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# Texture modeling by gaussian fields with prescribed local orientation

Kévin Polisano

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UNIVERSITÉ DE  
GRENOBLE

# TEXTURE MODELING BY GAUSSIAN FIELDS WITH PRESCRIBED LOCAL ORIENTATION

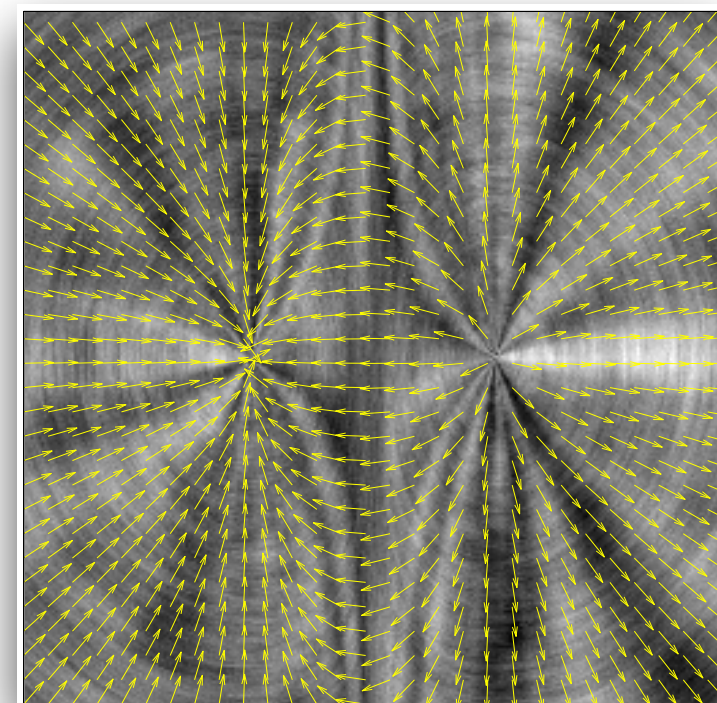
**Kévin Polisano** / [kevin.polisano@imag.fr](mailto:kevin.polisano@imag.fr)

joint work with

Marianne Clausel

Valérie Perrier

Laurent Condat



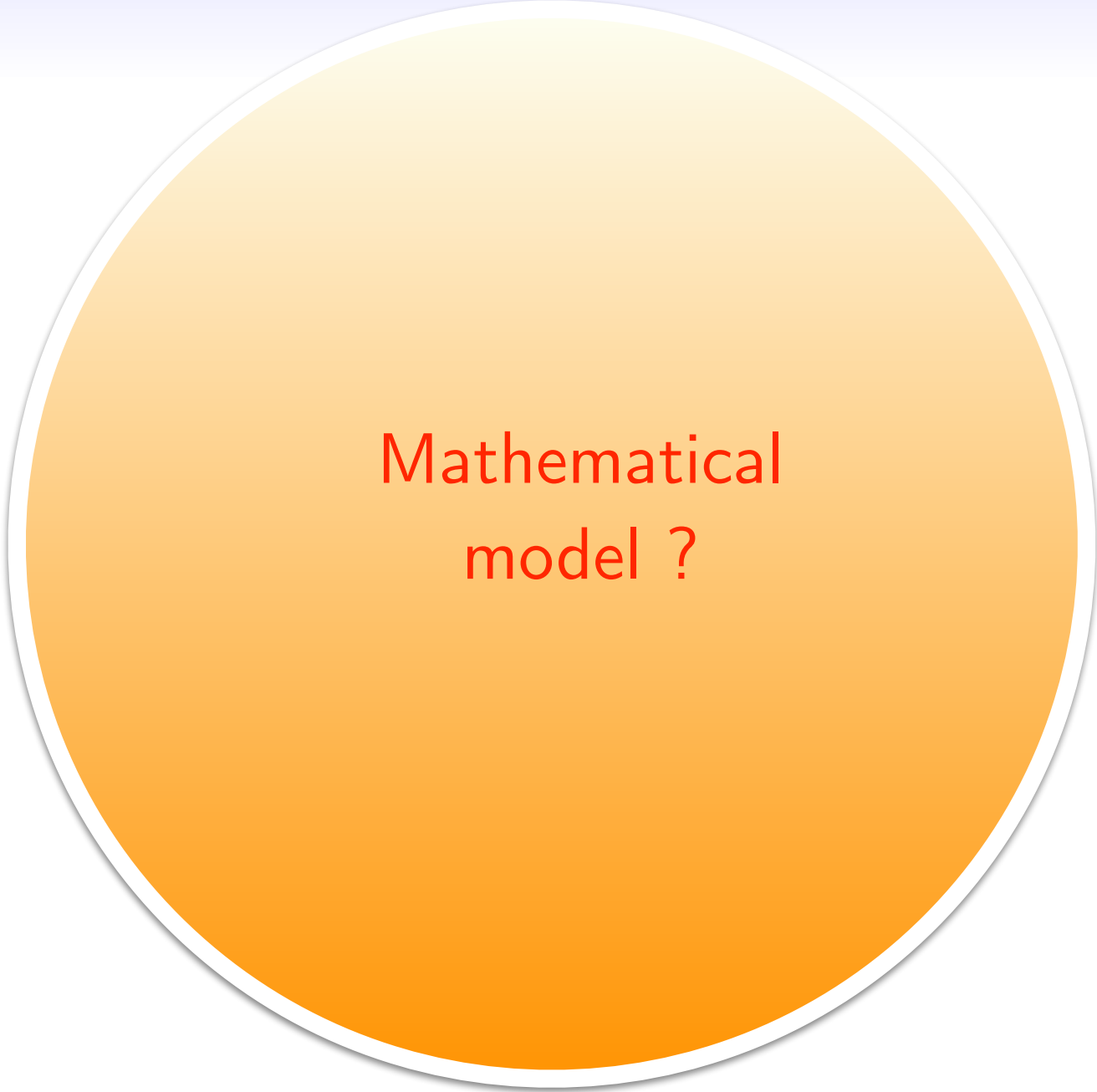
# Outline

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- Introduction
  - Motivation
  - General probabilistic framework
- Our new stochastic model
  - Definition: Locally Anisotropic Fractional Brownian Field
  - Notion of tangent field
- Synthesis methods
  - Tangent field simulation by a turning bands method
  - LAFBF simulation via tangent field formulation
- Numerical experiments
- Conclusion and future work

# How to synthesize natural random textures ?

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Mathematical  
model ?



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Randomness  
Self-similarity



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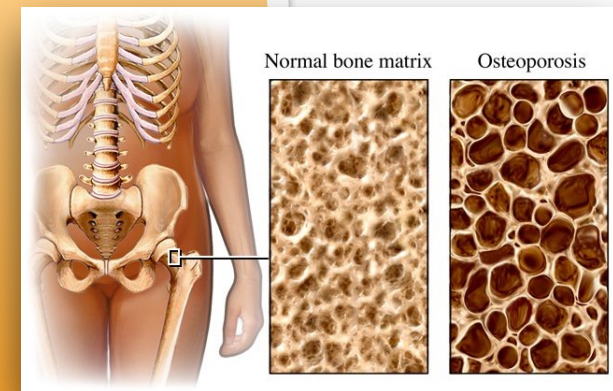
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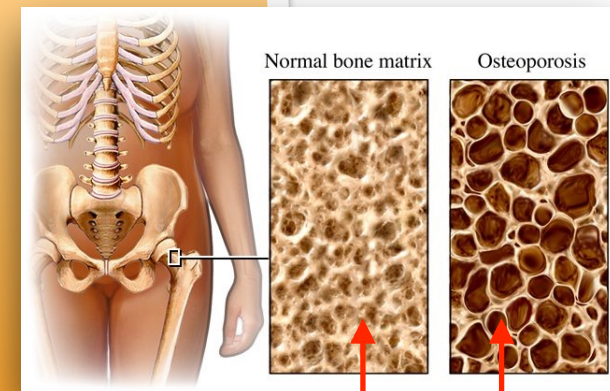


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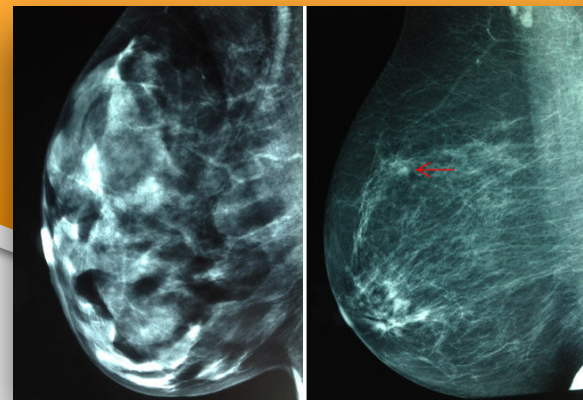
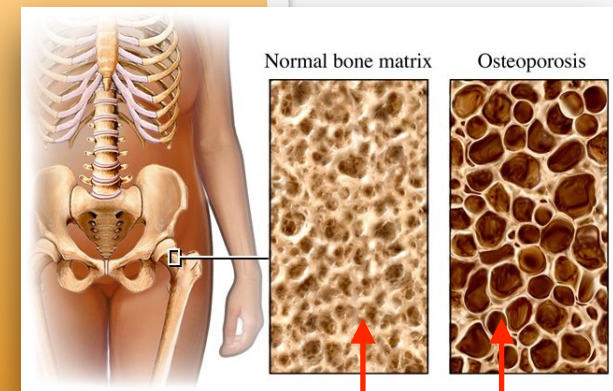
Roughness  
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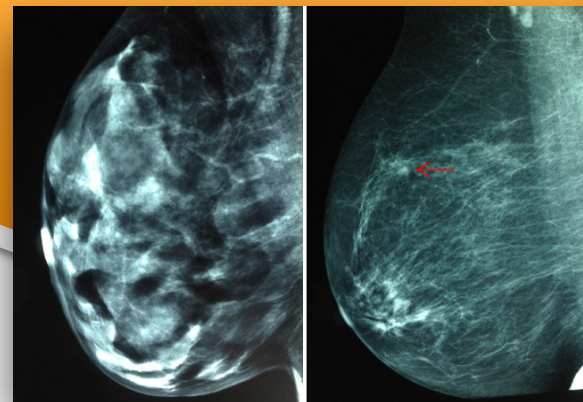
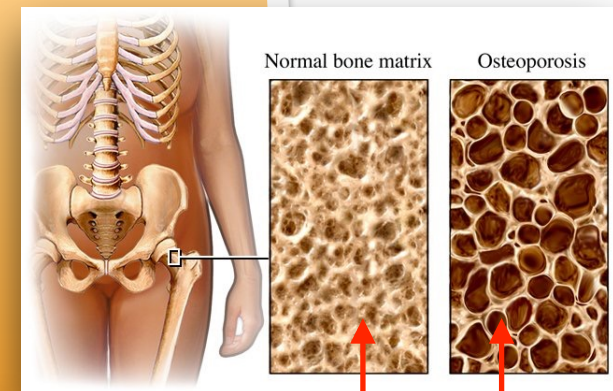


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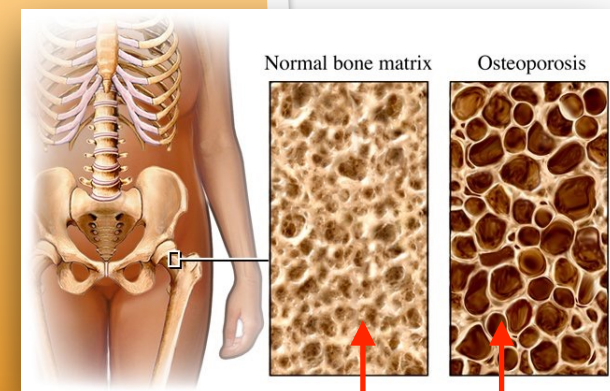
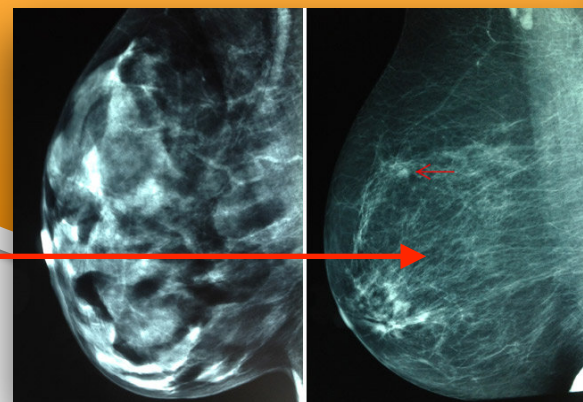
Randomness  
Self-similarity



Mathematical  
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Orientation  
and anisotropy



Roughness  
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# The basic component :

## Fractional Brownian Field (FBF)

■  $B^H$  FBF with Hurst index  $0 < H < 1$  [Mandelbrot, Van Ness, 1968]

■ **stationary increments** :  $B^H(\cdot + \mathbf{z}) - B^H(\mathbf{z}) \stackrel{\mathcal{L}}{=} B^H(\cdot) - B^H(0)$

■ **self-similar** :  $B^H(\lambda \cdot) \stackrel{\mathcal{L}}{=} \lambda^H B^H(\cdot)$

■ **isotropic** :  $B^H \circ R_\theta \stackrel{\mathcal{L}}{=} B^H$

■ The covariance is given by

$$\text{Cov}(B^H(\mathbf{x}), B^H(\mathbf{y})) = c_H(\|\mathbf{x}\|^{2H} + \|\mathbf{y}\|^{2H} - \|\mathbf{x} - \mathbf{y}\|^{2H})$$

# The basic component :

## Fractional Brownian Field (FBF)

### ■ Harmonizable representation

[Samorodnitsky, Taqqu, 1997]

$$B^H(\mathbf{x}) = \int_{\mathbb{R}^2} \frac{e^{i\mathbf{x}\cdot\xi} - 1}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

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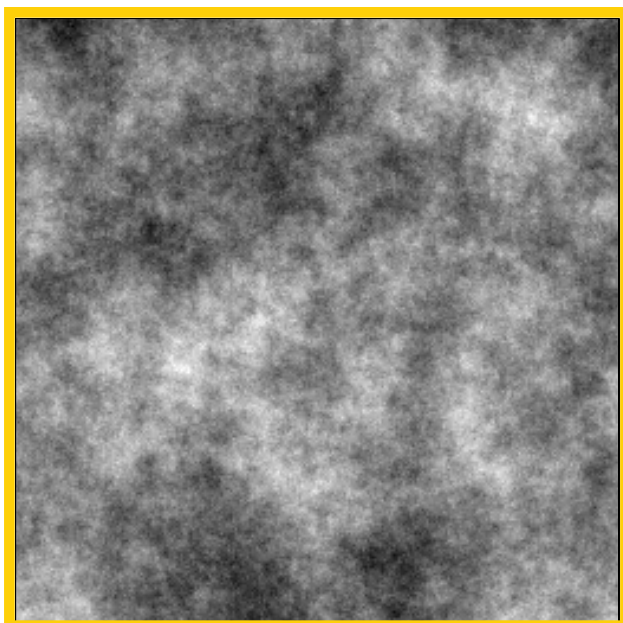
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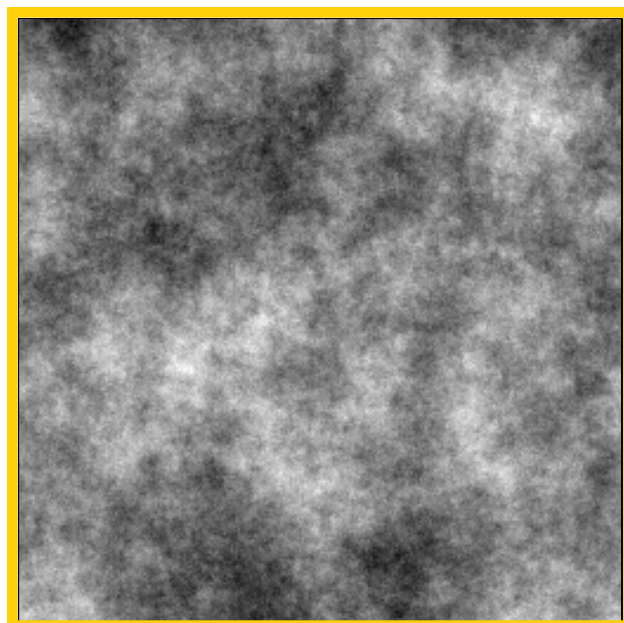
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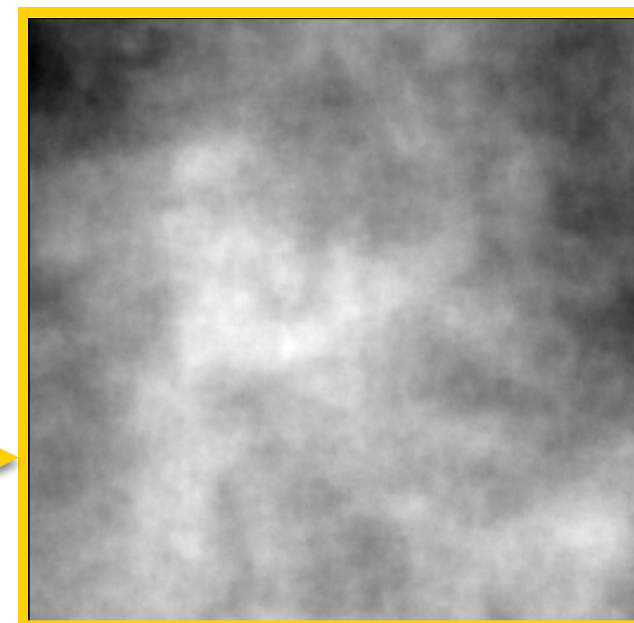
roughness indicator

complex Brownian measure



H=0.2

H=0.7



# General model :

## anisotropic self-similar Gaussian fields

$$X(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) f^{1/2}(\mathbf{x}, \xi) d\widehat{W}(\xi)$$

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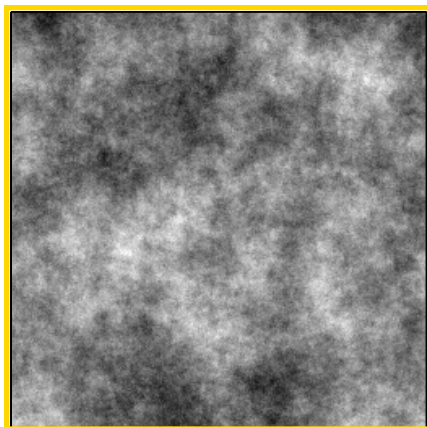
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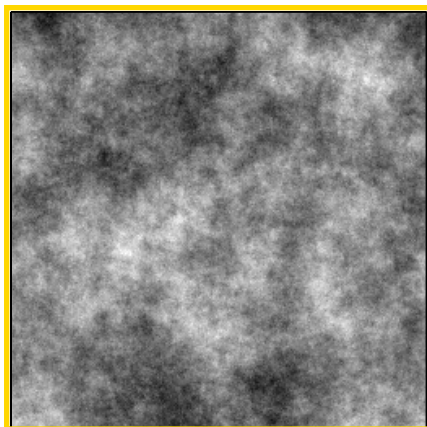
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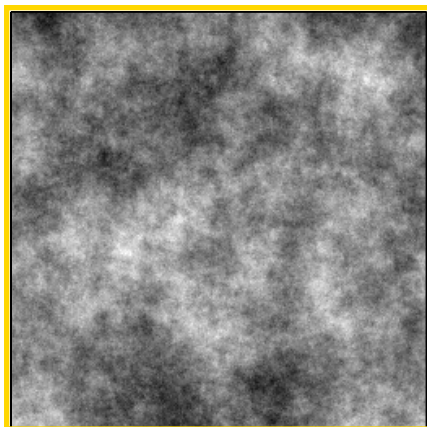
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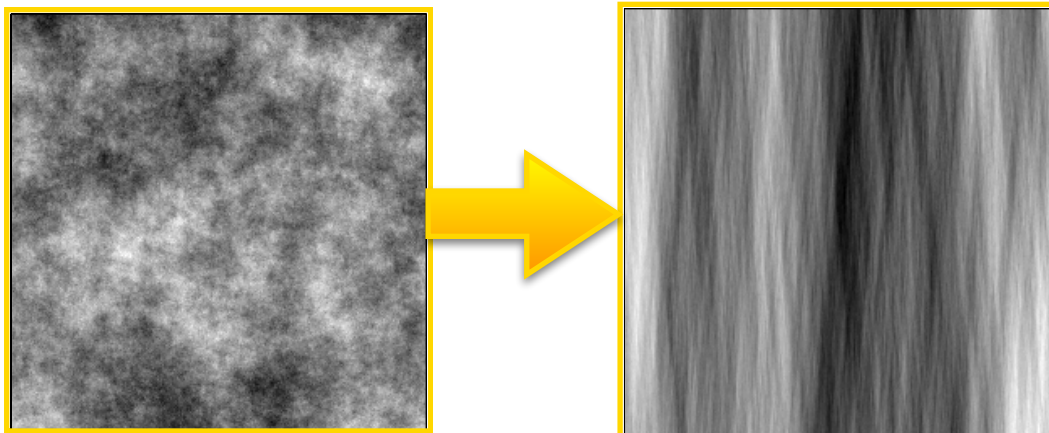
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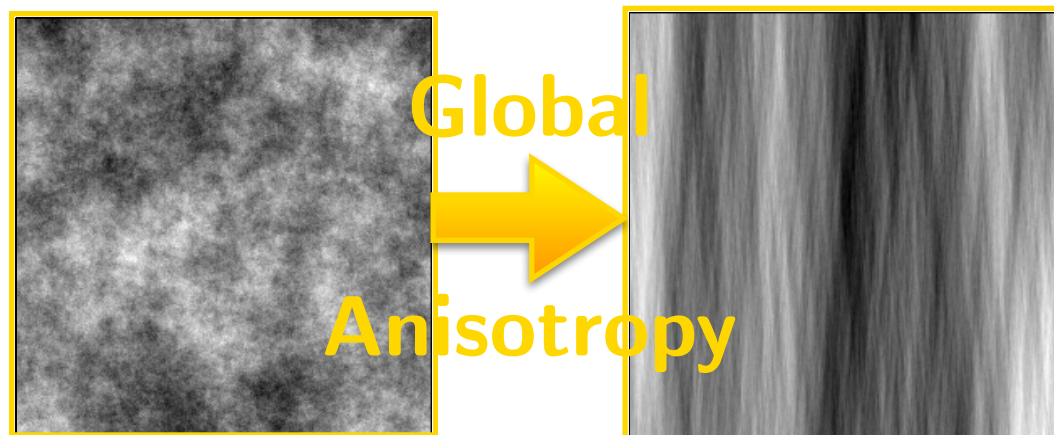
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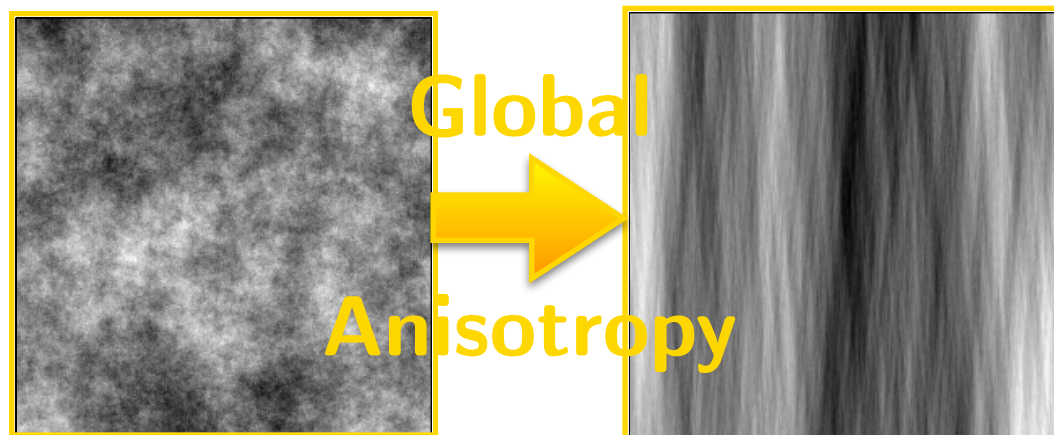
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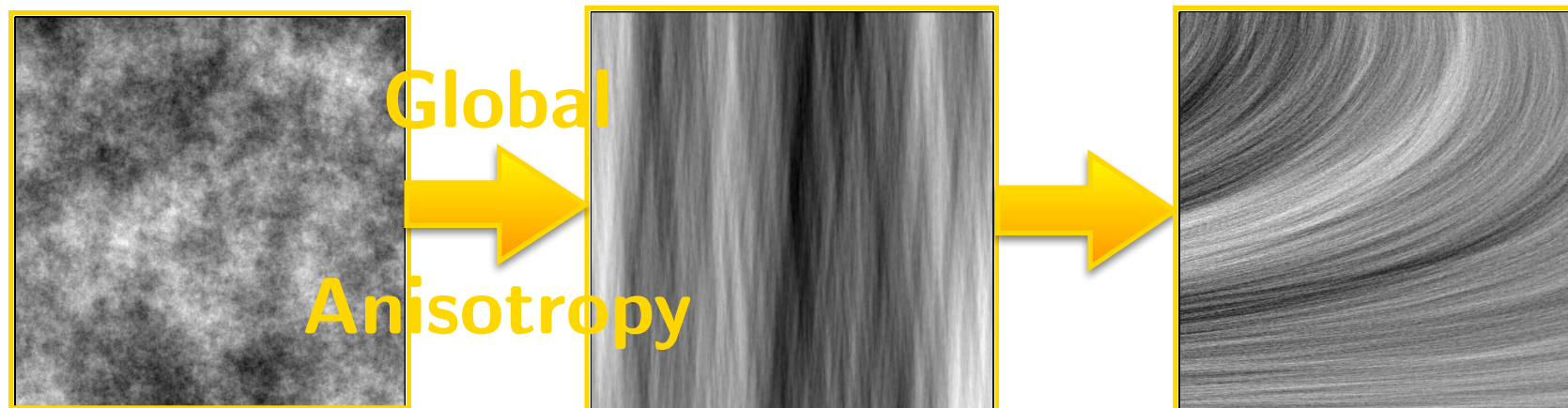
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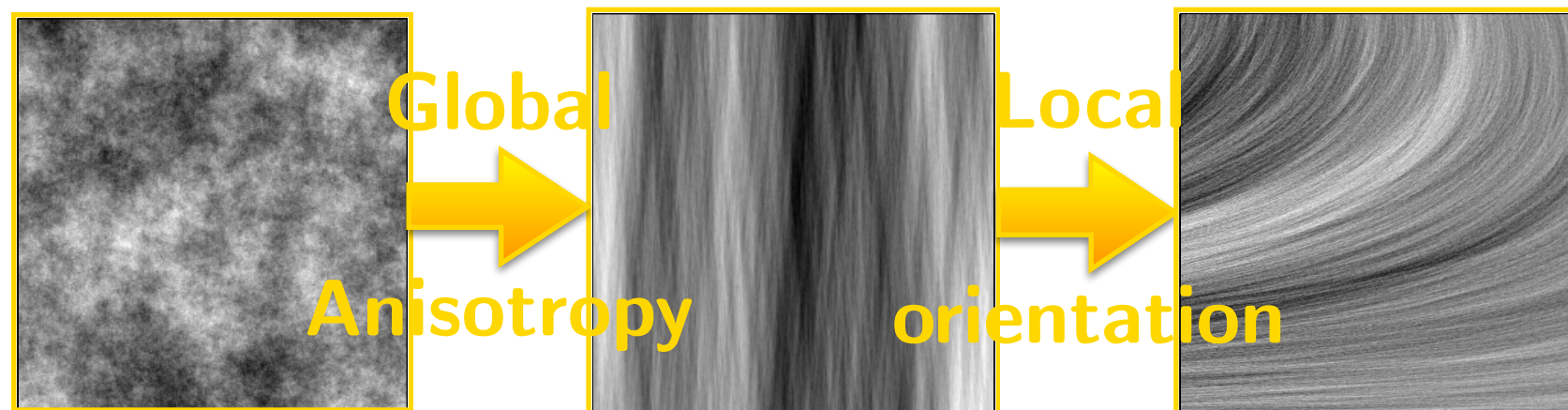
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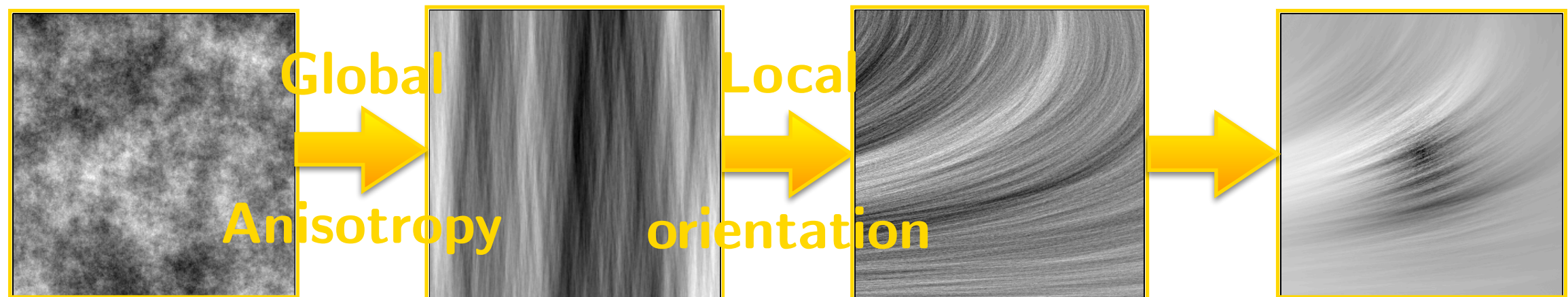
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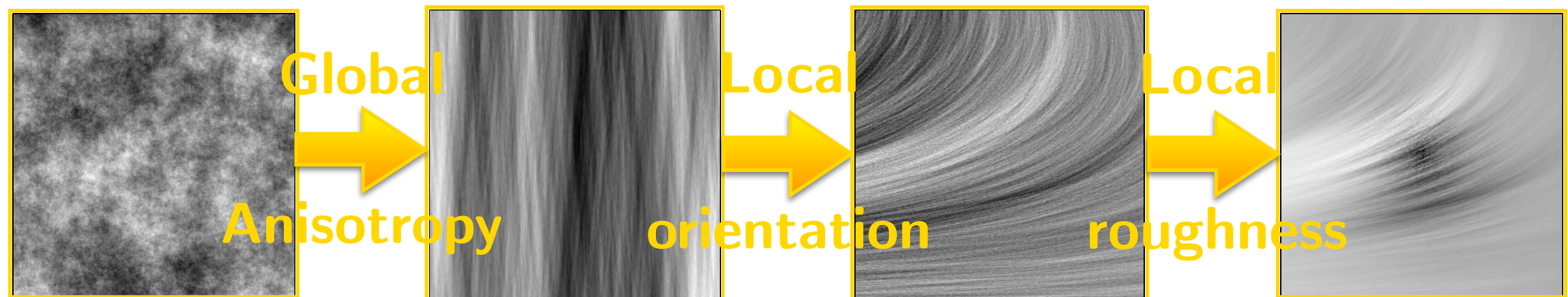
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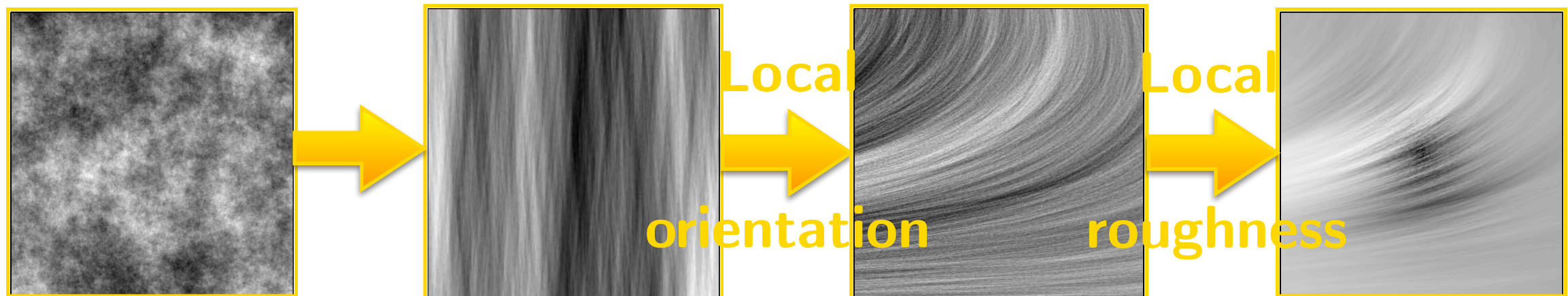
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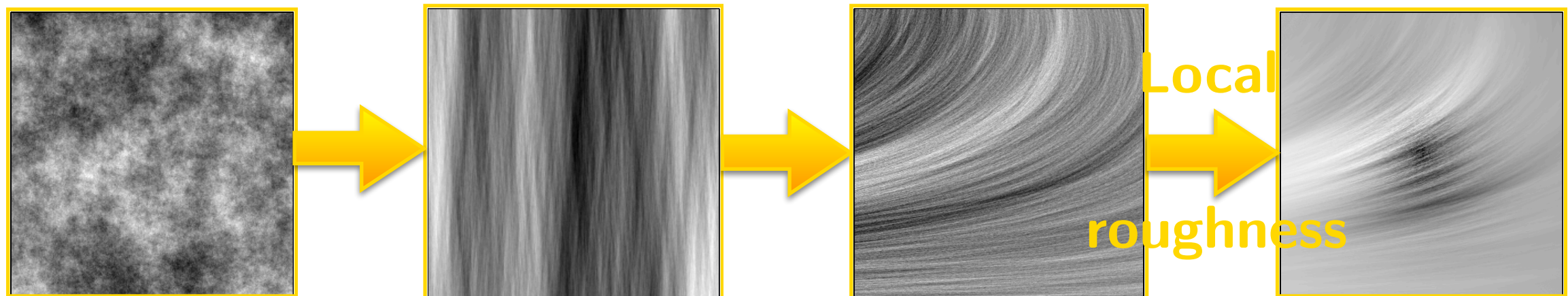
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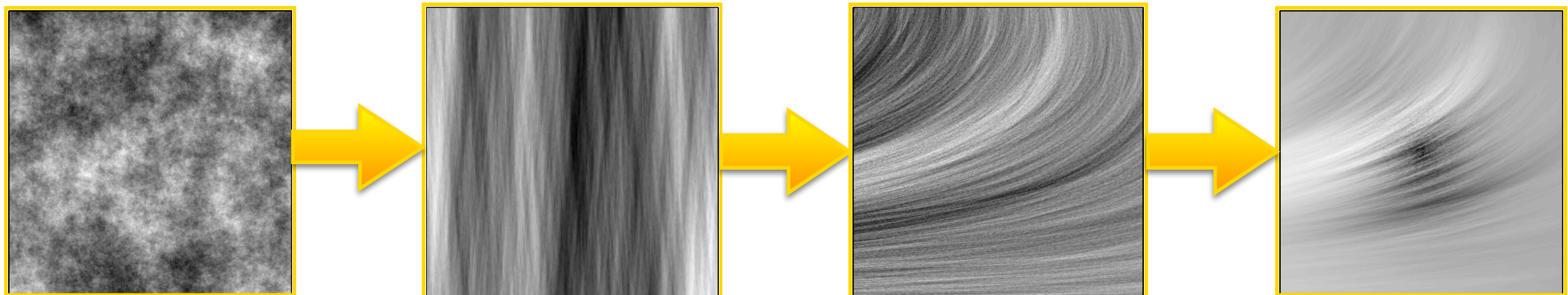
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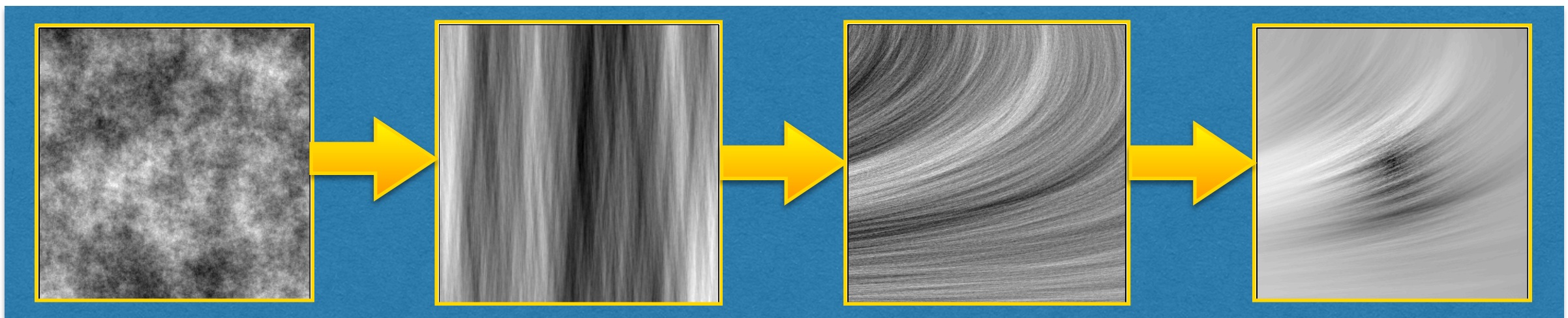
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spectral density

- $c(\mathbf{x}, \xi) \equiv 1$  and  $h(\mathbf{x}, \xi) \equiv H \Rightarrow X = B^H$  [Mandelbrot, Van Ness, 1968]
- $c(\mathbf{x}, \xi) \equiv c(\arg \xi)$  and  $h(\mathbf{x}, \xi) \equiv h(\arg \xi) \Rightarrow X = AFBF$  [Bonami, Estrade, 2003]
- Example : *elementary fields*  $c(\arg \xi) = \mathbb{1}_{[-\alpha, \alpha]}(\arg \xi - \alpha_0)$  [Bierme, Richard, Moisan, 2012]
- $c(\mathbf{x}, \xi) \equiv c(\mathbf{x}, \arg \xi)$  and  $h(\mathbf{x}, \xi) \equiv h(\mathbf{x})$  [Polisano, Clausel, Perrier, Condat, 2014]



# Locally Anisotropic Fractional Brownian Field (LAFBF)

- **Definition:** Our new Gaussian model LAFBF is a local version of the elementary field

$$B_{\alpha_0, \alpha}^H(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x} \cdot \boldsymbol{\xi}} - 1) \frac{\mathbb{1}_{[-\alpha, \alpha]}(\arg \boldsymbol{\xi} - \alpha_0(\mathbf{x}))}{\|\boldsymbol{\xi}\|^{H+1}} d\widehat{W}(\boldsymbol{\xi})$$

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The orientation may vary spatially.  $\alpha_0$  is now a differentiable function on  $\mathbb{R}^2$

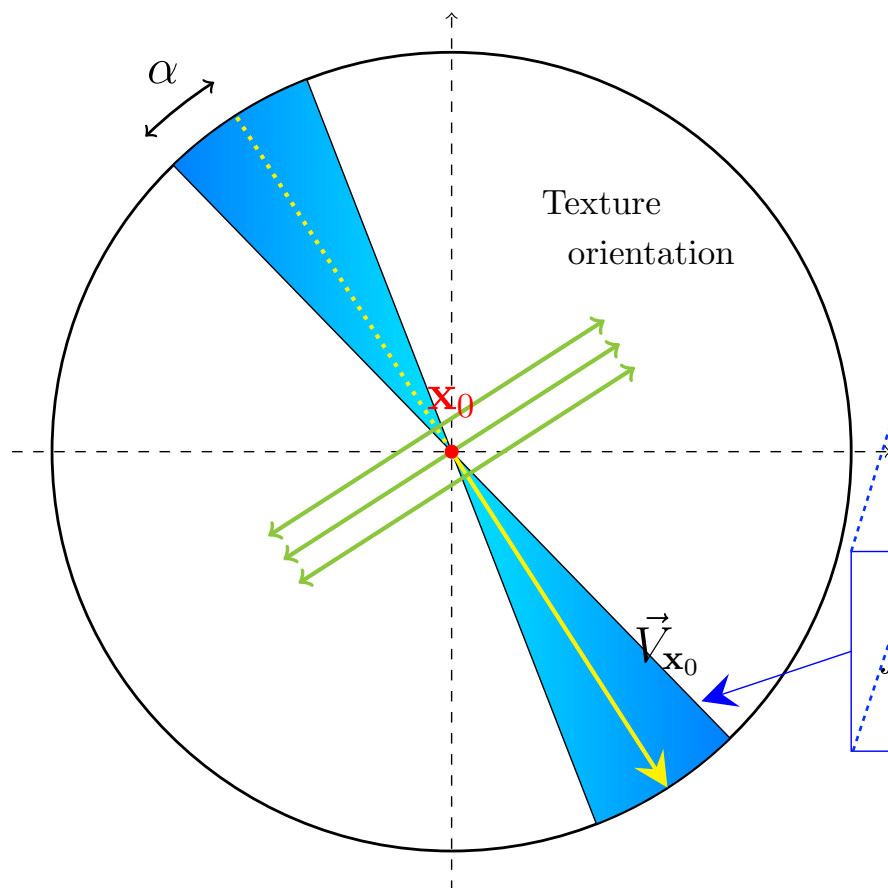


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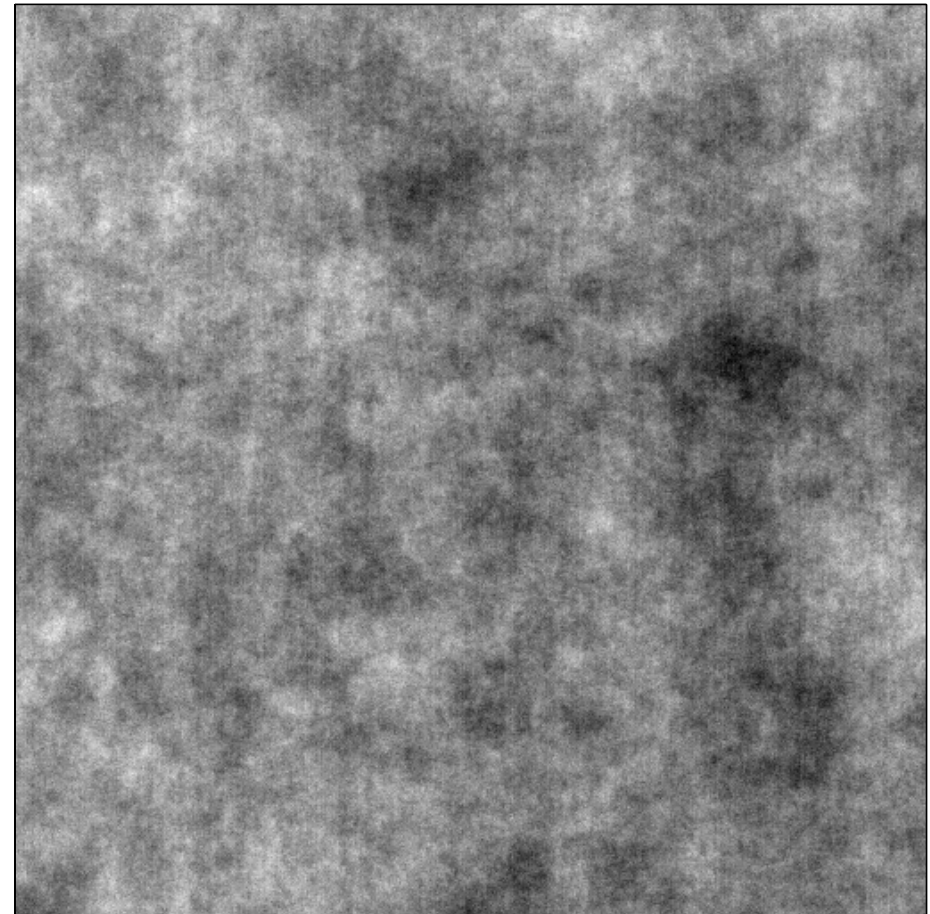
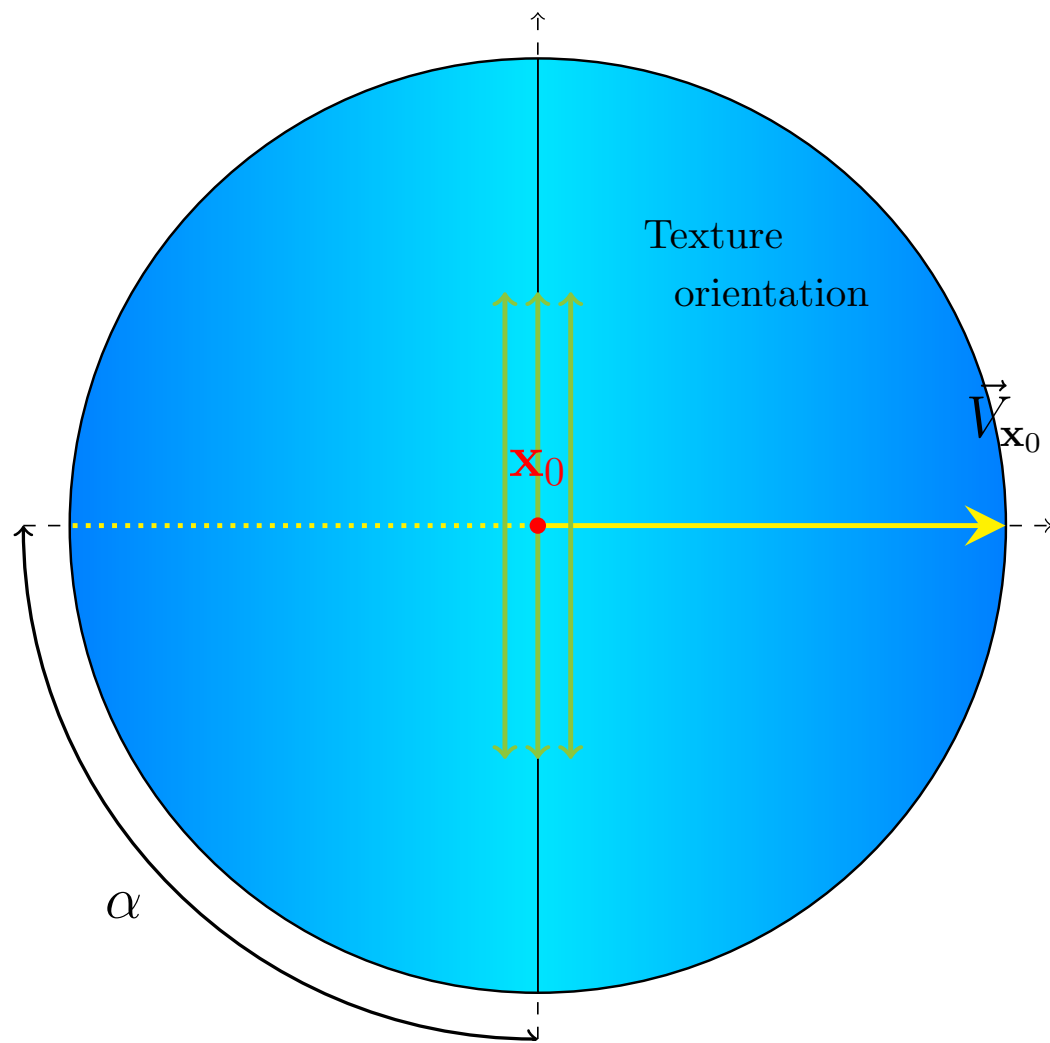


$$f^{1/2}(\mathbf{x}_0, \boldsymbol{\xi}) = \frac{c_{\alpha_0, \alpha}(\mathbf{x}_0, \arg \boldsymbol{\xi})}{\|\boldsymbol{\xi}\|^{H+1}}$$

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# Elementary field

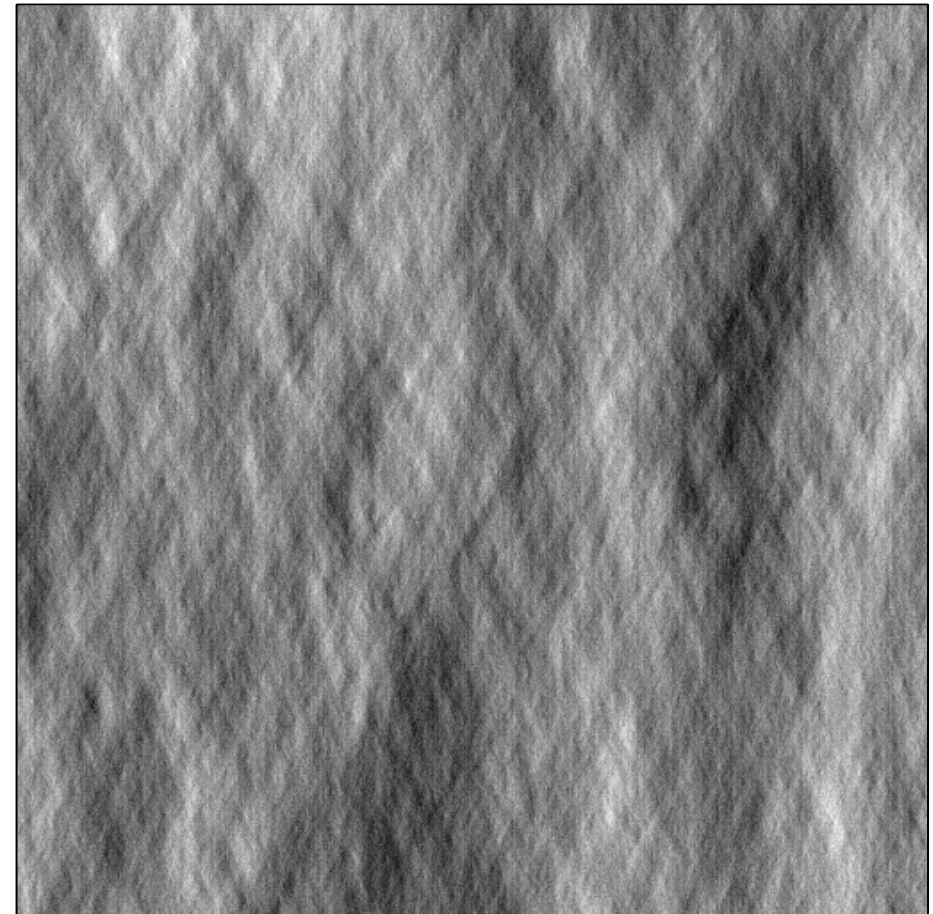
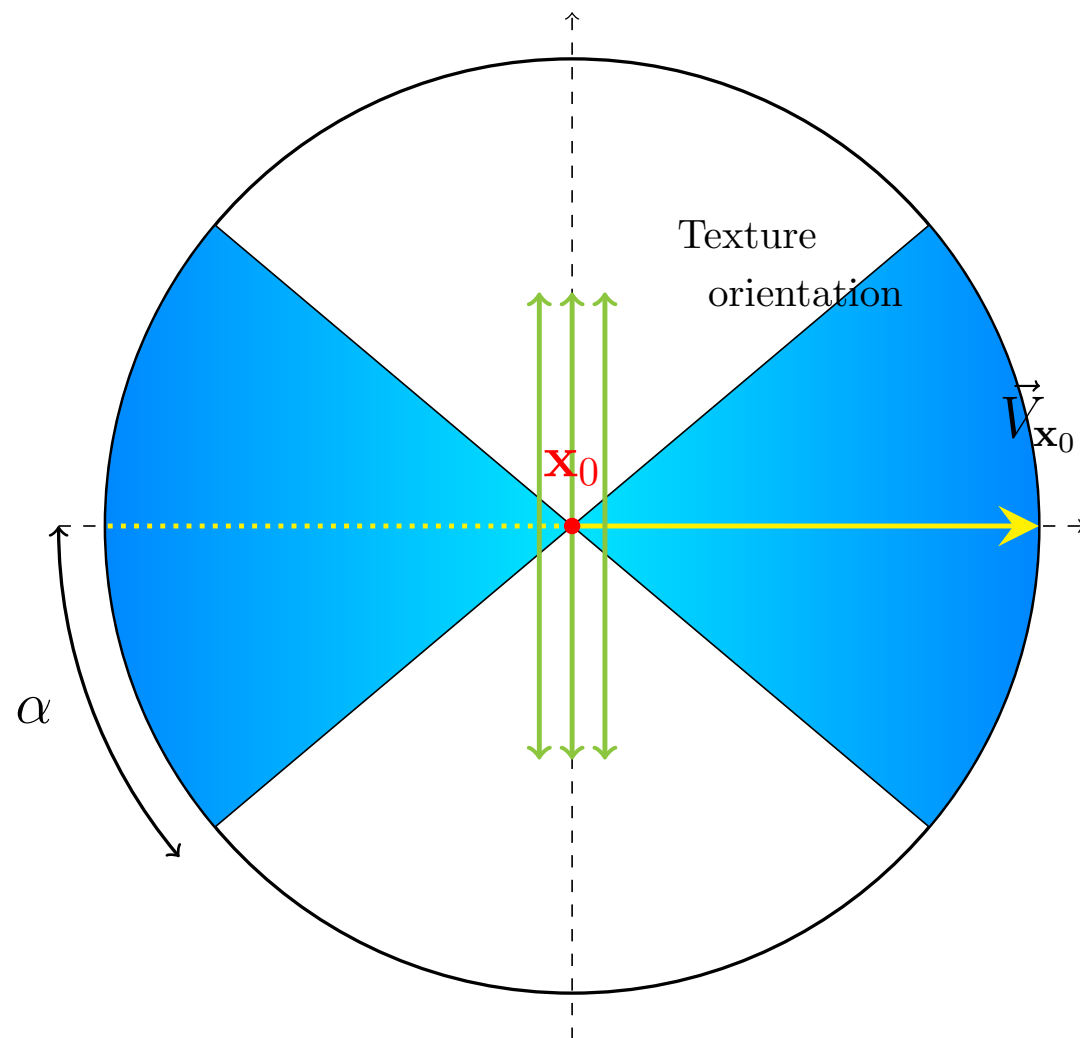
$$\alpha_0 = 0$$



$$\alpha = -\frac{\pi}{2}$$

# Elementary field

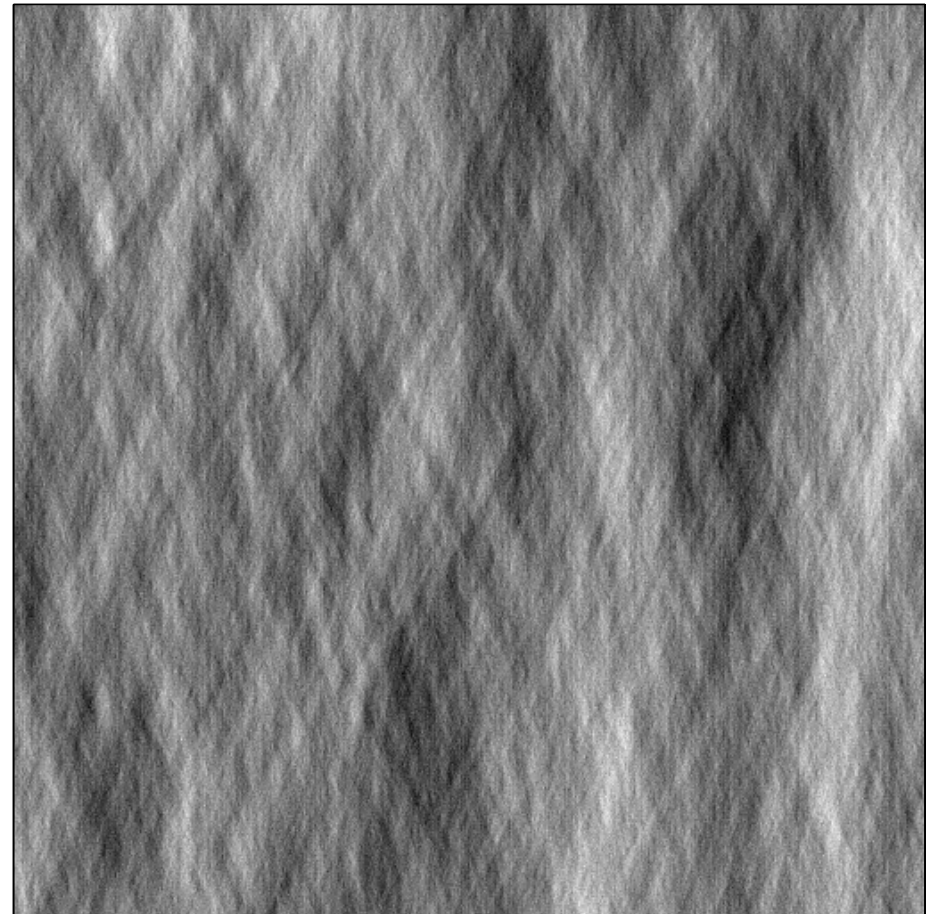
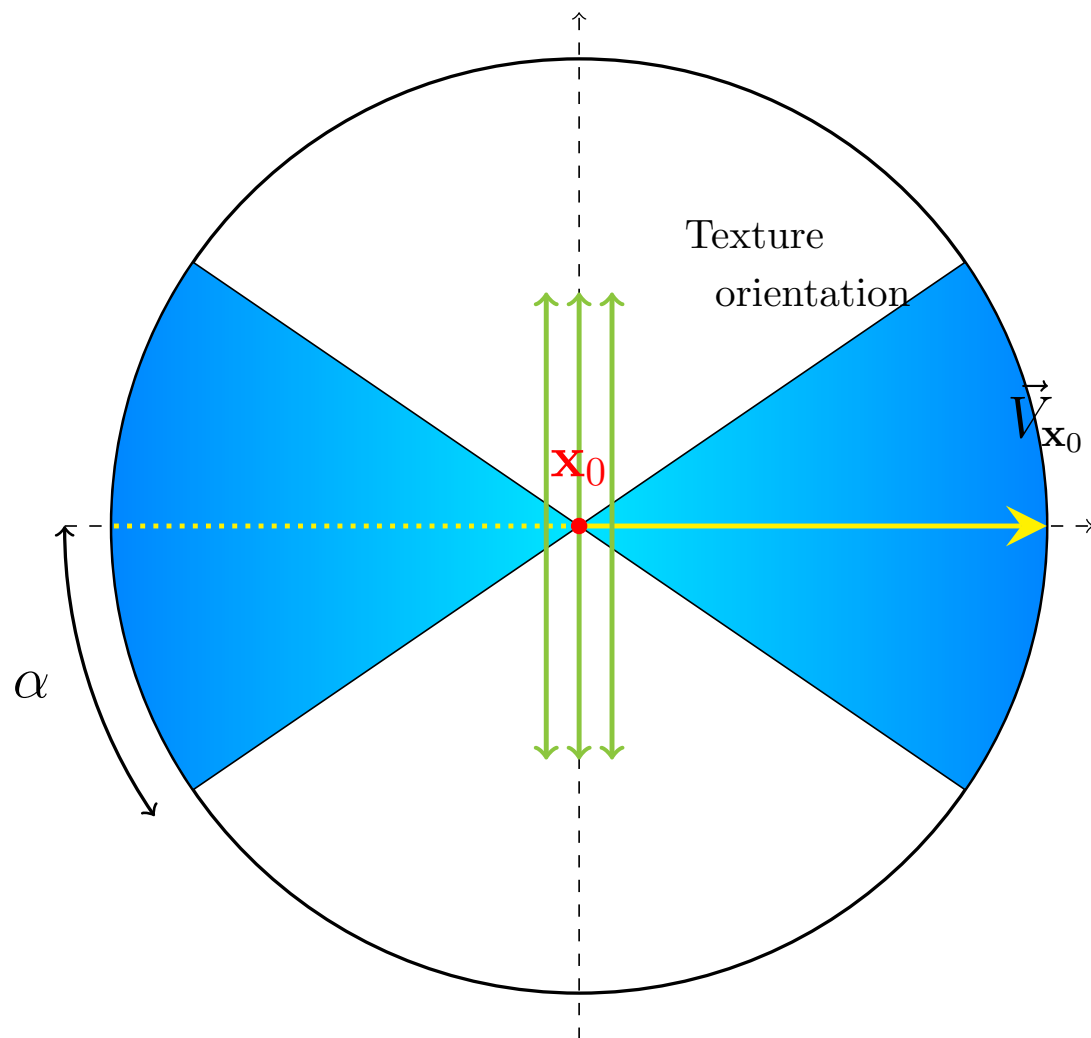
$$\alpha_0 = 0$$



$$\alpha = 0.7$$

# Elementary field

$$\alpha_0 = 0$$

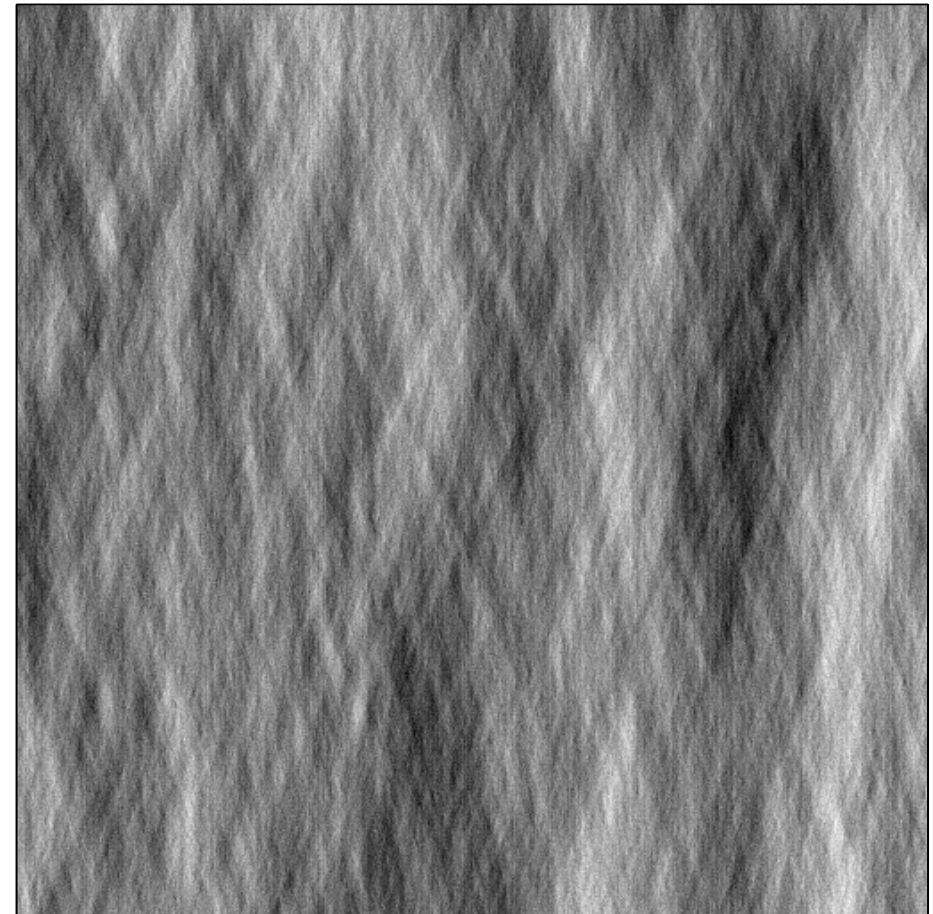
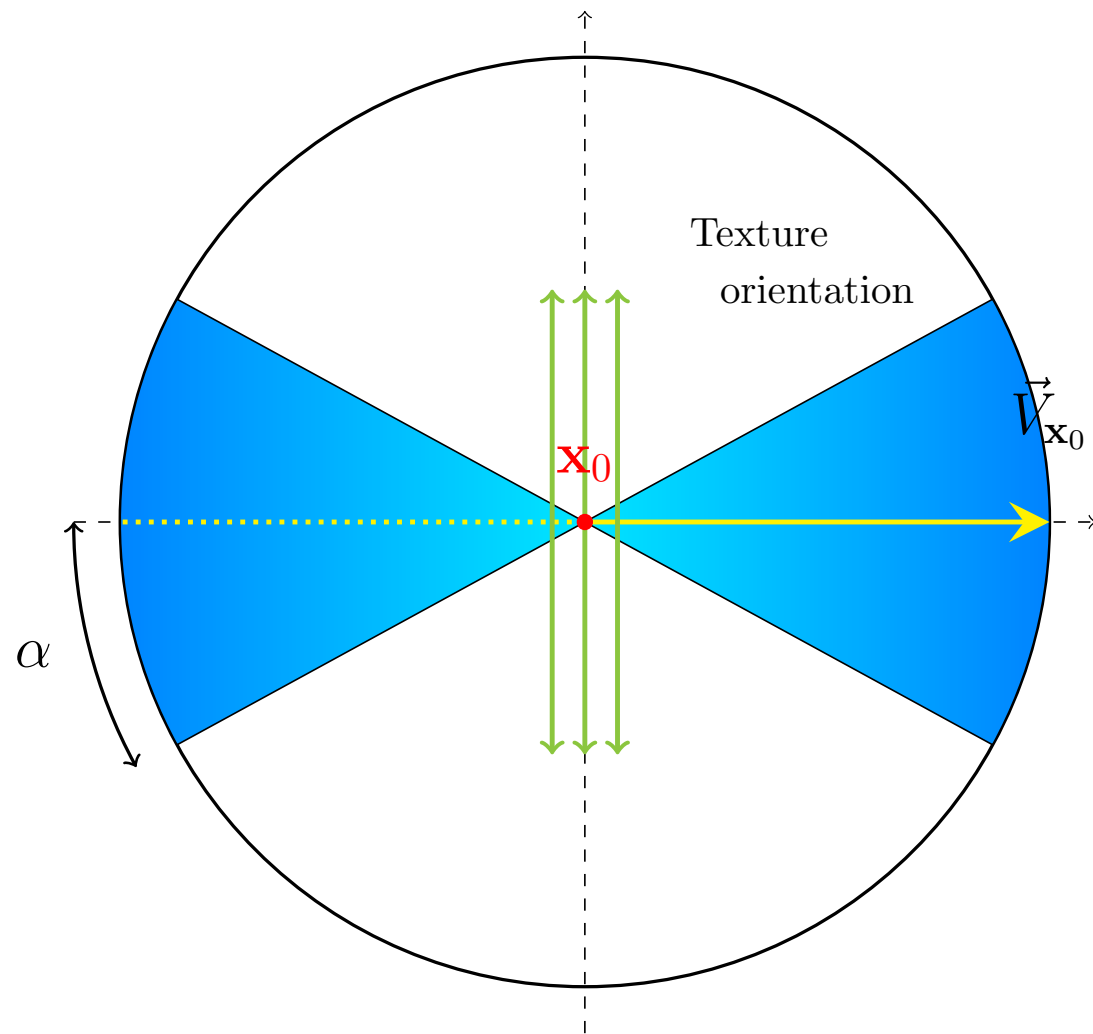


$$\alpha = 0.6$$



# Elementary field

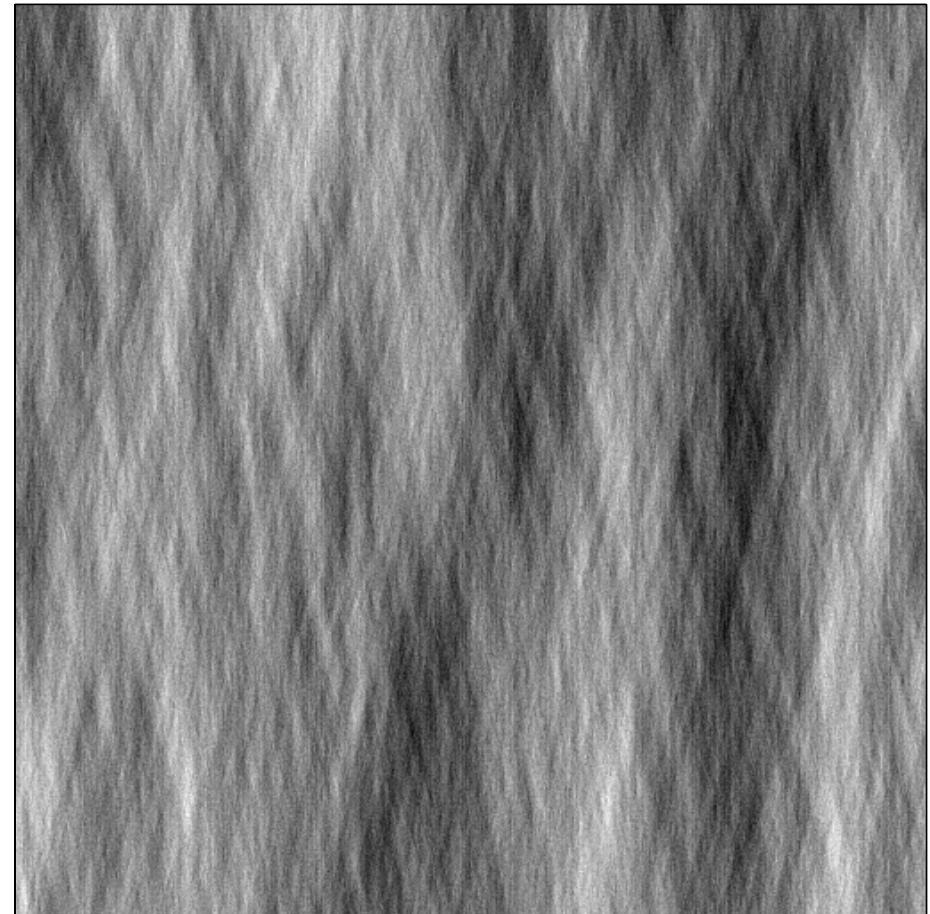
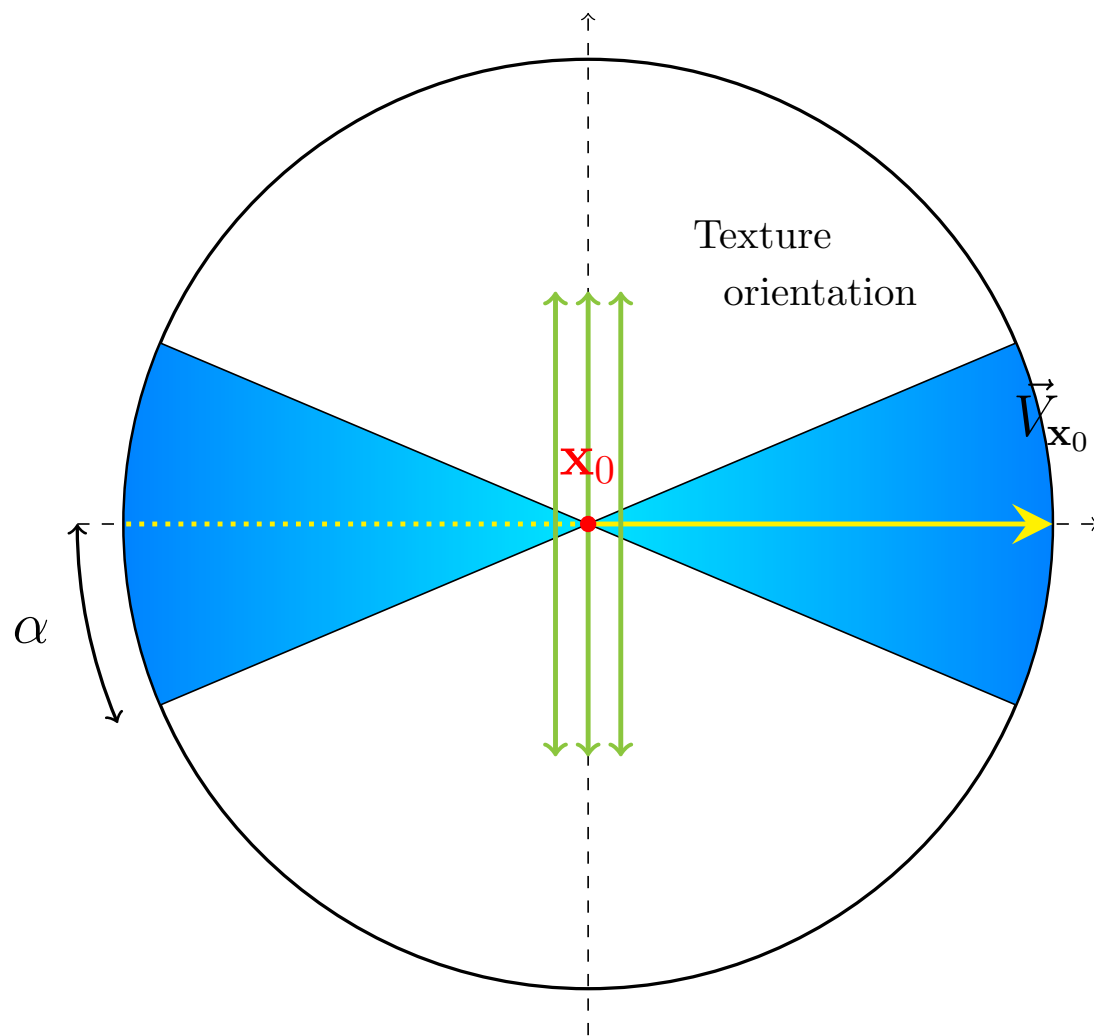
$$\alpha_0 = 0$$



$$\alpha = 0.5$$

# Elementary field

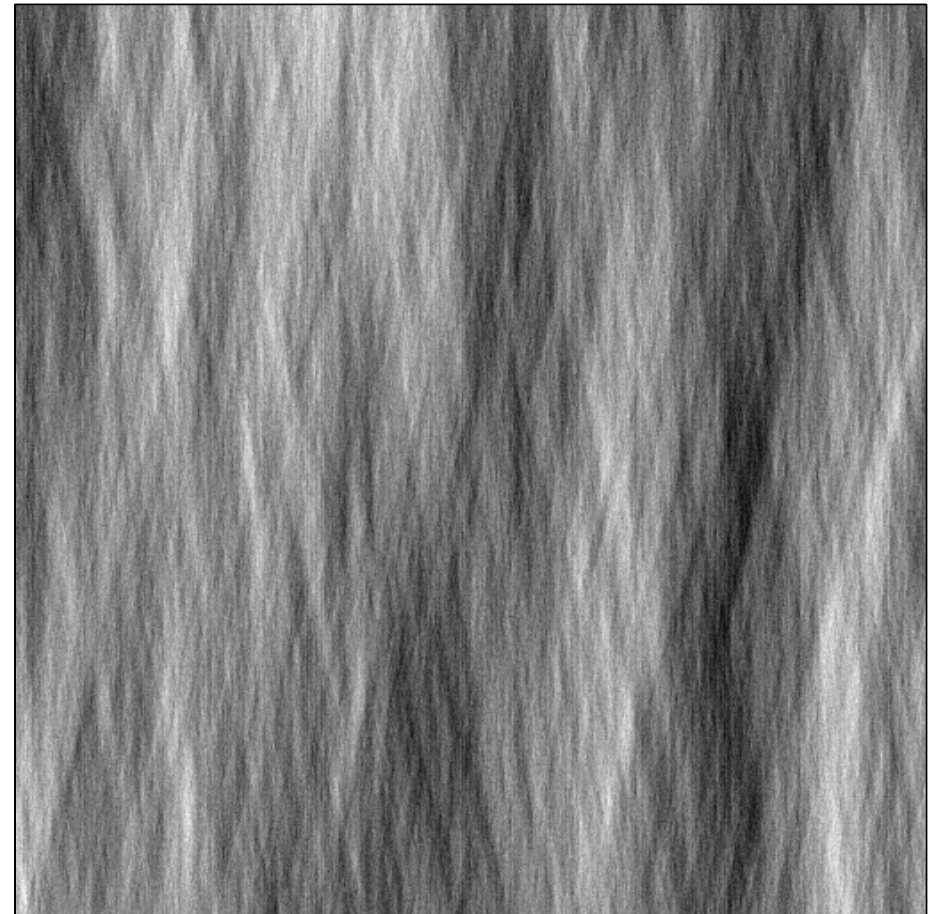
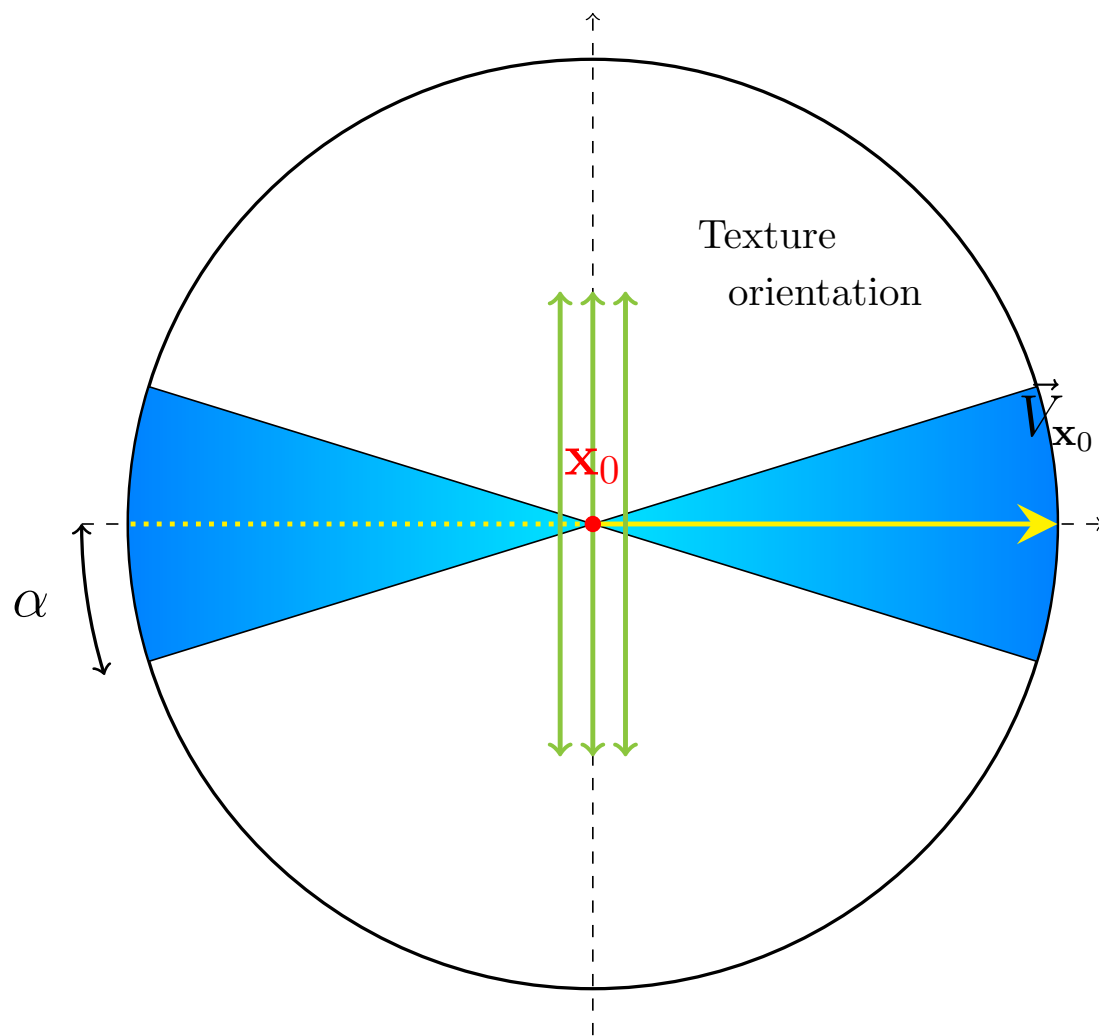
$$\alpha_0 = 0$$



$$\alpha = 0.4$$

# Elementary field

$$\alpha_0 = 0$$

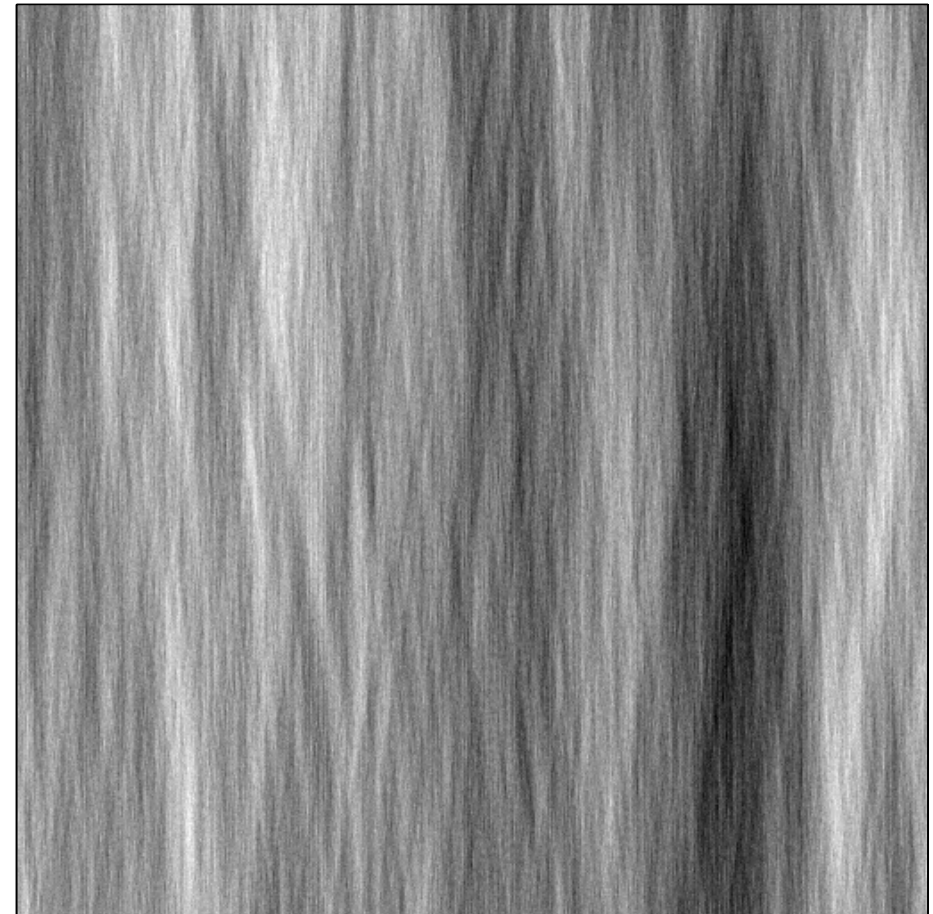
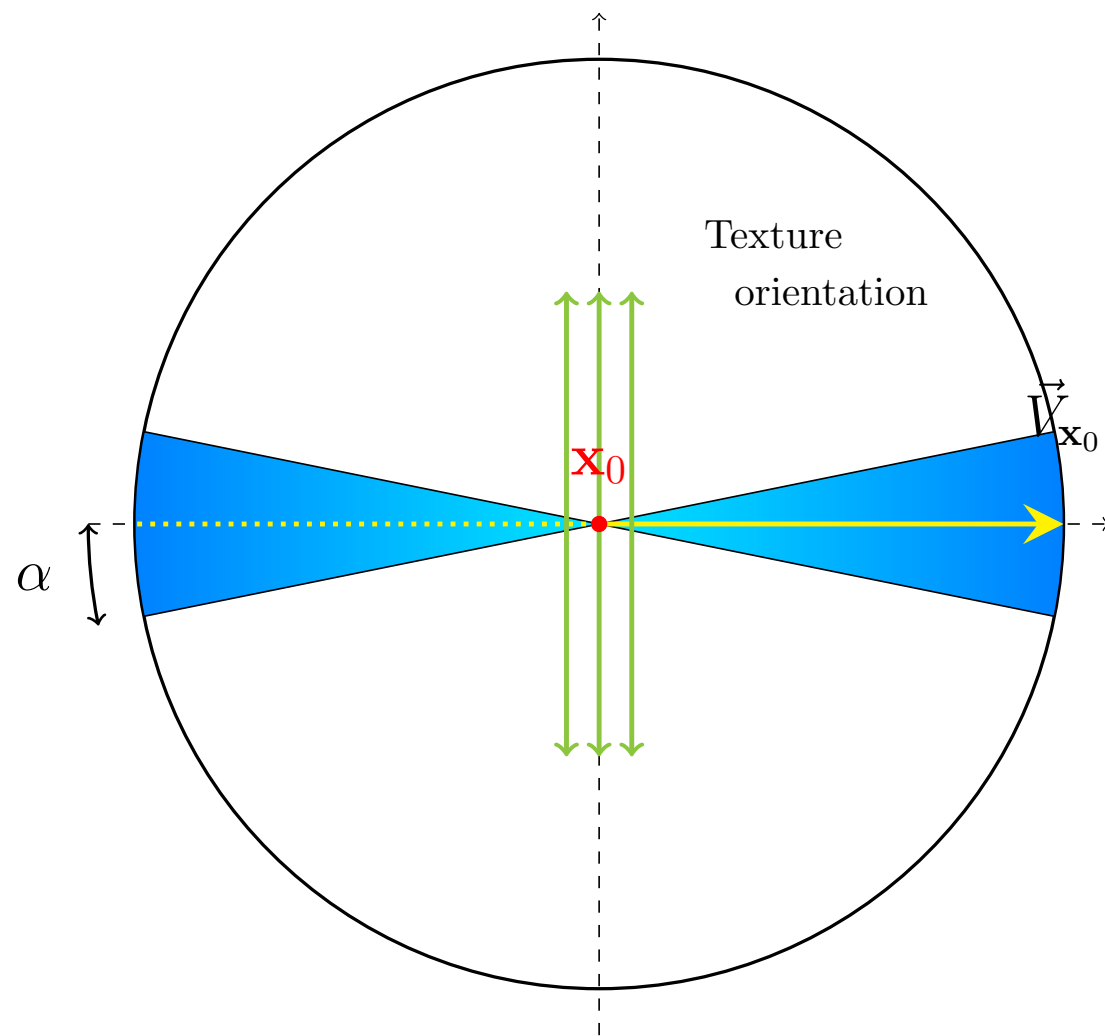


$$\alpha = 0.3$$



# Elementary field

$$\alpha_0 = 0$$

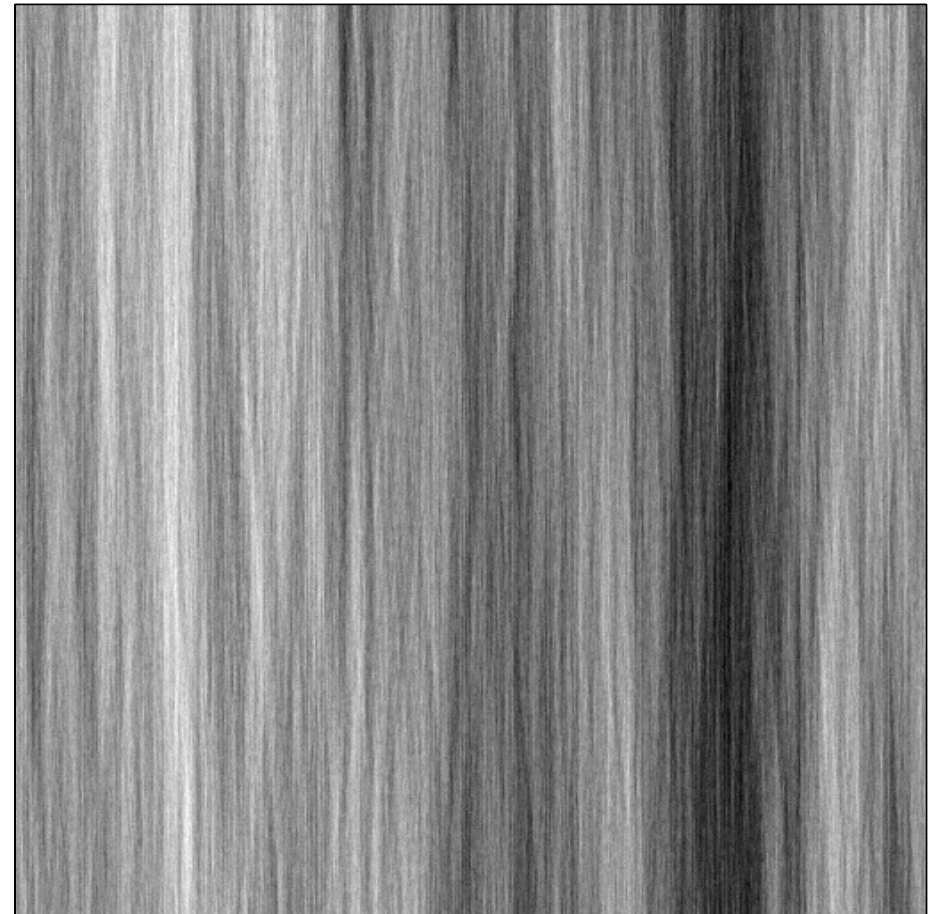
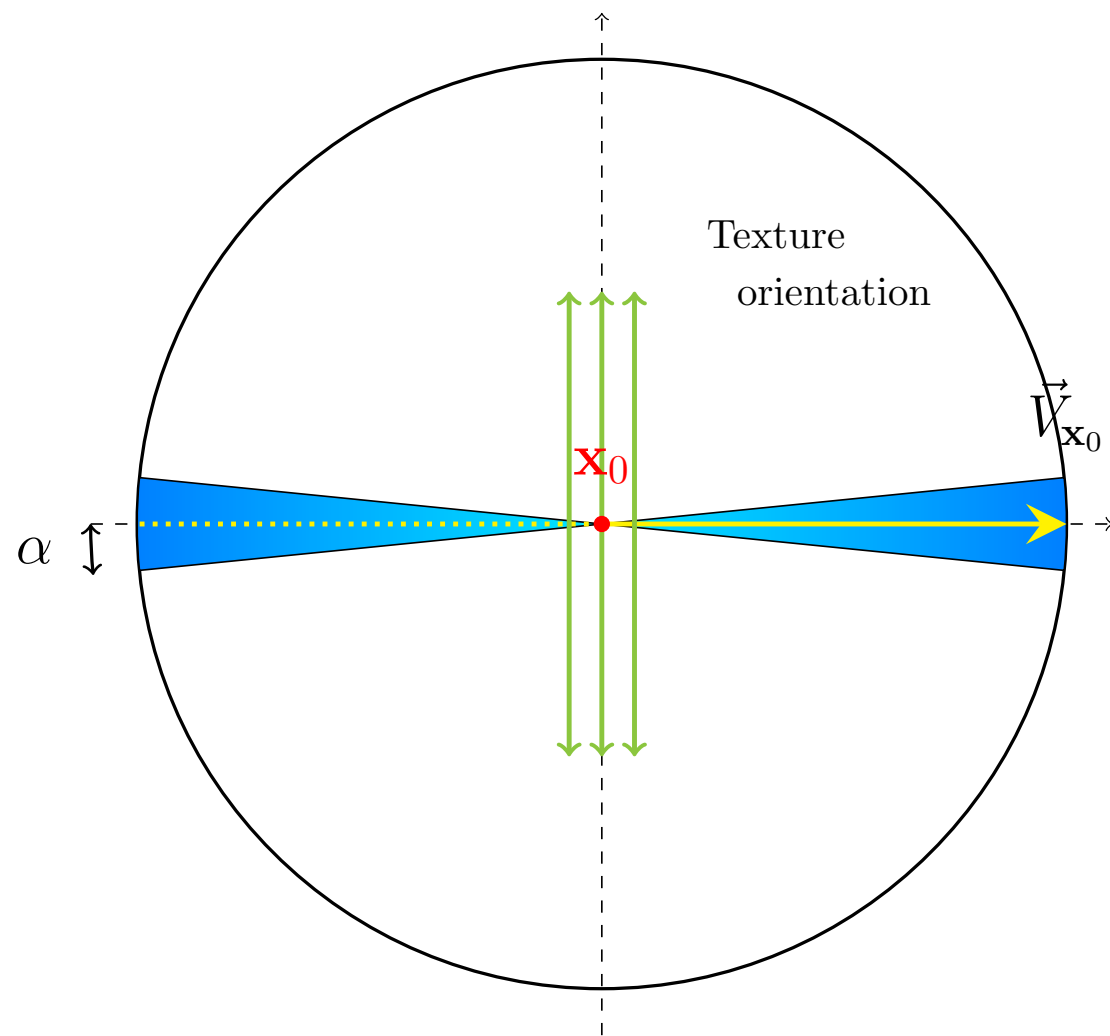


$$\alpha = 0.2$$



# Elementary field

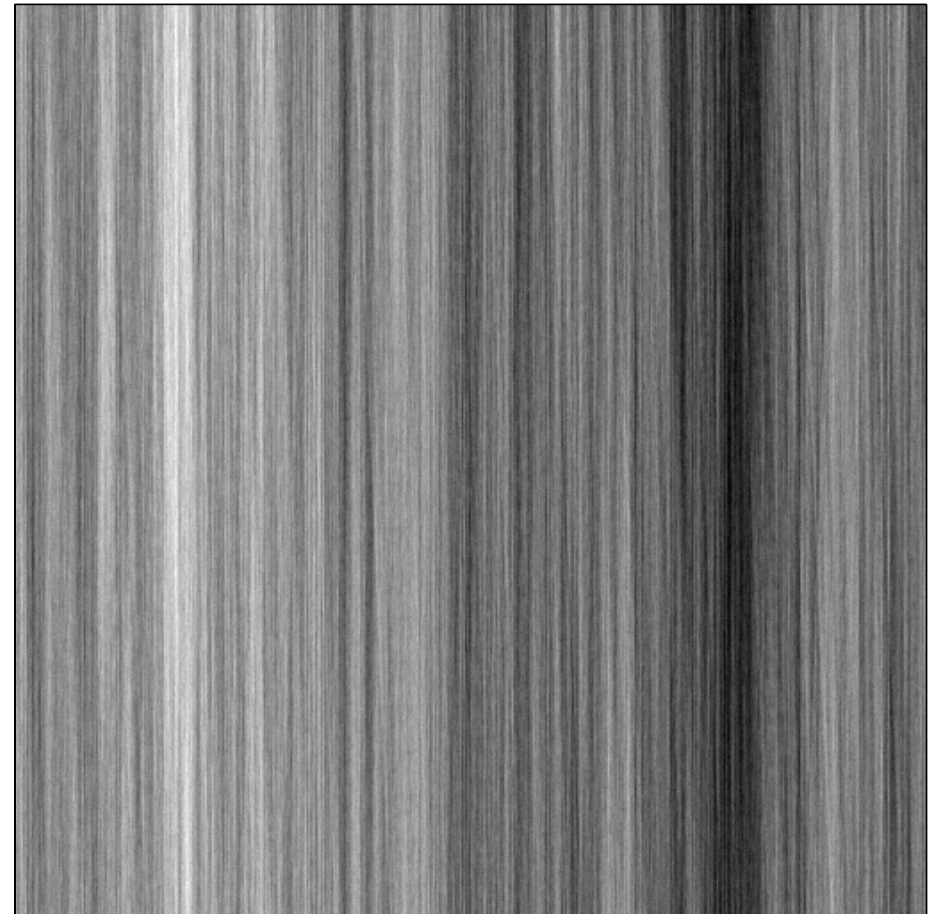
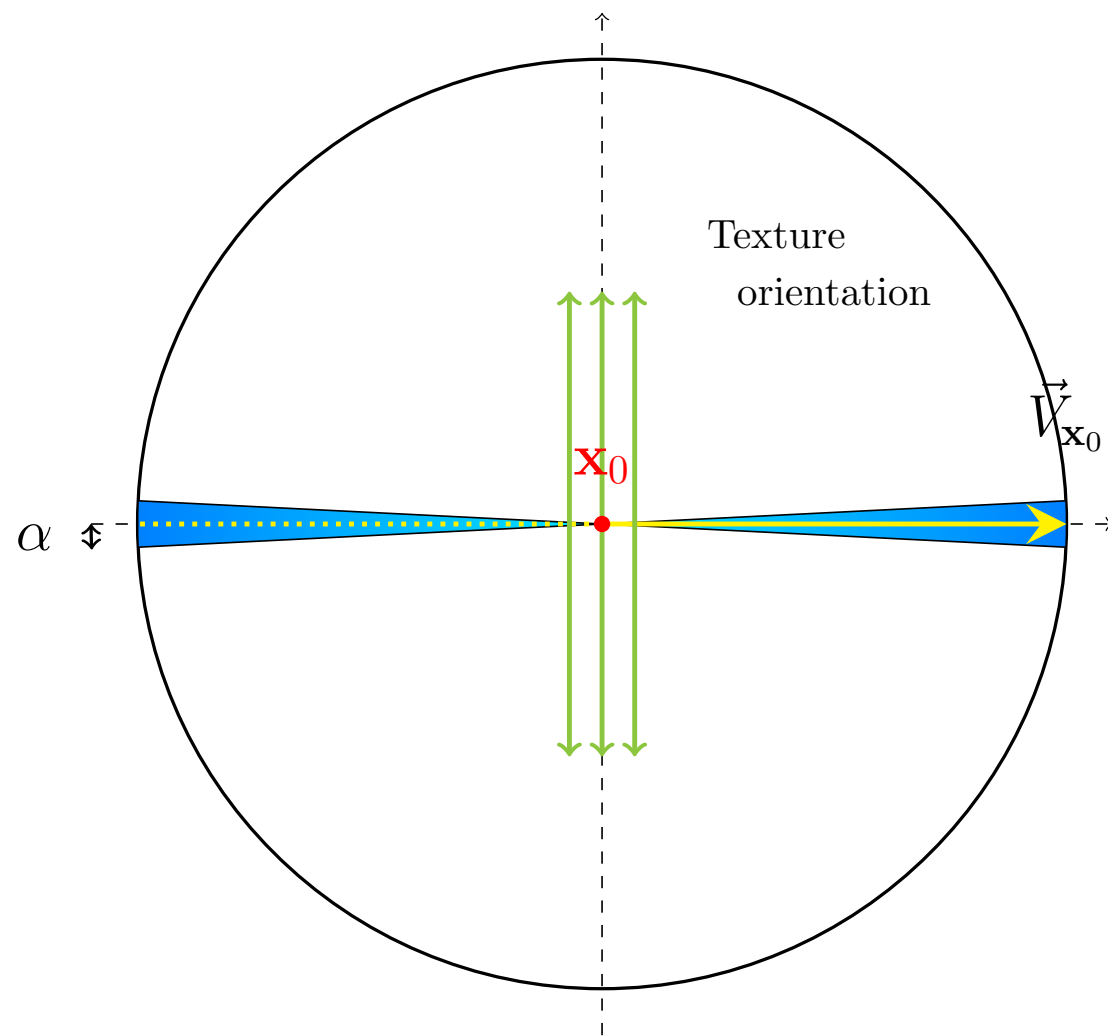
$$\alpha_0 = 0$$



$$\alpha = 0.1$$

# Elementary field

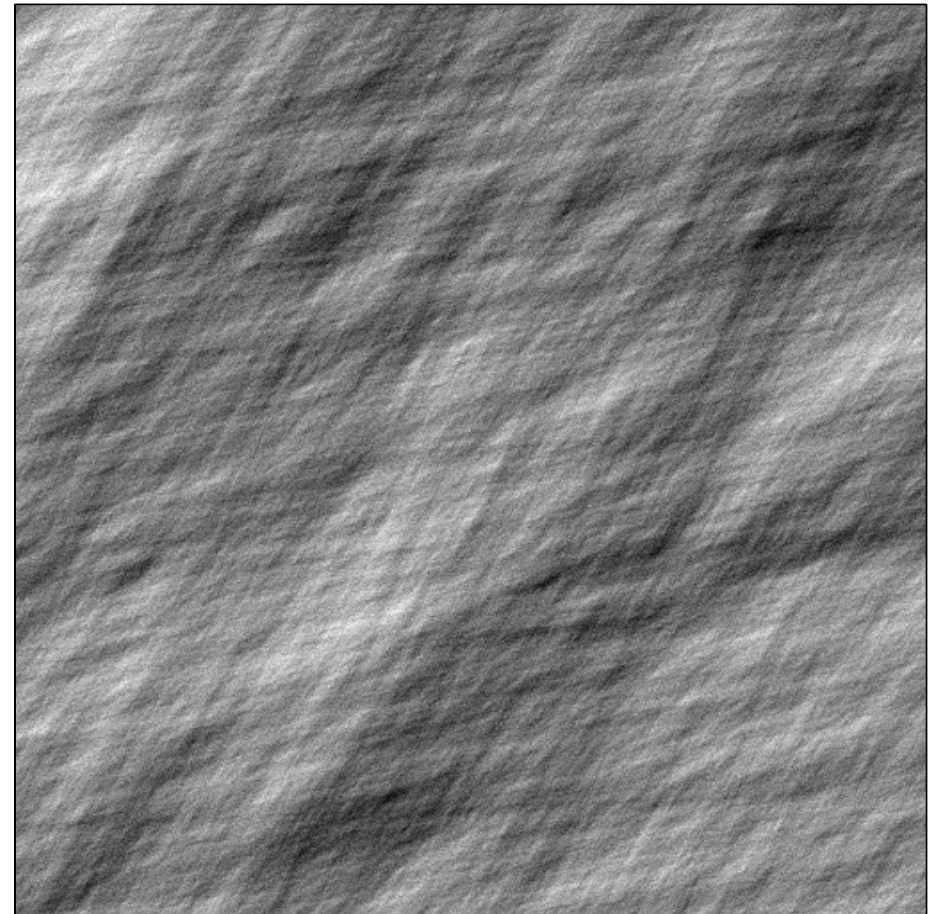
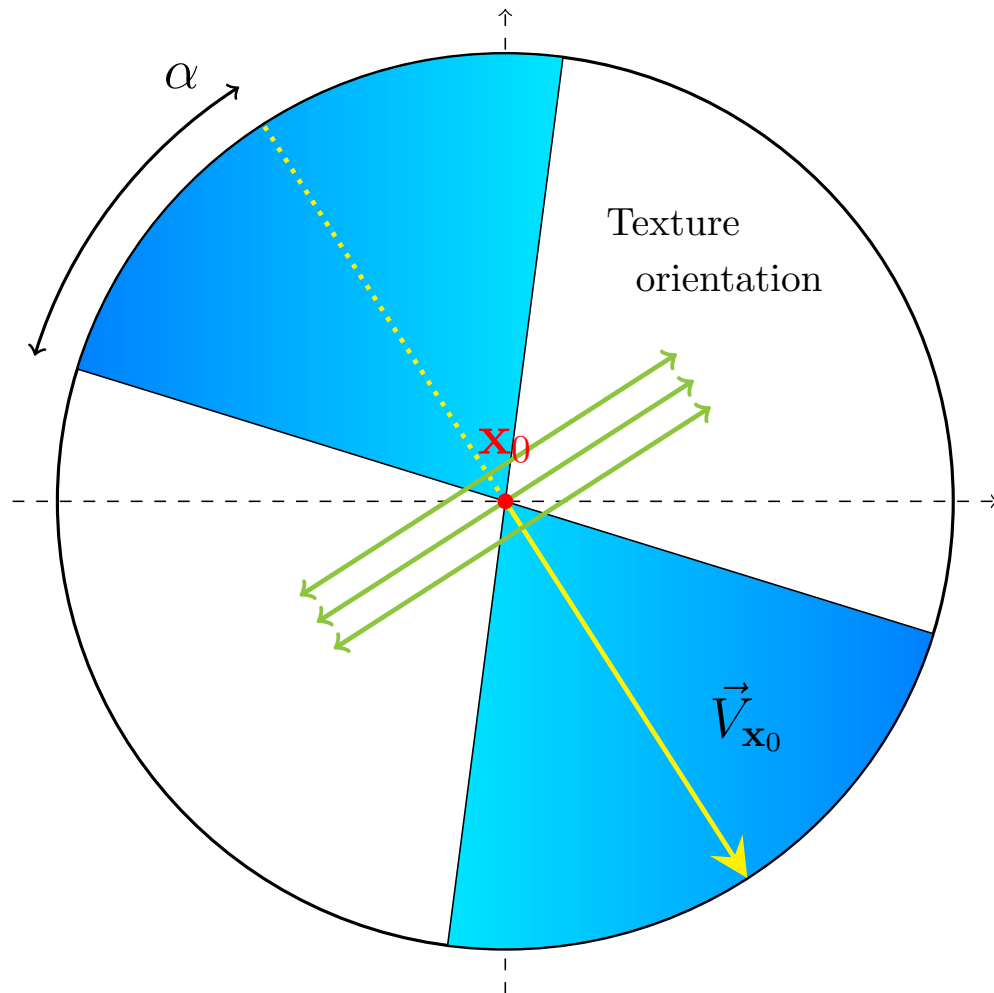
$$\alpha_0 = 0$$



$$\alpha = 0.05$$

# Elementary field

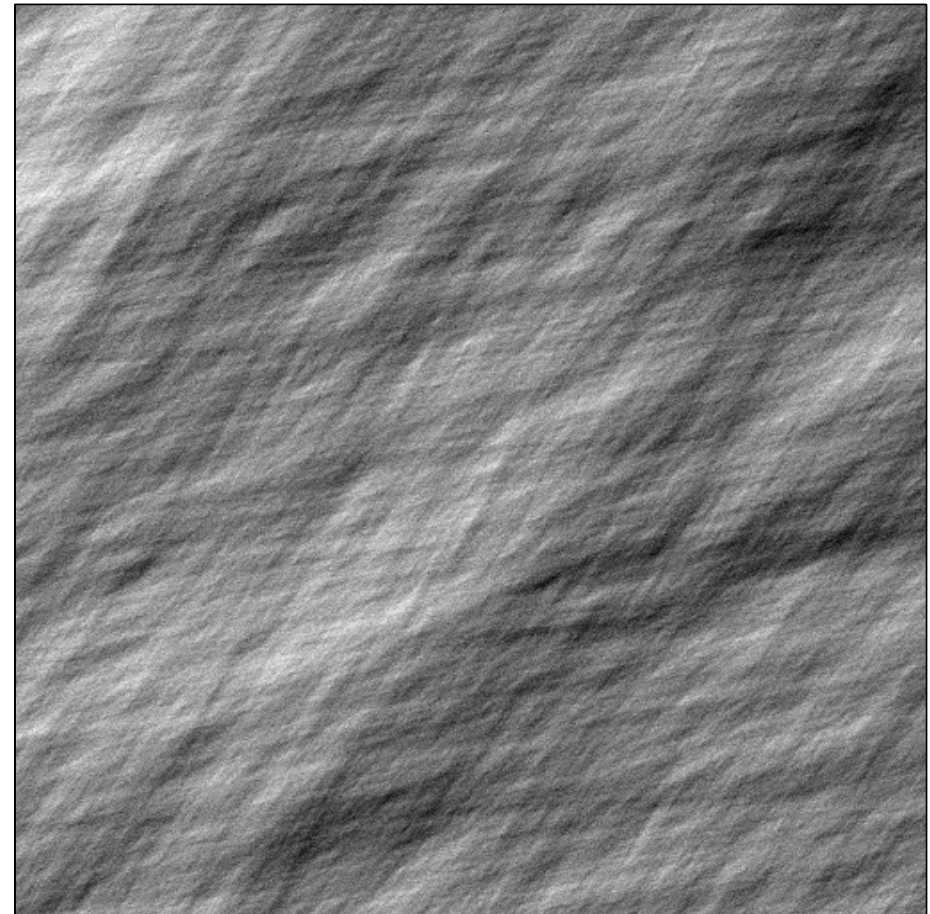
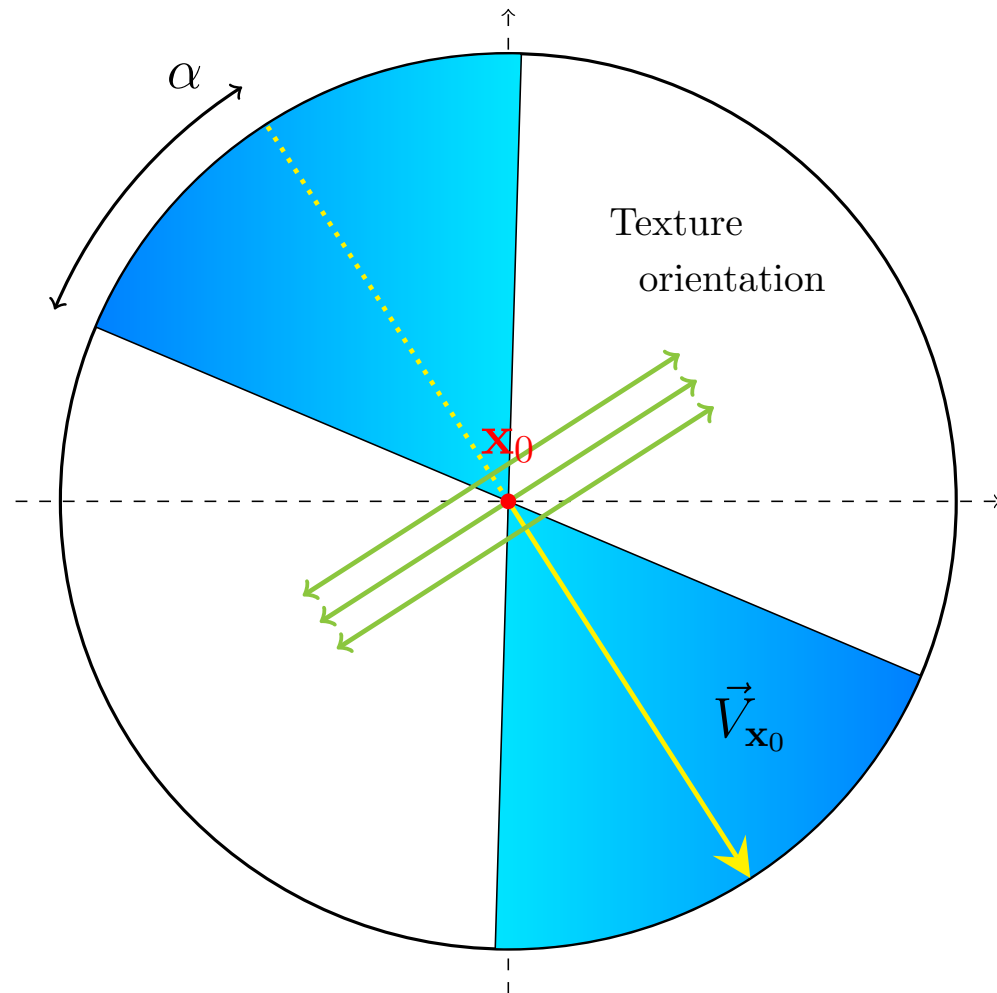
$$\alpha_0 = -\frac{\pi}{3}$$



$$\alpha = 0.7$$

# Elementary field

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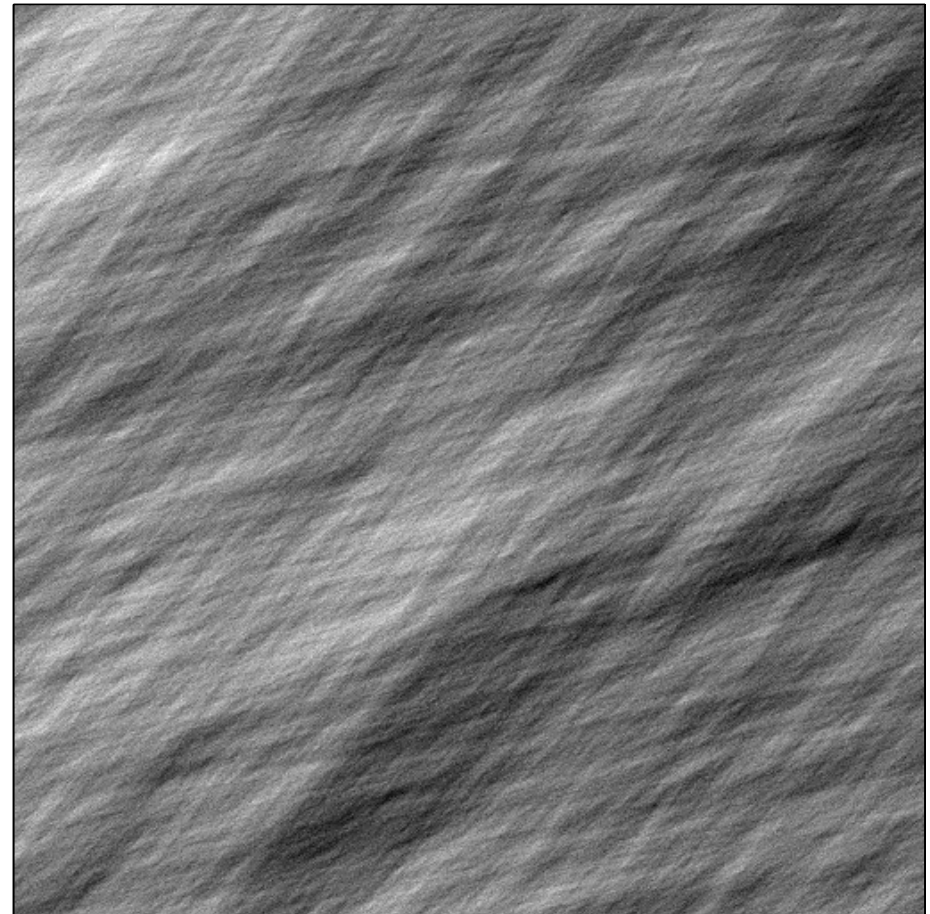
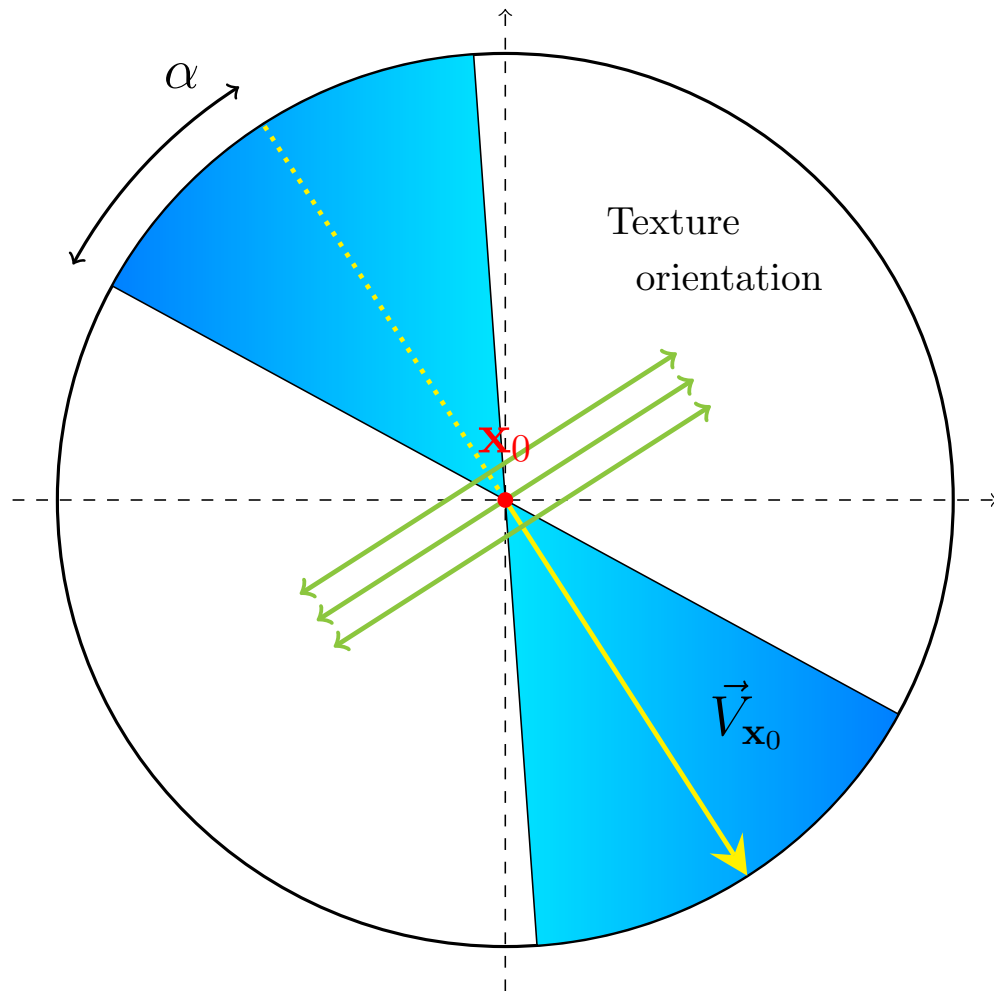


$$\alpha = 0.6$$



# Elementary field

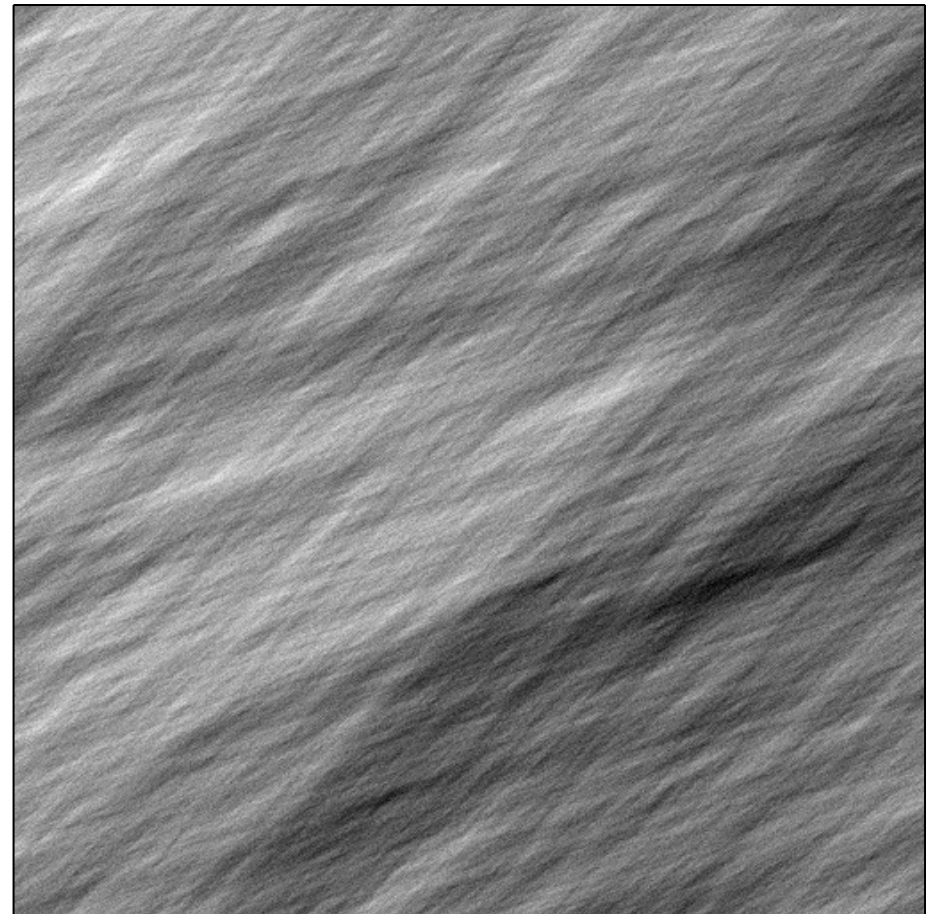
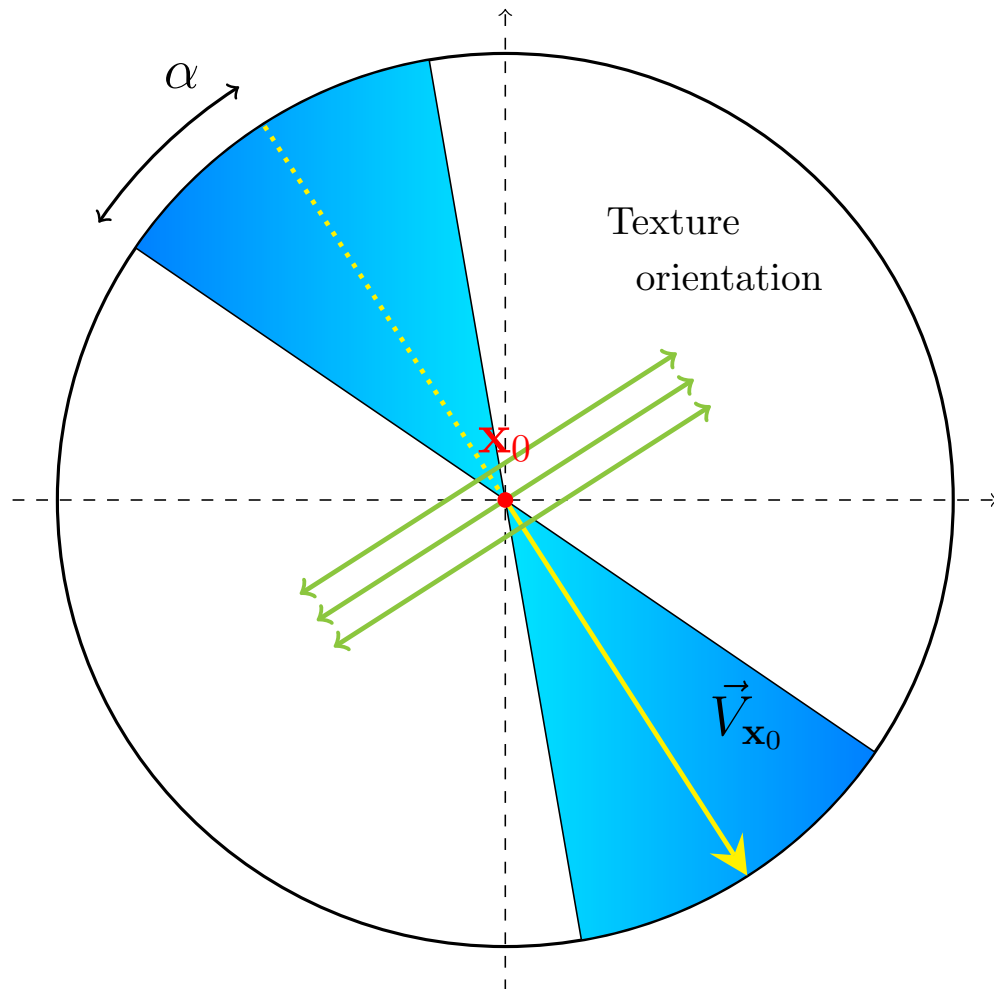
$$\alpha_0 = -\frac{\pi}{3}$$



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# Elementary field

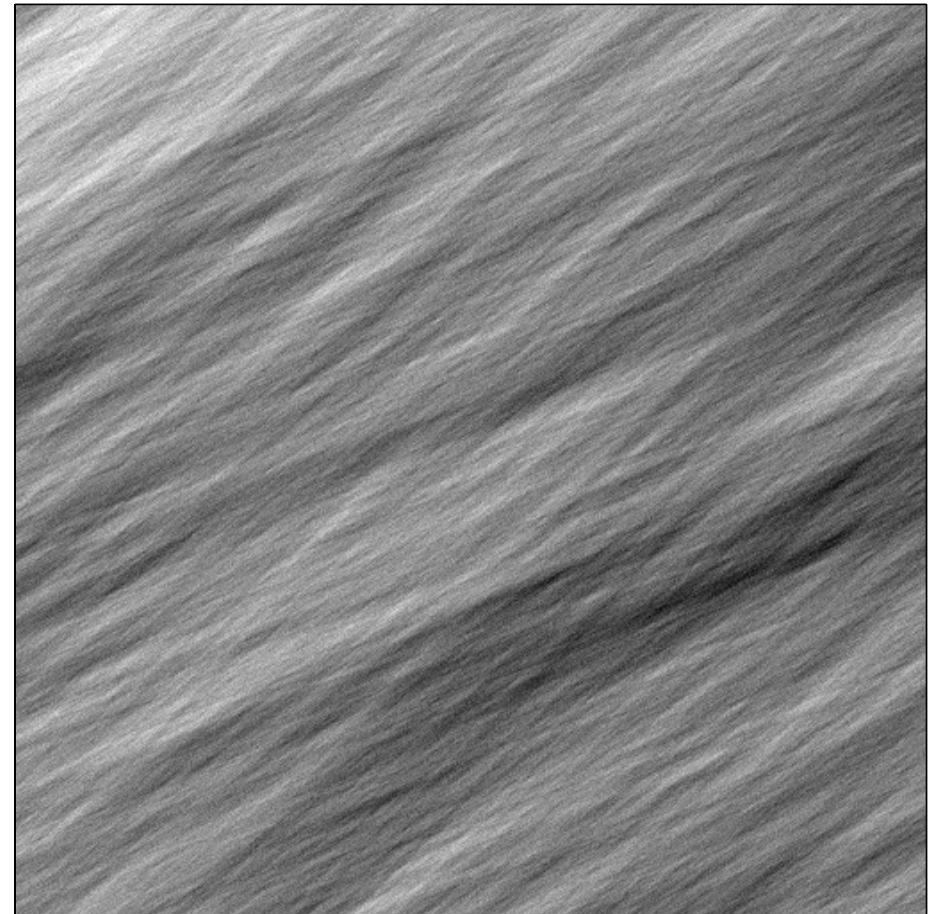
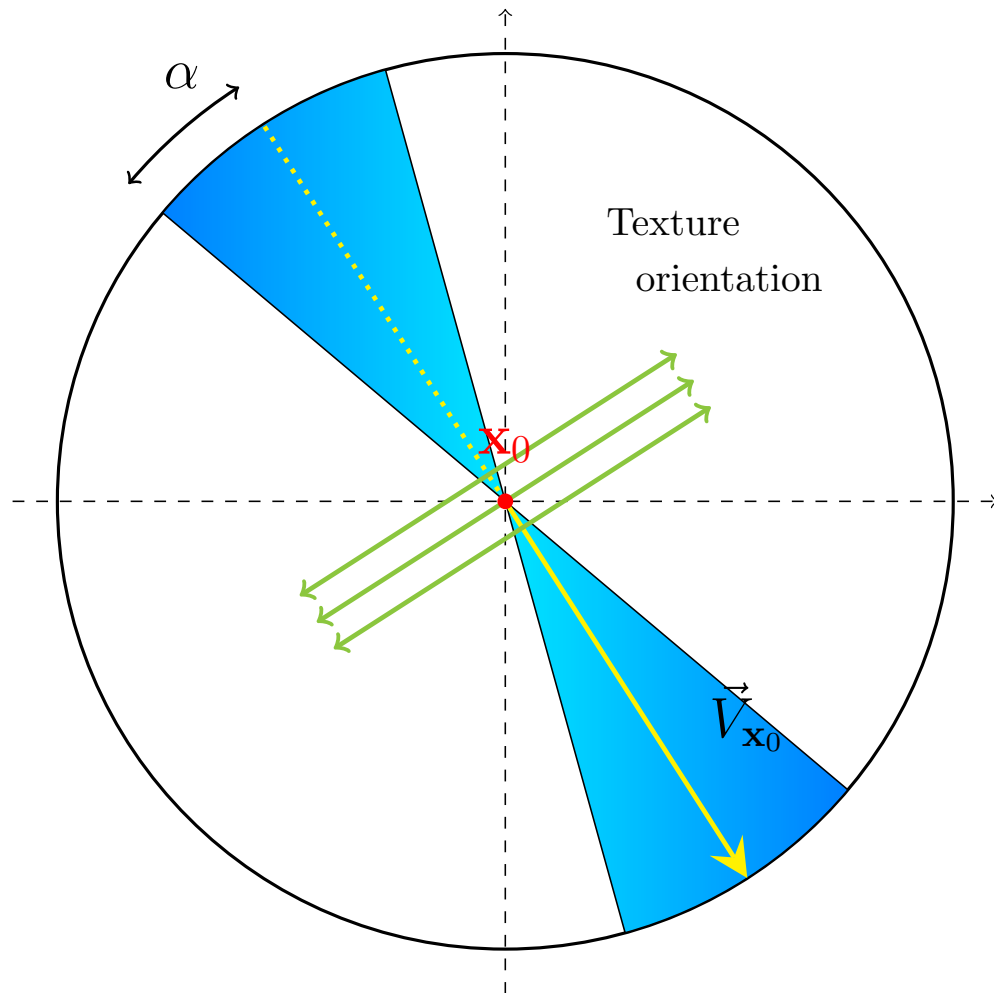
$$\alpha_0 = -\frac{\pi}{3}$$



$$\alpha = 0.4$$

# Elementary field

$$\alpha_0 = -\frac{\pi}{3}$$

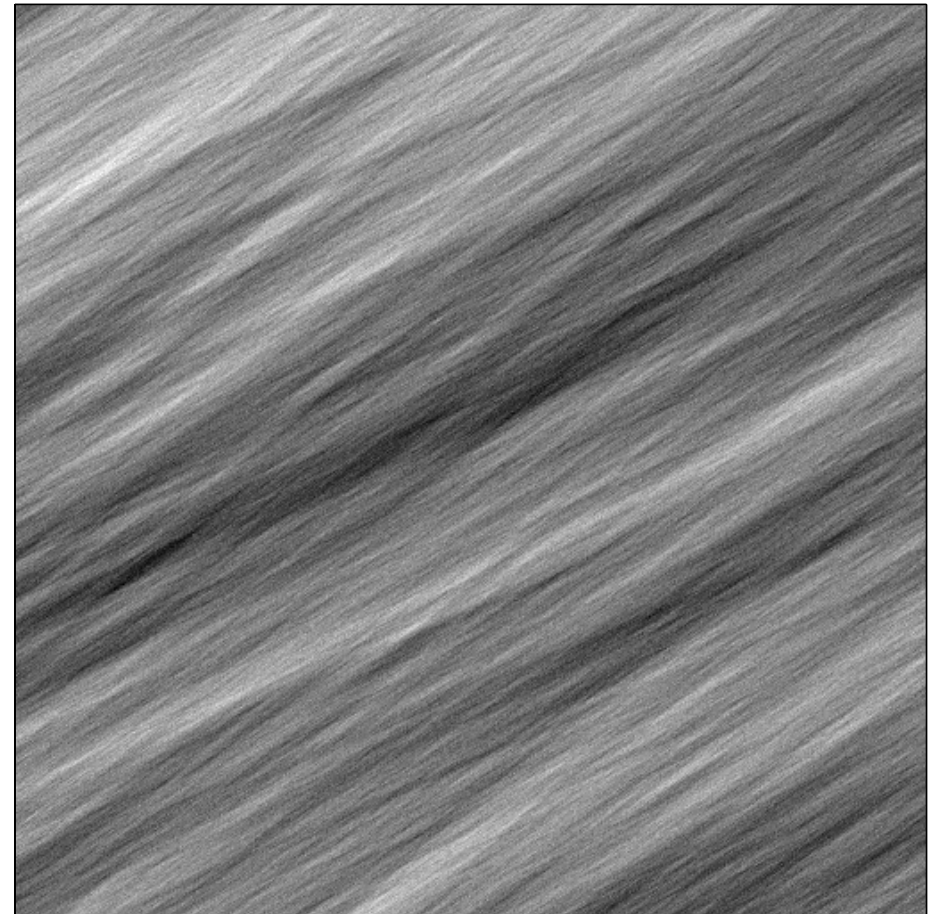
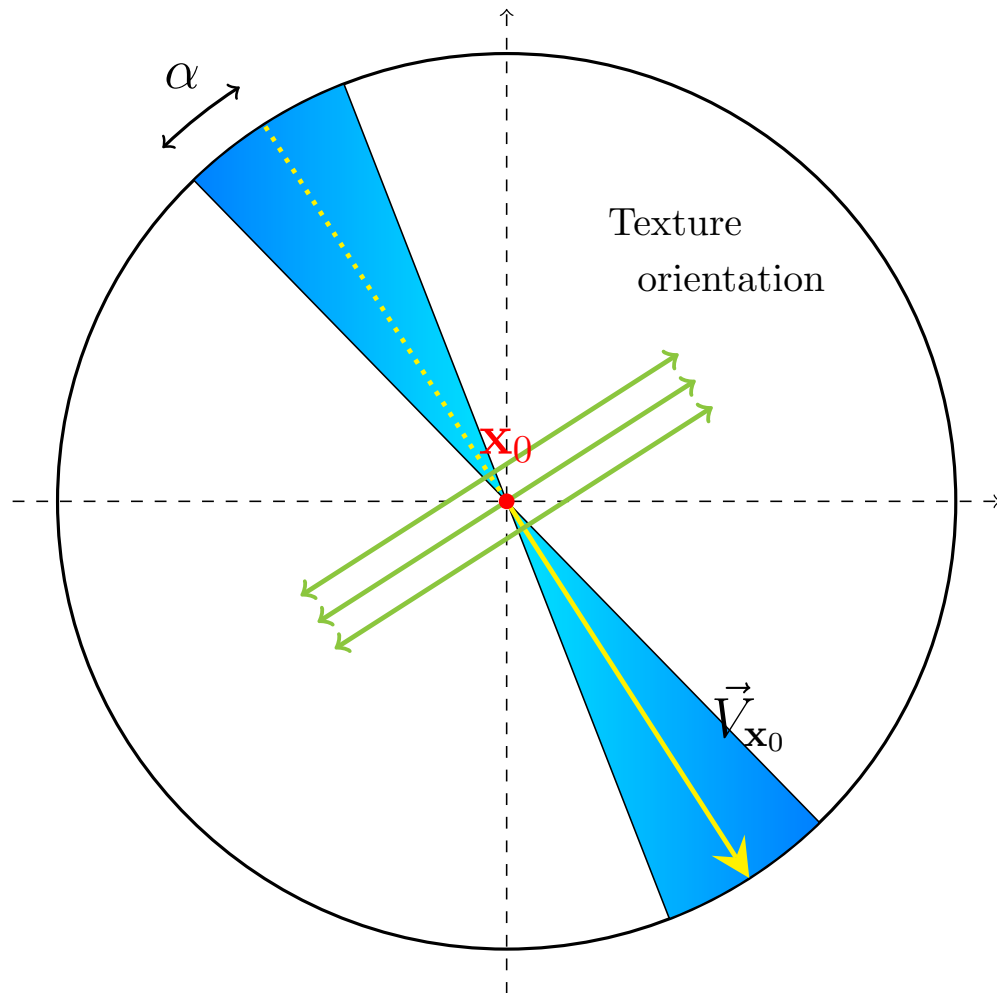


$$\alpha = 0.3$$



# Elementary field

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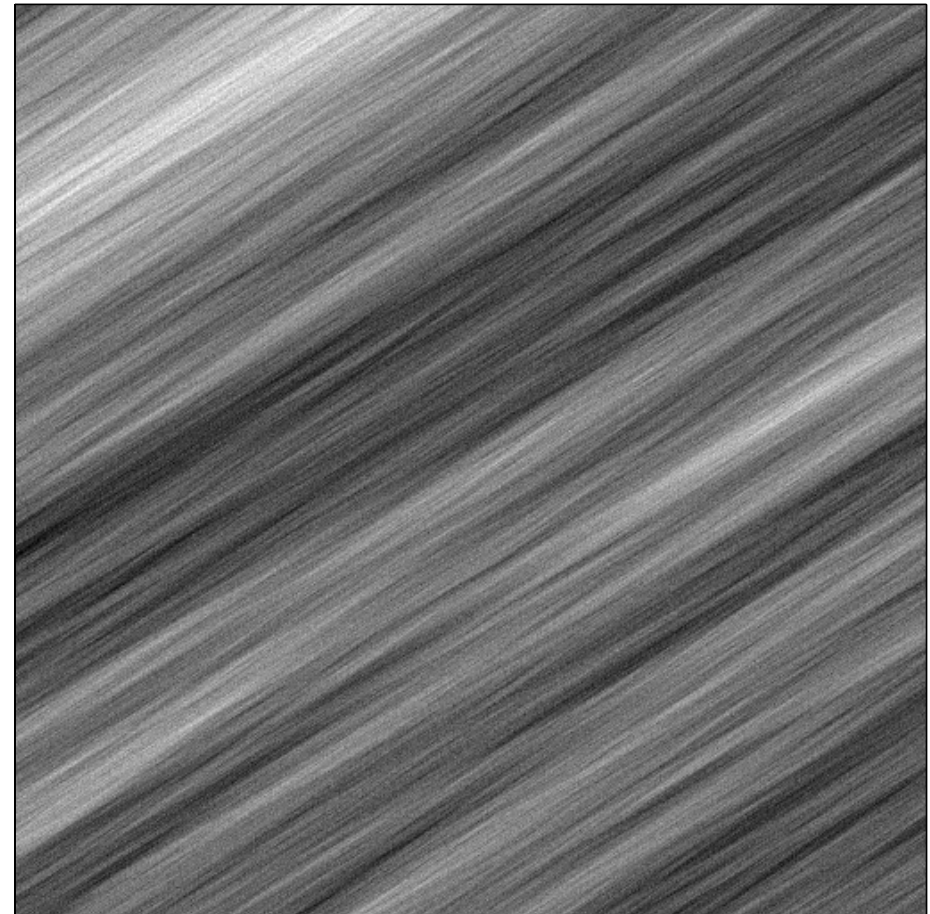
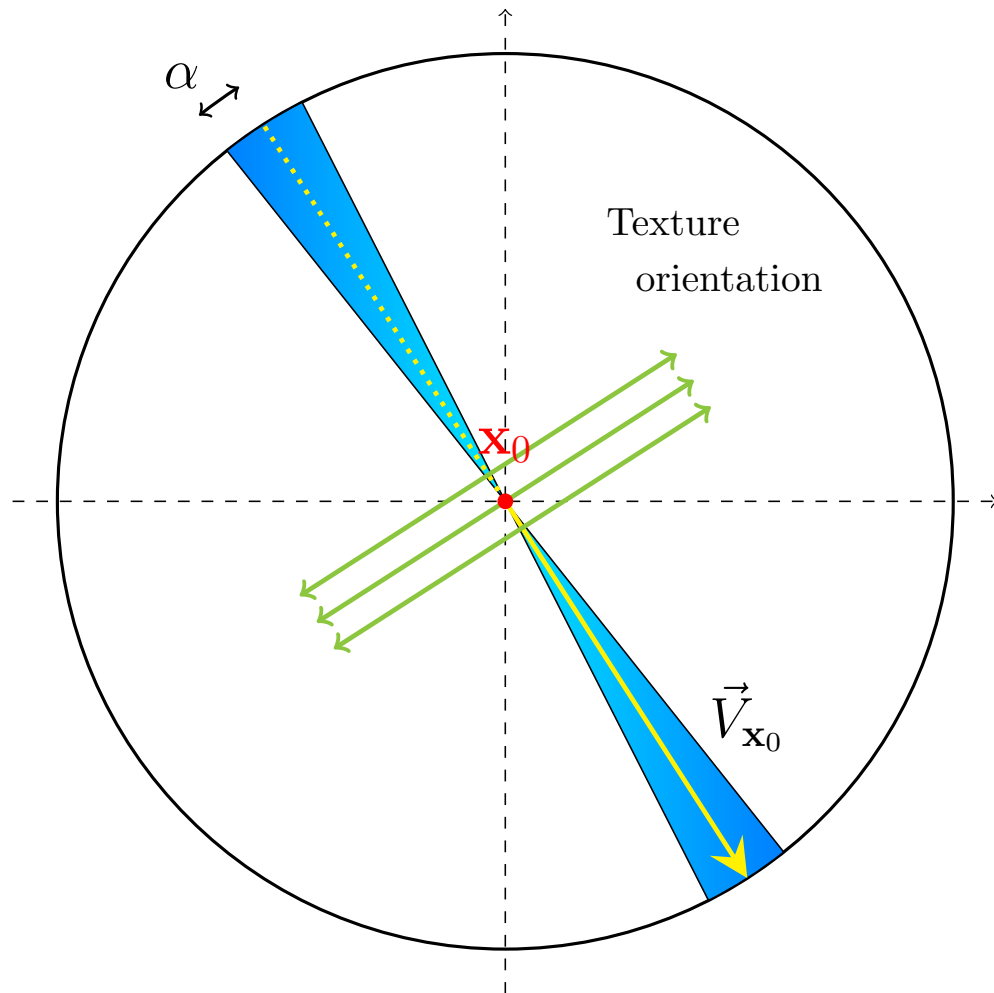


$$\alpha = 0.2$$



# Elementary field

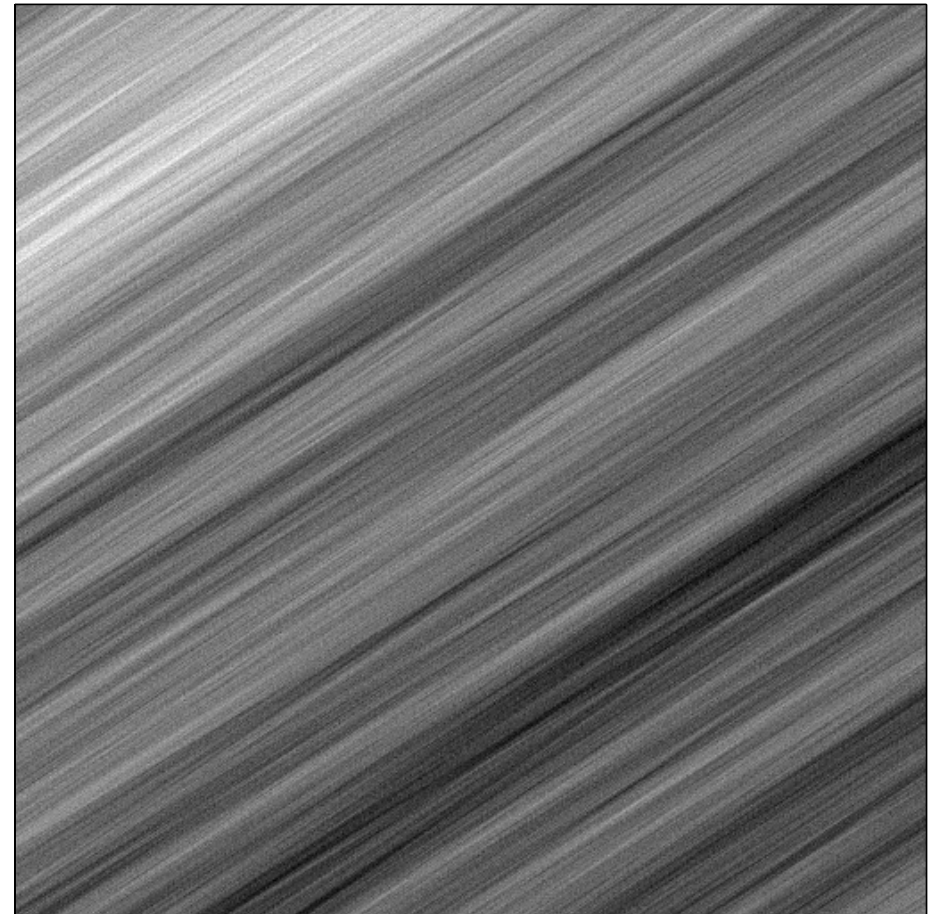
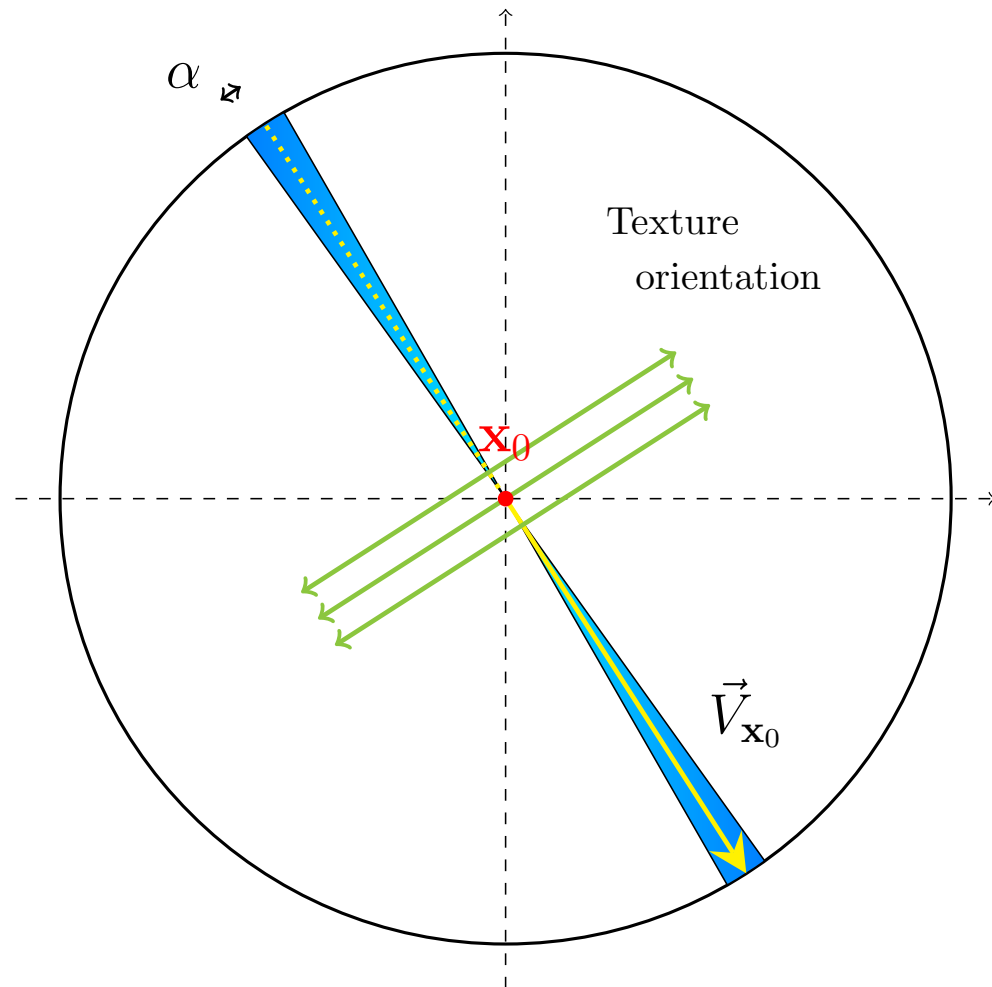
$$\alpha_0 = -\frac{\pi}{3}$$



$$\alpha = 0.1$$

# Elementary field

$$\alpha_0 = -\frac{\pi}{3}$$



$$\alpha = 0.05$$

# Tangent field

$$B_{\alpha_0, \alpha}^H(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x} \cdot \boldsymbol{\xi}} - 1) \frac{\mathbb{1}_{[-\alpha, \alpha]}(\arg \boldsymbol{\xi} - \alpha_0(\mathbf{x}))}{\|\boldsymbol{\xi}\|^{H+1}} d\widehat{W}(\boldsymbol{\xi})$$

## ■ Tangent field.

For a random field  $X$  locally asymptotically self-similar of order  $H$ ,

$$\frac{X(\mathbf{x}_0 + \rho \mathbf{h}) - X(\mathbf{x}_0)}{\rho^H} \xrightarrow[\rho \rightarrow 0]{\mathcal{L}} Y_{\mathbf{x}_0}$$

→  $Y_{\mathbf{x}_0}$  : tangent field of  $X$  at point  $\mathbf{x}_0 \in \mathbb{R}^2$

[Benassi,1997]  
[Falconer,2002]

Taylor's expansion



Tangent field

*Deterministic case*

*Stochastic case*

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constant

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# Tangent field

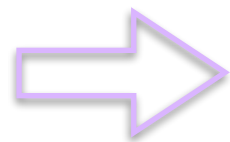
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$$B_{\alpha_0, \alpha}^H(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$$

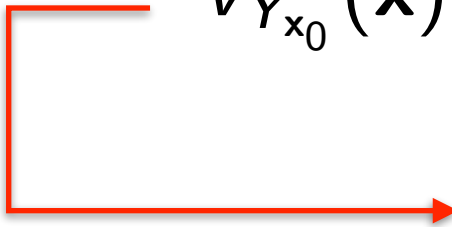
# Simulation of tangent fields

- **Continuous formulation.** Variogram of  $Y_{\mathbf{x}_0}$  : [Bierme, Richard, Moisan, 2012]

$$\begin{aligned}v_{Y_{\mathbf{x}_0}}(\mathbf{x}) &= \frac{1}{2} \int_{\mathbb{R}^2} |e^{i\mathbf{x} \cdot \boldsymbol{\xi}} - 1|^2 f(\mathbf{x}_0, \boldsymbol{\xi}) d\boldsymbol{\xi} \\ &= \frac{1}{2} \gamma(H) \int_{-\pi/2}^{\pi/2} c_{\alpha_0, \alpha}(\mathbf{x}_0, \theta) |\mathbf{x} \cdot \mathbf{u}(\theta)|^{2H} d\theta \\ &= \int_{-\pi/2}^{\pi/2} \tilde{v}_\theta(\mathbf{x} \cdot \mathbf{u}(\theta)) d\theta\end{aligned}$$

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$$\begin{aligned} \tilde{v}_\theta &= \frac{1}{2} \gamma(H) c_{\alpha_0, \alpha}(\mathbf{x}_0, \theta) \cdot |\cdot|^{2H} \\ \mathbf{u}(\theta) &= (\cos \theta, \sin \theta) \\ \gamma(H) &= \frac{\pi}{H \Gamma(2H) \sin(H\pi)} \end{aligned}$$

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in polar coordinates

variogram of a fractional brownian motion (FBM) of order H

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 \end{aligned}$$

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variogram of a fractional brownian motion (FBM) of order H

$Y_{\mathbf{x}_0}$   
 =  
 Infinite sum of independant rotating FBM of order H

$$\begin{aligned}
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# Simulation of tangent fields

## ■ Discrete formulation.

[Bierme, Richard, Moisan, 2012]

■  $(\theta_i)_{1 \leq i \leq n}$  are  $n$  bands orientations and  $\lambda_i = \theta_{i+1} - \theta_i$

■ The **turning band field** is defined as

$$Y_{\mathbf{x}_0}^{[n]}(\mathbf{x}) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^n \sqrt{\lambda_i c_{\alpha_0, \alpha}(\mathbf{x}_0, \theta_i)} B_i^H(\mathbf{x} \cdot \mathbf{u}(\theta_i))$$

■  $B_i^H$  are  $n$  independent FBM of order  $H$

■ Good approximation provided  $\max_i \lambda_i \leq \varepsilon$

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not equispaced



# Simulation of tangent fields

## ■ Simulation along particular bands.

[Bierme, Richard, Moisan, 2012]

- Discrete grid  $r^{-1}\mathbb{Z}^2 \cap [0, 1]^2$  with  $r = 2^k - 1, k \in \mathbb{N}^*$
- Choose  $(\theta_i)$  such that  $\tan \theta_i = \frac{p_i}{q_i}$  and  $\max_i \lambda_i \leq \epsilon$
- Then  $B_i^H(\mathbf{x} \cdot \mathbf{u}(\theta_i))$  becomes

$$\left\{ B_i^H \left( \frac{k_1}{r} \cos \theta_i + \frac{k_2}{r} \sin \theta_i \right) ; 0 \leq k_1, k_2 \leq r \right\} \stackrel{\mathcal{L}}{=} \left( \frac{\cos \theta_i}{r q_i} \right)^H \{ B_i^H(k_1 q_i + k_2 p_i); 0 \leq k_1, k_2 \leq r \}$$

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Dynamic programming

$$\left\{ B_i^H \left( \frac{k_1}{r} \cos \theta_i + \frac{k_2}{r} \sin \theta_i \right) ; 0 \leq k_1, k_2 \leq r \right\} \stackrel{\mathcal{L}}{=} \left( \frac{\cos \theta_i}{r q_i} \right)^H \{ B_i^H(k_1 q_i + k_2 p_i); 0 \leq k_1, k_2 \leq r \}$$



# Simulation of tangent fields

## ■ Simulation along particular bands.

[Bierme, Richard, Moisan, 2012]

■ Discrete grid  $r^{-1}\mathbb{Z}^2 \cap [0, 1]^2$  with  $r = 2^k - 1, k \in \mathbb{N}^*$

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self-similarity

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equispaced ✓

# Simulation of LAFBF

## using tangent fields

[Polisano et al.,2014]

■ **Algorithm.** For each pixel  $\mathbf{x}_0 = (k_1, k_2) \in \llbracket 0, r \rrbracket^2$

$$B_{\alpha_0, \alpha}^H((k_1, k_2)) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^n \sqrt{\lambda_i c_{\alpha_0, \alpha}((k_1, k_2), \theta_i)} \left( \frac{\cos \theta_i}{rq_i} \right)^H B_i^H(k_1 q_i + k_2 p_i)$$

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$B_{\alpha_0, \alpha}^H(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$

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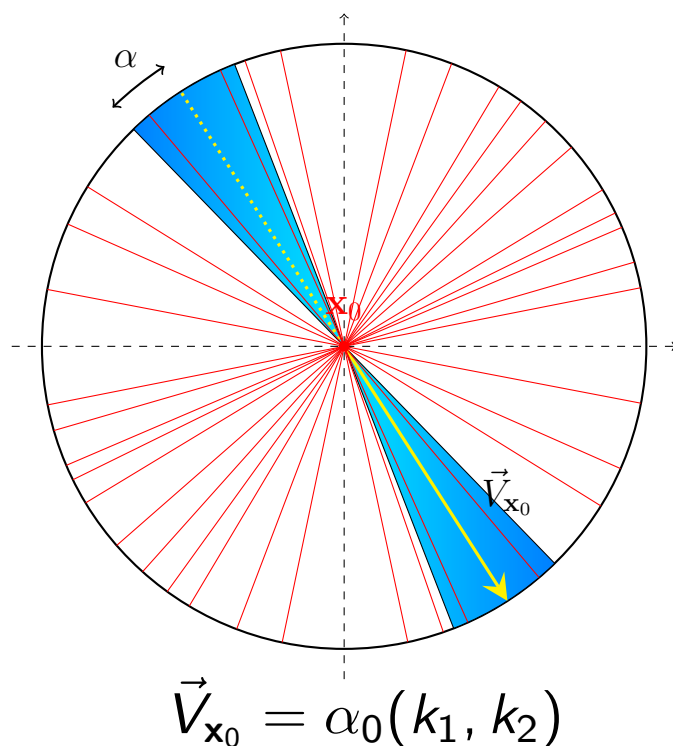
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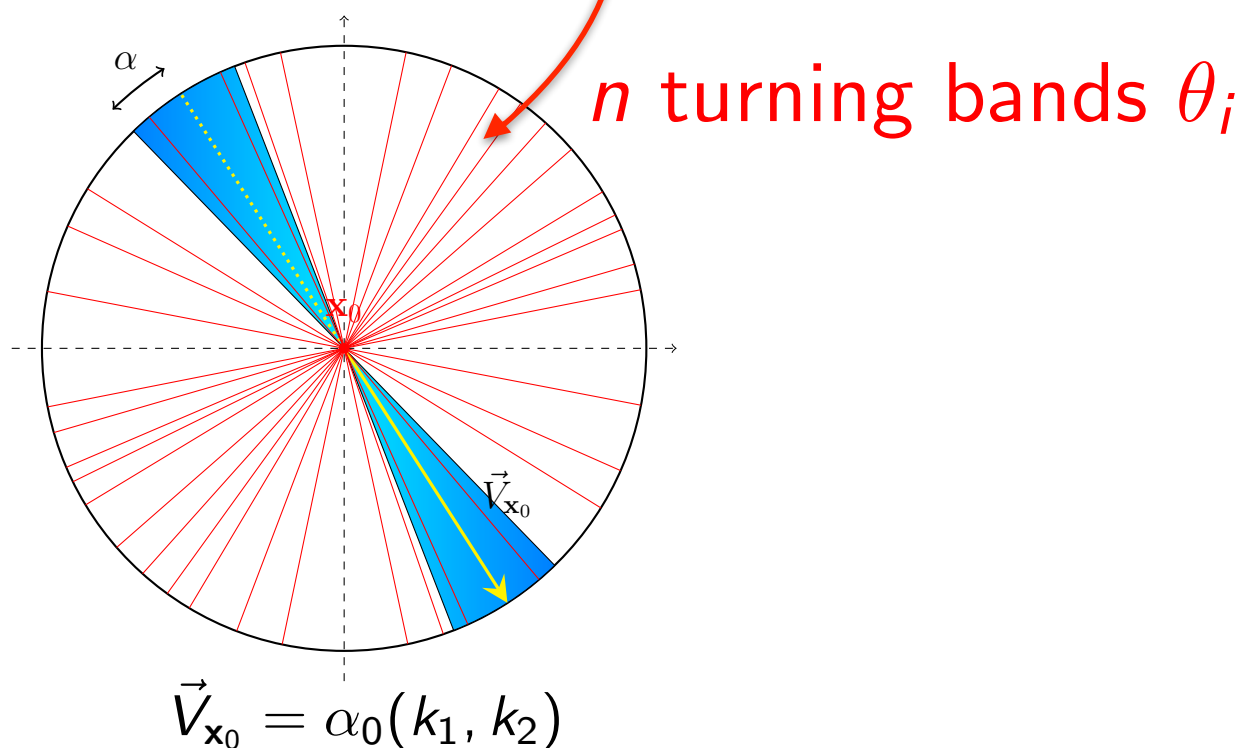
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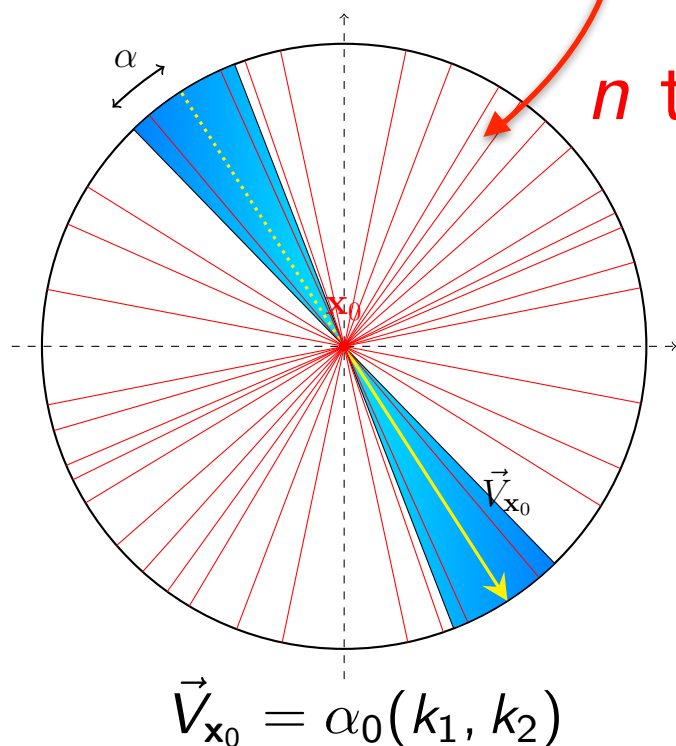
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$n$  turning bands  $\theta_i$

$$\mathbb{1}_{[-\alpha, \alpha]}(\theta_i - \alpha_0((k_1, k_2))) \neq 0 \Leftrightarrow |\theta_i - \alpha_0((k_1, k_2))| \leq \alpha$$

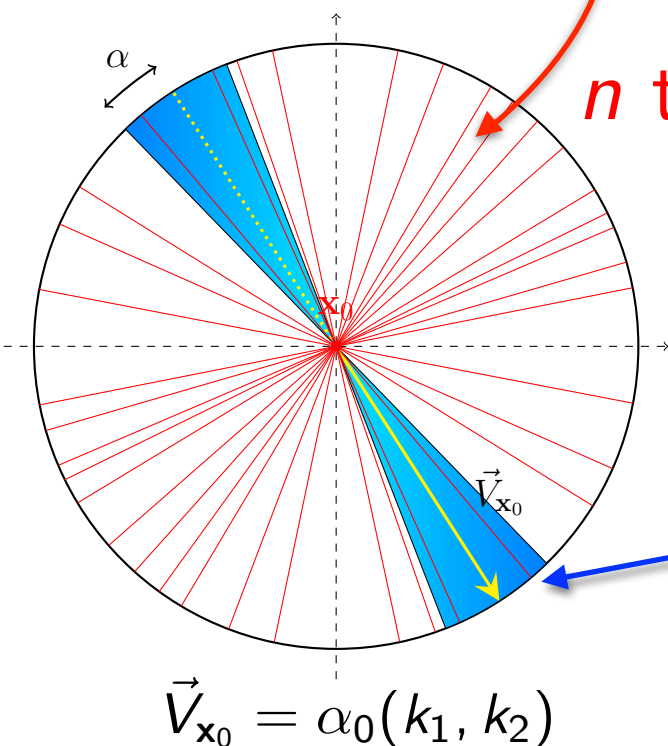
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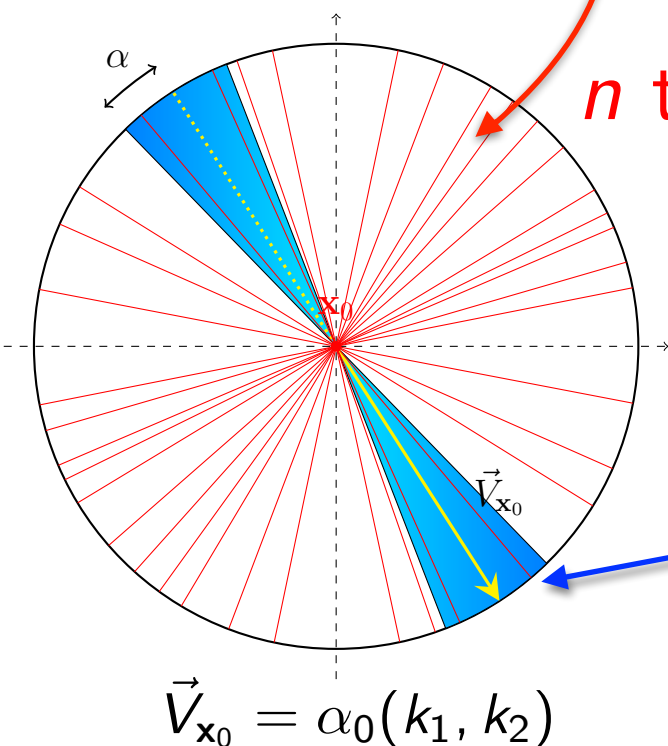
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few bands  
in the cone

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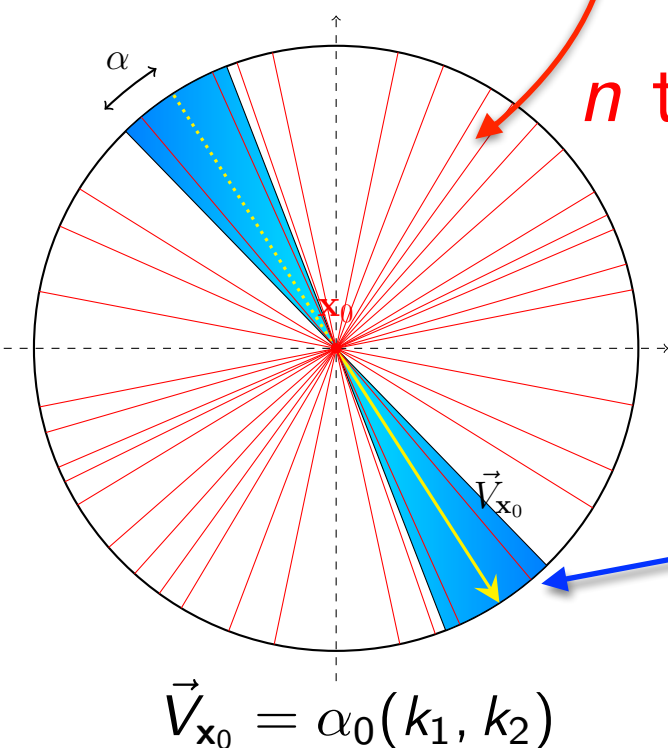
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$n$  turning bands  $\theta_i$

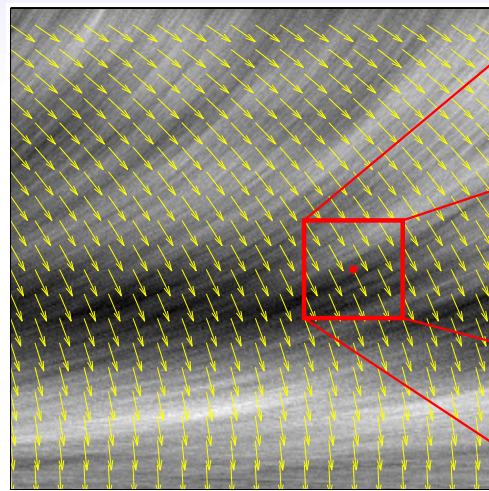
few bands  
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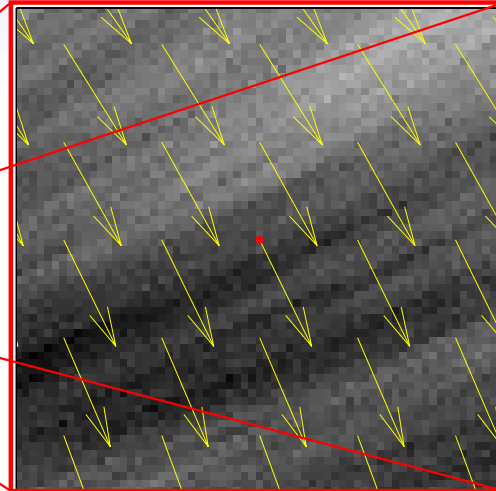
Complexity  $O(r^2 \log n)$

# Numerical experiments

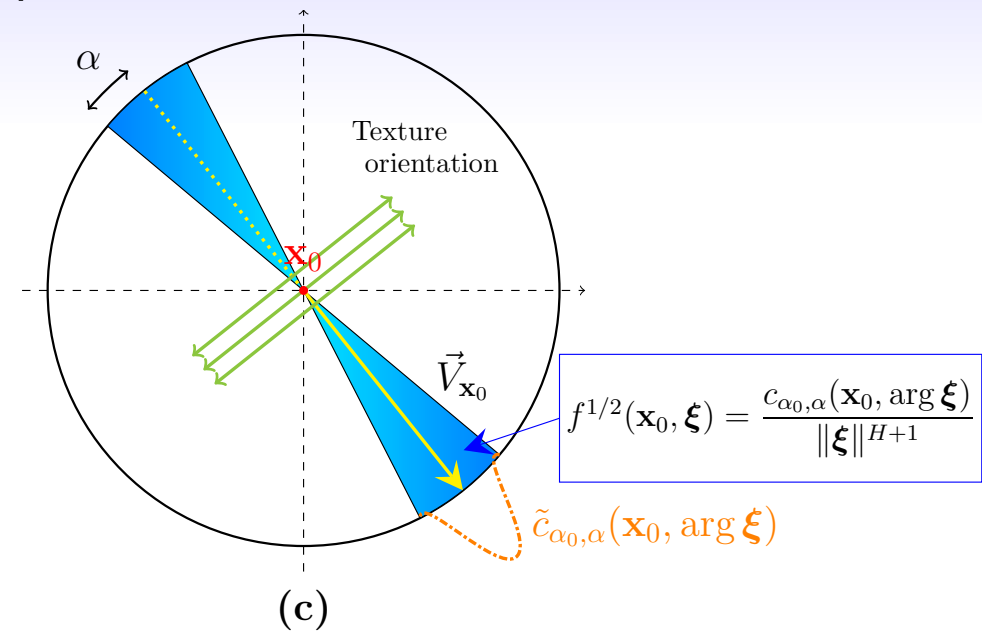
$$\vec{V}_{(x,y)}^1 = (\cos(-\pi/2 + y), \sin(-\pi/2))$$



(a)



(b)



(c)

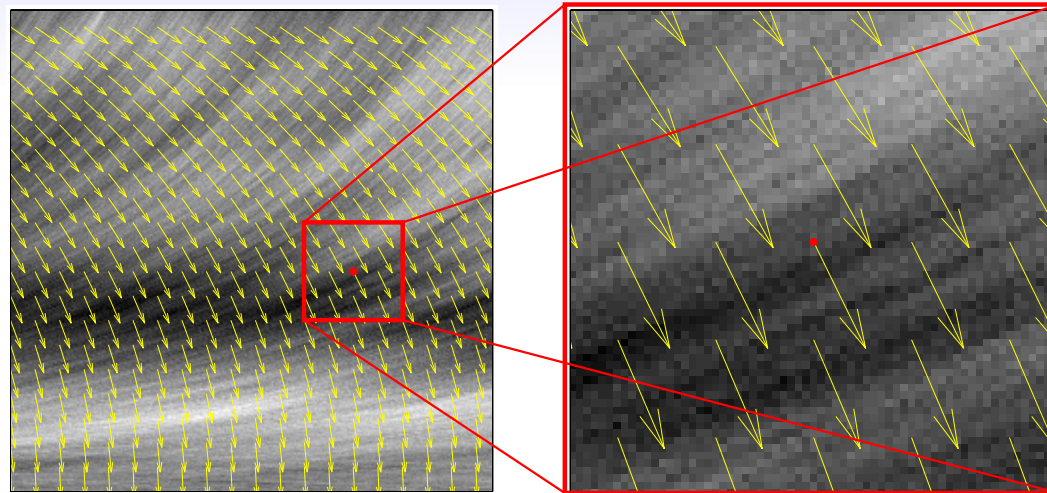
## Parameters

$$r = 255 \quad H = 0.2$$

$$\alpha = 10^{-1} \quad \epsilon = 10^{-2}$$

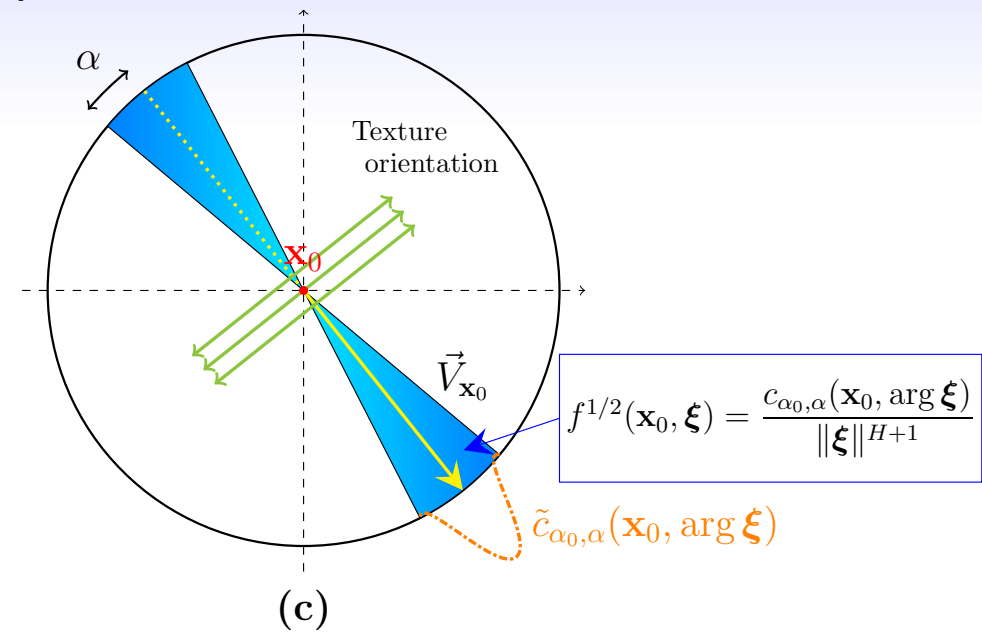
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(a)

(b)



(c)

Texture with prescribed local orientation at each point  $\mathbf{x}_0$  given by a vector field

$$\vec{V}_{\mathbf{x}_0} = \mathbf{u}(\alpha_0(\mathbf{x}_0))$$

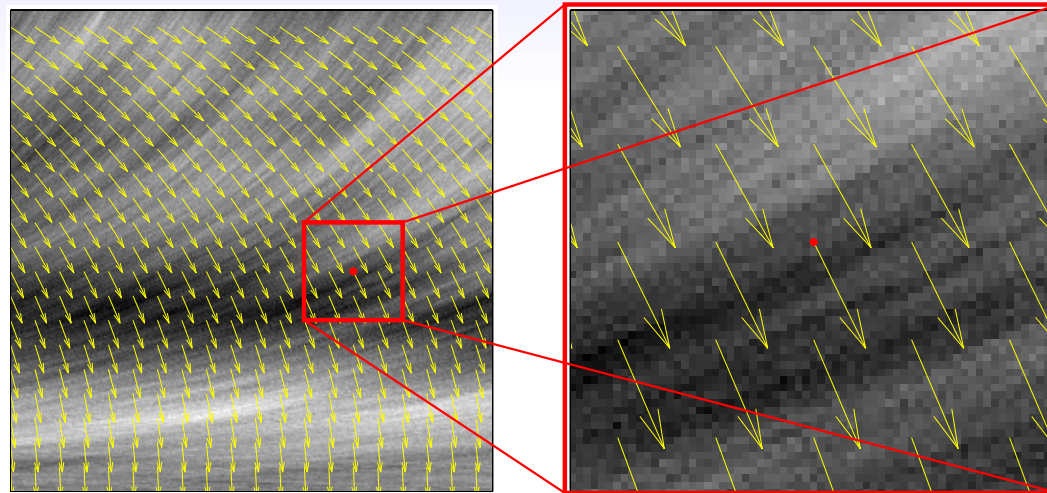
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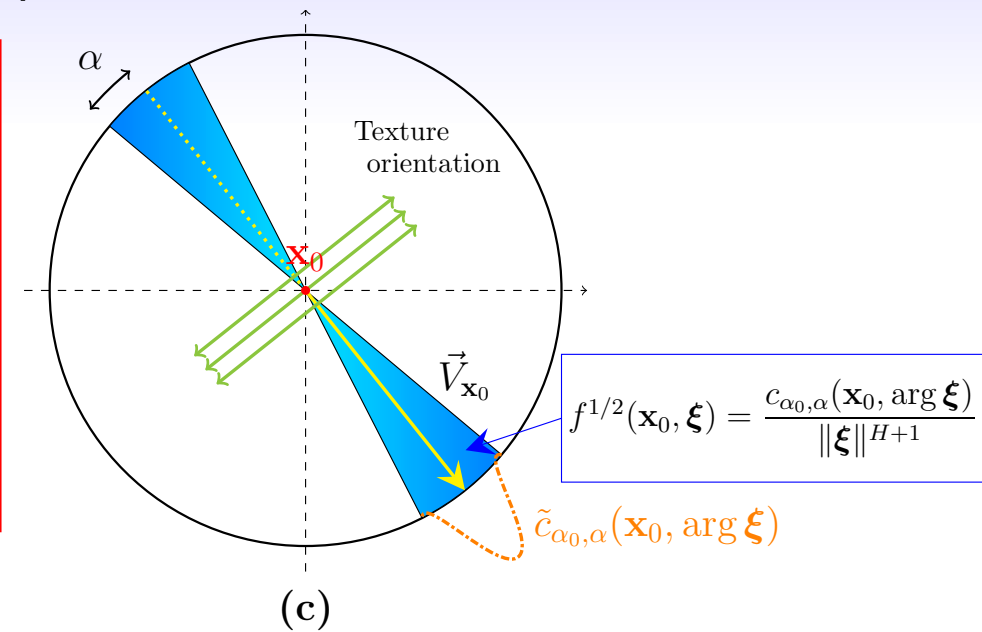
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A zoom around a point  $\mathbf{x}_0$  shows that locally a LAFBF behaves as an elementary field

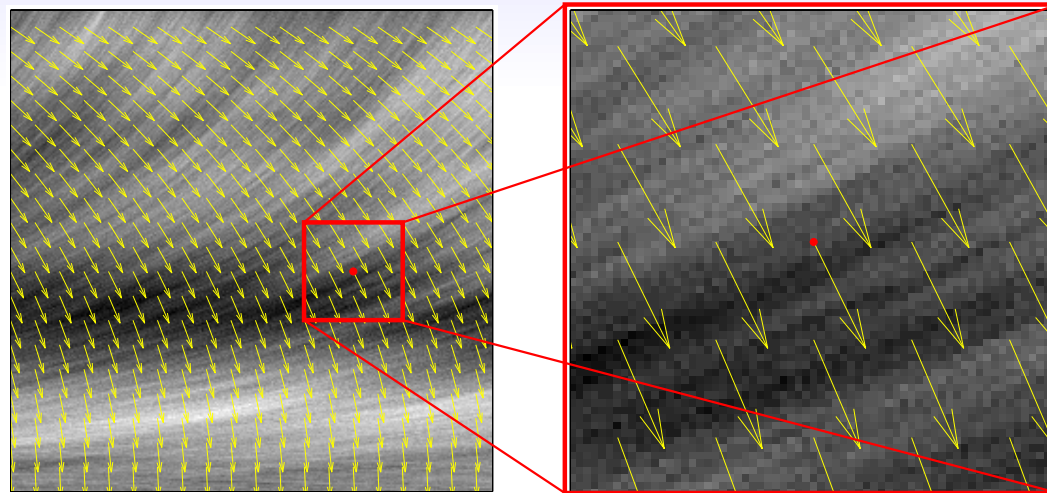
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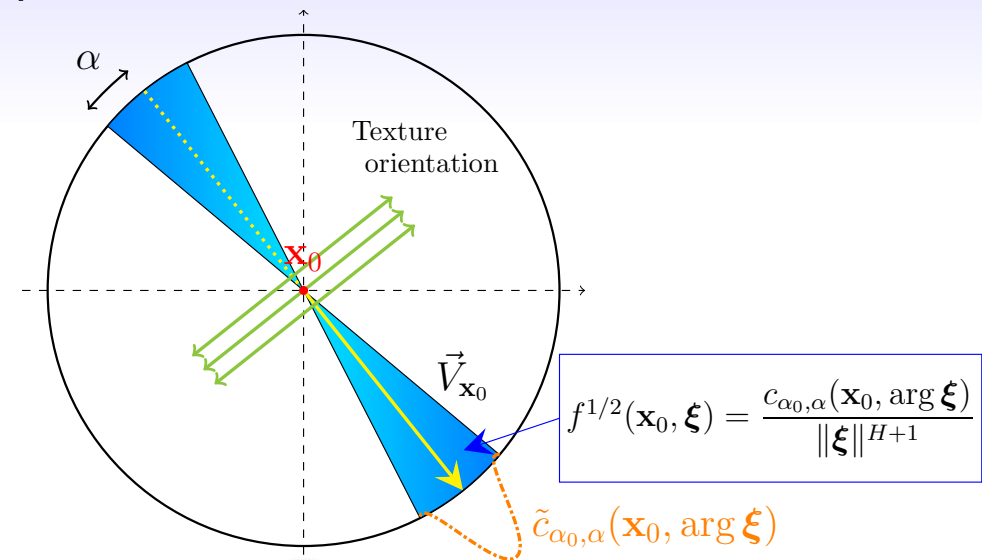
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Regularized version of the anisotropy function

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## Parameters

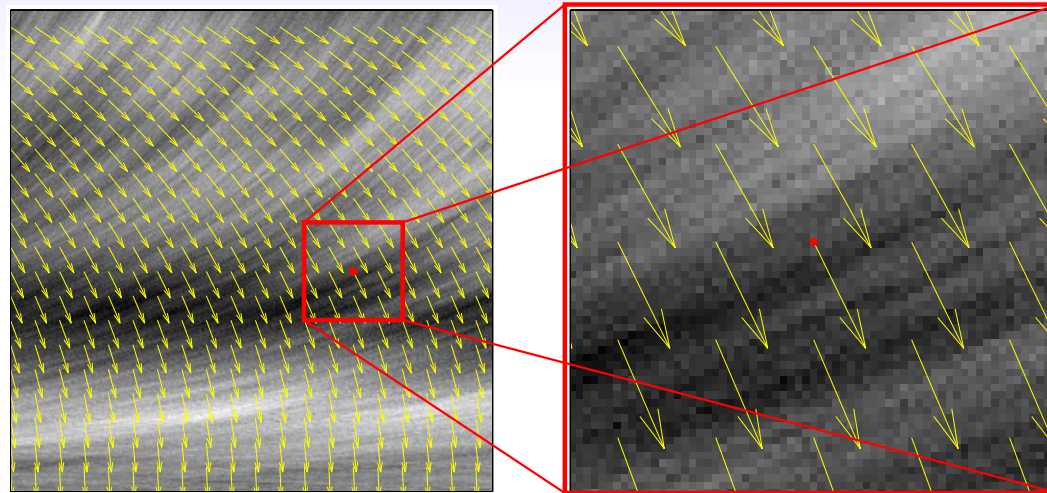
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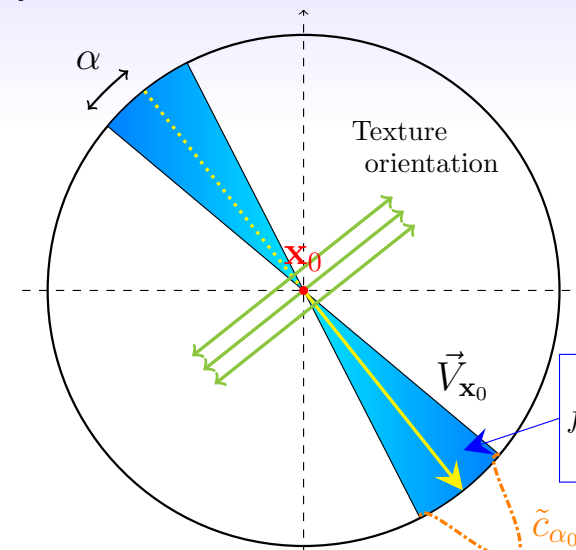
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(a)

(b)



(c)

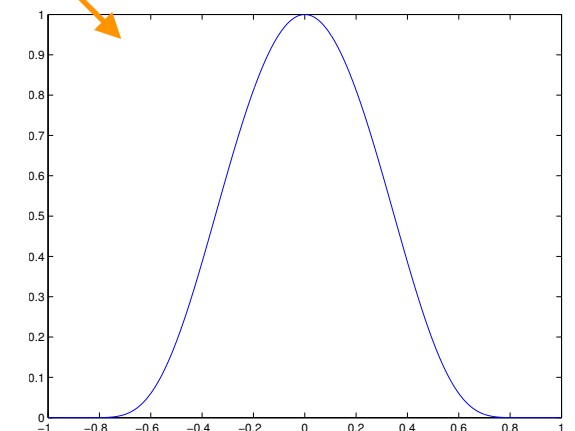
$$f^{1/2}(\mathbf{x}_0, \boldsymbol{\xi}) = \frac{c_{\alpha_0, \alpha}(\mathbf{x}_0, \arg \boldsymbol{\xi})}{\|\boldsymbol{\xi}\|^{H+1}}$$

$$\tilde{c}_{\alpha_0, \alpha}(\mathbf{x}_0, \arg \boldsymbol{\xi})$$

Texture with prescribed local orientation at each point  $\mathbf{x}_0$  given by a vector field

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Regularized version of the anisotropy function



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# Numerical experiments

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$$\vec{\mathbf{V}}_{(x,y)}^2 = (\cos(\cos(36xy)), \sin(\cos(36xy)))$$

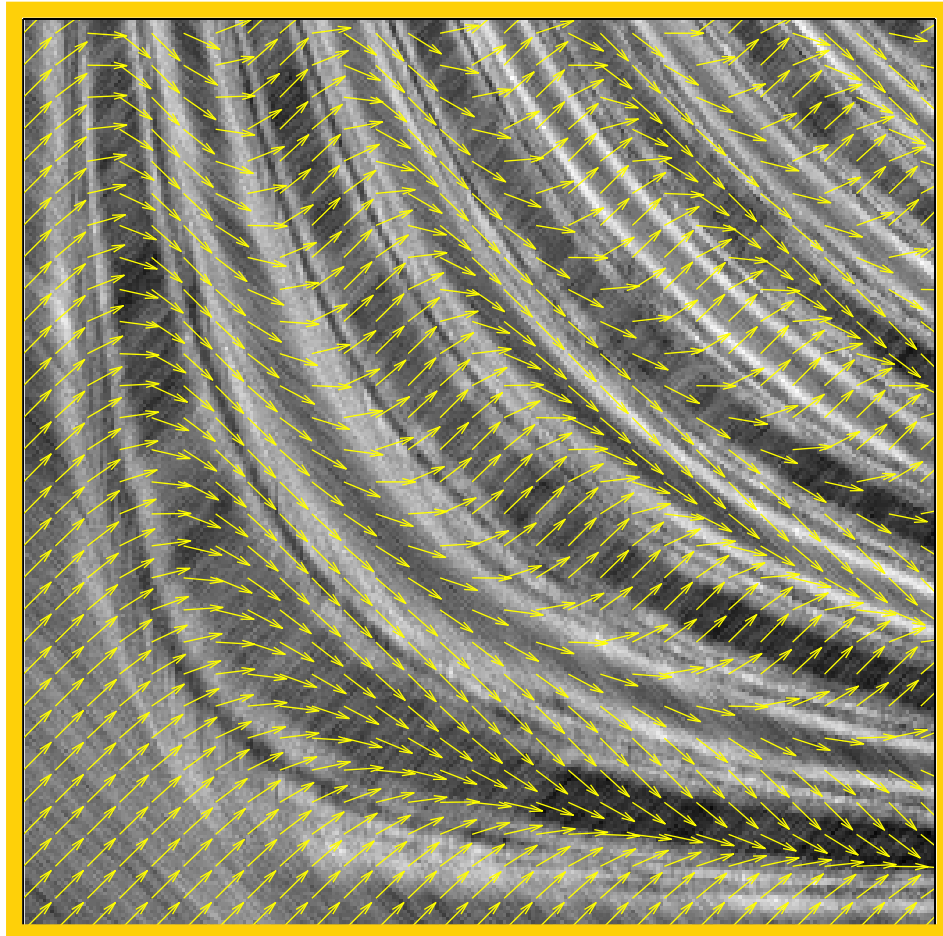
$$\vec{\mathbf{V}}_{(x,y)}^3 = \nabla F(x, y)$$

$$F(x, y) = (4x - 2)e^{-(4x-2)^2 - (4y-2)^2}$$

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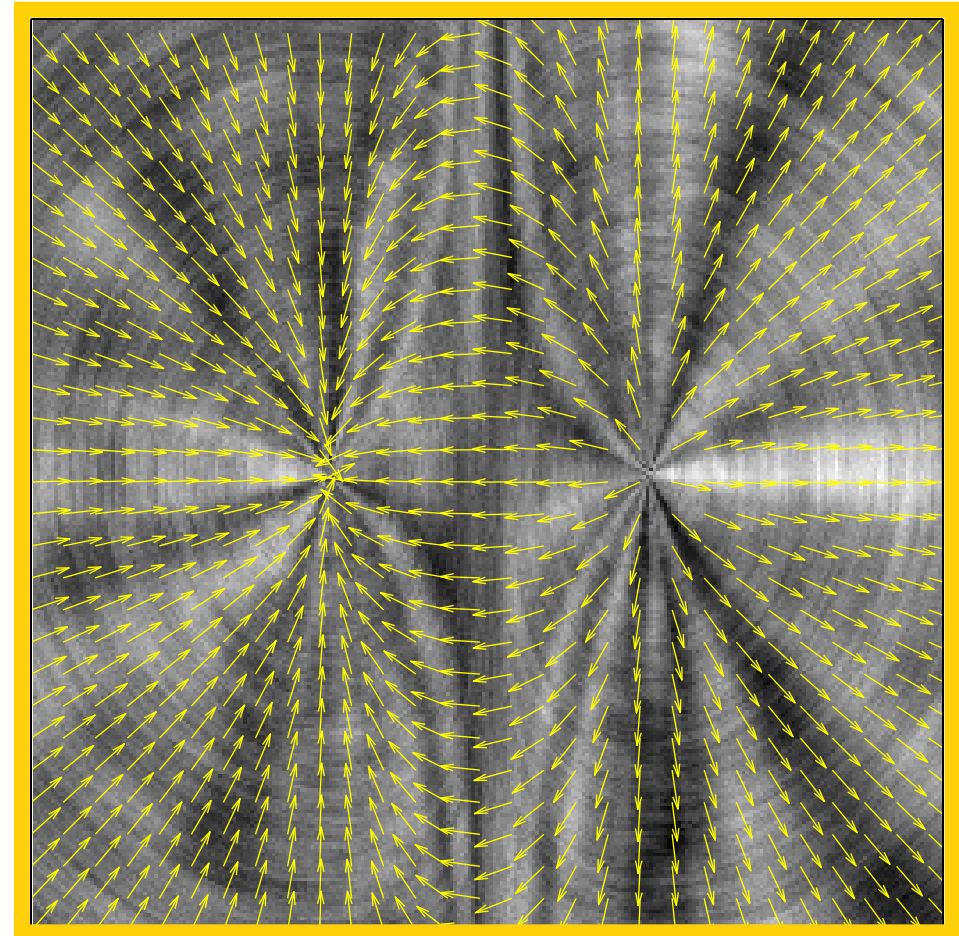
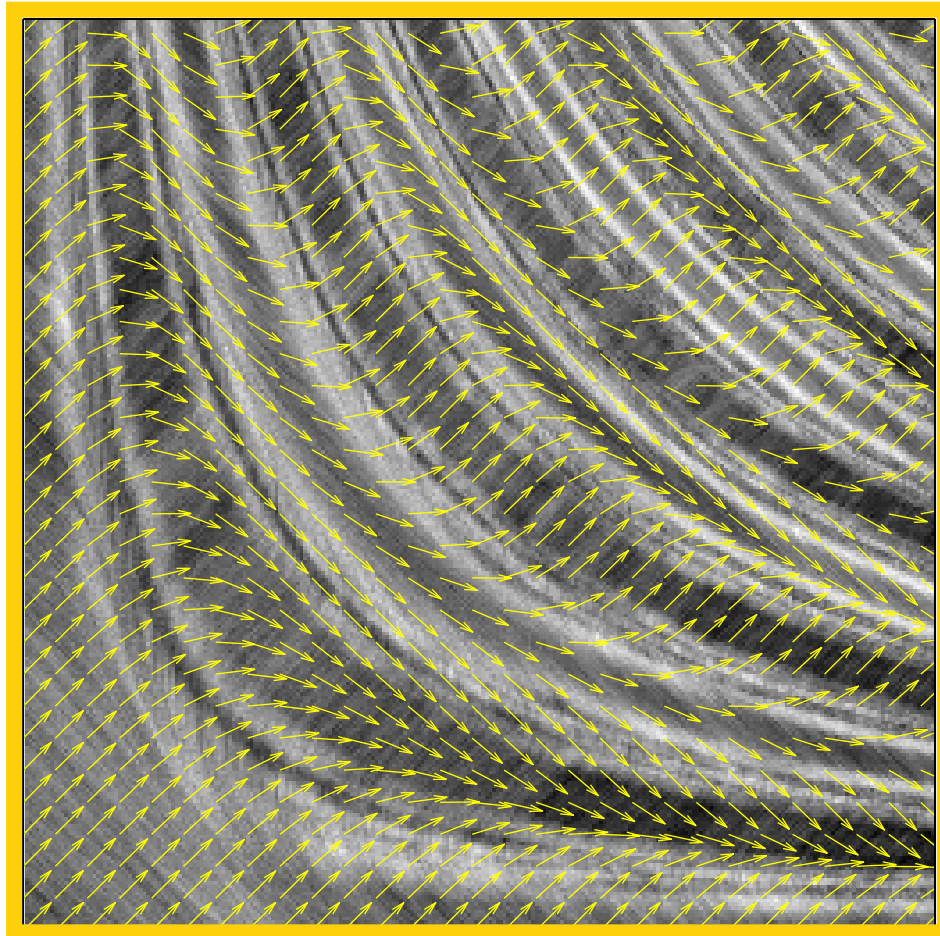


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H=0.2

H=0.5

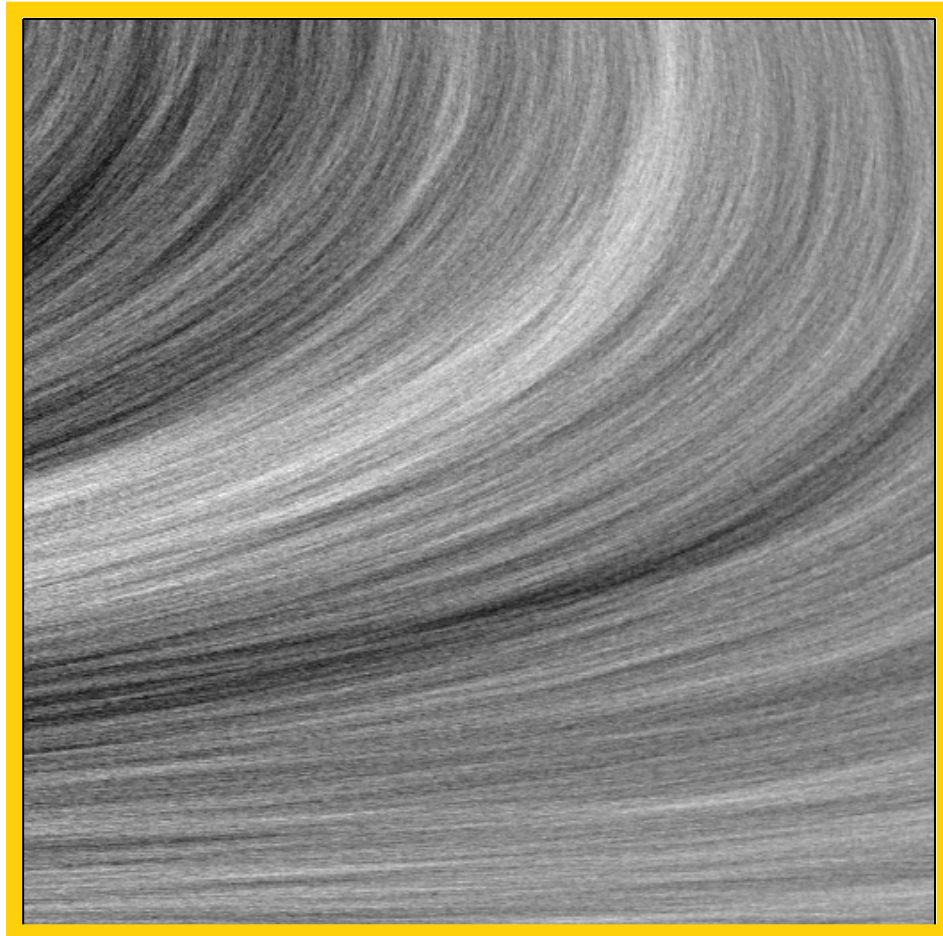
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# Numerical experiments

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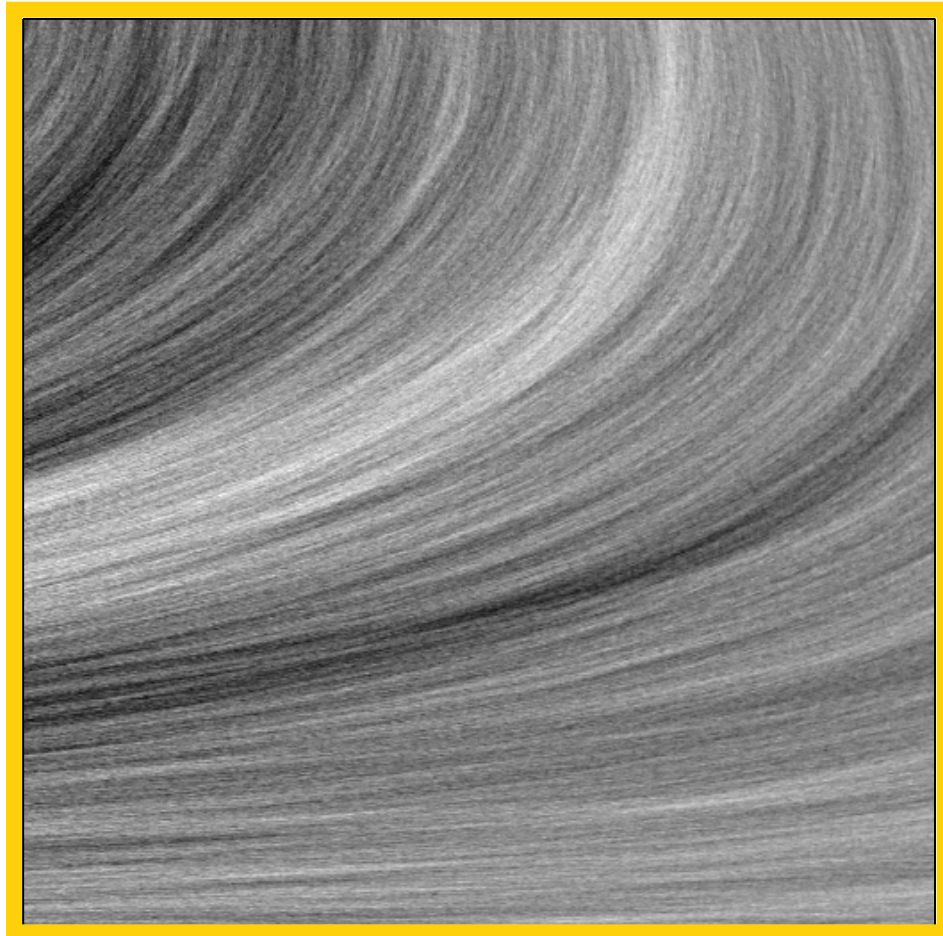
H=0.5



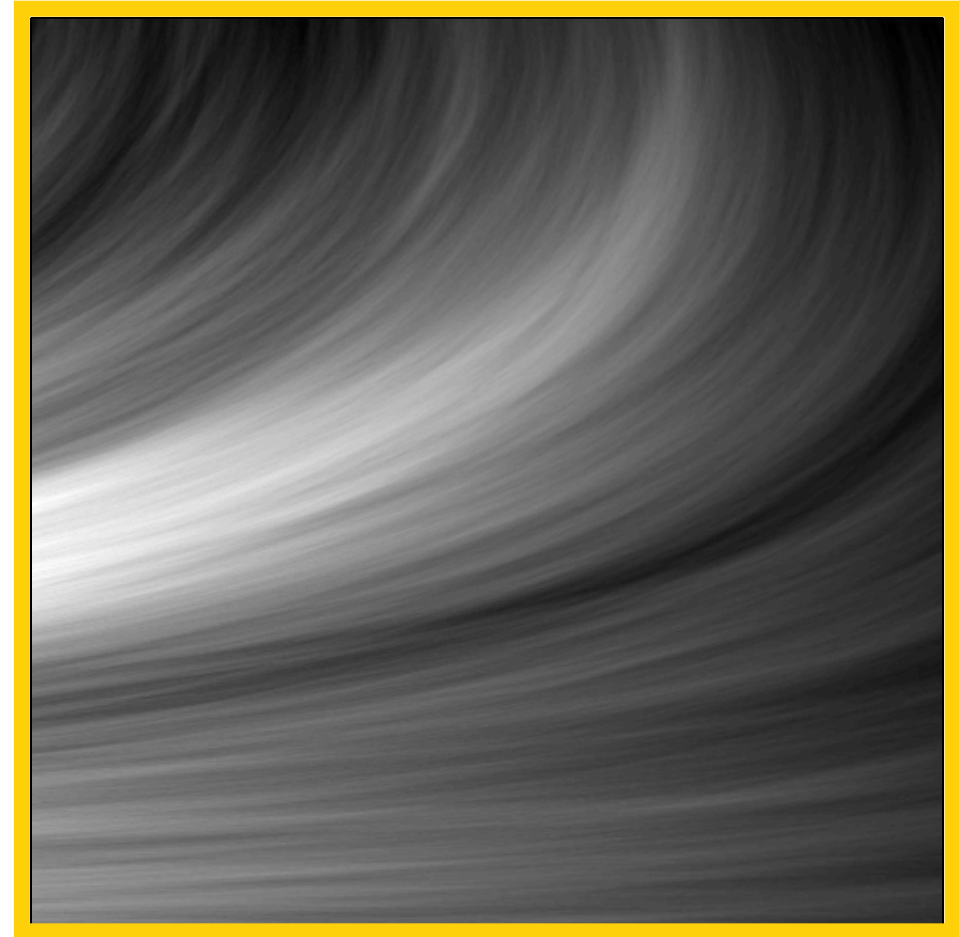
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# Conclusion and future work

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- Extensions of our model include Hurst index may **vary spatially**.

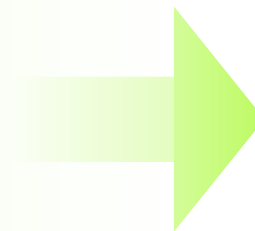
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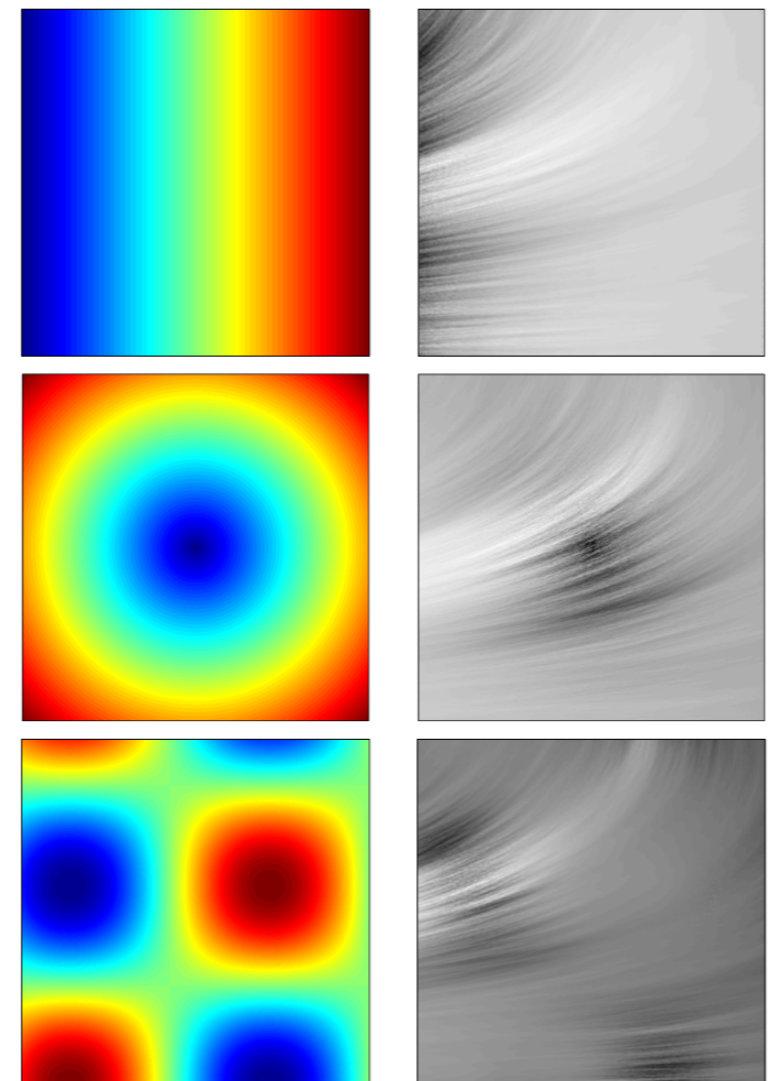
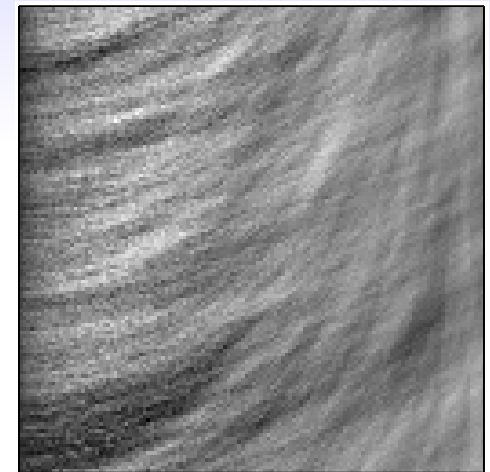
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# Bibliography

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## ■ Selected papers

- K. Polisano, M. Clausel, V. Perrier and L. Condat, "Texture modeling by Gaussian fields with prescribed local orientation", *IEEE ICIP*, 2014.
- A. Bonami and A. Estrade, "Anisotropic analysis of some Gaussian models", *Journal of Fourier Analysis and Applications*, vol. 9, no. 3, pp. 215–236, 2003.
- H. Bierme, L. Moisan, and F. Richard, "A turning-band method for the simulation of anisotropic fractional Brownian fields," *preprint MAP5 No. 2012-312012*, 2012.
- K.J. Falconer, "Tangent fields and the local structure of random fields," *Journal of Theoretical Probability*, vol. 15, no. 3, pp. 731–750, 2002.

# Questions ?

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# Questions ?

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**Thank you for your attention.**

■ **Dynamic programming.** The choice of the bands orientation  $(\theta_i)_{1 \leq i \leq n}$

is governed by the computational cost of the  $B_i^H$  's within dynamic programming.

■ Let the error  $\epsilon$  fixed. Taking  $N = \lceil \frac{1}{\tan \epsilon} \rceil$  consider the following set:

$$\mathcal{V}_N = \left\{ (p, q) \in \mathbb{Z}^2 / -N \leq p \leq N, 1 \leq q \leq N, p \wedge q = 1, -\frac{\pi}{2} < \arctan\left(\frac{p}{q}\right) < \frac{\pi}{2} \right\}$$

■ The aim is to find  $n$  pairs in the set  $\mathcal{V}_N$  which minimize the following global cost:

$$C(\Theta) = \sum_{k=1}^s C(r(|p_{i_k}| + q_{i_k}))$$

where  $C(\ell)$  is the cost of one FBM  $B_i^H$  in  $O(n \log n)$ , under the constraint  $\max_i (\theta_{i+1} - \theta_i) \leq \epsilon$