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Kévin Polisano

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UNIVERSITÉ DE
GRENOBLE

TEXTURE MODELING BY GAUSSIAN FIELDS WITH PRESCRIBED LOCAL ORIENTATION

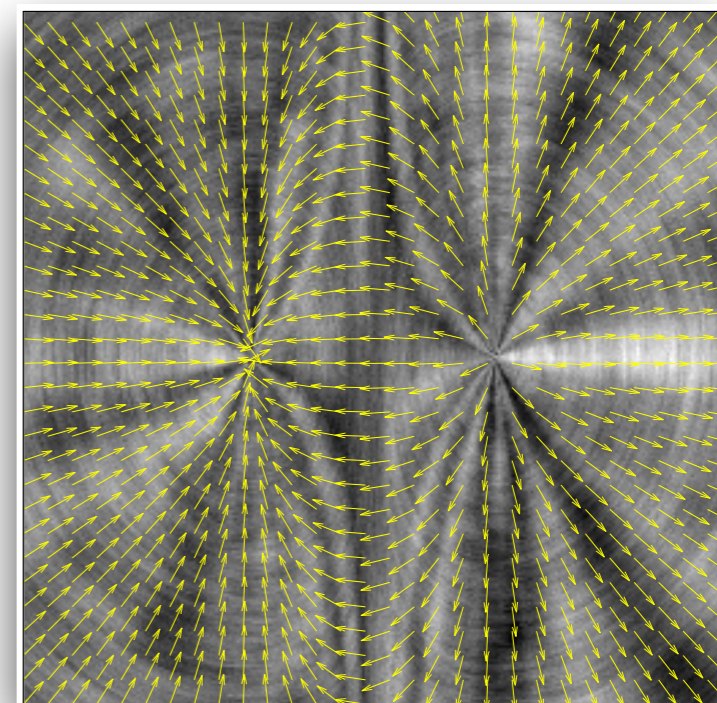
Kévin Polisano / kevin.polisano@imag.fr

joint work with

Marianne Clausel

Valérie Perrier

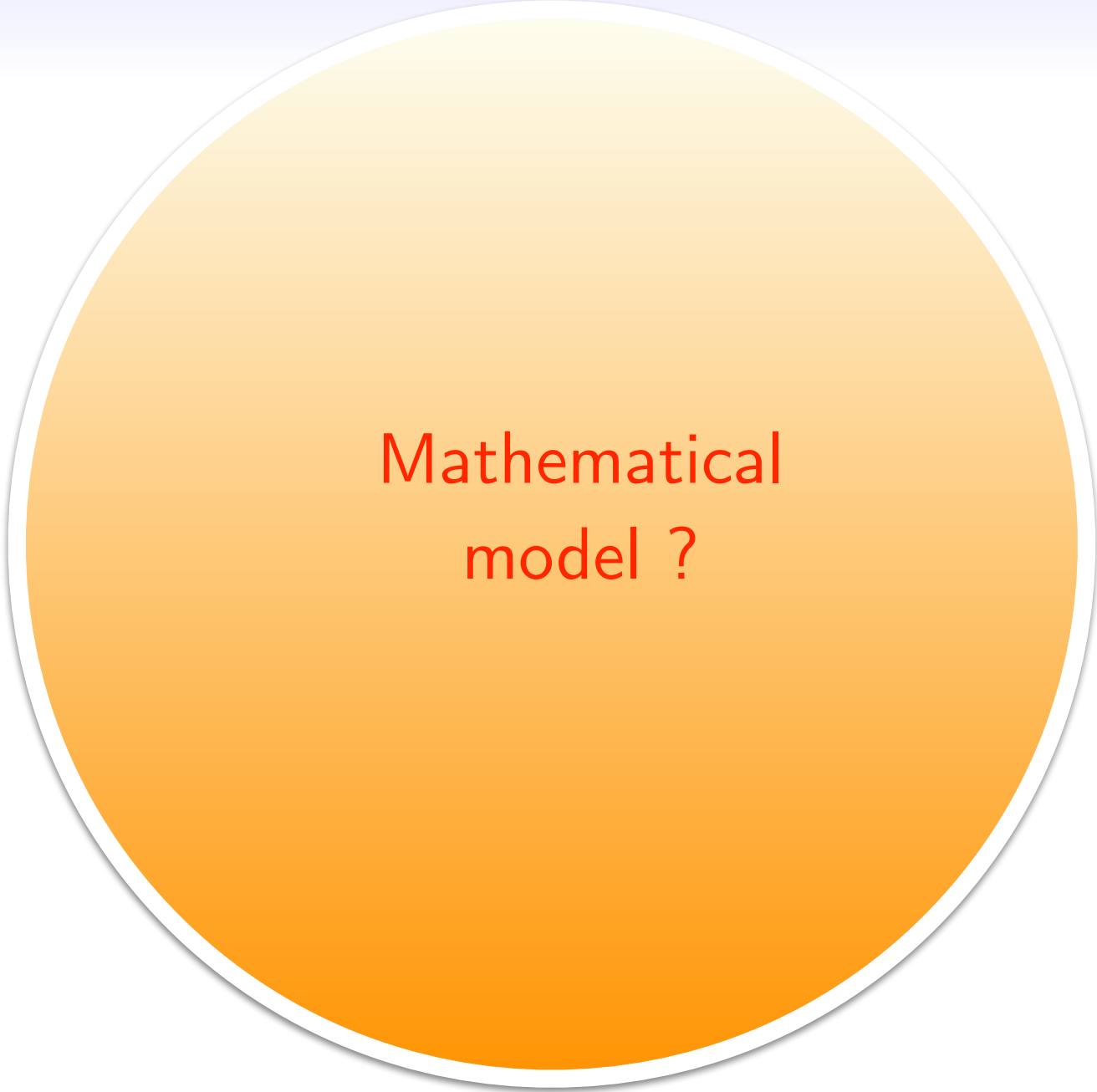
Laurent Condat



Outline

- Introduction
 - Motivation
 - General probabilistic framework
- Our new stochastic model
 - Definition: Locally Anisotropic Fractional Brownian Field
 - Notion of tangent field
- Synthesis methods
 - Tangent field simulation by a turning bands method
 - LAFBF simulation via tangent field formulation
- Numerical experiments
- Conclusion and future work

How to synthesize natural random textures ?



Mathematical
model ?

How to synthesize natural random textures ?



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How to synthesize natural random textures ?

Randomness
Self-similarity



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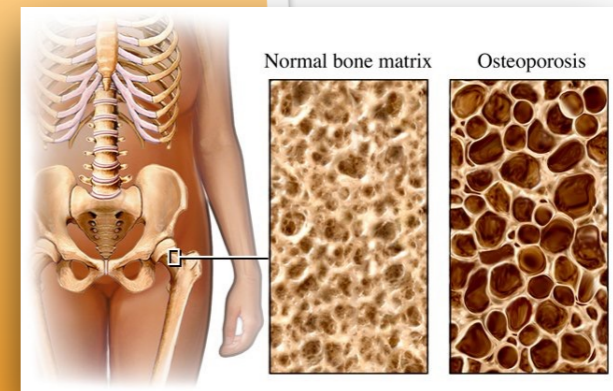
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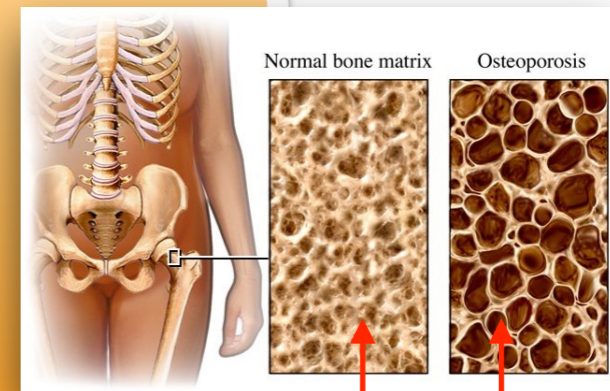


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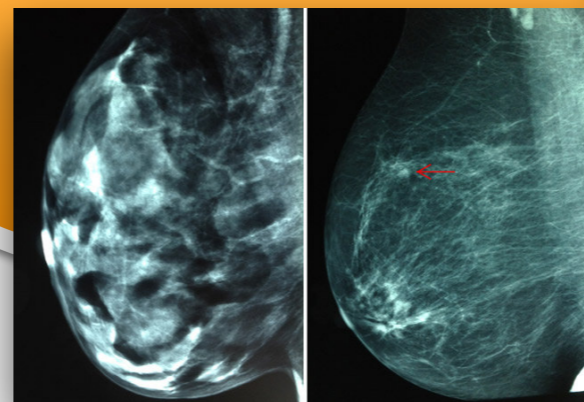
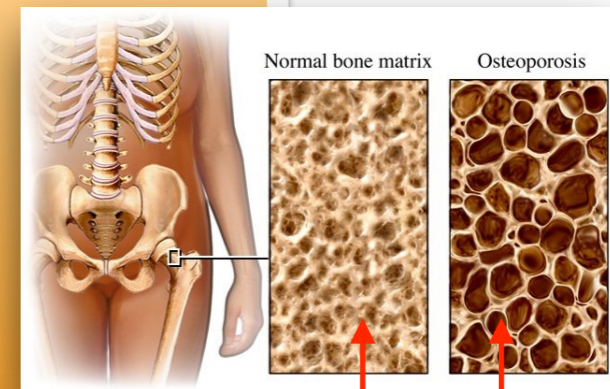
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and regularity

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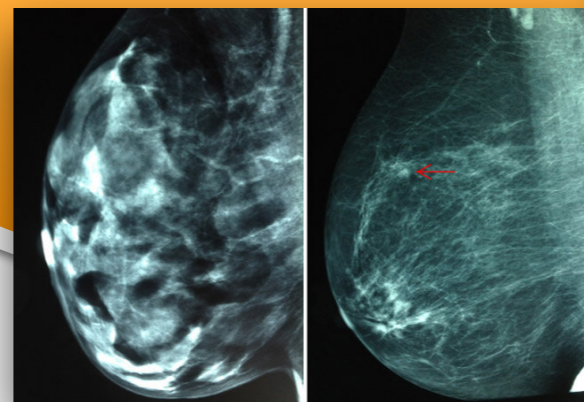
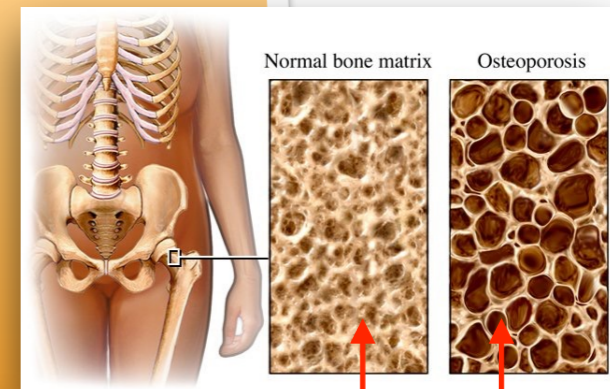
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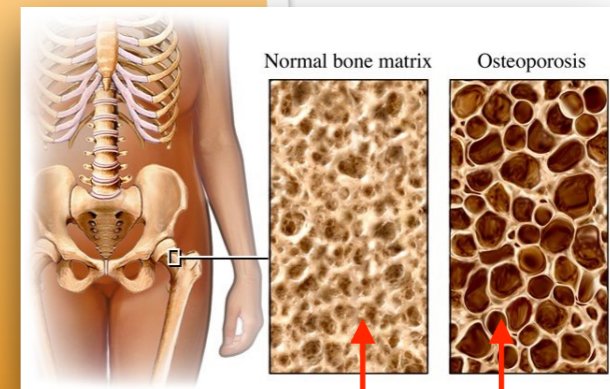
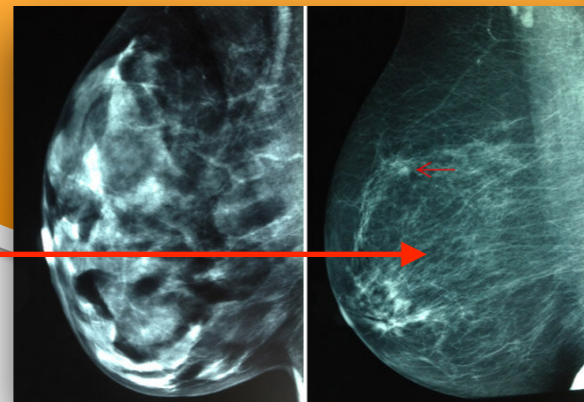
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Mathematical
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Orientation
and anisotropy



Roughness
and regularity

The basic component :

Fractional Brownian Field (FBF)

- B^H FBF with Hurst index $0 < H < 1$ [Mandelbrot, Van Ness, 1968]

- **stationary increments** : $B^H(\cdot + \mathbf{z}) - B^H(\mathbf{z}) \stackrel{\mathcal{L}}{=} B^H(\cdot) - B^H(0)$

- **self-similar** : $B^H(\lambda \cdot) \stackrel{\mathcal{L}}{=} \lambda^H B^H(\cdot)$

- **isotropic** : $B^H \circ R_\theta \stackrel{\mathcal{L}}{=} B^H$

- The covariance is given by

$$\text{Cov}(B^H(\mathbf{x}), B^H(\mathbf{y})) = c_H(\|\mathbf{x}\|^{2H} + \|\mathbf{y}\|^{2H} - \|\mathbf{x} - \mathbf{y}\|^{2H})$$

The basic component :

Fractional Brownian Field (FBF)

■ Harmonizable representation

[Samorodnitsky, Taqqu, 1997]

$$B^H(\mathbf{x}) = \int_{\mathbb{R}^2} \frac{e^{i\mathbf{x} \cdot \boldsymbol{\xi}} - 1}{\|\boldsymbol{\xi}\|^{H+1}} d\widehat{W}(\boldsymbol{\xi})$$

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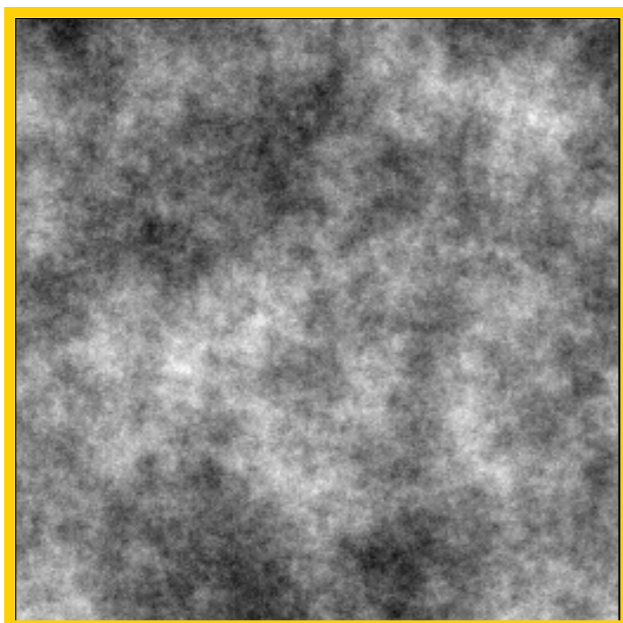
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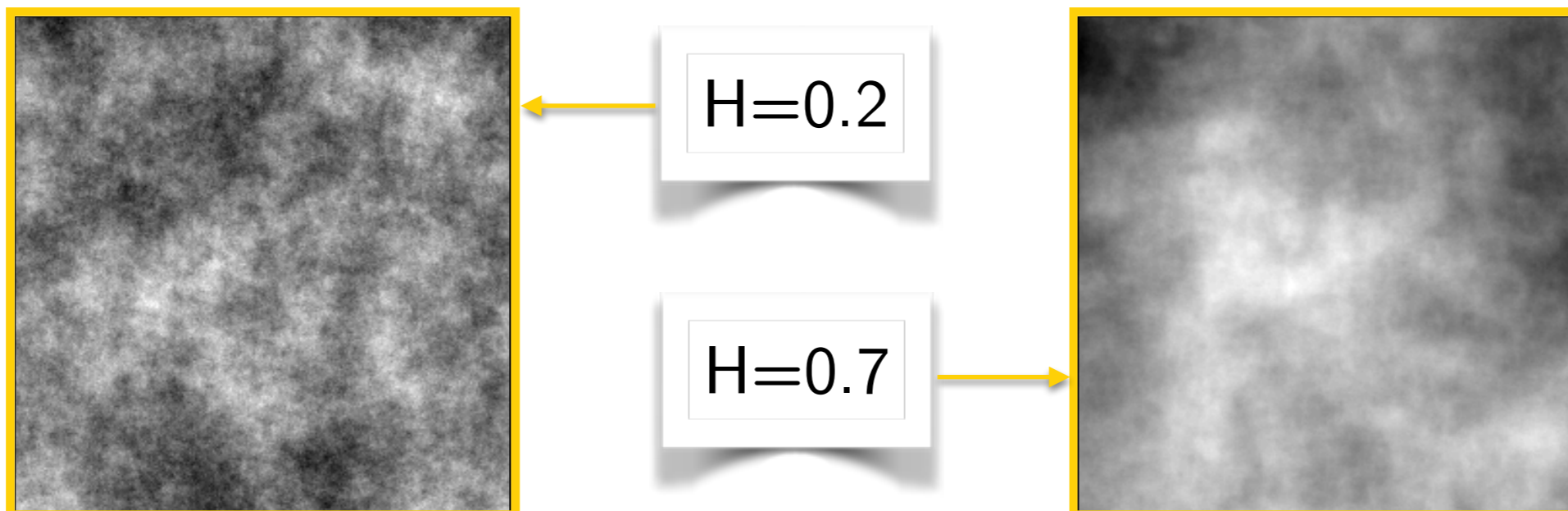
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General model :

anisotropic self-similar Gaussian fields

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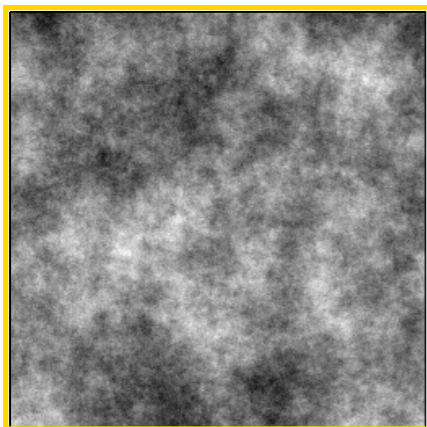
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[Mandelbrot, Van Ness, 1968]



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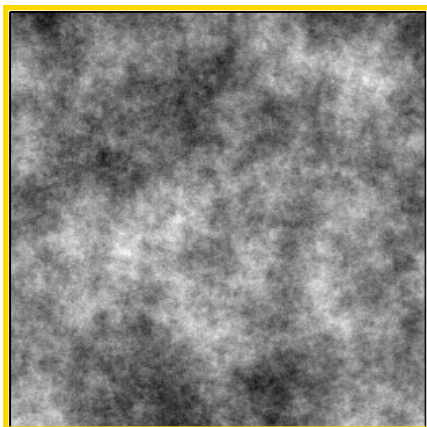
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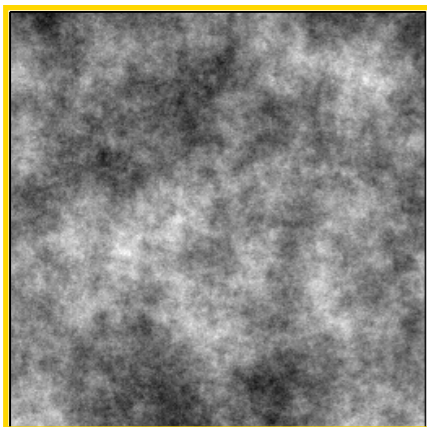
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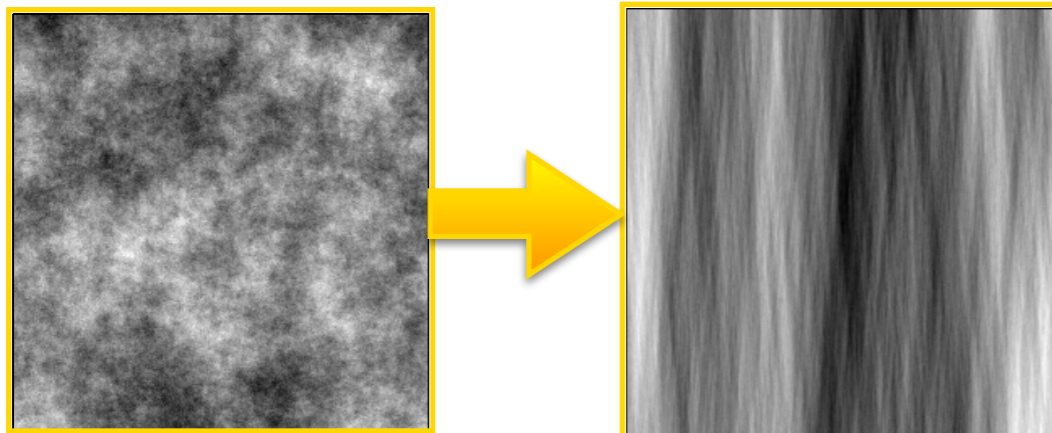
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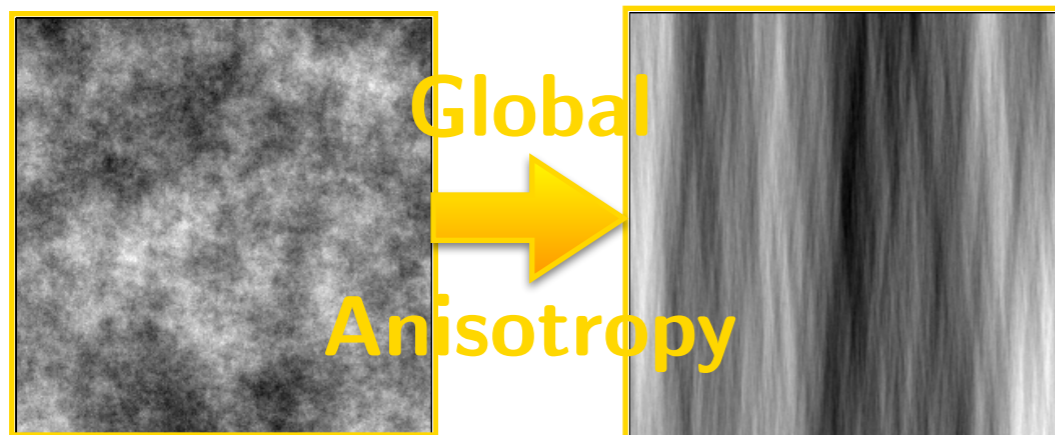
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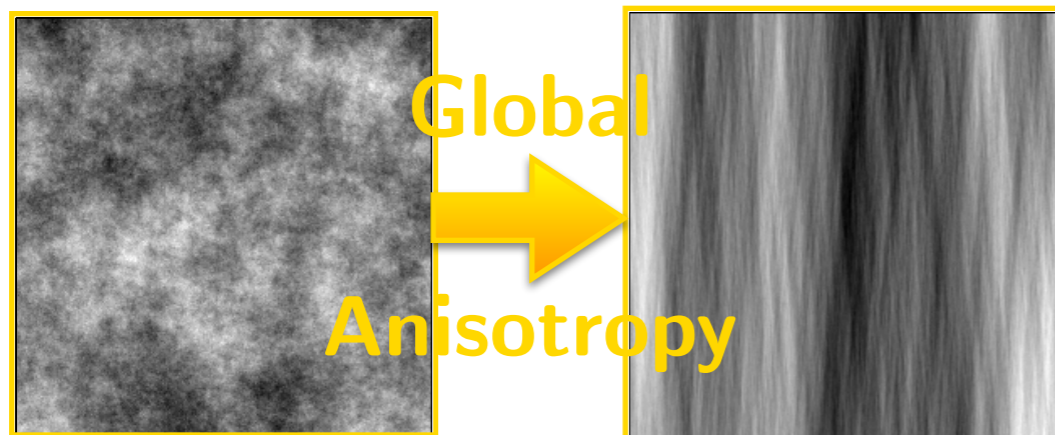
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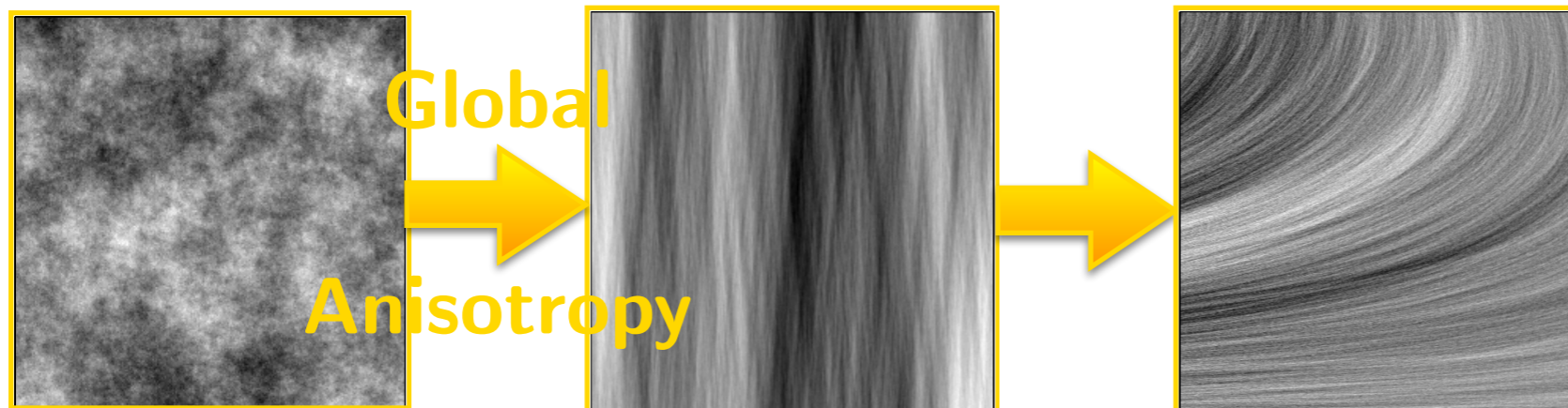
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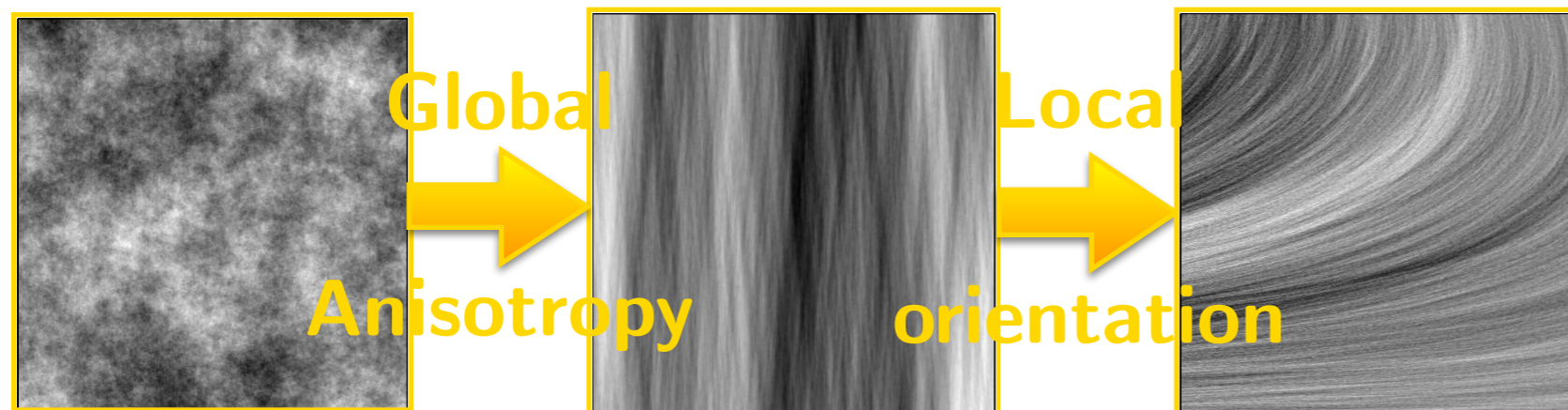
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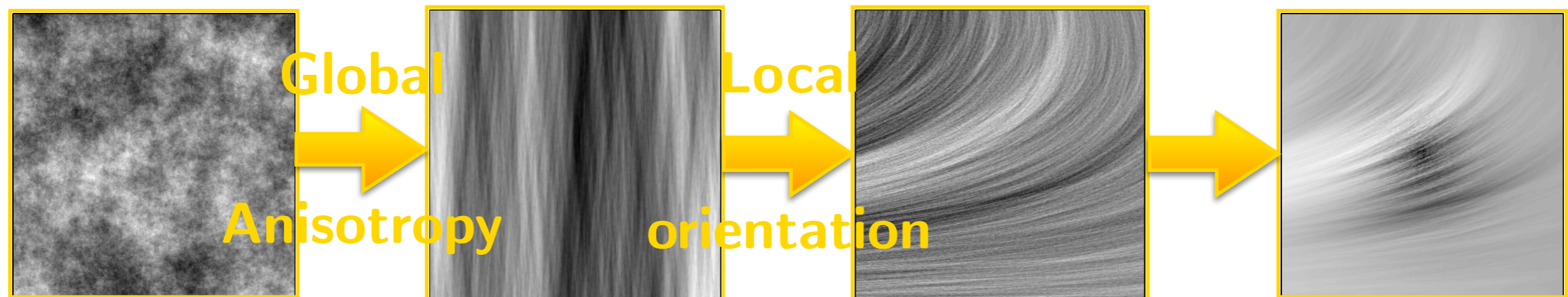
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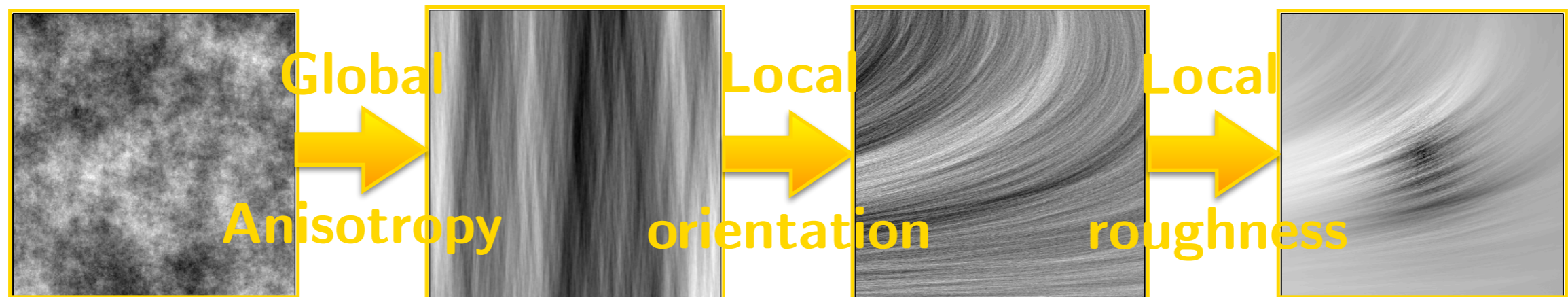
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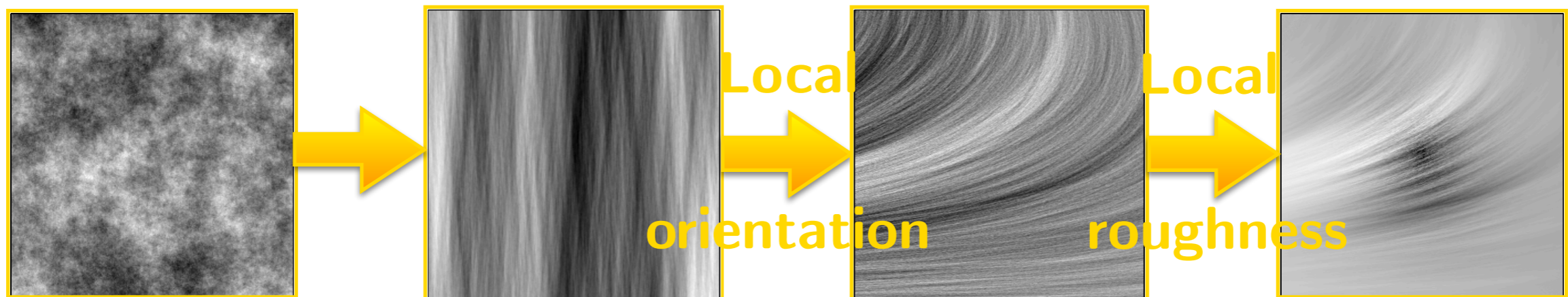
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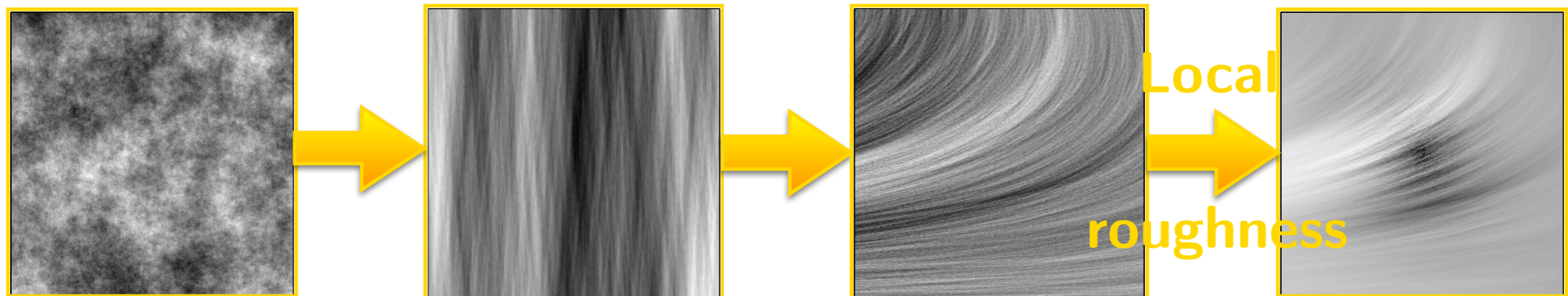
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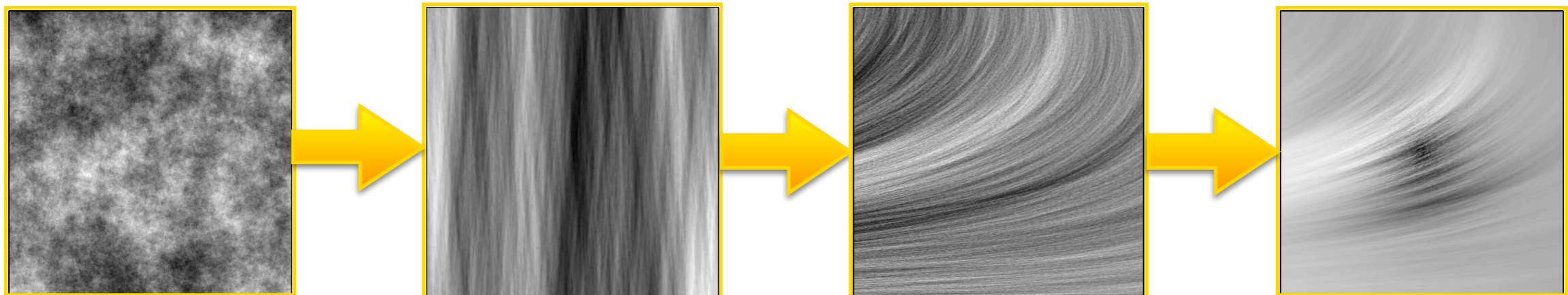
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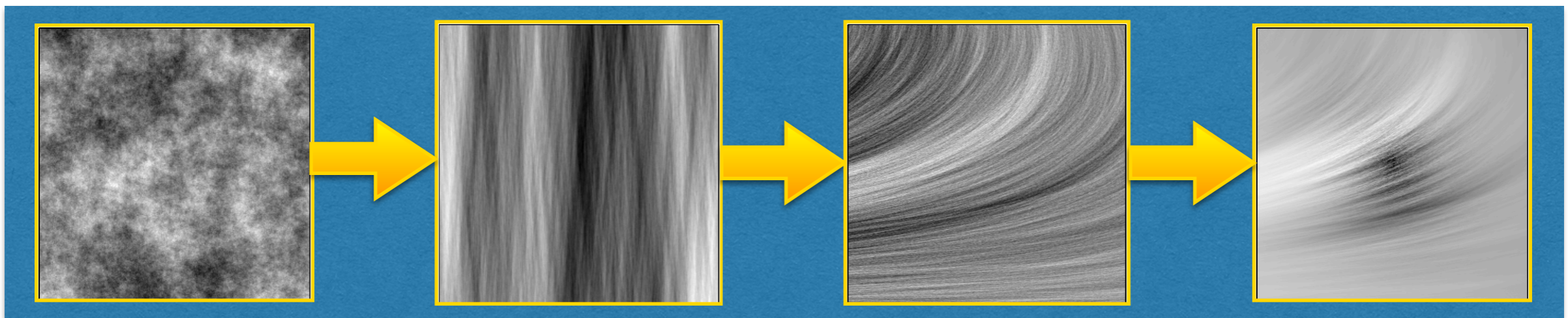
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Locally Anisotropic Fractional Brownian Field (LAFBF)

- **Definition:** Our new Gaussian model LAFBF is a local version of the elementary field

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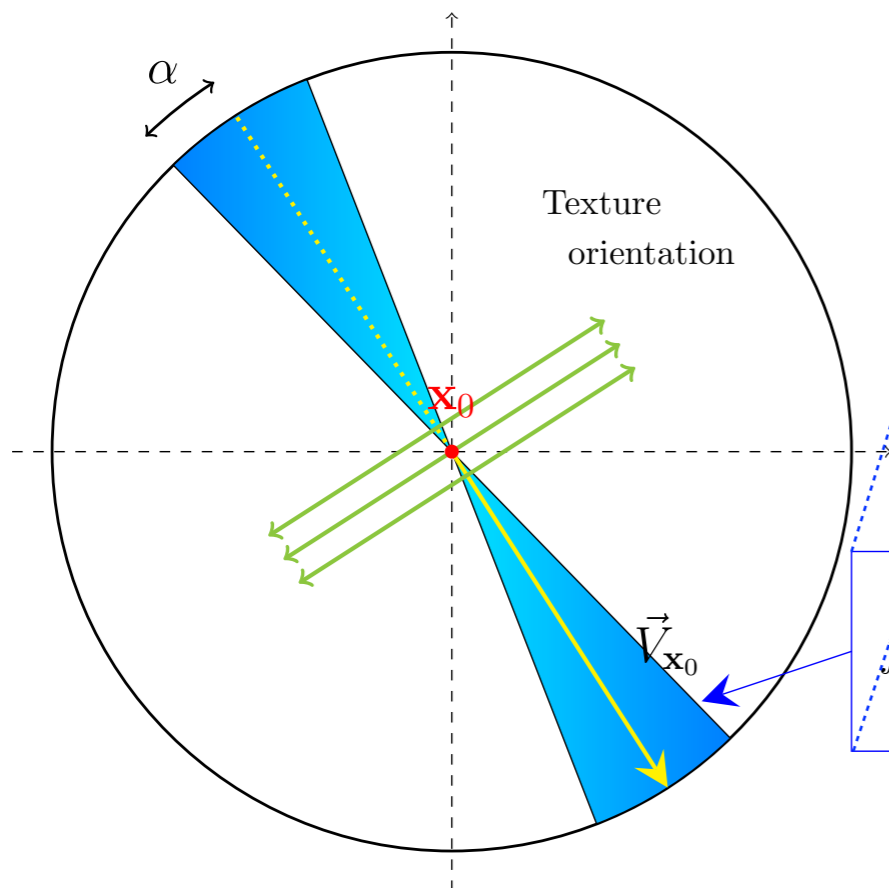
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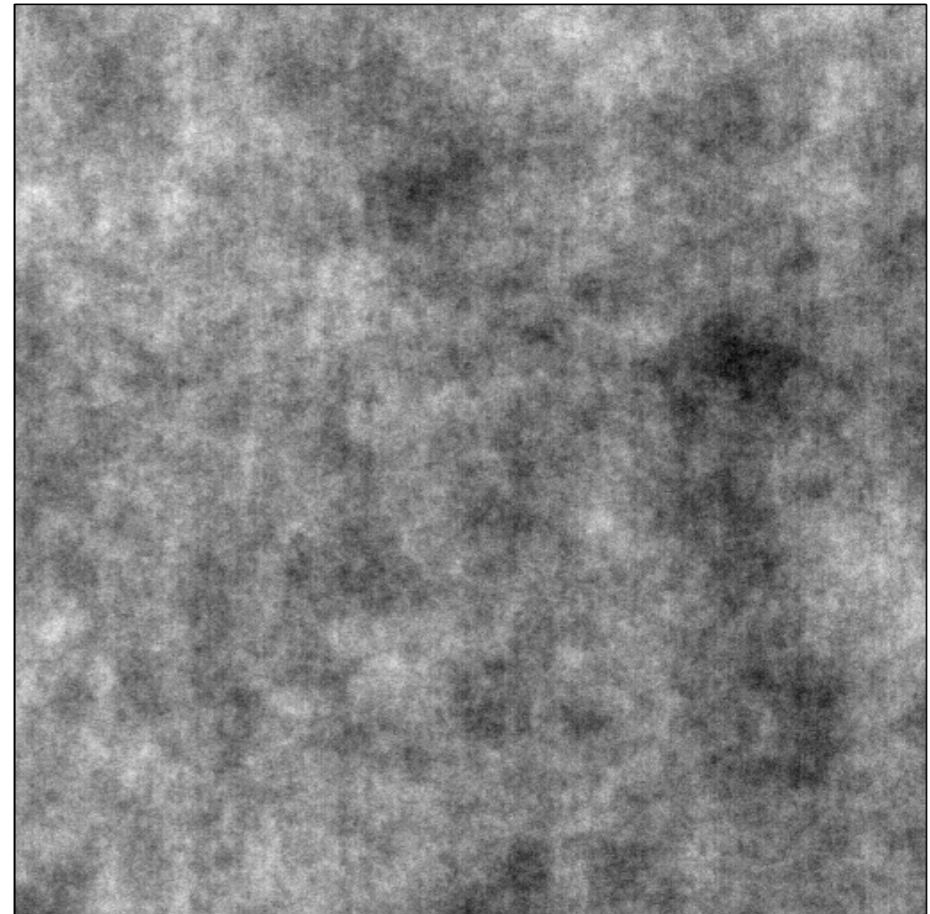
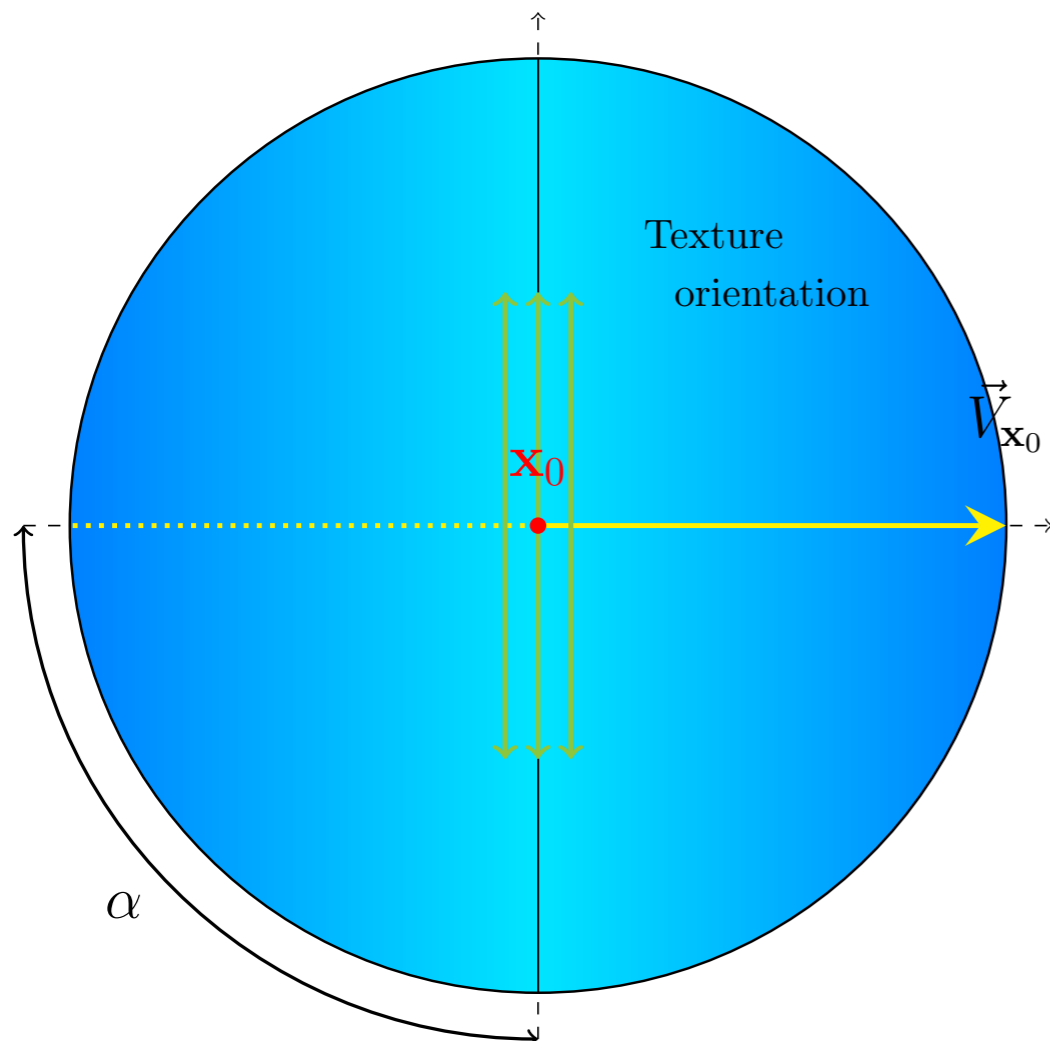


$$f^{1/2}(\mathbf{x}_0, \boldsymbol{\xi}) = \frac{c_{\alpha_0, \alpha}(\mathbf{x}_0, \arg \boldsymbol{\xi})}{\|\boldsymbol{\xi}\|^{H+1}}$$

The orientation may vary spatially. α_0 is now a differentiable function on \mathbb{R}^2

Elementary field

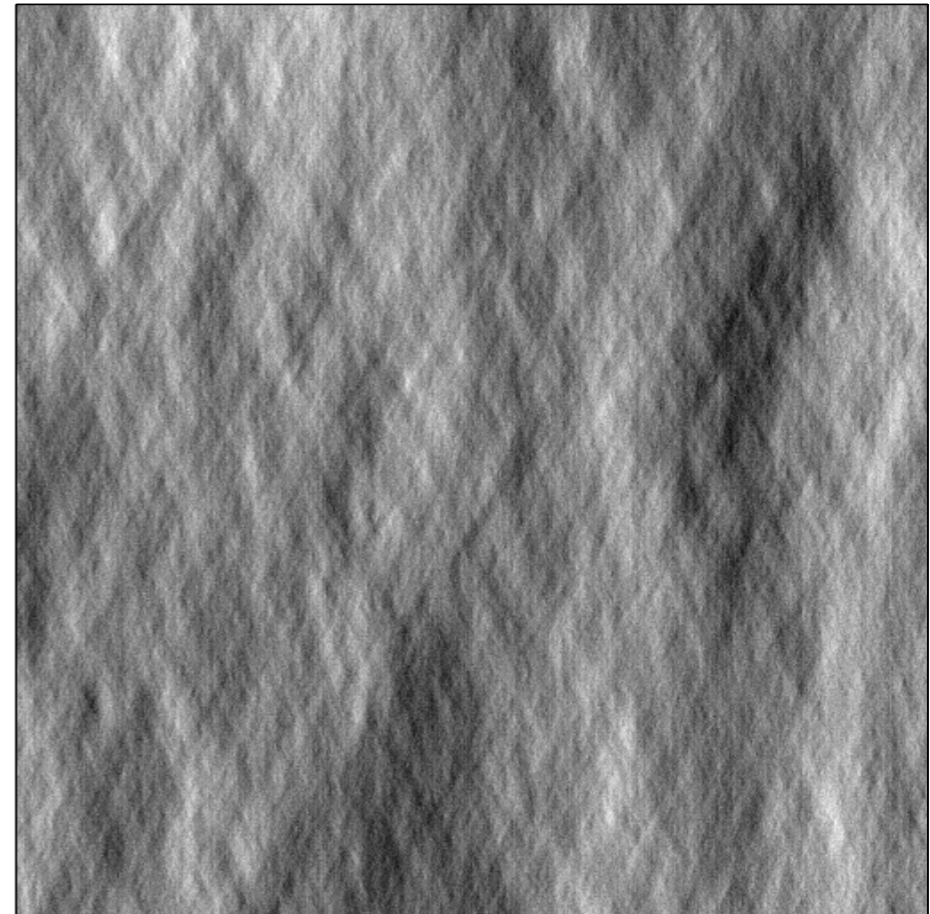
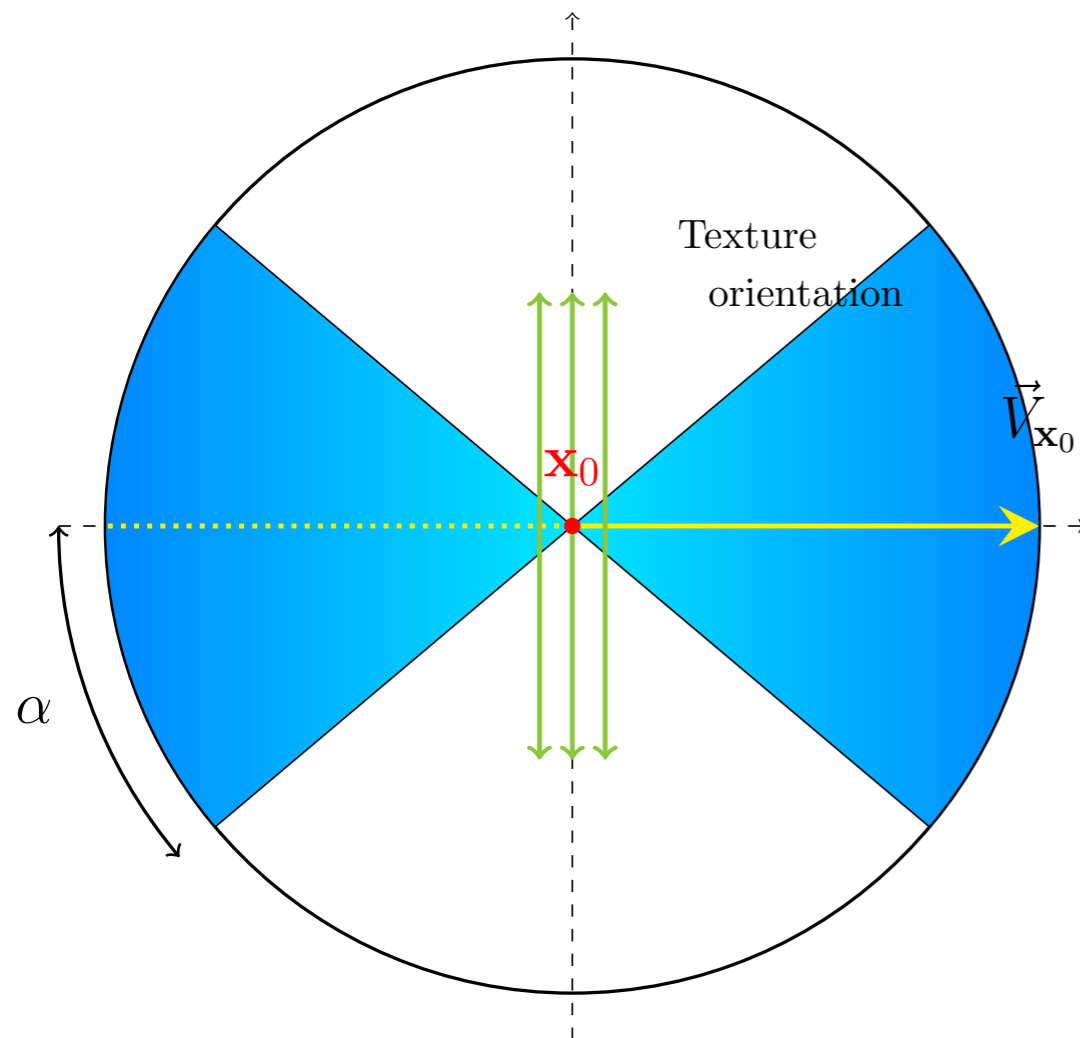
$$\alpha_0 = 0$$



$$\alpha = -\frac{\pi}{2}$$

Elementary field

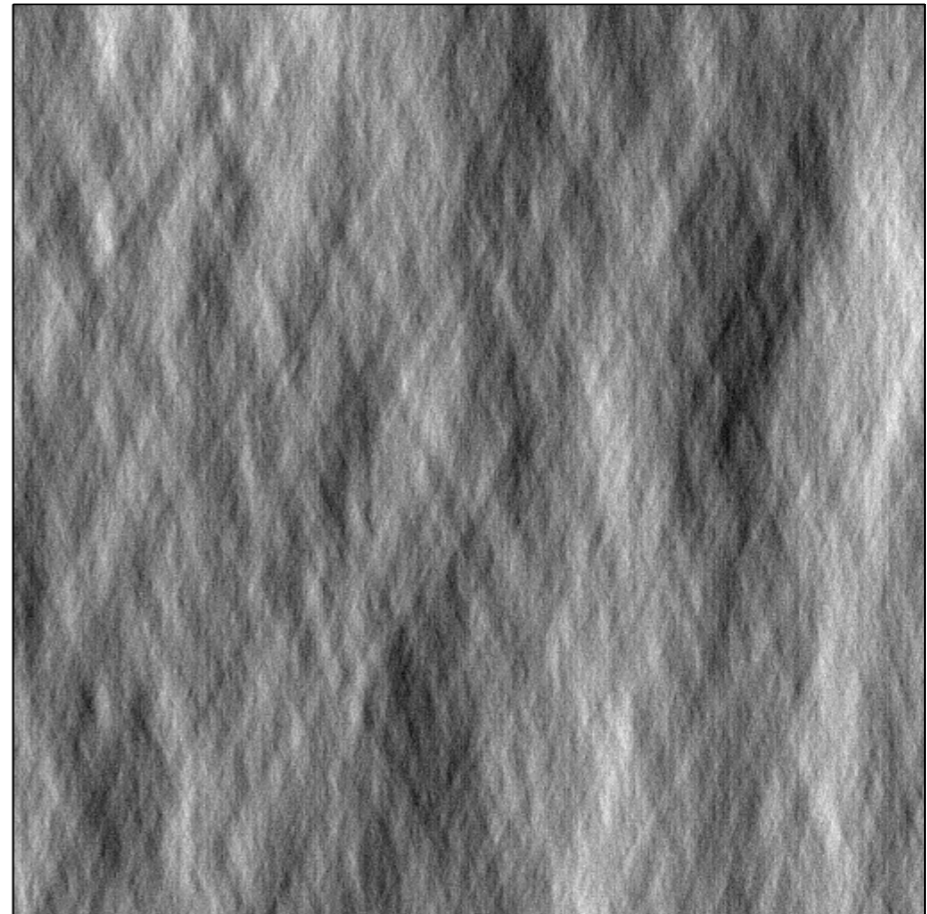
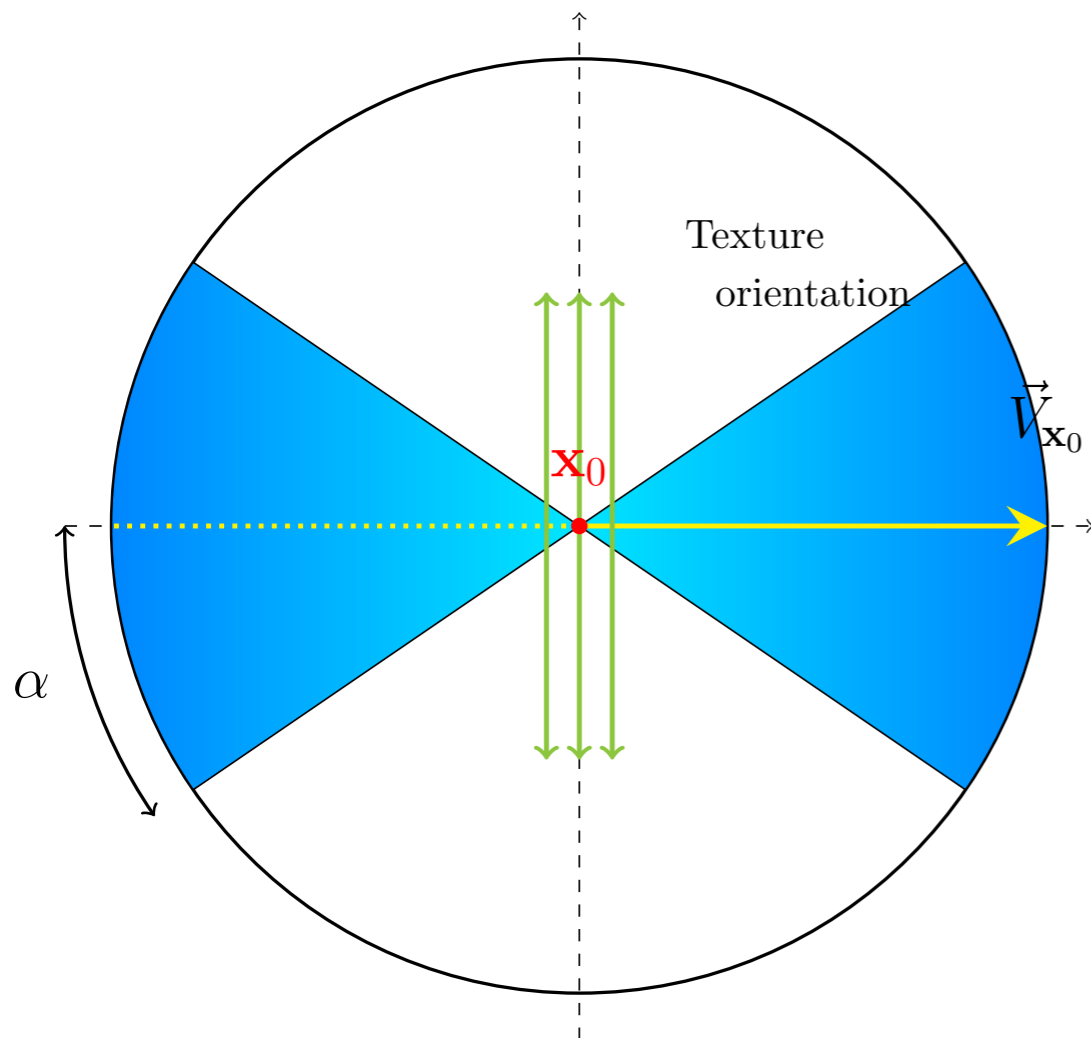
$$\alpha_0 = 0$$



$$\alpha = 0.7$$

Elementary field

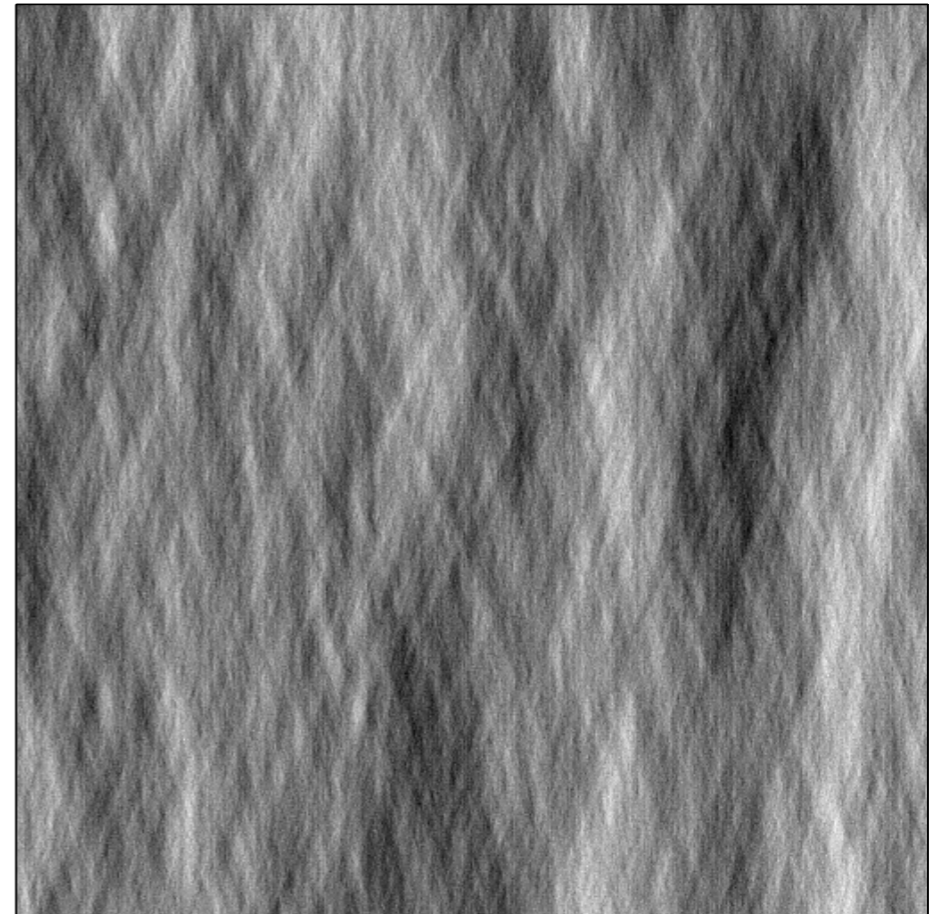
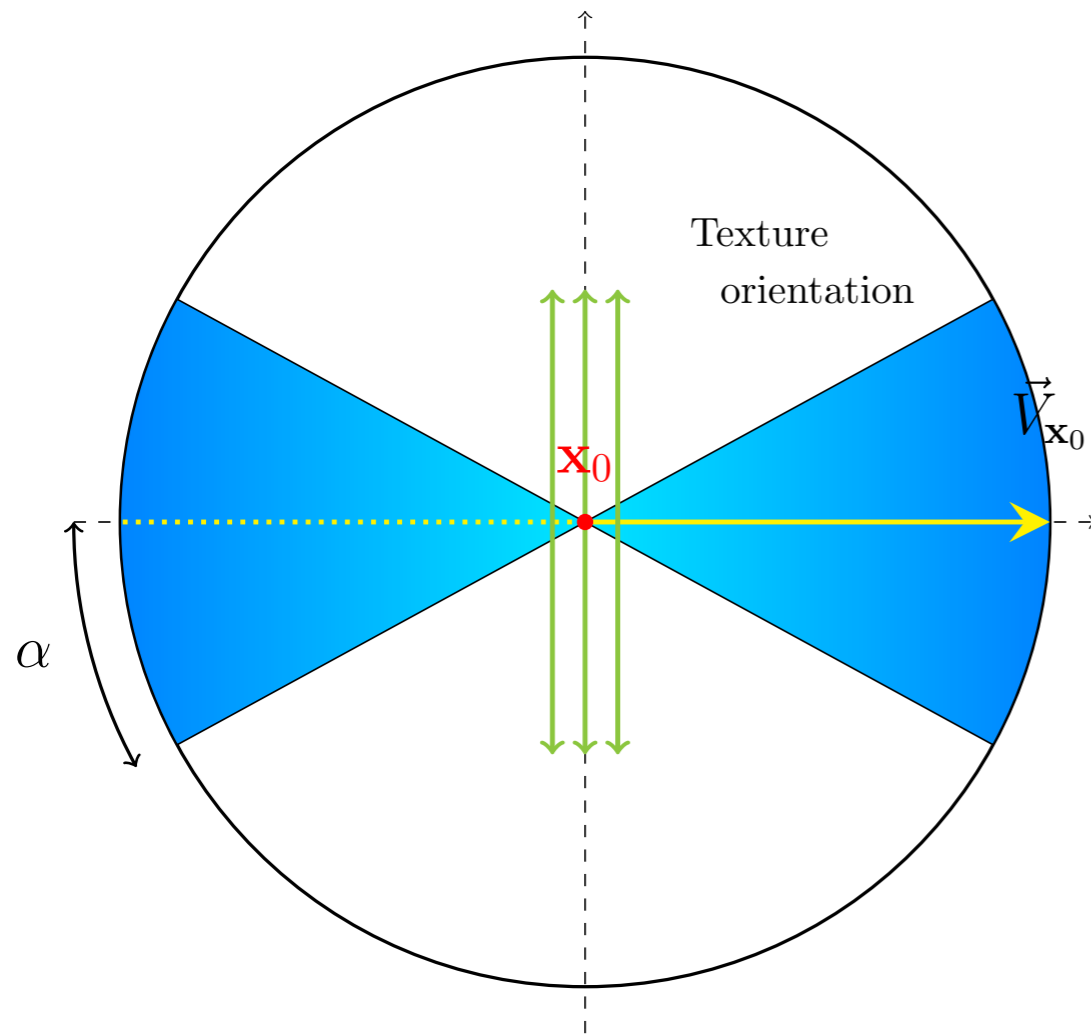
$$\alpha_0 = 0$$



$$\alpha = 0.6$$

Elementary field

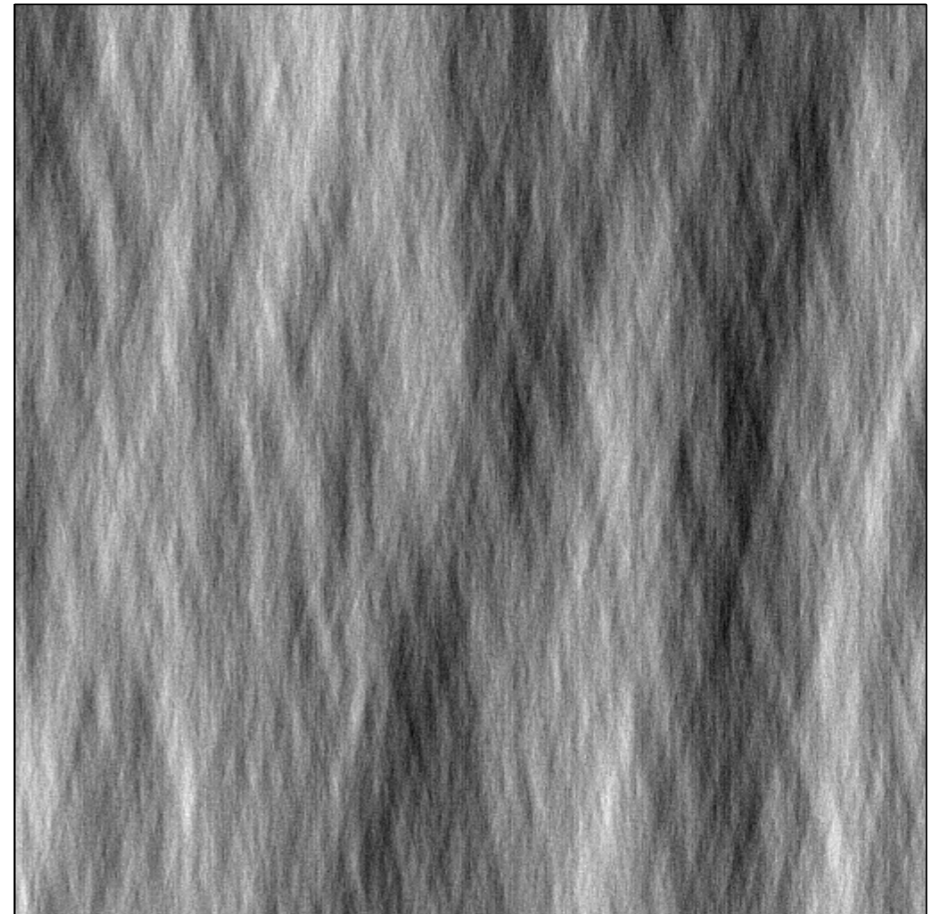
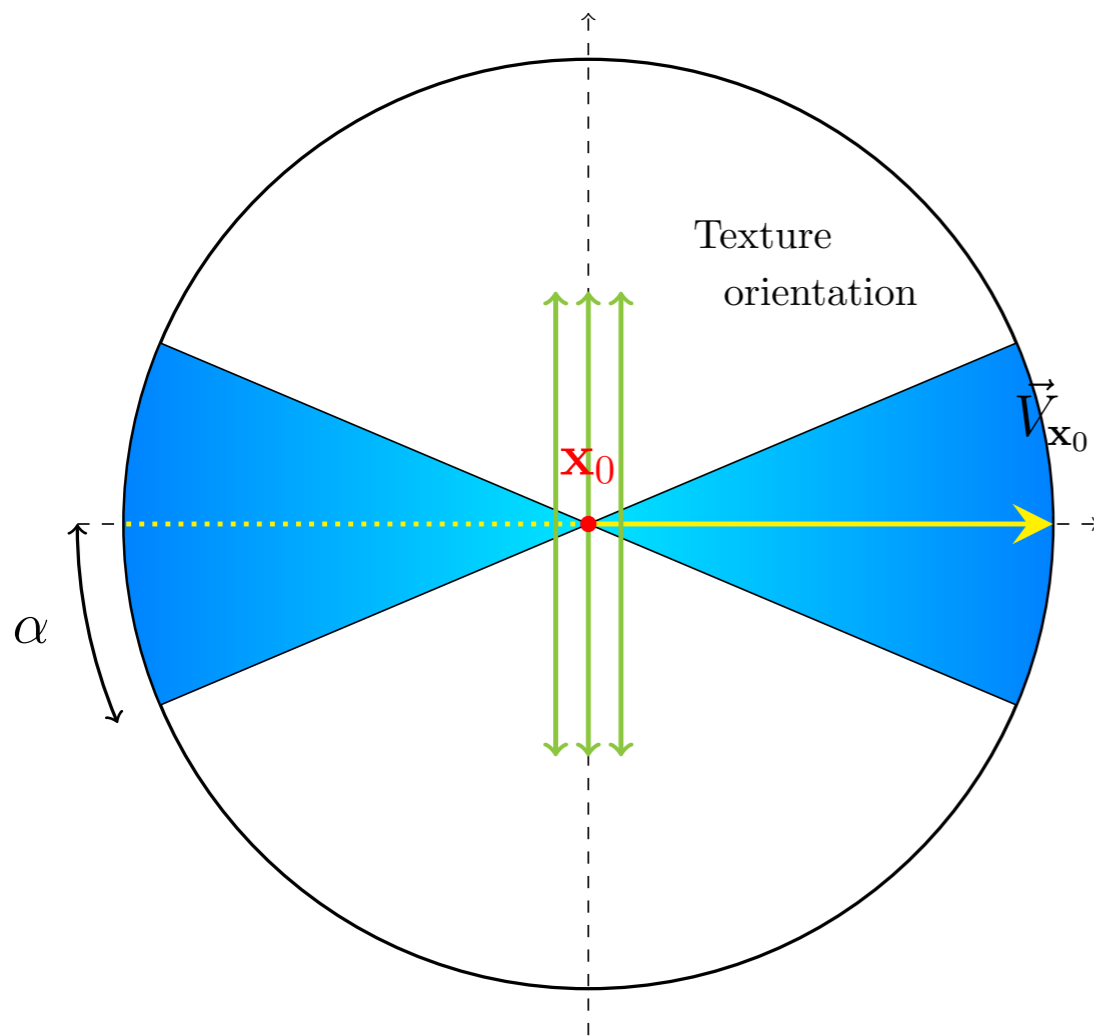
$$\alpha_0 = 0$$



$$\alpha = 0.5$$

Elementary field

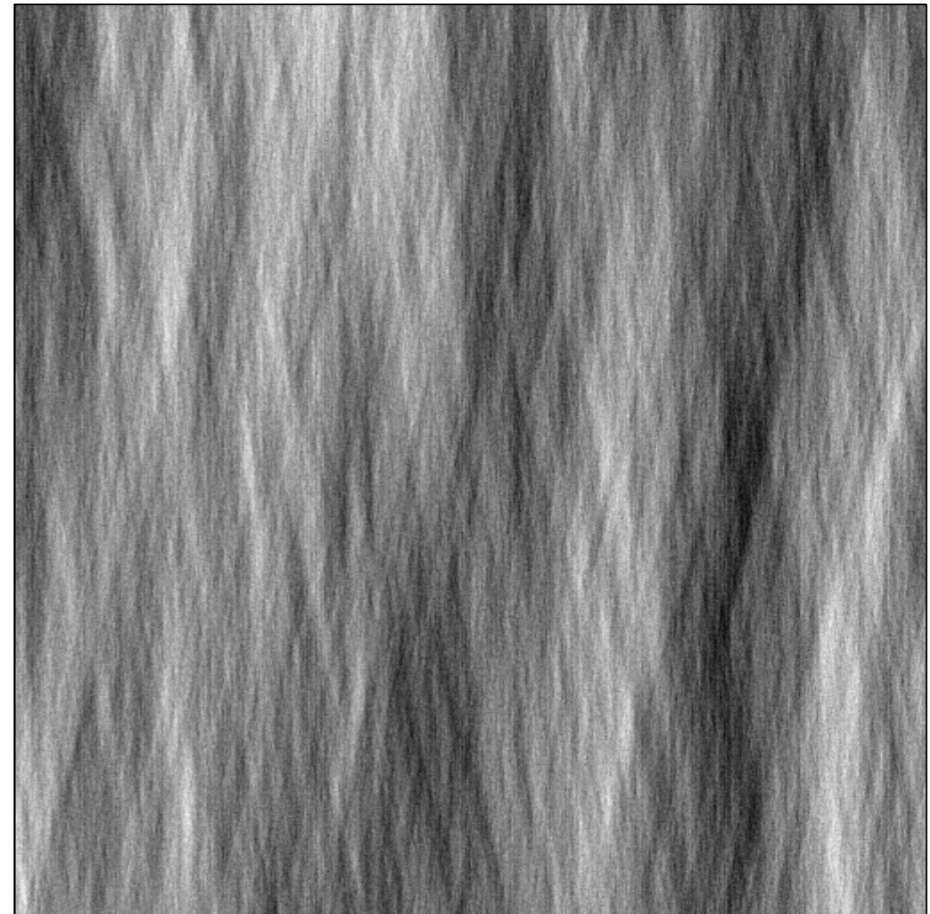
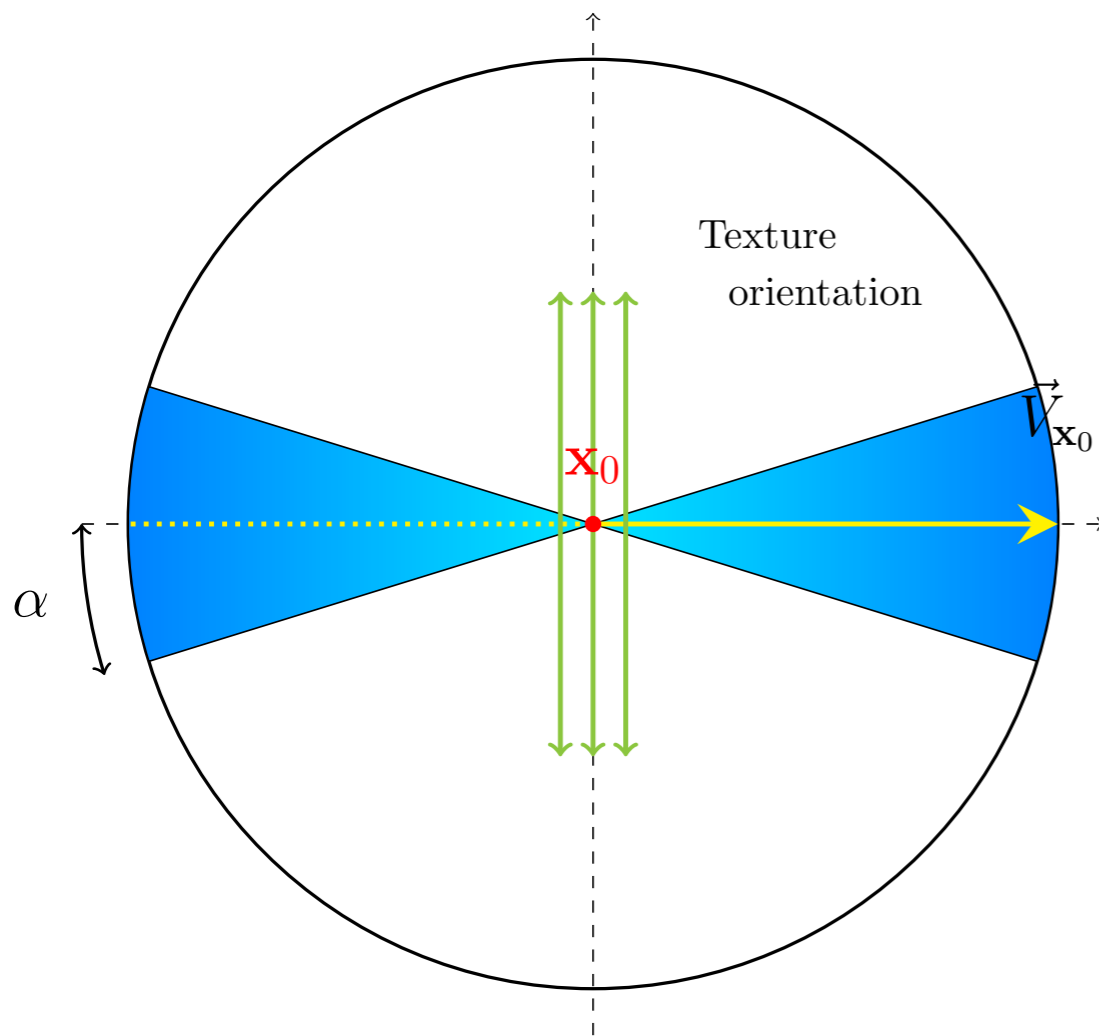
$$\alpha_0 = 0$$



$$\alpha = 0.4$$

Elementary field

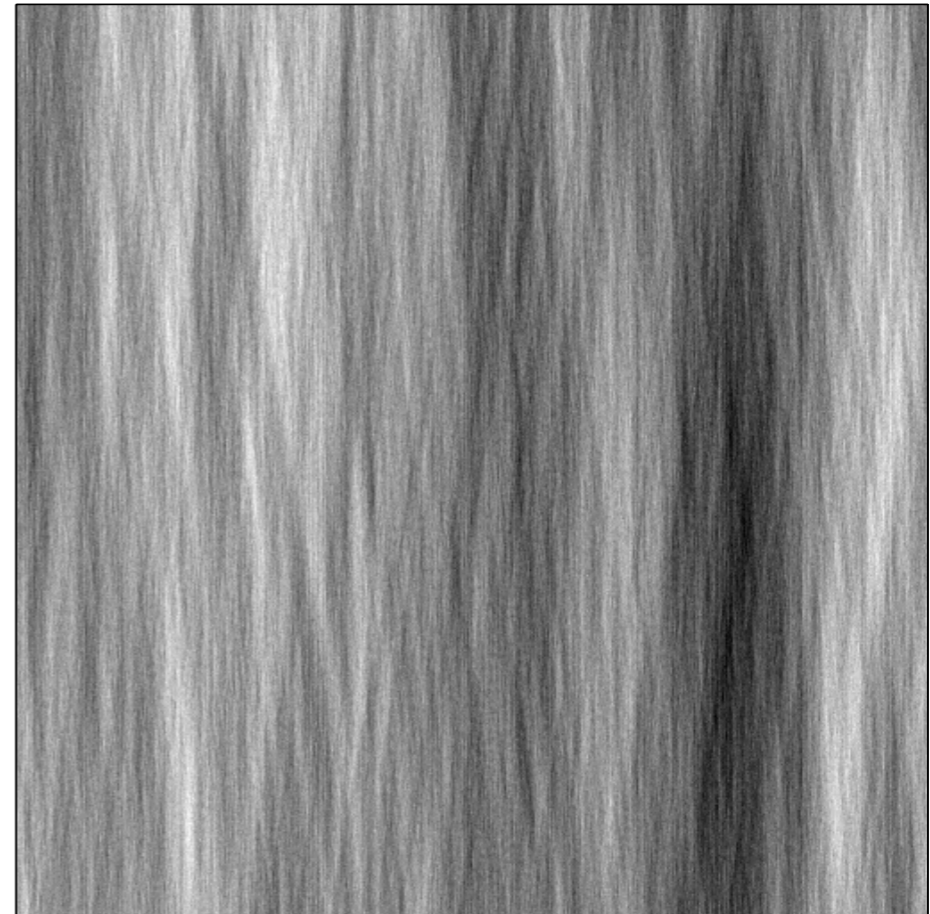
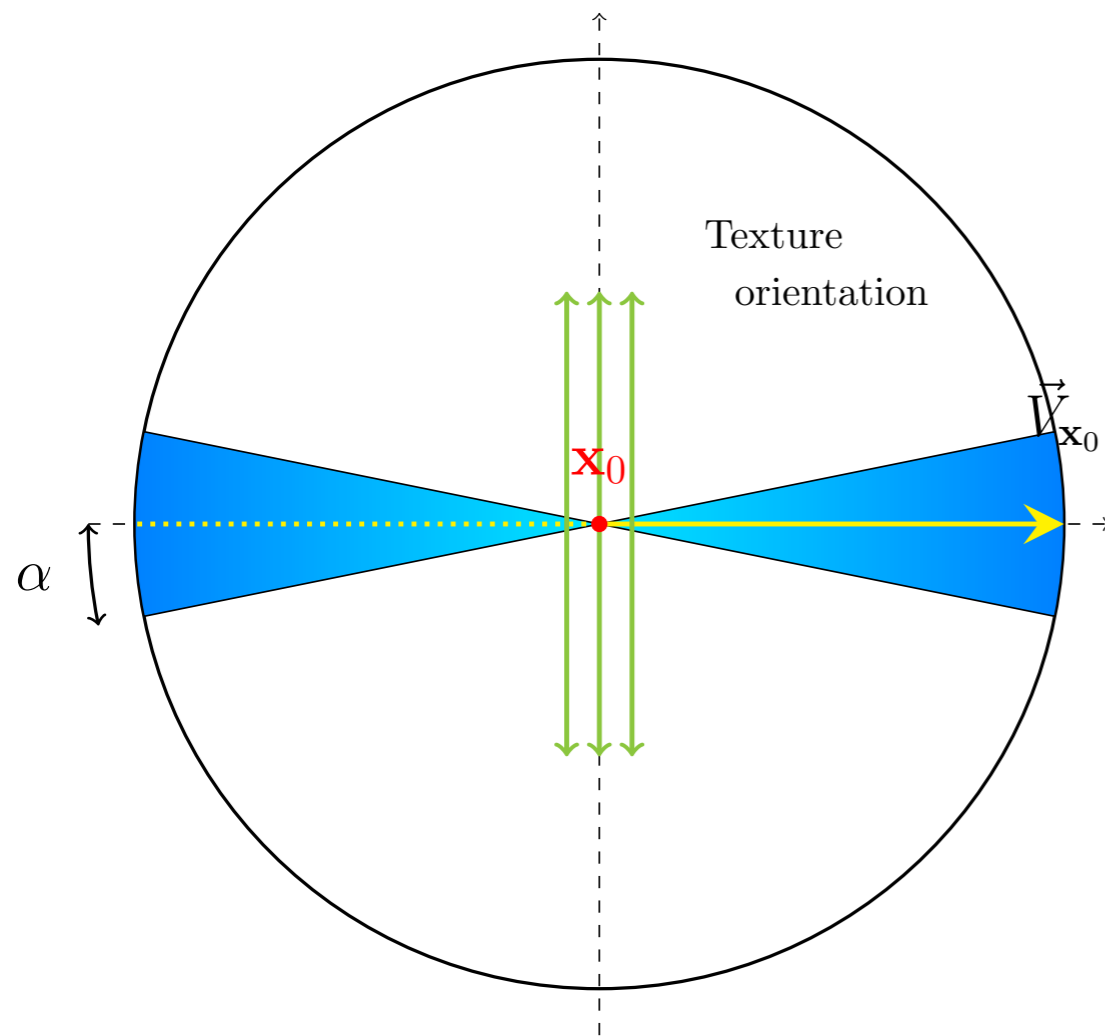
$$\alpha_0 = 0$$



$$\alpha = 0.3$$

Elementary field

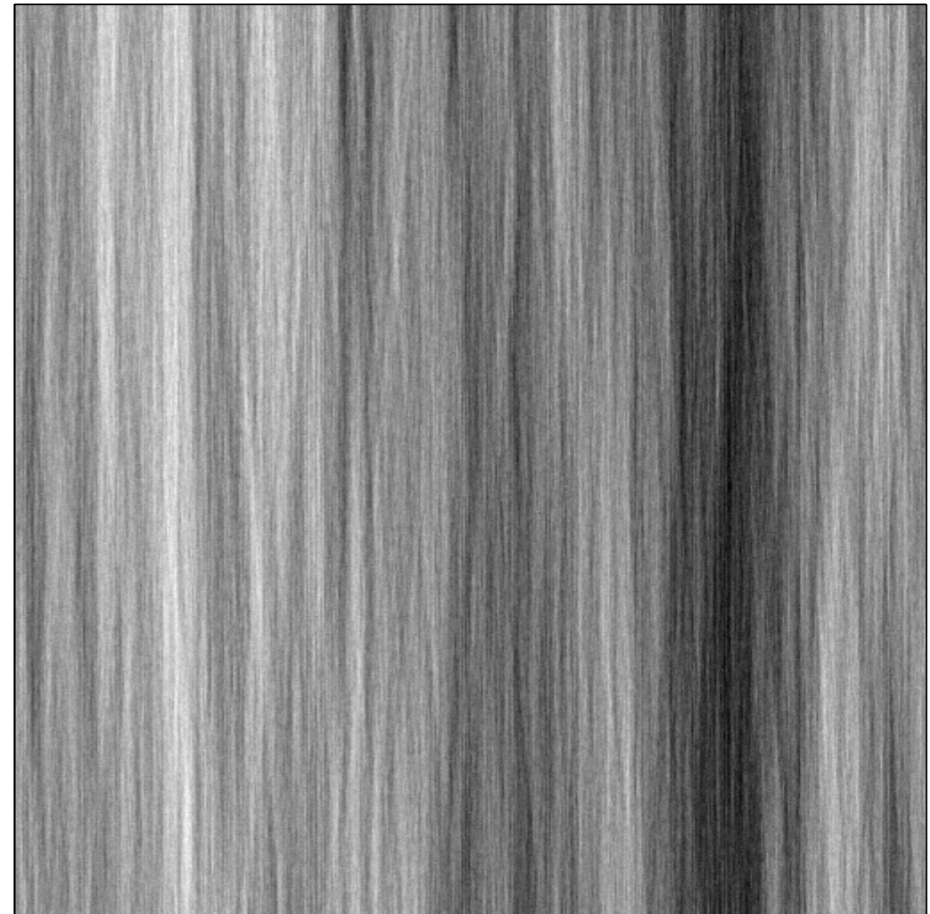
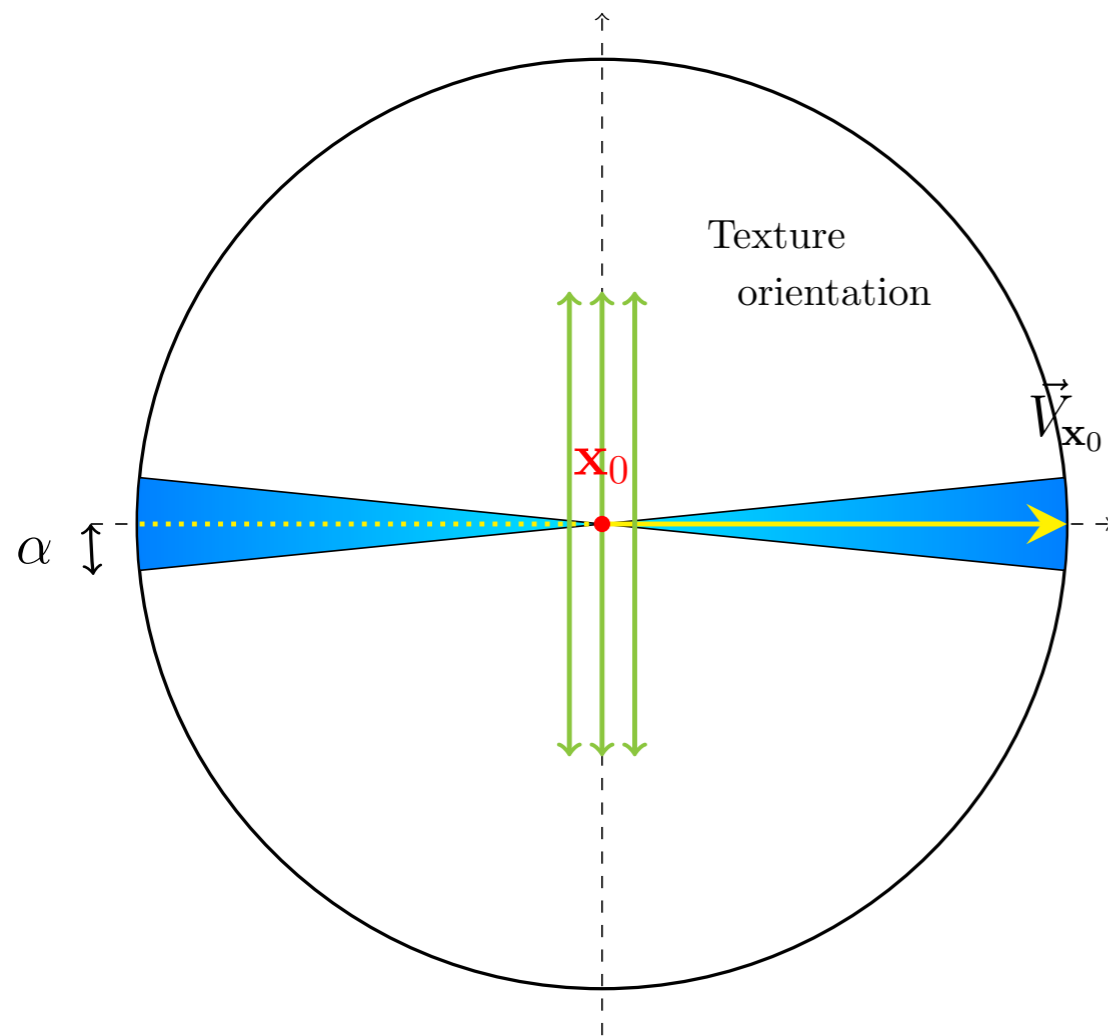
$$\alpha_0 = 0$$



$$\alpha = 0.2$$

Elementary field

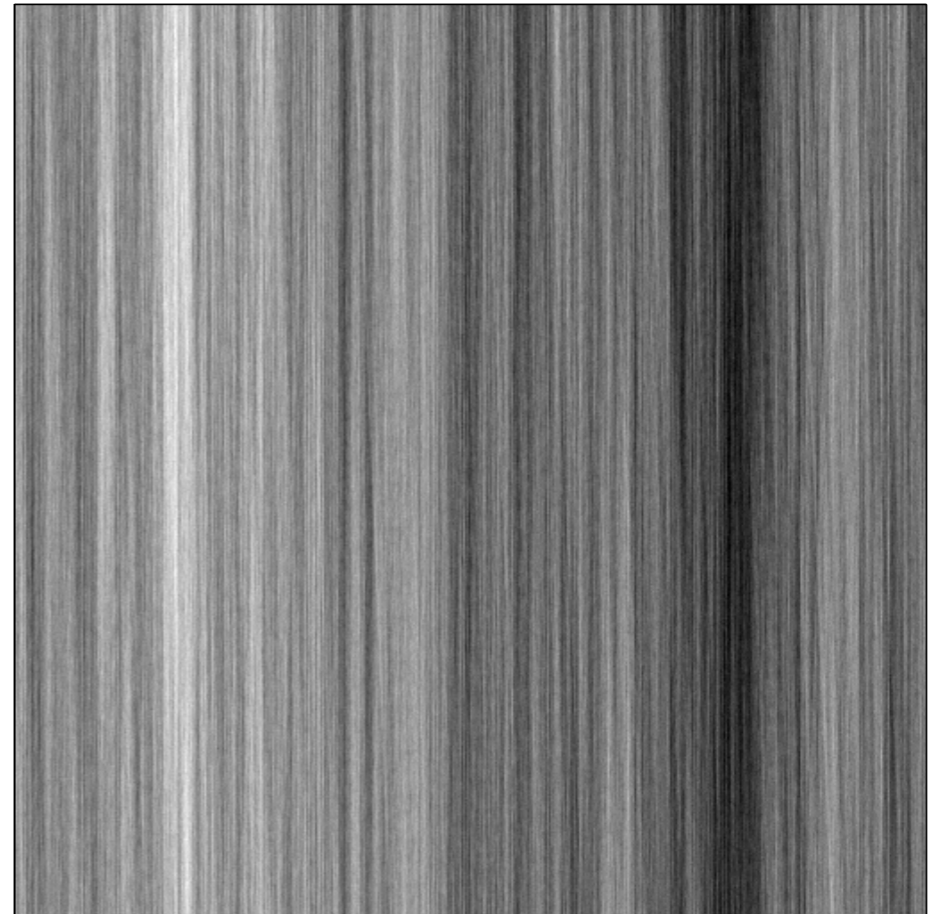
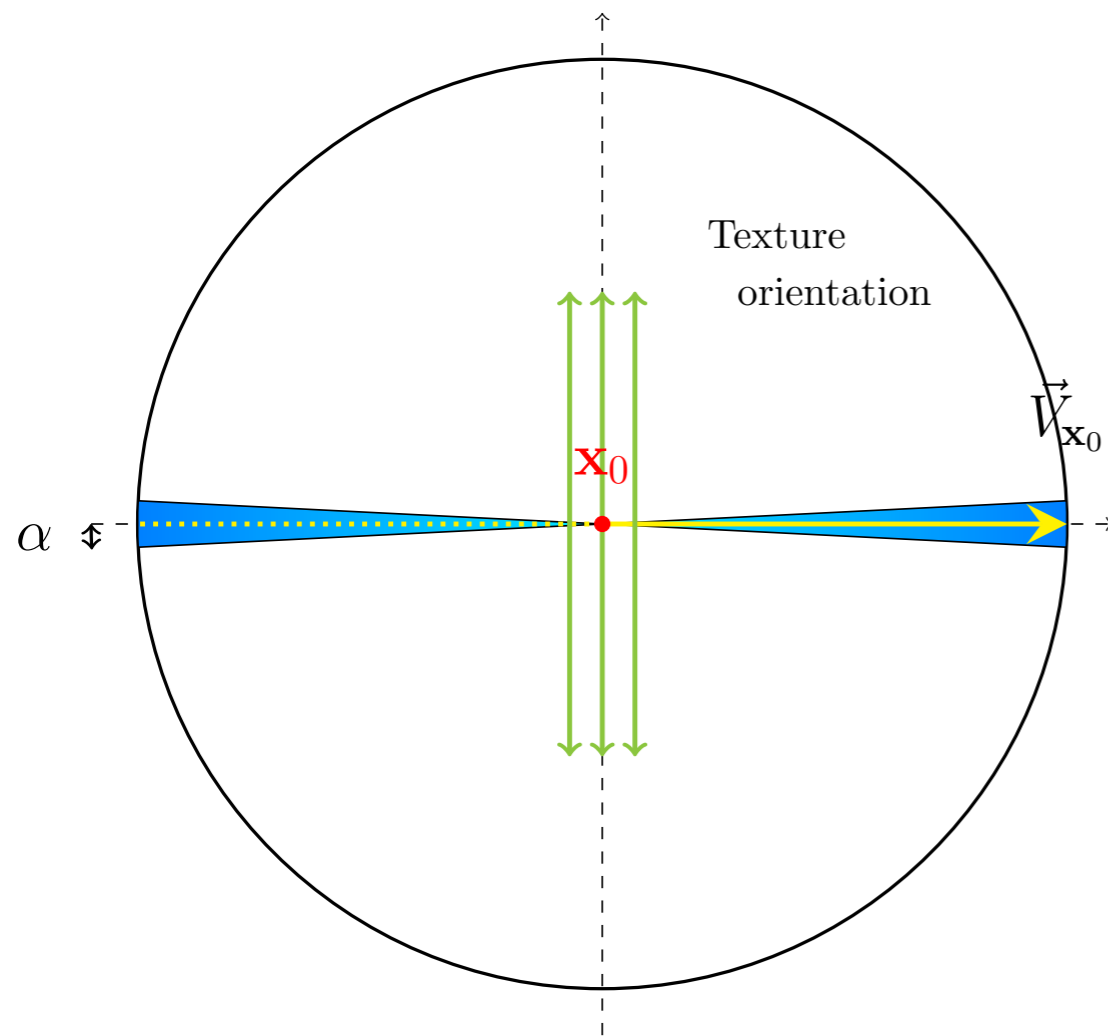
$$\alpha_0 = 0$$



$$\alpha = 0.1$$

Elementary field

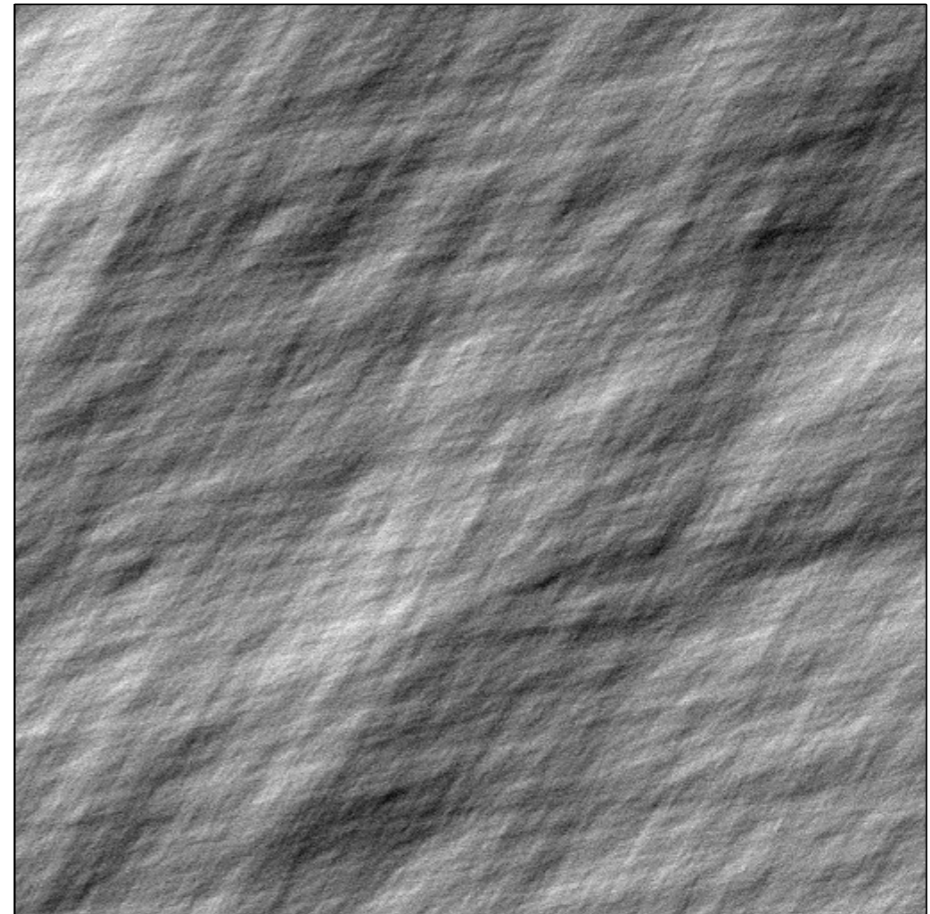
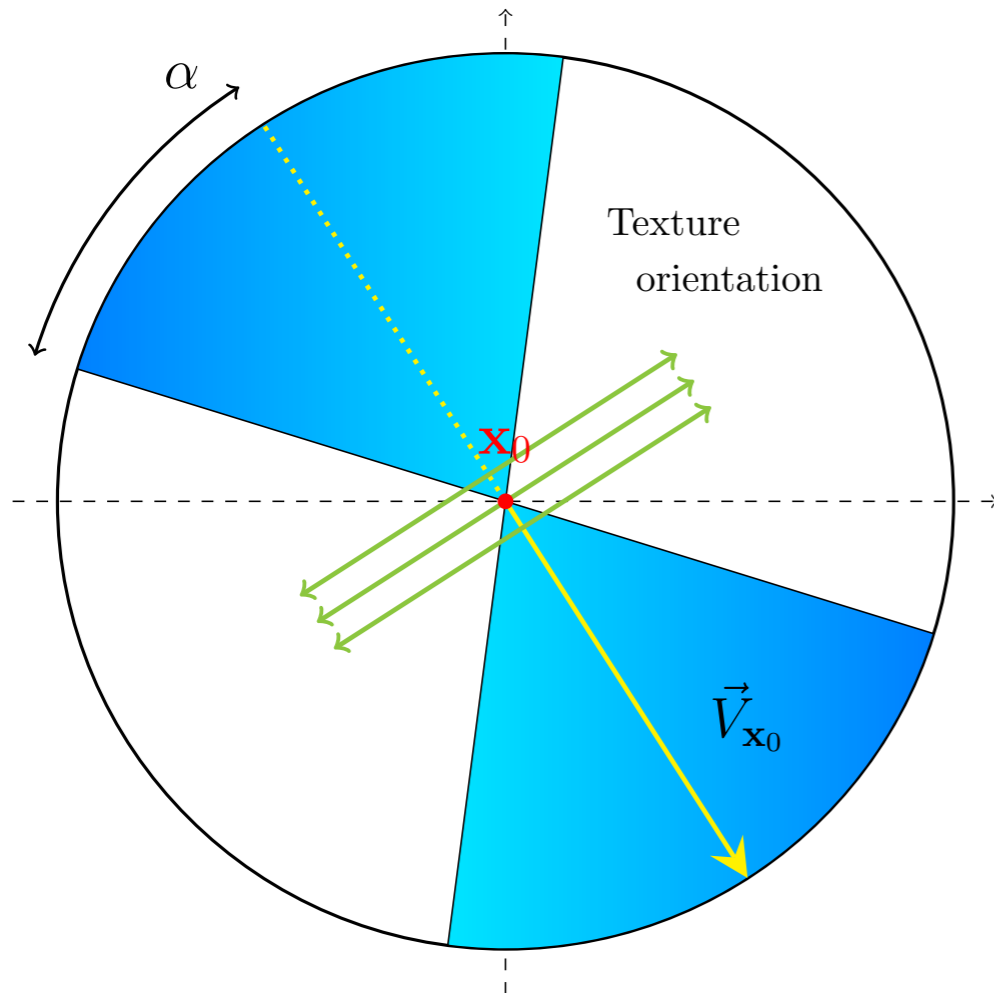
$$\alpha_0 = 0$$



$$\alpha = 0.05$$

Elementary field

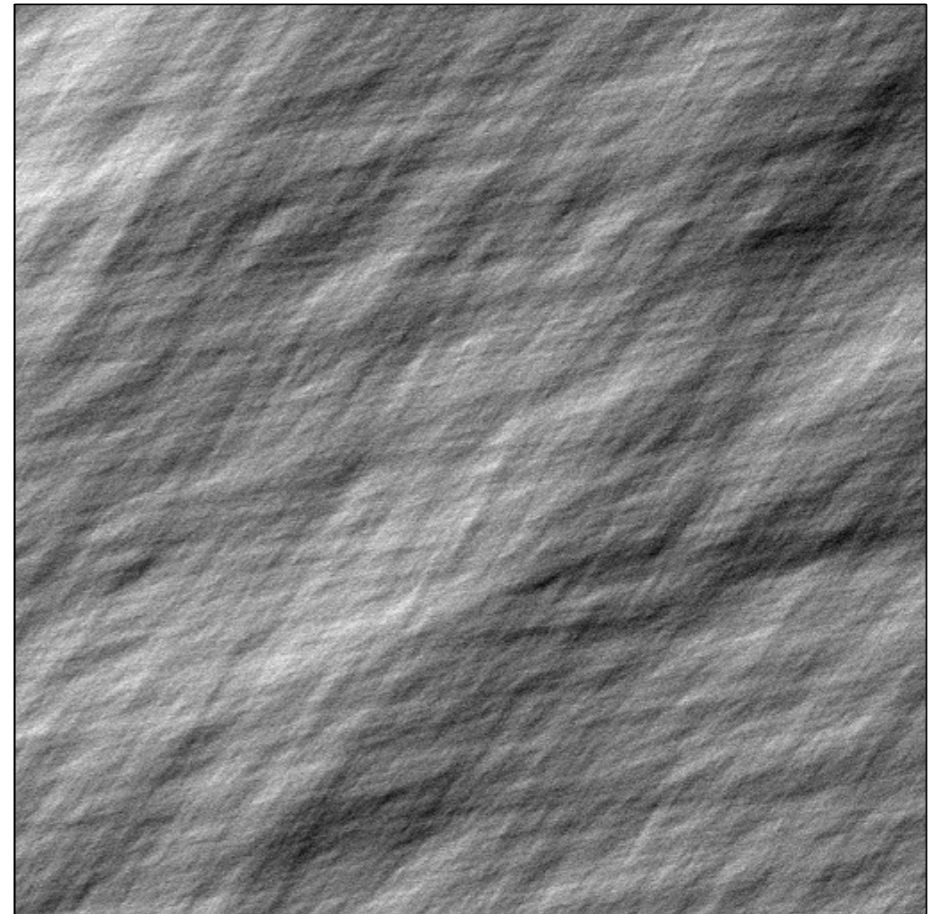
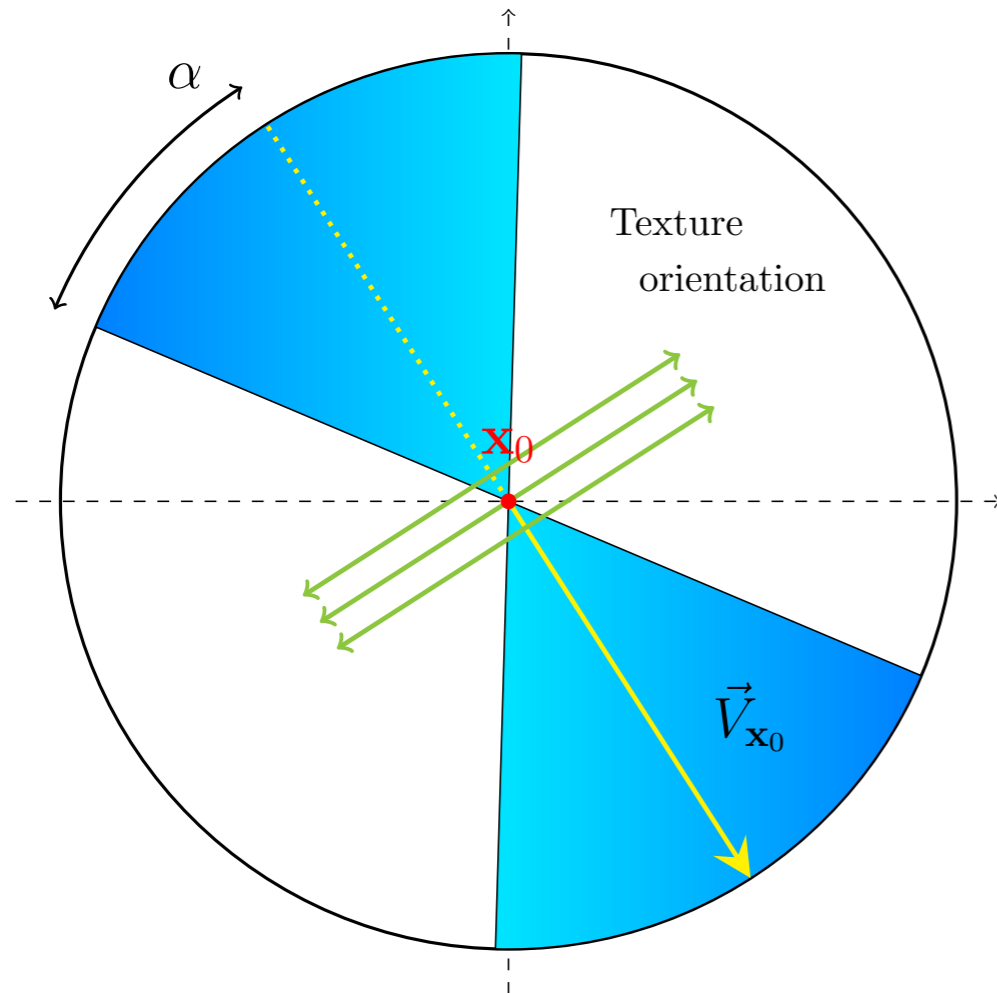
$$\alpha_0 = -\frac{\pi}{3}$$



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Elementary field

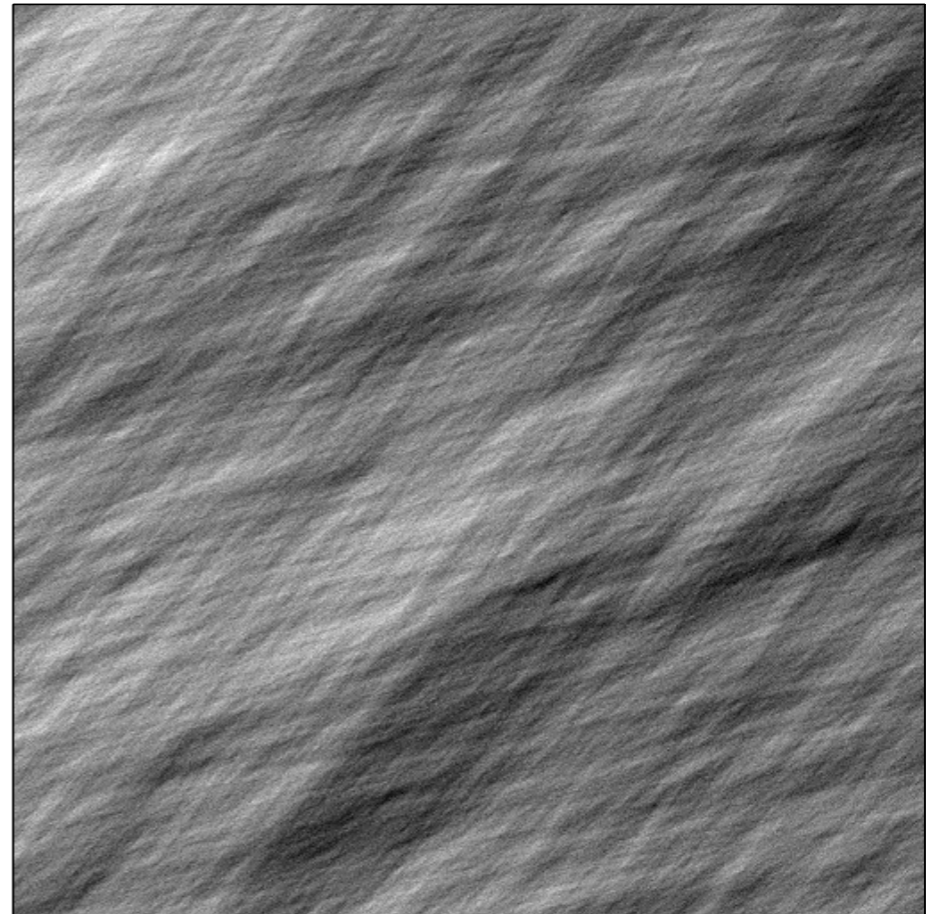
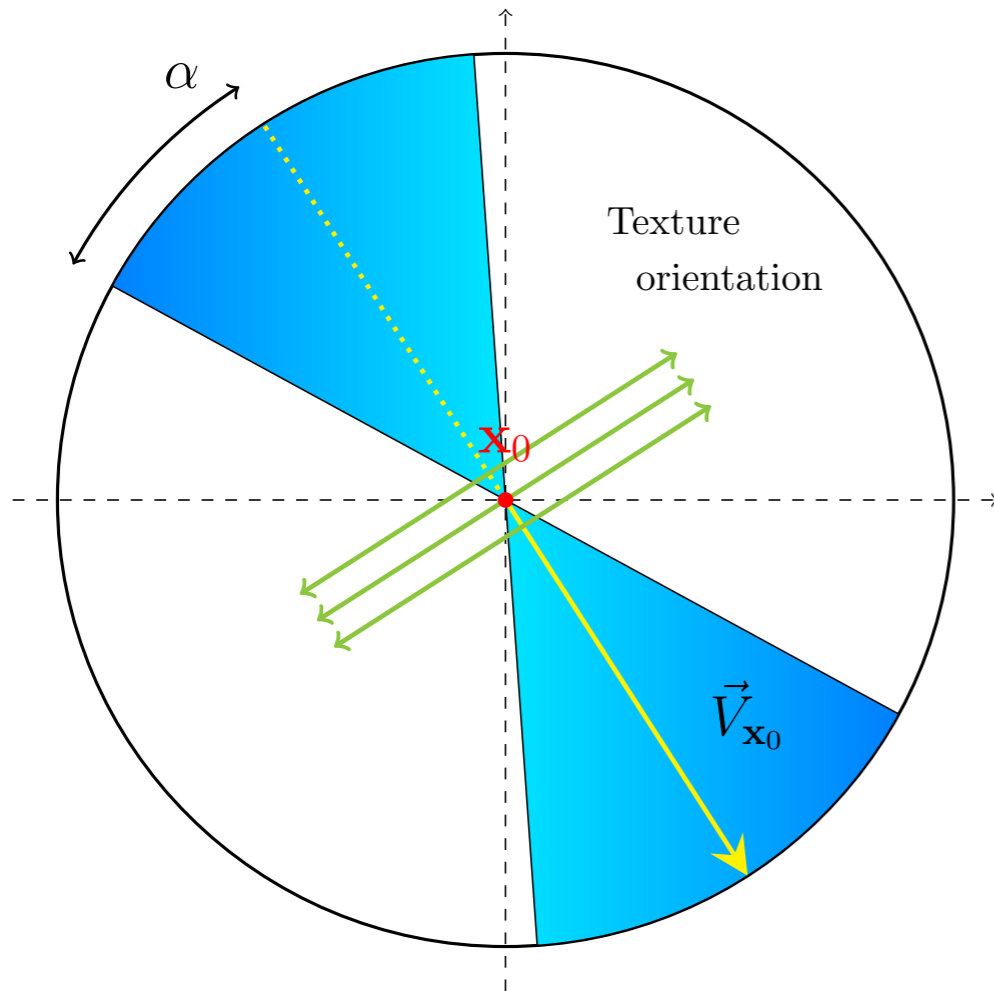
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Elementary field

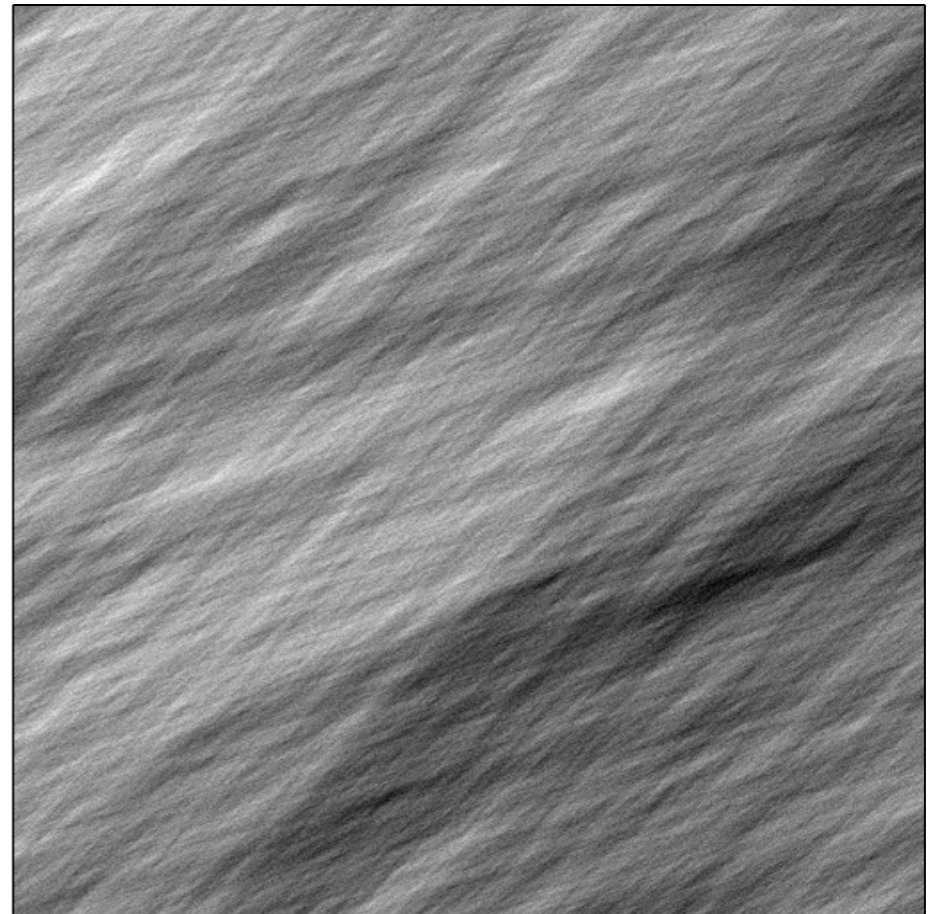
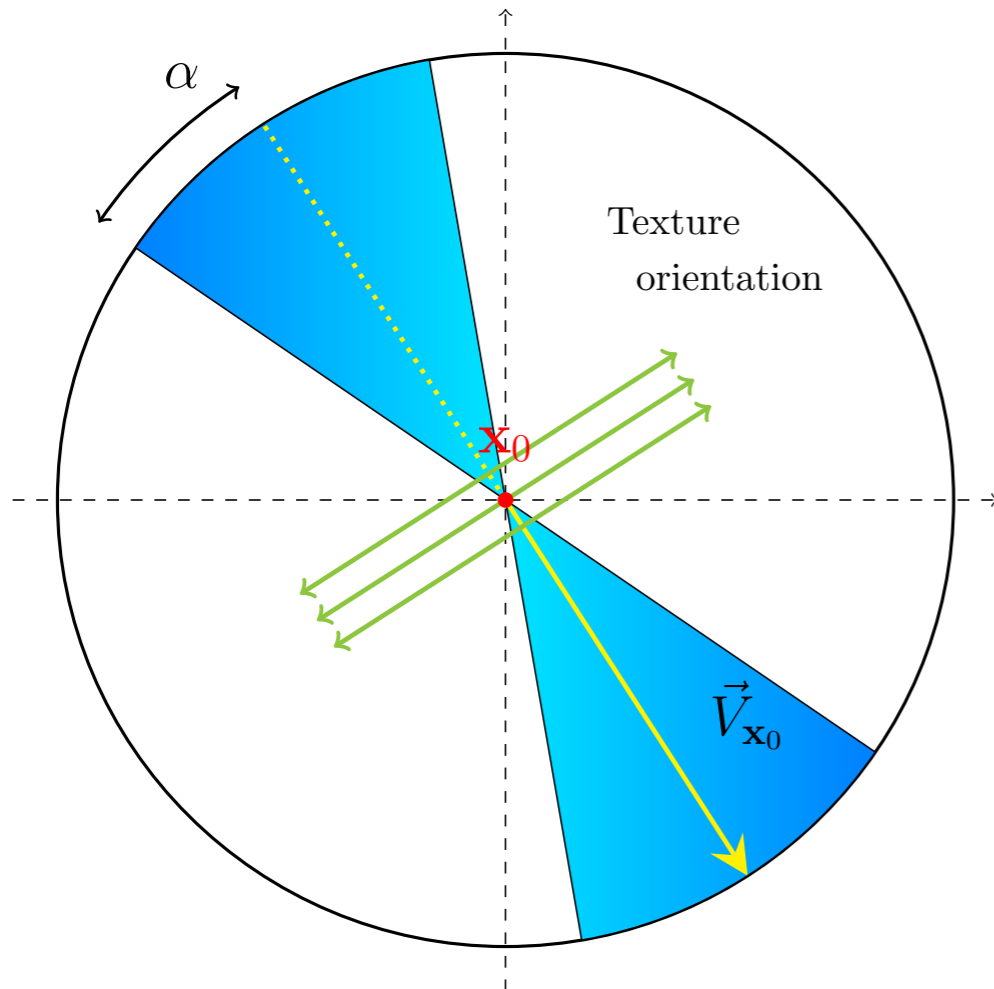
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Elementary field

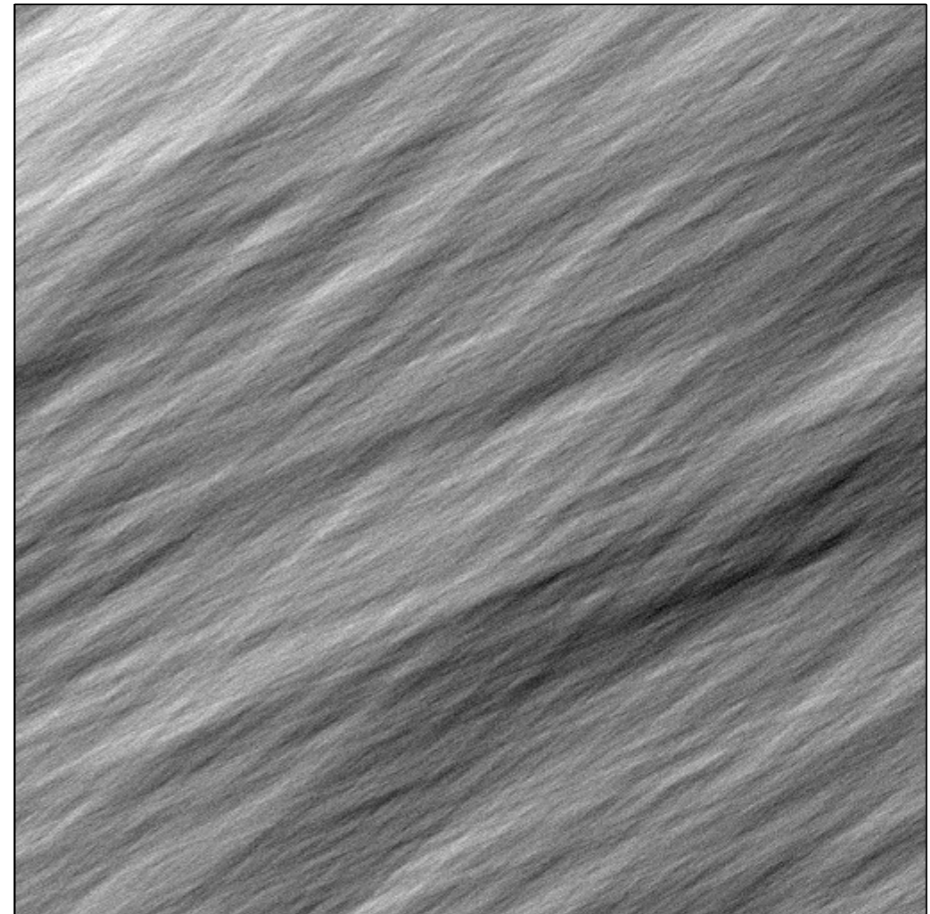
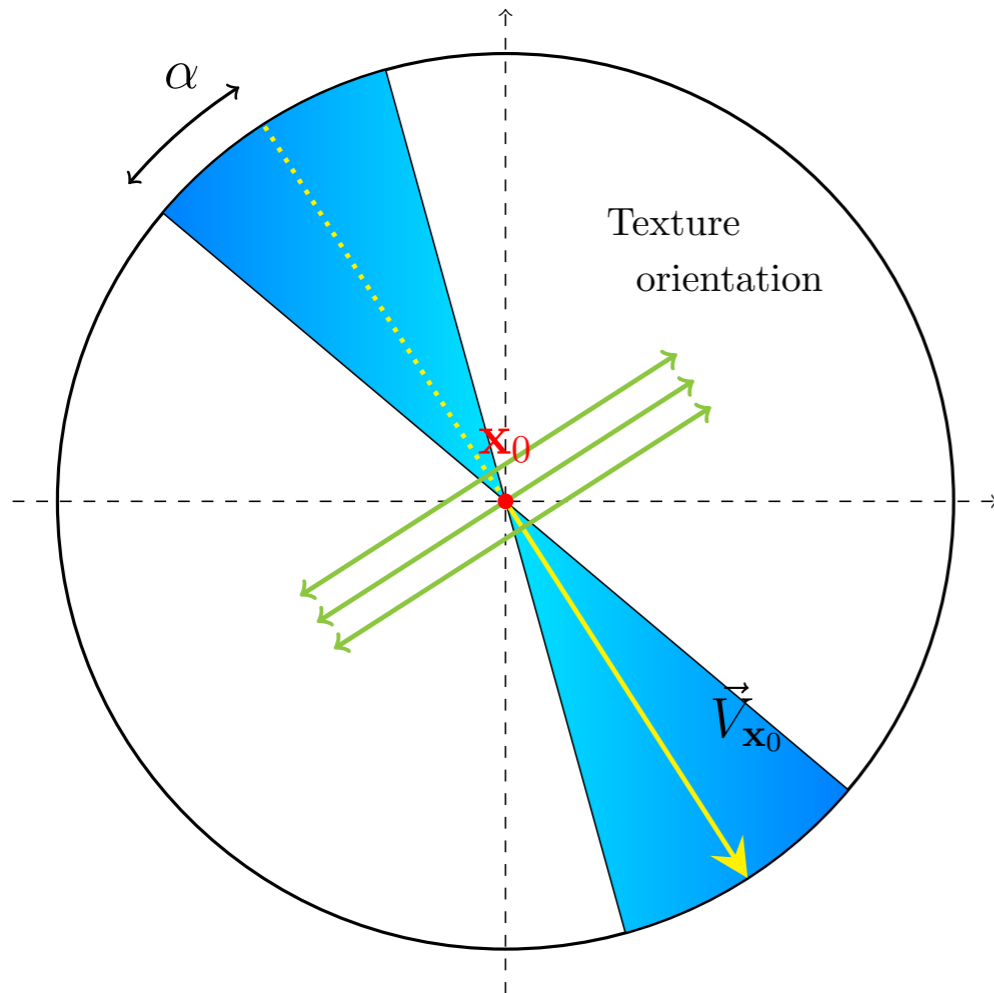
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Elementary field

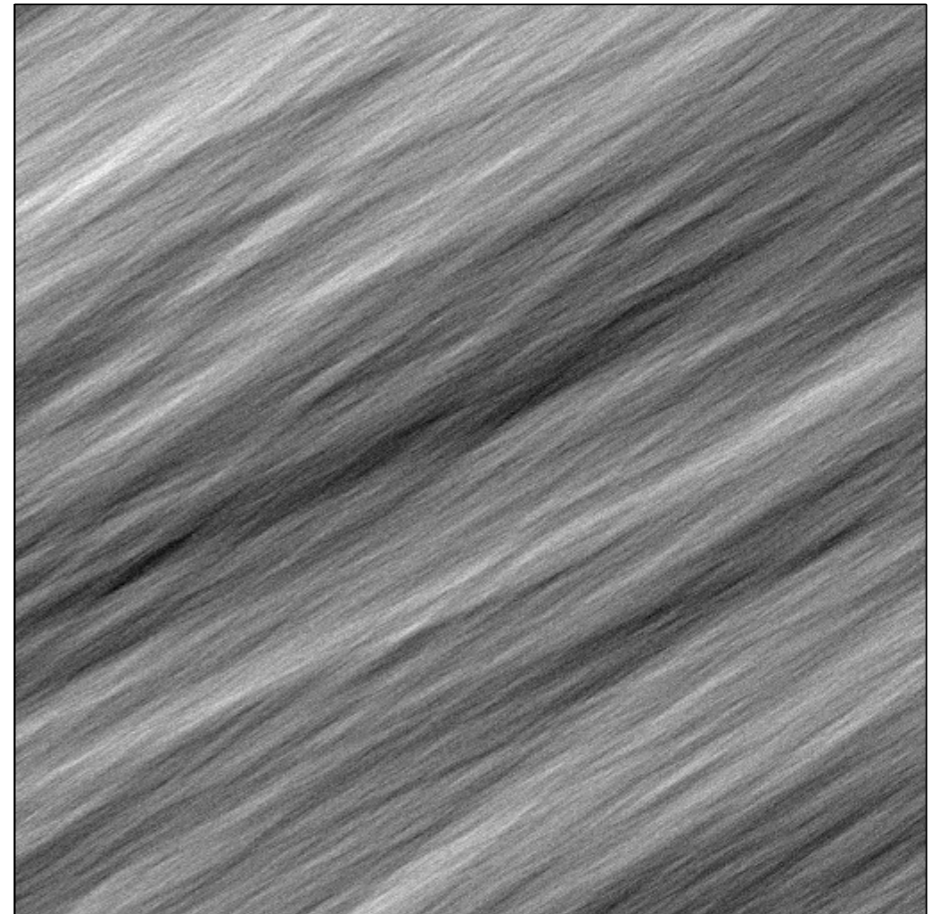
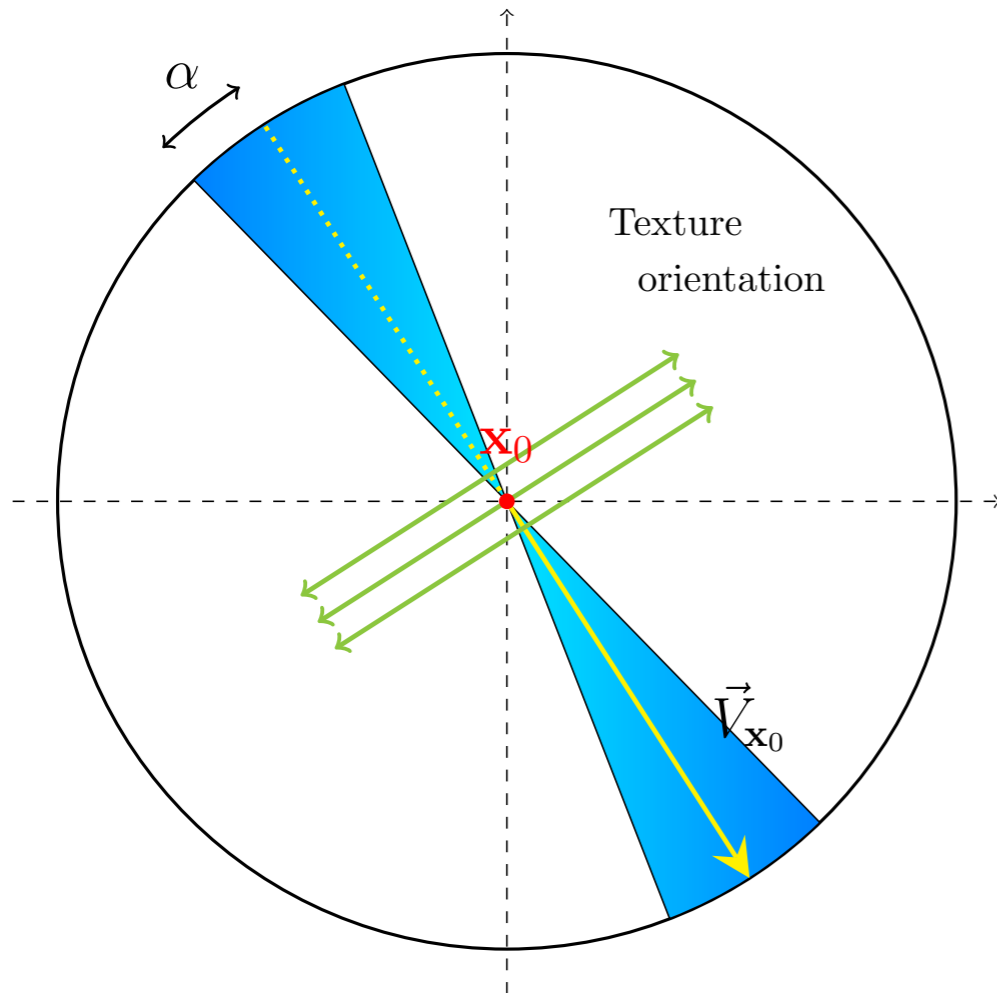
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Elementary field

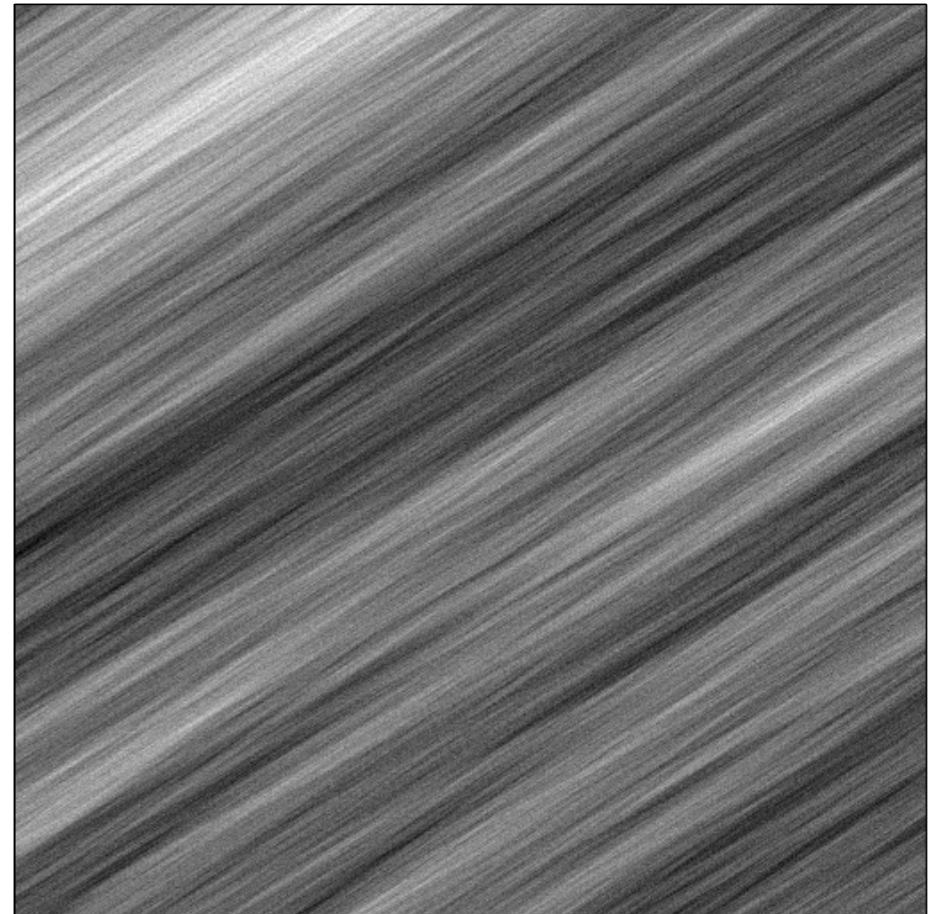
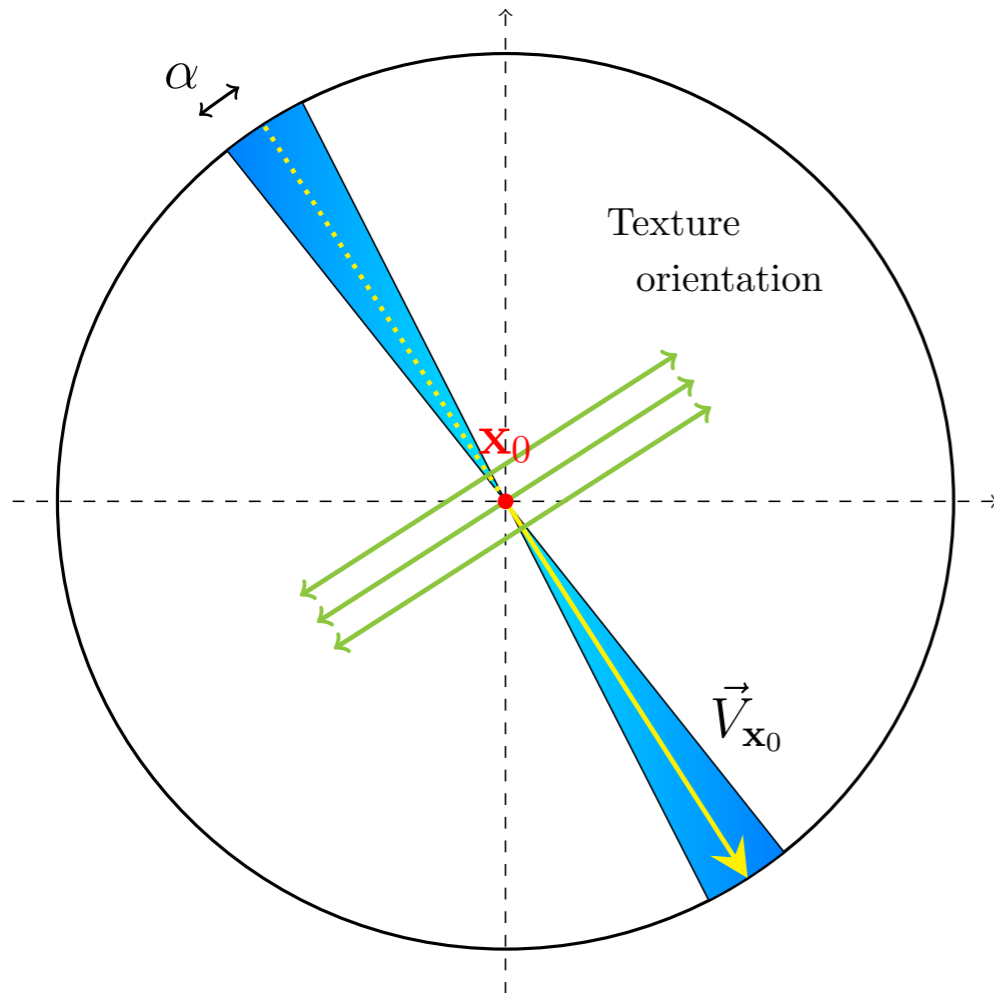
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Elementary field

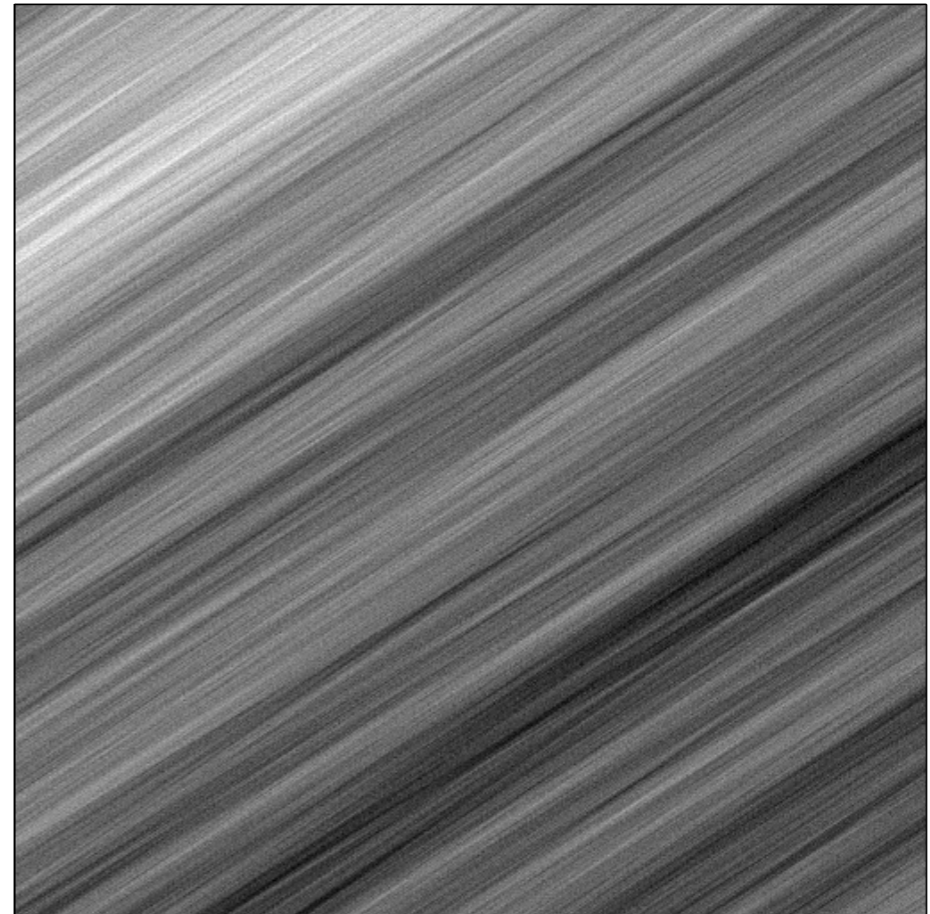
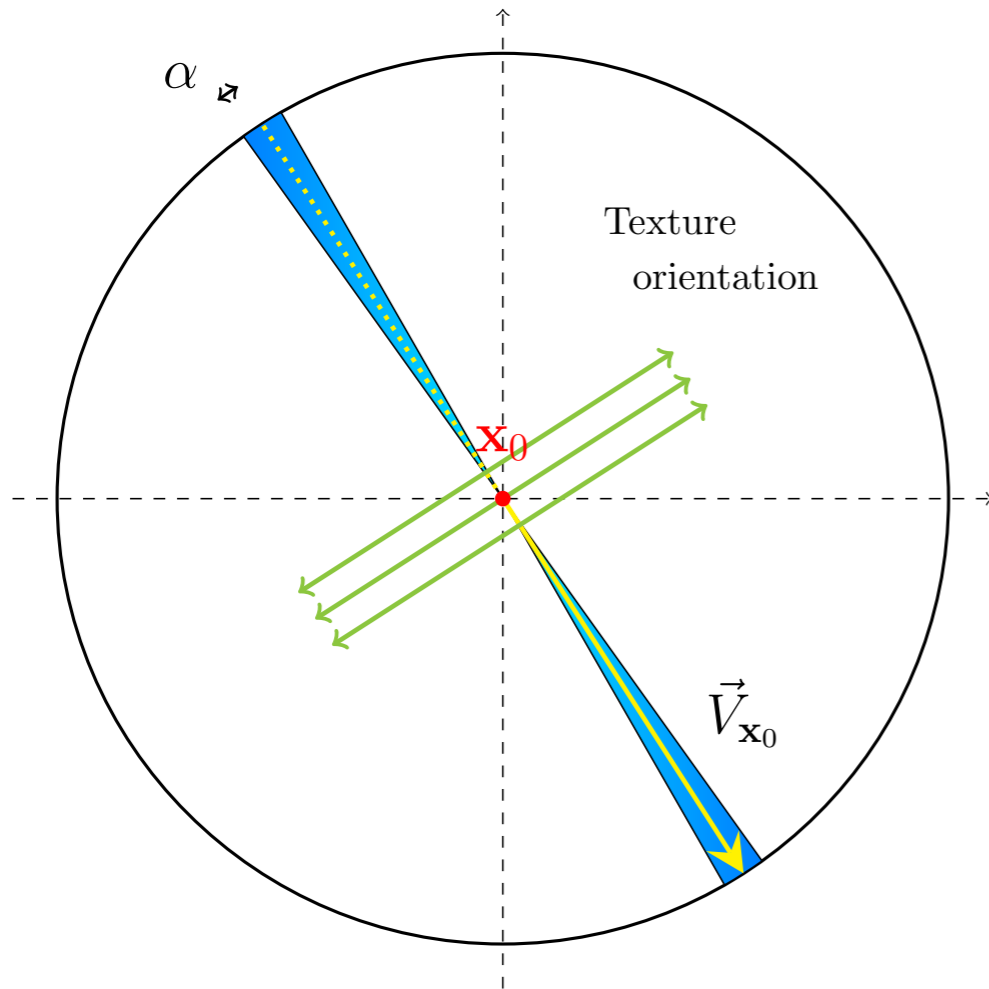
$$\alpha_0 = -\frac{\pi}{3}$$



$$\alpha = 0.1$$

Elementary field

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$$\alpha = 0.05$$

Tangent field

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■ Tangent field.

For a random field X locally asymptotically self-similar of order H ,

$$\frac{X(\mathbf{x}_0 + \rho \mathbf{h}) - X(\mathbf{x}_0)}{\rho^H} \xrightarrow[\rho \rightarrow 0]{\mathcal{L}} Y_{\mathbf{x}_0}$$

→ $Y_{\mathbf{x}_0}$: tangent field of X at point $\mathbf{x}_0 \in \mathbb{R}^2$

[Benassi,1997]
[Falconer,2002]

Taylor's expansion



Tangent field

Deterministic case

Stochastic case

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constant

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Tangent field

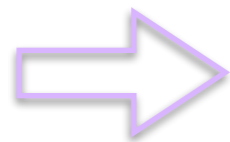
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$$B_{\alpha_0, \alpha}^H(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$$

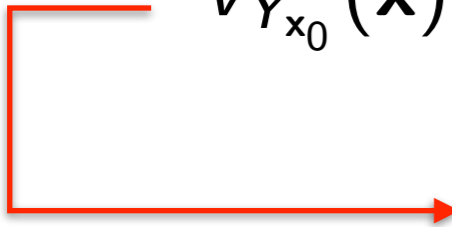
Simulation of tangent fields

- **Continuous formulation.** Variogram of $Y_{\mathbf{x}_0}$: [Bierme, Richard, Moisan, 2012]

$$\begin{aligned}v_{Y_{\mathbf{x}_0}}(\mathbf{x}) &= \frac{1}{2} \int_{\mathbb{R}^2} |e^{i\mathbf{x} \cdot \boldsymbol{\xi}} - 1|^2 f(\mathbf{x}_0, \boldsymbol{\xi}) d\boldsymbol{\xi} \\ &= \frac{1}{2} \gamma(H) \int_{-\pi/2}^{\pi/2} c_{\alpha_0, \alpha}(\mathbf{x}_0, \theta) |\mathbf{x} \cdot \mathbf{u}(\theta)|^{2H} d\theta \\ &= \int_{-\pi/2}^{\pi/2} \tilde{v}_\theta(\mathbf{x} \cdot \mathbf{u}(\theta)) d\theta\end{aligned}$$

Simulation of tangent fields

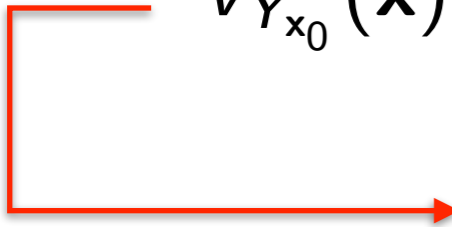
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$Y_{\mathbf{x}_0}$
 =
 Infinite sum of independant rotating FBM of order H

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Simulation of tangent fields

■ Discrete formulation.

[Bierme, Richard, Moisan, 2012]

■ $(\theta_i)_{1 \leq i \leq n}$ are n bands orientations and $\lambda_i = \theta_{i+1} - \theta_i$

■ The **turning band field** is defined as

$$Y_{\mathbf{x}_0}^{[n]}(\mathbf{x}) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^n \sqrt{\lambda_i c_{\alpha_0, \alpha}(\mathbf{x}_0, \theta_i)} B_i^H(\mathbf{x} \cdot \mathbf{u}(\theta_i))$$

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not equispaced 

Simulation of tangent fields

■ Simulation along particular bands.

[Bierme, Richard, Moisan, 2012]

- Discrete grid $r^{-1}\mathbb{Z}^2 \cap [0, 1]^2$ with $r = 2^k - 1, k \in \mathbb{N}^*$
- Choose (θ_i) such that $\tan \theta_i = \frac{p_i}{q_i}$ and $\max_i \lambda_i \leq \epsilon$
- Then $B_i^H(\mathbf{x} \cdot \mathbf{u}(\theta_i))$ becomes

$$\left\{ B_i^H \left(\frac{k_1}{r} \cos \theta_i + \frac{k_2}{r} \sin \theta_i \right) ; 0 \leq k_1, k_2 \leq r \right\} \stackrel{\mathcal{L}}{=} \left(\frac{\cos \theta_i}{rq_i} \right)^H \{ B_i^H(k_1 q_i + k_2 p_i); 0 \leq k_1, k_2 \leq r \}$$

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Dynamic programming

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Dynamic programming

self-similarity

$$B^H(\lambda \cdot) \stackrel{\mathcal{L}}{=} \lambda^H B^H(\cdot)$$

$$\left\{ B_i^H \left(\frac{k_1}{r} \cos \theta_i + \frac{k_2}{r} \sin \theta_i \right) ; 0 \leq k_1, k_2 \leq r \right\} \stackrel{\mathcal{L}}{=} \left(\frac{\cos \theta_i}{r q_i} \right)^H \{ B_i^H(k_1 q_i + k_2 p_i) ; 0 \leq k_1, k_2 \leq r \}$$

Simulation of tangent fields

■ Simulation along particular bands.

[Bierme, Richard, Moisan, 2012]

■ Discrete grid $r^{-1}\mathbb{Z}^2 \cap [0, 1]^2$ with $r = 2^k - 1, k \in \mathbb{N}^*$

■ Choose (θ_i) such that $\tan \theta_i = \frac{p_i}{q_i}$ and $\max_i \lambda_i \leq \epsilon$

■ Then $B_i^H(\mathbf{x} \cdot \mathbf{u}(\theta_i))$ becomes

Dynamic programming

self-similarity

$$B^H(\lambda \cdot) \stackrel{\mathcal{L}}{=} \lambda^H B^H(\cdot)$$

$$\left\{ B_i^H \left(\frac{k_1}{r} \cos \theta_i + \frac{k_2}{r} \sin \theta_i \right) ; 0 \leq k_1, k_2 \leq r \right\} \stackrel{\mathcal{L}}{=} \left(\frac{\cos \theta_i}{r q_i} \right)^H \{ B_i^H (k_1 q_i + k_2 p_i) ; 0 \leq k_1, k_2 \leq r \}$$

equispaced ✓

Simulation of LAFBF

using tangent fields

[Polisano et al.,2014]

■ **Algorithm.** For each pixel $\mathbf{x}_0 = (k_1, k_2) \in \llbracket 0, r \rrbracket^2$

$$B_{\alpha_0, \alpha}^H((k_1, k_2)) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^n \sqrt{\lambda_i c_{\alpha_0, \alpha}((k_1, k_2), \theta_i)} \left(\frac{\cos \theta_i}{rq_i} \right)^H B_i^H(k_1 q_i + k_2 p_i)$$

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$B_{\alpha_0, \alpha}^H(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$

Simulation of LAFBF

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[Polisano et al.,2014]

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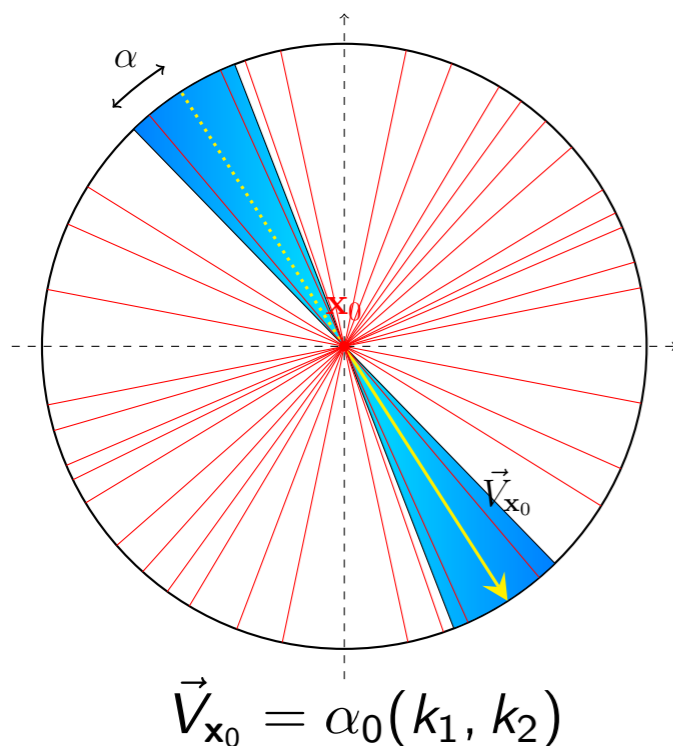
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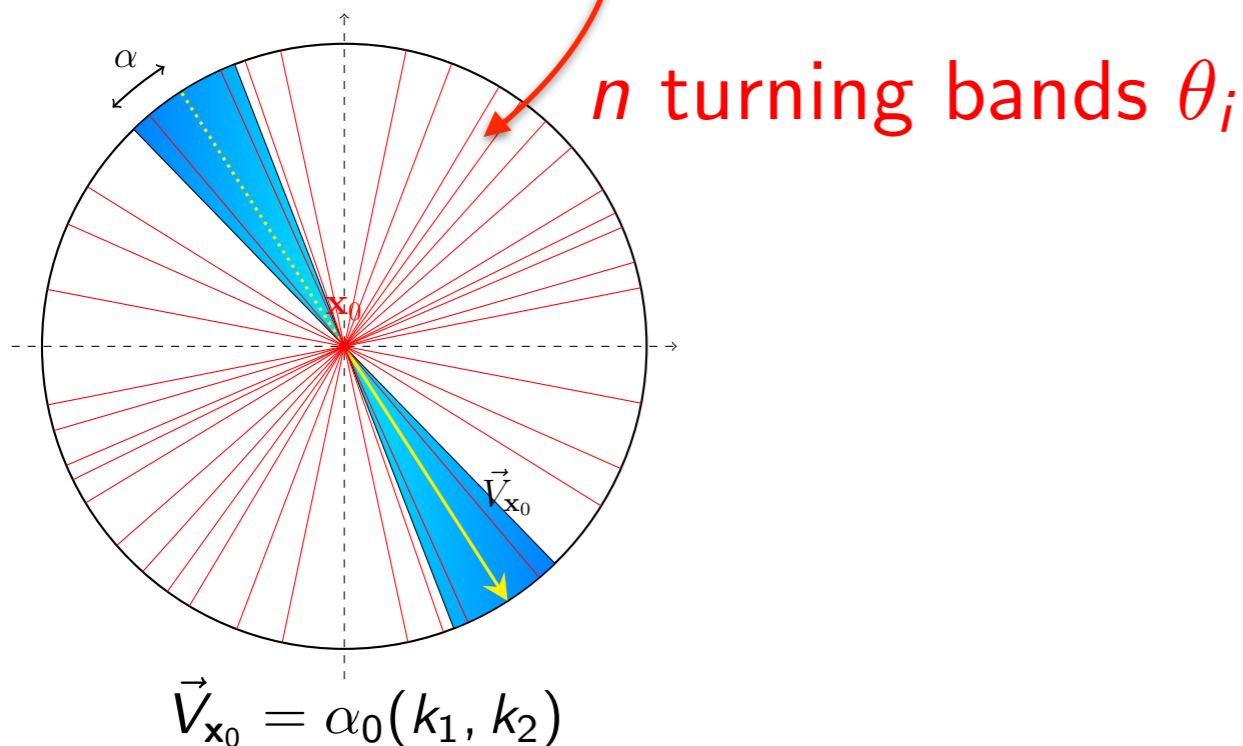
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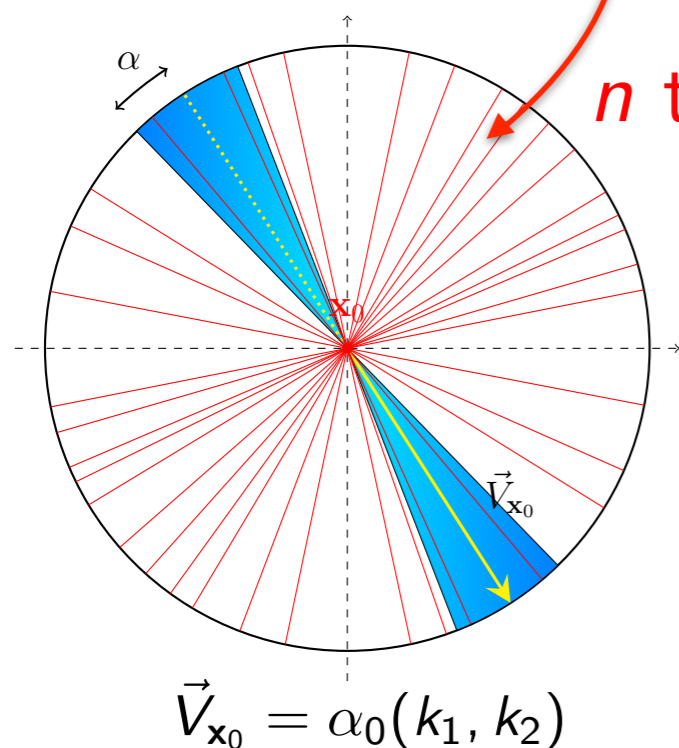
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n turning bands θ_i

$$\mathbb{1}_{[-\alpha, \alpha]}(\theta_i - \alpha_0((k_1, k_2))) \neq 0 \Leftrightarrow |\theta_i - \alpha_0((k_1, k_2))| \leq \alpha$$

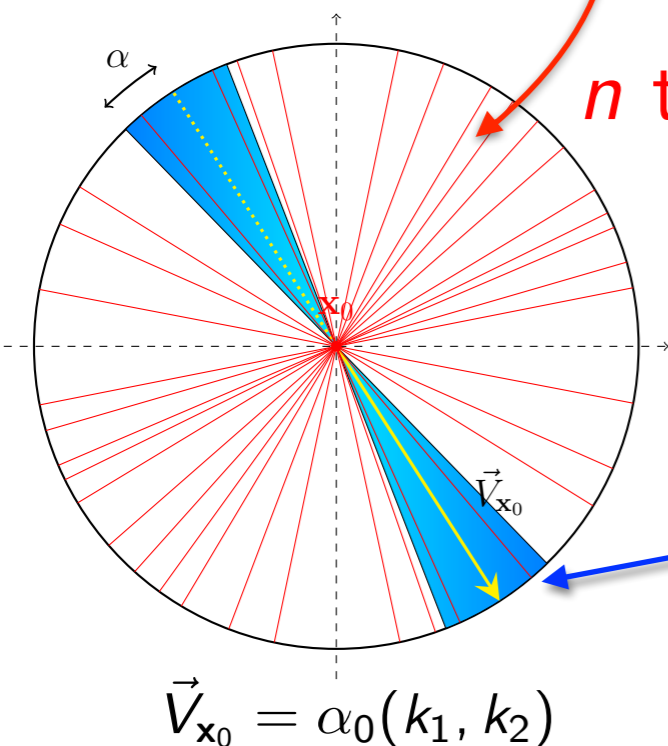
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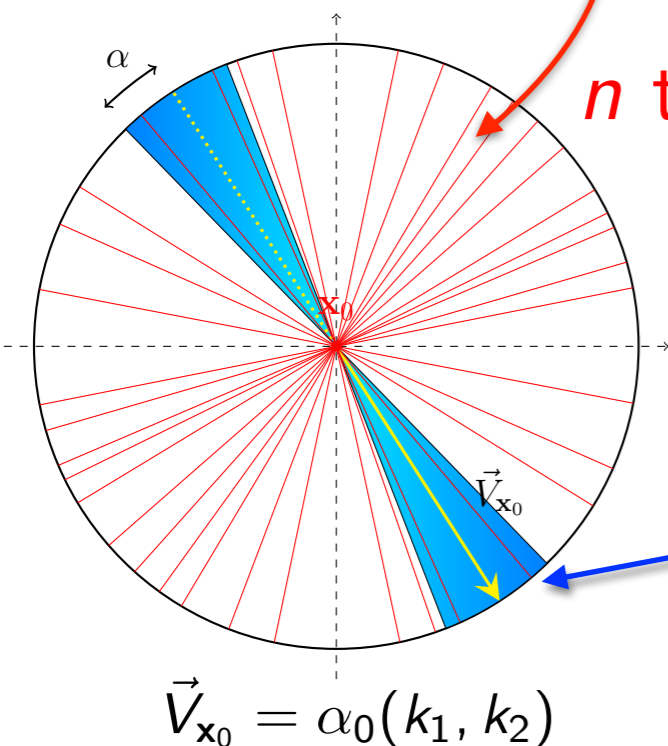
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n turning bands θ_i

few bands
in the cone

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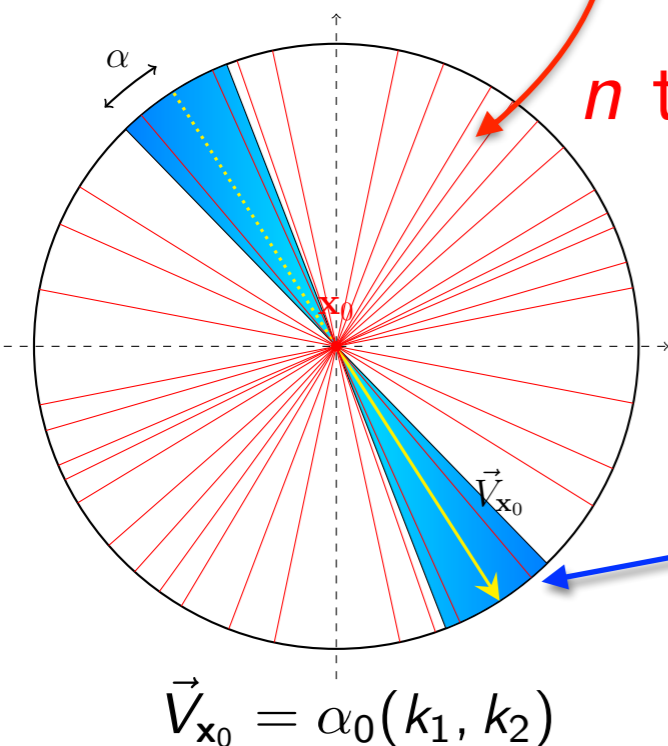
Simulation of LAFBF

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n turning bands θ_i

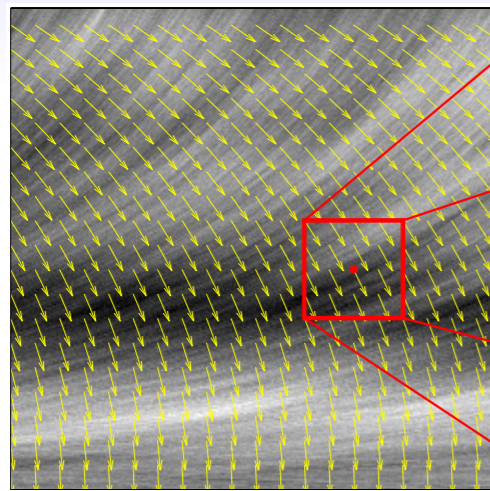
few bands
in the cone

$$\mathbb{1}_{[-\alpha, \alpha]}(\theta_i - \alpha_0((k_1, k_2))) \neq 0 \Leftrightarrow |\theta_i - \alpha_0((k_1, k_2))| \leq \alpha$$

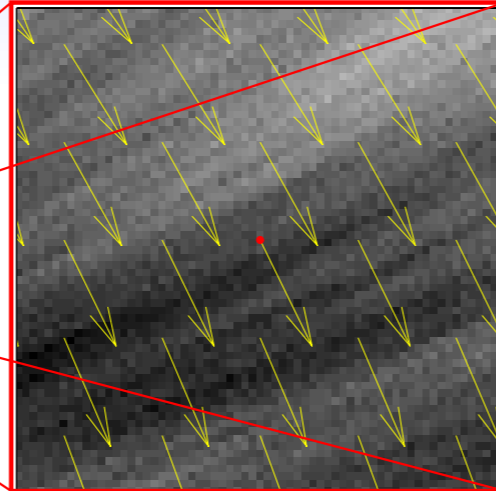
Complexity $O(r^2 \log n)$

Numerical experiments

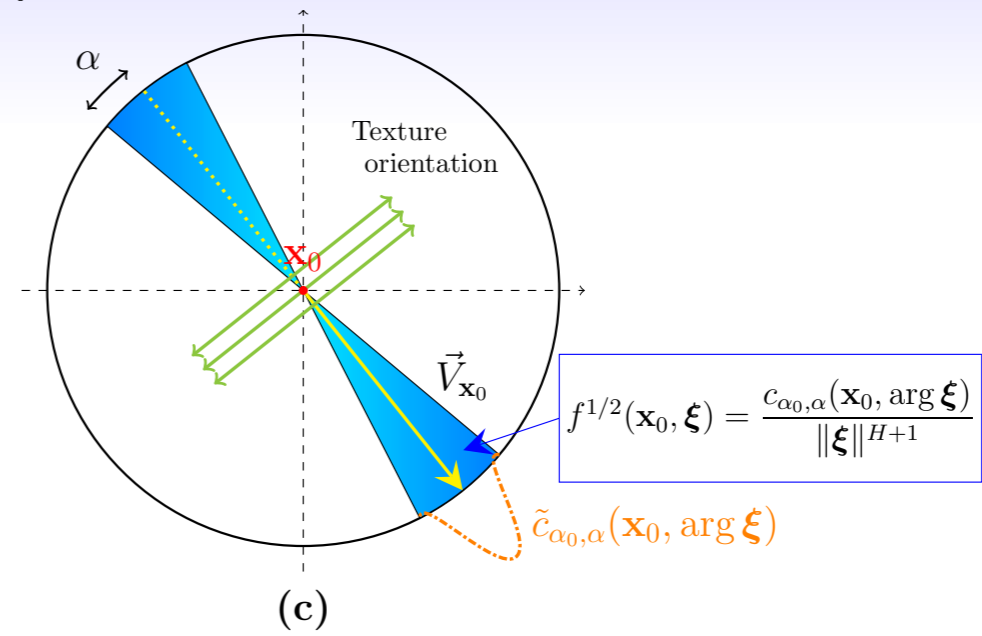
$$\vec{V}_{(x,y)}^1 = (\cos(-\pi/2 + y), \sin(-\pi/2))$$



(a)



(b)



(c)

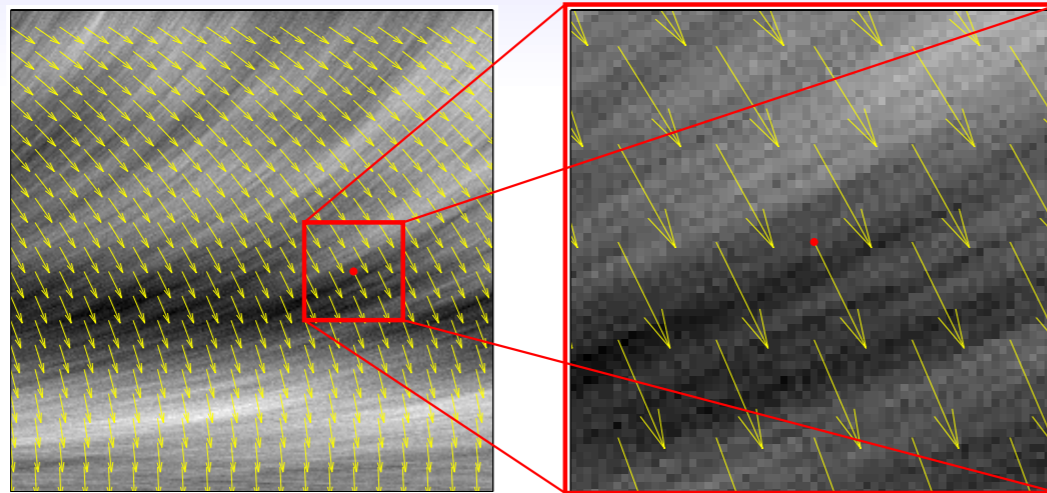
Parameters

$$r = 255 \quad H = 0.2$$

$$\alpha = 10^{-1} \quad \epsilon = 10^{-2}$$

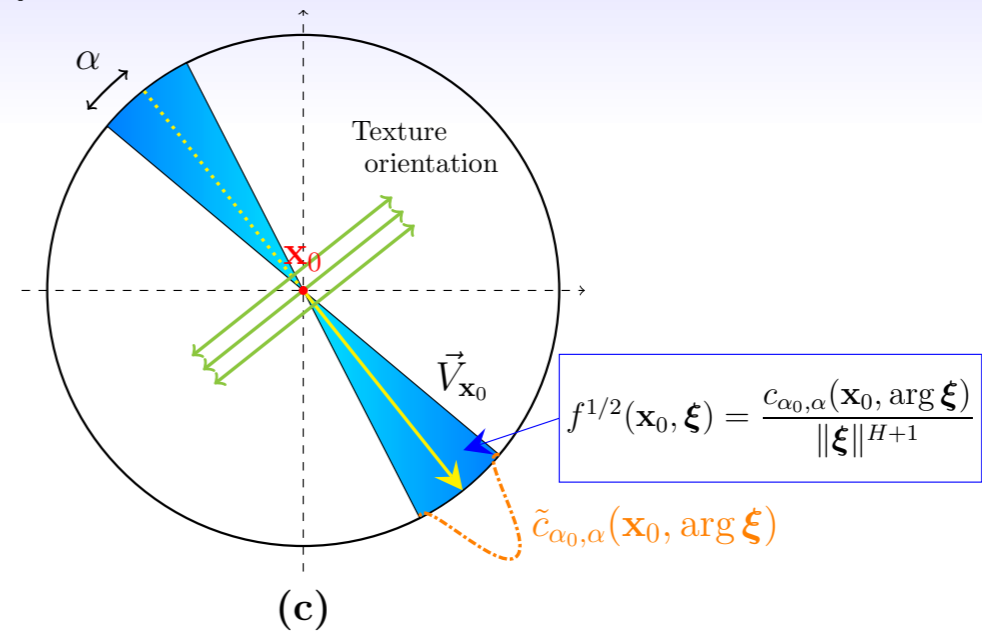
Numerical experiments

$$\vec{V}_{(x,y)}^1 = (\cos(-\pi/2 + y), \sin(-\pi/2))$$



(a)

(b)



(c)

Texture with prescribed local orientation at each point \mathbf{x}_0 given by a vector field

$$\vec{V}_{\mathbf{x}_0} = \mathbf{u}(\alpha_0(\mathbf{x}_0))$$

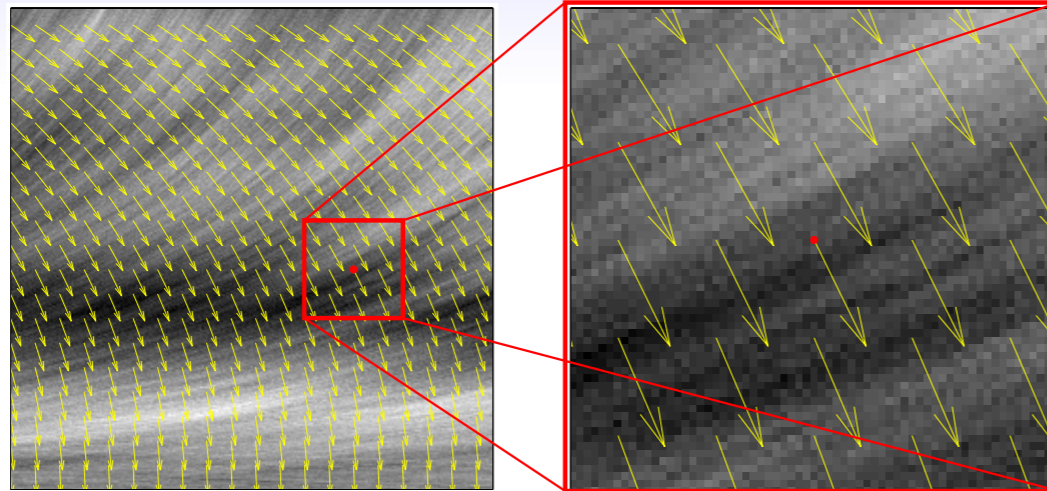
Parameters

$$r = 255 \quad H = 0.2$$

$$\alpha = 10^{-1} \quad \epsilon = 10^{-2}$$

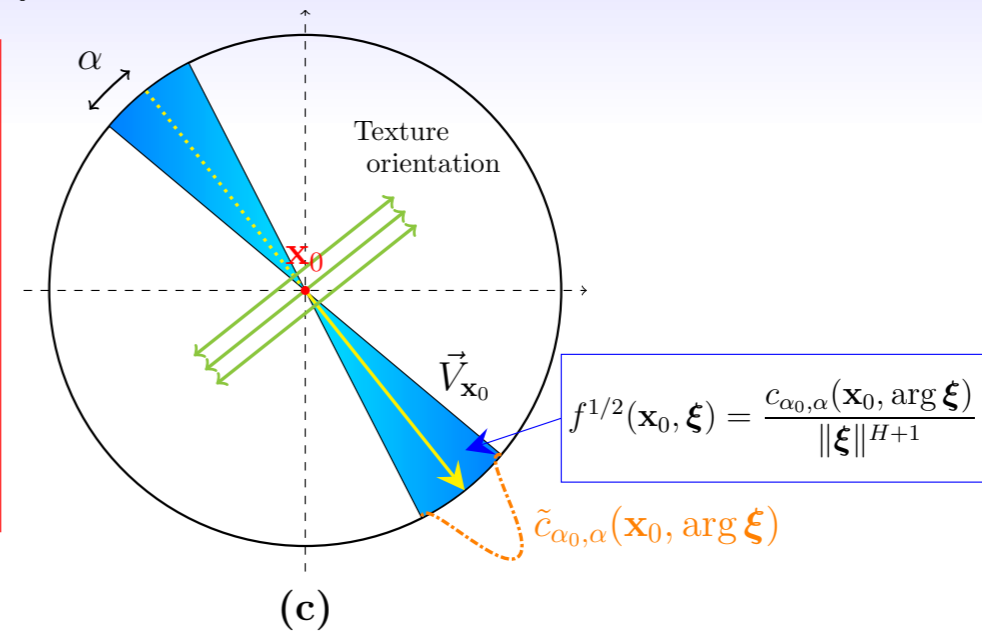
Numerical experiments

$$\vec{V}_{(x,y)}^1 = (\cos(-\pi/2 + y), \sin(-\pi/2))$$



(a)

(b)



(c)

Texture with prescribed local orientation at each point \mathbf{x}_0 given by a vector field

$$\vec{V}_{\mathbf{x}_0} = \mathbf{u}(\alpha_0(\mathbf{x}_0))$$

A zoom around a point \mathbf{x}_0 shows that locally a LAFBF behaves as an elementary field

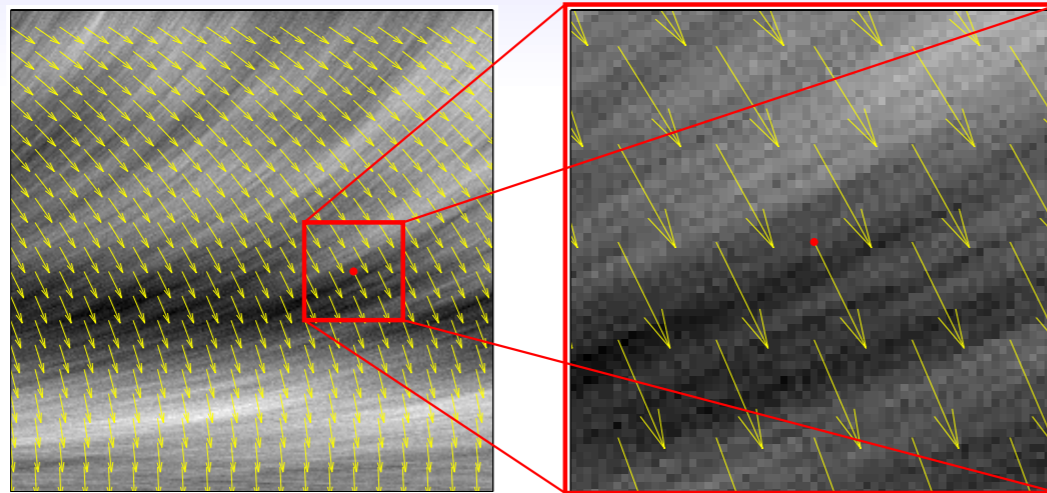
Parameters

$$r = 255 \quad H = 0.2$$

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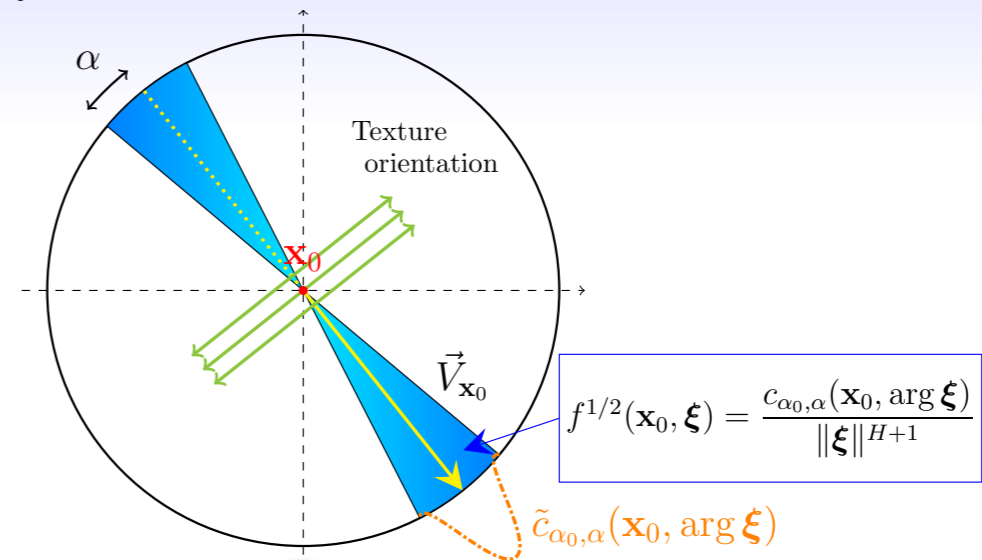
Numerical experiments

$$\vec{V}_{(x,y)}^1 = (\cos(-\pi/2 + y), \sin(-\pi/2))$$



(a)

(b)



(c)

Texture with prescribed local orientation at each point \mathbf{x}_0 given by a vector field

$$\vec{V}_{\mathbf{x}_0} = \mathbf{u}(\alpha_0(\mathbf{x}_0))$$

Regularized version of the anisotropy function

A zoom around a point \mathbf{x}_0 shows that locally a LAFBF behaves as an elementary field

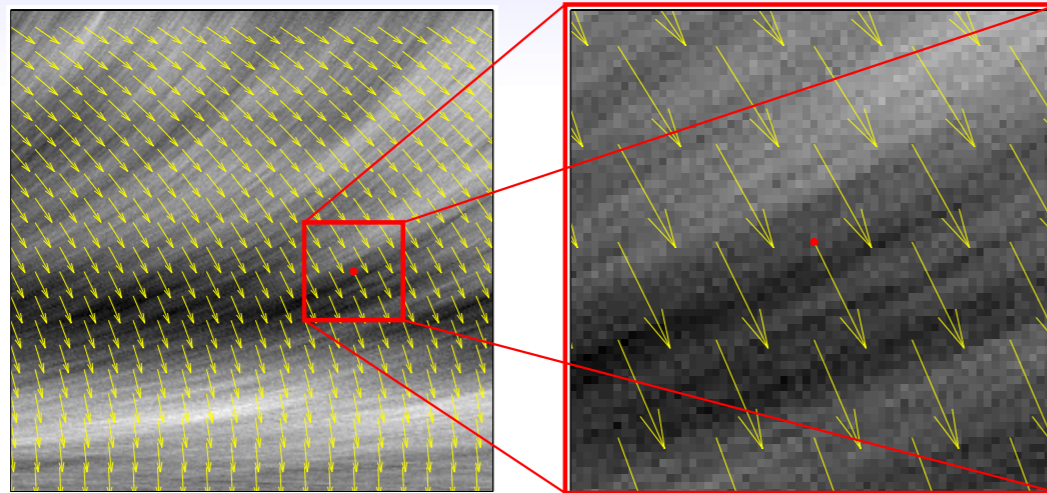
Parameters

$$r = 255 \quad H = 0.2$$

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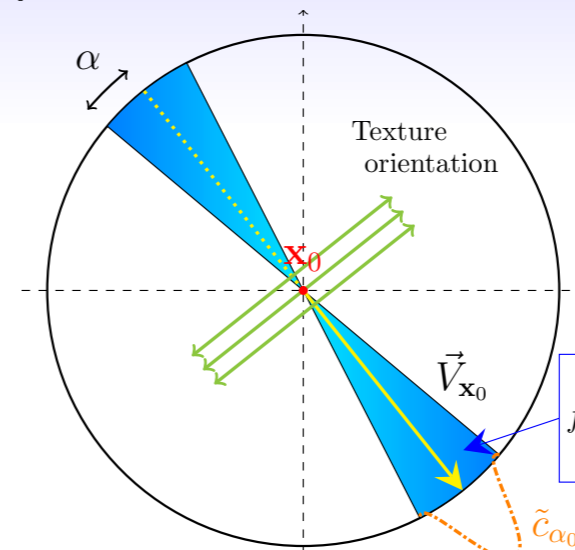
Numerical experiments

$$\vec{V}_{(x,y)}^1 = (\cos(-\pi/2 + y), \sin(-\pi/2))$$



(a)

(b)



(c)

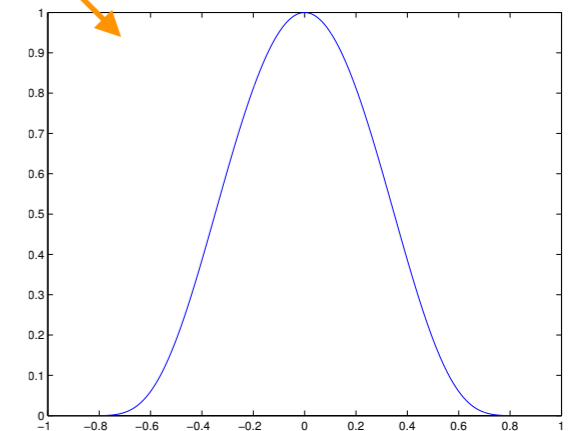
$$f^{1/2}(\mathbf{x}_0, \boldsymbol{\xi}) = \frac{c_{\alpha_0, \alpha}(\mathbf{x}_0, \arg \boldsymbol{\xi})}{\|\boldsymbol{\xi}\|^{H+1}}$$

$$\tilde{c}_{\alpha_0, \alpha}(\mathbf{x}_0, \arg \boldsymbol{\xi})$$

Texture with prescribed local orientation at each point \mathbf{x}_0 given by a vector field

$$\vec{V}_{\mathbf{x}_0} = \mathbf{u}(\alpha_0(\mathbf{x}_0))$$

Regularized version of the anisotropy function



A zoom around a point \mathbf{x}_0 shows that locally a LAFBF behaves as an elementary field

Parameters

$$r = 255 \quad H = 0.2$$

$$\alpha = 10^{-1} \quad \epsilon = 10^{-2}$$

Numerical experiments

$$\vec{V}_{(x,y)}^2 = (\cos(\cos(36xy)), \sin(\cos(36xy)))$$

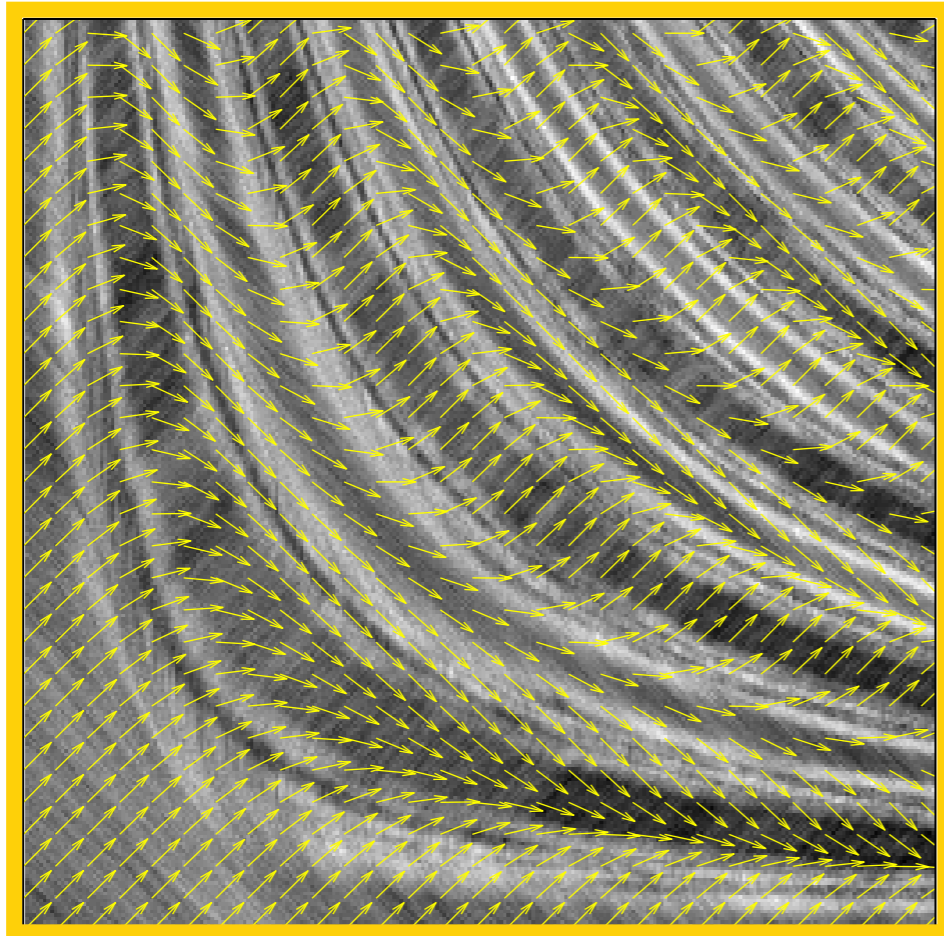
$$\vec{V}_{(x,y)}^3 = \nabla F(x, y)$$

$$F(x, y) = (4x - 2)e^{-(4x-2)^2 - (4y-2)^2}$$

Numerical experiments

$$\vec{V}_{(x,y)}^2 = (\cos(\cos(36xy)), \sin(\cos(36xy)))$$

$$\vec{V}_{(x,y)}^3 = \nabla F(x, y)$$

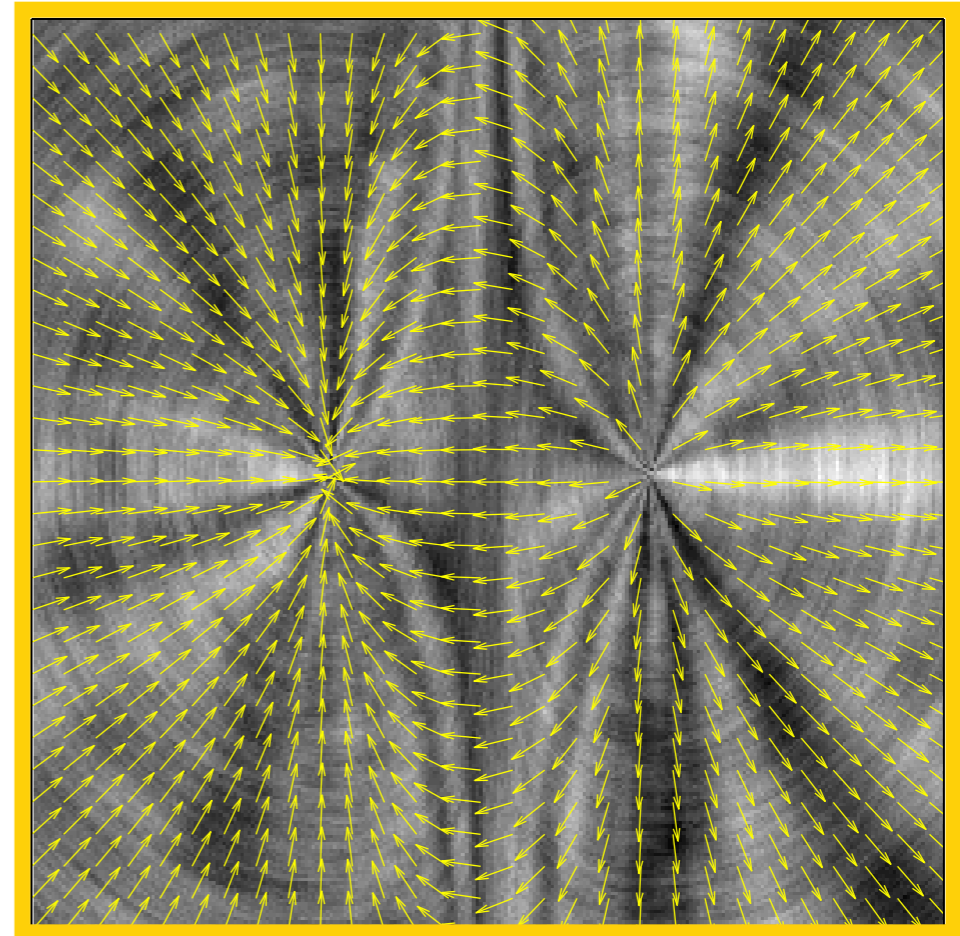
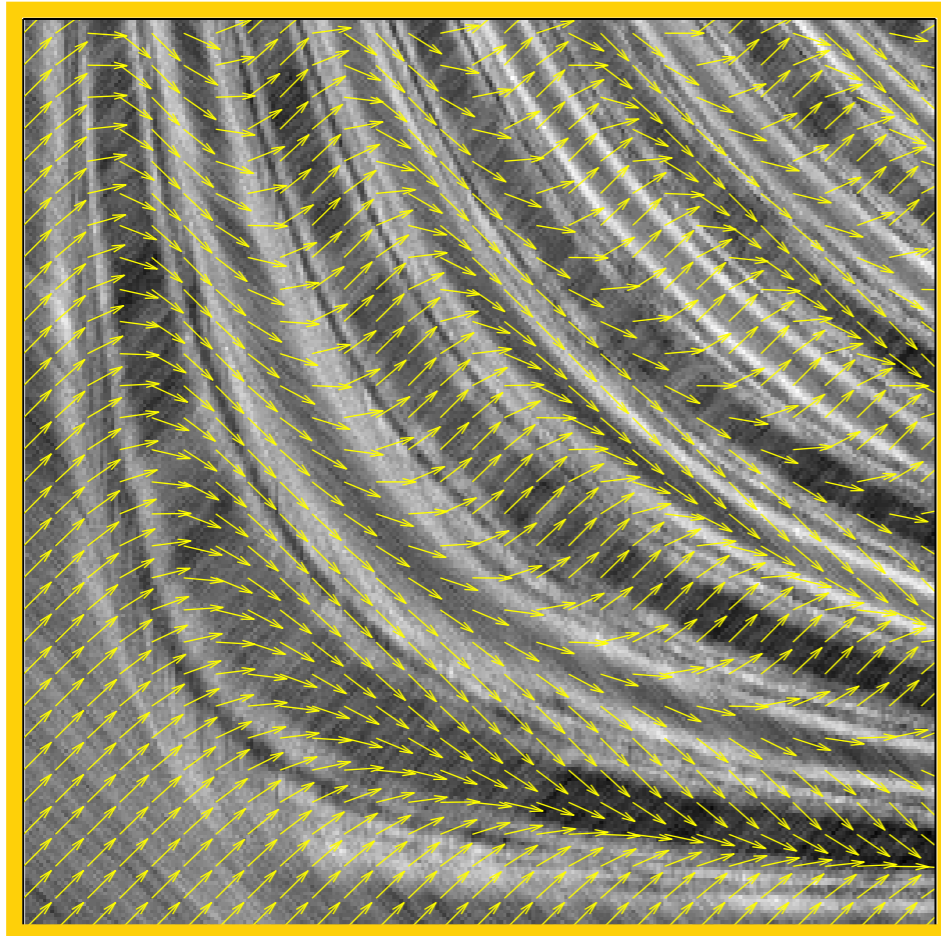


$$F(x, y) = (4x - 2)e^{-(4x-2)^2 - (4y-2)^2}$$

Numerical experiments

$$\vec{V}_{(x,y)}^2 = (\cos(\cos(36xy)), \sin(\cos(36xy)))$$

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Numerical experiments

H=0.2

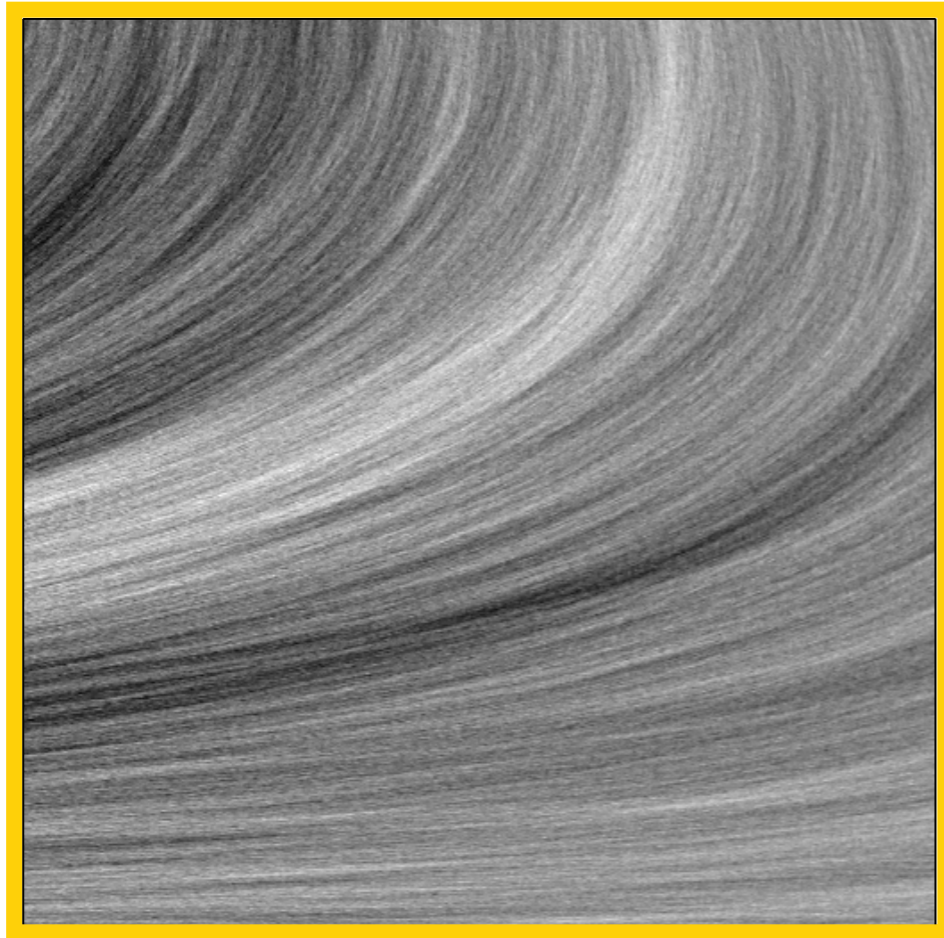
H=0.5

$$\vec{V}_{(x,y)}^1 = (\cos(-\pi/2 + y), \sin(-\pi/2))$$

Numerical experiments

H=0.2

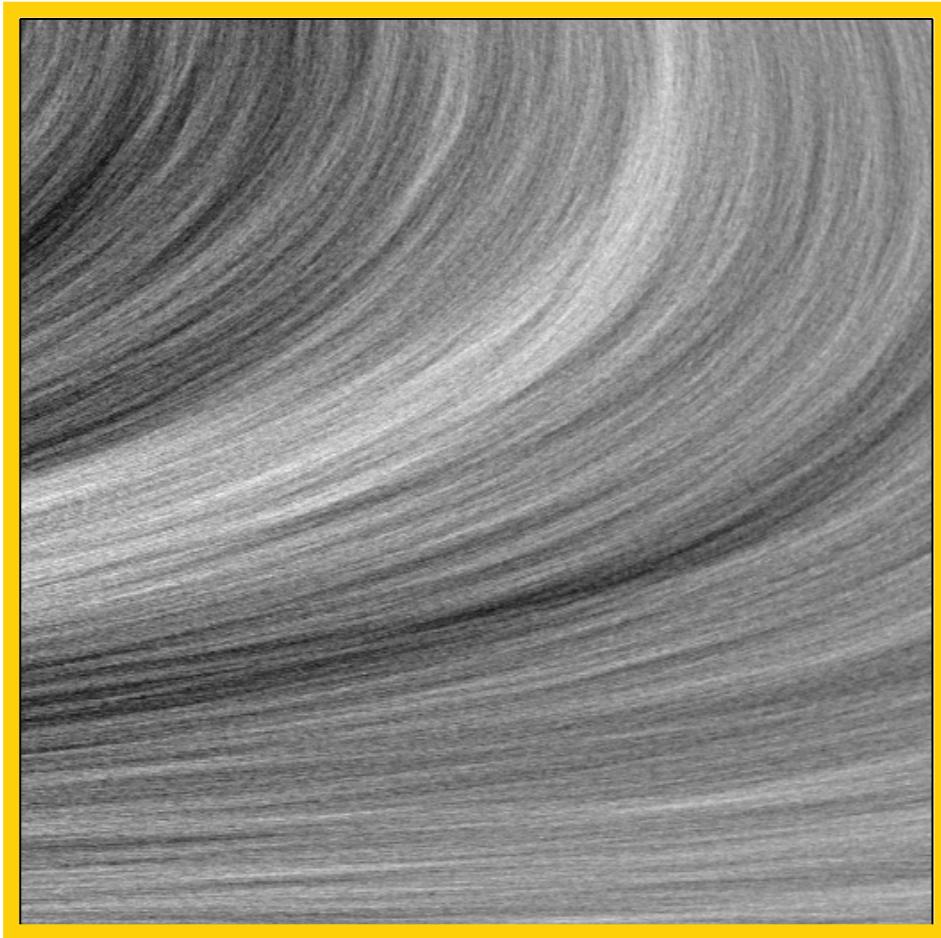
H=0.5



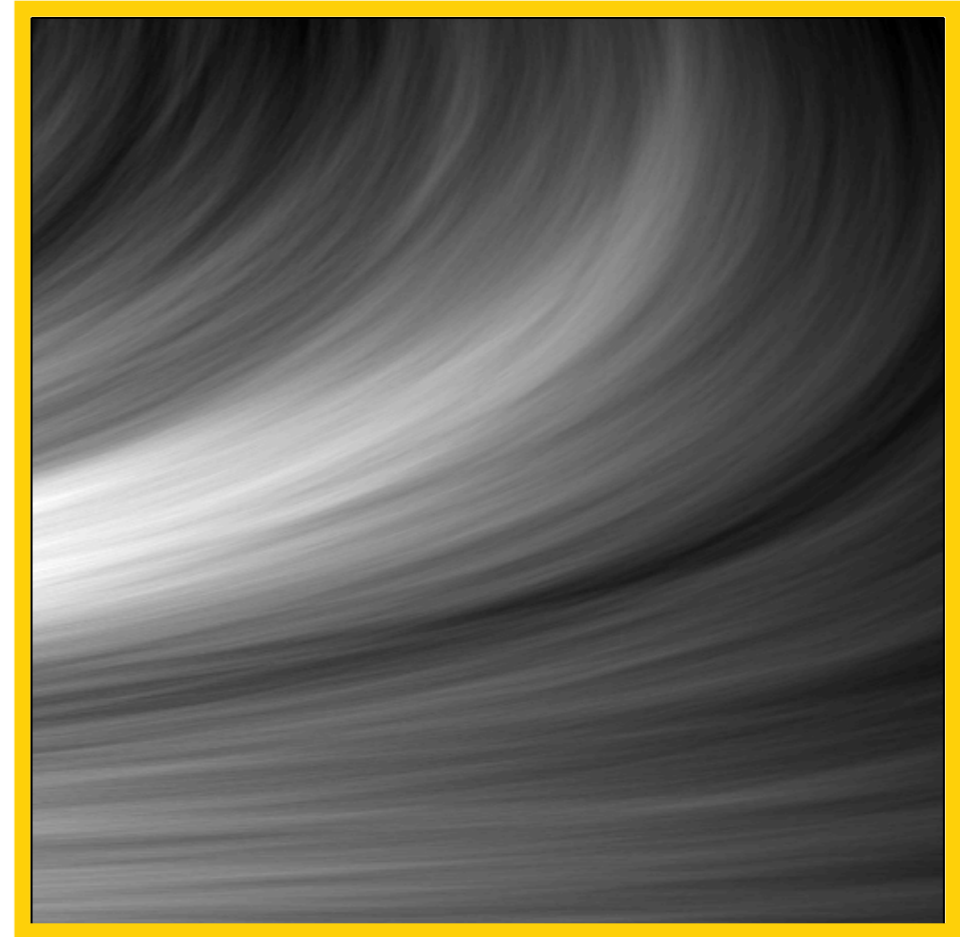
$$\vec{V}_{(x,y)}^1 = (\cos(-\pi/2 + y), \sin(-\pi/2))$$

Numerical experiments

H=0.2



H=0.5



$$\vec{V}_{(x,y)}^1 = (\cos(-\pi/2 + y), \sin(-\pi/2))$$

Conclusion and future work

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- Introduce a **new stochastic model** defined as a local version of an AFBF.

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■ Future work

- Extensions of our model include Hurst index may **vary spatially**.

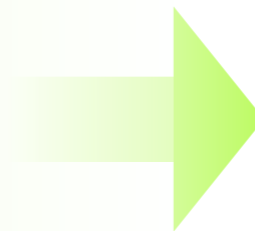
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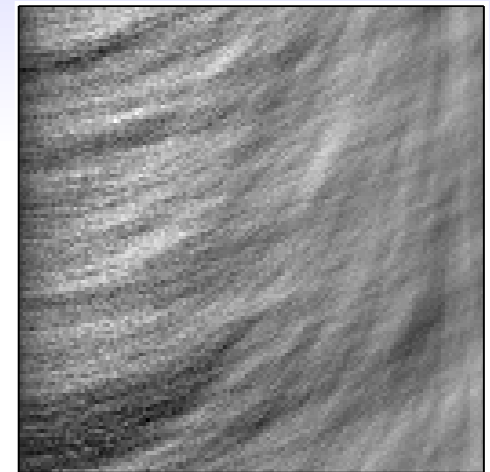
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Conclusion and future work

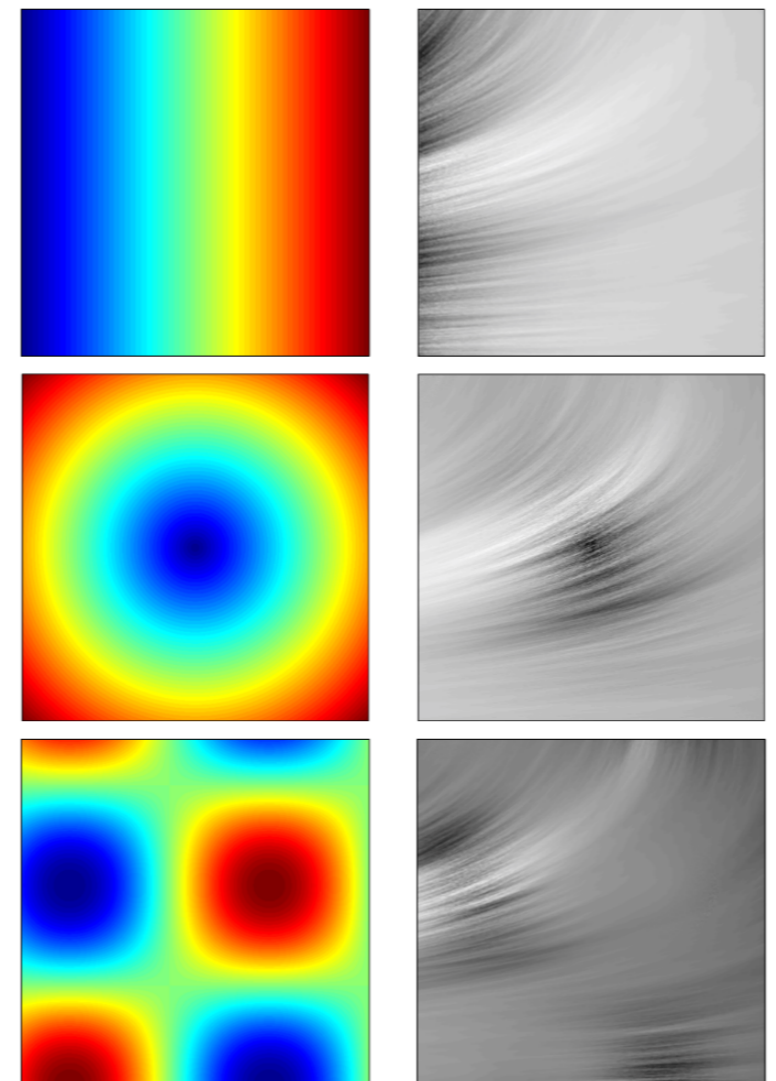
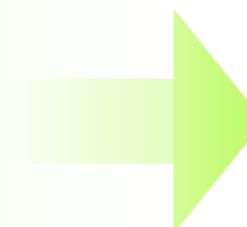
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Bibliography

■ Selected papers

- K. Polisano, M. Clausel, V. Perrier and L. Condat, "Texture modeling by Gaussian fields with prescribed local orientation", *IEEE ICIP*, 2014.
- A. Bonami and A. Estrade, "Anisotropic analysis of some Gaussian models", *Journal of Fourier Analysis and Applications*, vol. 9, no. 3, pp. 215–236, 2003.
- H. Bierme, L. Moisan, and F. Richard, "A turning-band method for the simulation of anisotropic fractional Brownian fields," *preprint MAP5 No. 2012-312012*, 2012.
- K.J. Falconer, "Tangent fields and the local structure of random fields," *Journal of Theoretical Probability*, vol. 15, no. 3, pp. 731–750, 2002.

Questions ?

Questions ?

Thank you for your attention.

- **Dynamic programming.** The choice of the bands orientation $(\theta_i)_{1 \leq i \leq n}$ is governed by the computational cost of the B_i^H 's within dynamic programming.

- Let the error ϵ fixed. Taking $N = \lceil \frac{1}{\tan \epsilon} \rceil$ consider the following set:

$$\mathcal{V}_N = \left\{ (p, q) \in \mathbb{Z}^2 / -N \leq p \leq N, 1 \leq q \leq N, p \wedge q = 1, -\frac{\pi}{2} < \arctan\left(\frac{p}{q}\right) < \frac{\pi}{2} \right\}$$

- The aim is to find n pairs in the set \mathcal{V}_N which minimize the following global cost:

$$C(\Theta) = \sum_{k=1}^s C(r(|p_{i_k}| + q_{i_k}))$$

where $C(\ell)$ is the cost of one FBM B_i^H in $O(n \log n)$, under the constraint $\max_i (\theta_{i+1} - \theta_i) \leq \epsilon$