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# Fast and Robust Flocking of Protesters on Street Networks

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## Abstract

We propose a simple model of protesters scattered throughout a city who want to gather into large and mobile groups. This model relies on random walkers on a street network that follow tactics built from a set of basic rules. Our goal is to identify the most important rules for fast and robust flocking of walkers. We explore a wide set of tactics and show the central importance of a specific rule based on alignment. Other rules alone perform poorly, but our experiments show that combining alignment with them enhances flocking, and that obtained groups are then remarkably robust.

*Keywords* : street networks, gathering, flocking, tactic design, robustness, protests

## 1 Introduction

Consider the following scenario. Protesters are scattered throughout a city and want to gather into groups large enough to perform significant actions. They face forces that may break up groups, block some places or streets and seize any communication devices protesters may be carrying. As a consequence, protesters only have access to local information on people and streets around them. Furthermore, formed protester groups must keep moving to avoid containment by adversary forces.

In this scenario, protesters need a distributed and as simple as possible protocol, that utilises local information exclusively and ensures the rapid formation of significantly large, mobile, and robust groups. We illustrate these objectives in Figure 1. Our goal in this paper is to identify the key building blocks for such protocols.

To do so, we model the city as a network of streets and intersections in Section 2. We then assume that protesters are biased random walkers on such a network. In Section 3, we present the set of basic rules that compose *tactics* walkers can use to move. We define, in Section 4, what flocking features groups must achieve and run extensive experiments to measure how effective each tactic is regarding those. We also explore the robustness of groups, measuring how they reform if adversary forces break them up while following an effective tactic. In Section 5, we compare our approach to related work on flocking and on formal analysis of protests. We emphasise the absence of a protocol that can solve our problem. Finally, we summarise our contributions in Section 6.

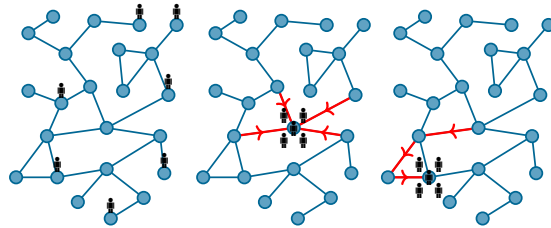


Figure 1: Walkers scattered on a street network (left) must rapidly gather (center) and subsequently flock (right).

## 2 Framework

We need a framework to simulate displacements of protesters in a city. We model cities as undirected graphs we call street networks. Protesters are then biased random walkers on this network. They can move from node to node with simple rules we introduce in next Section.

### 2.1 Street networks

In order to model real-world cities, we leverage OpenStreetMap [1] data and the OSMnx library [2]. For a given city, we use this library with its default settings to extract the graph  $G = (V, E)$  defined as follows: the nodes in  $V$  represent street intersections in this city and the links in  $E \subseteq V \times V$  represent pieces of streets between them. We take the undirected graph  $G$ , meaning there is no distinction between  $(u, v)$  and  $(v, u)$  in  $E$ . In addition, we denote by  $N(v) = \{u, (u, v) \in E\}$  the set of neighbors of any node  $v$  in  $V$ .

We performed experiments on a wide ranges of large worldwide cities of diverse sizes and structures. This led to no significant difference on obtained results. We therefore use a typical in-

stance, namely Paris, to present our work in this paper. This street network has 9 602 nodes, 14 974 links, leading to average degree 3.1. Its diameter is 83 hops and its average distance is 39.4 hops. The average street length is 99 meters, and the average distance is 5 552 meters.

## 2.2 Walkers

Given a network  $G = (V, E)$ , we consider a set  $W$  of walkers numbered from 1 to  $|W|$ . We denote the location of walker  $i$  at time  $t$  by  $x_i(t) = v$ , with  $v \in V$ . At each time step  $t$ , walker  $i$  moves to node  $x_i(t+1) \in N(v)$ . Rules we present in Section 3 determine the choice of  $x_i(t+1)$ .

For any link  $(u, v)$  in  $E$ , we also define the *flux* of walkers from  $u$  to  $v$  as  $J_{u \rightarrow v}(t) = |\{i, x_i(t-1) = u, x_i(t) = v\}|$ .

We call *group* the set of walkers at a given node  $v$  at a given time  $t$ :  $g_v(t) = \{i, x_i(t) = v\}$ . We denote by  $n_v(t) = |g_v(t)|$  the number of walkers located at node  $v$  at time  $t$ . We denote by  $g(t) = |\{g_v(t), v \in V, g_v(t) \neq \emptyset\}|$  the number of non-empty groups at step  $t$ .

## 2.3 Discretization

The links of a street network generally represent street segments of very heterogeneous lengths [3]. Then, moves from a node to another one may have very different duration. In order to model this, we use a classical discretization procedure [4] that we describe below. It consists in splitting each link of the street network into pieces connected by evenly spaced nodes, see Figure 2.

More formally, given a link  $(u, v)$  representing a piece of street of length  $L$  in a street network and a discretization step  $\delta$ , we split each undirected link  $(u, v)$  into  $k$  new links, with  $k = \lceil \frac{L}{\delta} \rceil$  as follows. Let us denote a set of new nodes  $V(u, v) = \{v_0, v_1, \dots, v_k\}$  with the nodes from the original network being  $v_0 = u$  and  $v_k = v$ . We then define a set of new links  $E(u, v) = \{(w_i, w_{i+1}), i = 0, 1, \dots, k-1\}$ . Then we define the set of all nodes  $V = \bigcup_{u,v} V(u, v)$  and the set of links  $E = \bigcup_{u,v} E(u, v)$ . Finally, we model the street network by the graph  $G = (V, E)$ .

In the obtained graph, each link represents a street slice of length close to  $\delta$ . Then, the walkers defined in previous section consistently make a move of length approximately  $\delta$  at each time step.

In this paper, we use  $\delta$  equal to 10 meters, leading to a network of  $N = 145000$  nodes and  $M = 300736$  links. It gives a sufficient precision in our context, and experiments with other reasonable values of  $\delta$  displayed no significant difference.

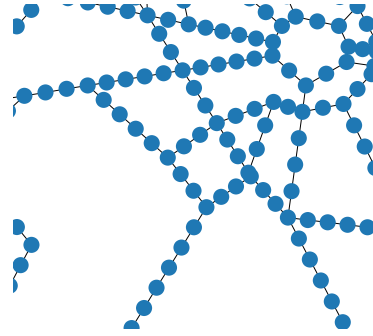


Figure 2: A piece of the discretized street network around Place de la Nation in Paris.

## 3 Walker model

At each time step  $t$ , each walker  $i$  moves from location  $x_i(t)$  to location  $x_i(t+1)$  in  $N(x_i(t))$ . This section presents how we choose the new location  $x_i(t+1)$ .

### 3.1 Available information

We assume that walkers have very limited capabilities: they have no access to the actual location of other walkers, even on neighbor nodes, and they have no long-term memory and no communication protocol. Instead, we assume they only have access to estimates of aggregated observables, such as the number of walkers on neighbor nodes and the flux intensity on surrounding links, and a short-term memory.

More formally, walker  $i$  may use the following information:

- its previous location  $x_i(t-1)$ ,
- the number  $n_v(t)$  of walkers on node  $v$  for all  $v$  in  $N(x_i(t)) \cup \{x_i(t)\}$ ,
- the flux of walkers arriving and leaving its current location  $v = x_i(t)$ , *i.e.*  $J_{u \rightarrow v}(t)$  and  $J_{v \rightarrow u}(t)$  for all  $u$  in  $N(v)$ .

Thanks to the information above, each walker  $i$  knows its previous location  $u = x_i(t-1)$ , its current location  $v = x_i(t)$ , and it has access to various criteria to decide its next location  $w = x_i(t+1)$ . Then, it needs a way to derive walking rules from criteria, and a way to combine these walking rules into a tactic.

### 3.2 Criteria

A criterion is a parameter from which we construct walking rules. We consider the following set of criteria  $\mathcal{C}$ .

- *Random*. The walker makes no difference between all possible neighbors: the criterion has value 1 for each of them.
- *Propulsion*. The walker never goes back to its previous location  $u$ , and otherwise makes random moves: the criterion has value 0 for  $u$  and value 1 for other neighbors of  $v$ .

- *Attraction.* The walker preferably moves to nodes where there are already many walkers: the criterion is equal to the number of walkers  $n_w(t)$ .
- *Follow.* The walker preferably follows the most popular moves of other walkers: the criterion has value  $J_{v \rightarrow w}(t)$ .
- *Alignment.* The walker takes into account the net flux in both directions: the criterion is equal to  $\Delta J_{v \rightarrow w}(t) = J_{v \rightarrow w}(t) - J_{w \rightarrow v}(t)$ .

### 3.3 Walking rules

Let us consider a criterion  $C_{u,v,w}(t)$ . We define the corresponding walking rule using the classical *logit rule*. It gives the probability  $\omega_{u,v,w}^C(t)$  that walker  $i$  moves from  $v$  to  $w$ , given the fact that it arrived at  $v$  from  $u$ :

$$\omega_{u,v,w}^C(t) = \frac{e^{\beta \cdot C_{u,v,w}(t)}}{\sum_{z \in N(v)} e^{\beta \cdot C_{u,v,z}(t)}} \quad (1)$$

Parameter  $\beta \geq 0$  is the intensity of choice: it quantifies the influence of the criterion on walker choices. If  $\beta = 0$ , the criterion has no influence and walkers make purely random choices. If  $\beta \rightarrow \infty$  walkers necessarily choose a neighbor among the ones that maximize the criterion.

This logit equation is widely used in the literature because it has many convenient features [5]. In particular, it ensures  $\omega$  is a positive and monotonic function of the criterion: the greater  $C_{u,v,w}(t)$ , the greater the chances to move to node  $w$  at time  $t$ . It is also compatible with criteria having both positive and negative values, like the flux-based one below.

We conducted experiments with other functions with similar properties to equation 1. They displayed no significant differences.

### 3.4 Tactics

A *tactic* is a linear combination of walking rules that defines the probability  $\pi_{u,v,w}(t)$  to move from  $v$  to one of its neighbors  $w$  when coming from  $u$ :

$$\pi_{u,v,w}(t) = \sum_{C \in \mathcal{C}} \alpha_C \cdot \omega_{u,v,w}^C(t) \quad (2)$$

where  $\alpha_C$  is the coefficient of criterion  $C$ , with  $\sum_{C \in \mathcal{C}} \alpha_C = 1$ . Therefore, a tactic is defined by a set of criteria and their coefficients.

We call *strict tactic* one that has all its coefficients, except one, set to zero and then always follows the same criterion.

In practice, at each time step, each walker selects a criterion  $C$  with probability  $\alpha_C$ . Then, it computes its probability  $\omega_{u,v,w}^C$  to go to each neighbor node  $w$  and selects its new location accordingly.

Notice that we may see a tactic as an inhomogeneous Markov chain, where transition probabilities are time-dependent. Still, at any given time, each walker makes a transition from its current state  $(u, v)$  giving its previous location  $u$  and its current location  $v$ , to its next state  $(v, w)$  giving its current location  $v$  and its next location  $w$ .

As transition probabilities at time  $t$  depend on previous non-deterministic moves, formal analysis of such processes is generally out of reach [6]. We will therefore explore possible tactics using simulations.

### 3.5 Baseline

In addition to the tactics above, we consider a reference baseline that easily achieves flocking thanks to collective decisions. This means that, at each time step, all walkers at a given node make the same choice. We then obtain reference results that we expect our walker models to reach or outperform, even though they are unable to make collective decisions.

More formally, for each node  $v$  at time  $t$ , a unique random neighbor  $u$  of  $v$  is chosen and all walkers  $i$  such that  $x_i(t) = v$  move to  $u = x_i(t+1)$ . This is equivalent to purely random walks of groups until they meet another group.

Since we consider non-bipartite connected graphs, it is well known that all groups will eventually merge. The number of needed steps is called the coalescence time [7]. Even though this number may be prohibitive, this means that the baseline successfully produces large groups. In addition, groups are mobile since, once formed, they perform purely random walks. As a consequence, this baseline makes a relevant reference for the success of flocking walkers.

## 4 Experiments

We seek to design good tactics for our walkers. They must produce flocking faster than the baseline presented in Section 3.5. Moreover, we want to obtain short time convergence, as our walkers model the action of protesters that can not walk forever. In this Section we define metrics to evaluate how well walkers flock. We then run simulations for a reasonable duration and evaluate which tactics have performed the best in this time interval.

### 4.1 Flocking metrics

We characterize *flocking* as a gathering of walkers exploring the network. With notations of Section 2.2, this leads to the two following *score* definitions for a given run of a given tactic.

**Definition 4.1** (cluster, gathering score  $\rho(t)$ ). A cluster is a maximal connected sub-graph with walkers on all its nodes. The gathering score  $\rho(t)$  is the average number of walkers in clusters.

For example, when every agents are in the same cluster,  $\rho(t) = |W|$ . If instead all agents are in different ones, it equals 1.

**Definition 4.2** (mobility score  $\mu(t)$ ). The mobility  $\mu_i(t)$  of walker  $i$  is the number of distinct nodes a walker has already visited at time  $t$ :  $\mu_i(t) = |\{v, \exists t' \leq t, x_i(t') = v\}|$ . The mobility score  $\mu(t)$  is the average walker mobility:  $\mu(t) = \frac{1}{|W|} \sum_{i \in W} \mu_i(t)$ .

Notice that the mobility score is monotonically non-decreasing with time:  $\mu(t+1) \geq \mu(t)$ . In addition, if all walkers move to a node they already visited then  $\mu(t+1) = \mu(t)$ .

We are interested in tactics with high gathering and mobility scores. Indeed, the high gathering score ensures that walkers form significant groups. In addition, the high mobility score implies that walkers continue to move. However, the fact that the gathering score remains high shows that walkers stay grouped. This means that the tactic successfully achieves flocking.

In order to gain more insight on group structure and dynamics, we introduce an additional metric.

**Definition 4.3** (sprawling score  $\sigma(t)$ ). The sprawling score  $\sigma(t)$  is the average number of nodes in clusters.

If the sprawling score is 1, all groups are isolated from each other: whenever walkers are at a node, there is no walker on neighbor nodes. If instead the sprawling score is high, walkers form large clusters of neighbor groups. Its largest possible value is the total number of nodes in the network, meaning that there are walkers on each node.

We are interested in tactics with low sprawling score, meaning that they succeed in merging neighbor groups.

## 4.2 Behaviors on a single street

Our networks are essentially composed of streets, modeled as lines of nodes by the discretization presented in Section 2.3 (Figure 2). This plays an important role in walker dynamics, therefore we investigate this special case in this Section.

Figure 3 shows *space-time* diagrams of three different strict tactics, *i.e.* the ones using only one rule, for 100 walkers on a line of 100 nodes. These space-time diagrams display the evolution of walker locations over time. The horizontal axis represents the nodes, numbered from 1 to 100. The vertical axis represents time steps,

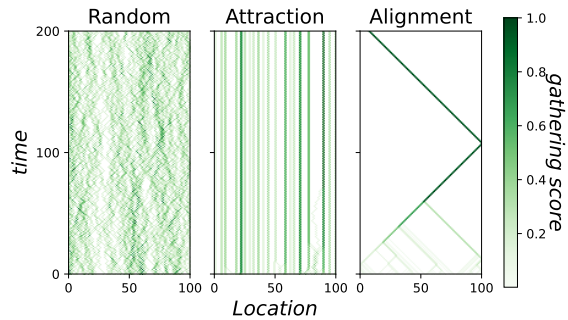


Figure 3: Space-time diagrams of three tactics on a line using only (from left to right) the Random, Attraction, and Alignment rule. The gathering score is expressed relatively to the maximum number of walkers in a given run.

numbered from 1 to 200. Then, the color at coordinates  $(x, y)$  represents the number of walkers at node  $x$  after  $y$  time steps, namely  $n_x(y)$ . The color is white if this number is 0, and it is the darkest if this number is the maximal value for the considered run. Walkers are initially uniformly distributed on all nodes.

With the Random and with the Attraction strict tactic (Figure 3, left and middle), the largest obtained groups during the run both always contain less than 10 walkers (respectively 6 and 8 on the Figure) which never makes up to 5% of all walkers. However, the two tactics have distinct behaviors: while the Random one leads to walker constantly moving between groups, the Attraction one leads to stable groups. These groups fail to move along the line, though.

With Alignment (Figure 3, right), a more interesting pattern emerges: all walkers gather into a unique cluster that achieves flocking since it moves all along the line.

In conclusion, the Alignment strict tactic is sufficient to obtain flocking on a street. However, the groups we obtain eventually reach the street extremities, which raise the question of their behavior at street intersections.

## 4.3 Extensive exploration of tactics

In this Section, we follow an extensive method to explore the wide set of possible tactics on an entire city network:

- we consider the Paris street network discretized with parameter  $\delta = 10$  meters, leading to a network of  $N = 145000$  nodes and  $M = 300736$  links,
- we consider  $N$  walkers initially distributed uniformly at random in the network,
- we perform 1000 time steps, thus considering reasonably short walks of approximately 10 kilometers,
- we set  $\beta$  large enough to ensure that each

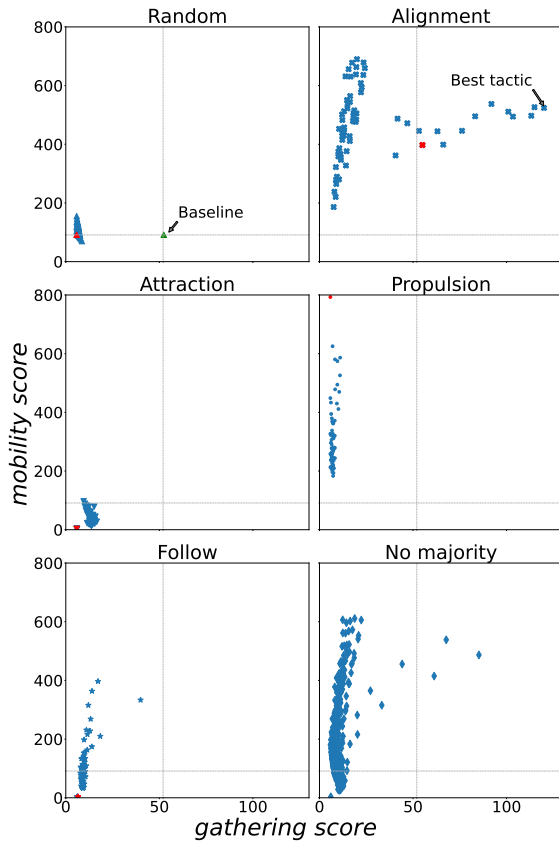


Figure 4: *Gathering and mobility of all tactics.* Each dot corresponds to the average last step value from ten runs of a tactic. The horizontal axis gives the gathering score, the vertical one gives the mobility score. From left to right and from top to bottom: tactics based mostly on the Random, Alignment, Attraction, Propulsion and Follow rule, respectively. On each of these plots, we indicate the strict tactic, that exclusively uses the corresponding rule. The bottom-right plot corresponds to tactics with no prevailing rule.

walking rule strictly follows its criterion,

- we finally consider all tactics obtained as combinations of  $\alpha_C$  parameter values from 0 to 1 by steps of 0.1 such that  $\sum_{C \in \mathcal{C}} \alpha_C = 1$ .

With this setup, we obtain 1001 different tactics. We run 10 simulations of each tactic and we plot the average mobility and gathering scores in Figure 4. In these plots, each dot corresponds to a tactic defined by a set of  $\alpha_C$  parameter values. We split these tactics into six plots: we display a set of tactics on the same plot if they all have  $\alpha_C > 0.5$  for the same criterion  $C$ , and we display on the last plot the set of all other tactics.

We also display in each plot of Figure 4 a vertical and an horizontal line that indicate baseline results. Then, the tactics achieving the best flocking performances are the ones in the upper right corner: they obtain groups bigger and are more mobile than the baseline.

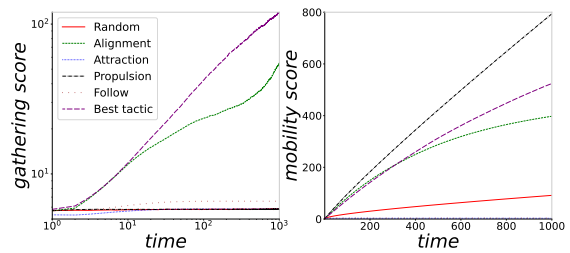


Figure 5: Plots showing the evolution of gathering and mobility scores for the strict tactics and the best tactic. Gathering score is in log scale.

We pay particular attention to strict tactics, which performances are spotted by red dots in Figure 4. We also highlight what we identify as the best tactic regarding our scores. It is the tactic with the highest gathering score among tactics that outperform our baseline. We display the evolution of mobility and gathering scores over time for these tactics in Figure 5.

All strict tactics have very poor scores, except the Alignment one. This is a consequence from our previous Section results, where it was the only to generate large and mobile groups in a street, and therefore fail on an entire city.

Attraction, Follow and Random strict tactic have almost no dynamic as walkers do not explore much the network and do not form large groups. Propulsion however displays an excellent mobility score with an almost linear growth on Figure 5, as walkers never go back to their previous location.

Figure 4 clearly shows that Alignment-based tactics (top center plot) outperform others. All other sets of tactics perform poorly, except a few tactics for which no rule weights up more than 50% (bottom right plot). These tactics actually also use Alignment rule, to a lesser extent. This identifies the Alignment rule as a key building block for flocking tactics.

#### 4.4 Best tactics

We now focus on the two main tactics that achieve flocking: the Alignment strict tactic and the best tactic (the one that corresponds to the rightmost dot on Figure 4). Figure 5 displays their scores over time in green and purple colors, respectively.

The plots show that the mobility score of both tactics first rapidly grow, and that this growth significantly decreases over time. Even if these tactics are not as good as the Propulsion strict tactic regarding mobility, they have comparable performances for this metric.

Notice that the Propulsion strict tactic has very low gathering scores (Figure 5, left), which makes it an irrelevant tactic despite its mobil-

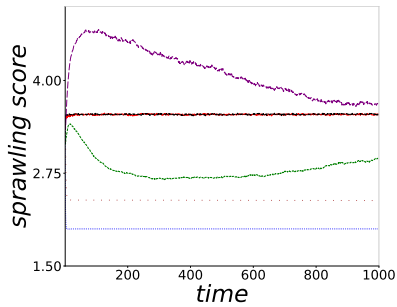


Figure 6: Plot showing the evolution of the sprawling scores for the strict tactics and the best tactic. Legend is the same as in Figure 5.

ity score. Instead, both the Alignment strict tactic and the best static quickly reach excellent gathering scores. The best tactic significantly outperforms the Alignment strict tactic and has a linear gathering score plot in log-log scale. This means that its evolution has a polynomial growth (of exponent below 1), indicating a fast growth, but also that the evolution of group size tends to flatten over time. This is due to groups reaching a state where all clusters of groups are in distant regions of the network. Then, it takes longer for groups to meet other groups, merge, and grow in size.

Finally, Figure 6 displays the sprawling score for all considered tactics. We observe that the best tactic produces a greater sprawling score than Alignment. The sprawling, for those two tactics, is due to groups following each others when they detect another group on a neighbor node, without necessarily merging with it.

With the Alignment strict tactic, the sprawling of groups first very quickly increases, then decreases and stabilizes. This is because this tactic forms groups immediately at the beginning, mostly as lines of walkers following each others. The sprawling reduces as groups reach intersections and split, until the aggregation and splitting dynamics reach an equilibrium.

For the best tactic, groups aggregate into lines for a longer time period, resulting into a much higher sprawling score. It then slowly linearly decreases until the end of the run. This is because the Attraction rule, when chosen in the best tactic, will make the front group wait for the groups behind it, leading to less sprawled clusters.

As explained in Section 4.1, an efficient tactic should have a low sprawling score. The sprawling of the baseline is 1 (up to the third digit), thanks to the collective decision. This is the optimum value.

Our walkers do not have access to collective decision. When a cluster of groups arrives at an intersection, at the end of a street, it can split into multiple groups. In the case of a lone group

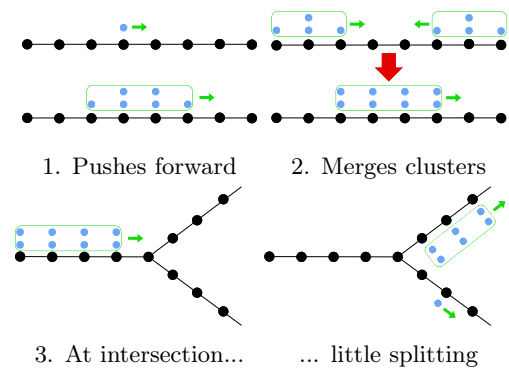


Figure 7: Schematic configurations that walkers, groups and clusters of groups achieve with the best tactics.

(a cluster with no sprawling) it will split into multiple groups, with equal number of walkers on average. This reduces the gathering score.

However, with a larger sprawling, a cluster keeps most of its groups unchanged after an intersection. Indeed, the first of its groups to reach the intersection node might split into multiple groups, as described above. However, if using an Alignment-based tactic, next walkers will then all follow the biggest of the new groups that came from the split. This shows an effective tactic can benefit from a little bit of sprawling to maintain its clusters size.

## 4.5 Interpretation

Recall that the best tactic corresponds to the rightmost dot among those in the upper right corner of plots in Figure 4. It is defined by the following parameters:  $\{\alpha_{follow} = 0.1, \alpha_{align} = 0.8, \alpha_{attr} = 0.1\}$  and a null weight for other rules. This tactic produces groups of 121 walkers on average, and walkers explore on average 540 distinct nodes during their 1000 steps. These scores are more than twice and five times more than what the baseline gets, respectively.

Figure 7 illustrates the behavior of this tactic. First, Alignment imposes walkers to move forward, may they be alone or part of a cluster, as shown in the first configuration of Figure 7. Indeed, a walker  $i$  alone at location  $x_i(t) = u$  and  $x_i(t-1) = v$  will measure a negative flux  $\Delta J_{u \rightarrow v}(t) = -1$  at time  $t$ , while it will be  $\Delta J_{u \rightarrow w}(t) = 0$  for all  $w \neq v$ . This implies the walker never goes back. This effect is left unchanged with multiple walkers in a cluster.

Second, this same rule guarantees that, if two groups cross path, they then merge in a single cluster in which all walkers will follow the same path. Indeed when a group  $u$  cross path with a smaller group  $v$ , we have  $x_i(t+1) = x_j(t)$  and  $x_j(t+1) = x_i(t)$  for all  $i \in g_u(t)$  and all  $j \in g_v(t)$ . The net flux is then  $\Delta J_{u \rightarrow v}(t) = n_u(t) - n_v(t) > 0$  for all walkers, which drive them all

in the same direction: the smaller group goes back towards  $v$  (where it came from) while the trajectory of the largest group is left unchanged.

However, this rule alone is not perfectly effective at avoiding splitting. The walkers in the first group in a cluster will not always all choose the same node at an intersection, as illustrated in the two bottom pictures of Figure 7, as the different possible nodes all have a flux equal to 0.

In this context, the Attraction rule allows walkers that split in the least chosen direction at an intersection to go backward and avoid losing sight of the cluster.

Finally, the Follow rule improves the gathering. Indeed, clusters of groups tend to sprawl when walkers use the Alignment rule. In such chain of groups following each others, the Follow rule allows walkers in the front group to move backwards, merging with the group behind them, while it forces walkers in other groups to move forward to catch up the leading group.

This equilibrium between those three rules gives an outcome where groups flock very efficiently.

Figure 7 also shows why the baseline is not good enough: walkers do not move forward at every step. However we could obtain this with a *collective propulsion* (a collective decision with the Propulsion rule instead of the Random one), such tactic neither guaranty groups merge when they cross path. That is the strength of Alignment rule: walkers merge not only if they fall on the same cell, but also if they simply exchange their position.

## 4.6 Group robustness

It is crucial for protesters to form groups that resist adversary forces that may break them up. In order to explore this robustness, we model break ups as walkers suddenly following the Random strict tactic for one step. In this way, a group located at a given node splits into smaller groups that move to neighbor nodes, in a way similar to a group of protesters targeted by adversary forces.

More formally, we perform the following experiment: we run the Alignment strict tactic for 100 steps, then we run the Random strict tactic for one step, and we finally run the Alignment strict tactic again. We perform the same experiment with the best tactic (Section 4.5) instead of the strict Alignment one.

Figure 8 displays the average observed scores over ten runs of each experiment, together with results of the same experiments without the Random strict tactic step. Since the mobility score is trivially increased by these experiments, which brings no useful insight, we do not display this metric.

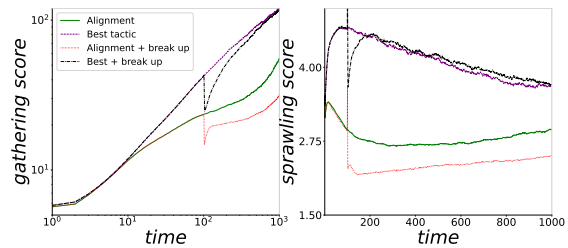


Figure 8: Plots of robustness experiments, similar to Figure 5. We display the sprawling and gathering scores for the Alignment strict tactic and the best tactic, as well as for the experiments where all walkers perform a random move at time 100.

The break up simulation clearly appears on both plots. The gathering score plots show that large groups reappear shortly after break ups. The two tactics do not have the same behaviour, though. The strict Alignment tactic needs some time to reach gathering scores close to its pre-break up ones (green curve in Figure 8). Instead, the best tactic is strikingly efficient in quickly recovering from break ups. Indeed, large groups do not only reappear shortly after the break up, they do so even faster than before. This brings its gathering score (black curve in Figure 8) to quickly catch up with the case with no break up (purple curve).

We also observe that, before going back to normal, the sprawling score experiences a sharp increase immediately followed by a sharp decrease. The increase is due to the fact that groups in each cluster first split into several neighbor groups, thus forming a larger cluster. Then, the new groups tend to go forward, due to the Alignment rule, and so they move away from each other forming separate clusters. Once again, this metric shows in Figure 8 that the best tactic recovers faster than the Alignment strict tactic.

## 5 Related work

First notice that our work is not concerned with the modeling of actual pedestrian behaviours and trajectory planning [8, 9, 10]. Instead, our goal is to design simple tactics that lead to targeted features, like flocking.

Likewise, our approach is different from usual protest models based on thresholds [11] or agent-based [12, 13, 14] approaches. Indeed, these works focus on how people decide to participate to a protest; they do not deal with protester mobility.



Models	Network	Lattice	Flocking	Fast
Random walk [15]	●	●		
UTS/UXS [16, 17]	●	●		
EAW [18]	●	●		●
SIP [19]		●		
MIPS [20]				●
Boids [21]			●	●
Cucker-Smale [22]			●	●
Robot swarm [23, 24]		●	●	●
SVM [25, 26]		●	●	●
Our work	●	●	●	●

Table 1: Important models of collective walks found in literature that provide solutions to problems similar to ours. The ● specifies if they succeed at solving one of the issues we deal with.

## 5.1 Walks on networks

Instead, a number of works deal with flocking behaviors of walkers in a variety of situations. We summarize their relevant features in Table 1. It shows the limits of these works from our perspective. We discuss them in the following.

In distributed systems, computer scientists proposed solutions to gathering problems where walkers follow a common distributed algorithm to meet on any connected graph [27, 28, 29]. In a sense, the main algorithms (UTS and UXS in Table 1) [16, 17] from this community are similar tactics to our baseline, except they are deterministic.

It is important to note that labeling nodes, and letting walkers have the global knowledge to know all labels, are essential in their protocols. Moreover those studies only look for guaranties regarding gathering at long time. Unlike those, we had to ensure our walkers gather sufficiently fast and with no need for hard computations.

## 5.2 Flocking in free space

Flocking is a collective dynamic where groups of walkers move spontaneously in the same *direction* [22, 30]. Ranging from swarms to pedestrian crowds, numerous articles explored this behavior, in *free space*, since the Boids model [21]. The work of Vicsek [25] notably settled that alignment and noise were sufficient to produce flocking in the plan (SVM in Table 1). The combination of alignment and attraction is also an effective way to get gathering and flocking [31], as attraction avoids sprawling. Robot swarms also achieve flocking and similar objectives to ours [23]. However, that last example often uses the *local communication model* that exceeds our walkers capabilities.

In our case, trajectories of walkers are network constrained. As we are not in free space we could not, in a simple manner, define a notion of direction to describe walkers motion in a structure as loose as a graph (except for a lattice where you can define pairs of right/left, top/bottom direc-

tion). That is why we introduced instead the mobility score in Section 4.1.

## 5.3 Flocking and networks

Research on flocks *and* networks mostly do not study walks *on* graphs. They study the *proximity graph* of walkers: the graph of which walkers are in sight of another walker [32]. Researchers then study guaranties regarding the connectivity of this graph to evaluate if walkers achieve flocking.

However, a few articles explore flocking on simple networks. The motivation, in the context of statistical physics, is to use lattices as an approximation of free space to study active particles [20]. For example, in [26] the authors implement rules similar to ours to study the possible outcomes of their combinations on a line. They get results very similar to ours from Section 2.1. We also note that our baseline from Section 3.5 is very similar to the *simple inclusion process* (SIP in Table 1) [19].

We can also cite algorithms that solve problems very close to ours on a network. There is [33] that studies the reciprocal problem of completely scattering walkers over a graph, while they are all gathered on a single node at the beginning. There is also the *edge ant walk* (EAW in Table 1) [18], a bio-inspired model, where walkers communicate by leaving a trace of their passage, that can be used to patrol a graph.

Still, to our knowledge, we are the first to experiment rules for flocking that we can apply on any network. Such gap exists in the literature as such dynamic is not meaningful on many complex networks. Those often exhibit small-world and hub properties, making it very easy to gather fast when simply hopping from a high degree node to another at each step. Moreover, as phenomena in free spaces inspire flocking models, existing algorithms are then usually designed for application in free space and not on networks.

## 6 Conclusion

This paper proposes a framework for exploring what basic rules are crucial for walkers to gather and flock in a street network. Our findings indicate that the Alignment strict tactic is sufficient for flocking on a network. Alignment-based tactics are the most effective, and obtained groups are robust to break ups.

This framework is very general and may be extended in several ways. First, the city modelling may be improved with more urban data. For instance, one may use street width information to estimate the number of protesters needed to block each street. One may also use such data,

as well as point-of-interest (POI) information, such as the presence of restaurants, stores, or theaters around protesters, to design more effective walking rules. Last but not least, this work opens the way to studying how obtained walks impact the underlying street network and traffic taking place on it.

### Code and data.

We provide an implementation of our models in C, and a Python code result analysis: [https://github.com/K-avi/protesting\\_on\\_graphs](https://github.com/K-avi/protesting_on_graphs). We describe how to use it, including how to generate the obtained dataset, on the companion site: [https://k-avi.github.io/protesting\\_on\\_graphs](https://k-avi.github.io/protesting_on_graphs).

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