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Abdellah Bouchendouka, Zine El Abiddine Fellah, Erick Ogam, Mohamed Fellah, Claude L. Depollier. Non-Integer Dimensional Analysis of Ultrasonic Wave Propagation in Fractal Porous Media. *Journal of Physics: Conference Series*, 2024, 2768, 10.1088/1742-6596/2768/1/012003 . hal-04613200

HAL Id: hal-04613200

<https://hal.science/hal-04613200v1>

Submitted on 15 Jun 2024

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Non-Integer Dimensional Analysis of Ultrasonic Wave Propagation in Fractal Porous Media

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Abstract. This paper explores the acoustics of porous media characterized by fractal, or self-similar, structures. Employing a fractal approach, we use differential operators in non-integer dimensional spaces to address the fundamental equations of acoustics in such media. The primary aim is to examine the transmission of ultrasonic waves within a fractal porous medium. Our findings reveal that the fractal dimension significantly influences wave transmission. In fractal porous materials, waves travel along more complex and intricate paths, resulting in increased tortuosity and attenuation. We introduce the concept of an effective path length l_{eff} , which is dependent on the fractal dimension D_x , to describe the actual trajectory of wave propagation. Additionally, we define an effective tortuosity α_{D_x} , directly proportional to l_{eff}^2 , to quantify the additional tortuosity brought about by the fractal structure. The insights gained from this study are crucial, as they enhance our understanding of wave behavior in self-similar porous media, which are prevalent in various natural settings and have multiple practical applications, including sound insulation and the design of acoustic materials. Furthermore, understanding the impact of fractal dimensions on wave behavior is vital for developing more efficient acoustic solutions. This research also sets the stage for further theoretical and experimental work on applying fractal geometry to analyze wave propagation in porous structures.

1. Introduction

Benoit Mandelbrot [1] introduced the term "fractal" to describe objects that are not whole or continuous in either spatial or temporal dimensions. These objects can be broken down into smaller, similar pieces in a predictable or random manner, known for a characteristic called self-similarity. This quality of fractals means they seem to have an unending intricacy, no matter the scale at which they're observed [2, 3]. In the realm of spatial fractals, rather than temporal ones like signals or processes, we see natural and man-made formations that approximate fractal behavior. These include the ragged edges of coastlines, the porous structures within various materials, the pattern of cracks forming, the swirling of fluids in turbulence, the bolt of a lightning strike, the complexity of cerebral configurations, the delicate design of snowflakes,

and the transforming states of water to ice. These examples represent only a fraction of the exhaustive list seen in nature and documented in resources like the book "Fractals Everywhere" [4]. Fractals serve as ideal models in mathematics for studying phenomena that have a certain scale range. Objects that are not quite fractals but exhibit similar characteristics within specific size limits are referred to as 'pre-fractals'. Fractal geometry provides accurate modeling of scale invariant systems exhibiting irregularities at many scales. However, functions defined on fractal domains are almost nowhere differentiable in the classical sense [1, 5, 6, 7]. Consequently, the ordinary differential calculus cannot be used to describe dynamic processes in a fractal system. This stimulates development of alternative approaches to deal with problems on fractal domains [8].

The concept of integration within Non-Integer Dimensional Spaces (NIDS) has been explored in various studies [9, 10, 11]. Stillinger [9] laid the groundwork for integrating over non-integer dimensional spaces, and also suggested a variation of the Laplace operator for NIDS. This approach was further expanded by Palmer and Stavrinou [11], who applied it to functions with multiple variables, emphasizing its utility solely for scalar fields in spaces with non-integer dimensions. Notably, the application of first-order differential operators like gradient, divergence, and vector Laplacian in the context of NIDS was not addressed in these works. Limiting the scalar Laplacian to NIDS as described by Stillinger significantly narrows the scope for applying continuous models to fractal media and materials. This limitation becomes evident when we try to apply Stillinger's Laplacian to vector fields in elasticity or when modeling electric and magnetic fields within fractal media using NIDS. These restrictions are overcome in other research [12], where gradient, divergence, and curl operations are not merely seen as approximations of the Palmer-Stavrinou version of the Laplace operator. This advancement in generalizing differential vector operators to non-integer dimensions supports the use of continuous models for fractal media in NIDS [13, 14]. The NIDS calculus developed from this allows for the description of both isotropic and anisotropic fractal media, and has been instrumental in advancing the study of fractal hydrodynamics [15, 16, 17, 18, 19], fractal electrodynamics, the elasticity of fractal materials, and acoustics of fractal porous media [20, 21, 22].

This study aims to apply the NIDS operators referenced in prior research to model ultrasonic wave propagation through a fractal porous medium. It has been documented in the literature [23, 24], that fractal patterns in porous materials exist. Katz and Thompson [24] might have been pioneers in demonstrating that the pore spaces in certain porous sandstone samples exhibit fractal characteristics, being self-similar across three to four orders of magnitude in size, ranging from 10 Å to 100 meters. Therefore, fractal geometry could be crucial for analyzing flow and transport properties within these media [23, 25, 26]. A key feature of these fractal porous materials is their fractal dimension D , which quantifies the level of self-similarity in the material. Unlike uniform structures, fractal porous materials display a non-uniform and complex pore network. This study seeks to understand how these intricate and irregular patterns influence wave propagation, using a macroscopic approach.

The structure of the rest of this paper is outlined as follows: Section 2 provides a concise review of the fundamental equations that govern the acoustics of porous media. Building upon these principles, Section 3 details how we have applied the NIDS operators to address the equations discussed earlier, culminating in the analytical resolution of the equation describing wave propagation in the fractal case. Section 4 delivers a thorough analysis of wave transmission through a fractal porous medium. Finally, Section 5 concludes the paper by summarizing our results and offering an overall evaluation of the research conducted.

2. Mathematical Modeling of Acoustic Waves in Porous Structures

In the case of a rigid solid matrix, and wave propagation occurs solely within the fluid (without any vibration of the frame), a specific application of Biot theory is utilized. In this context, the porous material is treated as an effective medium [27]. This approach involves the representation of traditional medium properties through two linear response functions: one representing an effective density and another for an effective bulk modulus [28, 29, 30, 31]. These functions are critical in describing the fluid-structure interactions at the microscale, which are primarily defined by visco-inertial and thermal exchanges. The functional expressions of these response functions are derived from the Johnson-Champoux-Allard model in the high-frequency limit [32, 33], and are presented as follows:

$$\rho(\omega) = \rho_0 \alpha_\infty \left(1 + \frac{2}{\Lambda} \sqrt{\frac{\eta}{i\omega\rho_0}} \right), \quad K(\omega) = K_f \left(1 + \frac{2(\gamma-1)}{\Lambda'} \sqrt{\frac{\eta}{i\omega\rho_0 P_r}} \right)^{-1}, \quad (1)$$

where η is the dynamic viscosity, ρ_0 the fluid density, K_f the fluid bulk modulus, γ the adiabatic index, P_r is Prandtl number, and $i = \sqrt{-1}$. The parameter α_∞ is called the ideal fluid tortuosity, which describes the tortuous path of pores, and is always greater than 1 or equal to 1 for straight cylindrical pores. The ideal fluid tortuosity is defined as follows:

$$\alpha_\infty = \frac{\langle \vec{E} \cdot \vec{E} \rangle}{\langle \vec{E} \rangle \cdot \langle \vec{E} \rangle}, \quad (2)$$

where \vec{E} can be interpreted as the electrical field of a conductive fluid flowing through a porous medium with insulating solid matrix, or the velocity of a potential flow [34, 35, 36]. The parameters Λ and Λ' are called, respectively, the viscous and thermal characteristic lengths, and are defined as follows:

$$\frac{2}{\Lambda} = \frac{\int_S E^2 dS}{\int_V E^2 dV}, \quad \frac{2}{\Lambda'} = \frac{\int_S dS}{\int_V dV}. \quad (3)$$

From these definitions, it becomes evident that both lengths essentially represent the ratio of the pore volume to the pore surface area. However, for Λ this ratio is modified by the influence of the electrical field or velocity \vec{E} [32, 33], which implies that Λ' is invariably greater than Λ . The ratio Λ'/Λ typically ranges between 2 and 3 for various porous materials. An exception occurs with straight cylindrical pores, where $\Lambda' = \Lambda = R$, with R being the pore radius. From the density operator we can define the dynamic viscous tortuosity $\alpha(\omega)$ as $\rho(\omega)/\rho_0$, and from the bulk modulus operator, we define the dynamic compressibility $\beta(\omega)$ as $K_f/K(\omega)$.

In the case of a macroscopically uniform porous medium that is saturated with a visco-thermal fluid, and assuming a simple geometric structure where fluid motion at the pore scale is divergence-free, the governing equations that describe the behavior of acoustic waves within the frequency domain can be expressed as follows:

$$i\omega\rho_0\alpha(\omega)\vec{v} = -\vec{\nabla}\hat{p}, \quad i\omega\hat{p} = -\frac{K_f}{\beta(\omega)}\vec{\nabla}\cdot\vec{v}, \quad (4)$$

where \vec{v} is the macroscopic fluid velocity, and \hat{p} the macroscopic fluid pressure, obtained by averaging the microscopic velocity and pressure fields over a representative elementary volume. Note that $\partial/\partial t = i\omega$ is used for Equations (4).

Using Equations (4), we can easily obtain the following Helmholtz equation:

$$\vec{\nabla}^2 \hat{p} + k(\omega)^2 \hat{p} = 0, \quad (5)$$

where $k(\omega)$ is the effective wave number, which can be written as:

$$k(\omega) = k_0(\alpha(\omega)\beta(\omega))^{1/2}. \quad (6)$$

where $k_0 = \omega/c_0$ is the wave number in absence of the porous medium, with $c_0 = \sqrt{K_f/\rho_f}$ the speed of sound in air.

Equation (5) fundamentally characterizes the wave propagation within the porous material, taking into consideration the visco-thermal interactions occurring at the pore level.

3. Fractal porous material

Materials with a fractal porous structure display a complex, self-similar geometric pattern. Such fractal geometries can profoundly influence the material's physical and mechanical characteristics, including permeability, porosity, and tortuosity. Unlike the typical operators employed to predict the waves transmitted or reflected through a porous medium, this study utilizes operators defined within a non-integer dimensional space specific to fractal porous materials [13]. The definition of the gradient operator in this context is provided as follows:

$$\vec{\nabla}_{D_i} \varphi = \frac{c_i^{-1} \partial \varphi}{\partial x_i} \vec{e}_i, \quad (7)$$

where φ denotes a scalar field, and c_i is defined as $c_i = \partial \delta_i / \partial x_i$. The function δ_i is associated with the Cartesian coordinates x_i in the following manner:

$$\delta_i = \epsilon L_0^{1-D_i} x_i^{D_i}, \quad (8)$$

where ϵ is a geometrical constant, and L_0 is a characteristic size of elementary components which constitute the fractal structure. The exponent D_i represents a fractal dimension, which can be a non-integer value. This dimension describes the configuration of a line along a specific direction i , ranging from 0 when the line exhibits high self-similarity, to 1 when it is a straight line devoid of fractal patterns. It is posited that the distance within a fractal continuum between point A and point B can be articulated using Cartesian coordinates in the embedding Euclidean space as:

$$\delta_i(A, B) \propto |x_i(A)^{D_i} - x_i(B)^{D_i}|. \quad (9)$$

In a fractal porous medium, the pore structure exhibits a complex, self-similar pattern characterized by a non-integer fractal dimension. This indicates that the fractal architecture of the pores cannot be captured by traditional Euclidean geometry, which relies on integer dimensions. Consequently, it is more practical to employ fractional differential operators (i.e., differential operators in non-integer dimensional spaces) for analyzing wave behavior in such media [6]. By utilizing the previously defined gradient operator (7), the Helmholtz equation (5) can be expressed as:

$$\vec{\nabla}_{D_i}^2 \hat{p} + k(\omega)^2 \hat{p} = 0. \quad (10)$$

where $\vec{\nabla}_{D_i}^2$ represents the scalar Laplace operator, derived easily using Equation (7). The primary distinction between Equation (5) and Equation (10) lies in the consideration for the

latter of the complex structure inherent in the porous material. This complexity is notably advantageous as it can be described succinctly by the fractal dimension D_i .

The Helmholtz equation (10), in the case of a planar wave propagating in the x direction of a unit amplitude has the following form:

$$\left[\frac{c_1^{-1} \partial}{\partial x} \left(\frac{c_1^{-1} \partial}{\partial x} \right) + k^2(\omega) \right] \hat{p} = 0. \quad (11)$$

Using the boundary conditions $\hat{p} = 1$ when $x \rightarrow 0$, and $\hat{p} = 0$ when $x \rightarrow \infty$, the solution to Equation (11) is:

$$\hat{p} = e^{-ik(\omega)\lambda x^{D_x}}. \quad (12)$$

where D_x is a fractal dimension that describes the fractal nature of a line in the x direction, and the factor λ is expressed as follows:

$$\lambda = \frac{\epsilon L_0^{1-D_x}}{D_x}. \quad (13)$$

Expression (12) outlines how a wave propagates through a fractal porous medium. According to the expression, the wave travels a distance that is proportional to x^{D_x} . This proportional relationship is crucial for grasping the complex path traversed by the wave. We will explore the significance and consequences of this relationship, which reflects the intricate paths within the fractal structure, in further detail later on.

4. Transmission in fractal porous media

To obtain the transmission coefficient for the case of a fractal porous medium, we use the conventional acoustic wave propagation problem depicted in Figure 1. The figure represents a slab of a fractal porous medium, of thickness L, which interacts with an incident wave p^i . A portion of the wave p^r is reflected, and another portion of the wave p^t is transmitted.

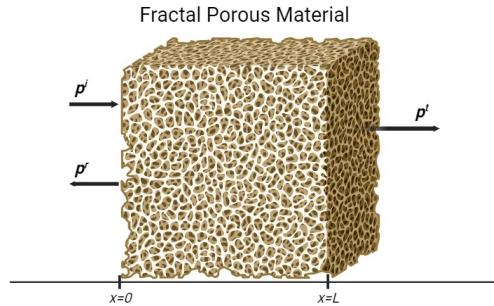


Figure 1. Geometry of the problem

Using the continuity condition of the pressure \hat{p} , and the conservation of flow rate at the boundaries $x = 0$ and $x = L$, as well as the solution provided by Equation (12) we obtain the following expression for the transmission coefficient:

$$T = \frac{2Z}{2Z \cosh(ik(\omega)l_{eff}) + (1 + Z^2) \sinh(ik(\omega)l_{eff})}, \quad (14)$$

here $Z = \phi(\beta(\omega)/\alpha(\omega))^{1/2}$ with ϕ the porosity, and l_{eff} is the effective path the wave takes when traveling through the fractal medium, and it is defined as follows:

$$l_{eff} = \lambda L^{D_x}, \quad (15)$$

where λ is defined by Equation (13) and represents a scaling factor. The definition of the effective path length l_{eff} in this way aligns with the property (9). Essentially, this means that the distance l_{eff} corresponds to the actual path traveled by the transmitted wave as it moves through the fractal medium with thickness L . This effective path length in a fractal medium accounts for the numerous twists and turns characteristic of the fractal structure. From the definition of l_{eff} , it is evident that this path is longer than the straight-line distance L , as it navigates the fractal's geometry.

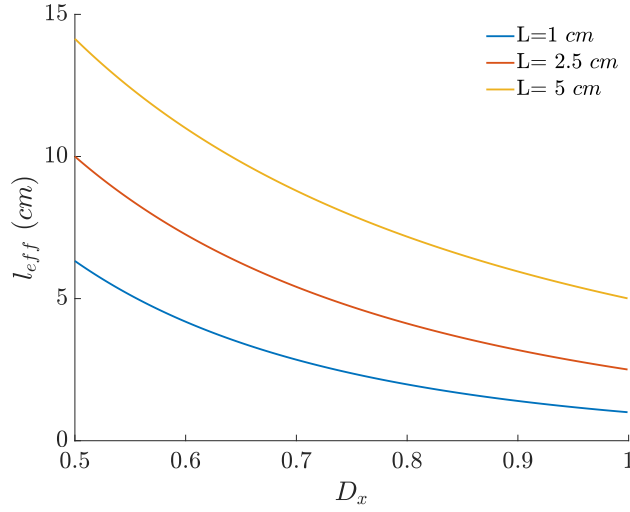


Figure 2. Plot of the effective path length l_{eff} with respect to the fractal dimension D_x for different values of L , with $\alpha_\infty = 1.1$.

Figure 2, which displays the effective path length l_{eff} as outlined in Equation (15) in relation to the fractal dimension D_x , reveals two significant findings. First, by analyzing the fractal pattern of the path traversed by the wave, we deduce that it indeed covers a greater distance, because l_{eff} increases with $D_x < 1$. Second, the actual path depicted by the effective path length l_{eff} contributes to the tortuosity of the medium. This observation prompts the definition of an effective tortuosity α_{D_x} , which may be expressed as follows:

$$\alpha_{D_x} = \alpha_\infty \tau. \quad (16)$$

where

$$\tau = \left(\frac{l_{eff}}{L} \right)^2 \quad (17)$$

is a tortuosity attributed to the fractal pattern of the path that the wave traverses. The definition of τ in this context is supported by various sources [37, 38]. Other researchers conceptualize tortuosity as $\tau = L_t/L$ [37, 38] or $\tau = (L/L_t)^2$ [37], where L_t represents the actual length of the flow path. However, it is widely acknowledged that determining tortuosity presents significant challenges due to the highly intricate microstructures found in porous media. Despite these difficulties, employing fractal dimensions to assess the tortuous streamlines/capillaries in fractal

porous media has proven effective in accurately describing the complex patterns observed within such media. This approach is well illustrated by the Wheatcraft and Tyler formula [39]:

$$L_t = \epsilon^{1-D_t} L^{D_t} \quad (18)$$

where ϵ represents the scale of the measuring unit, and $1 \leq D_t < 2$ denotes the fractal dimension of a curved or tortuous line in two dimensions. It is important to note that the expression for L_t cited in existing literature differs from the expression for l_{eff} introduced in this paper, which is novel and based on the use of operators in non-integer dimensional spaces.

The relationship between D_t and the fractal dimension employed in this paper is expressed as $D_x = 2 - D_t$. Although both dimensions yield similar information, they do so in slightly different ways. For example, the tortuosity of the flow path in a porous medium is influenced by the presence of disconnected solid particles within the medium (see Figure 3). Consequently, the disconnected line in direction x is characterized by the dimension D_x , and the resulting tortuosity is captured by the dimension D_t . Additionally, various other formulas exist that link the effective path length L_t to the straight line path L through a power-law relationship $L_t \propto L^D$, where the definition of the exponent D varies depending on the specific approach considered [40, 41, 42].

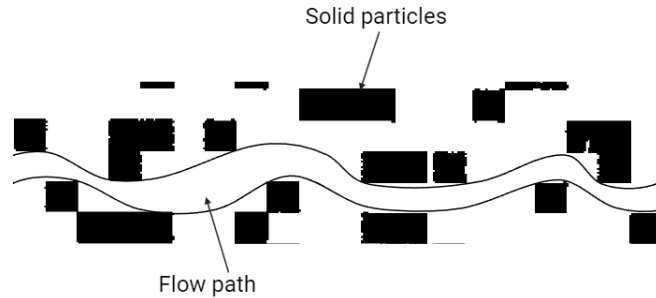


Figure 3. Schematic of the tortuous flow path in a porous medium.

The effective tortuosity of the medium, α_{D_x} , is essentially the product of the conventional tortuosity α_∞ , which describes the bending of the electrical field \vec{E} or the streamlines of potential flow velocity, and the fractal tortuosity τ , which captures the fractal and complex pattern of the path. α_{D_x} represents the cumulative effect of both conventional and fractal tortuosity, each contributing significantly and distinctly to the overall resistance to flow. This relationship is depicted in Figure 4, which illustrates the effective tortuosity α_{D_x} in relation to the fractal dimension D_x across various values of L , with α_∞ set at 1.1. The graph highlights how the tortuosity is substantially influenced by the fractal pattern ($D_x < 1$), emphasizing the critical importance of the correction provided by the fractal dimension D_x .

With this definition of α_{D_x} , we can identify four distinct scenarios:

Straight Path Case: For a straight path with no fractal pattern and straight velocity streamlines, $\alpha_{D_x} = \tau = \alpha_\infty = 1$. **Fractal Pattern, Straight Streamlines Case:** When there are straight velocity streamlines but a fractal pattern is present, $\alpha_{D_x} = \tau$ and $\alpha_\infty = 1$. **Conventional Case:** In the absence of fractal patterns, $\alpha_{D_x} = \alpha_\infty$ and $\tau = 1$. **General Case:** In more complex scenarios where both conventional and fractal elements contribute, $\alpha_{D_x} = \alpha_\infty \tau$, with $\alpha_\infty > 1$ and $\tau > 1$.

Figure 5 illustrates simulated transmitted signals for a fractal dimension of $D_x = 1$ and $D_x = 0.7$, within a material of thickness $L = 2.5$ cm, porosity $\phi = 0.98$, conventional tortuosity $\alpha_\infty = 1.1$, and characteristic length scale $\Lambda = 250 \mu m$. The results indicate that for $D_x = 0.7$,

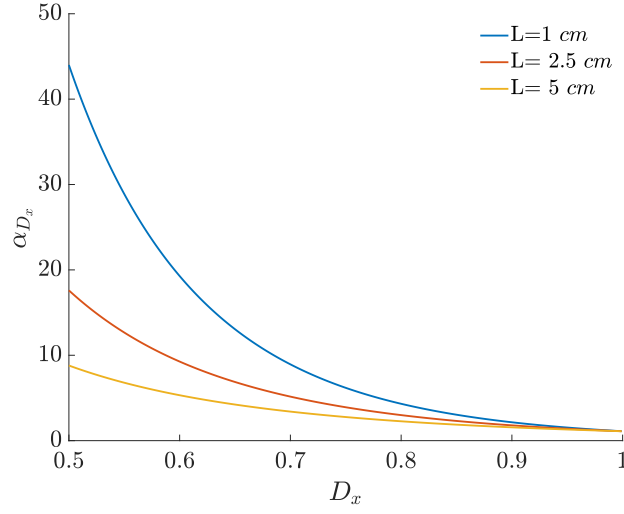


Figure 4. Plot of the effective tortuosity α_{D_x} with respect to the fractal dimension D_x for different values of L , with $\alpha_\infty = 1.1$.

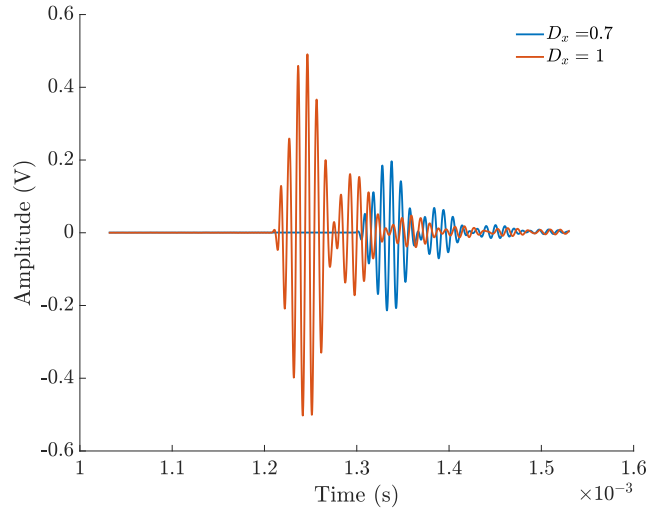


Figure 5. Plot of simulated transmitted signals using Equation (14), for $D_x = 1$ and $D_x = 0.7$, with $L = 2.5 \text{ cm}$, $\phi = 0.98$, $\alpha_\infty = 1.1$, $\Lambda = 250 \text{ }\mu\text{m}$.

corresponding to a fractal porous medium, the transmitted wave is received significantly later than a wave transmitted through a porous medium without a fractal pattern ($D_x = 1$). This delay suggests that the wave velocity for $D_x = 0.7$ is much slower compared to $D_x = 1$, due to its traversal along a longer and more complex path. This increased complexity and path length are quantified using α_{D_x} , where for $D_x = 0.7$, the effective tortuosity α_{D_x} is calculated to be 5.15, whereas for $D_x = 1$, it remains at $\alpha_\infty = 1.1$.

Additionally, the simulation reveals that the fractal pattern also influences the wave amplitude. Specifically, transitioning from $D_x = 1$ to $D_x = 0.7$ results in a 60% decrease in amplitude. This change in amplitude is expected given the increased path complexity and tortuosity in the medium with a lower fractal dimension.

5. Conclusion

The fractal approach is utilized to analyze wave propagation through a fractal porous medium. Central to this analysis is the fractal dimension, denoted as D_x , which plays a pivotal role in determining the trajectory of the wave. As D_x approaches 1, the path of the wave becomes less fractal, embodying more straightforward trajectories. Conversely, a D_x value less than 1 indicates that the wave follows a path that is more fractal and convoluted. This increase in complexity is quantified by the effective path length, l_{eff} , which subsequently elevates the overall tortuosity of the medium.

To encapsulate the combined effects of both the inherent path complexity and the structural attributes of the medium, an effective tortuosity, α_{D_x} , is defined. This measure integrates the conventional tortuosity α_∞ —which describes the bending of the electrical field or the streamlines of potential flow velocity within the pores—with the fractal tortuosity $\tau = (l_{eff}/L)^2$, reflecting the intricate nature of the wave's route. This foundational research sets the stage for further theoretical and experimental exploration into the application of Non-Integer-Dimensional-Space operators and fractal geometry to better understand wave propagation in porous media.

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