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# Retro-information implies quantum unitary violation

Philippe Jacquet  
Inria  
Saclay, France  
philippe.jacquet@inria.fr

Veronique Joly  
ONERA Alumni  
Palaiseau, France  
vjoly58@icloud.com

**Abstract**—We prove simple theorems around the impossibility of sending information backward in time without the violation of the principle of quantum unitarity in physics. The result also extends to faster than light information transfer. The theorem is comparable to the "no-cloning" theorem, which we generalize. This result is a kind of reciprocal of the author former result which proved that a repetitive unitary violation would make possible retro-information.

**Index Terms**—quantum unitarity, relativity.

## I. INTRODUCTION

Time travel has been a popular topic among science fiction fans. Traveling forward in time is an every day experience. However traveling backward in time would be impossible, to the most popular belief in the scientific community mostly due to the apparent trap of time paradoxes or causality violations. However by toying with general relativity many astrophysicist has demonstrated the possibility of closed time curves (CTC), also called time loops. The General Relativity theoretically allows CTCs. In fact Gödel, the father of the mathematic incompleteness theory, was able to show that the equations of general relativity accept some solutions which describe a cyclic universe [3]. Tipler has also imagined that a infinitely long rotating cylinder will create CTCs [5]. These solutions have been criticised by the fact that they would need very unrealistic initial conditions in order to materialize.

The concept of faster than light travels based on warp drive within negative energy field [1] seems less ambitious and attracts less criticism, but in fact it should not, since two consecutive faster than light travels could result in a time backward travel just by adjusting the Lorentzian referentials at their ends. This will be developed in more detail in the last part of our paper. Figure 1, illustrates this fact and shows the convention that the time axis is vertical, the top oriented to the future. This convention will be implicit in the following figures.

Our aim is to investigate the possibility of sending information backward in time (coined here by the term "retro-information") which is a much weaker concept than time travel. Contrary to time travel, retro-information is conceptually not against causality, because the so-called time paradoxes would only result in transmission errors.

In [7] we have shown that a repetitive unitarity violation of rate  $e^\nu$  for  $\nu \neq 0$  would imply the possibility of a retro-information channel of capacity rate  $\frac{\nu}{\log 2}$ . Here we show the reversed result, namely that sending information backward in

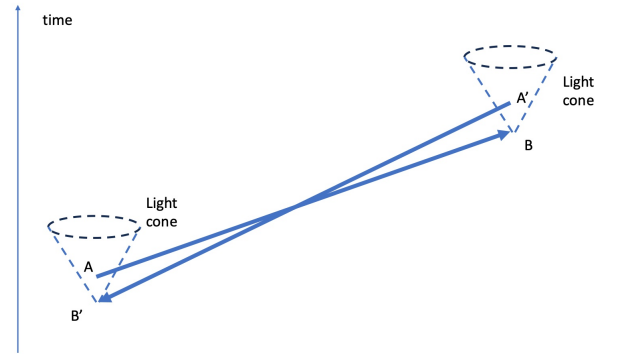


Fig. 1. Two concatenated faster than light segments send information backward in time.

time with non zero information rate would result in unitarity violation:

**Theorem 1 (Our contribution):** The capability to transmit information backward in time implies a violation of quantum unitarity

The consequence of this theorem is that compared to the problem of time travel, the problem of transmitting information backward in time, now switches from the status "impossible" to the status "very difficult", since unitarity is one of the foundation of quantum mechanics.

Our paper is organized as follow. In the next section we present a brief (and simplified) description of the principle of unitarity. Then we present the proof of the no-cloning theorems [10] which tells that an unknown quantum state cannot be mechanically duplicated without violating unitarity. Those proofs bear some resemblance with our theorem although not being strictly on the information theory plan. We also present the corollary about the unitarity violation of faster than light information transfer.

## II. THE PRINCIPLE OF UNITARITY

### A. Schrödinger equation, first quantification

The quantum physics are based on the hypothesis that the state of a massive particle is given at time  $t$  by a "wave" function  $|\Psi(t)\rangle$  by using the expression bra-ket of Paul Dirac. The function is on  $\mathbb{R}^3 \rightarrow \mathbb{C}$  [9] and the space of such functions is an Hilbert space with a metric. The dot product of this Hilbert metric on an arbitrary pair of wave functions  $|\Psi\rangle$  and

$|\phi\rangle$  is defined by  $\langle\phi|\Psi\rangle = \int_{\mathbb{R}^3} \phi^*(\mathbf{z})\Psi(\mathbf{z})d\mathbf{z}^3$ . The quantity  $\phi^*(\mathbf{z})$  is the conjugate complex value of the quantity  $\phi(\mathbf{z})$ .

The time evolution of the state function  $|\Psi(t)\rangle$  is given by the Schrödinger equation [9]:

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle \quad (1)$$

where  $i$  is the complex number  $\sqrt{-1}$ ;  $\hbar$  is the reduced Planck constant;  $\hat{H}$  is the Hamiltonian operator.

The Hamiltonian operator is a self adjoint operator whose eigenvalues are the energy levels attainable by the particle. By self-adjoint we mean that for all wave functions  $\Psi$  and  $\phi$  we have the identity  $\langle\hat{H}\phi|\Psi\rangle = \langle\phi|\hat{H}\Psi\rangle$ .

In general

$$\hat{H}\Psi(t, \mathbf{z}) = -\frac{\hbar^2}{2m}\Delta_{\mathbf{z}}\Psi(t, \mathbf{z}) + V(t, \mathbf{z})\Psi(t, \mathbf{z}) \quad (2)$$

where  $m$  is the mass of the particle,  $\Delta_{\mathbf{z}}$  is the Laplacian operator on the variable  $\mathbf{z}$  and  $V(t, \mathbf{z})$  is the potential function acting on the particle. The potential may depend on time. The equation is then too complicated to accept an analytic solution and must be resolved via approximation or numerical methods (except in some simple models including the hydrogen atom).

### B. Eigenvectors, unitarity

When the Hamiltonian is independant of time, the eigenvalues are the energy levels  $E_n$  of the particle and may have discrete values, thus  $n \in \mathbb{N}$ . If  $|\phi_n\rangle$  is an eigenvector, we have

$$\hat{H}|\phi_n\rangle = E_n|\phi_n\rangle. \quad (3)$$

The set of the states  $|\phi_n\rangle$  when  $n$  varies makes an orthogonal set, since the Hamiltonian is self-adjoint. It is possible that the eigenspace corresponding to a given energy value  $E_n$  be of dimension greater than 1, in this case the energy level is said to have degeneracies. An orthonormal base of such eigenspace would be  $|\phi_{n,j}\rangle$ . An arbitrary wave function  $|\Psi(t)\rangle$  can have the decomposition

$$\text{at } t = 0: |\Psi(0)\rangle = \sum_{n,j} \alpha_{n,j} |\phi_{n,j}\rangle \quad (4)$$

$$\text{and } \langle\Psi(0)|\Psi(0)\rangle = \sum_{n,j} |\alpha_{n,j}|^2.$$

$$\text{And at time } t: |\Psi(t)\rangle = \sum_{n,j} \alpha_{n,j} e^{-iE_n t/\hbar} |\phi_{n,j}\rangle. \quad (5)$$

We see that  $\langle\Psi(t)|\Psi(t)\rangle$  is invariant and equal to  $\langle\Psi(0)|\Psi(0)\rangle$  which is set at 1: this is the principle of unitarity in quantum mechanics.

We notice that  $\langle\Psi(t)|\Psi(t)\rangle = \int_{\mathbb{R}^3} |\Psi(t, \mathbf{z})|^2 d\mathbf{z}^3$ , and the quantity  $|\Psi(t, \mathbf{z})|^2$  is interpreted as the probability density of the particle on  $\mathbf{z}$  at time  $t$ .

When the Hamiltonian depends on the time, *i.e.*  $\hat{H} = \hat{H}(t)$ , the time variation of the solutions of the Schrödinger equation are more difficult to make analyse. However since the Schrödinger equation is fundamentally linear, its resolution goes through a linear operator which depend on  $t$ :  $|\Psi(t)\rangle =$

$\mathcal{U}(t)|\Psi(0)\rangle$ . The operators  $\mathcal{U}(t)$  are unitary in the sense that they preserve the Hilbertian dot product.

Indeed if  $|\phi(t)\rangle$  and  $|\Psi(t)\rangle$  are both solution of the Schrödinger equation, then  $\langle\phi(t)|\Psi(t)\rangle = \langle\phi(0)|\Psi(0)\rangle$ . Indeed:

$$\begin{aligned} \frac{d}{dt} \langle\phi(t)|\Psi(t)\rangle &= \left\langle \frac{d}{dt} \phi(t) \middle| \Psi(t) \right\rangle + \left\langle \phi(t) \middle| \frac{d}{dt} \Psi(t) \right\rangle \\ &= \frac{i}{\hbar} \left\langle \hat{H}(t) \phi(t) \middle| \Psi(t) \right\rangle - \frac{i}{\hbar} \left\langle \phi(t) \middle| \hat{H}(t) \Psi(t) \right\rangle \\ &= 0 \end{aligned} \quad (6)$$

since  $\hat{H}(t)$  is self adjoint. Consequently we still have  $\langle\Psi(t)|\Psi(t)\rangle = \langle\Psi(0)|\Psi(0)\rangle = 1$  as the expression of the unitarity principle.

### C. Klein-Gordon equation, special and general relativity

The previous description is valid for the non relativistic Schrödinger equation where is it possible to separate the time variable  $t$  with the space variables  $\mathbf{z}$  in the wave function  $\Psi(t, \mathbf{z})$ . In relativistic setting it is not possible to make such a separation since the variables can be mixed via any Lorentzian transforms. In this case the Schrödinger equation must be changed into the Klein-Gordon equation [6]:

$$\hbar^2 \partial_t^2 \Psi(t, \mathbf{z}) = \hbar^2 c^2 \Delta_{\mathbf{z}} \Psi(t, \mathbf{z}) - m^2 c^4 \Psi(t, \mathbf{z}) \quad (7)$$

where  $\partial_t$  is the partial derivative with respect to the time variable. When the energy levels  $E_n$  are small compared to the rest energy  $mc^2$ , the solution we would find back the Schrödinger equation at first order:

$$\hbar \partial_t^2 \Psi(t, \mathbf{z}) = -\hbar^2 \Delta_{\mathbf{z}} \Psi(t, \mathbf{z}) - m^2 c^4 \Psi(t, \mathbf{z}) \quad (8)$$

the term  $m^2 c^4$  reminding that the relativistic vector impulsion-energy must have a Minkowski norm equal to  $m^2 c^4$ . The potential in the Hamiltonian may have a formulation which depends on  $|\Psi\rangle$ .

The unitarity have a different expression here since one must consider *probability current*  $\mathbf{J}(t, \mathbf{z}) = \text{Im}(\Psi^*(t, \mathbf{z}) \nabla_4 \Psi(t, \mathbf{z}))$  where  $\nabla_4$  is the 4-gradient operator:  $(-\partial_t, c\nabla_{\mathbf{z}})$ . Thus  $\mathbf{J} = \text{Im}(\Psi^* \partial_t \Psi, c\Psi^* \nabla_{\mathbf{z}} \Psi)$ . This is the flow of the probability current through an space like hyper-surface which determines the density of probability at point  $(t, \mathbf{z})$  with the form  $\mathbf{s} \cdot \mathbf{J}(t, \mathbf{z})$  where  $\mathbf{s}$  is a vector orthonormal to the hyper-surface. The probability density may locally be negative, which calls for the second quantization (not addressed here).

The unitarity condition with the Klein-Gordon equation is equivalent to the conservation of the flow of probability, that is the 4-divergence of the 4-vector  $\mathbf{J}$  is null. Consequently the difference of the flow of the probability current between two hyper-surfaces is equal to the integral of the 4-divergence in the separating volume.

$$\begin{aligned} \nabla_4 \mathbf{J} &= \text{Im}(\Psi^* \partial_t^2 \Psi - \Psi^* c^2 \Delta_{\mathbf{z}} \Psi) \\ &\quad + \text{Im}(\partial_t \Psi^* \partial_t \Psi) + c^2 \text{Im}(\nabla_{\mathbf{z}} \Psi^* \cdot \nabla_{\mathbf{z}} \Psi) \end{aligned} \quad (9)$$

Since  $\partial_t \Psi^* \partial_t \Psi$  is real, thus  $\text{Im}(\partial_t \Psi^* \partial_t \Psi) = 0$ , also  $\text{Im}(\nabla_{\mathbf{z}} \Psi^* \cdot \nabla_{\mathbf{z}} \Psi) = 0$ . Since  $\partial_t^2 \Psi = c^2 \Delta_{\mathbf{z}} \Psi - \frac{m^2 c^4}{\hbar^2} \Psi$  we have

$$\nabla_4 \mathbf{J} = \frac{m^2 c^4}{\hbar^2} \text{Im}(\Psi^* \Psi) = 0. \quad (10)$$

which is the principle of unitarity in relativistic quantum mechanics. We can still define the dot product between kets  $\langle \phi | \Psi \rangle$  when focusing on specific parameters such that spins or polarization, via a suitable algebra similar to the one introduced by Paul Dirac where unitarity is preserved.

In general relativity, the space-time has a non Euclidian geometry with a curvature function of the density of mass and energy. The Klein-Gordon equation becomes

$$\frac{1}{\sqrt{\det(\mathbf{g})}} \sum_{ij} \partial_i (g^{ij} \sqrt{\det(\mathbf{g})} \partial_j \Psi) + \frac{m^2 c^2}{\hbar^2} \Psi = 0 \quad (11)$$

where  $\mathbf{g}$  is the  $4 \times 4$  matrix of the non euclidian metric.  $g_{ij}$  (resp.  $g^{ij}$ ) is the coefficient  $(i, j)$  of the matrix  $\mathbf{g}$  (resp. of the matrix  $\mathbf{g}^{-1}$ ). The conservation of unitarity between two hyper-surface still holds as long as the metric does not show singularity in the separating volume. This never occurs when the metric is flat, but it may occur when we consider the metric close to a black hole, where the matrix  $\mathbf{g}$  is singular on the event horizon as we know for the Schwartzschild deSitter metric [8]. In this case the unitarity might be violated [11].

#### D. Multi-particle systems, second quantification

The wave function can be made more complicated by defining it in  $\mathbb{R}^D \times \mathcal{S} \rightarrow \mathbb{C}^d$  where  $D \geq 3$  and  $d \geq 1$  and  $\mathcal{S}$  being a symmetry group, and this is just talking of first quantification, the second quantification talks of creation and annihilation of particles operators (thus  $d$  would be infinite). In its simplest setting,  $\mathbb{R}^3$  stands for the spatial position of the particle and  $\mathbb{C}$  stands for the intensity of the wave function. The wave function summarizes the probabilistic properties of the particle.

For example in a two state system with a binary outcome  $u$  and  $v$  after a measurement, e.g a spin measurement, then an arbitrary wave function  $|\Psi\rangle$  has the decomposition  $|\Psi\rangle = \alpha|u\rangle + \beta|v\rangle$  where  $|u\rangle$  and  $|v\rangle$  are respectively the pure states corresponding to the outcome  $u$  and  $v$ . The states  $|u\rangle$  and  $|v\rangle$  being an orthonormal base then the unitarity condition tells that every time  $|\alpha|^2 + |\beta|^2 = 1$ . In fact as for the spin measurement of a particle with angular momentum, the pure states could be respectively the tensor products of the wave function  $|\phi_u\rangle$  (resp.  $|\phi_v\rangle$ ) of the position of the particle in  $\mathbb{R}^3$  with  $|\Sigma_u\rangle$  (resp.  $|\Sigma_v\rangle$ ) the wave function of the angular momentum. Thus  $|\Psi\rangle = \alpha|\phi_u\rangle \otimes |\Sigma_u\rangle + \beta|\phi_v\rangle \otimes |\Sigma_v\rangle$  would still results in the identity  $|\alpha|^2 + |\beta|^2 = 1$ . In this simple framework to prove an unitarity violation it suffices to show that for a particle it is impossible to have everytime  $\langle \Psi | \Psi \rangle = 1$ .

In the case of a system, made of two distinct particles  $A$  and  $B$  corresponding to a pair of positions, each in  $\mathbb{R}^3$ , the resulting wave function will operate in  $\mathbb{R}^6$ . If the particles are independent,  $A$  will have wave function  $|\Psi_A\rangle$  and  $B$  wave function  $|\Psi_B\rangle$ , the combined wave function will be the tensor  $|\Psi_A\rangle \otimes |\Psi_B\rangle$  and its value at  $(\mathbf{z}, \mathbf{z}') \in \mathbb{R}^6$  is  $\Psi_A(\mathbf{z})\Psi_B(\mathbf{z}')$ . Of course we can generalize to an arbitrary number of particles. The unitarity violation will be revealed if it at a given time is impossible to have both  $\langle \Psi_A | \Psi_A \rangle = 1$  and  $\langle \Psi_B | \Psi_B \rangle = 1$ .

### III. THE NO-CLONING THEOREM

What follow is the first proof of impossibility without unitarity violation. To simplify our proofs we will only consider pure quantum states with no degeneracy. Degeneracy won't change the result but would need to involve state projection, or other operations which would complicate the presentation.

*Theorem 2:* It is not possible to make two copies of an unknown wave function  $|\Psi\rangle$  without violating unitarity.

*Proof:* To simplify we consider two originally independent particles  $A$  and  $B$ . The particle  $A$  has the wave function  $|\Psi\rangle_A$ , supposed to be unknown (not fabricated, so that duplication would be trivial), and the particle  $B$  has the "blank" wave function  $|e\rangle_B$  which is supposed to be the vehicle of the copy  $|\Psi\rangle$ . Thus the original system wave function the tensor product  $|\Psi\rangle_A \otimes |e\rangle_B$  (to simplify we remove the sign " $\otimes$ " since it is only a multiplication. We want to find a process which leads to the tensor product  $|\Psi\rangle_A \otimes |\Psi\rangle_B$ ).

The process cannot involve a measurement, because it would destroy  $|\Psi\rangle_A$ . Thus the evolution should be completely unitary. Let  $\mathcal{U}$  be the unitary operator which maps  $|\Psi\rangle_A \otimes |e\rangle_B$  into  $e^{-i\delta} |\Psi\rangle_A \otimes |\Psi\rangle_B$  (we can allow an extra phase shift by an arbitrary real number  $\delta$ ). We know that  $\mathcal{U}^* \mathcal{U} = 1$  by property of unitary operators which must keep invariant the dot products.

We suppose two possible candidate wave functions  $|\Psi_1\rangle_A$  and  $|\Psi_2\rangle_A$ . We have  $\mathcal{U} |\Psi_1\rangle_A |e\rangle_B = e^{i\delta_1} |\Psi_1\rangle_A |\Psi_1\rangle_B$  and  $\mathcal{U} |\Psi_2\rangle_A |e\rangle_B = e^{i\delta_2} |\Psi_2\rangle_A |\Psi_2\rangle_B$  for some real numbers  $\delta_1, \delta_2$ . We have

$$\begin{aligned} \langle \Psi_1 | \Psi_2 \rangle \langle e | e \rangle &= \langle \Psi_1 |_A \langle e |_B \mathcal{U}^* \mathcal{U} | \Psi_2 \rangle_A | e \rangle_B \\ &= e^{-i\delta_1 + i\delta_2} \langle \Psi_1 |_A \langle \Psi_1 |_B | \Psi_2 \rangle_A | \Psi_2 \rangle_B \\ &= e^{-i\delta_1 + i\delta_2} (\langle \Psi_1 | \Psi_2 \rangle)^2 \end{aligned} \quad (12)$$

Since  $\langle e | e \rangle = 1$ , we get the identity  $|\langle \Psi_1 | \Psi_2 \rangle| = |\langle \Psi_1 | \Psi_2 \rangle|^2$ , which should hold for any arbitrary pair of wave functions. This would imply that either  $|\Psi_1\rangle = 0$  or  $|\Psi_2\rangle = 0$  thus contradicting  $\langle \Psi_1 | \Psi_1 \rangle = \langle \Psi_2 | \Psi_2 \rangle = 1$ . Therefore the operator  $\mathcal{U}$  cannot be unitary. ■

This theorem is originally due to James Park in 1970 [10]. We should be careful not to confuse the no-cloning theorem with the fact one can fabricate particles with the same wave function (for example multiple photon with the same polarization, through a polarizer with a fixed angle). But in this case the wave function  $|\Psi\rangle$  is fully known and the argument above with an arbitrary pair  $(|\Psi_1\rangle, |\Psi_2\rangle)$  won't hold. But there would be no information to obtain by measuring  $|\Psi\rangle$  since we already know all its parameters. The theorem also applies on partial wave function, i.e. wave functions restricted to one particle parameter, e.g. the spin, as long as the isolated parameter evolves unitarity.

Let  $\epsilon > 0$ , we call " $\epsilon$ -approximate" cloning the mapping of an arbitrary wave function  $|\Psi\rangle$  into a wave function  $|f_\epsilon(\Psi)\rangle$  such that for all  $|\Psi\rangle$ :  $|\langle \Psi | f_\epsilon(\Psi) \rangle| > \epsilon$ . Notice that  $\epsilon = 1$  is equivalent to perfect cloning (*modulo* an phase factor) We have the easy follow-up theorem:

*Theorem 3 (our contribution):* For all  $\epsilon > 0$   $\epsilon$ -approximate cloning would violate unitarity.

*Proof:* We assume that the transition from  $|\Psi\rangle_A |e\rangle_B$  to  $|\Psi\rangle_A |f_\epsilon(\Psi)\rangle$  is unitary, *i.e.* there exist an unitarity operator  $\mathcal{U}_\epsilon$  such that  $\mathcal{U}_\epsilon |\Psi\rangle_A |e\rangle_B = e^{-i\delta} |\Psi\rangle_A |f_\epsilon(\Psi)\rangle$  for some real number  $\delta$ . Taking two arbitrary wave functions  $|\Psi_1\rangle$  and  $|\Psi_2\rangle$  and using the conservation of the dot product of  $|\Psi_1\rangle_A |e\rangle_B$  with  $|\Psi_2\rangle_A |e\rangle_B$  we get

$$\begin{aligned} \langle \Psi_1 | \Psi_2 \rangle &= \langle \Psi_1 |_A \langle e |_B | e \rangle_B | \Psi_2 \rangle_A \\ &= \mathcal{U}_\epsilon^* \langle \Psi_1 |_A \langle e |_B \mathcal{U}_\epsilon | e \rangle_B | \Psi_2 \rangle_A \\ &= e^{-i\delta_1 + i\delta_2} \langle \Psi_1 |_A \langle f_\epsilon(\Psi_1) |_B | \Psi_2 \rangle_A | f_\epsilon(\Psi_2) \rangle_B \\ &= e^{-i\delta_1 + i\delta_2} \langle \Psi_1 | \Psi_2 \rangle \langle f_\epsilon(\Psi_1) | f_\epsilon(\Psi_2) \rangle \quad (13) \end{aligned}$$

Thus we would have  $|\langle f_\epsilon(\Psi_1) | f_\epsilon(\Psi_2) \rangle| = 1$  as in general  $\langle \Psi_1 | \Psi_2 \rangle \neq 0$ . This would implies that all the  $|f_\epsilon(\Psi)\rangle$  are co-aligned with an unitary vector  $|\mathbf{w}_\epsilon\rangle$ . But in this case the approximate cloning of a wave function  $|\Psi\rangle$  orthogonal to  $\mathbf{w}$  will result in  $\langle \Psi | f_\epsilon(\Psi) \rangle = 0$  which would contradict the  $\epsilon$  approximation condition. ■

An  $\epsilon$ -approximate clone of  $\Psi$  for  $\epsilon > 0$  shares information with the original wave function. This information theory point of view is an excellent transition to our main theorem.

Besides the  $\epsilon$ -approximate cloning should not be confused with the approximate cloning of quantum system in general. The later can be obtained via unitary operations (see [2]) but with a non zero failure probability, or in other words the result of an 0-approximate cloning.

#### IV. OUR THEOREM PROOF

*Proof:* To prove the theorem we assume a retro-channel whose reception is at a space time point  $B$ . Since receiving data is always based on physical measurements, thus necessarily quantum measurements, we assume a binary outcome 0 or 1 at point  $B$  which is affected by some quantum states at space-time point  $A$  located in the future of the reception measurement (*i.e.* in the light cone of  $B$ ). We assume two transmit states  $|A_0\rangle$  or  $|A_1\rangle$  which can be located at point  $A$ , so that

- if the transmit state is  $|A_0\rangle$  then the wave function of the reception state is  $|\Psi_0\rangle = \alpha_0 |0\rangle + \beta_0 |1\rangle$  with  $|\alpha_0|^2 + |\beta_0|^2 = 1$ ;
- if the transmit state is  $|A_1\rangle$  then the wave function of the reception state is  $|\Psi_1\rangle = \alpha_1 |0\rangle + \beta_1 |1\rangle$  with  $|\alpha_1|^2 + |\beta_1|^2 = 1$ ;
- $|\alpha_0|$  and  $|\alpha_1|$  differ (and so also  $|\beta_0|$  and  $|\beta_1|$ ) thus there is an information transfer from the transmission space-time point  $A$  to the reception space-time point  $B$ .

The selection of the state  $|A_0\rangle$  and  $|A_1\rangle$  is made on a point  $E$  which may not be in the light cone of  $B$  (see figure 2) in order to outline the fact that there might be no direct signal from the source of information to the receiver located in the space-time point  $B$ .

Till now, the story shows nothing special with respect to the direction of the time arrow. But now we introduce one specific aspect of time backward information transmission, that is the

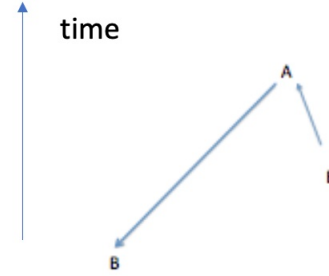


Fig. 2. A retro-channel from  $A$  to  $B$

reception can interfere with the emission, since the later is in the future of the former. We call *forward coupling* the effect of interference of the reception outcome with the emission setting. We imagine two forward coupling scenarios:

- the scenario ”+”: if the reception outcome is 0 then the emission setting is  $|A_0\rangle$ , otherwise if the reception outcome is 1, the emission setting is  $|A_1\rangle$ ;
- the scenario ”-” is the reverse: if the reception outcome is 0 then the emission setting is  $|A_1\rangle$ , otherwise if the reception outcome is 1, the emission setting is  $|A_0\rangle$ ;

We denote  $|\Psi_+\rangle$  (resp.  $|\Psi_-\rangle$ ) the reception state function when the coupling is ”+” (resp. ”-”):  $|\Psi_+\rangle = \alpha_+ |u\rangle + \beta_+ |v\rangle$  (resp.  $|\Psi_-\rangle = \alpha_- |u\rangle + \beta_- |v\rangle$ ). If we show that  $|\alpha_+|^2 + |\beta_+|^2 \neq |\alpha_-|^2 + |\beta_-|^2 \neq 1$  then we would have proven an unitarity violation.

We imagine that at point  $E$  the mode of coupling is decided by a quantum measurement between two state  $|E_+\rangle$  and  $|E_-\rangle$  with uniform probability, *i.e.* the coupling state is  $\frac{1}{\sqrt{2}} |E_+\rangle + \frac{1}{\sqrt{2}} |E_-\rangle$ . Let a point  $C$  in the light cones of both  $B$  and  $E$  and such that  $A$  is in the light cone of  $C$ . We suppose that the state of the forward coupling is decided at point  $C$  as a result of the outcome of the measure at  $B$  and the measure  $\pm$  at  $E$  jointly received at  $C$  (see figure 3).

The wave function at point  $B$  and  $E$  before measurement is  $|\Psi_{BE}\rangle = \frac{1}{\sqrt{2}} |\Psi_+\rangle \otimes |E_+\rangle + \frac{1}{\sqrt{2}} |\Psi_-\rangle \otimes |E_-\rangle$ .

The joint wave function at space-time points  $B$  and  $E$  involves the ancestor of the states  $|A_0\rangle$  and  $|A_1\rangle$  which we assume to be located at space-time  $B$  to simplify. We take advantage of the fact that the uniform weights between  $|E_+\rangle$  and  $|E_-\rangle$  is like a XOR (addition *modulo* 2) between the binary sequence made of measurements on  $B$  and the random uniform binary sequence made of the measurements on  $E$ , assuming the state  $+$  is interpreted as a ”0”, and  $-$  as a ”1”. The consequence is that the dependencies inside the non uniform sequence are lost. Therefore the wave function at  $B$  is exactly as if as the states on  $A$ ,  $|A_0\rangle$  or  $|A_1\rangle$ , were chosen randomly independently of the outcome 0 or 1 on  $B$ . Therefore

the joint wave function an the space-time point  $B$  is exactly

$$\begin{aligned} |\Psi_B\rangle &= \frac{1}{\sqrt{2}} |\Psi_0\rangle \otimes \mathcal{U}^{-1} |A_0\rangle + \frac{1}{\sqrt{2}} |\Psi_1\rangle \otimes \mathcal{U}^{-1} |A_1\rangle \\ &= \frac{\alpha_0}{\sqrt{2}} |0\rangle \mathcal{U}^{-1} |A_0\rangle + \frac{\beta_0}{\sqrt{2}} |1\rangle \mathcal{U}^{-1} |A_0\rangle \\ &\quad + \frac{\alpha_1}{\sqrt{2}} |0\rangle \mathcal{U}^{-1} |A_1\rangle + \frac{\beta_1}{\sqrt{2}} |1\rangle \mathcal{U}^{-1} |A_1\rangle \end{aligned} \quad (14)$$

where  $\mathcal{U}^{-1}$  is the reverse of the operator  $\mathcal{U}$  which maps the states  $|A\rangle$  from the ancestor  $B$  to the point  $A$ . If  $\mathcal{U}$  is unitary the state  $\mathcal{U}^{-1} |A_0\rangle$  and  $\mathcal{U}^{-1} |A_1\rangle$  forms an orthonormal basis. The couplings  $+$  and  $-$  give a distinct subset of allowed states:

$$\begin{aligned} \Psi_{BE} &= \frac{\alpha_0}{\sqrt{2}} |0\rangle \mathcal{U}^{-1} |A_0\rangle |E_+\rangle + \frac{\beta_0}{\sqrt{2}} |1\rangle \mathcal{U}^{-1} |A_0\rangle |E_-\rangle \\ &\quad + \frac{\alpha_1}{\sqrt{2}} |0\rangle \mathcal{U}^{-1} |A_1\rangle |E_-\rangle + \frac{\beta_1}{\sqrt{2}} |1\rangle \mathcal{U}^{-1} |A_1\rangle |E_+\rangle. \end{aligned} \quad (15)$$

Collecting the states  $|E_+\rangle$  and  $|E_-\rangle$  we get the expression for  $|\Psi_+\rangle$  and  $|\Psi_-\rangle$ :

$$\begin{cases} |\Psi_+\rangle &= \alpha_0 |0\rangle + \beta_1 |1\rangle \\ |\Psi_-\rangle &= \alpha_1 |0\rangle + \beta_0 |1\rangle \end{cases} \quad (16)$$

where clearly  $|\alpha_0|^2 + |\beta_1|^2 \neq |\alpha_1|^2 + |\beta_0|^2$  yielding an unitarity violation and hinting that the operator  $\mathcal{U}$  from  $B$  to  $A$  is not unitary. ■

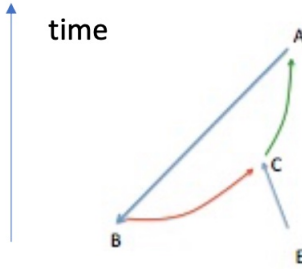


Fig. 3. The retro-channel with forward coupling

*Corollary 1 (Our contribution):* Faster than light information transmission implies an unitarity violation

*Proof:* We build the proof under the framework of special relativity. Let assume that a communication device transmits information from a space time point  $A$  to a space-time point  $B$ . We denote the space-time 3-vector  $\vec{AB} = (t, \mathbf{z})$ , where  $t$  is the time component and  $\mathbf{z}$  is a 3-vector. We suppose that the vector  $\vec{AB}$  is a space kind vector, i.e.  $\frac{\|\mathbf{z}\|}{t} = w > c$  where  $c$  is the speed of light (see figure 1).

We suppose that the property of faster than light information transmission can be repeated independently of symmetry, rotation and translation by uniform speed of the vector  $\vec{AB}$ . In other words a copy of the channel over a vector  $\vec{A'B'}$  obtained from  $\vec{AB}$  after an arbitrary symmetry, dilatation and uniform translation will too be prone to faster than light transmission.

We consider that  $\vec{A'B'}$  in a second Lorentzian referential is equal to  $\vec{BA}$  in the Lorentzian referential of the first Bell system. We will show as an exercise that there exists a speed of translation (slower than light) which makes this possible. In

other words we can make  $A' = B$  and  $B' = A$  in the absolute space-time: the second Bell system makes possible to transfer information backward from  $B$  to  $A$ .

We take the relative translation speed of the vector  $\vec{A'B'}$  parallel to the spatial component  $\vec{AB}$ . The problem becomes a two dimensional problem: a spatial component and a time component. By virtue of the Lorentzian transformation, a vector  $(t, x)$  by a change of referential with uniform speed  $v$ , ( $|v| < c$ ) is mapped in the new vector  $(t', x')$ :

$$t' = \left(t - \frac{vx}{c^2}\right)\gamma \quad (17)$$

$$x' = (x - vt)\gamma, \quad (18)$$

with  $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ . If the vector  $(t, x)$  is the components of the vector  $\vec{AB}$  then  $w = \frac{x}{t}$ . It turns out that with  $v = \frac{2w}{\frac{w^2}{c^2} + 1}$  (which is smaller than the speed of light  $c$ ).

Therefore if  $\vec{AB} = (t, x)$  then  $(t, -x)$  is the spatial flipped version of  $\vec{AB}$  and our main hypotheses assume that we can build a faster than speed of light channel over this vector. If this device is moved with a speed  $w' = \frac{2w}{\frac{w^2}{c^2} + 1}$  along the  $x$  abscissa, then the resulting vector is  $(-t, -x)$  which is  $\vec{BA}$ , therefore the new channel returns information from  $B$  to  $A$ . Of course the nature of the information transmission may differ since the Lorentzian transform would modify the spin of the particle if they are used to carry information.

Via a small perturbation of the Lorentzian referential, the transmission can be made from a point  $A'$  slightly in the future of  $B$  to a point  $B'$  slightly in the past of point  $A$  (see figure 1), thus it would be possible to transmit information from  $A$  to  $B$  then to forward it from from  $B$  to  $A'$  and it back to  $B'$  in the past of  $A$ . This would imply an unitarity violation thanks to theorem 1. ■

A consequence of this corollary is that an entangled pair of particles in Bell state [4] cannot be used to transmit information faster than light although the spooky interaction is assumed to happen faster than light. The reason of this impossibility comes from the fact that the theory of Bell states is build in a full quantum unitary framework.

## V. CONCLUSION

We have presented two contributions. (i) We show that for any  $\epsilon > 0$  an  $\epsilon$ -approximate quantum state cloning would imply a quantum unitarity violation. This is proved via as a simple extrapolation of the no-cloning theorem. (ii) We also show that any transfer of information backward in time or faster than light would also lead to unitarity violation. The later contribution is more surprising because an information transfer does not *a priori* refer to quantum mechanics. In fact it does, because receiving data always implies to physical measurements. Fundamentally the violation of unitarity might not be impossible, but it is certainly very difficult. Therefore everytime a person, inspired like Nostradamus, pretends to foresee a fragment of the future, there should be a great perturbation of the quantum physics as a consequence.

## REFERENCES

- [1] Zohuri, Bahman, and Masoud J. Moghaddam. "The Quest for Warp Drive and Faster-Than-Light (FTL)."
- [2] Scarani, Valerio, Sofyan Iblisdir, Nicolas Gisin, and Antonio Acin. "Quantum cloning." *Reviews of Modern Physics* 77, no. 4 (2005): 1225.
- [3] Gödel, K., "An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation", *Rev. Mod. Phys.* 21, 447, published July 1, 1949.
- [4] Bell, John S. "On the einstein podolsky rosen paradox." *Physics Physique Fizika* 1.3 (1964): 195.
- [5] Tipler, Frank J. "Rotating cylinders and the possibility of global causality violation." *Physical Review D* 9.8 (1974): 2203.
- [6] Kragh, Helge. "Equation with the many fathers. The Klein–Gordon equation in 1926." *American Journal of Physics* 52.11 (1984): 1024-1033.
- [7] Jacquet, P., and V. Joly. "Capacity of retro-information channels." 2000 IEEE International Symposium on Information Theory (Cat. No. 00CH37060). IEEE, 2000
- [8] McInnes, Brett. "De Sitter and Schwarzschild-de Sitter according to Schwarzschild and de Sitter." *Journal of High Energy Physics* 2003.09 (2003): 009.
- [9] Cohen-Tannoudji, Claude, Bernard Diu, and Franck Laloë. *Quantum mechanics, volume 3: fermions, bosons, photons, correlations, and entanglement.* John Wiley & Sons, 2019.
- [10] Wootters, William K., and Wojciech H. Zurek. "The no-cloning theorem." *Physics Today* 62.2 (2009): 76-77.
- [11] Jacquet, Philippe, and Véronique Joly. "Unitarity excess in Schwarzschild metric." *International Conference on Geometric Science of Information.* Cham: Springer Nature Switzerland, 2023.