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Matching Saleh and Volterra series model

Arthur Louchart

Centre for Digital Systems

IMT Nord Europe, Institut Mines-Télécom, Univ. Lille

Lille, France

arthur.louchart@imt-nord-europe.fr

Inbar Fijalkow

ETIS, UMR 8051

CY Cergy Paris University, ENSEA, CNRS

Cergy, France

inbar.fijalkow@ensea.fr

Abstract—Signal processing for wireless communications often use a third order Volterra series to model the nonlinear amplifier. Nevertheless, the Saleh model corresponds better to power amplifier device. We optimize the parameters of the Volterra series and derive their analytical expression using the moments of functions of exponential variables. We show the optimal parameters are accurate, but are function of the power amplifier mean in power.

Index Terms—Signal processing for communications, nonlinear power amplifier, statistical signal processing

I. INTRODUCTION

The signal to be transmitted in wireless communications systems undergoes a radio frequency frontal including a digital to analog converter, a high power amplifier (HPA) and a mixer. When the bandwidth is wide or when the HPA efficiency is high for having a high transmitted power, the HPA is not ideal, i.e. it is a nonlinear device with memory. This is in particular the case for satellite transmissions and even 5G systems when one wants to reduce power consumption.

Many HPA models exist in the literature. Some result from the components modeling, such as the clipping, Saleh, Rapp [1], with or without memory and are considered for the future transmission standards [2]. The polynomial or Volterra series models [3], [4] are usually considered in wireless communications and signal processing studies because they allow tractable signal processing, see for instance [5]. In this paper, we focus on two well-established nonlinear memoryless models, namely the Saleh [1] for travelling wave tube (TWT) amplifier and the order three Volterra series models. Notice that models for TWT amplifier onboard satellite communication systems regain interest with non-terrestrial networks (NTN) trend.

We address the matching between these two models in the mean square error (MSE) sense so that the parameters of the polynomial model will have meaningful values. If solving the complex valued MSE minimization problem is not too complicated, evaluating the nonlinear moments involved in the solution is difficult and implies generalized exponential integral functions. Both are original contributions, to the best of our knowledge. In [6], [7], the authors approximate HPA models with polynomial functions, and evaluate numerically the coefficients using the classical least square method.

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II. SYSTEM MODEL

We consider a HPA designed to amplify an input signal $x_a(t)$. In this paper, we assume that this signal is a complex cyclostationary circularly-symmetric process, where μ_2 , μ_4 and μ_6 define its real-valued second, fourth and sixth moments as $\mu_k = \mathbb{E} [|x_a(t)|^k]$, for $k = 2, 4, 6$.

Let $y_a(t)$ be the output analytic signal of the HPA, which is ideally a scalar-multiplied version of the input analytic signal $x_a(t)$. We denote by $y_a^S(t)$ the output signal of the HPA using the Saleh model for TWT amplifier [1], given by

$$y_a^S(t) = \mathbf{g}(r)e^{j(\Phi(r)+\phi)} \quad (1)$$

where $r = |x_a(t)|$ and $\phi = \arg(x_a(t))$. The functions $\mathbf{g}(\cdot)$ and $\Phi(\cdot)$ represent the AM/AM and AM/PM conversion respectively. These transfer characteristics are given by

$$\mathbf{g}(r) = \frac{\alpha_r r}{1 + \beta_r r^2}, \quad \text{and} \quad \Phi(r) = \frac{\alpha_\phi r^2}{1 + \beta_\phi r^2}. \quad (2)$$

where the coefficients α_r , β_r , α_ϕ and β_ϕ are real-valued parameters and characterize the nonlinear distortion of the HPA [1]. Finally the output signal of the Saleh model is

$$\begin{aligned} y_a^S(t) &= \frac{\alpha_r |x_a(t)|}{1 + \beta_r |x_a(t)|^2} e^{j\left(\frac{\alpha_\phi |x_a(t)|^2}{1 + \beta_\phi |x_a(t)|^2} + \arg(x_a(t))\right)} \\ &= \frac{\alpha_r x_a(t)}{1 + \beta_r |x_a(t)|^2} e^{j\left(\frac{\alpha_\phi |x_a(t)|^2}{1 + \beta_\phi |x_a(t)|^2}\right)}. \end{aligned} \quad (3)$$

The polynomial model is commonly used in wireless communications [4], [6]. We denote by $y_a^V(t)$ the output signal of the HPA using the order three Volterra series, given by

$$\begin{aligned} y_a^V(t) &= \gamma_1 x_a(t) + \gamma_3 x_a(t)x_a(t)\overline{x_a(t)} \\ &= \left(\gamma_1 + \gamma_3 |x_a(t)|^2\right) x_a(t), \end{aligned} \quad (4)$$

where $\overline{\cdot}$ stands for the complex-conjugate. The coefficients γ_1 and γ_3 are complex-valued parameters and characterize the nonlinear distortion of the HPA [4].

The goal of this paper is to find a link between the two above-mentioned models.

III. MEAN SQUARE ERROR BETWEEN SALEH AND POLYNOMIAL MODEL

In this section, the match between Saleh and third order polynomial models is studied. We first express the mean square

error between the output signals of the Saleh and polynomial models, defined as

$$\begin{aligned} J(\gamma_1, \gamma_3) &= \mathbb{E} \left[\left| y_a^S(t) - y_a^V(t) \right|^2 \right] \\ &= \mathbb{E} \left[\left| y_a^S(t) \right|^2 \right] + \mathbb{E} \left[\left| y_a^V(t) \right|^2 \right] \\ &\quad - 2\Re \left\{ \mathbb{E} \left[y_a^S(t) \overline{y_a^V(t)} \right] \right\} \end{aligned} \quad (5)$$

where the mean expectation is taken over the input signal $x_a(t)$ distribution. The minimization of the MSE with respect to γ_1 and γ_3 is

$$\begin{aligned} (\gamma_1^*, \gamma_3^*) &= \arg \min_{\gamma_1, \gamma_3} J(\gamma_1, \gamma_3) \\ &= \arg \min_{\gamma_1, \gamma_3} \mathbb{E} \left[\left| y_a^V(t) \right|^2 \right] - 2\Re \left\{ \mathbb{E} \left[y_a^S(t) \overline{y_a^V(t)} \right] \right\} \end{aligned} \quad (6)$$

where

$$\mathbb{E} \left[\left| y_a^V(t) \right|^2 \right] = |\gamma_1|^2 \mu_2 + |\gamma_3|^2 \mu_6 + 2\Re \{ \gamma_1 \overline{\gamma_3} \} \mu_4$$

and

$$\begin{aligned} \mathbb{E} \left[y_a^S(t) \overline{y_a^V(t)} \right] &= \overline{\gamma_1} \mathbb{E} \left[\frac{\alpha_r |x_a(t)|^2}{1 + \beta_r |x_a(t)|^2} e^{j \left(\frac{\alpha_\phi |x_a(t)|^2}{1 + \beta_\phi |x_a(t)|^2} \right)} \right] \\ &\quad + \overline{\gamma_3} \mathbb{E} \left[\frac{\alpha_r |x_a(t)|^4}{1 + \beta_r |x_a(t)|^2} e^{j \left(\frac{\alpha_\phi |x_a(t)|^2}{1 + \beta_\phi |x_a(t)|^2} \right)} \right]. \end{aligned}$$

We introduce the following notations,

$$\begin{aligned} \nu_2 &= \mathbb{E} \left[\frac{\alpha_r |x_a(t)|^2}{1 + \beta_r |x_a(t)|^2} e^{j \left(\frac{\alpha_\phi |x_a(t)|^2}{1 + \beta_\phi |x_a(t)|^2} \right)} \right], \\ \nu_4 &= \mathbb{E} \left[\frac{\alpha_r |x_a(t)|^4}{1 + \beta_r |x_a(t)|^2} e^{j \left(\frac{\alpha_\phi |x_a(t)|^2}{1 + \beta_\phi |x_a(t)|^2} \right)} \right]. \end{aligned}$$

Then, the optimization problem becomes

$$(\gamma_1^*, \gamma_3^*) = \arg \min_{\gamma_1, \gamma_3} f(\gamma_1, \gamma_3),$$

with

$$\begin{aligned} f(\gamma_1, \gamma_3) &= |\gamma_1|^2 \mu_2 + |\gamma_3|^2 \mu_6 + 2\Re \{ \gamma_1 \overline{\gamma_3} \} \mu_4 \\ &\quad - 2\Re \{ \overline{\gamma_1} \nu_2 \} - 2\Re \{ \overline{\gamma_3} \nu_4 \}. \end{aligned}$$

To derive the optimal solution, we first express the partial derivatives

$$\begin{aligned} \frac{\partial f}{\partial \Re \{ \gamma_1 \}} &= 2\Re \{ \gamma_1 \} \mu_2 + 2\Re \{ \gamma_3 \} \mu_4 - 2\Re \{ \nu_2 \} \\ \frac{\partial f}{\partial \Re \{ \gamma_3 \}} &= 2\Re \{ \gamma_3 \} \mu_6 + 2\Re \{ \gamma_1 \} \mu_4 - 2\Re \{ \nu_4 \} \\ \frac{\partial f}{\partial \Im \{ \gamma_1 \}} &= 2\Im \{ \gamma_1 \} \mu_2 + 2\Im \{ \gamma_3 \} \mu_4 - 2\Im \{ \nu_2 \} \\ \frac{\partial f}{\partial \Im \{ \gamma_3 \}} &= 2\Im \{ \gamma_3 \} \mu_6 + 2\Im \{ \gamma_1 \} \mu_4 - 2\Im \{ \nu_4 \}, \end{aligned}$$

then we find the solutions that cancel them, leading to the following linear system,

$$A \begin{pmatrix} \gamma_1 \\ \gamma_3 \end{pmatrix} = \mathbf{b}, \quad \text{where } A = \begin{pmatrix} \mu_2 & \mu_4 \\ \mu_4 & \mu_6 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} \nu_2 \\ \nu_4 \end{pmatrix}.$$

So the optimal solution is obtained when A is invertible, i.e., $\mu_2 \mu_6 \neq \mu_4^2$. In that case, we get

$$\begin{cases} \gamma_1^* = \frac{\mu_6 \nu_2 - \mu_4 \nu_4}{\mu_2 \mu_6 - \mu_4^2} \\ \gamma_3^* = \frac{\mu_2 \nu_4 - \mu_4 \nu_2}{\mu_2 \mu_6 - \mu_4^2} \end{cases}. \quad (7)$$

We notice that the derived optimal solution is valid for non-constant modulus signals for which $\mu_2 \mu_6 \neq \mu_4^2$. For constant modulus signals, all $|x_a(t)|^2 = \mu_2$, so that any γ_3 could be satisfactory, in particular $\gamma_3^* = 0$ and $\gamma_1^* = \nu_2 / \mu_2$. It simplifies to $\gamma_1^* = \frac{\alpha_r}{1 + \beta_r \mu_2} e^{j \frac{\alpha_\phi \mu_2}{1 + \beta_\phi \mu_2}}$ and encompasses entirely for the non-linearity deterministically.

IV. DERIVATION OF THE CROSS-CORRELATION TERMS

In this section, we focus on the derivation of the terms ν_2 and ν_4 . We take an additional assumption on $x_a(t)$ to have a Gaussian distribution with variance μ_2 . This assumption is usually implicit when Shannon rates are analyzed, even if we acknowledge it is an approximation for QAM modulations. Therefore, $\mu_4 = 2\mu_2^2$ and $\mu_6 = 6\mu_2^3$ [8], so that $\mu_2 \mu_6 \neq \mu_4^2$.

We also denote $\rho = |x_a(t)|^2$. The random variable ρ therefore follows an exponential distribution with rate parameter λ , given by

$$\mathbb{E} [\rho] = \frac{1}{\lambda} = \mu_2.$$

A. Expression for ν_2

We have to derive

$$\nu_2 = \mathbb{E} \left[\frac{\alpha_r \rho}{1 + \beta_r \rho} e^{j \left(\frac{\alpha_\phi \rho}{1 + \beta_\phi \rho} \right)} \right].$$

First we use the alternative form $\frac{\alpha \rho}{1 + \beta \rho} = \frac{\alpha}{\beta} \left(1 - \frac{1}{1 + \beta \rho} \right)$, to obtain

$$\begin{aligned} \nu_2 &= \frac{\alpha_r}{\beta_r} e^{j \frac{\alpha_\phi}{\beta_\phi}} \mathbb{E} \left[\left(1 - \frac{1}{1 + \beta_r \rho} \right) e^{-j \frac{\alpha_\phi}{\beta_\phi} \frac{1}{1 + \beta_\phi \rho}} \right] \\ &= \frac{\alpha_r}{\beta_r} e^{j \frac{\alpha_\phi}{\beta_\phi}} \left(\mathbb{E} \left[e^{-j \frac{\alpha_\phi}{\beta_\phi} \frac{1}{1 + \beta_\phi \rho}} \right] - \mathbb{E} \left[\frac{e^{-j \frac{\alpha_\phi}{\beta_\phi} \frac{1}{1 + \beta_\phi \rho}}}{1 + \beta_r \rho} \right] \right). \end{aligned}$$

Then we decompose the exponential function in series, leading to

$$\begin{aligned} \nu_2 &= \frac{\alpha_r}{\beta_r} e^{j \frac{\alpha_\phi}{\beta_\phi}} \sum_{n \in \mathbb{N}} \frac{1}{n!} \left(-j \frac{\alpha_\phi}{\beta_\phi} \right)^n \\ &\quad \times \left(\mathbb{E} \left[\frac{1}{(1 + \beta_\phi \rho)^n} \right] - \mathbb{E} \left[\frac{1}{(1 + \beta_r \rho)(1 + \beta_\phi \rho)^n} \right] \right). \end{aligned}$$

If we assume that $\beta_r = \beta_\phi$, we have

$$\begin{aligned} \nu_2 &= \frac{\alpha_r}{\beta_r} e^{j \frac{\alpha_\phi}{\beta_\phi}} \sum_{n \in \mathbb{N}} \frac{1}{n!} \left(-j \frac{\alpha_\phi}{\beta_r} \right)^n \\ &\quad \times \left(\mathbb{E} \left[\frac{1}{(1 + \beta_r \rho)^n} \right] - \mathbb{E} \left[\frac{1}{(1 + \beta_r \rho)^{n+1}} \right] \right). \end{aligned}$$

The difficulty is now the evaluation of the expectation.

We define the random variable Z as

$$Z = \frac{1}{1 + \beta_r \rho}.$$

In the following, we try to express the moments of the random variable Z , given by

$$M_n = \mathbb{E}[Z^n] = \mathbb{E}\left[\frac{1}{(1 + \beta_r \rho)^n}\right].$$

Going back to the definition of the expectation, we have

$$M_n = \int_0^{+\infty} \frac{1}{(1 + \beta_r x)^n} \lambda e^{-\lambda x} dx.$$

First we use the change of variable $t = 1 + \beta_r x$ to obtain

$$M_n = \frac{\lambda}{\beta_r} \int_1^{+\infty} \frac{1}{t^n} e^{-\lambda \frac{t-1}{\beta_r}} dt.$$

Then we use a second change of variable $u = \frac{\lambda}{\beta_r} t$ to obtain

$$M_n = \left(\frac{\lambda}{\beta_r}\right)^n e^{\frac{\lambda}{\beta_r}} \int_{\frac{\lambda}{\beta_r}}^{+\infty} \frac{1}{u^n} e^{-u} du.$$

In order to simplify the expression, we introduce the generalized exponential integral function [9] defined by

$$E_n(x) = x^{n-1} \int_x^{+\infty} \frac{e^{-u}}{u^n} du, \quad \forall x > 0, n \in \mathbb{Z}.$$

The final expression of the moments of Z is

$$M_n = \frac{\lambda}{\beta_r} e^{\frac{\lambda}{\beta_r}} E_n\left(\frac{\lambda}{\beta_r}\right).$$

Going back to the initial problem, the expression of ν_2 is

$$\begin{aligned} \nu_2 = \frac{\alpha_r}{\beta_r} e^{j \frac{\alpha_\phi}{\beta_r}} \sum_{n \in \mathbb{N}} \frac{1}{n!} \left(-j \frac{\alpha_\phi}{\beta_r}\right)^n \frac{\lambda}{\beta_r} e^{\frac{\lambda}{\beta_r}} \\ \times \left(E_n\left(\frac{\lambda}{\beta_r}\right) - E_{n+1}\left(\frac{\lambda}{\beta_r}\right)\right). \end{aligned} \quad (8)$$

B. Expression for ν_4

We have to derive

$$\nu_4 = \mathbb{E}\left[\frac{\alpha_r \rho^2}{1 + \beta_r \rho} e^{j \left(\frac{\alpha_\phi \rho}{1 + \beta_\phi \rho}\right)}\right].$$

First, we use the alternative expression $\frac{\alpha_r \rho^2}{1 + \beta_r \rho} = \frac{\alpha_r}{\beta_r} \rho \left(1 - \frac{1}{1 + \beta_r \rho}\right)$, to obtain

$$\nu_4 = \frac{\alpha_r}{\beta_r} \mathbb{E}\left[\rho e^{j \left(\frac{\alpha_\phi \rho}{1 + \beta_\phi \rho}\right)}\right] - \frac{1}{\beta_r} \nu_2.$$

We focus on the first term and using the alternative expression $\frac{\alpha_\phi \rho}{1 + \beta_\phi \rho} = \frac{\alpha_\phi}{\beta_\phi} \left(1 - \frac{1}{1 + \beta_\phi \rho}\right)$, we obtain

$$\mathbb{E}\left[\rho e^{j \left(\frac{\alpha_\phi \rho}{1 + \beta_\phi \rho}\right)}\right] = e^{j \frac{\alpha_\phi}{\beta_\phi}} \mathbb{E}\left[\rho e^{-j \frac{\alpha_\phi}{\beta_\phi} \left(\frac{1}{1 + \beta_\phi \rho}\right)}\right].$$

Then we decompose the exponential function in series to obtain

$$\mathbb{E}\left[\rho e^{j \left(\frac{\alpha_\phi \rho}{1 + \beta_\phi \rho}\right)}\right] = e^{j \frac{\alpha_\phi}{\beta_\phi}} \sum_{n \in \mathbb{N}} \frac{1}{n!} \left(-j \frac{\alpha_\phi}{\beta_\phi}\right)^n \mathbb{E}\left[\frac{\rho}{(1 + \beta_\phi \rho)^n}\right].$$

The difficulty is located in the derivation of the expectation.

We define the random variable Z_n as

$$Z_n = \frac{\rho}{(1 + \beta_\phi \rho)^n}.$$

The expectation of Z_n is

$$\mathbb{E}[Z_n] = \int_0^{+\infty} \frac{x}{(1 + \beta_\phi x)^n} \lambda e^{-\lambda x} dx.$$

With the change of variable $t = 1 + \beta_\phi x$, we obtain

$$\begin{aligned} \mathbb{E}[Z_n] &= \frac{\lambda}{\beta_\phi^2} \int_1^{+\infty} \frac{t-1}{t^n} e^{-\frac{\lambda}{\beta_\phi}(t-1)} dt \\ &= \frac{\lambda}{\beta_\phi^2} \int_1^{+\infty} \frac{1}{t^{n-1}} e^{-\frac{\lambda}{\beta_\phi}(t-1)} dt \\ &\quad - \frac{\lambda}{\beta_\phi^2} \int_1^{+\infty} \frac{1}{t^n} e^{-\frac{\lambda}{\beta_\phi}(t-1)} dt. \end{aligned}$$

If we assume that $\beta_r = \beta_\phi$, the two terms have been already derived previously, leading to

$$\mathbb{E}[Z_n] = \frac{1}{\beta_r} (M_{n-1} - M_n)$$

Finally, we have

$$\nu_4 = \frac{\alpha_r}{\beta_r^2} e^{j \frac{\alpha_\phi}{\beta_r}} \sum_{n \in \mathbb{N}} \frac{1}{n!} \left(-j \frac{\alpha_\phi}{\beta_r}\right)^n (M_{n-1} - 2M_n + M_{n+1}). \quad (9)$$

V. NUMERICAL RESULTS

The input signal $x_a(t)$ has a complex circularly-symmetric Gaussian distribution, with variance μ_2 . The number of realization for the input signal $x_a(t)$ is 10^6 . The coefficients values for the Saleh model are $\alpha_r = 1$, $\alpha_\phi = 0.26$ and $\beta_r = \beta_\phi = 0.25$. The coefficients values for the matched Volterra model are computed with (7). The output signals $y_a^S(t)$ and $y_a^V(t)$ are evaluated using (3) and (4) respectively.

A. Validation of ν_2 and ν_4

We first want to validate the expressions of ν_2 and ν_4 in (8) and (9) respectively. In Fig. 1, we plot the Euclidean distance between numerical and analytical evaluation of ν_2 and ν_4 in solid and dotted line respectively, versus the number of terms in the summation, i.e. the order of the power series decomposition for the exponential function.

In Fig. 2, we plot the Euclidean distance between numerical and analytical evaluation of ν_2 versus the second moment μ_2 , for different numbers of considered terms of the summation series in (8).

In Fig. 3, we plot the Euclidean distance between numerical and analytical evaluation of ν_4 versus the second moment μ_2 , for different numbers of considered terms of the summation series in (9). In order to be sufficiently accurate whatever the second moment μ_2 , the analytical evaluations must consider at least the first 5 terms for the summation. In the sequel, we consider the 10th-order power series decomposition of the exponential function in (8) and (9).

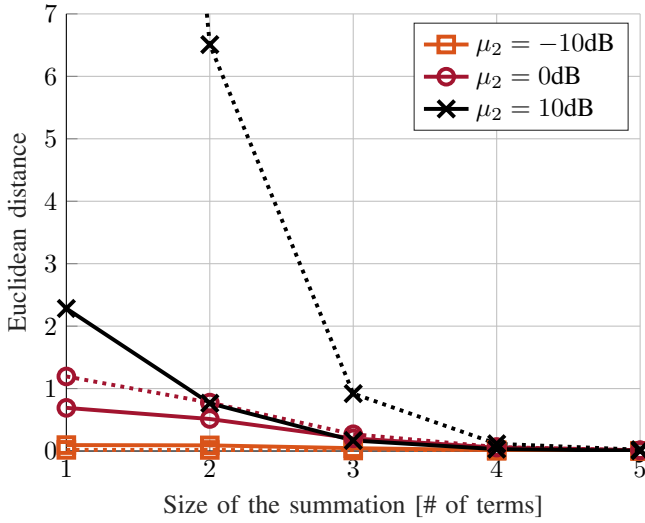


Fig. 1. Euclidean distance between analytical and numerical evaluation of ν_2 and ν_4 , in solid and dotted line respectively, versus number of considered terms of the power series in the analytical evaluation, for three values of the second moment μ_2 .

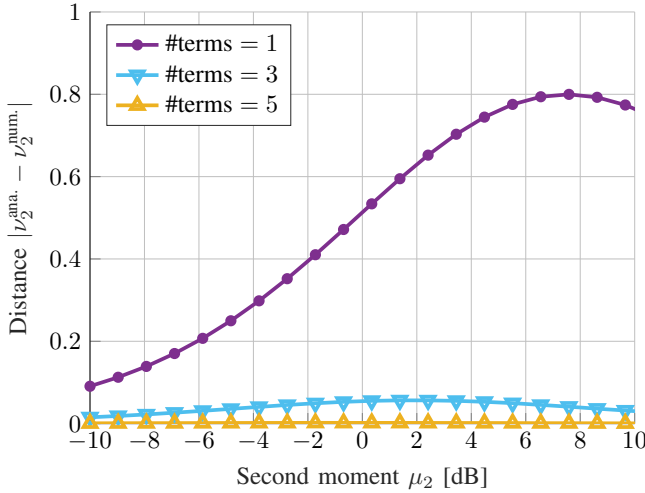


Fig. 2. Euclidean distance between analytical and numerical evaluation of ν_2 versus the second moment μ_2 , for different numbers of considered terms of the power series in the analytical evaluation of the second moment μ_2 .

In Fig. 4, we plot the Euclidean distance between analytical evaluations of ν_2 and ν_4 , in solid and dotted line respectively, with the true second moment μ_2 and altered version, versus the second moment μ_2 . The error on μ_2 is uniformly distributed with a standard deviation ϵ . The analytical expression of ν_4 is more sensitive to a mismatch estimation of the second moment of the input signal.

B. Comparison between Saleh and Volterra

In Tab. I, we display the optimal values γ_1^* and γ_3^* for a given second moment μ_2 . We observe that the coefficients values are complex and are modified when the second moment μ_2 of the input signal changes. To our best knowledge, this is never considered in the state-of-the-art.

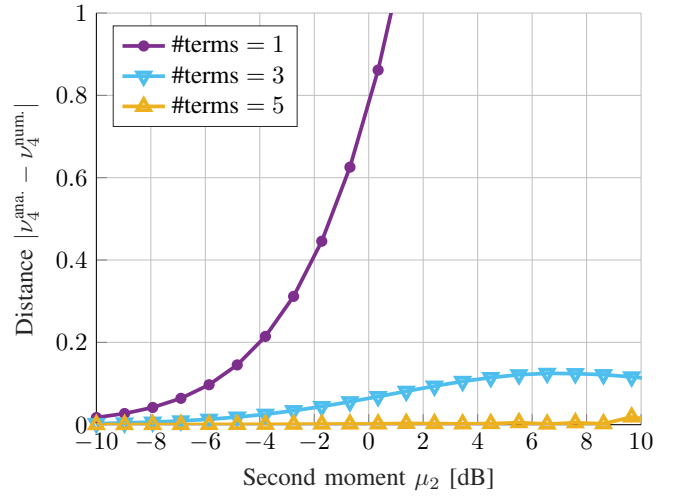


Fig. 3. Euclidean distance between analytical and numerical evaluation of ν_4 versus the second moment μ_2 , for different numbers of considered terms of the power series in the analytical evaluation of the second moment μ_2 .

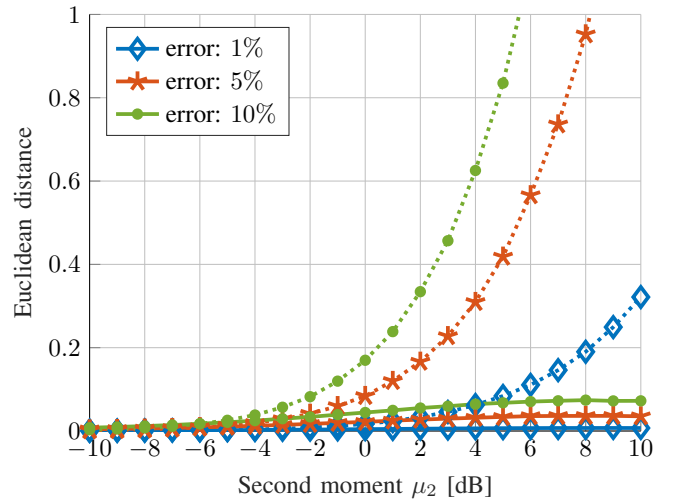


Fig. 4. Euclidean distance between analytical evaluations of ν_2 and ν_4 with true and altered variance $\epsilon \in \{1, 5, 10\}$ %, in solid and dotted line respectively, versus the second moment μ_2 .

Next, we compare the amplified signal for the Saleh model $y_a^S(t)$ and for the matched Volterra model $y_a^V(t)$ with the values of γ_1^* and γ_3^* corresponding to the second order moment of μ_2 . In Fig. 5, we plot the second moment of $y_a^S(t)$ and $y_a^V(t)$ versus the second moment μ_2 . Fig. 5 corresponds to the AM/AM curves showing a linear zone for $\mu_2 < -2\text{dB}$ and a saturation zone above (note that the HPA is not used for $\mu_2 > 6\text{dB}$). The matching of the models in the saturation zone is satisfactory for $\mu_2 < 4\text{dB}$, the Volterra series model being more severe than the Saleh model.

In Fig. 6, we plot the expectation of the relative phase of the output signal, for the Saleh model and for the matched Volterra model versus the second moment μ_2 of the input signal. Fig. 6 corresponds to the AM/PM distortion.

In Fig. 7, we plot the MSE given by (5) between Saleh

TABLE I
VALUES OF γ_1 AND γ_3 FOR A GIVEN SECOND MOMENT μ_2 .

μ_2 [dB]	γ_1^*	γ_3^*
-10	0.9907 + 0.0122i	-0.2237 + 0.1896i
0	0.8801 + 0.1386i	-0.1088 + 0.0282i
10	0.3145 + 0.2206i	-0.0070 - 0.0036i

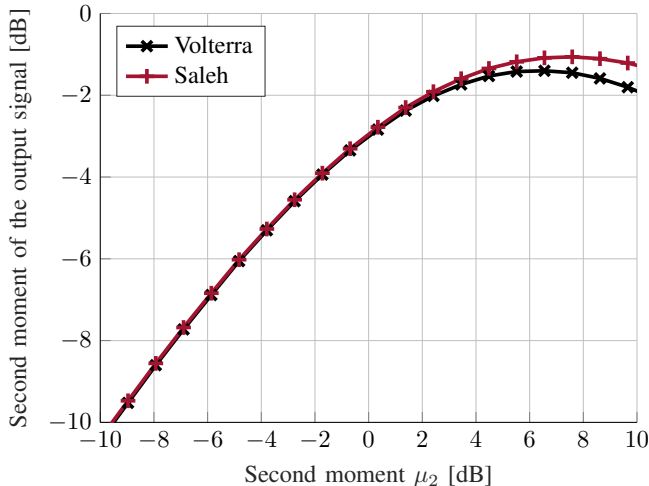


Fig. 5. Second moment of the output signal versus second moment μ_2 of the input signal, for the Saleh model and the matched Volterra model.

model and matched Volterra model versus the second moment μ_2 for the optimal values γ_1^* and γ_3^* . The MSE is very low for $\mu_2 < 0$ dB, and below 2% for $\mu_2 < 4$ dB.

VI. CONCLUSION & PERSPECTIVES

We have derived the analytical expression of the third order Volterra series that approximates at best a signal non-linearly amplified by a HPA following a Saleh model. The resulting terms have complicated expression of the signal statistics.

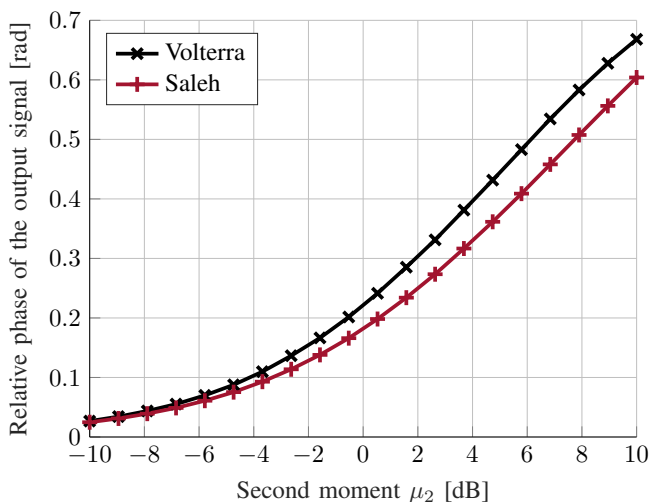


Fig. 6. Second moment of the output signal versus second moment μ_2 of the input signal, for the Saleh model and the matched Volterra model.

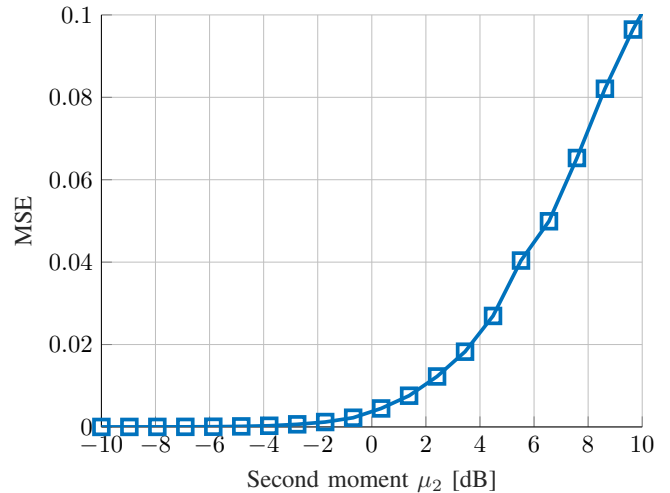


Fig. 7. Second moment of the output signal versus second moment μ_2 of the input signal, for the Saleh model and the matched Volterra model.

Under a Gaussian approximation, they can be expressed with respect to sums of generalized exponential integrals that can be numerically approximated by their first terms. These approximations are validated numerically and show that the usage of the Volterra series should be function of the input signal mean power.

We use these expressions to perform improved power allocation in a satellite transmission context [10].

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