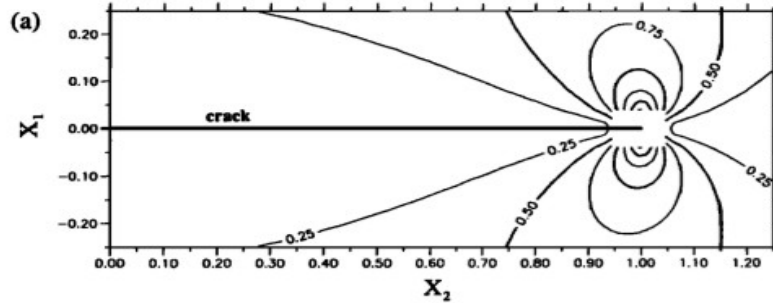
Pressure boundary condition



Displacement boundary condition : kinematic models

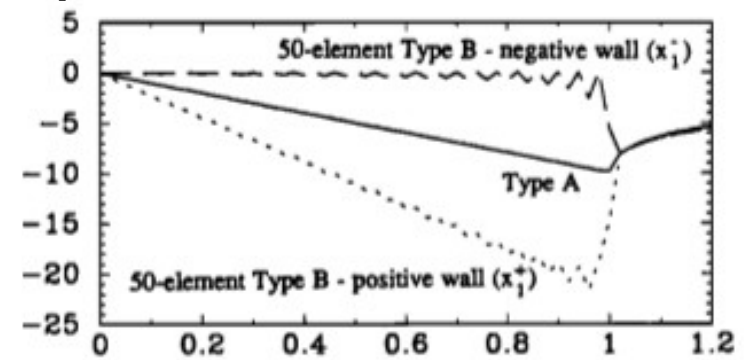
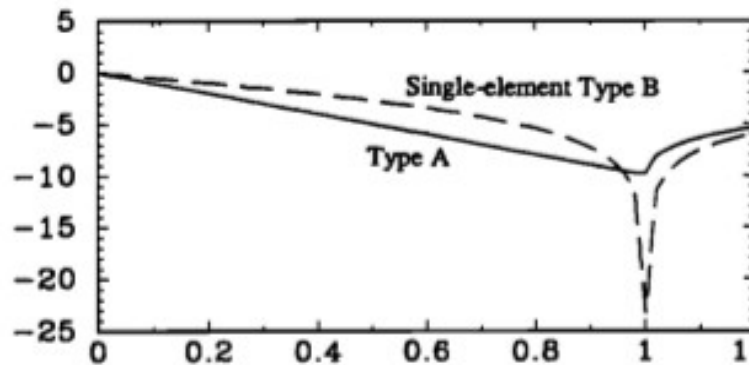
Less stress singularities

Fracture stress



Fracture shear displacement

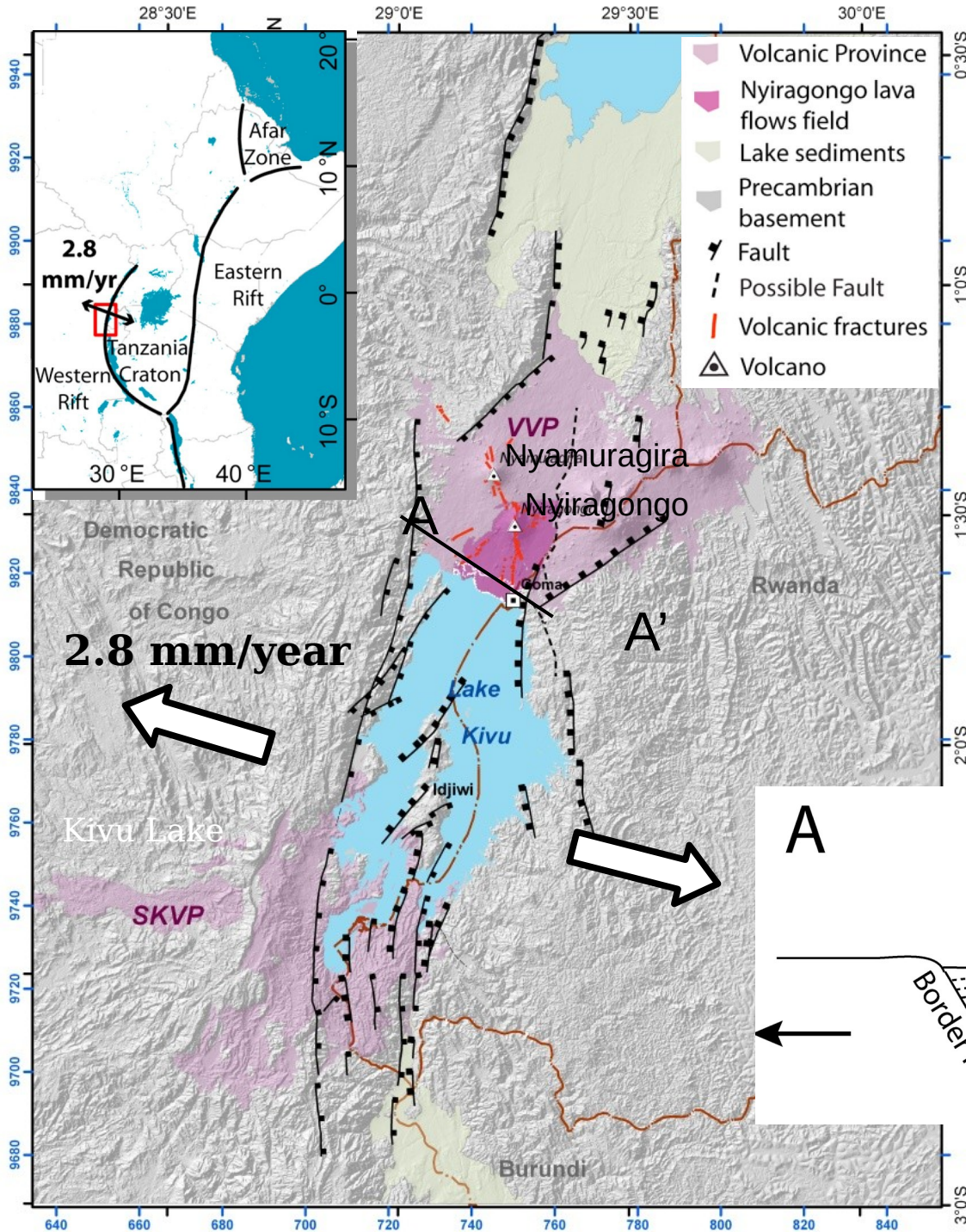
Shear displacements are determined



Zeller and Pollard, JGR, 1992

Stress change inversion as crustal stress gauge

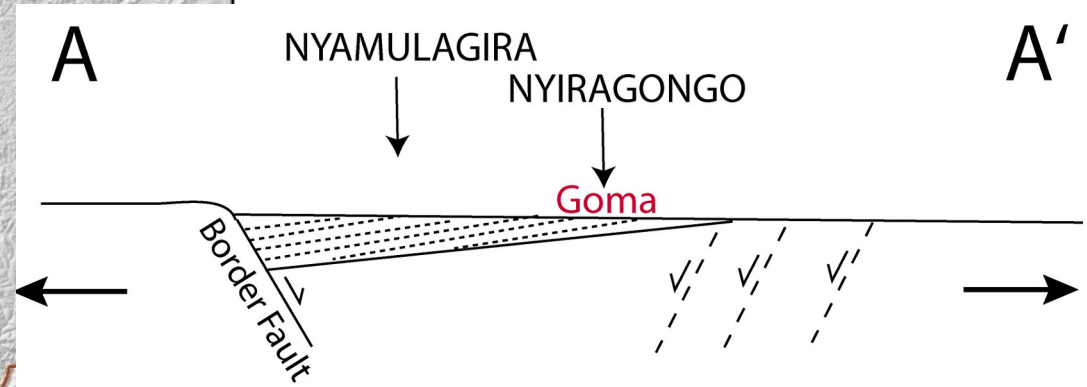
What drives and accommodates rift extension in Kivu ?



The Virunga volcanic Province (VVP)



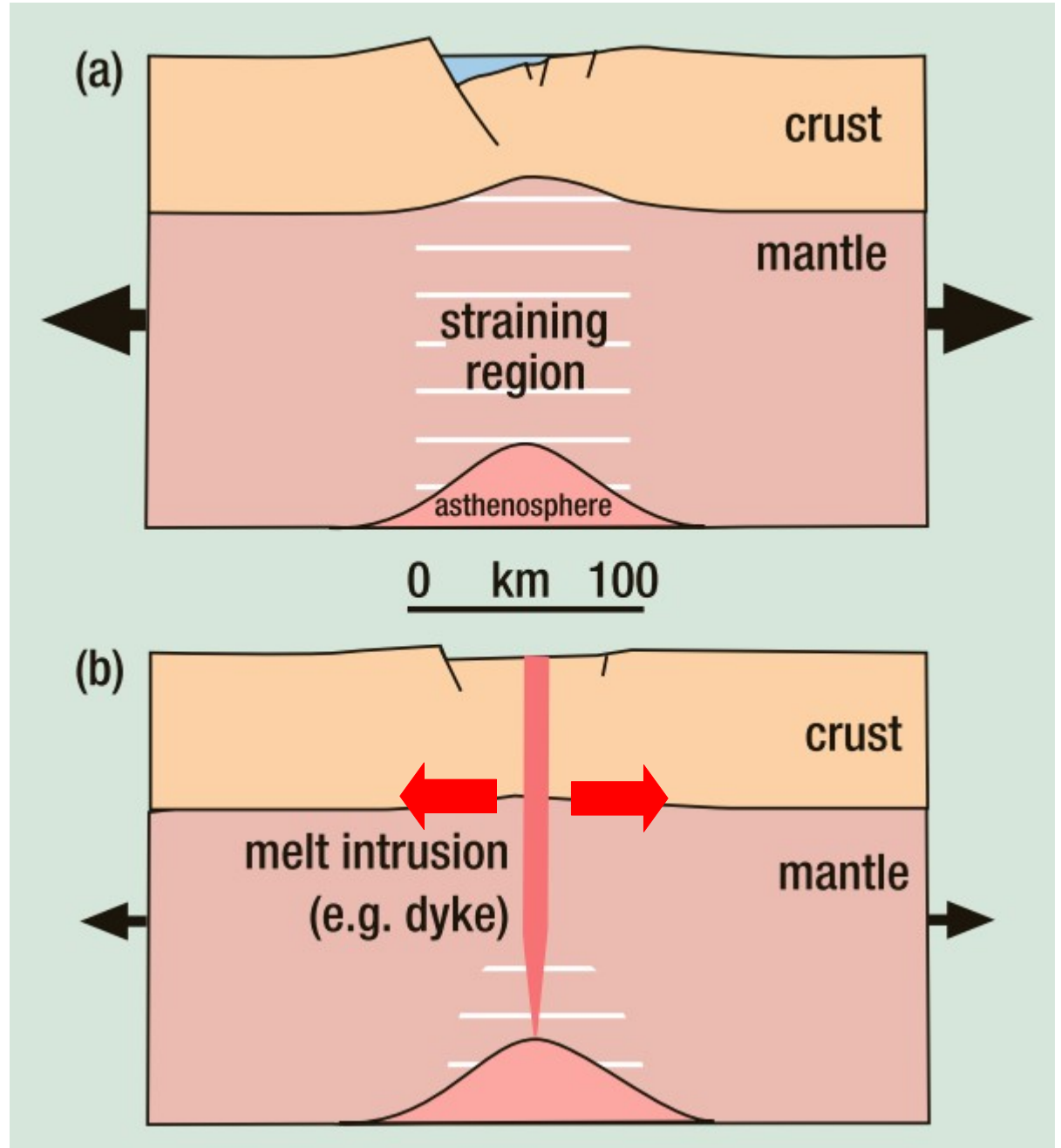
Nyiragongo, Goma and its suburb of 2,1 M inhabitants



Wauthier et al., JGR, 2012

What drives and accommodates rift extension in Kivu ?

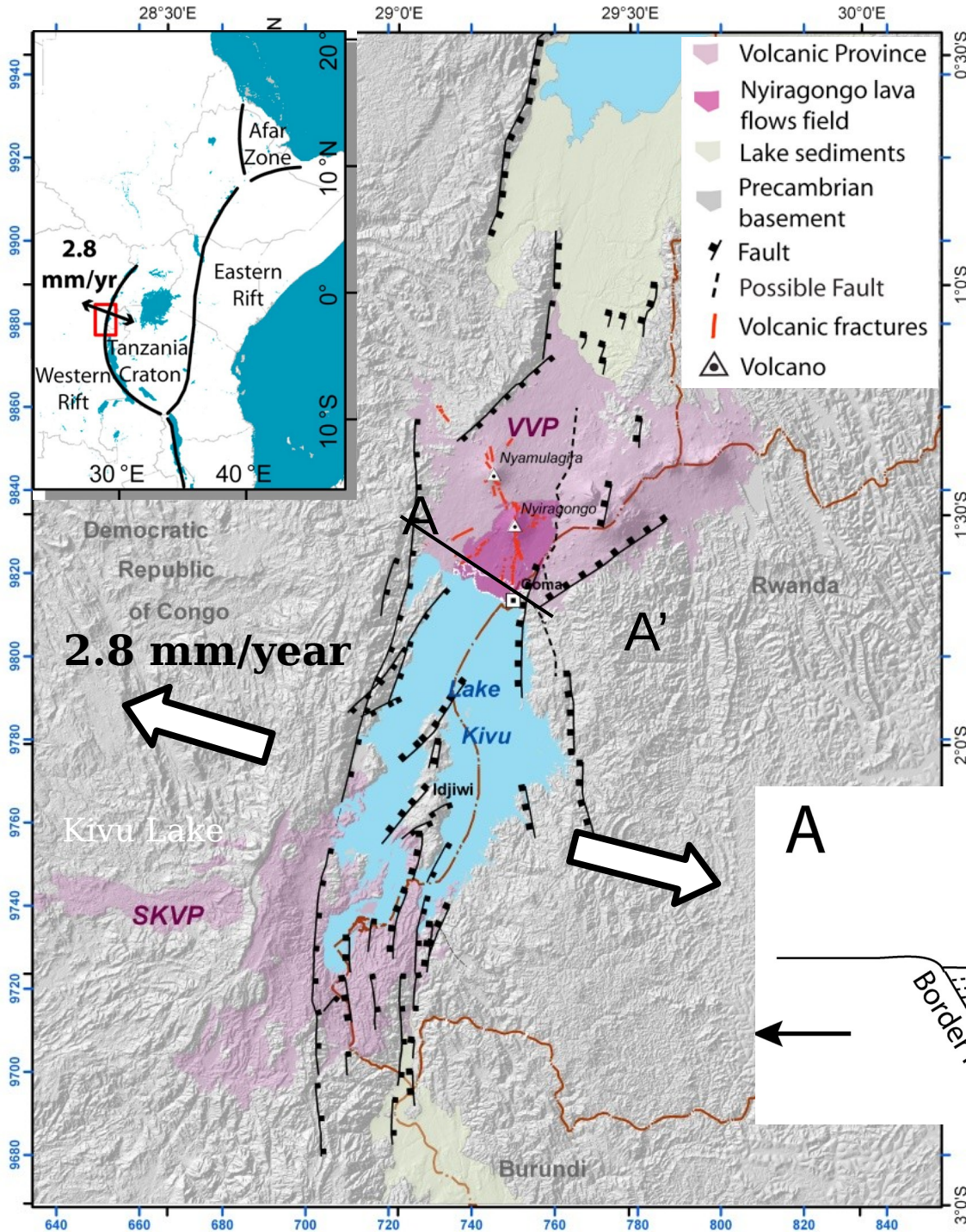
Tectonic stresses and
Faulting assisted extension ?



Magma assisted extension ?

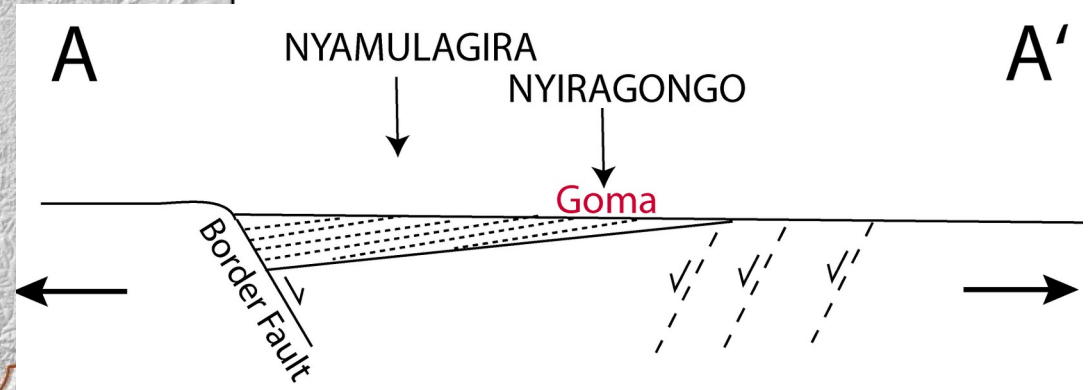
(Ebinger, *Astronomy and Geophysics*, 2005)

What drives and accommodates rift extension in Kivu ?



In the Virunga Basin:

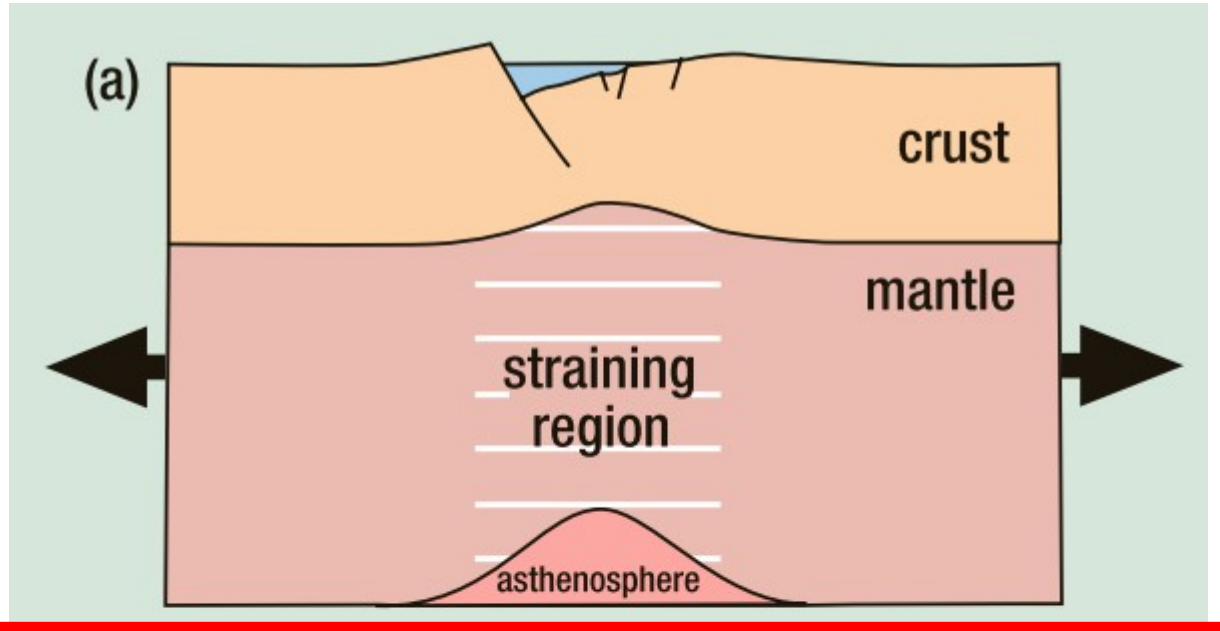
- Only 15% of crustal extension
- Extension is accommodated by western border detachment faults, Ebinger, Geol. Soc. Am. Bull, 1989



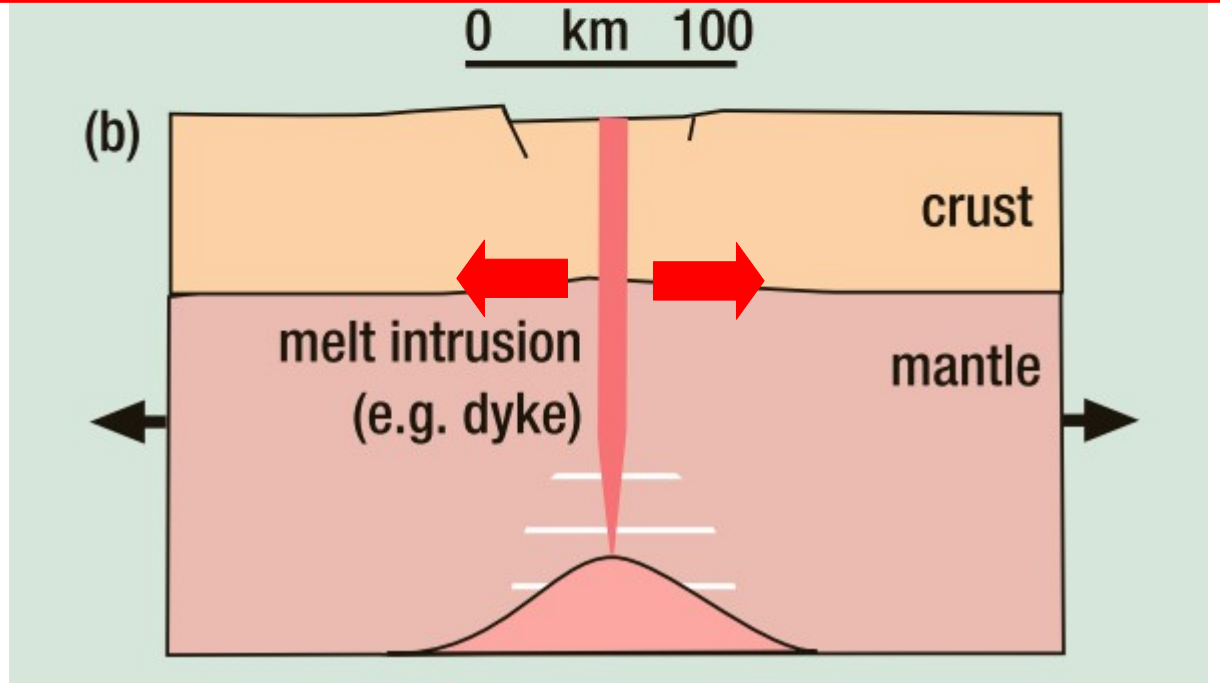
Wauthier et al., JGR, 2012

What drives and accommodates rift extension in Kivu ?

Tectonic stresses ?

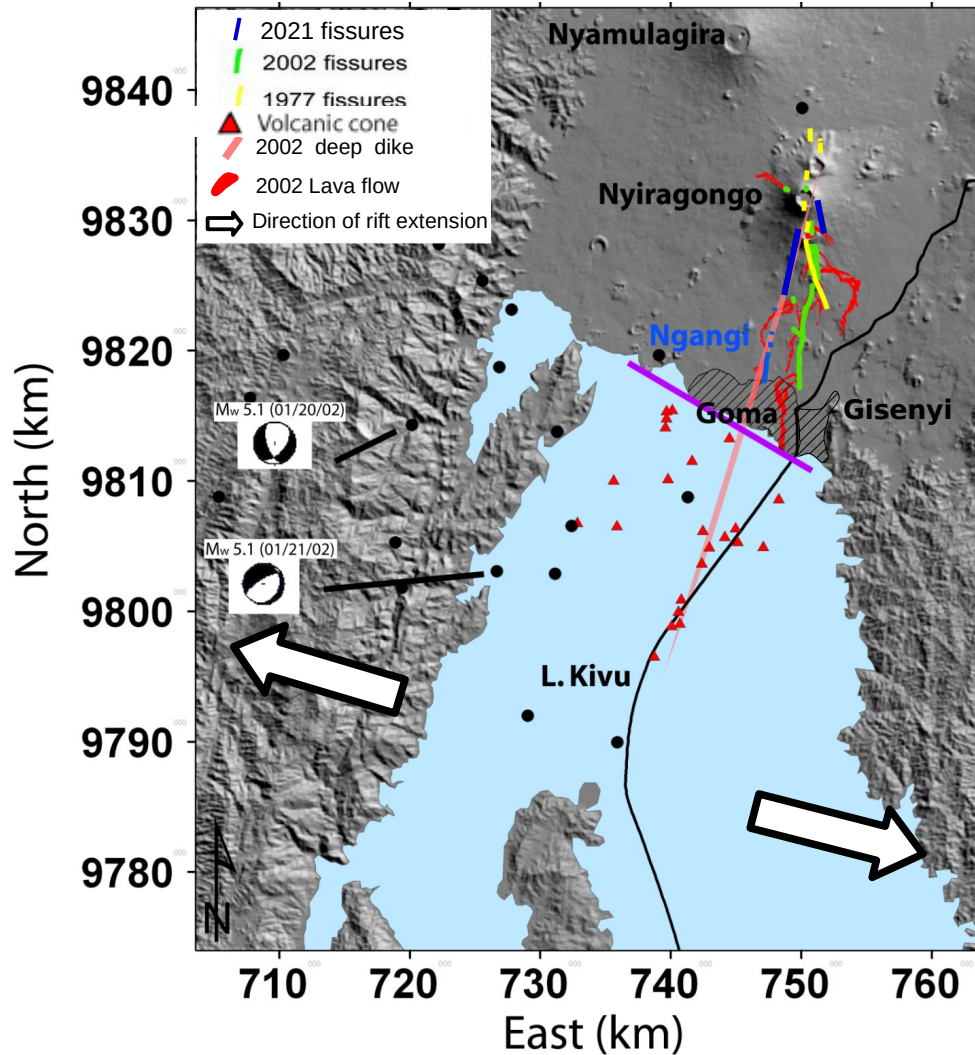


Magma assisted extension ?



(Ebinger, *Astronomy and Geophysics*, 2005)

Nyiragongo 2002 and 2021 eruption Democratic Republic of the Congo

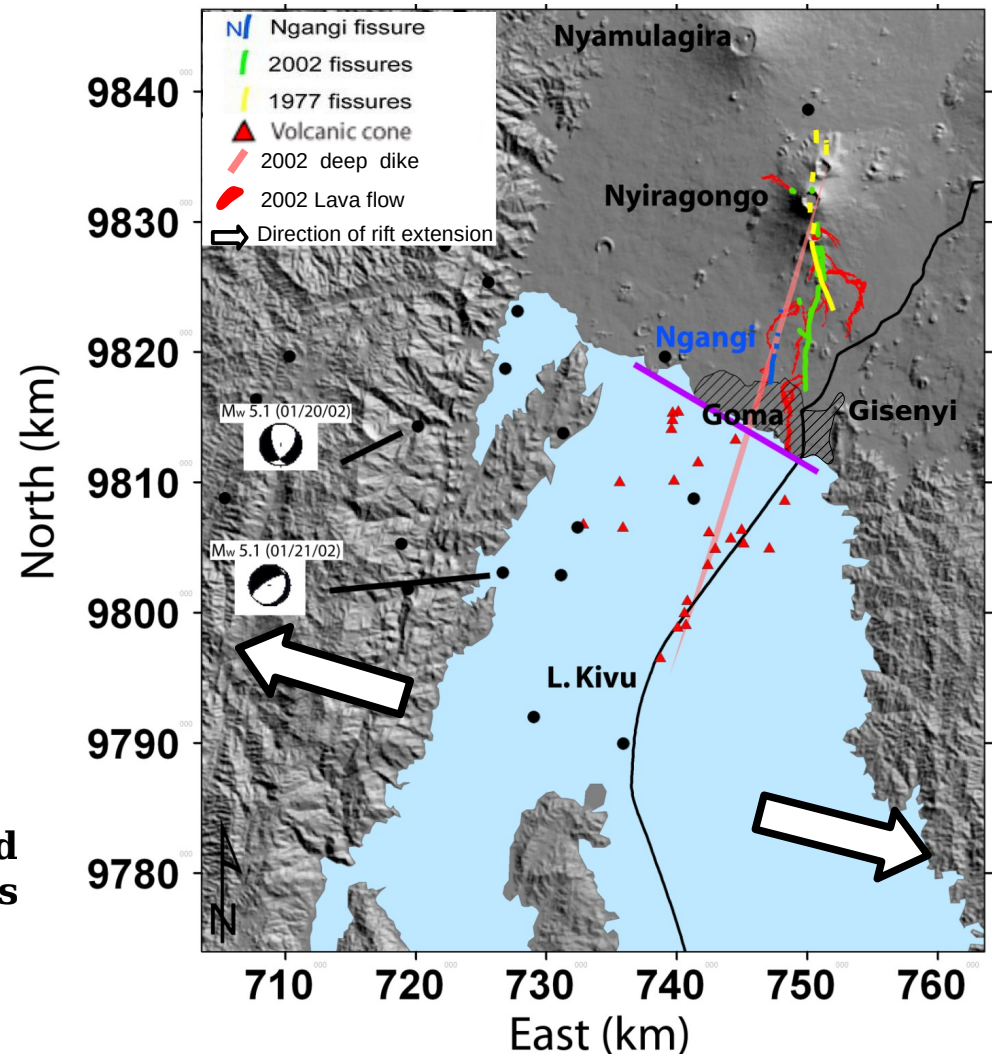
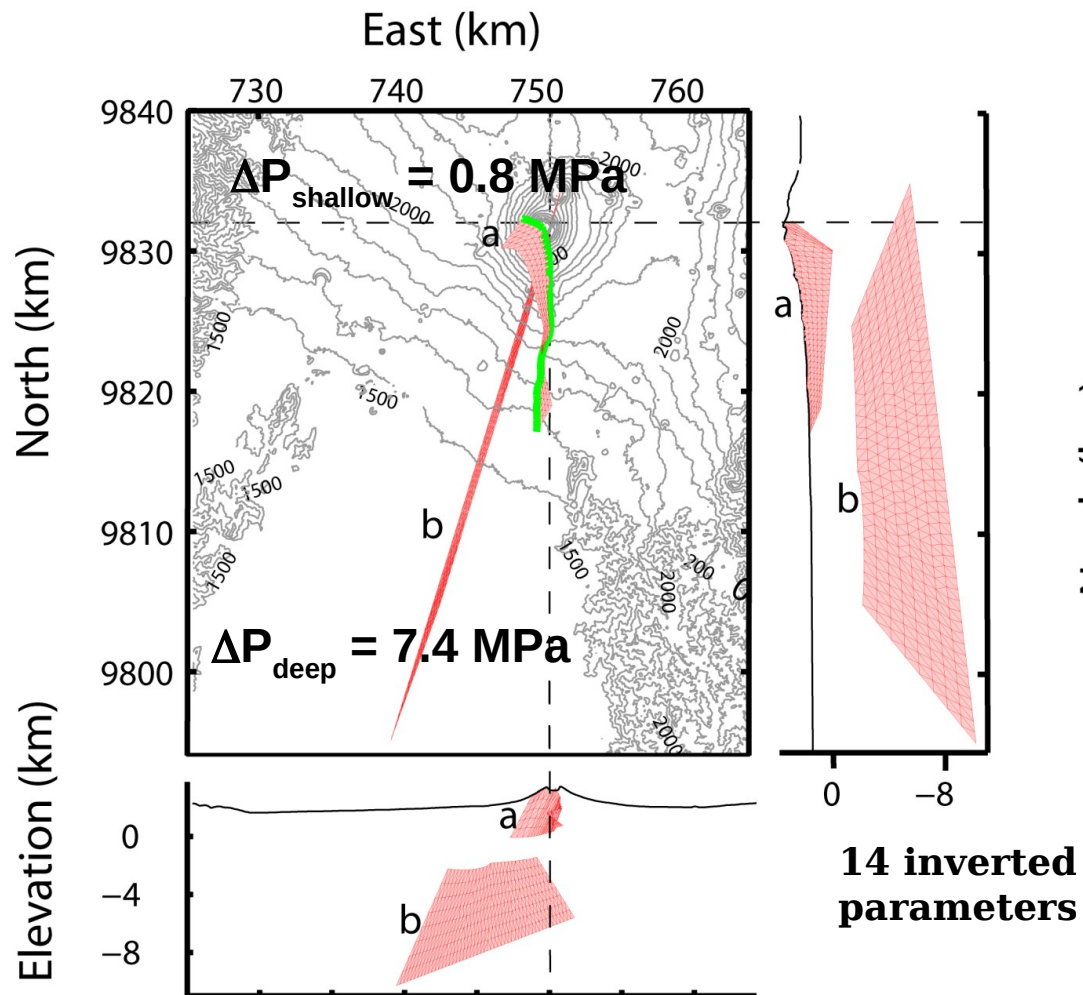


- A strato-volcano with a crater lava lake



- Three historical eruptions in 1977, 2002 and 2021
- Associated fissures trend NS

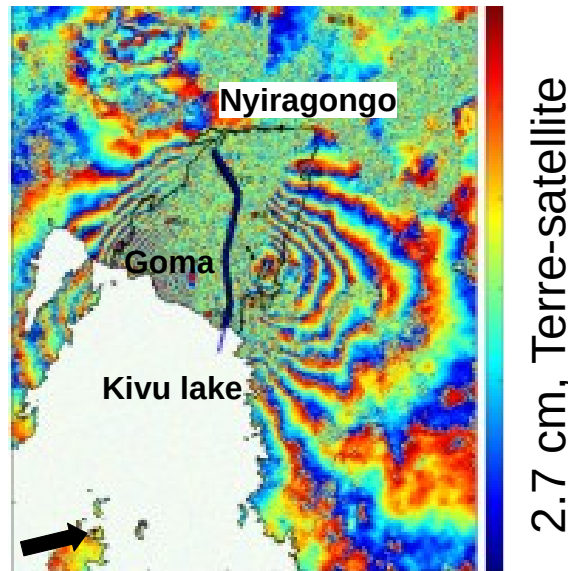
Model for Nyiragongo 2002 eruption (Wauthier et al., JGR, 2012)



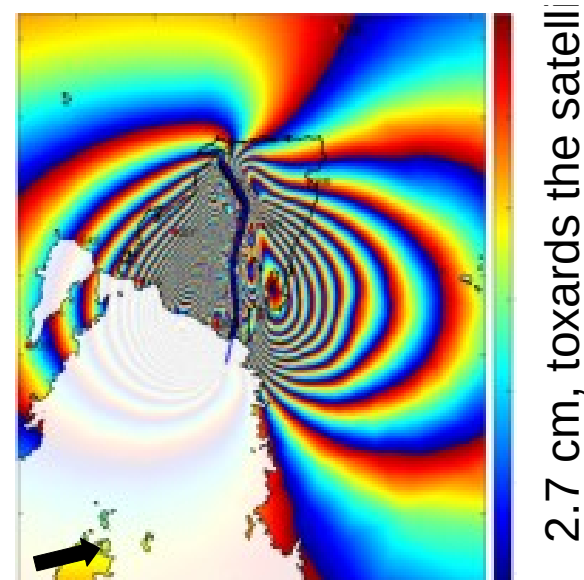
- The deep dike is perpendicular to the rift extension direction → Injection direction guided by the rift extension

The may 2021 eruption confirms the small overpressure

InSAR data

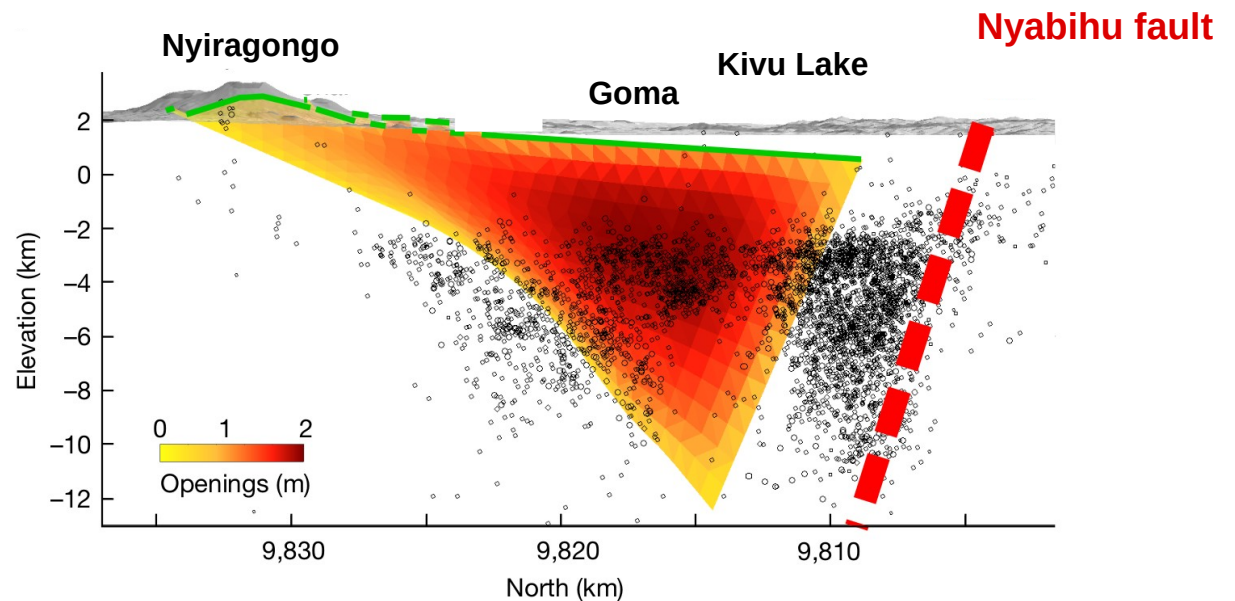


Model



7 inverted parameters

$$\Delta P_{\text{shallow}} = 0.8 \text{ MPa}$$

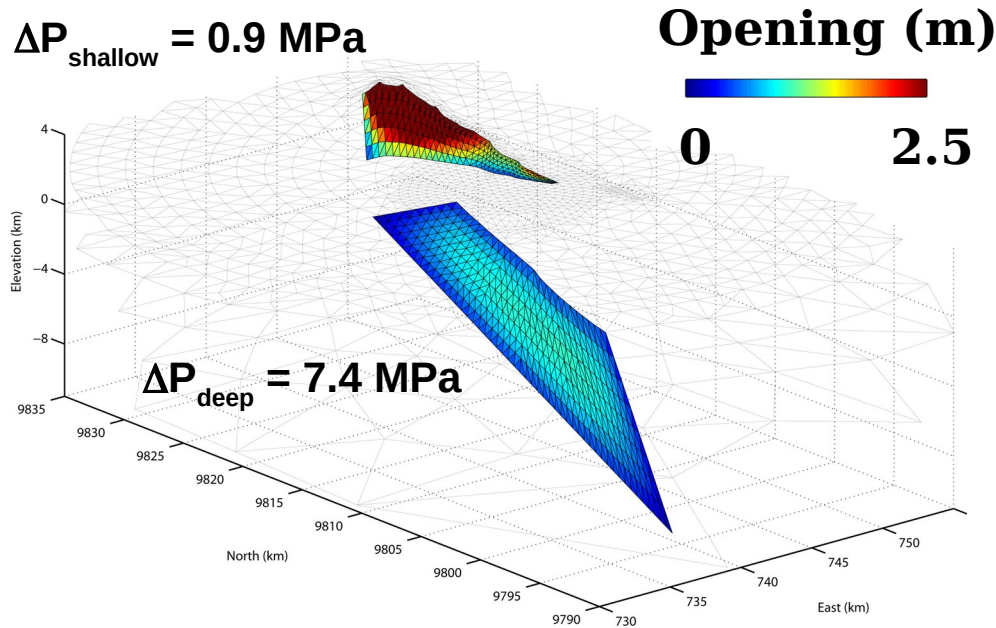


Smittarello et al., Nature, 2022

A magma-assisted rift rextension

Overpressure from InSAR data inversion:

Wauthier et al., JGR, 2012

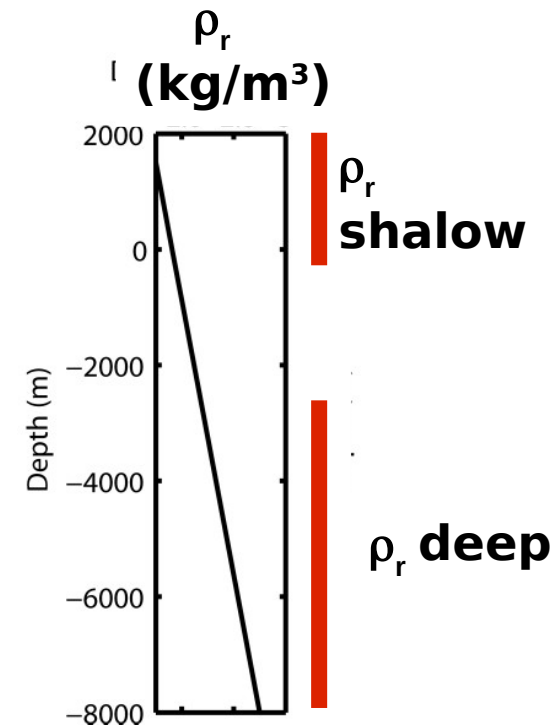


Overpressure theoretical model:

Assumption: crustal stresses are lithostatic ($\sigma_h \sim \sigma_v \sim P_{\text{rock}}$),

$$\Delta P(z_{\text{dike}}) = P_{\text{magma}} - P_{\text{rock}} = \int (\rho_m - \rho_r(z)) g dz,$$

with:



we get $\Delta P_{\text{Shallow}} \sim 1 \text{ MPa}$ and $\Delta P_{\text{Deep}} = 4.5 \text{ MPa}$



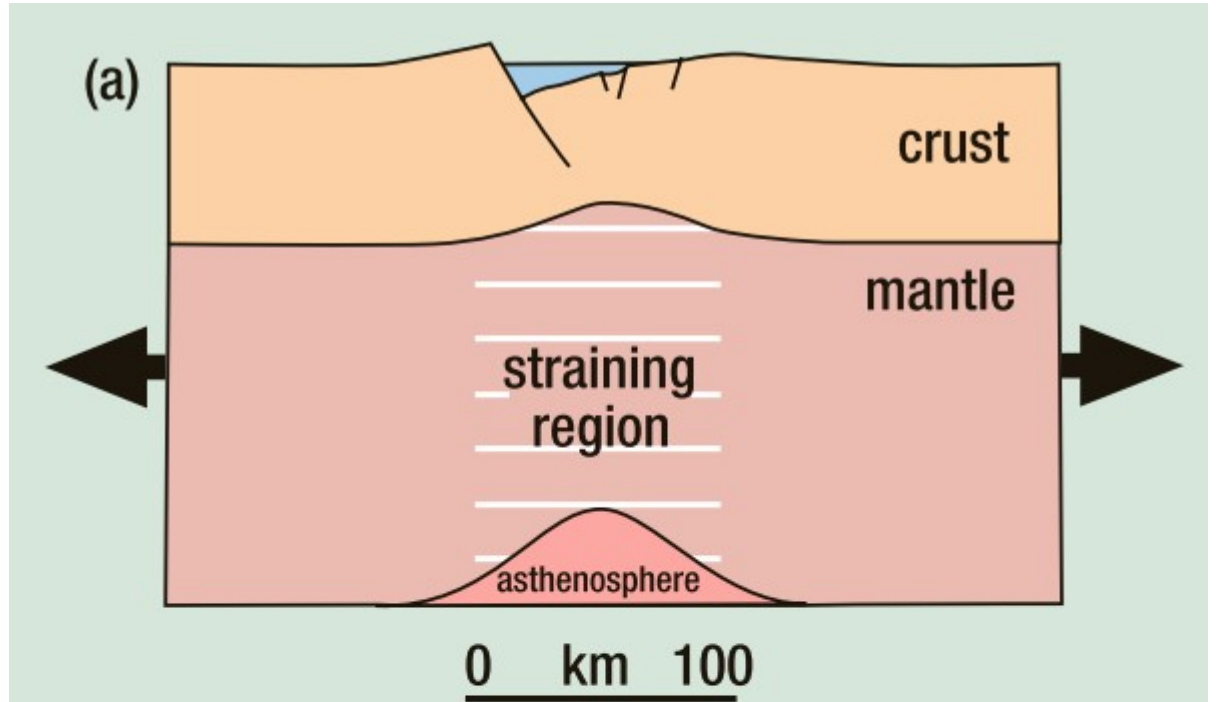
**The crust is at a lithostatic stress state :
Unconsistent with a rift extension driven by plate separation**



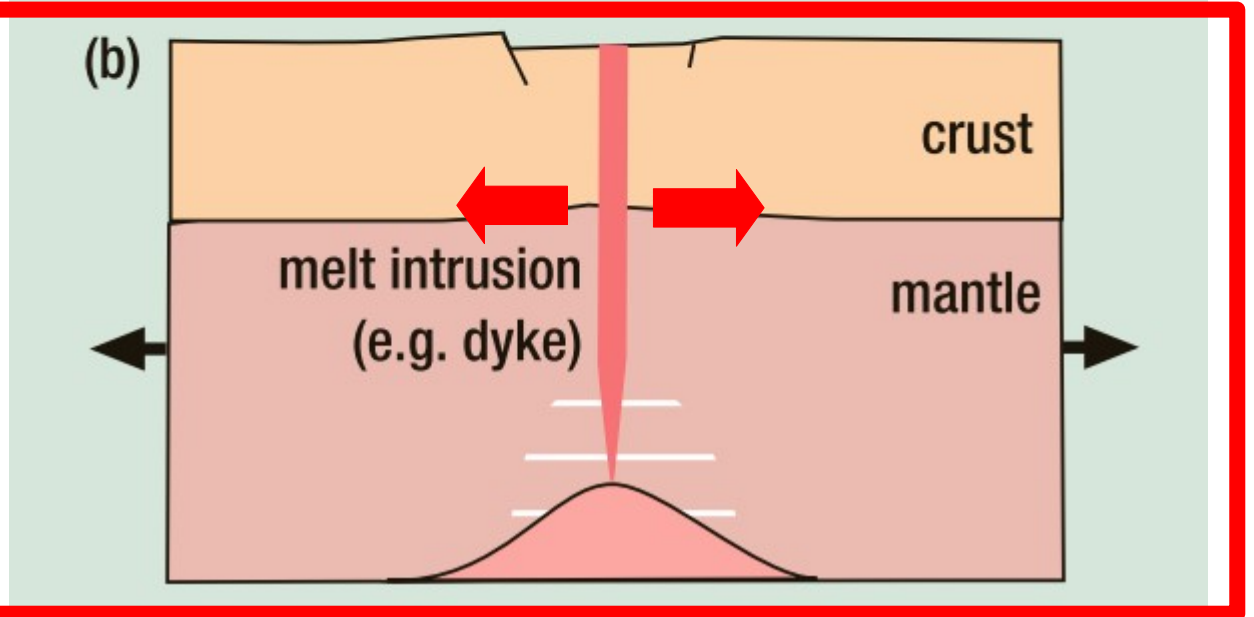
The rift extension is driven by the magmatic activity

What drives and accommodates rift extension in Kivu ?

Tectonic stresses ?



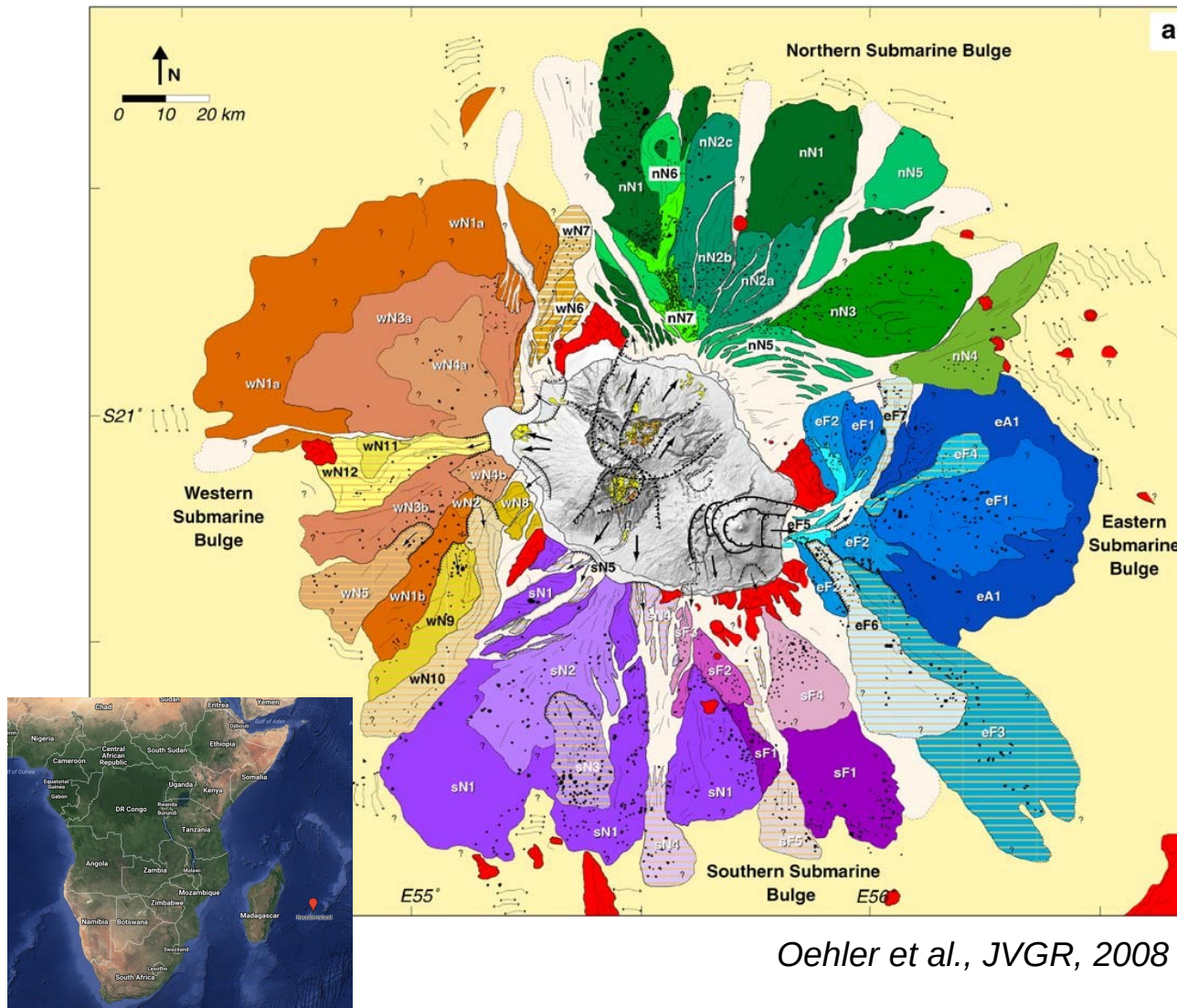
Magma assisted extension ?



Stress change inversion for flank failure mechanisms

Flank failures at Réunion Island

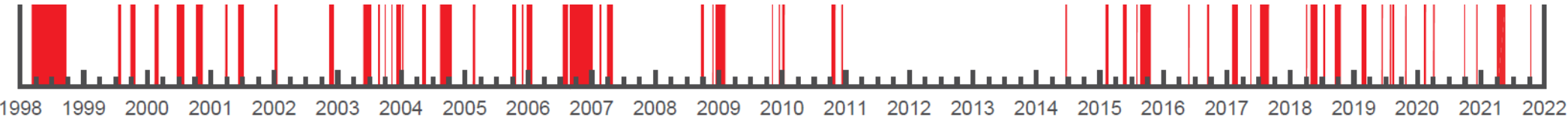
- Induce 24 % of volcano casualties (tsunamis and large earthquakes)
- Ubiquitous at Réunion Island



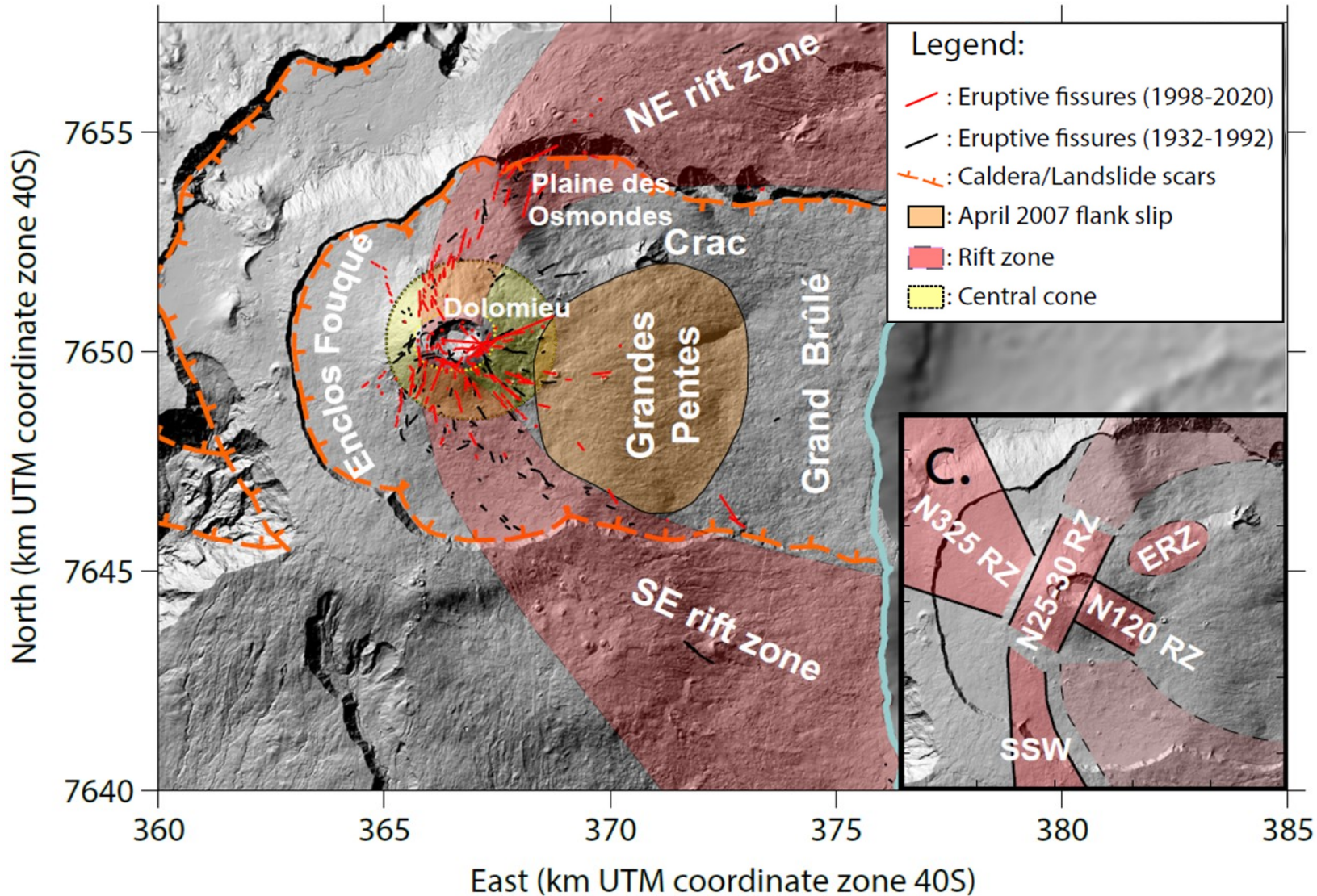
47 flank failure events
Largest 100 km³
Oldest 2 My

Oehler et al., JVGR, 2008

Piton de la Fournaise is very active: 59 intrusions since 1998 (2.3/year)

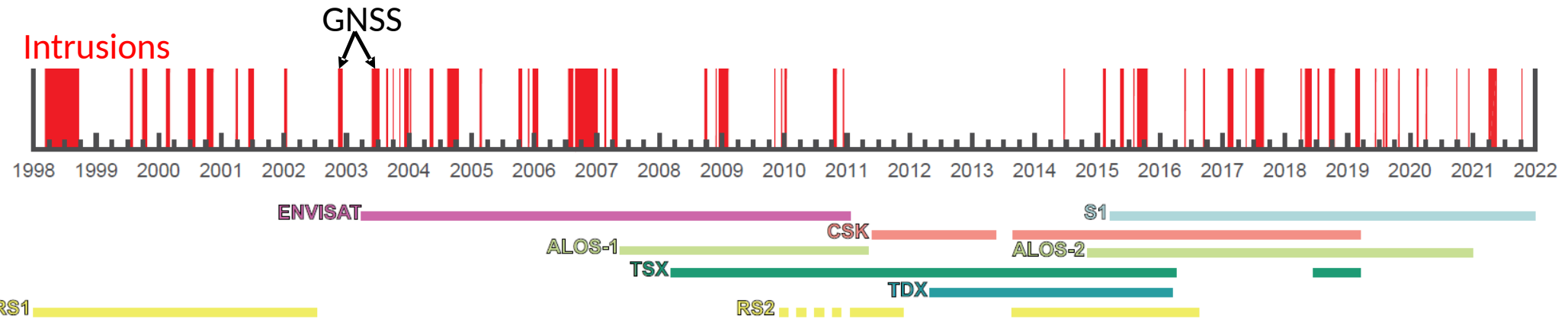


Eruptive fissures : 1932 - 2020

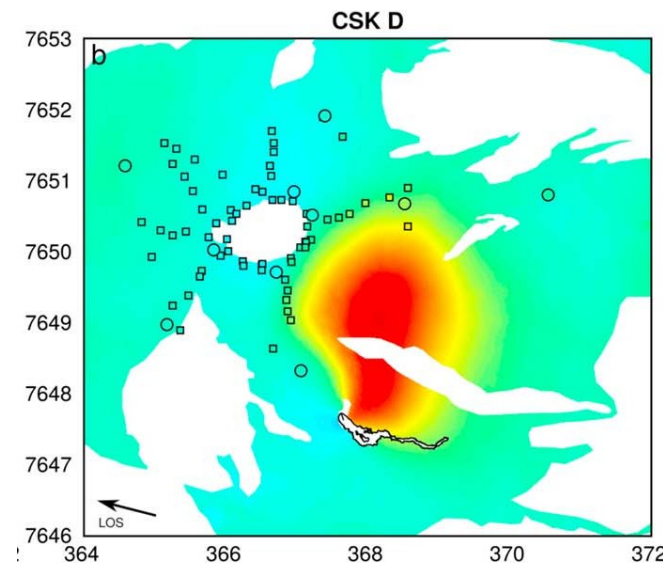


Piton de la Fournaise is one of the best monitored volcanoes

- since 1998, 57/59 intrusions imaged by at least one InSAR data

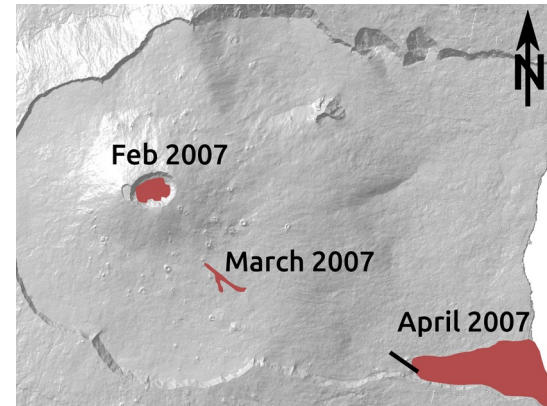
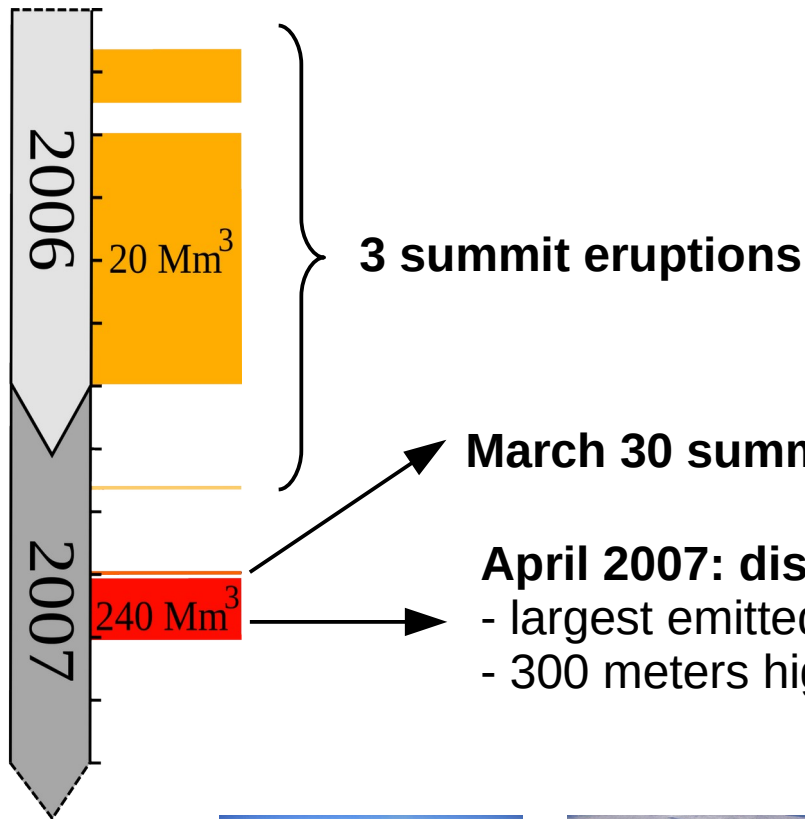


- GNSS campaign + continuous data can be used for non imaged eruptions



- Campaign GNSS
- Continuous GNSS

An unusual flank displacement in 2007



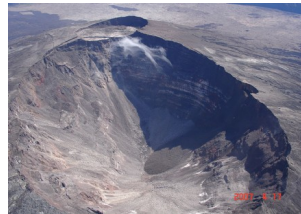
March 30 summit eruption

April 2007: distal eruption

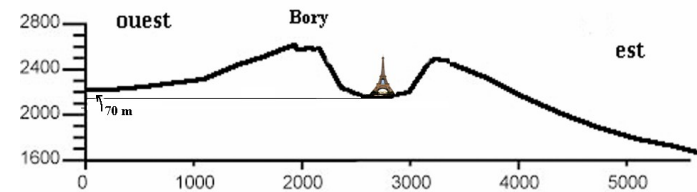
- largest emitted volume of XX, XXI century = 240 Mm³
- 300 meters high caldera collapse on April 6



September 2006

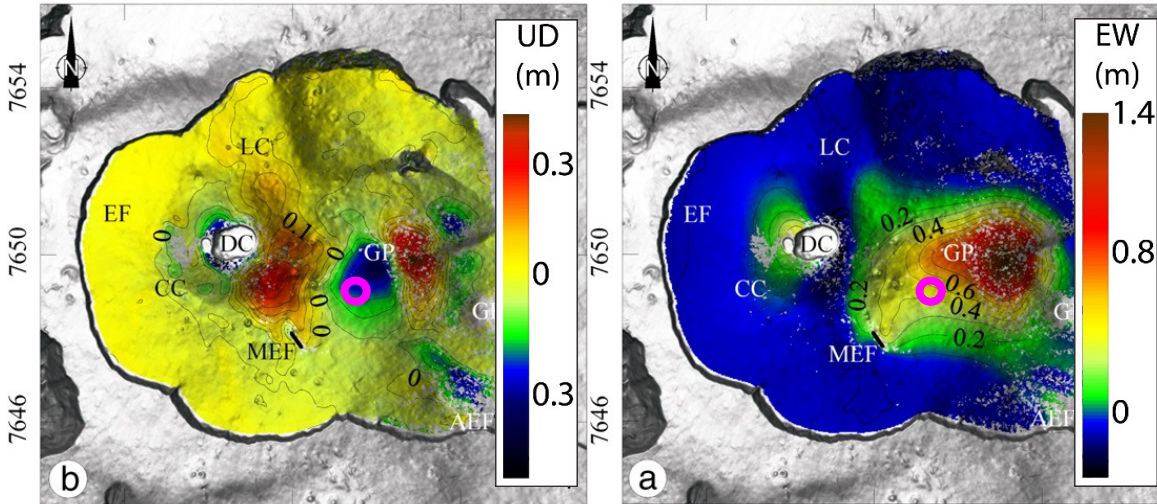


April 2007

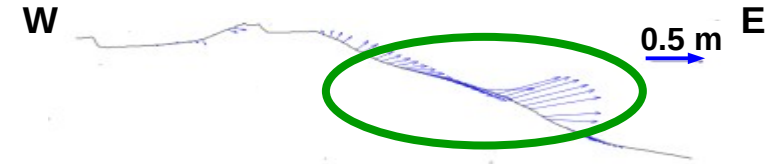


An unusual flank displacement in 2007

Co-eruptive displacement

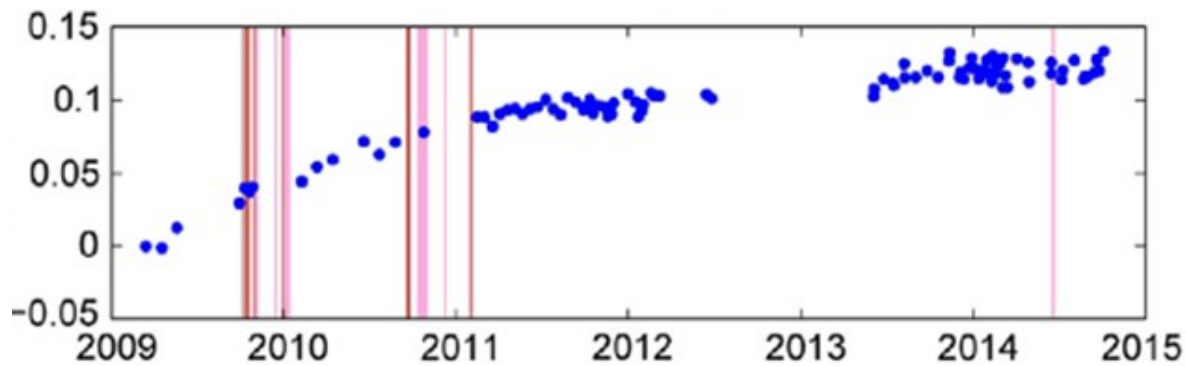


Froger et al., JVGR, 2015



1.4 m eastward / 0.37 m uplift

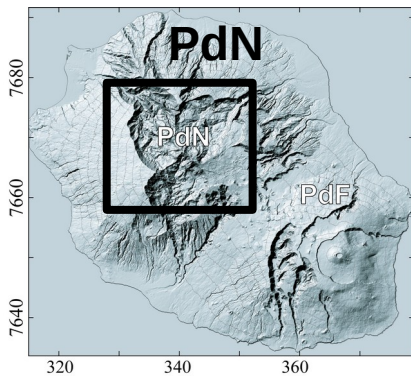
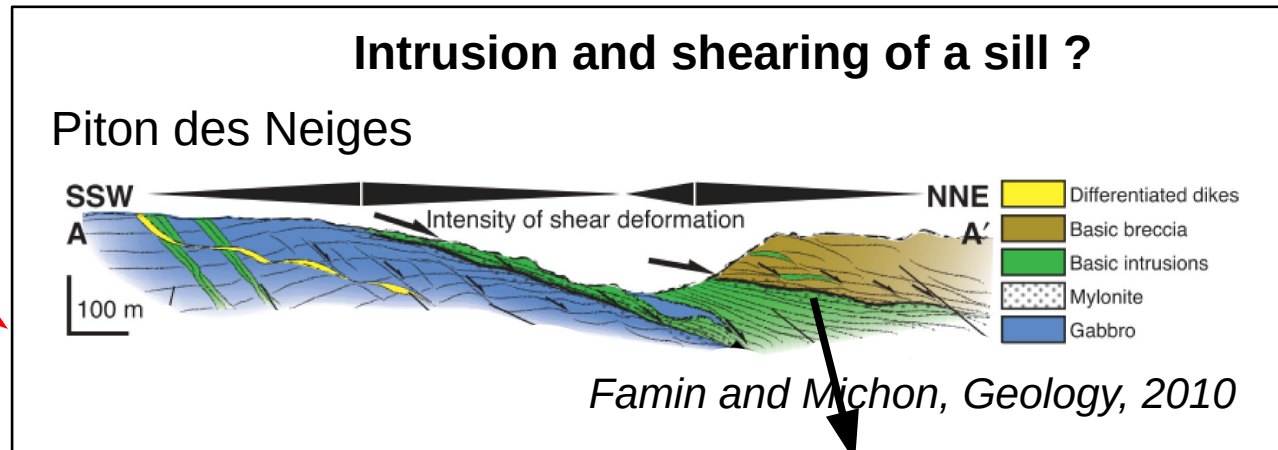
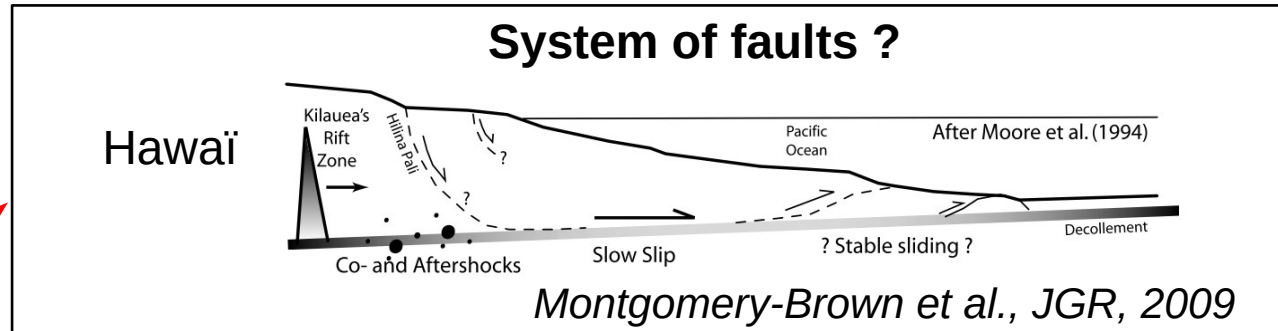
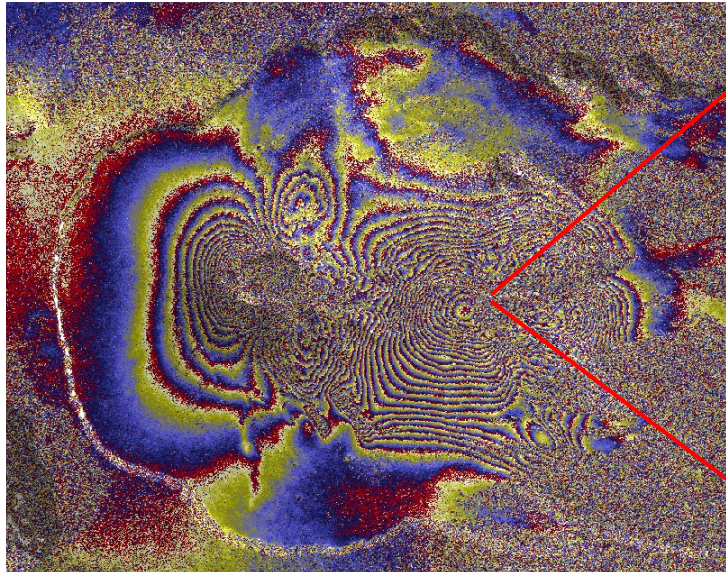
Long-term EW displacement



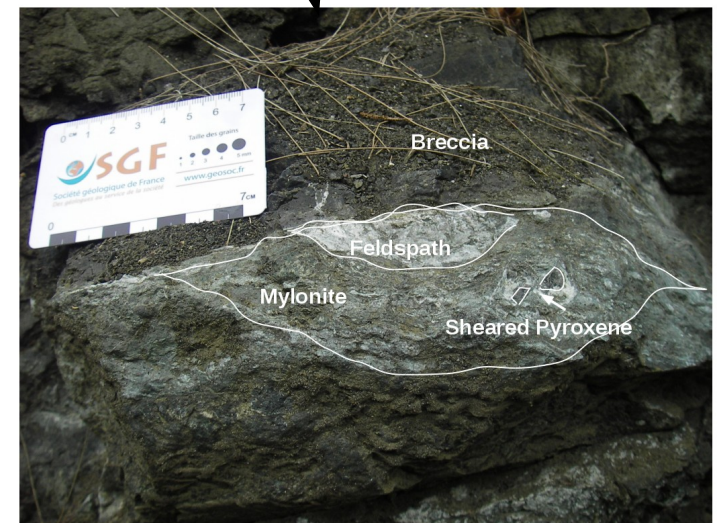
1-2 cm/yr eastward and subsidence

Chen et al., Rem. Sens. Envir., 2017

Origin of the 2007 flank displacement



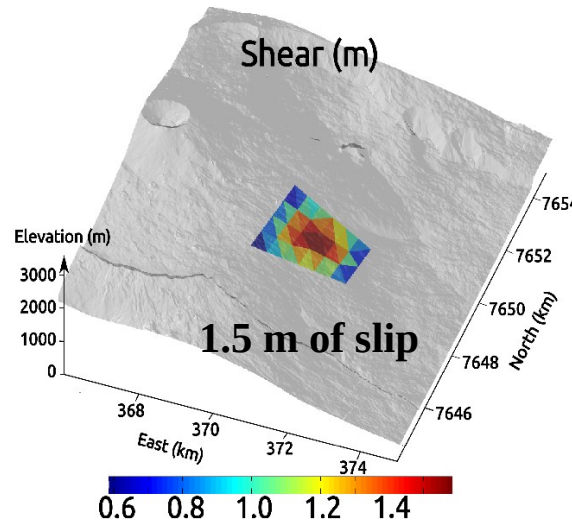
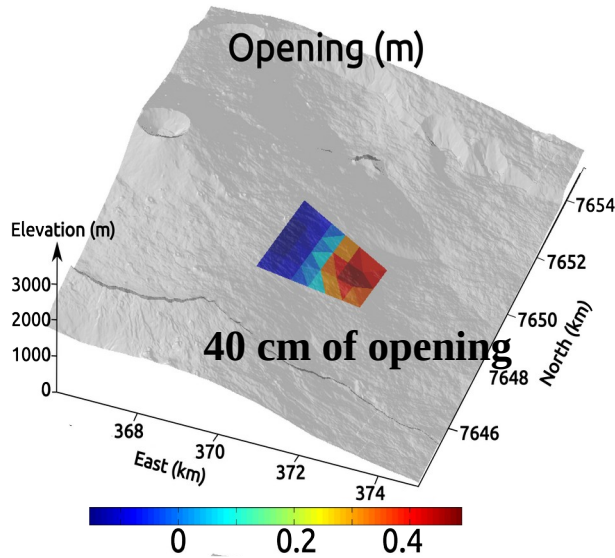
Evidence of shear ductile and brittle deformation



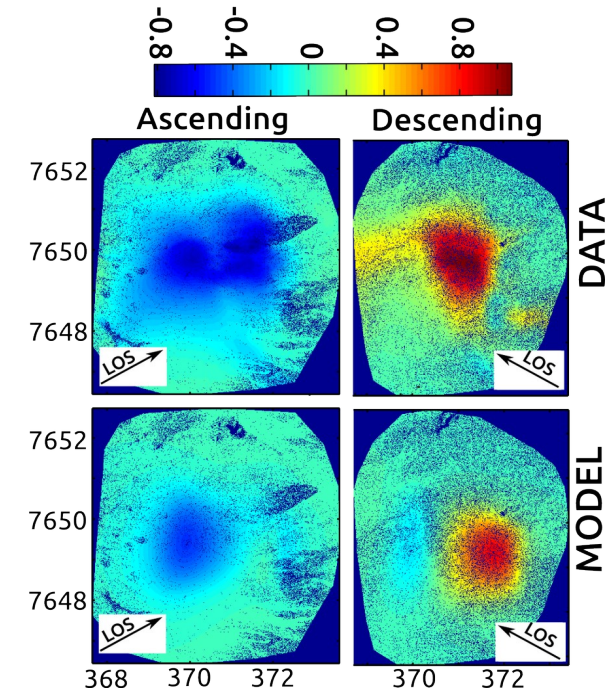
Inversion of geometry and stress changes for the co-eruptive displacement

Tridon et al., IAVCEI, 2017

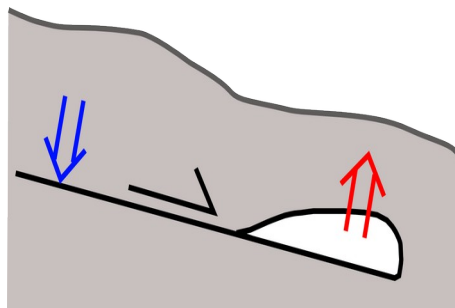
Inversion of 8 geometrical parameters + a pressure and a shear stress change



Displacement in LOS direction (m)

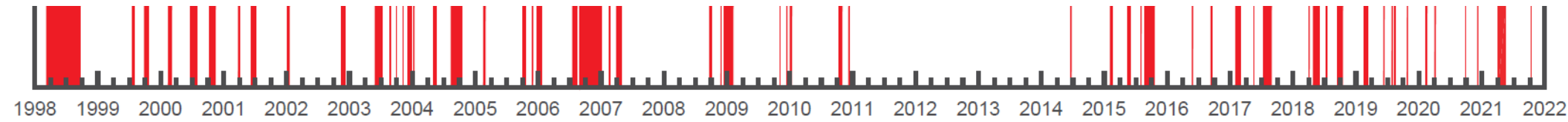


A FAULT !

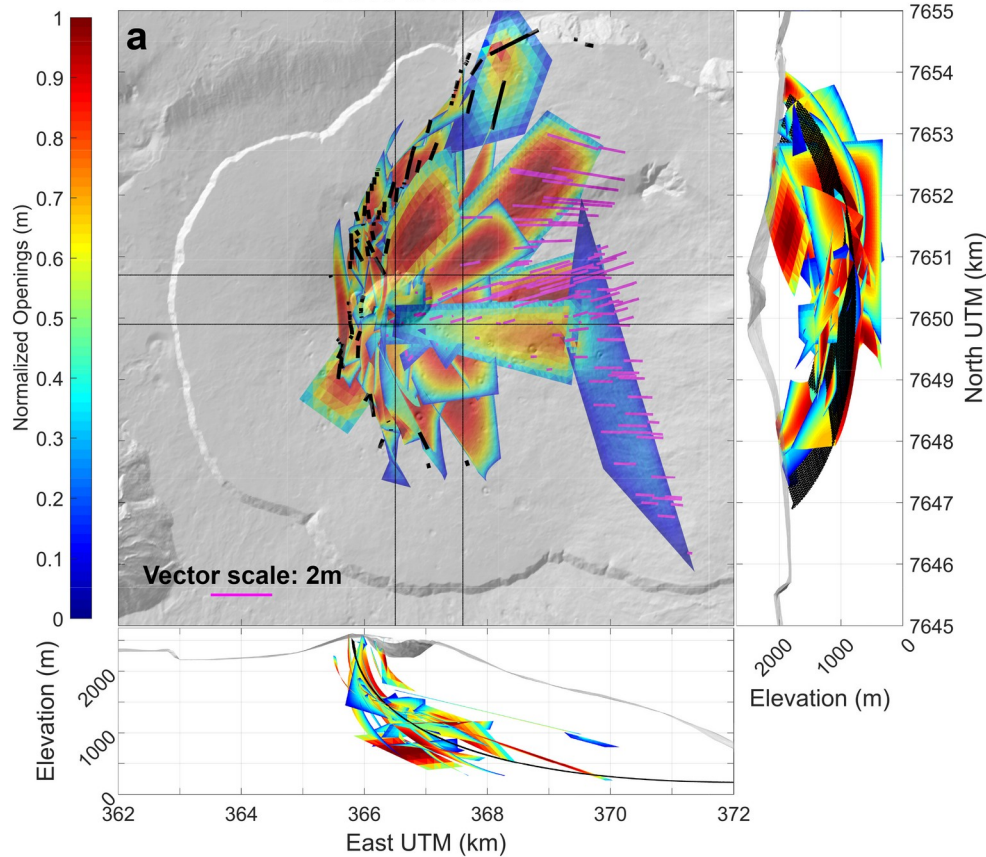


Displacements are characteristic of a detachment fold

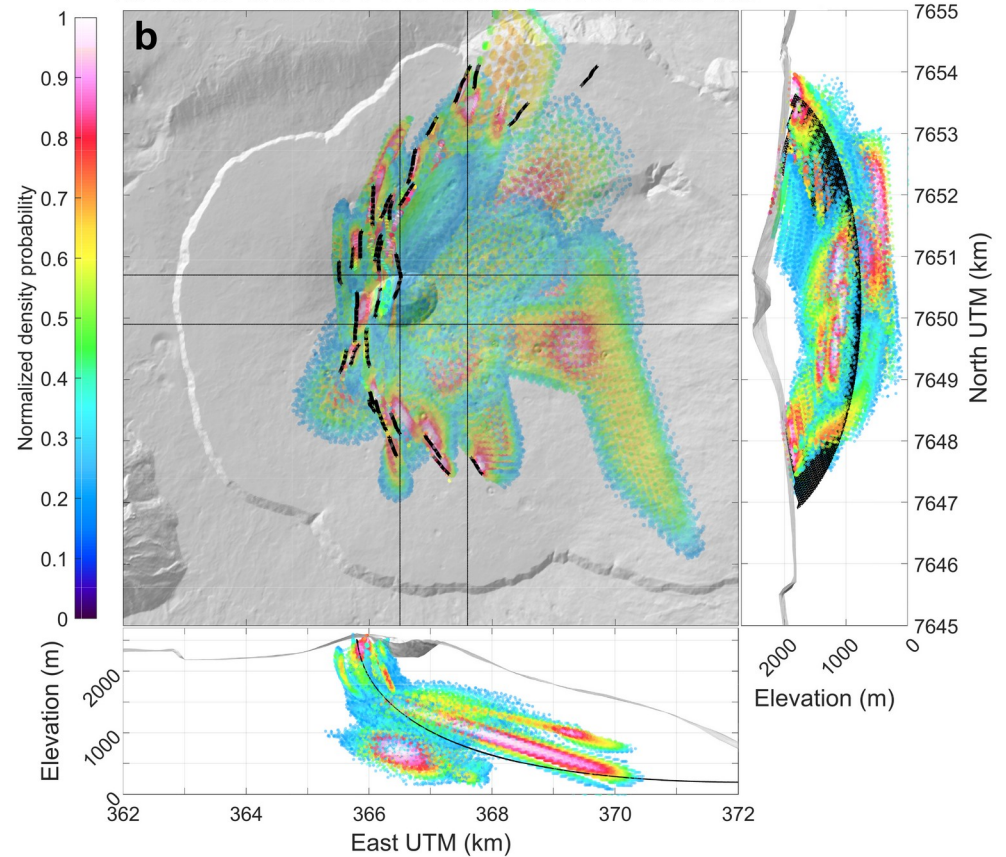
Inverse modeling of 22 years of InSAR and GNSS data



Best models



Models within 95% confidence interval

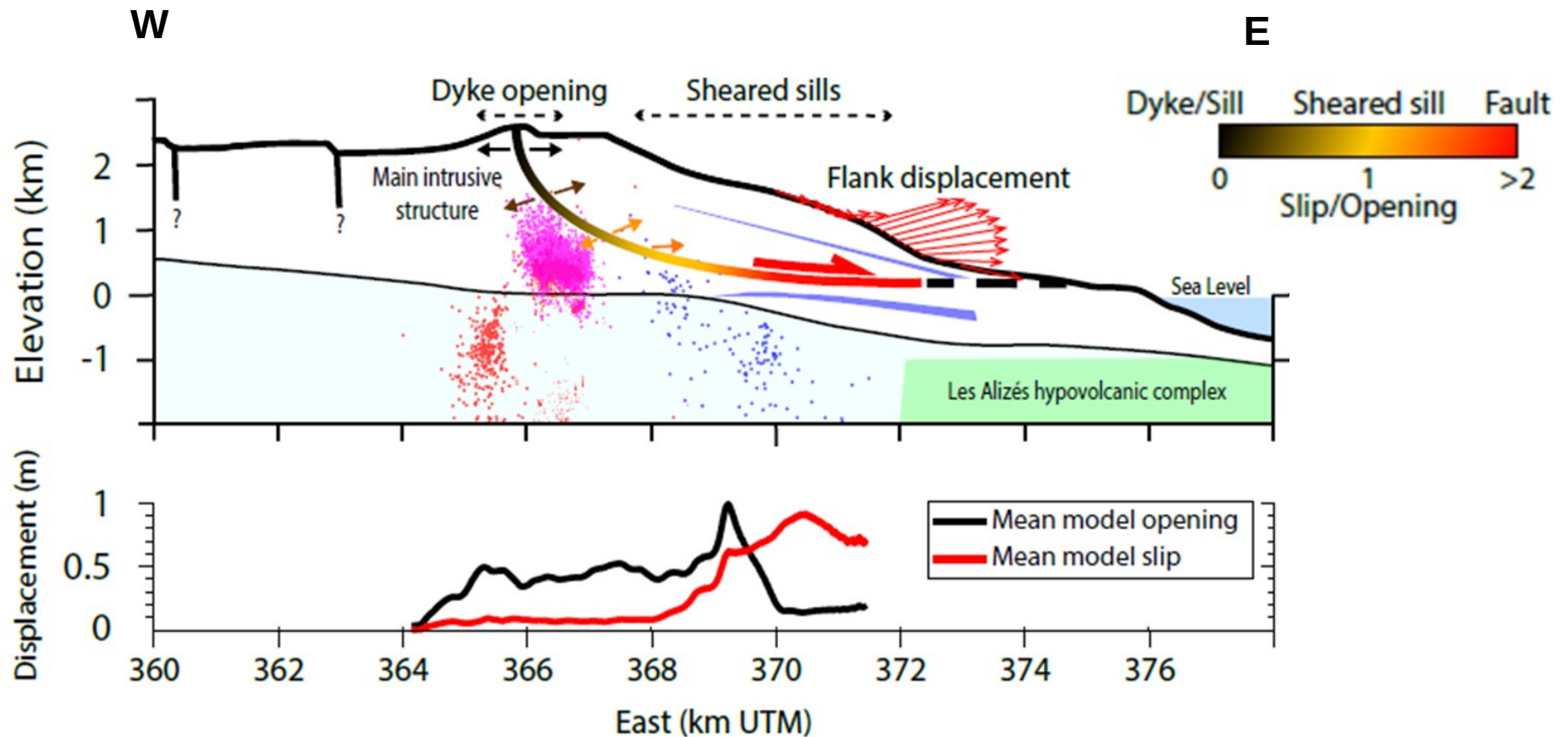


Dumont et al., Nature Communication, 2023 ; EPSL, 2024

➔ 80% of the magma intrudes in a spoon-shaped collapse structure

A major spoon-shaped collapse structure

Dumont et al., Nature Communication, 2023 ; EPSL, 2024



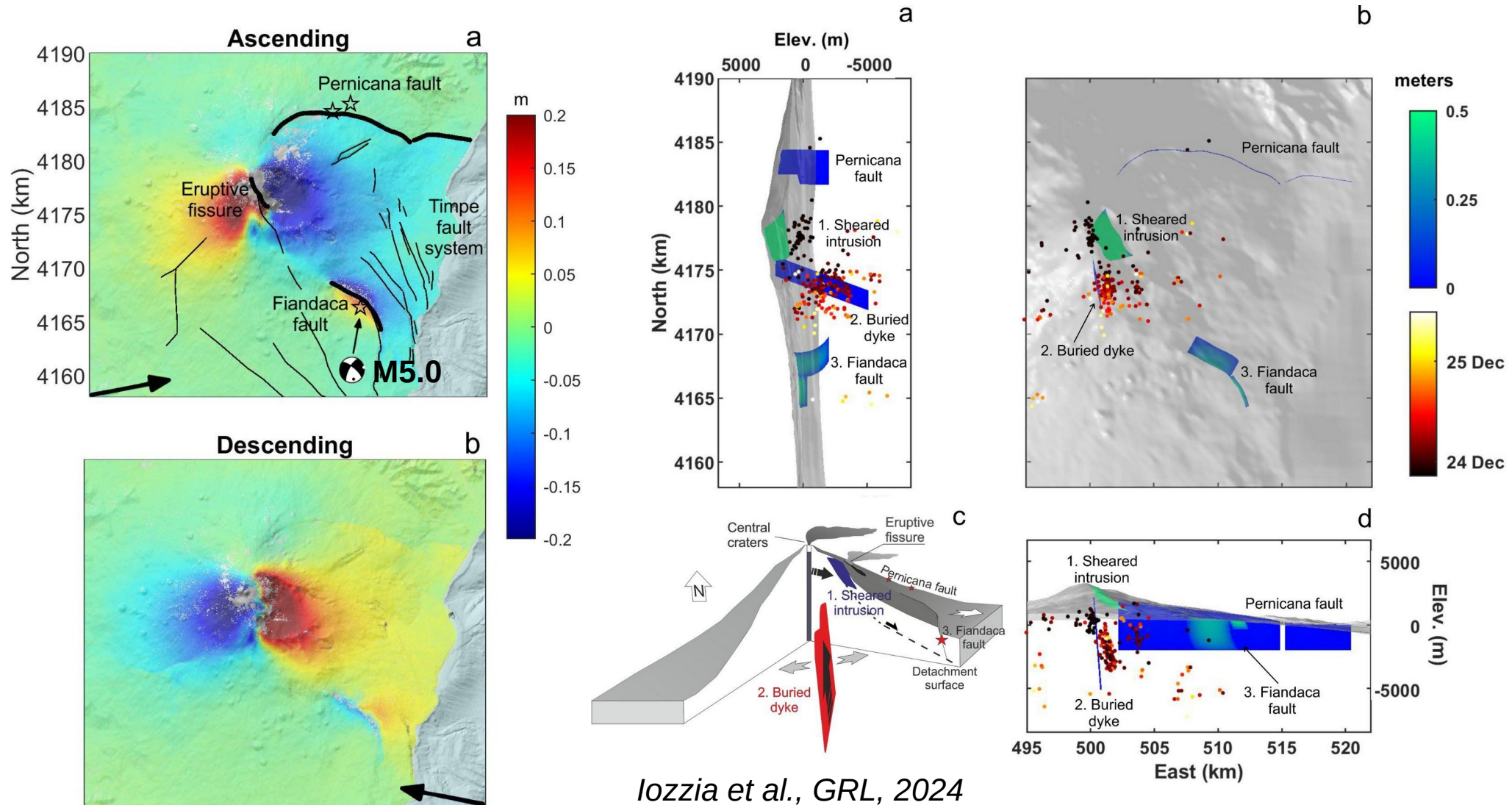
Continuum of displacements from west to east:

- **Pure opening** of subvertical curved dykes
- **Curved sheared sills**
- **Fault slip** in the easternmost part (in 2007)

➡ Hybrid between previously assumed models;

➡ Could accommodate flank failure

A similar structure may be active at Etna as evidence by the 2018 Christmas event

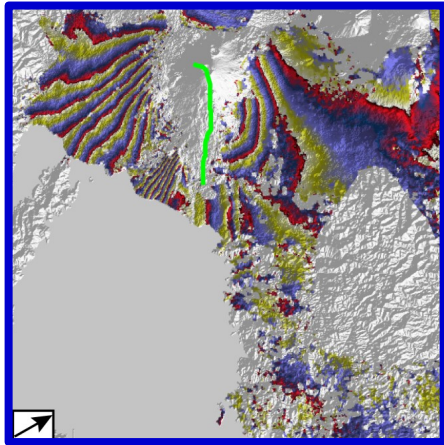


- A **curved sheared intrusion** and a **buried dyke** explain displacement close to the summit;
- Pernicana fault responded passively; Fiandaca fault released accumulated stress;

What can be learnt from the Inverse modelling of InSAR data ?

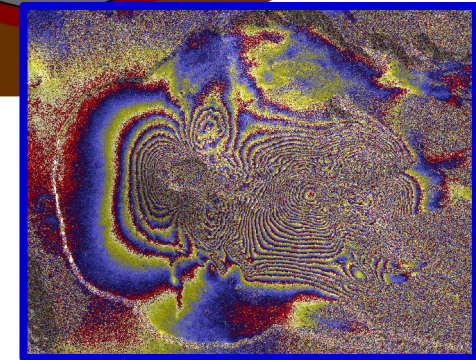
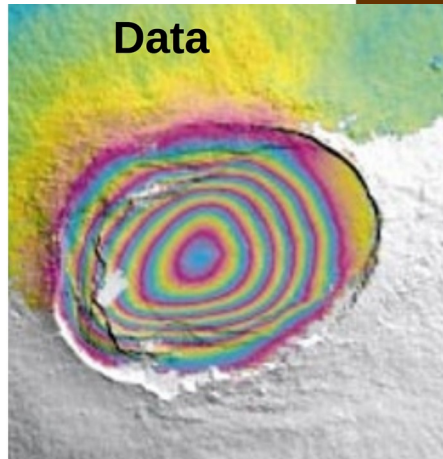
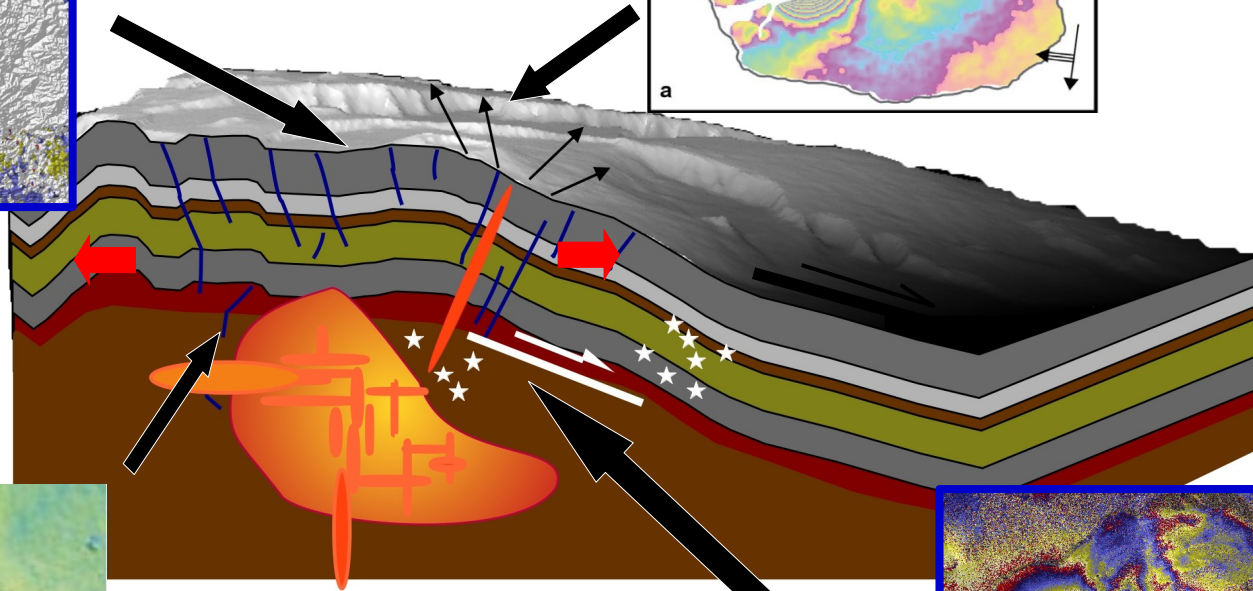
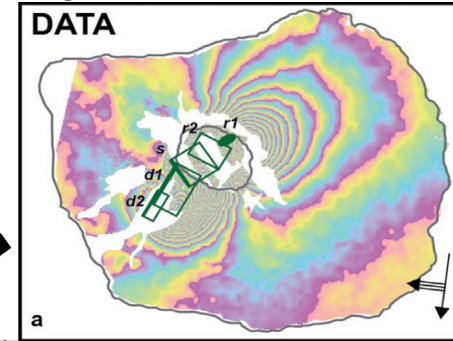
Rift extension drive

Wauthier et al., 2012



Intrusion pathways

Bagnardi et al., EPSL, 2013



Flank slip mechanism

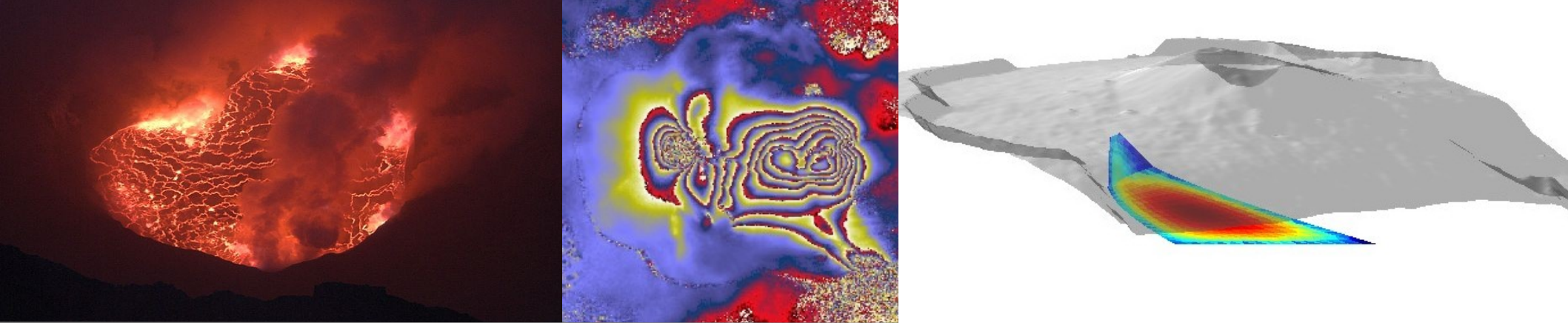
Tridon et al., JGR, 2016;
Dumont et al., Nat. comm., 2022.

Characteristics of reservoirs

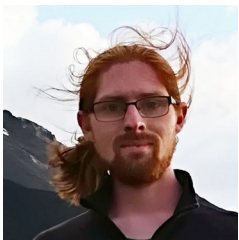
Amelung et al., Science, 2000

Conclusions

- Inverting for **stress changes**:
 - is **more physical** than kinematic inversions;
 - leads to more **likely** models;
 - is more **informative**.
- In the **Virunga Volcanic Province**, the **rift extension** is driven by **magmatic activity** rather than plate extension;
- At **Piton de la Fournaise**, we find a continuum of fracture displacement: **dike** intrusion -> **sheared intrusions** -> **fault** slip that accommodates magma intrusions;
- Sheared intrusions also seem to be active at **Etna**.
- Sheared intrusions should be searched at other shield volcanoes with evidence of flank slip



Thank you for your attention !



Quentin Dumont



Christelle Wauthier



Yo Fukushima



Gilda Currenti



Jean-Luc Froger



Adriana Iozzia



Marine Tridon



PennState
College of Earth
and Mineral Sciences

