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► **To cite this version:**

Caroline Pascal, Pierre Marchand, Alexandre Chapoutot, Olivier Doaré. Automated far-field sound field estimation combining robotized acoustic measurements and the boundary elements method. 2024. hal-04601337

HAL Id: hal-04601337

<https://hal.science/hal-04601337>

Preprint submitted on 4 Jun 2024

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**Automated far-field sound field estimation combining robotized acoustic
measurements and the boundary elements method**

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1 The identification and reconstruction of acoustic fields radiated by unknown struc-
2 tures is usually performed using either Sound Field Estimation (SFE) or Near-field
3 Acoustic Holography (NAH) techniques. The latter turns out to be especially useful
4 when data is only available close to the source, but information throughout the whole
5 space is needed.

6 Yet, the lack of amendable and efficient implementations of state-of-the-art solu-
7 tions, as well as the laborious and often lengthy deployment of acoustic measurements
8 continue to be significant obstacles to the practical application of such methods.

9 The purpose of this work is to address both problems. First, a completely auto-
10 mated metrology setup is proposed, in which a robotic arm is used to gather extensive,
11 yet accurate geometric and acoustic data without any human intervention. The im-
12 pact of the robot on acoustic pressure measurements has been cautiously estimated,
13 and proved to remain negligible within a defined validity frequency range.

14 The sound field prediction is then tackled using the Boundary Element Method
15 (BEM), and implemented using the *FreeFEM++* BEM library. Numerically simu-
16 lated measurements have allowed us to assess the method accuracy, which matches
17 theoretically expected results, and robustness against positioning inaccuracies, pro-
18 vided that the robot is carefully calibrated.

19 The overall solution has been successfully tested using actual robotized measure-
20 ments of an unknown loudspeaker, with a reconstruction error of less than 30 % on
21 the previously defined validity frequency range.

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22 I. INTRODUCTION

23 The reconstruction of the acoustic pressure field radiated by an arbitrary source, using
24 only a limited set of actual microphone measurements, is a common task in experimental
25 acoustics. In particular, *Sound Field Estimation* (SFE) aims to accurately predict the far-
26 field properties of a studied source based on near-field measurements. In that way, it is
27 possible to study the impact of the source in its environment, without having to perform
28 wide range measurements at a large distance from the source. This method has proven to
29 be relevant for loudspeaker characterization (Klippel, 2022), but also for the study of other
30 noise sources, such as aircraft (Martinus *et al.*, 2005), UAVs (Bi *et al.*, 2021), cars (Wu and
31 Wu, 1998) (Gade *et al.*, 2014) (Tao and Ren, 2010) or trains (Wang and Bei, 2017) to cite
32 a few.

33 On the other hand, *Near-field Acoustic Holography* (NAH) objective is to estimate the
34 surfacic acoustic features of a radiating source using near-field measurements. Unlike SFE,
35 NAH tries to solve an inverse problem, and requires an additional regularization step. Yet,
36 NAH algorithms may also be used to solve SFE problems (Salin and Kosteev, 2020), and
37 will equivalently be considered in the following.

38 The foundations of SFE and NAH rely on two main aspects: the practical acoustic
39 measurements, and the reconstruction method.

40 *a. Acoustic measurements.* One of the main challenges encountered while performing
41 SFE acoustic measurements is the need to perform numerous accurately positioned, near-
42 field measurements.

43 Traditionally, this issue has been tackled by the use of microphone arrays, consisting of
44 a rigid assembly of microphones, which can be fixed at a given distance from the source.
45 Such arrays come in various geometries - linear (Gur, 2014), planar (Hald *et al.*, 2008) (Li
46 *et al.*, 2021), (Wang and Bei, 2017), cylindrical (Norris, 1997) (Su *et al.*, 2017) (Gur, 2017),
47 spherical (Bi *et al.*, 2021) (Behler *et al.*, 2012) - but often lack of versatility, as their cost
48 makes it is not foreseeable to create a new array for each studied object. Furthermore, the
49 simultaneous operation of many microphones requires adapted, expensive hardware, as well
50 as a careful calibration of the array (Sugiyama *et al.*, 2023).

51 In order to allow variable resolution and large-scale measurements, the idea of mobile
52 arrays have been quickly introduced: rigid arrays are mounted on one (Havranek *et al.*,
53 2015), two (De La Croix *et al.*, 2005) or three (Szczodrak *et al.*, 2016) (Virovlyansky and
54 Deryabin, 2019) prismatic joints, or one revolute joint (McBride *et al.*, 2020) (Vold *et al.*,
55 2010), and moved around the source to gather the required data. Yet, depending on the
56 axes' layout, some positions and orientations of the microphones remain unreachable.

57 Recent work have proposed to combine motion tracking (Luo *et al.*, 2021) (Fernan-
58 dez Comesana *et al.*, 2015), 2D (Legg and Bradley, 2013) and 3D (Meyer and Döbler, 2006)
59 (Li *et al.*, 2021) (Legg and Bradley, 2014) localization methods with acoustic measurements,
60 so that the microphone, or microphone array, is continuously located during the process. If
61 such approaches lead to a greater versatility and the ability to perform measurements at a
62 larger scale, they are still limited by occlusion issues, and does not enable fully autonomous
63 measurements.

64 Eventually, the use of multiple degrees of freedom robots for acoustic measurements has
65 been considered, with for instance the robotized NAH solution proposed by Klippel [Klippel](#)
66 [and Bellmann \(2016\)](#), using a cylindrical robot with 3 *Degrees Of Freedom* (DOF).

67 The use of full 6 DOF robots in acoustic however still remains scarce, with few applications
68 in non-destructive testing ([Bernhardt et al., 2020](#)) ([Walker and Foged, 2018](#)), room acoustics
69 ([Nolan et al., 2019](#)), and NAH ([Kefer and Lu, 2016](#)) ([Knežević et al., 2018](#)). However, most
70 of these procedures still limit themselves to planar measurements, and are not taking the
71 full benefits of the maneuverability and autonomy of the robot. Furthermore, the actual
72 impact of the robot on the measurements, both in terms of positioning accuracy ([Pascal](#)
73 [et al., 2023](#)) and acoustic footprint, remains to be assessed.

74 *b. Sound field estimation.* In its most generic form, SFE allows for a temporal as well
75 as a spatial estimation of the sound field, that is, the sought acoustic quantity is expressed
76 as a function of both space and time. In our case however, we will assume that the studied
77 source is stationary, and that the sound field is only a function of space, with a harmonic
78 time dependency.

79 As for most identification problems, SFE relies on two major steps: modeling and iden-
80 tification. The modeling step essentially translates the way the sound field is represented.
81 Two options were mainly investigated in the literature: the sound field may be written as
82 an expansion of elementary solutions of the Helmholtz equation, or as a solution of this
83 equation, identified over a given discretized function space.

84 The first approach was originally investigated by [Maynard et al. \(1985\)](#), using a *Spatial*
85 *Fourier Transform* (SFT) method. It however suffers from a lack of flexibility, as the el-

86 elementary solutions are limited to a few simple separable geometries, and is impacted by
87 aliasing effects.

88 The expansion approach has then been intensively investigated in order to cope with SFT
89 issues. The *Statistically Optimized NAH* (SONAH) method proposed by [Hald \(2009\)](#), dealt
90 with the frequential aliasing issue with a continuous approach of the wave-number space.
91 The *Helmholtz Equation Least Squares* (HELs) method proposed by [Wu \(2000\)](#) and [Wu](#)
92 [\(2015\)](#) opted for spheroidal elementary solutions as the expansion set. The *Equivalent Source*
93 *Method* (ESM) ([Valdivia, 2019](#)) proposed a more tangible modeling approach, using isolated
94 and distributed ([Semenova and Wu, 2005](#)) artificial sources as elementary solutions. In
95 particular, the spherical harmonics expansion ([Klippel, 2022](#)) uses punctual acoustic sources,
96 defined by a finite sum of spherical harmonics, which allows for an accurate reconstruction
97 of the sound field with a limited expansion set. Yet, in the end, all these methods require *i*)
98 a clever choice of expansion set ([Canclini et al., 2017](#)) ([Antoni, 2012](#)) and *ii*) a discretization
99 choice on this set to ensure a good trade-off between reconstruction accuracy, computational
100 resources and the risk of over-fitting ([Chardon et al., 2013](#)) ([Semenova and Wu, 2005](#)).

101 The identification of the expansion coefficients is often performed using a least-squares
102 approach, combined with a regularization scheme to ensure the reconstruction regularity,
103 especially when solving NAH problems ([Williams, 2001](#)) ([Martinus et al., 2007](#)). Recent
104 developments, benefitting from the compressive sensing theory, have proposed a sparse iden-
105 tification of the expansion coefficients ([Hald, 2016](#)), in order to avoid the energy spreading
106 effect caused by an L^2 norm minimization. An L^0 or L^1 norm minimization ([Chardon et al.,](#)
107 [2012](#)) ([Hald, 2018](#)) indeed leads to a concentration of the energy on a few expansion func-

108 tions, or artificial sources, hence allowing for a better reconstruction accuracy far from the
109 measurements.

110 On the other hand, several work opted for a direct resolution of the Helmholtz equation,
111 using an element-based approach. Whereas the *Finite Elements Method* (FEM) (Ungnad
112 and Sachau, 2019) received small attention, mainly because of its impractical adaptation
113 to infinite domains and pollution effects, most efforts were put on the *Boundary Elements*
114 *Method* (BEM) (Bai, 1992) (Kim and Ih, 1996) (Martinus *et al.*, 2007) (Herrin *et al.*, 2010)
115 (Bi *et al.*, 2019) (Luo *et al.*, 2021). In comparison to the expansion approach, BEM offers
116 a higher flexibility, as the only requirement is that measurements are to be performed on
117 the nodes of a surfacic mesh. In particular, the impact of the choice of the discretized
118 function space can be predicted *a priori*, and the method is not impacted by frequential
119 aliasing, or by the placement of artificial sources. However, such flexibility comes at a cost,
120 as computations are inherently more complex, and resources consuming.

121 It should be noted that the recent emergence of learning-based methods in physics-based
122 problems have led to the use of convolutional (Alguacil *et al.*, 2021) and physics informed
123 (Chen *et al.*, 2023) (Olivieri *et al.*, 2021) (Shigemi *et al.*, 2022) neural networks to solve the
124 SFE problem. Although these methods have shown promising results, they are still limited
125 by the need of a large amount of data, and the lack of physical guarantees and interpretations
126 of the results.

127 Despite their great diversity, all these methods still remain scarcely implemented, es-
128 pecially in open-source software, and often prove to be poorly coupled with the actual
129 measurements step.

130 *c. Problem statement.* The practical setup of SFE tools is hindered by two major
131 drawbacks: the lack of efficient implementation of the existing numerical methodologies,
132 and the time-consuming and tedious roll-out of acoustic measurements.

133 *d. Contributions.* We present a novel acoustic sound field estimation methodology,
134 handling both the measurements and reconstruction steps. The measurements are performed
135 using a 7 DOF arm robot, allowing for numerous and versatile measurements, and the
136 reconstruction is performed using an optimized parallel implementation of BEM (Hecht,
137 2012)(Marchand, 2020). Bearing in mind that the quality of a metrology system depends
138 on the impact of each item in the acquisition chain, the actual impact of the robot on the
139 measurements is assessed, and the underlying effects on the reconstructed sound field are
140 discussed.

141 II. ACOUSTIC ROBOTIZED MEASUREMENTS

142 One of the main contribution of this paper is the introduction of a robotic arm to per-
143 form accurate tri-dimensional acoustic measurements in full autonomy. The details of the
144 experimental setup, and the impact of the robot on the measurements are presented in this
145 section.

146 A. Measurements setup

147 Drawing inspiration from previous works (Walker and Foged, 2018) (Nolan *et al.*, 2019)
148 (Knežević *et al.*, 2018), we designed a complete measurement setup (*cf.* Fig. 1) combining a
149 Franka Robotics Panda serial robot, equipped with a numerically manufactured microphone



FIG. 1. Robotized acoustic measurements setup.

150 prop, allowing the microphone to be held at a distance from the robot while performing the
 151 measurements. The 7 degrees of freedom of the serial robot allow for an increased maneu-
 152 verability and a simplified collision avoidance, by offering multiple possible configurations
 153 for the same microphone position and orientation.

154 The handling of the robot inverse kinematics and motion planning computation is over-
 155 seen by the Moveit (Coleman *et al.*, 2014) library implemented in *Robot Operating System*
 156 (ROS) (Quigley *et al.*, 2009), which hand fully bypasses the impractical proprietary soft-
 157 ware. This library also offers off-the-shelf tools for self and external collision avoidance, and
 158 supports the addition of geometric constraints during inverse kinematics resolution. The
 159 latter asset was successfully exploited to free an additional degree of freedom associated to
 160 the rotation around the microphone axis, which does not impact the measurement.

161 On the microphone side, the support tool was designed to fit most Bruel & Kjaer and
 162 GRAS microphone pre-amplifiers. The acoustic measurements are handled by the *measpy*
 163 (Doaré, 2024) Python library, which generically handles signals provided by classical sound
 164 cards, and National Instruments acquisition devices. In our current set up, a Bruel & Kjaer

165 4190 free-field microphone mounted on a GRAS 26AJ preamplifier were used, and recorded
166 using a Behringer UMC404HD sound card.

167 All features were finally bundled up in a single ROS package *robot_arm_acoustics* (Pascal,
168 2024), allowing to easily define the robotic cell collisions objects, and plan autonomous
169 measurements routines along generic trajectories (circles, spheres, lines, etc.).

170 B. Robotized measurements validity hypothesis

171 The advantages in versatility and autonomy of the robotized measurements setup comes
172 at the cost of a more complex measurement processing and inherently bulkier installation.
173 To ensure the measurements quality, three hypotheses are derived, and assessed.

174 *a. Hypothesis 1: Time-invariant sound source.* In comparison to usual microphone-
175 array-based measurements setups, where all acquisitions are performed simultaneously, robo-
176 tized acoustic measurements are performed sequentially. As our work focuses on the study of
177 electroacoustic measurements, where the response to an input signal is measured, this lack
178 of synchronicity is not a problem as long as the transfer function between these correlated
179 signals is computed.

180 In particular, our focus is set on the estimation of the sound field radiated by a JBL
181 FLip 2, which was monaurally fed on its left channel with a white noise signal of 10 seconds,
182 containing frequencies between $f_{min} = 10$ Hz and $f_{max} = 20$ kHz. The transfer functions
183 between the recorded acoustic pressure (in Pa), and the signal generated by the sound card
184 (in V) were then computed using the Welch (1967) method, and smoothed over 1/12 octave
185 bands.

186 Additionally, the behavior of the studied sound source must remain the same throughout
 187 the whole characterization process, by generating the same output signal for a given input
 188 signal. In other words, any measurement at a given location must generate the same results,
 189 regardless to the time at which it was performed. As this hypothesis heavily depends on the
 190 studied source, it was assessed experimentally by performing a series of three consecutive
 191 measurements with the robot fixed in the same configuration (*cf.* first configuration on
 192 Fig. 3).

The obtained transfer functions and the corresponding modulus errors are displayed on
 Fig. 2. The error between two complex transfer functions H_1 and H_2 is expressed as the
 logarithm of the modulus and phase of their ratio:

$$\epsilon(H_1, H_2) = \left(20 \log \left(\frac{|H_1|}{|H_2|} \right), \arg(H_1 - H_2) \right),$$

193 where H_2 is the reference transfer function, and in our case, the first transfer function of the
 194 series.

195 As expected from an industrial product, the loudspeaker features a good time invari-
 196 ability, with an average error of 0.25 dB over the whole frequency range. Consequently, the
 197 hypothesis of a time-invariant sound source is considered verified for the studied loudspeaker.

198 *b. Hypothesis 2: Low robot acoustic footprint.* As the robot is not an acoustic transpar-
 199 ent element, it will inherently impact the acoustic measurements. Although parasitic noise
 200 generated by the robot can be filtered during the transfer function computation, thanks
 201 to averaged measurements over a sufficiently long period of time (Welch, 1967), the robot
 202 acoustic footprint is more complex to handle, as it depends on the robot angular configura-
 203 tion.

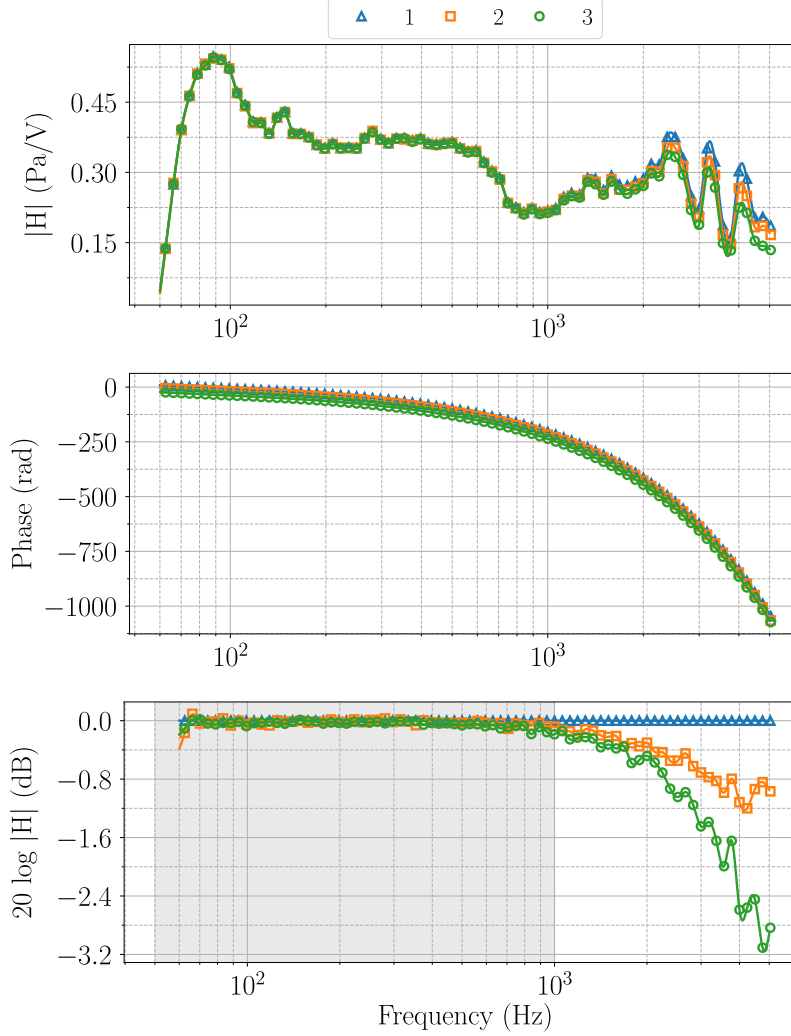


FIG. 2. Time-invariability results obtained for the JBL FLip 2, with the robot set in the first control configuration of Fig. 3. The first two plots represent the computed transfer functions, and the last one the modulus error relative to the first (1) transfer function. The shaded zone highlights the frequency range where the absolute value of the error remains below 0.25 dB.

204 As for the time invariability, our aim was to experimentally assess that the robot acoustic
 205 footprint is actually low enough to be neglected in the following. To achieve so, we performed
 206 two successive series of acoustic measurements, with the microphone rigidly held in the same
 207 position using pliers. For the first series, the robot is moved aside, so that its presence

208 does not impact the acquisition. For the second one, it is placed as if it was holding the
209 microphone in the very same position. As the microphone and its fixations are not shifted
210 between the two series of measurements, their difference will only depend on the acoustic
211 impact of the robot.

212 These two-fold series of measurements were performed at six so-called control configura-
213 tions located around a JBL Flip 2, and are displayed on Fig. 3.

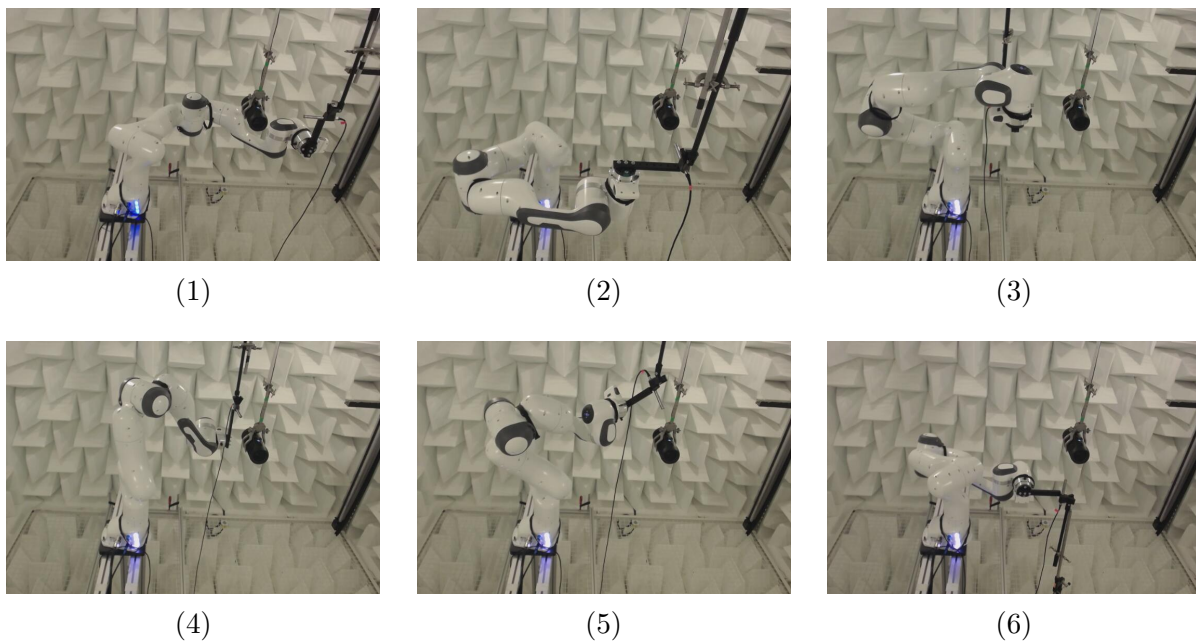


FIG. 3. Robot control configurations for sound source time invariance and robot acoustic footprint assessment.

214 For each control configuration, the error between the transfer functions obtained with
215 and without the robot is displayed on Fig. 4.

216 Unlike the time-invariability results, the robot acoustic footprint has a non-negligible
217 impact on the measurements, especially at higher frequencies. Yet, such behavior was ex-
218 pected: at lower frequencies, and larger wavelengths, the robot becomes small enough not

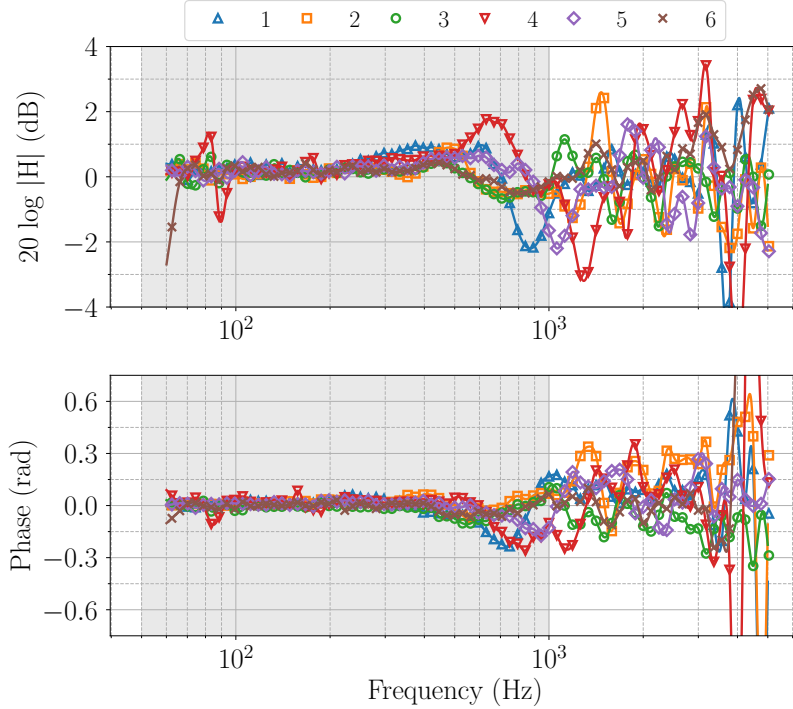


FIG. 4. Robot acoustic footprint errors obtained for the six robot control configurations, relative to the robot-less transfer function. The shaded zone highlights the frequency range where the absolute value of the modulus error remains below 2.25 dB.

219 to disturb the sound propagation. On the other hand, at higher frequencies and smaller
 220 wavelengths, the robot becomes a significant obstacle, and generates reflections of the in-
 221 coming waves towards the microphone. As seen on Fig. 4, this behavior becomes globally
 222 noticeable above $f = 500$ Hz, which corresponds to a wavelength of $\lambda = 0.68$ m, and is
 223 consistent with the robot dimensions. More specifically, configurations (1) and (4) appear
 224 to be the most impactful, as the robot is the closest and most exposed to the sound source.
 225 Inversely, configurations (2) and (6) have a lower impact on the measurements, as the robot
 226 is further away from the source, and exposes a smaller surface to the incoming waves.

227 Nevertheless, the deterioration caused by the robot induced reflections remain under an
228 acceptable level for frequencies below $f = 1$ kHz, with an amplitude error below 2.25 dB and
229 a phase difference of 0.25 rad at its maximum, as shown on Fig. 4. As the sounds generated
230 by the loudspeaker become actually significant only above $f = 50$ Hz, the frequency range
231 $[50 \text{ Hz}, 1 \text{ kHz}]$ will be defined as the validity frequency range for our setup, within which
232 measurements will be considered exploitable.

233 *c. Hypothesis 3: Low robot positioning error.* Although robotic arms are known for
234 their high repeatability and maneuverability, their flawed accuracy is often disregarded when
235 used for metrology purposes. In this context however, an accurate sensor placement is of
236 paramount importance (Martinus *et al.*, 2007), as it directly impacts the quality of the
237 measurements: a misplaced microphone will fail to correctly capture the spatial variations
238 of high frequency sound fields.

239 In order to guard us against this pitfall, an extended work has been carried out to ensure
240 that the positioning accuracy of the Franka Robotics Panda used for our measurements
241 remains lower than 2 mm (Pascal *et al.*, 2023).

242 If not completely removed, the actual impact of the remaining positioning errors on
243 acoustic measurements cannot simply be assessed as for the previous hypothesis, and will
244 be considered verified for the time being.

245 III. THE BOUNDARY ELEMENTS METHOD FOR SOUND FIELD ESTIMA- 246 TION

247 Following on the introduction of our robotized acoustic measurement setup, we will now
248 focus on the theoretical background and numerical resolution of the sound field estimation
249 problem using BEM.

250 A. Previous work

251 The boundary elements method is a numerical method used to solve partial differential
252 equations, that is particularly well suited for problems where a solution is sought over an
253 infinite domain (Sauter and Schwab, 2011) (Kirkup, 2019). The equations derived from
254 BEM are solely defined on the boundary of the domain. Hence, the solution at any field
255 point can be obtained with no need for a domain wide discretization.

256 This boundary-wise approach made BEM a perfect candidate for the resolution of the
257 Helmholtz equation in wave scattering problems (Chandler-Wilde *et al.*, 2012), and in NAH
258 (Bai, 1992) (Luo *et al.*, 2019) (Valdivia and Williams, 2004) (Bi *et al.*, 2019) and SFE
259 problems (Kim and Ih, 1996) (Herrin *et al.*, 2010) (Luo *et al.*, 2021). In the latter case,
260 the practical use of BEM with a free-field hypothesis is often hindered by the required
261 boundary conditions, which consist in a set of acoustic measurements performed on closed
262 mesh surrounding the studied source. However, the use of robotized measurements should
263 conveniently ensure the reachability of the targeted mesh nodes.

264 Most BEM based SFE resolution methods employ collocation techniques (Bai, 1992) (Bi
265 *et al.*, 2019) (Herrin *et al.*, 2010) (Luo *et al.*, 2021), where the boundary integrals are
266 computed by a simple point-wise approximation. If these methods benefits from a simple
267 implementation, and does not require a proper meshing of the measurement surface, it does
268 not take full advantage of BEM variational form.

269 On the contrary, the *Variational Boundary Element Method* (VBEM) approach (Valdivia
270 and Williams, 2004) (Schuhmacher *et al.*, 2003) (Raveendra *et al.*, 1998), solves NAH prob-
271 lems by increasing BEM discretization complexity both spatially and functionally. First, the
272 boundary surfaces are described using higher order elements, allowing for a better descrip-
273 tion of the local curvature, and second, the solution is sought using the Galerkin method with
274 Lagrange polynomial elements. Despite its higher computational complexity, this method
275 offers better overall convergence guarantees.

276 In the remainder of this section, the application of the VBEM to the SFE problem will
277 be detailed, with a particular focus on its theoretical reconstruction errors estimates

278 **B. Problem statement and indirect resolution method**

279 The SFE problem can be formulated as an exterior Dirichlet problem for the Helmholtz
280 equation. We seek a solution of the free-field acoustic equation in the infinite 3D space,
281 knowing its value on a bounded surface.

282 As pictured on Fig. 5, let us denote by $\partial\Omega$ the surface on which the measurements are
283 performed, and by Ω the exterior domain over which the complex sound field p is studied.
284 Unlike SFT based methods, no particular constraint is set on the shape of $\partial\Omega$, but it must

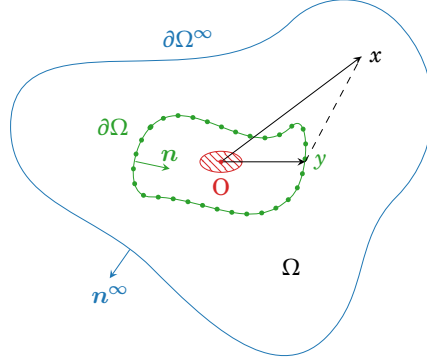


FIG. 5. Dirichlet exterior problem for the Helmholtz equation.

285 define a smooth closed surface around the studied source. In this situation, the Helmholtz
 286 equation, for a constant wave number $k \in \mathbb{R}$, and under the harmonic hypothesis, is written

287 as

$$\left\{ \begin{array}{ll} \Delta p + k^2 p = 0 & \text{in } \Omega, \\ p = p_0 & \text{on } \partial\Omega, \\ \lim_{\partial\Omega^\infty \rightarrow \infty} \left(\frac{\partial}{\partial|x|} - ik \right) p(x) = 0. & \end{array} \right. \quad (1)$$

288 The infinitely away boundary $\partial\Omega^\infty$ is used to introduce the Sommerfeld radiation condi-
 289 tion, which ensures that the solution p is physically acceptable, and that the overall problem
 290 is well-posed.

291 The Green function G associated to Eq. (1) is given by

$$\forall x, y \in \mathbb{R}^3, x \neq y, G(x, y) = \frac{e^{ik|x-y|}}{4\pi|x-y|}, \quad (2)$$

292 and allows us to define the single-layer potential S , and double-layer potential D as (Sauter
 293 and Schwab, 2011)

$$\forall u : \partial\Omega \rightarrow \mathbb{C}, \forall x \in \mathbb{R}^3 \setminus \partial\Omega, S(u)(x) = \int_{\partial\Omega} G(x, y) u(y) d\sigma(y), \quad (3)$$

294 and

$$\forall u : \partial\Omega \rightarrow \mathbb{C}, \forall x \in \mathbb{R}^3 \setminus \partial\Omega, D(u)(x) = \int_{\partial\Omega} \frac{\partial G(x, y)}{\partial \mathbf{n}(y)} u(y) d\sigma(y), \quad (4)$$

295 with u regular enough to be integrated on $\partial\Omega$ and $d\sigma$ defining the surface measure on $\partial\Omega$.

296 The boundary elements method is then built on the proof that both potentials are in-
 297 finitely differentiable solutions of Eq. (1) on $\Omega \setminus \partial\Omega$, and the only requirement to extend it
 298 over all Ω is to find a boundary density u_S or u_D on $\partial\Omega$ such that the boundary conditions
 299 are satisfied. Introducing the Dirichlet trace operator $\gamma_0^{\partial\Omega}$, which extends on $\partial\Omega$ sufficiently
 300 smooth functions defined on $\Omega \setminus \partial\Omega$, these problems can be written as

$$\exists u_S : \partial\Omega \rightarrow \mathbb{C}, \gamma_0^{\partial\Omega} [S(u_S)] = V(u_S) = p_0, \quad (5)$$

301 and

$$\exists u_D : \partial\Omega \rightarrow \mathbb{C}, \gamma_0^{\partial\Omega} [D(u_D)] = \frac{u_D}{2} + K(u_D) = p_0, \quad (6)$$

302 where V and K denote the *Boundary Integral Operators* (BIOs) arising from the Dirichlet
 303 traces of the potentials S and D on $\partial\Omega$.

304 Both BIOs contain singular surfacic integrals, involving the ill-defined evaluation of the
 305 Green function on $\partial\Omega$. Yet, their calculations are still possible but require u_S and u_D to
 306 belong in a particular space of functions with sufficient regularity guarantees, and significant
 307 computational efforts (*e.g.* specific quadrature formulas for singular integrals computation).

308 Eventually, Eqs. (5) and (6) define two *Boundary Integral Equations* (BIEs), which can be
 309 solved using a Galerkin approximation, where u_S and u_D are sought in a finite dimensional
 310 subspace V_h

$$\exists u_S \in V_h, \forall v \in V_h, \int_{\partial\Omega \times \partial\Omega} V(u_S)(y) v(x) d\sigma(x, y) = \int_{\partial\Omega} p_0(x) v(x) d\sigma(x), \quad (7)$$

and

$$\exists u_D \in V_h, \forall v \in V_h, \int_{\partial\Omega \times \partial\Omega} \left[\frac{u_D(y)}{2} + K(u_D)(y) \right] v(x) d\sigma(x, y) = \int_{\partial\Omega} p_0(x) v(x) d\sigma(x). \quad (8)$$

311 The values of u_S (*resp.* u_D) on $\partial\Omega$ are then obtained by solving the fully-populated
 312 linear system associated to Eq. (7) (*resp.* Eq. (8)), and can directly be fed to the potential
 313 formulations of Eq. (3) (*resp.* Eq. (4)) to reconstruct the sound field p over the whole domain
 314 Ω . As the sought quantities are computed using an intermediate potential, this approach of
 315 the boundary elements method is often referred to as the indirect formulation.

316 C. Spurious resonances and hybrid formulations

317 Even though the single and double layer potentials both provide solutions to Eq. (1),
 318 it can be shown that for certain wave numbers, the BIEs Eqs. (7) and (8) are ill-posed,
 319 and will not lead to a unique solution (Kreß and Spassov, 1983) (Valdivia, 2019). This
 320 phenomenon is known as spurious resonances and is specific to the boundary integral formu-
 321 lation. The issue is either solved by adding over determining equations to the BIEs, as for
 322 *Combined Helmholtz Integral Equation Formulation* (CHIEF) method (Schenck, 1968), or by
 323 using a hybrid formulation of BEM, combining both the single and double layer potentials
 324 (Chandler-Wilde *et al.*, 2012) (Marburg and Wu, 2008)

$$\forall u : \partial\Omega \rightarrow \mathbb{C}, \forall x \in \mathbb{R}^3 \setminus \partial\Omega, C(u)(x) = D(u)(x) - i\eta S(u)(x), \quad (9)$$

325 where η is a complex parameter, chosen equal to the studied wave number k . Applied to the
 326 indirect formulation, this combined potential is known as the Brakhage-Werner formulation
 327 (Brakhage and Werner, 1965) and has now become a reference in BEM literature. As for

328 the two other potentials, it provides an infinitely differentiable solution of Eq. (1) on $\Omega \setminus \partial\Omega$,
 329 and may be extended on $\partial\Omega$ through the following BIE

$$\exists u : \partial\Omega \rightarrow \mathbb{C}, \quad \gamma_0^{\partial\Omega} [C(u)] = \frac{u}{2} + K(u) - ikV(u) = p_0. \quad (10)$$

Finally, Eq. (10) leads to the Galerkin equation

$$\begin{aligned} \exists u \in V_h, \forall v \in V_h, \quad & \int_{\partial\Omega \times \partial\Omega} \left[\frac{u(y)}{2} + K(u)(y) - ikV(u)(y) \right] v(x) d\sigma(x, y) \\ & = \int_{\partial\Omega} p_0(x) v(x) d\sigma(x), \quad (11) \end{aligned}$$

330 which can be solved for u , and then fed to Eq. (9) to reconstruct the sound field p over the
 331 whole domain Ω .

332 D. Convergence and error estimates

As mentioned earlier, the VBEM approach provides *a priori* error and convergence estimates on the reconstructed sound field. In particular, [Sauter and Schwab \(2011\)](#) focused on the case where the boundary surface is approximated by surface elements of order ℓ (geometric elements), and the solution approximated by Lagrange elements of order m (algebraic elements). As a consequence of the boundary surface approximation, the BIOs are no longer computed on the boundary $\partial\Omega$ itself, but on a mesh of size h , denoted by $\partial\Omega_h$. Introducing Ψ , the orthogonal projector from $\partial\Omega_h$ to $\partial\Omega$, the boundary conditions of Eq. (1) becomes

$$\forall x \in \partial\Omega, \quad p(x) = p_{0_h}(x) = p_0(\Psi^{-1}(x)).$$

333 Denoting by p the exact solution of Eq. (1) and p_h its computed approximation using the
 334 combined and indirect BEM formulation, we have the following convergence behavior

$$\forall y \in \Omega, |p(y) - p_h(y)| \lesssim C_A h^{2(m+1)} + C_G h^\ell + C_F \|p_0 - p_{0_h}\|_{L^2(\partial\Omega)} \quad (12)$$

The estimate given by Eq. (12) shows the crucial impact of geometric approximation on the overall estimation error. For instance, if the surface is approximated by planar triangles, *i.e.*, $\ell = 1$, the estimation error will decrease at worst as fast as $\mathcal{O}(h)$, regardless of the order of Lagrange elements m . The geometric approximation also partially appears in the term involving the difference between the exact and approximated boundary conditions, through the projection operator Ψ . It can be shown (Ciarlet and Raviart, 1972) that this term also decrease with the mesh size, such that

$$\|p_0 - p_{0_h}\|_{L^2(\partial\Omega)} \lesssim C_R h^{n+1},$$

335 with n increasing with the regularity of the approximated boundary conditions. In particu-
 336 lar, when the boundary conditions are C^∞ , n is high enough so that this term do not impact
 337 the convergence order.

338 As for the resolution of Eq. (1), the estimate of Eq. (12) is only valid for a given value of
 339 the wave number k , which must remain such that $kh \leq 1$ for the result to still hold. This
 340 constraint can be viewed as a spatial equivalent of the Shannon criteria: it is not possible
 341 to correctly reconstruct a given signal of wavelength λ if it is not spatially sampled at a
 342 larger resolution. In practice, this constraint is commonly included by choosing between 3
 343 (Martinus *et al.*, 2007) to 6 (Marburg, 2002) measurements points per studied wavelength.

344 IV. IMPLEMENTATION AND NUMERICAL SIMULATIONS

345 As mentioned in previous section, the versatility and convergence guarantees of BEM
346 come at the cost of complex and resources consuming singular integral computations, and
347 dense matrices factorization. However, benefiting from the rise of high performance paral-
348 lel computing, *FreeFEM++* (Hecht, 2012) proposes a competitive implementation of BEM,
349 relying on the hierarchical matrices' computation tool HTool (Marchand, 2020), and the
350 parallel solvers proposed by PETSc (Balay *et al.*, 1998). Even though *FreeFEM++* handles
351 Lagrange element of order 0 to 2, it currently only supports linear surface elements, hence
352 bounding the expected resolution performances. Despite the limitation identified in sub-
353 section III D, the following results were obtained using P_1 Lagrange elements, as they offer
354 a good trade-off between accuracy and computational cost, while allowing for a simplified
355 mesh generation.

356 A numerical simulation of our BEM based SFE tool was thus implemented in order to
357 assess our implementation agreement with the theoretically expected behaviors, as well as
358 to infer good practices for our measurements setup. This simulation was also used as a
359 mean to evaluate the actual impact of the robotic arm on the estimated acoustic pressure
360 field. The acoustic measurements are simulated using elementary sound sources - the acous-
361 tic infinitesimal dipole and regular dipole - whose analytical pressure fields expressions are
362 sampled over a surrounding spheric mesh of varying diameter and size. Each mesh is gener-
363 ated using a geodesic polyhedron primitive to ensure a uniform distribution of the vertices
364 over the sphere.

365 The estimated sound field is then computed on a circular mesh of diameter 1 m, located
 366 in the $z = 0$ plane, and containing 100 nodes, for frequencies distributed between 100 Hz and
 367 5000 Hz. The presented reconstruction errors are defined as the relative L^2 norm between
 368 the exact and estimated sound fields, computed over the whole circular mesh

$$\epsilon_{L^2,rel} = \frac{\sum_{i=1}^{N_{nodes}} |p_i^{estimated} - p_i^{exact}|^2}{\sum_{i=1}^{N_{nodes}} |p_i^{exact}|^2} \quad (13)$$

369 **A. Agreement with theoretical error estimates**

370 Given the choices of geometric and algebraic elements, the convergence rate of the com-
 371 puted solution towards the exact solution is theoretically expected to lay below $\mathcal{O}(h)$. This
 372 behavior was confirmed and even superseded by our numerical simulations, as a quadratic
 373 convergence rate was actually observed. The corresponding results are shown on on Fig. 6,
 374 where the reconstruction error is plotted against the product of the mesh size h and the
 375 wave number k .

376 As was nonetheless expected, the convergence tends to deteriorate when hk nears 10
 377 (Sauter and Schwab, 2011), while the reconstruction error reaches 10 % as hk increases
 378 towards 1. Enforcing a maximum estimation error of this magnitude, we indeed fall back
 379 on previously mentioned thumb rule of 6 measurements points per wavelength.

380 This thumb rule turns out to be even more crucial for the regular dipole, whose directivity
 381 pattern features sharpening lobes as the frequency increases. In this situation, a finer size
 382 mesh h is required to correctly sample the thin spatial variations of the sound field, and to
 383 avoid a worn out, smoothed reconstruction as seen on Fig. 7.

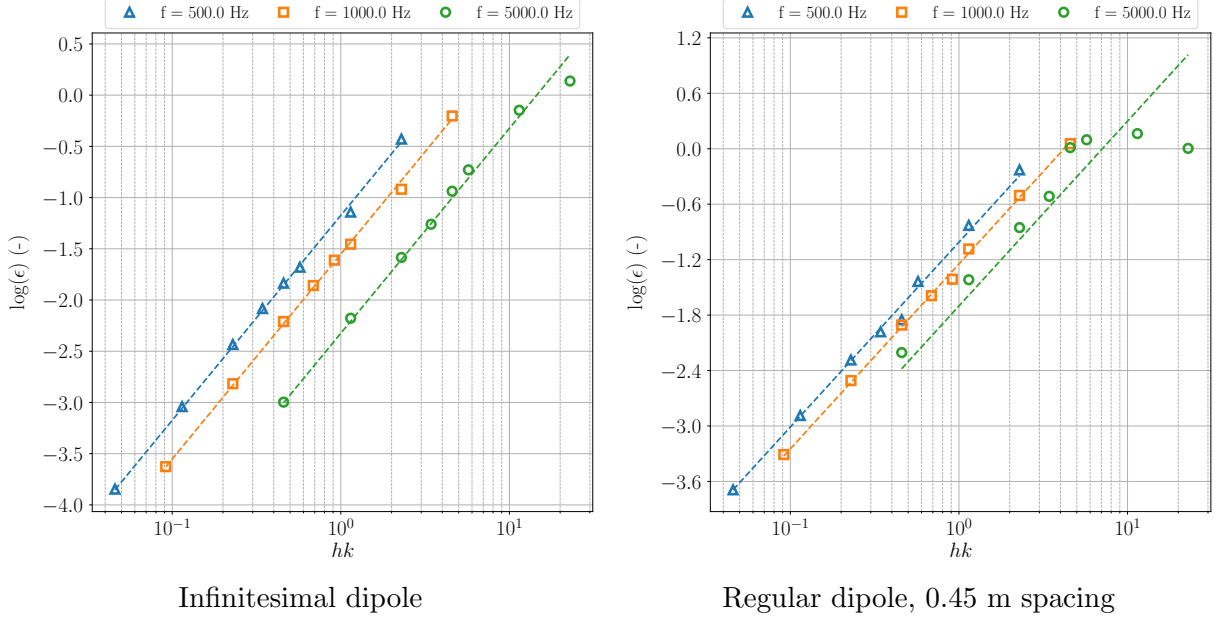


FIG. 6. Reconstruction error obtained for measurements performed on a sphere of diameter $D = 0.5$ m, with increasing mesh sizes h . The dashed line represents the actually observed $\mathcal{O}(h^2)$ convergence rate.

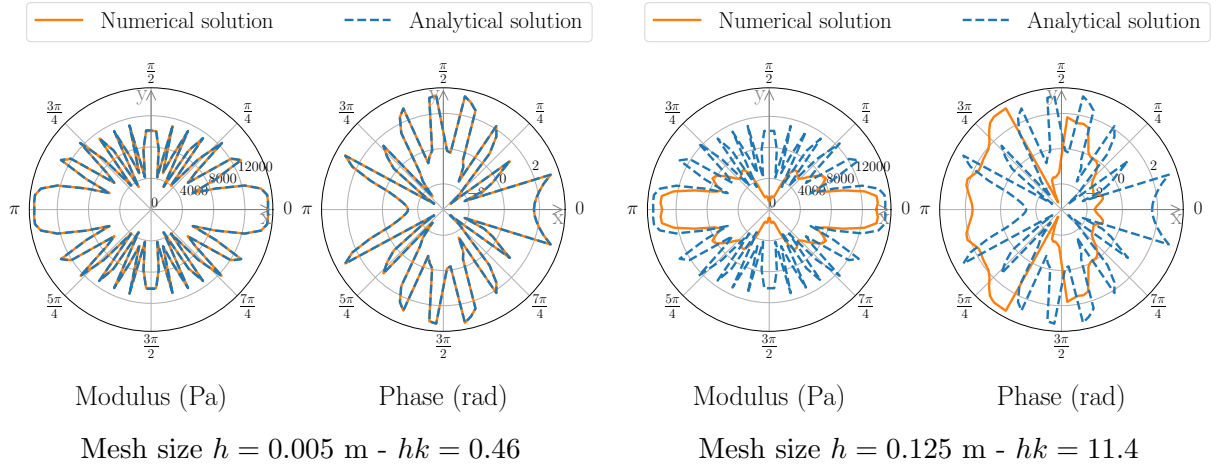


FIG. 7. Reconstructed directivity pattern obtained at $f = 5$ kHz for measurements of a regular dipole with a 0.45 m spacing performed on a sphere of diameter $D = 0.5$ m.

384 Despite the numerous advantages of a smaller mesh size, its practical value remains
385 bounded by the minimal distance between two microphones placements, and is *de facto*
386 constrained by the accuracy of the robot. A calibrated robot (Pascal *et al.*, 2023), with
387 smaller positioning errors, will then allow for a finer mesh size, and a better reconstruction
388 of the sound field, as will be further discussed in subsection IV C.

389 B. Practical guidelines for the measurements setup

390 Whereas the agreement of our SFE tool with the expected error estimates provided an
391 insight for the measurements density, their distance relative to the studied source remains
392 to be discussed.

393 Provided that the mesh size satisfies $kh \leq 1$, a larger mesh will provide a more spatially
394 complete sampling of the sound field (Martinus *et al.*, 2007), and eventually reduce the
395 estimation error. This hypothesis was confirmed by our simulations, as seen on figure Fig. 8,
396 with the reconstruction error decreasing with the diameter of the mesh D when the mesh
397 size h remains constant.

398 Nevertheless, one should bear in mind that this simple numerical simulation does not
399 take into account the physical aspects of real measurements, which heavily depend on the
400 placement of microphones relative to the source. A microphone placed too far from the
401 source will suffer from a high noise to signal ratio, and will be more likely to record reflected
402 sound waves.

403 In practice, general guidelines (Wu, 2000) advise to place the microphones on surfaces
404 conformal to the studied object (*i.e.*, at a similar distance to the studied object), at a

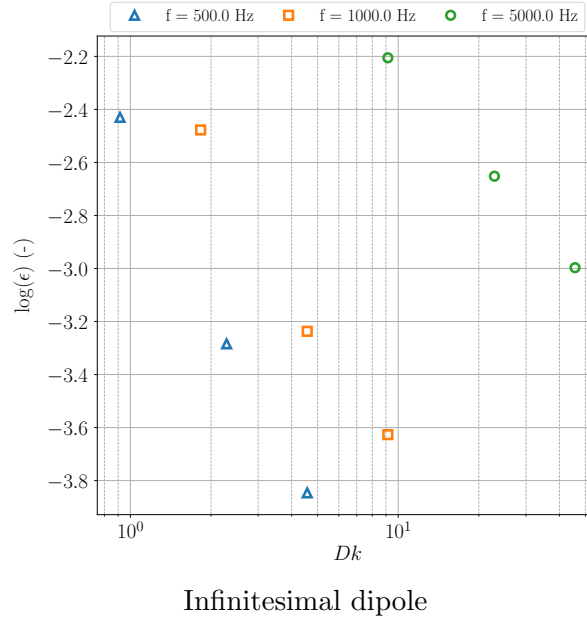


FIG. 8. Reconstruction error obtained for measurements performed on spheres of increasing diameters D , with a mesh size $h = 0.005$ mm. The measurements are assumed to be independent of the distance to the sound source.

405 distance i) small in comparison to the studied object and ii) small in comparison to the
 406 studied wavelength. It is also advised to perform measurements uniformly around the studied
 407 source, so that the radiated around field is completely sampled (Martinus *et al.*, 2007).

408 Finally, from a time-performance point of view, it should be recalled that both the di-
 409 mensions and size of the measurements mesh directly impact the size of the linear system to
 410 solve, and thus the computational cost of the reconstruction algorithm. As shown in Table I,
 411 the monitored computation time is found to increase quadratically with the ratio between
 412 the diameter of the mesh D and its size h , *i.e.*, linearly with the number of measurements.
 413 For the lowest mesh size and highest mesh diameter, the reconstruction of the sought sound
 414 field may still take up to several minutes.

TABLE I. Monitored computation times for the reconstruction of the sound field generated by an infinitesimal dipole at 5 kHz, with increasing ratios between the mesh diameter and size $\frac{D}{h}$.

Computations were run on the 4 cores of an Intel Core i5-4430 @ 3.00 GHz.

$\frac{D}{h}$	5	10	20	50	100
Computation time (s)	1.80	4.38	16.38	244.46	1664.62

415 C. Impact of robotized measurements

416 In section II, we reconned the strong hypothesis that the robot inaccuracies would only
 417 have a negligible impact on the acoustic acquisitions, but with no actual measurements to
 418 back up this claim. In order to comfort our decision, and to tackle this scarcely address
 419 topic, the following section aims to study the impact of a misplacement of the robot on the
 420 sound field estimation error.

421 *a. Introducing the robot inaccuracies.* The robot positioning inaccuracy is embedded
 422 in our simulation in the same way it impacts actual measurements, that is, not by modifying
 423 the boundary mesh itself.

424 Indeed, even with a preliminary calibration, the actual error made by the robot at a given
 425 pose cannot be estimated *a posteriori*, and prevents any correction of the boundary mesh.
 426 Computations are therefore carried on the theoretical mesh, although the measurements are
 427 not exactly performed on the mesh nodes. Consequently, the positioning error is introduced
 428 by adding a Gaussian noise of standard deviation σ_P to each coordinate of the points where

429 the analytical acoustic pressure field is sampled. The obtained noisy measurements are then
 430 assigned to the corresponding unmodified mesh elements, as pictured on Fig. 9.

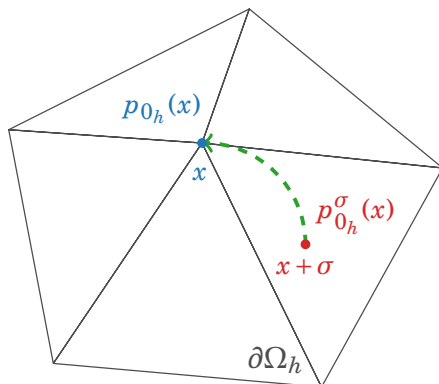


FIG. 9. Illustration of the positioning noise as formulated in Eq. (14). For each node x of the mesh $\partial\Omega_h$, the initial noise-free boundary condition $p_{0_h}(x)$ is replaced by $p_{0_h}^\sigma(x)$.

431 It should be noted that this approach only tackles the position inaccuracies of the robot,
 432 and not the orientation ones. In practice, the latter are expected to have a lower impact on
 433 the measurements, provided that the deployed microphone maintains the same properties
 434 at low incidence angles. In particular, this property is ensured by the Bruel & Kjaer 4190
 435 microphone used in our experimental setup.

436 The procedure was repeated 20 times for various values of σ_P ranging from $0.1h$ to $2h$,
 437 using an infinitesimal dipole as studied source.

b. Impact on the error estimate. Considering our description of the positioning noise,
 the boundary conditions on $\partial\Omega$ are now given by

$$\begin{aligned} \forall x \in \partial\Omega, p(x) &= p_{0_h}^\sigma(x) = p_0^\sigma(\Psi^{-1}(x)) \\ &= p_0(\Psi^{-1}(x) + \sigma) \end{aligned} \tag{14}$$

438 where σ is a random vector containing three random variables, each following a Gaussian
 439 distribution of standard deviation σ_P .

440 Substituting $p_{0_h}^\sigma$ for p_{0_h} in Eq. (12), the error estimate becomes

$$\forall y \in \Omega, |p(y) - p_h(y)| \lesssim C_A h^{2(m+1)} + C_G h^\ell + C_F \|p_0 - p_{0_h}^\sigma\|_{L^2(\partial\Omega)} \quad (15)$$

441 It should be noted that the approximations of p appearing in Eq. (12), p_h , and Eq. (15),
 442 p_h^σ , are not derived from the same boundary density. Yet, the other terms involved in both
 443 estimates remain the same, as the perturbations are added to the measurements directly,
 444 and not to the boundary mesh.

Provided that the value of σ_P is small in comparison to the mesh size h and to the wave-
 length λ , the L^2 norm involving the boundary conditions in Eq. (15) may be approximated
 by

$$\begin{aligned} \|p_0 - p_{0_h}^\sigma\|_{L^2(\partial\Omega)} &\simeq \|p_0 - p_{0_h} - (\nabla p_0 \circ \Psi^{-1}) \cdot \sigma\|_{L^2(\partial\Omega)} \\ &\lesssim \|p_0 - p_{0_h}\|_{L^2(\partial\Omega)} + |\sigma| \|\nabla p_0 \circ \Psi^{-1}\|_{L^2(\partial\Omega)}. \end{aligned}$$

The error estimate of Eq. (15) then becomes

$$\begin{aligned} \forall y \in \Omega, |p(y) - p_h(y)| &\lesssim C_A h^{2(m+1)} + C_G h^\ell \\ &+ C_F \left(\|p_0 - p_{0_h}\|_{L^2(\partial\Omega)} + |\sigma| \|\nabla p_0 \circ \Psi^{-1}\|_{L^2(\partial\Omega)} \right) \quad (16) \end{aligned}$$

445 with $|\sigma|$ increasing linearly with σ_P .

446 The last part of the right-hand term of Eq. (16) translates the two-fold impact of place-
 447 ment inaccuracies on the reconstruction error. First, the error will now grow linearly with
 448 the value of σ_P , and second, the previously expected $\mathcal{O}(h)$ convergence rate will no longer
 449 hold, as the last term does not decrease with the mesh size h anymore.

450 The two aspects are comforted by the numerical simulations, regarding both the highest
451 and the average estimation errors obtained over the 20 draws. As shown on Fig. 10, these
452 errors are indeed found to broadly follow a linear trend, provided that the values of σ_P
453 remains small in comparison to the mesh size h . The estimation errors obtained with
454 $\sigma_P = 0.0125$ m, which is more than two times the mesh size $h = 0.005$ m, are indeed found
455 to provide off trend results.

456 For reasonable values of σ_P , but at higher frequencies, the reconstruction error seems to
457 be more sensitive to the value of σ_P as well, with a more likely supra-linear trend. This
458 observation highlights the increasing impact of positioning inaccuracies with the frequency,
459 where a small misplacement of the microphone will lead to a large error in the sampling of
460 the highly spatially varying sound field.

461 On the other hand, Fig. 11 illustrates the degradation of the convergence rate, by re-
462 porting the average slope value of the reconstruction error curve with respect to the mesh
463 size. As expected for both the average and highest estimation errors, the computed values
464 are found to gradually decrease from a quadratic trend to a sublinear one as the value of
465 σ_P increases. Here too, the position noise sensitivity at higher frequencies is visible, as the
466 decrease in the corresponding slopes values is seemingly steeper.

467 Studying the impact of the robot accuracy on the sound field estimation error is also an
468 opportunity to highlight another benefit from a preliminary calibration of the robot (Pascal
469 *et al.*, 2023). Indeed, as shown on Fig. 10, the expected error with an un-calibrated Franka
470 Robotics Panda robot, *i.e.*, $\sigma_P = 7.6$ mm, is on average and in the worst case scenario, almost
471 4 times higher than the expected error with a calibrated robot, *i.e.*, $\sigma_P = 1.6$ mm. Assuming

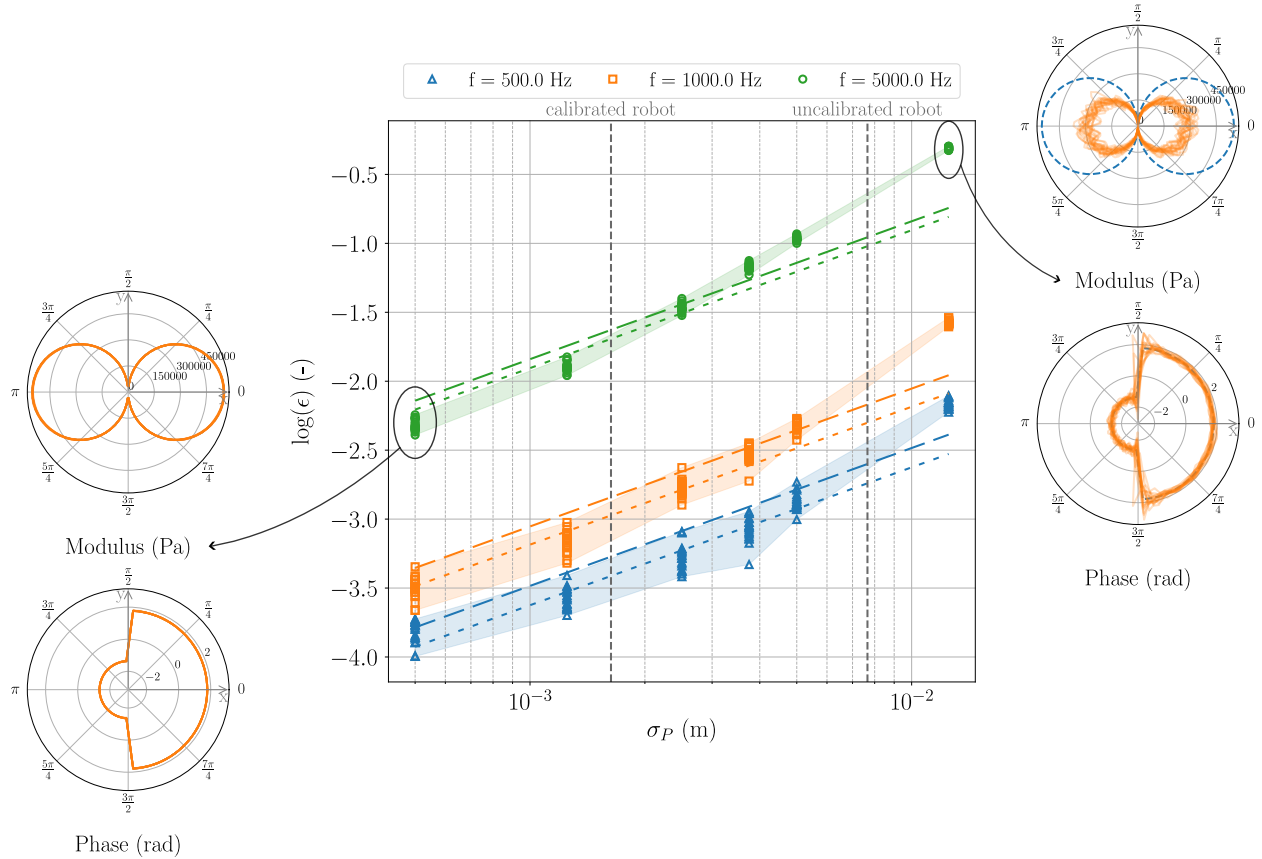


FIG. 10. Reconstruction error obtained for measurements performed on a sphere of diameter 0.5 m with a mesh size of 0.005 m and increasing values of positioning noise σ_P . The dashed and dotted lines represents the expected $\mathcal{O}(\sigma_P)$ convergence rate for the highest and average reconstruction errors, respectively. The directivity patterns illustrate the results of the 20 draws at 5 kHz, for the smallest (left) and largest (right) values of σ_P .

472 a linear decrease of the reconstruction error with the mesh size h , using a calibrated robot
 473 has roughly the same impact as dividing the mesh size by 4.

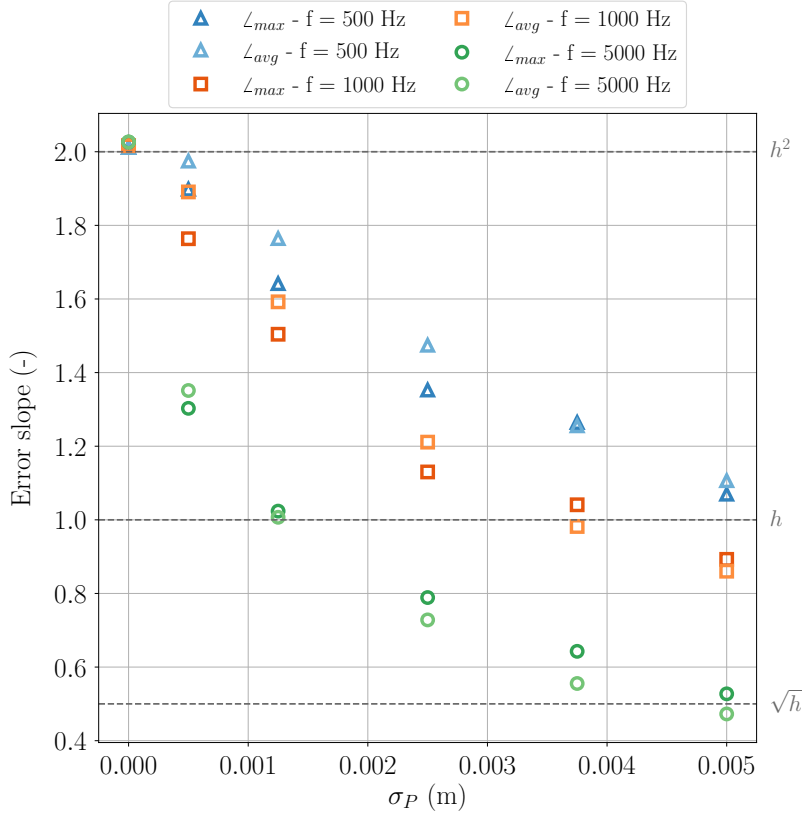


FIG. 11. Evolution of the average slope value of the highest and average reconstruction error curves with respect to the mesh size, for measurements performed on a sphere of diameter 0.5 m and increasing values of σ_P . The dashed lines represent the corresponding quadratic, linear and sublinear trends.

474 **V. EXPERIMENTAL RESULTS**

475 In order to assess the correct operation and performances of our robotized sound field
 476 estimation methodology, we applied the complete procedure to the acoustic characterization
 477 of an un-modeled JBL Flip 2 loudspeaker.

478 A. Experimental setup and Robotized measurements

479 Prior to the deployment of acoustic measurements, and to avoid any collisions between
480 the robot and the loudspeaker, a 3D point cloud of the loudspeaker was recovered using an
481 Intel RealSense D435 depth camera mounted on the robot flange. This point cloud allowed
482 us to extract both a simple geometric description of the loudspeaker for collision avoidance,
483 and its position and orientation relative to the robot base.

484 Based on the gathered geometric data, and in concordance with the guidelines described
485 in subsection IV B, measurements were planned on a sphere of diameter 0.35 m, centered
486 around the loudspeaker. At its closest point, the microphone will be located at a distance
487 of roughly 0.05 m from the loudspeaker, which is small in comparison to the studied source
488 dimensions, and to the minimal wavelength of the validity range identified in subsection II B,
489 $\lambda_{min} = 0.34$ m. A mesh size of 0.025 m was chosen for the triangulation of the sphere, such
490 that hk remains below 1 over the validity range.

491 The measurements were then autonomously performed according to the acquisition pro-
492 cedure of section II, by programming the robot to place the microphone successively at each
493 vertex of the mesh, and align it with the local inward normal to the sphere.

494 Finally, the acquisition was concluded by 20 additional verification measurements, located
495 on a circular mesh of diameter 0.5 cm, lying in the horizontal $z = 0$ plane of the loudspeaker.
496 The total duration as well as the number of measurements are reported in Table II.

TABLE II. Detailed acquisition parameters for the JBL Flip 2 loudspeaker measurements.

Number of mesh vertices	2220
Number of measurements	2153
Missing measurements	3.0%
Overall acquisition duration	16 h 28
Planning & motion duration	9 h 53

497 It should be noted that the extended acquisition duration is mainly caused by the motion
 498 planning computations and executions, which both turned out to be slowed down because
 499 of the robot reduced speed, and complex collision avoidance trajectories.

500 The difference between the actual number of measurements, and the expected number of
 501 vertices is mostly caused by collision avoidance with the loudspeaker, or singular, unreach-
 502 able robot poses. The missing measurements were recovered using an iterative hole-filling
 503 procedure, where each missing value is replaced by the averaged data of its direct neighbors.
 504 A statistical outliers' removal step was also applied, assuming a normal distribution of the
 505 measured phase and amplitude over each vertex direct neighbors.

506 B. Sound field estimation results

507 Aiming to assess the conclusions of subsection IV A, two additional meshes were uniformly
 508 sub-sampled out of the initial measurements mesh. The detailed features of the three meshes

509 are reported in Table III, and an outlook of the corresponding measurements is given on
 510 Fig. 12.

TABLE III. Details of the three meshes used for the sound field prediction.

	Coarse mesh	Intermediate mesh	Fine mesh
Size (m)	0.14	0.07	0.025
kh at 1000 Hz	2.56	1.28	0.46
$kh = 1$ frequency	390 Hz	780 Hz	2184 Hz
Number of vertices	54	226	2220

511 The far-field acoustic pressure field was then estimated with the three measurements sets,
 512 using the BEM-based procedure presented in section III and the circular verification mesh
 513 as the reconstruction domain. A sample of the detailed results obtained for the three meshes
 514 is shown on Fig. 13.

515 At first sight, the reconstruction seems to correctly fit the measurements, with a faithful
 516 rendering of the left monaural directivity pattern of the loudspeaker. However, the SFE
 517 appears to have a smoothing effect on both the modulus and phase reconstructions, especially
 518 in comparison with the sharper verification measurements. Yet, this observation is not fully
 519 reliable, as the varying measurements may also be caused by faulty acquisitions, or by the
 520 acoustic perturbations of the robot. Comparing the results obtained with the three meshes,
 521 it is actually not clear whether a finer resolution is beneficial, as the reconstructions slightly

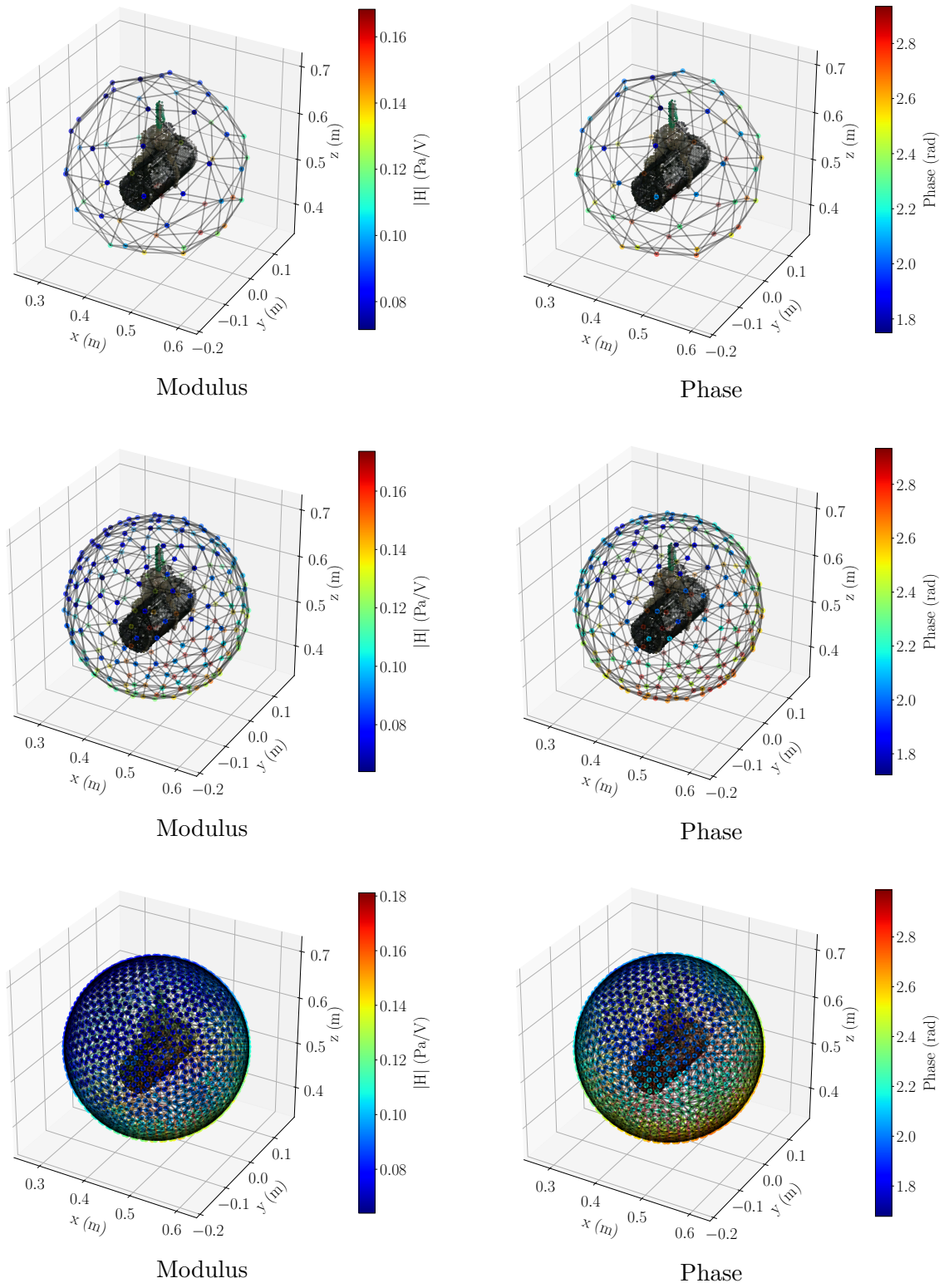
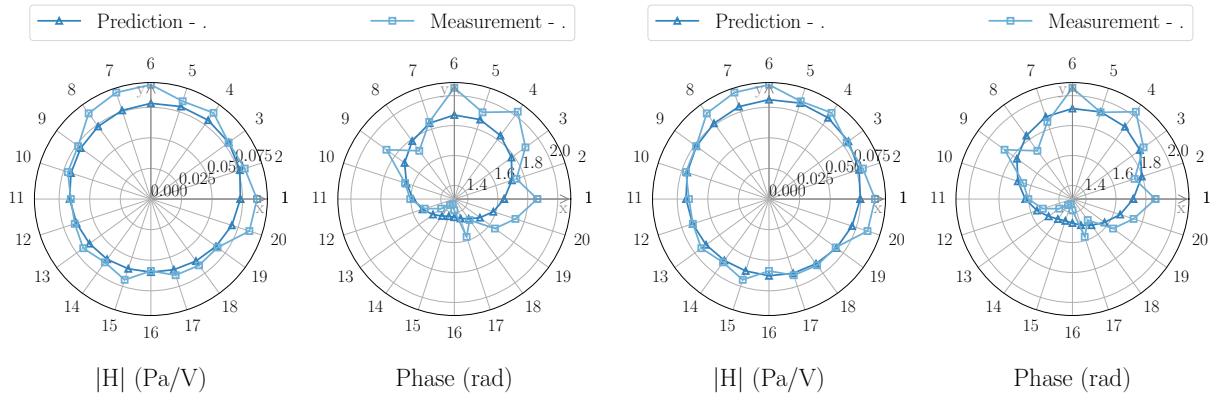


FIG. 12. Example of post-processed measurements at $f = 500$ Hz, for the coarse, intermediate and fine meshes.

Pressure/Input signal TFE - $f = 500$ Hz

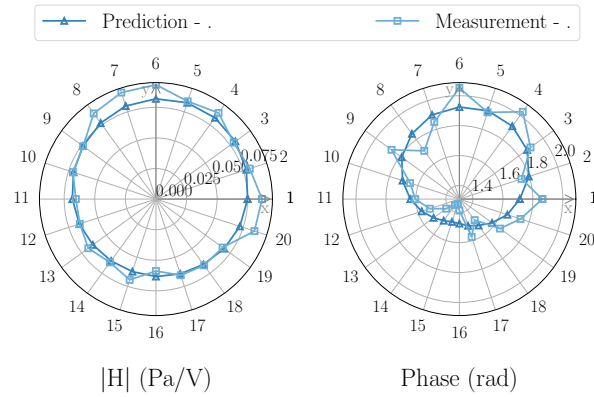
Pressure/Input signal TFE - $f = 500$ Hz



Coarse mesh

Intermediate mesh

Pressure/Input signal TFE - $f = 500$ Hz



Fine mesh

FIG. 13. Predicted and measured acoustic pressure at $f = 500$ Hz, for the coarse, intermediate and fine meshes.

522 differ from one another, except for the coarse mesh phase prediction, which more clearly
 523 undervalued.

524 Taking a closer look at the evolution of the reconstruction errors with the frequency
 525 on Fig. 14, the previous observation is confirmed: the three measurements sets provide
 526 very similar results, especially when comparing the intermediate and fine meshes, which are

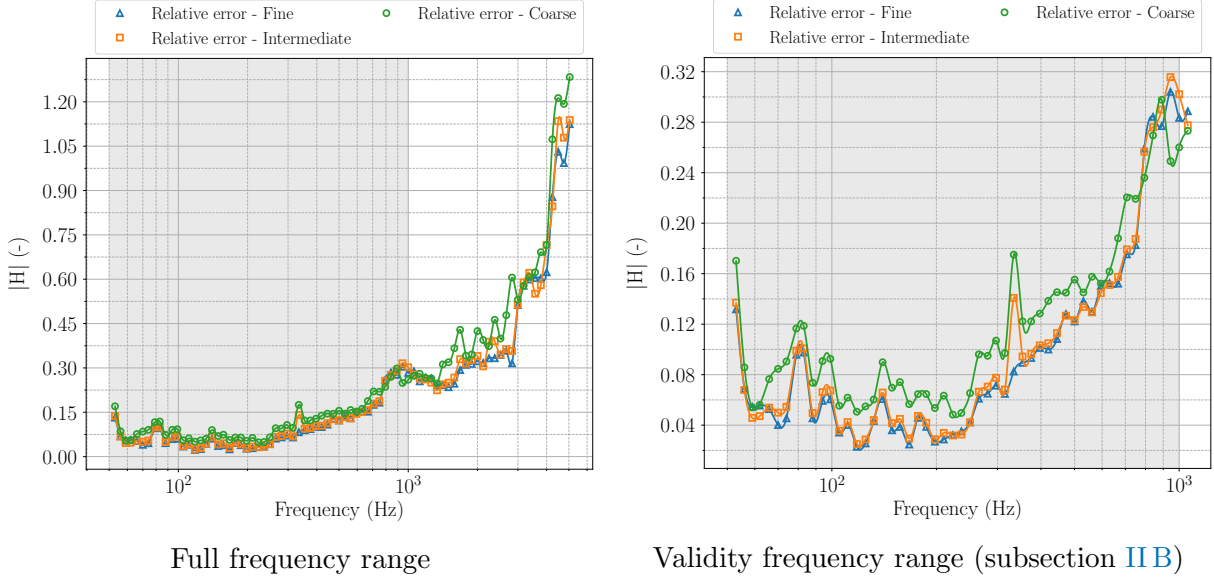


FIG. 14. Reconstruction error, as defined in Eq. (13), for the coarse, intermediate and fine meshes.

527 hardly distinguishable, even on the close-up view. The coarse mesh results appear slightly
 528 less accurate, whether within or outside the validity frequency range, but the difference
 529 still remains far from the predicted 4 times increase for a doubling of the mesh size (*cf.*
 530 section IV A).

531 Considering the fine mesh, and as was expected with measurements performed at a fixed
 532 distance from one another, the reconstruction error globally increases with the frequency.
 533 In detail, the error remains below 15 % for frequencies below 1 kHz but quickly rise to
 534 30 % around 1 kHz, and towards 100 % when the validity frequency range is outreached,
 535 and the frequency reaches 5 kHz. Whereas the explosion of the estimation error at high
 536 frequencies, when hk exceeds 1, is in line with the expected behavior of the SFE tool, the
 537 sudden increase around 1 kHz is less so. It seems however to match the sharpening impact
 538 of the robot acoustic footprint at these frequencies (*cf.* Fig. 4), signaling a weakness in our
 539 second hypothesis.

540 By extension, the ill-compensated acoustic perturbations of the robot, combined with a
 541 still perfectible microphone placement could also explain the excessive similarity between
 542 the three measurements sets.

TABLE IV. Monitored computation times for the reconstruction of the sound field generated by the JBL Flip 2 at 5 kHz, for the coarse, intermediate and fine meshes. Computations were run on the 4 cores of an Intel Core i5-4430 @ 3.00 GHz.

	Coarse mesh	Intermediate mesh	Fine mesh
Computation time (s)	1.97	4.51	32.44

543 In the time being, and considering both the SFE results and the corresponding computa-
 544 tion times reported in Table IV, the intermediate mesh seems to provide the best compromise
 545 between reconstruction accuracy and computational cost. Yet, the fine mesh is eventually
 546 expected to provide more accurate results, granted that the robot acoustic perturbations
 547 are properly accounted for.

548 VI. FUTURE WORK AND CONCLUSION

549 Throughout this paper, we tackled the problem of sound field estimation considering
 550 both the measurements and reconstruction aspects. The requirements for spatially dense
 551 and versatile acoustic measurements were met using a robotic arm, which also allows for a
 552 fully autonomous acquisition of the data. The impact of the robot on the measurements

553 was assessed, and found to be acceptable provided that the studied source is sufficiently
554 repeatable, the robot properly calibrated, and the studied frequencies remain below 1 kHz.

555 The reconstruction was performed using a resonance-free VBEM-based method, bene-
556 fitting from *FreeFEM++* parallelization capabilities. This approach was found to be com-
557 putationally efficient, and to provide accurate results, as the expected reconstruction error
558 decreases quadratically with the mesh size, and linearly with the number of measurements.
559 The estimation errors caused by the robot positioning inaccuracies were also studied, and
560 despite lowering the expected convergence rate to the square root of the mesh size, their
561 impact was found to remain acceptable thanks to the robot calibration.

562 The overall procedure was finally tested on a JBL Flip 2 loudspeaker, around which more
563 than 2000 measurements were autonomously performed in less than 17 hours. The estimated
564 acoustic pressure was found to be in good agreement with the measured data, with a recon-
565 struction error below 30% for frequencies below 1 kHz. However, the expected quadratic
566 increase in the estimation error with the measurements spacing was not clearly observed,
567 and erratic behaviors were noticed around 1 kHz, where the robot acoustic perturbations
568 reach significant levels.

569 *a. Perspectives in robotized acoustic measurements and sound field estimation.* As
570 mentioned in subsection VB, the deviation of experimental results from theoretical expecta-
571 tions highlights a failure in our hypothesis, especially regarding the robot acoustic footprint.
572 In this matter, both physical and numerical solutions can be considered.

573 On one hand, the application of foam on the robot surface could help dampen reflections
574 at high frequencies, while increasing the distance between the robot body and the studied

575 source during the acquisition could also reduce the amplitude of the reflected waves. Fol-
576 lowing the general guidelines provided by Wu (2000) and Martinus *et al.* (2007), building a
577 measurement mesh close and conformal to the studied object should also help to reduce the
578 noise to signal ratio.

579 On the other hand, filtering techniques such as the wave separation method, or the
580 time windowing technique (Klippel, 2022), should be able to partially remove the robot
581 perturbations, thanks to additional, but easily performed measurements. In particular, (Gao
582 *et al.*, 2022) (Langrenne *et al.*, 2008) investigated promising BEM based implementations of
583 the wave separation method, using double layered measurements.

584 Concluding on the SFE part, *FreeFEM++* BEM implementation presented in section IV,
585 could evidently benefit from the use of higher surface elements, which would allow to take the
586 full benefits from higher order Lagrange elements. Furthermore, deepening the possibilities
587 offered by PETSc (Balay *et al.*, 1998), the use of the minimization based solvers makes us
588 confident in the possibility to tackle SFE dual NAH problem, whose inverse-problem nature
589 might help to reduce SFE smoothing effect.

590 On a more theoretical level, the quadratic convergence rates observed in our simulations
591 call for a sharper reconstruction error estimate than Eq. (12), which could benefit from the
592 finer geometric arguments of Nedelec (1976).

593 ACKNOWLEDGMENTS

594 Authors warmly thanks Thibault Toralba and Clément Yver, and Clément Savaro for
595 their help setting up the robotized measurements setup.

596 This work was partially supported by Carnot TSN, Carnot Mines, and *Agence de*
597 *l'Innovation de Défense* – AID – via *Centre Interdisciplinaire d'Études pour la Défense*
598 *et la Sécurité* – CIEDS – (project 2022 - *Aéroacoustique des systèmes multi-PROpulseurs*
599 *pour les drones* (APRO)).

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