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# Conditioning Public Pensions on Health: Effects on Capital Accumulation and Welfare\*

Giorgio Fabbri<sup>†</sup>    Marie-Louise Leroux<sup>‡</sup>    Paolo Melindi-Ghidi<sup>§</sup>    Willem Sas<sup>¶</sup>

## Abstract

This paper develops an overlapping generations model which links a public health system to a pay-as-you-go (PAYG) pension system. It relies on two assumptions. First, the health system directly finances curative health spending on the elderly. Second, public pensions partially depend on health status by introducing a component indexed to society's average level of old-age disability. Reducing disability then lowers pension benefits as the need to finance long-term care services also drops. We study the effects of introducing such a 'comprehensive' Social Security system on individual decisions, capital accumulation, and welfare. We first show that health investments can boost savings and capital accumulation under certain conditions. Second, if individuals are sufficiently concerned with their health when old, it is optimal to introduce a health-dependent pension system, as this will raise social welfare compared to a system where pensions are not tied to the society's average level of old-age disability. Our analysis thus highlights an important policy recommendation: making PAYG pension schemes partially health-dependent can be beneficial to society.

**JEL Classification:** H55, I15, O41.

**Keywords:** Curative Health Investments, PAYG Pension System, Disability, Overlapping Generations, Long-term Care.

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# 1 Introduction

Because of impressive health care improvements over the last fifty years, longevity has risen substantially in all developed countries. This has not come without policy challenges, however. In 2019, more than two out of five EU individuals aged 65 or older reported having some kind of functional limitation (Eurostat, 2023). These can range from sensory, physical and mental limitations to limitations in personal care and household activities, and dramatically increase with age. In many OECD countries the importance of Long-term care (LTC) has therefore grown – and will continue to grow – together with its costs (Siciliani, 2013).<sup>1</sup>

Different solutions have been discussed over the years to guarantee sufficient financial leeway for the elderly in case they become dependent. A first solution would be to incentivize the purchase of private LTC insurance, but this has been shown to be relatively ineffective. Individuals often fail to insure themselves against the risk of becoming dependent at a later age, despite the substantial costs associated with the loss of autonomy.<sup>2</sup> A second solution would be to let the family take care of their elderly through either formal (in cash) or informal (in time) care, but this may come at the expense of reduced labour supply of informal caregivers, higher psychological costs, and possibly lower aggregate welfare (De Donder and Leroux, 2017).

The last solution, and this is the one we explore in this paper, is to let the government provide financial support to the elderly when they become dependent. More specifically, we propose a ‘comprehensive’ Social Security system that combines both a health system and a pension system, which are closely interconnected. On the one hand, the government invests in public health to reduce the dependency rate of the elderly. On the other hand, pension benefits are partially indexed to society’s average level of old-age disability.<sup>3</sup> We thus extend the standard Diamond (1965) overlapping generations (OLG) model by ‘augmenting’ the

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<sup>1</sup>We follow Cremer (2014) in defining long-term care as: “the provision of assistance and services to people who, because of disabling illnesses or conditions, have limited ability to perform daily activities such as bathing and preparing meals.” LTC is mainly targeted toward the elderly, with needs arising from various chronic diseases (mostly diabetes and -increasingly- cancer), Alzheimer or other forms of dementia.

<sup>2</sup>Multiple explanations have been suggested for this ‘LTC insurance puzzle’, both on the supply side (e.g. adverse selection leading to high loading costs) as the demand-side (e.g. myopia, bequest motives). See, among others, Pauly (1990), Pestieau and Ponthière (2012), Brown and Finkelstein (2009), Brown and Finkelstein (2007), Lockwood (2018) and Boyer et al. (2020).

<sup>3</sup>In Québec in 2015, the implementation of an additional benefit to retirees above a given age was discussed in order to cope with the potential additional costs due to a loss of autonomy at older age (Hébert, 2016). Because of a change in government majority, this so-called ‘*assurance autonomie*’ was never implemented.

universal PAYG pension benefit with a component that is health-dependent (which we will call a ‘disability-augmented’ pension benefit). This modulation of the pension system then partially ties pension benefits to health status, which compensates the elderly for spending more on costly LTC services in case they would grow more dependent.

We are interested in such a comprehensive Social Security system because it offers three advantages when investing in the elderly’s health. First, better health will reduce the average disability rate in the economy and hence increase individual welfare. Second, and for a given average longevity, better health decreases financial pressure on the social security system. This is because when old-age dependency is reduced, our comprehensive system allows pension benefits to follow suit since the need to finance long-term care services decreases. Third, and perhaps most surprisingly, we show that this process can boost saving for old age as pension contribution rates decrease, and consumption is smoothed across time. This sparks off a general equilibrium effect which nudges up capital accumulation, growth, wages and ultimately aggregate welfare. The latter positive feedback loop is set in motion when people are sufficiently concerned with their health when old, and the level of health taxation is not too high.

Importantly, since the health care pillar of our comprehensive system aims to directly reduce the dependency rate of the elderly, we focus on *curative* health spending to improve the quality of elderly life. A few examples are: building nursing homes, providing more day-to-day care, medications and therapies, or extending other existing medical services. In a recent report, the OECD (2021) indicates that a vast majority of public health spending is directed toward curative care rather than disease prevention and health promotion. In particular, curative and rehabilitative care services account for 60% of total health spending in OECD countries through inpatient and outpatient treatments (see Figure 7.15 in the OECD report).

Overall, our modeling of a comprehensive Social Security system offers a more encouraging outlook for the future of social protection in general, and pension systems in particular. Indeed, in many countries facing the consequences of population aging, public discourse often spreads the idea that health improvements are one of the reasons why PAYG pension systems are under pressure.<sup>4</sup> However, our results suggest that by investing in curative health care

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<sup>4</sup>This is inherent to the design of PAYG pension systems, where working individuals pay for the pensions of retired individuals. If healthier people live longer there will logically be fewer working contributors for each pensioner, a downward trend which is projected to accelerate (Pecchenino and Pollard, 2005). See also Cigno and Werding (2007) on increasing age-dependency ratios.

and hence, reducing the dependency of the elderly, the need for additional LTC spending can be reduced which, in turn, boosts saving for old age and increases overall welfare. This paper thus aims to show how public health investments targeted toward the elderly may be useful in designing more efficient pension schemes.

More specifically, our model features a representative individual who lives two periods of fixed length with certainty. He derives utility from consumption in both periods and from being autonomous during old age. In the first period, he works and contributes to the health system as well as to the Social Security system. He also saves for old-age consumption. In the second period, he consumes the proceeds of his savings and the pension benefit. Public intervention is twofold. First, using the health contributions, the government invests in public health by making curative expenditures. Contributions of workers in a given period are directly invested to increase the health condition of the old in the same period. This directly increases the elderly's utility since they value better health during old age. Second, individuals contribute to a PAYG pension system in the first period and receive a pension benefit in the second. As already mentioned, we will model an augmented version of a standard pension system by allowing the amount of the pension benefit to vary with the average level of old age disability in the economy. If health during old age deteriorates (equivalently, the intensity of dependency is higher in the society), the elderly will receive higher pension benefits so as to cope with extra LTC expenditures. We can then compute the steady state level of capital as a function of health and pensions contribution rates.

Before going further, let us note that we abstract from the potential impact of health investment on increased longevity, which in turn would affect the pension budget constraint. This way, we focus exclusively on the consequences of improved health of the elderly, regardless of lifespan changes. To our knowledge, no other study considers the effect of curative (rather than preventive) health investments in explaining capital accumulation and welfare via a health-dependant PAYG pension scheme. To do so, however, we had to shut down other channels. We believe that studying the effect of curative health spending together with health-dependent pensions is sufficiently novel to study it in isolation.

To analyze our comprehensive system in a context that would match reality more closely, we run an analysis assuming that Social Security tax rates are close to those observed in OECD countries and look at how higher public health spending impacts health and capital

accumulation. Second, we study the welfare effects of introducing a disability-augmented component to the pension benefit. Doing so, we fix a minimum PAYG pension contribution and find the optimal levels of both the public health tax and the additional Social Security tax rate, which would finance the disability-augmented part of the pension benefit. Importantly, we relate the optimal tax rate levels and the welfare effects to the individual preference for being healthier during old age.

Our main results are twofold. First, we show that, if the social security contribution rates are close to those observed in OECD countries, higher public health expenditures foster capital accumulation and growth as long as the contribution rate to the health system is not too large. This is the result of two opposing forces. On the one hand, increasing the health tax increases average health (i.e. decreases average disability) so it decreases pension benefits in the second period, and thus increases the willingness to save for old age to smooth consumption across periods. On the other hand, a higher health tax decreases disposable income and thus, savings. If the health tax is not too big, the first effect dominates the second: savings increase, and so do capital and output per capita.

Our second contribution concerns the welfare effects of introducing a disability-augmented pension benefit. We show that if individuals are sufficiently concerned with their health during old age, it is welfare-improving to introduce a system where pensions are tied to society's average level of old-age disability, as compared to a standard system where this is not the case. In addition, as the individual's preference for health increases, the welfare gains of augmenting the PAYG system become larger.

Relating these findings with what we observe in reality, it is reasonable to think that in the last decades, individuals have become more concerned with their health as a consequence of first, population aging (and with it, an increasing probability to become dependent) and second, of our economies becoming richer and more informed about health issues. Our results indicate it could indeed be welfare-improving to introduce a health-augmented pension system under these circumstances.

## **Related Literature**

By linking up the health system with the pension system, our interest in a comprehensive Social Security system also adds to the existing literature on two fronts.

First, since we investigate the effect of health investment on growth and welfare, we extend the findings of a strand of the literature that followed Chakraborty (2004). He incorporates public health expenditures in an overlapping generation model (OLG) to endogenously determine longevity, finding that increased taxes for public health investments improve elderly survival chances and boost long-term capital accumulation.<sup>5</sup> While the latter approach endogenises *longevity* via preventive health care investment, we endogenise *health* during old age via curative public investment. Our primary interest is thus to determine the extent to which a reduction in morbidity – and not an increase in longevity – resulting from increased health investment can promote capital accumulation and economic growth. In this sense, our paper is first to fill in the other side of the coin of health care investment. In doing so, moreover, we examine the interaction between public health programs and PAYG pension systems, demonstrating how their combined design influences capital accumulation, growth and ultimately welfare. This aspect has not been explored in the existing OLG literature.<sup>6</sup>

Second, our proposed indexation of PAYG pension benefits to health status adds to the many studies on rising LTC aging costs and its impact on social welfare. Most of the existing literature has concentrated on the reasons behind the LTC insurance puzzle (see footnote 2), however, and relies on partial equilibrium models.<sup>7</sup> Our general equilibrium analysis of LTC needs and its potential impact on saving decisions and welfare is hence fairly novel, barring the following three contributions. Marchiori and Pierrard (2022) consider the link between endogenous health investment, saving, growth, LTC and welfare. They find that there is a need for public preventive health care to internalize the negative externalities of private under-investment. Kopecky and Koreshkova (2014) calibrate a life-cycle model where individuals face risk on earnings, survival and LTC expenses, but can partially insure through public programs such as Medicaid and Social Security. They show that a large part of wealth is devoted to planning for old-age risks of survival and of needing LTC. They further show that all young generations would benefit from making these public programs more generous.

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<sup>5</sup>See for instance, Blackburn and Cipriani (2002), Bhattacharya and Qiao (2007), de la Croix and Ponthiere (2010), Jouvét et al. (2010), de la Croix and Licandro (2013), Fanti and Gori (2014), Zhao (2014) and Davila and Leroux (2015).

<sup>6</sup>Grossmann and Strulik (2019) come closest, by studying the effect of medical progress on health spending, pension savings for old age and the retirement age. They show that increasing medical progress is in conflict with reducing health inequalities. Contrary to us, they have no private savings and the pension system does not include an ‘augmented’ disability component.

<sup>7</sup>See e.g. Pestieau and Sato (2008), Canta and Pestieau (2013), De Donder and Leroux (2014), De Donder and Leroux (2017) and De Donder and Pestieau (2017).

Finally, Canta et al. (2016) study the dynamics of capital accumulation when individuals face a probability to become dependent during old age and when the existence of family norms can influence the level of informal care provided by the family. They allow for social insurance covering LTC risk but take it as given.

Differently from these papers, we model a public pension system where any changes in old-age disability are affecting the pension benefits to be distributed, and we focus on curative public health investment. Furthermore, we provide a normative analysis and show that introducing a health component in the pension benefit formula is welfare-improving. As such, we propose that both public programs should not be considered in isolation, contrary to what is mostly assumed in the above-cited literature. This, in a sense, also bridges the two strands of literature we discussed here.

The paper proceeds as follows. Section 2 presents the model and establishes the equilibrium. Section 3 describes the effect of a rise in health investment on steady state capital accumulation. Section 4 combines our findings to shed light on the potential welfare ramifications brought about by the kind of mixed pension system we propose. Section 5 concludes.

## 2 The Model

We consider a closed economy populated by a continuum of perfectly foresighted and identical individuals whose lifespan is finite and divided into two periods: youth (the working period) and old age (the retirement period). We assume no uncertainty in lifespan, and each period duration is normalized to one.

At every date  $t$ , a mass  $N_t$  of newly born individuals (generation) overlaps with the previous one.  $N_t$  grows at an exogenous rate of  $n \in (-1; +\infty)$ , so that  $N_t = (1 + n)N_{t-1}$ .

When young, the representative agent born at  $t$  inelastically supplies one unit of labor and earns the competitive wage rate  $w_t$ . When retiring, agents consume their accumulated savings as well as the pension benefits (which we formally define below) provided by the government.

The public system consists of two separate pillars: a pension system and a health care system. These two pillars are detailed below.

## 2.1 Public Health Investment

We assume that individuals have perfect health (normalized to 1) in the first period of their life, while in the second (old age) period, their health has deteriorated. Their health status in old age, or equivalently their level of autonomy, is denoted by  $d_t$  and depends on public health investment (denoted by  $h_t$ ), so that

$$d_t(h_t) = \frac{d_0 + h_t}{1 + h_t}, \quad (1)$$

with  $d_0 \in [0, 1)$ . In particular  $d_t$  is an increasing and concave function of  $h_t$  with  $d(0) = d_0$ ,  $\lim_{h \rightarrow \infty} d(h) = 1$  and  $\lim_{h \rightarrow 0} d'(h) = 1 - d_0$ . We remain agnostic about the significance of  $d_0$ , which corresponds to the minimum health status in the absence of public health expenditures. It could be a fixed health status component inherited from a younger age or any other component influencing health status when old.

Note that since the economy is populated by a continuum of identical individuals,  $d_t$  also represents the *average* health status at time  $t$  in this economy.

Quite importantly, observe that the time index of  $h_t$  in equation 1 is the same as the time index of  $d_t$  so that health expenditures are made in the same period as the benefits on health are experienced.<sup>8</sup> This happens because the objective of our paper is to consider curative rather than preventive health expenditures, as is the case for public health investment in quality of life during old-age. As already discussed in the introduction, such investments range from building nursing homes to providing day-to-day care, medications and therapies, or quite simply extending existing medical services.<sup>9</sup>

In order to finance public investments in the health of the elderly generation  $h_t$ , we assume that the government levies a health tax  $\tau_h \in [0, 1]$  on the labor income of the current working generation. For the sake of simplicity, we set the proportional tax on gross wages to be constant over time and, assuming that at each period the health care budget needs to be balanced, we have

$$h_t = \tau_h w_t. \quad (2)$$

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<sup>8</sup>This specification is used by Chakraborty (2004) for the survival probability. A translated version of this function is also used by Blackburn and Cipriani (2002) or by Fanti and Gori (2014).

<sup>9</sup>Although the literature has already modeled the impact of preventive health investment in OLG models with PAYGO pensions (see Chakraborty, 2004 or Fanti and Gori, 2011a), to the best of our knowledge, no model has studied the effect of curative health investments in such a framework.

The objective of this paper is to understand the effect of *public* health investments in the health of the elderly on capital accumulation and welfare. For this reason, we voluntarily abstract from modeling private investments in health.<sup>10</sup>

## 2.2 Health-Dependent Pensions

We assume a PAYG pension system where young adults pay at period  $t$  for the pension benefits received by retirees in the same period. In order to finance the pension system, a proportional contribution rate,  $\tau_{p,t} \in [0, 1]$  is paid by the workers, so that

$$\tau_{p,t} = \tau_0 + \tau_1(1 - d_t), \quad (3)$$

where  $\tau_0, \tau_1 \in (0, 1)$  are exogenous constants. The contribution rate paid by workers is then twofold. The first part  $\tau_0$  does not depend on the current health status in society; the second one,  $\tau_1(1 - d_t)$  does. This second (health-related) contribution part constitutes the main difference with respect to existing models of pensions and LTC, which always consider pensions and LTC systems as two separate systems and never condition the taxes to be paid on the current average level of old-age disability in society. The idea here is to make workers pay more whenever the health status of the old in the society deteriorates (i.e.  $d_t$  is smaller).<sup>11</sup> For that reason, the contribution rate to the pension system depends on time  $t$ , through  $d_t$  (or  $h_t$ ).

Assuming that the pension system is balanced at each time period  $t$ , each retiree will receive a pension benefit  $p_t$  of the following form:

$$p_t = (1 + n)(\tau_0 + \tau_1(1 - d_t))w_t \quad (4)$$

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<sup>10</sup>In reality, the individual health condition, even in old age, depends on many factors such as for example, personal behavior and lifestyle, as well as health investments (see among others, Balia and Jones, 2008, Contoyannis and Jones, 2004, Ettner, 1996, Kaplan et al., 1987). Modeling private investments in health would require to make further assumptions regarding complementarity or substitutability between public and private health spending. In any case, we would find some crowding-out effect of private health spending by public ones, the extent to which would depend on the relative return of private versus public spending, on the form of the health production function and on the levels of the pension contributions and benefits. This would complicate our model without further qualitative results. At the extreme end, and for some parameter values, it may even be the case that private spending is fully crowded out by public spending, which brings us back to the special case in our model with only public spending.

<sup>11</sup>This is comparable to the “Journée de la solidarité” (i.e. “solidarity day”) which was implemented in France in 2004. This policy measure consisted in suppressing one vacation day so as to finance LTC at the old age and disability programs. This can equivalently be seen as imposing an additional contribution based on the average old-age disability level in the society.

where the left-hand side accounts for all pension benefits given to retirees and the right-hand side accounts for total contributions to the pension system.

The first part of the above equation, that is,  $(1+n)\tau_0w_t$ , denotes the *traditional* contribution part of the PAYG pension benefit. The second part, i.e.  $(1+n)\tau_1(1-d_t)w_t$ , constitutes the novelty of our approach with respect to standard retirement models: it is the ‘*disability-augmented*’ part of the pension benefit. If health were perfect (i.e., individuals remained fully autonomous in old age),  $d_t = 1$  the second part would vanish. In that case, retirees would obtain a lump sum benefit,  $p_t = \tau_0(1+n)w_t$ , independent from the health status in society. Inversely, whenever health is imperfect, that is,  $d_t < 1$ , workers face an additional tax contribution which is redistributed to retirees. The better (resp. the worse) the health condition in old age in society (through higher -resp. lower- public investment in health), the lower (resp. the higher) would be the tax rate and thus the pension benefits to be served. This additional benefit, indexed to the average level of disability in society, can then be used to finance the additional LTC expenditures caused by a lower level of autonomy during old age.<sup>12</sup>

Note finally that we assume here that the financing of the pension system and of the health system are separated so that no cross-subsidisation is possible between the two systems. We do so for several reasons. First, it is quite realistic to assume two separate public systems financed through different contributions. Second, allowing for cross-subsidisation would complicate the model substantially, and we may not be able to have an explicit relationship between the health status and the pension benefit (as in (4)). Finally, allowing for cross-subsidisation is not necessary to establish the impacts on capital accumulation and on welfare of linking pension benefits to health status.

### 2.3 Individuals

The lifetime welfare of the representative individual of generation  $t$  is modeled through a homothetic and separable utility function  $U_t$ , defined over consumption of a private good in young and old ages, and over the health status in old age:

$$U_t = \ln(c_{1,t}) + \beta[\ln(c_{2,t+1}) + \gamma \ln(d_{t+1})], \quad (5)$$

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<sup>12</sup>We implicitly assume here that becoming dependent in old age entails higher overall financial needs than remaining autonomous. On this, see De Donder and Leroux (2021).

with  $c_{1,t}$  and  $c_{2,t+1}$ , denoting private consumption respectively when young and when old (i.e. retired), and  $d_{t+1}$  is the health status in old-age which, as we detailed in Section 2.1, depends on public health investment. The parameter  $0 < \beta \leq 1$  accounts for time preferences, while  $\gamma > 0$  is an individual parameter for health preference. A higher  $\gamma$  corresponds to a higher weight attributed to health in old age, and as we show below, it will motivate higher public investment in health and higher health-related tax rates. As a preference for desired health status during old age, this parameter can also be thought of as a health-oriented expression of ‘future orientation’.<sup>13</sup> Except for the parameter  $\gamma$ , this formulation of the individual’s utility is similar to Marchiori and Pierrard (2022).

Assuming separability is standard in the OLG and the health economics literature. Indeed we implicitly assume here that there is no complementarity or substitutability between health (or disability) and consumption in old age, or equivalently that the marginal utility of old-age consumption does not depend on the health status (i.e. cross derivatives between consumptions and health are null). As argued by Hall and Jones (2007) and De Donder and Leroux (2021), whether substitutability and complementarity should be assumed is an empirical (and debatable) question. It crucially depends on the type of consumption goods considered, whether it includes traveling, food, restaurants or, conversely, medical expenditures and the adaptation of the house to limitations in daily-life activities. In the former case, consumption and good health are likely to be complements; in the latter case, they are likely to be substitutes. In order to keep our analysis tractable, we decided to abstract from these considerations.

At time  $t$ , young individuals join the workforce and offer one unit of labour to firms, and receive the competitive wage  $w_t$ . This salary is taxed to finance both public health investment and the pension system. Therefore, the budget constraint of the young agent at time  $t$  is given by

$$c_{1,t} + s_t = w_t(1 - \tau_h - \tau_{p,t}), \tag{6}$$

where  $s_t$  are the individual’s savings. Savings are deposited in a mutual fund accruing at a

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<sup>13</sup>In psychology and related fields, ‘future orientation’ is broadly defined as the extent to which an individual thinks about the future, anticipates future consequences, and plans ahead before acting. In a survey covering French respondents, Apouey (2018) explores this broader notion of time preferences over the future, and finds correlational evidence linking it to preparation for old age (showing, for example, in saving behaviour, home adaptation and ownership, but *not* in LTC insurance). Similarly to Reckers-Droog et al. (2021), this preference parameter can then be seen as representing the marginal willingness to pay for health, but then specifically oriented towards future old age.

gross return of  $r_{t+1}$ . Note that the overall tax rate needs to be below unity:  $(\tau_{p,t} + \tau_h) \in (0, 1]$ .

During old age, consumption  $c_2$  is financed out of savings and pension benefits. The budget constraint of an old agent born at time  $t$  then writes as

$$c_{2,t+1} = s_t(1 + r_{t+1}) + p_{t+1}, \quad (7)$$

with  $p_{t+1}$  the pension benefit as defined by (4). Substituting equations (4), (6) and (7) into (5) and maximizing the individual's utility function  $U_t$  w.r.t. savings, it can be shown that the optimal saving decision  $s_t$  of an individual born in period  $t$  is equal to

$$s_t = \frac{(1 + r_{t+1})w_t(1 - \tau_h - \tau_{p,t})\beta - (1 + n)\tau_{p,t+1}w_{t+1}}{(1 + r_{t+1})(1 + \beta)}. \quad (8)$$

The second term in the numerator represents the pension benefit,  $p_{t+1}$  the individual expects to receive at the old age. The higher the pension benefit, the smaller the amount of savings,  $s_t$ . Hence, if the government increases public health spending  $h_{t+1}$ , this will improve the health status at the old age ( $d_{t+1}$ ), but at the same time it will decrease the amount of pension received. In turn, the individual will choose to save more.

## 2.4 Firms

At every period  $t$ , firms produce a final good whose price is normalised to unity, for an amount  $Y_t = F(K_t, L_t)$ , that can be consumed, saved or invested in public health. Since in our model, every young agent works,  $L_t = N_t$  and assuming a standard Cobb Douglas production function, we have

$$F(K_t, N_t) = AK_t^\alpha N_t^{1-\alpha}, \quad (9)$$

with  $\alpha \in (0, 1)$  and where  $A > 0$  accounts for the exogenous technology productivity or, equivalently, total factor productivity.

We define the production function in *per capita* terms:

$$y_t = \frac{Y_t}{N_t} = f(k_t) = Ak_t^\alpha,$$

with  $k_t$  is capital per unit of labor.

Perfect competition in the goods market implies that both capital and labor are paid at

their respective marginal productivity, so that the wage is  $w_t = \frac{\partial F(K_t, N_t)}{\partial N_t}$  and the rental rate of capital is  $R_t = \frac{\partial F(K_t, N_t)}{\partial K_t}$ . Assuming (in the latter) full depreciation of capital, we get

$$w_t = (1 - \alpha)Ak_t^\alpha \quad \text{and} \quad R_t \equiv 1 + r_t = \alpha Ak_t^{\alpha-1}. \quad (10)$$

Again, in the situation of complete depreciation of the capital at each period, we obtain the following relationship between capital and savings:

$$k_{t+1} = \frac{s_t N_t}{N_{t+1}}. \quad (11)$$

## 2.5 Equilibrium

Given the initial level of capital stock  $k_0$ , the competitive equilibria are characterized by a sequence of  $\{k_t\}$  that jointly satisfy the savings condition defined in (8), the full depreciation capital equation (11), the prices equation (10) and, the health status and health expenditures equations (1) and (2) respectively. A steady state equilibrium is an equilibrium in which the values of  $k_t$  (and thus, of all the other variables) are constant over time. The following proposition provides an existence and a uniqueness result for the steady state equilibria in the case where  $\alpha \in (0, \frac{1}{2}]$ .<sup>14</sup>

**Proposition 1.** *Suppose that  $\tau_h + \tau_0 + \tau_1 < 1$  and  $\alpha \in (0, \frac{1}{2}]$ . Then, for a given initial level of capital  $k_0 > 0$ , the dynamics  $\{k_t\}$  of the per capita stocks of capital over time is well defined. The system admits two steady states: the trivial steady state where  $k^* = 0$  which is unstable and a positive asymptotically stable steady state  $k^* > 0$ .*

*Proof.* See Appendix A.1. □

For further use, we denote the steady state values of consumption in both periods as well as the health status associated to  $k^* > 0$ , by  $c_1^*$ ,  $c_2^*$  and  $d^*$ .

## 3 Public Health Investment and Capital Accumulation

Having established the equilibrium, we now focus on the long-term welfare effects of public health investment when the pension system is made conditional on the old-age level of dis-

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<sup>14</sup>Values for  $\alpha$  typically range between 0.3 and 0.4 (see for example Gollin, 2002). In the case where  $\alpha$  is greater than 1/2, we may have multiplicities of stable equilibria.

ability in society (that is,  $p_t$  depends on  $d_t$ ). We first analyse the effect of the health tax  $\tau_h$  on capital accumulation. This is crucial since the level of capital determines long-term outcomes such as wages, interest rates, and ultimately welfare (see Section 4).

Whether  $k^*$  increases or decreases in  $\tau_h$  depends on several forces going in opposite directions. On the one hand, a higher health tax reduces disposable income, in turn discouraging savings and eroding capital stock. On the other hand, a higher health tax increases health in old age (through higher public health investments) so that the contribution rate to finance pensions decrease, which increases disposable income, increases the willingness to save and fosters capital accumulation. Nonetheless, depending on parameter values, the latter effect may dominate and we may obtain that as  $\tau_h$  increases, the steady state level of capital,  $k^*$ , increases. Proposition 2 below gives an explicit condition ensuring that this is the case.

**Proposition 2.** *Suppose that the assumptions in Proposition 1 are verified. Consider an economy where  $\tau_h$  is initially equal to 0. The steady state stock of capital,  $k^*$  increases with a small  $\tau_h$  as long as*

$$\Gamma(\alpha, n, \beta, A, \tau_0, \tau_1) := (1 - d_0)\tau_1(1 - \alpha) [(1 + n)k_h + A\beta\alpha k_h^\alpha] - \alpha\beta > 0 \quad (12)$$

where  $k_h$  is the steady state level of capital when  $\tau_h = 0$  and is equal to

$$k_h = \left( \frac{(1 - \tau_0 - (1 - d_0)\tau_1)\alpha(1 - \alpha)}{\tau_0 + (1 - d_0)\tau_1 + \alpha(1 + \beta - \tau_0 - (1 - d_0)\tau_1)} \frac{\beta A}{(1 + n)} \right)^{1/(1-\alpha)}. \quad (13)$$

*Proof.* See Appendix A.1. □

Note that the expression on the left-hand side of condition (12) is increasing in the level of capital per capita  $k_h$  so that this condition is more likely to be verified in developed economies with an aging population. That being said,  $k_h$  is not exogenous in our model but depends on specific values of the parameters as in formula (13). In the next proposition, we derive some comparative statics and see how  $\Gamma$  varies with  $A$ ,  $\tau_0$ , and  $\tau_1$ .

**Proposition 3.** *Suppose that the assumptions of Proposition 1 are verified. The function  $\Gamma$  in (12) increases in  $A$ , decreases in  $\tau_0$ , but is ambiguous in  $\tau_1$  and  $d_0$ .*

*Proof.* See Appendix A.1. □

The above proposition shows that for higher levels of the technological parameter  $A$ , condition (12) is more likely to be verified. Hence, the introduction of a health tax is more likely to foster capital accumulation when  $A$  is high. This is then more likely to be the case in developed economies.

Conversely, the effect of  $\tau_0$  on  $\Gamma$  is driven by the negative effect of  $\tau_0$  on capital accumulation. Reducing disposable income of households reduces their saving capacity, which in turn decreases capital accumulation, and decreases the likelihood that the threshold condition is satisfied. Hence, a higher level of  $\tau_0$  is more likely to make condition (13) not being satisfied and to lead to a lower level of  $k^*$  when  $\tau_h$  increases. Finally, when  $\tau_1$  increases, the same effect on  $\Gamma$  is at play: a higher  $\tau_1$  decreases disposable income and pushes toward lower savings and a lower level of  $k^*$ . Yet, there is now an additional competing effect. Increasing  $\tau_1$  amplifies the impact of increasing  $\tau_h$  (through  $d_t$ ) on the contribution rate. Indeed, the degree to which increasing  $\tau_h$  also increases  $d_t$  and hence reduces the contribution rate, will be higher for higher levels of  $\tau_1$ . This, in turn, increases disposable income, savings and capital accumulation. This is why the overall impact of  $\tau_1$  on  $\Gamma$  is ambiguous.

To illustrate how the steady state level of capital responds to an increase of the health tax, we perform a simple numerical analysis in Figure 1. To this end, we set the parameter values at the following levels. First, we assume that each representative young adult individual (whose life starts at 20 years old) lives two periods of 40 years each. Assuming an annual discount rate equal to 1.25%, we find that  $\beta = 0.6$ , which is similar to the value of  $\beta$  in Žamac (2007).

We use a capital-output elasticity of  $\alpha = 0.33$  as in de la Croix and Michel (2002). We then set the contribution rate at  $\tau_0 = 8\%$  and  $\tau_1 = 7.5\%$ , because a majority of the OECD countries have rates within this range.<sup>15</sup> Following de la Croix and Michel (2002), we consider the replacement rate in a single-parent model over a period of 40 years, so that  $(1 + 0.025)^{40} = 1 + n$ , which gives an exogenous population growth rate of  $n = 1.685$ . Finally, we set the scale parameter  $A$  equal to 100 and the minimum health level  $d_0$  to 0.25.<sup>16</sup> We

<sup>15</sup> See for instance the graphs available at <https://data.oecd.org/tax/social-security-contributions.htm>. Over the period 2000-2020, Social Security contribution rates in the OECD countries varied around an average of 8%.

<sup>16</sup> Our results are robust to alternative levels of  $d_0$ . Yet, for higher levels of  $d_0$ , our results, although qualitatively similar, are quantitatively less strong. To see this, note that eq. (1) can be rewritten as  $d_t(h_t) = d_0 + (1 - d_0)h_t/(1 + h_t)$  where the last part is an increasing function of  $h_t$ . This implies that the larger  $d_0$ , the smaller the marginal impact of  $h_t$  on  $d_t(h_t)$ .

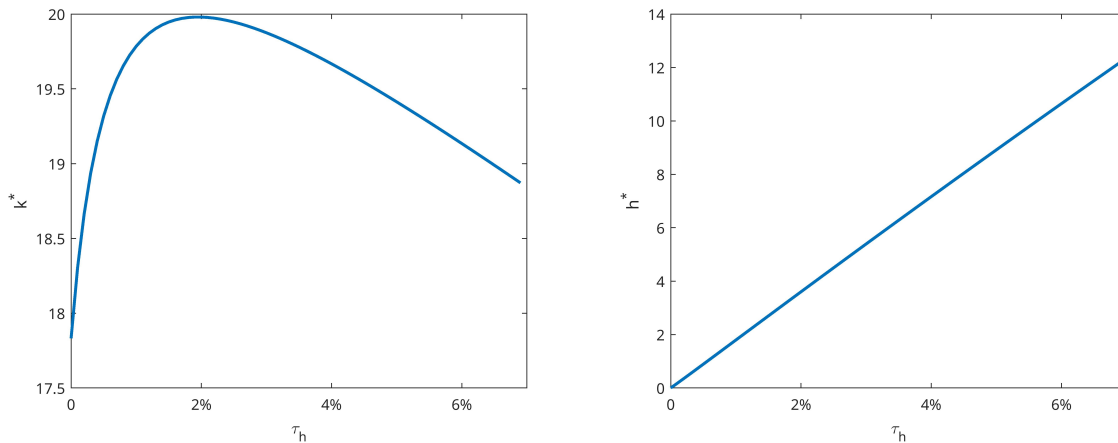


Figure 1: Numerical analysis: the effect of  $\tau_h$  on  $k^*$  and on  $h^*$

then perform a numerical exercise considering a range of health tax rate  $\tau_h$  between 0% to 7%.<sup>17</sup>

When  $\tau_h$  is set to zero, the corresponding steady state level of capital is equal to  $k^* = 17.8$ . Increasing the health tax, we find that the steady state level of capital increases up to 20.0 when  $\tau_h = \bar{\tau}_h \simeq 2.0\%$ , we find that increasing public health taxation further would reduce capital accumulation. The behavior we observe in Figure 1 (and which is robust to the different simulations we have performed) highlights the existence of a threshold  $\bar{\tau}_h$ .

When health taxation is low, that is,  $\tau_h < \bar{\tau}_h$ , we find that the steady state level of capital increases following an increase in  $\tau_h$ . In that situation, the increase in the health tax decreases the pension contribution rate (this corresponds to the second part in (3)) which in turn increases disposable income and the willingness to save for old age. This effect is found to dominate the direct negative impact of a higher  $\tau_h$  on disposable income of the young. The overall effect is an increase in savings and thus, in capital accumulation  $k^*$ . This, in turn, results in higher wages, and into even higher health investment and a better health condition of the elderly. This sparks off an indirect general equilibrium feedback effect which further encourages saving, and fosters capital accumulation. As a result, the steady state output per worker increases.<sup>18</sup>

However, this multiplier effect is only triggered for small values of  $\tau_h$ . For higher tax

<sup>17</sup>The code for generating figures and conducting numerical simulations in the paper is accessible at the following link: <https://sites.google.com/site/giorgiofabri1979/code>.

<sup>18</sup>A similar mechanism where health investment bears on economic growth can be found in Chakraborty (2004), Fanti and Gori (2011b) and Fanti and Gori (2014).

levels, the direct effect (reduction of savings due to higher taxation and reduced disposable income) outmatches the positive effect of reduced health-dependent pensions as a result of the improvement of elderly health.

All in all, this section shows that the observed positive relationship between increased public health expenditures and capital accumulation (when  $\tau_h$  is not too high) is driven by the young individuals anticipating a better health status in old age and thus lower public pensions. Obviously, this would not occur under a standard (non-augmented) PAYG pension system. When  $\tau_1 \rightarrow 0$ , there is no effect of increasing health expenditures on pension benefits, and thus increasing public health taxation always reduces capital accumulation.

Finally, we find that the effect of  $\tau_1$  on the steady state level of capital is monotonic: increasing  $\tau_1$  always decreases capital accumulation  $k^*$  at the steady state.<sup>19</sup> To understand this, it is sufficient to note that savings and pension benefits are substitutes in transferring resources from the young age to the old age period. Increasing  $\tau_1$  (or equivalently  $\tau_0$ ) would unambiguously increase  $p_t$  which, in turn, would decrease the willingness to save for old age. In addition, higher first-period taxation would decrease disposable income and as such the saving capacity. These two effects go in the same direction with respect to decreasing savings and thus, to decreasing capital accumulation in equilibrium.

## 4 Welfare

In this section, we first derive the optimal tax rates  $\tau_1$  and  $\tau_h$ , taking  $\tau_0$  as given. We start by modeling an exogenous universal pay-as-you-go pension system, by fixing  $\tau_0 > 0$  and assuming that this tax rate can take different exogenous values (see below). Indeed, in reality and for different political, historical and economic reasons (e.g. insurance motive, myopia), which we do not model here, public pension systems do exist. We then explore the implications of adding a health-dependent pension component to this existing pension system. Therefore, we consider a *second-best* problem and show first to what extent the pension scheme should be augmented. Second, we quantify the increase in welfare associated with enhancing the standard pension system.

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<sup>19</sup>To prove this, we use the implicit function theorem on  $k^*$  similar to the comparative statics derivation with respect to  $\tau_h$  (see proof of Proposition 2 in Appendix A.1).

## 4.1 Optimal Taxation

In this section, we derive the second-best tax rate levels,  $\tau_1^*$  and  $\tau_h^*$ , postulating the existence of a pension system that would provide benefits in old age to the individual (as is the case in most developed countries). To do so, we exogenously fix  $\tau_0 \geq 0$  and find the values of  $\tau_1$  and  $\tau_h$  that would maximize the utility of the representative agent at the steady state when varying the parameter of health preference  $\gamma$ . More precisely, we study the impact of varying  $\gamma$  on the (non-trivial) steady state equilibrium characterized in Proposition 1.

In our general equilibrium framework, choosing the optimal values for  $\tau_1$  and  $\tau_h$  consists in maximizing the utility of the representative agent, eq. (5), taking into account that  $c_1^*$ ,  $c_2^*$  and  $d^*$  (or  $h^*$ ) depend both directly on these tax rates as well as indirectly on them (through  $k^*$  in the pension benefits, wages and interest rates). Table 1 reports the optimal values of the tax rates as a function of the preference parameter  $\gamma$ .<sup>20</sup> In this table, we consider different possible values of  $\tau_0$  ranging from 0% to 15%. The assumed values for the  $\tau_0$  rates correspond to a range of what is usually observed in OECD countries (see footnote 15 about this).<sup>21</sup>

First note that the level of the contribution to the health system,  $\tau_h^*$ , is always strictly positive, whatever the levels of  $\tau_0$  and  $\tau_1^*$ . This is directly related to the fact that individuals value health in their utility function so that  $h^* > 0$  is always optimal. On the contrary,  $\tau_1^*$  becomes positive only above some  $\gamma$ -threshold which will depend positively on the level of  $\tau_0$ .<sup>22</sup> This is due to the fact that both  $\tau_0$  and  $\tau_1$  put pressure on financial resources available to the individual in the first period, so that when  $\tau_0$  increases, the minimum  $\gamma$  level for which  $\tau_1^*$  becomes positive must also increase.

Let us now concentrate on the role of  $\gamma > 0$  on the optimal pension design. An increase in the health-preference parameter always increases the optimal levels of the tax rates  $\tau_1$  and  $\tau_h$ , for any given level of  $\tau_0$ .<sup>23</sup> Indeed, when people exhibit a higher preference for health, it is optimal for a government who seeks to maximise welfare to increase health-related taxes.

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<sup>20</sup>In a situation where agents do not value their health (i.e.  $\gamma = 0$ ), we find that it is optimal to set  $\tau_h^* = \tau_1^* = 0$  for two reasons. On the one hand, individuals do not obtain direct utility from investing in health and, on the other hand, the agents' savings (and eventually the fixed part of the pension system) are already sufficient to ensure consumption smoothing.

<sup>21</sup>We use the same parametrization as in the previous section:  $\alpha = 0.33$ ,  $\beta = 0.6$ ,  $A = 100$ ,  $d_0 = 0.25$  and  $n = 1.685$ . In the simulations, we also find that the level of health  $d^*$  is always comprised between 0.90 and 0.96.

<sup>22</sup>Equivalently, the minimum  $\gamma$  level at which  $\tau_1^*$  becomes positive increases with the level of  $\tau_0$ .

<sup>23</sup>Note that the optimal tax rates increase nonetheless at a lower rate when  $\gamma$  becomes higher, which is due to the concavity of the utility function in consumption.

Table 1: Optimal values of the tax rates varying  $\gamma$ 

	$\tau_0 = 0\%$			$\tau_0 = 8\%$		
	$\tau_1^*$	$\tau_h^*$	$\tau_1^*(1-d^*)$	$\tau_1^*$	$\tau_h^*$	$\tau_1^*(1-d^*)$
$\gamma = 1$	0.0%	3.1%	0.00%	0.0%	3.1%	0.00%
$\gamma = 2$	0.0%	4.5%	0.00%	0.0%	4.5%	0.00%
$\gamma = 3$	0.0%	5.5%	0.00%	0.0%	5.5%	0.00%
$\gamma = 4$	0.0%	6.4%	0.00%	0.0%	6.4%	0.00%
$\gamma = 5$	0.1%	7.1%	0.01%	0.0%	7.1%	0.00%
$\gamma = 6$	1.3%	7.8%	0.06%	0.0%	7.8%	0.00%
$\gamma = 7$	2.4%	8.5%	0.10%	0.0%	8.4%	0.00%
$\gamma = 8$	3.3%	9.1%	0.13%	0.7%	8.9%	0.03%
$\gamma = 9$	4.1%	9.6%	0.16%	1.7%	9.5%	0.07%
$\gamma = 10$	4.8%	10.1%	0.18%	2.5%	10.0%	0.10%
$\gamma = 15$	7.8%	12.4%	0.24%	5.9%	12.2%	0.20%
$\gamma = 20$	10.0%	14.2%	0.27%	8.4%	14.0%	0.25%

	$\tau_0 = 5\%$			$\tau_0 = 15\%$		
	$\tau_1^*$	$\tau_h^*$	$\tau_1^*(1-d^*)$	$\tau_1^*$	$\tau_h^*$	$\tau_1^*(1-d^*)$
$\gamma = 1$	0.0%	3.1%	0.00%	0.0%	3.1%	0.00%
$\gamma = 2$	0.0%	4.5%	0.00%	0.0%	4.4%	0.00%
$\gamma = 3$	0.0%	5.5%	0.00%	0.0%	5.5%	0.00%
$\gamma = 4$	0.0%	6.4%	0.00%	0.0%	6.3%	0.00%
$\gamma = 5$	0.0%	7.1%	0.00%	0.0%	7.0%	0.00%
$\gamma = 6$	0.0%	7.8%	0.00%	0.0%	7.7%	0.00%
$\gamma = 7$	0.6%	8.4%	0.03%	0.0%	8.3%	0.00%
$\gamma = 8$	1.6%	9.0%	0.07%	0.0%	8.8%	0.00%
$\gamma = 9$	2.6%	9.6%	0.10%	0.0%	9.3%	0.00%
$\gamma = 10$	3.4%	10.1%	0.13%	0.7%	9.8%	0.03%
$\gamma = 15$	6.6%	12.3%	0.22%	4.4%	12%	0.16%
$\gamma = 20$	9.0%	14.1%	0.26%	7.0%	13.8%	0.23%

Conversely, the optimal level of  $\tau_1$  becomes positive only for a sufficiently high level of  $\gamma$ . The mechanism behind this result runs as follows. When  $\gamma$  increases,  $\tau_h$  increases which decreases savings in the first period as well as the overall return individuals can obtain during old age. Appendix A.2 shows that this is effectively the case. When  $\gamma$  is low, pension benefits together with savings are still high enough to guarantee smoothed consumption in old age. Yet, when  $\gamma$  increases further, saving decreases and consumption smoothing is not possible anymore. It then becomes desirable to introduce the health-dependent component of the pension benefit,  $\tau_1 > 0$  so as to ensure enough resources are effectively transferred for old age.

All in all, this section shows that while it is always optimal to have a health system ( $\tau_h^* > 0$ ), introducing a health component in the pension benefit (i.e. an augmented pension benefit with  $\tau_1^* > 0$ ) becomes desirable only above some high enough level of  $\gamma$ , in order to compensate for lower savings.

Relating these findings with what we observe in reality, it is reasonable to assume that in recent years individuals have become more concerned with their health (in all stages of life) as a consequence of first, population aging (and with it, an increasing probability to become dependent) and second, of our economies becoming richer and more informed about health issues. The above results then demonstrate that it would indeed be optimal to implement a health-dependent pension system in this context.

## 4.2 Quantification of Welfare

We now concentrate on quantifying the welfare effect of implementing an augmented pension system, that is the welfare effect of allowing the pension benefits to be conditioned on the average disability level in society. To do so, we set  $\tau_0 = 8\%$  as in the average of OECD countries and we compare the welfare of the representative individual in steady state when  $\tau_1 = 0$  (that is, when the pension system is a “traditional” one) with his welfare when  $\tau_1^* > 0$  is set at its optimal steady state level (i.e. the pension benefit is “disability-augmented”). In both cases,  $\tau_h$  is chosen optimally (i.e. as a solution of the individual’s problem). We then quantify by how much society’s welfare is increased when augmenting the standard PAYG pension system by its disability-related component. As we showed in the previous section, whether the optimal level of  $\tau_1^*$  is strictly positive depends on the size of the health-preference parameter. In order to be able to compare welfare in the two pension situations (whether it is augmented or not), we will then concentrate on values of  $\gamma$  between 5 and 25 ensuring that  $\tau_1^* > 0$ .

Our results are reported in Table 2, which is divided in three parts. In the first four columns, we report the values of  $c_1^*$ ,  $c_2^*$ ,  $h^*$  (and  $\tau_h^*$ ) when  $\tau_0$  is fixed to 8%,  $\tau_1$  is exogenously fixed to 0% but  $\tau_h$  maximises the steady state utility of the representative agent. In the second part of the table (columns six, seven, eight and nine), we report the values of  $c_1^*$ ,  $c_2^*$ ,  $h^*$  (and  $\tau_h^*$ ) when  $\tau_0$  is still fixed to 8% but both  $\tau_1$  and  $\tau_h$  are (jointly) chosen to maximize the utility of the representative agent at the steady state.<sup>24</sup> The last column reports the relative variation in the utility between a standard PAYG system ( $\tau_1 = 0\%$ ) and the augmented PAYG system ( $\tau_1^* > 0$ ).

Table 2 clearly shows that, given the existence of a universal PAYG pension system,

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<sup>24</sup>We do not report the values of  $\tau_1^*$  here, but these correspond to the values reported in the part of Table 1 where  $\tau_0 = 8\%$ .

augmenting the system with a health-dependent pension increases welfare when  $\tau_1$  and  $\tau_h$  are set at their optimal levels and, provided that  $\gamma$  is sufficiently high. The effects of augmenting the pension system may look small when considering variations in utility, but this is due to the concavity of the log-utility function: for large values of  $c$ , the utility function is not very sensible to variations of  $c$ . Still, we can observe a relevant change in the consumption behavior of the agents.<sup>25</sup>

Table 2: The welfare effect of augmenting the PAYG system when  $\tau_0 = 8\%$

	$\tau_1 = 0\%$				$\tau_1^* > 0$				% variation in $U$
	$c_1^*$	$c_2^*$	$d^*$	$\tau_h^*$	$c_1^*$	$c_2^*$	$d^*$	$\tau_h^*$	
$\gamma = 5$	99.3	272.5	0.9449	7.1%	99.3	272.5	0.9449	7.1%	+0.000%
$\gamma = 10$	94.4	268.1	0.9589	9.9%	94.1	268.0	0.9593	10.0%	+0.004%
$\gamma = 15$	91.0	264.8	0.9652	11.9%	90.1	264.4	0.9658	12.2%	+0.025%
$\gamma = 20$	88.2	262.1	0.9690	13.6%	87.0	261.5	0.9697	14.0%	+0.053%
$\gamma = 25$	85.9	259.8	0.9715	15.0%	84.4	258.9	0.9723	15.5%	+0.086%

## 5 Concluding Remarks

This paper shows that introducing a Social Security system which combines both a public health system and a health-dependent pension system to account for average old-age disability in the economy, can be beneficial.

We do so by studying an OLG problem that includes both a health system and a PAYG pension scheme in which pension benefits are augmented by a health-related component, indexed to the average level of disability during old age. We show how such modification of the pension system affects capital accumulation and welfare.

Our analysis produces several insightful predictions regarding the effects of Social Security taxation on individual decisions, capital accumulation, and welfare. First, an increase in public health investment through an increase in the health tax can boost the steady state stock of capital. This result emerges because health taxation (financing curative health care) improves health during old age, mitigating average disability in society, which in turn allows to reduce taxation for financing pensions. This increases the willingness to save for old age, as a result of consumption smoothing. Second, given the existing PAYG pension system, introducing a disability-linked component to the pension benefit allows for welfare improvements. This will

<sup>25</sup>These results are robust to other values of  $\tau_0$ .

even be more the case if the preference for health in society is high enough.

In a context where LTC expenditures are projected to rise dramatically in the near future, and where society's preference for health improvements has been increasing over the last decades, our model highlights the possible positive spillovers of linking the effects of public health investment (reducing old-age dependency) to the size of public pension schemes. Indeed, under the conditions we have identified, investment in curative health programs is shown to increase social welfare if pension are made health-dependent.

Lastly, let us note that in order to obtain clear results we voluntarily abstracted from modeling preventive health expenditures, which may also reduce the extent of dependency in old age. Doing so would require to make several further hypotheses, in particular whether such spending is decided publicly or results from private decisions. In addition, we would need to make assumptions regarding the relationship between preventive and curative health spending, in particular whether these are substitutes or complements. Depending on the preferred assumptions, we may then obtain different conclusions. We believe this would be an interesting extension left for future work.

## Declarations

*Ethical Approval:* not applicable.

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# A Appendix

## A.1 Proofs of Propositions

*Proof of Proposition 1.* Using (8) and (10) one can rewrite (11) as

$$k_{t+1}(1+n) = \frac{(\alpha Ak_{t+1}^{\alpha-1})(1-\alpha)Ak_t^\alpha(1-\tau_h-\tau_{p,t})\beta - (1+n)\tau_{p,t+1}(1-\alpha)Ak_{t+1}^\alpha}{(\alpha Ak_{t+1}^{\alpha-1})(1+\beta)}$$

where  $\tau_{p,t+1}$  is defined by (3). Replacing further for equations (1), (2) and (3), we obtain after some manipulations,

$$\begin{aligned} k_{t+1}(1+n)(1+\beta) + \frac{(1-\alpha)k_{t+1}(1+n)(\tau_0 + (1-d_0)\tau_1 + Ak_{t+1}^\alpha\tau_0\tau_h(1-\alpha))}{\alpha + Ak_{t+1}^\alpha\tau_h(1-\alpha)\alpha} \\ = (1-\alpha) \left( 1 - \frac{(\tau_0 + (1-d_0)\tau_1 + \tau_h + Ak_t^\alpha\tau_h(\tau_0 + \tau_h)(1-\alpha))}{1 + Ak_t^\alpha\tau_h(1-\alpha)} \right) Ak_t^\alpha\beta. \end{aligned} \quad (14)$$

Denoting by  $G(k_{t+1})$  (respectively,  $F(k_t)$ ) the left- (resp. right-) hand side of the above condition, we have

$$G(k_{t+1}) = F(k_t), \quad (15)$$

where  $F$  and  $G$  are functions that map  $\mathbb{R}_+$  in  $\mathbb{R}_+$  (for  $F$ , it is obvious while for  $G$ , we need to use the hypothesis  $\tau_1 + \tau_0 + \tau_h < 1$ ). Computing  $G'(k)$ , we obtain

$$G'(k) = (1+n) \left( 1 + \tau_0 \left( \frac{1}{\alpha} - 1 \right) + \frac{(1-d_0)\tau_1 \left( (1-\alpha)^2 A\tau_h k^\alpha + 1 \right) (1-\alpha)}{\alpha \left( (1-\alpha) A\tau_h k^\alpha + 1 \right)^2} + \beta \right) > 0,$$

moreover  $G(0) = 0$  and  $\lim_{k \rightarrow +\infty} G(k) = +\infty$ . Hence, given an initial level of capital  $k_0 > 0$ , equation (15) uniquely defines a sequence of  $\{k_t\}$  of positive *per capita* stocks of capital over time.

The values of  $k_t$  in steady state correspond to the  $k^*$  satisfying  $G(k^*) = F(k^*)$ . Since  $F(0) = G(0) = 0$ , we find that  $k^* = 0$  is an (obvious) steady state.

If we compute the derivative  $F'$ , we get

$$F'(k) = \frac{(1-\alpha)\alpha A\beta k^{\alpha-1} \left( (1-\tau_h) \left( (1-\alpha) A\tau_h k^\alpha + 1 \right)^2 - (1-d_0)\tau_1 - \tau_0 \left( (1-\alpha) A\tau_h k^\alpha + 1 \right)^2 \right)}{\left( (1-\alpha) A\tau_h k^\alpha + 1 \right)^2}$$

From the explicit expressions of  $G'$  and  $F'$ , we have that (since  $1 - \tau_h - \tau_1 - \tau_0 > 0$ )

$\lim_{k \rightarrow 0} F'(k) = +\infty$ ,  $\lim_{k \rightarrow 0} G'(k) = \frac{\alpha(1+\beta)(1+n)+(1+n)(\tau_0+(1-d_0)\tau_1)(1-\alpha)}{\alpha} \in (0, +\infty)$ . This ensures, since we know that  $F(0) = G(0) = 0$ , that  $F(k) > G(k)$  for small  $k$  (different from 0). Moreover, since  $\lim_{k \rightarrow +\infty} F'(k) = 0$ ,  $\lim_{k \rightarrow +\infty} G'(k) = \frac{\alpha(1+\beta)(1+n)+(1+n)(1-\alpha)\tau_0}{\alpha} \in (0, +\infty)$ ,  $F(k) < G(k)$  for large values of  $k$ . All in all, this ensures that there must exist at least a strictly positive solution to  $F(k) = G(k)$  i.e. a steady state,  $k^* > 0$ .

To prove the uniqueness of the strictly positive steady state one can observe that, after some computation, the condition  $F(k) = G(k)$  for  $k > 0$  is equivalent to

$$\begin{aligned} & \frac{(1+n)}{\beta A} k^{1-\alpha} \\ = & \frac{[(1-\tau_0 - (1-d_0)\tau_1 - \tau_h) + (1-\alpha)Ak^\alpha\tau_h(1-\tau_0 - \tau_h)]\alpha(1-\alpha)}{\tau_0 + (1-d_0)\tau_1 + \alpha(1+\beta - \tau_0 - (1-d_0)\tau_1) + Ak^\alpha\tau_h(1-\alpha)(\tau_0(1-\alpha) + \alpha(1+\beta))} \end{aligned} \quad (16)$$

Multiplying each side by

$$\tau_0 + (1-d_0)\tau_1 + \alpha(1+\beta - \tau_0 - (1-d_0)\tau_1) + Ak^\alpha\tau_h(1-\alpha)(\tau_0(1-\alpha) + \alpha(1+\beta))$$

we obtain an equation in the form

$$a_1 k^{1-\alpha} + a_2 k = a_3 + a_4 k^\alpha.$$

with  $a_i > 0$ . Dividing both sides by  $k^{1-\alpha}$ , we get

$$a_1 + a_2 k^\alpha = a_3 k^{-1+\alpha} + a_4 k^{2\alpha-1}$$

Given that  $\alpha < 1/2$ , the left hand side of the above equation is an increasing function of  $k$  while the right hand side is a decreasing function of  $k$ . So the strictly positive steady state is unique. We denote it by  $k^*$ .

Since  $G(k) > F(k)$  for all  $k > k^*$  and  $G(k) < F(k)$  for all  $0 < k < k^*$  then  $k^*$  is globally asymptotically stable (and the null steady state is unstable).

□

*Proof of Proposition 2.* The unique strictly positive steady state  $k^*$  is defined by  $0 = H(k^*)$  with  $H(k^*) := G(k^*) - F(k^*)$  (where  $F$  and  $G$  are defined in (14)). Using the implicit function

theorem, we have that  $\frac{dk^*}{d\tau_h} = -\left(\frac{\partial H}{\partial k}\right)^{-1} \left(\frac{\partial H}{\partial \tau_h}\right)$  where

$$\begin{aligned} \frac{\partial H}{\partial \tau_h} &= \frac{(1-\alpha)Ak^\alpha \left( k(1+n)(1-d_0)\tau_1(1-\alpha) - \alpha\beta + Ak^\alpha((1-d_0)\tau_1 - 2\tau_h)(1-\alpha)\alpha\beta \right)}{\left( (1-\alpha)Ak^\alpha\tau_h + 1 \right)^2 (\alpha + \alpha\beta)} \\ &\quad - \frac{(1-\alpha)Ak^\alpha \left( A^2k^{2\alpha}\tau_h^2(1-\alpha)^2\alpha\beta \right)}{\left( (1-\alpha)Ak^\alpha\tau_h + 1 \right)^2 (\alpha + \alpha\beta)} \\ \text{and } \left( \frac{\partial H}{\partial k} \right)^{-1} &= -\frac{k \left( (1-\alpha)Ak^\alpha\tau_h + 1 \right)^2 (\alpha + \alpha\beta)}{(1+n)k \left( (1-\alpha)Ak^\alpha\tau_h + 1 \right)^2 (\alpha + \alpha\beta) + (1-\alpha)J} \end{aligned}$$

with

$$\begin{aligned} J &= k(1+n)(\tau_0 + (1-d_0)\tau_1) \\ &\quad + Ak^{1+\alpha}(1+n)\tau_h((1-d_0)\tau_1(1-\alpha) + 2\tau_0)(1-\alpha) + A^2k^{1+2\alpha}(1+n)\tau_0\tau_h^2(1-\alpha)^2 \\ &\quad - Ak^\alpha(1-\tau_0 - (1-d_0)\tau_1 - \tau_h)\alpha^2\beta - 2A^2k^{2\alpha}\tau_h(1-\tau_0 - \tau_h)(1-\alpha)\alpha^2\beta \\ &\quad - A^3k^{3\alpha}\tau_h^2(1-\tau_0 - \tau_h)(1-\alpha)^2\alpha^2\beta \quad (17) \end{aligned}$$

Hence,

$$\frac{dk^*}{d\tau_h} = -\left(\frac{\partial H}{\partial k}\right)^{-1} \left(\frac{\partial H}{\partial \tau_h}\right) = \frac{(1-\alpha)Ak^{1+\alpha}I}{(1+n)k \left( (1-\alpha)Ak^\alpha\tau_h + 1 \right)^2 (\alpha + \alpha\beta) + (1-\alpha)J} \quad (18)$$

with

$$I = k(1+n)(1-d_0)\tau_1(1-\alpha) - \alpha\beta + Ak^\alpha((1-d_0)\tau_1 - 2\tau_h)(1-\alpha)\alpha\beta - A^2k^{2\alpha}\tau_h^2(1-\alpha)\alpha\beta.$$

When  $\tau_h = 0$ , (18) simplifies to

$$\begin{aligned} &\frac{(1-\alpha)Ak_h^{1+\alpha} \left( k_h(1+n)(1-d_0)\tau_1(1-\alpha) - \alpha\beta + Ak_h^\alpha(1-d_0)\tau_1(1-\alpha)\alpha\beta \right)}{(1+n)k_h(\alpha + \alpha\beta) + (1-\alpha) \left[ k_h(1+n)(\tau_0 + (1-d_0)\tau_1) - Ak_h^\alpha(1-\tau_0 - (1-d_0)\tau_1)\alpha^2\beta \right]} \\ &= \frac{(1-\alpha)Ak_h^{1+2\alpha} \left( k_h(1+n)(1-d_0)\tau_1(1-\alpha) - \alpha\beta + Ak_h^\alpha(1-d_0)\tau_1(1-\alpha)\alpha\beta \right)}{(1+n)k_h^{1-\alpha} [(\alpha + \alpha\beta) + (1-\alpha)(\tau_0 + (1-d_0)\tau_1)] - (1-\alpha)A(1-\tau_0 - (1-d_0)\tau_1)\alpha^2\beta} \quad (19) \end{aligned}$$

with  $k_h$  the solution to  $G(k) = F(k)$  when  $\tau_h = 0$ :

$$k_h = \left( \frac{(1 - \tau_0 - (1 - d_0)\tau_1)\alpha(1 - \alpha)}{\tau_0 + (1 - d_0)\tau_1 + \alpha(1 + \beta - \tau_0 - (1 - d_0)\tau_1)} \frac{\beta A}{(1 + n)} \right)^{1/(1-\alpha)}.$$

The denominator of (19) is positive if and only if

$$\begin{aligned} (1 + n) \left( \frac{(1 - \tau_0 - (1 - d_0)\tau_1)\alpha(1 - \alpha)}{\tau_0 + (1 - d_0)\tau_1 + \alpha(1 + \beta - \tau_0 - (1 - d_0)\tau_1)} \frac{\beta A}{(1 + n)} \right) \\ \times [(\alpha + \alpha\beta) + (1 - \alpha)(\tau_0 + (1 - d_0)\tau_1)] \\ > (1 - \alpha)A(1 - \tau_0 - (1 - d_0)\tau_1)\alpha^2\beta \quad (20) \end{aligned}$$

or equivalently, if and only if

$$\begin{aligned} (1 + \beta) + \frac{(1 - \alpha)}{\alpha}(\tau_0 + (1 - d_0)\tau_1) \\ > \tau_0 + (1 - d_0)\tau_1 + \alpha(1 + \beta - \tau_0 - (1 - d_0)\tau_1) = \alpha(1 + \beta) + (1 - \alpha)(\tau_0 + (1 - d_0)\tau_1) \quad (21) \end{aligned}$$

which is always verified under  $\alpha \in (0, 1)$ . So (19) is positive if and only if the numerator is positive i.e. if and only if

$$(1 - d_0)\tau_1(1 - \alpha) [(1 + n)k_h + A\beta\alpha k_h^\alpha] > \alpha\beta$$

This yields condition (12). □

*Proof of Proposition 3.* Observe first that

$$\frac{\partial k_h^{1-\alpha}}{\partial A} = \frac{(1 - \alpha)\alpha\beta(1 - \tau_0 - (1 - d_0)\tau_1)}{(n + 1)(\alpha(\beta + 1) + (1 - \alpha)\tau_0 + (1 - \alpha)(1 - d_0)\tau_1)} > 0$$

and

$$\begin{aligned} \frac{\partial k_h^{1-\alpha}}{\partial \tau_0} &= \frac{1}{1 - d_0} \frac{\partial k_h^{1-\alpha}}{\partial \tau_1} = -\frac{1}{\tau_1} \frac{\partial k_h^{1-\alpha}}{\partial d_0} \\ &= -\frac{(1 - \alpha)\alpha A\beta(\alpha\beta + 1)}{(n + 1)(\alpha\beta + \alpha(1 - \tau_0 - (1 - d_0)\tau_1) + \tau_0 + (1 - d_0)\tau_1)^2} < 0 \quad (22) \end{aligned}$$

so  $\frac{\partial k_h}{\partial A}, \frac{\partial k_h}{\partial d_0} > 0$  and  $\frac{\partial k_h}{\partial \tau_0}, \frac{\partial k_h}{\partial \tau_1} < 0$ .

We compute the partial derivatives of  $\Gamma$  w.r.t. the various parameters. We get

$$\frac{\partial \Gamma}{\partial A} = (1 - \alpha)(1 - d_0)\tau_1 \left[ \alpha\beta k_h^\alpha + \frac{\partial k_h}{\partial A} (1 + n + A\beta\alpha^2 k_h^{\alpha-1}) \right] > 0$$

On the other hand,  $\Gamma$  only depends on  $\tau_0$  through its effect on  $k_h$ . Since  $k_h$  decreases with  $\tau_0$  and  $\Gamma$  increases with  $k_h$ , we obtain that  $\Gamma$  decreases with  $\tau_0$ .

Finally, the effect of  $\tau_1$  on  $\Gamma$  is ambiguous because it is the sum of two terms of opposite signs:

$$\frac{d\Gamma}{d\tau_1} = \frac{\partial \Gamma}{\partial \tau_1} + \frac{\partial \Gamma}{\partial k_h} \frac{\partial k_h}{\partial \tau_1}.$$

We already showed that  $\frac{\partial k_h}{\partial \tau_1} < 0$  and one has  $\frac{\partial \Gamma}{\partial \tau_1} = (1 - d_0)(1 - \alpha)((1 + n)k_h + A\beta k_h^\alpha) > 0$  and  $\frac{\partial \Gamma}{\partial k_h} = (1 - d_0)(1 - \alpha)\tau_1((1 + n) + \alpha A\beta k_h^{\alpha-1}) > 0$ .  $\square$

## A.2 Values of $s^*$ and $r^*$ when $\gamma$ varies

In the table below, we use the following parametrization:  $\alpha = 0.33$ ,  $\beta = 0.6$ ,  $A = 100$ ,  $d_0 = 0.25$  and  $n = 1.685$ . We also set  $\tau_0 = 8\%$ .

Table 3: Values of the interest rate and saving at the optima described in Table 1 for  $\tau_0 = 8\%$

	$r$	$s$	$s(1+r)$
$\gamma = 1$	3.3705	54.8738	239.8264
$\gamma = 2$	3.4389	53.6166	237.9992
$\gamma = 3$	3.4917	52.6789	236.6174
$\gamma = 4$	3.5364	51.9063	235.4666
$\gamma = 5$	3.5759	51.2391	234.4634
$\gamma = 6$	3.6117	50.6463	233.5648
$\gamma = 7$	3.6448	50.1091	232.7443
$\gamma = 8$	3.6817	49.5207	231.8388
$\gamma = 9$	3.7190	48.9369	230.9334
$\gamma = 10$	3.7537	48.4051	230.1021