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# Damage evaluation and tracking using kriging approaches on wave-guide dispersion curves

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**ABSTRACT:** The calculation of the wave numbers is sometimes a prerequisite as well as a potential indicator of the evolution of the structure's state. On the one hand, when addressing medium- and high-frequency monitoring signals, even Wave Finite Elements Method's calculations can take a long time to obtain the dispersion curves. The present work proposes the kriging based metamodelling for both the prediction and the measurement of dispersion curves in waveguides. The main idea is to predict the dispersive curves by using a small number of WFEM based simulations. This study hence analyses the reliability and validity of the kriging based metamodelling when used for the prediction and the evaluation of the dispersion curves for a waveguide structure.

## 1 PROPERTIES OF WAVEGUIDES FOR STRUCTURAL HEALTH MONITORING

The importance of determining properties of guided waves such as the dispersion curves is well established in numerous fields relating in particular to Structural Health Monitoring (SHM). Indeed, for a given structure, the dispersion curves permit the characterizing of the velocity-frequency relationship for all the modes which may propagate in a given structure, which, in the defects and damage detection with non-destructive approaches, is necessary to obtain insight about the efficiency of the wave transmission. Then, predicting the dispersion curves has been addressed few decades ago and numerous approaches have been addressed in literature (Keane 1997, Fahy 1994, Yong 1989). The most popular classification of these approaches is that dividing them into two main groups namely analytical or exact methods and numerical methods (Bartoli 2006, Sorohan 2011).

When based on propagative approaches, the health monitoring techniques require that the behavior of the structure shall be somehow understood in order to get an accurate evaluation of the damage level. The calculation of the wave numbers is sometimes a prerequisite as well as a potential indicator of the evolution of the structure's state. Indeed, it permits to getting insight about how wavemodes are propagating and interacting in structures.

On the one hand, when addressing medium- and high-frequency monitoring signals, even Wave

Finite Elements Method's calculations can take a long time to obtain the dispersion curves.

On the other hand, when time has come to obtain the experimental dispersion curves to identify the occurrence of a damage, the question arises about the selection the measurement points. The kriging tools are then suited for this.

The present work proposes the kriging based metamodelling for both the prediction and the measurement of dispersion curves in waveguides. The main idea is to predict the dispersive curves by using a small number of WFEM based simulations. This way is expected to drastically reduce the above-mentioned numerical difficulties and thus enhance the efficiency. The distribution and the number of the measurement points can also be derived.

This study hence analyses the reliability and validity of the kriging based metamodelling when used for the prediction and the evaluation of the dispersion curves for a waveguide structure. Different parameters of kriging metamodells are hence considered. The main objective is to search for the kriging parameters that maximize the accuracy of the kriging based predictions of the dispersive curves while ensuring an acceptable computing time.

## 2 WAVE FINITE ELEMENT METHOD FOR PERIODIC WAVEGUIDE

The Wave Finite Element Method (WFEM) is widely used to study free and forced vibration in

periodic structures. Basically, this method helps for the describing of waves travelling in left and right directions in a periodic structure by using a finite element model of only a single cell with a small number of degrees of freedom, instead of considering the whole periodic structure (Droz 2018, Mal-louli 2019, Zhou 2015).

Let's consider an elastic waveguide structure which consists of  $H$  identical substructures of length  $d$  as shown in Figure 1. Both the left and right boundaries of the substructure contain the same number of  $n$  degrees of freedom.

The dynamic equilibrium of the substructure is formulated and then, by using the Bloch's theorem associated to the augmented state vector, the wave modes that propagate along the whole structure are obtained from the solutions of the corresponding eigen-solution equations.

The eigenvectors associated to the eigenvalues characterize the incident waves and reflected waves which also represents the nodal displacement mode and the force mode on one edge of the substructure. This matrix can be expressed by considering the incident and reflective wave parts (Zhou 2015).

For wave guides with damping (namely dissipative waveguide), the associated wavenumbers are complex and are related to decaying waves. The dispersion curve can be determined from the relationship between wavenumbers and frequency like in Figure 2.

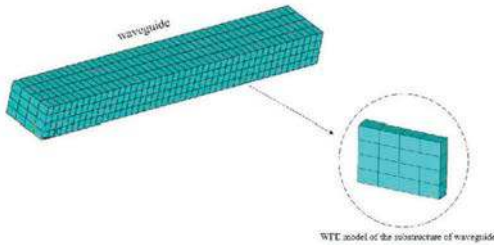


Figure 1. FE model of a substructure.

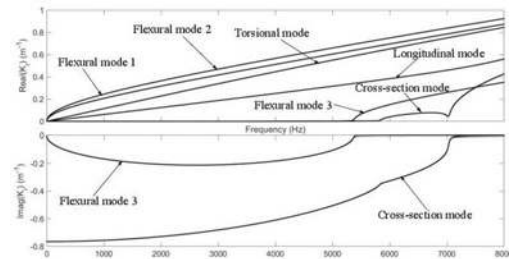


Figure 2. Dispersion curve of a substructure.

### 3 KRIGING BASED META-MODELLING FOR THE PREDICTION OF THE DISPERSION CURVE

#### 3.1 Advantage of the kriging based metamodelling

The key idea and advantage in kriging based meta-modelling is related to the using of only a small size data set of suitable input/output samples in order to mimic and predict the behavior of an unknown function. To acquire this mimicry capacity, a kriging metamodel exploits information about spatial correlations between sample points by considering the associated distance. This is performed by using pre-defined correlation models. The latter should well represent the most likely relationships between samples. In other words, models are built such as two infinitely distant points be not correlated while two identical points have a unitary correlation. In the here-reported works, the main principles are used to build a kriging metamodel in order to approximate and predict the dispersion curves.

#### 3.2 Calculation of the dispersions curves

For this goal, we first start with a set of frequency samples for which the corresponding set of  $n$ -dimensional vector of wave-numbers is available. The corresponding optimal scaling parameter is obtained from a specific equation. Hence, from the Kriging theory, the wave-number  $k$  can be expressed as follows:

$$k(\omega) = F(\omega)^T \beta + Z(\omega) \quad (1)$$

where  $F$  is a vector of  $m$  basis polynomial functions that are weighted by the unknown  $\beta$  coefficients defining the mean behavior of the considered wave-number. The predicted value of the wavenumber is then updated and corrected by the second part  $Z$  which represents a realization of a normally distributed random process. The latter is assumed to be with a zero mean value and with a covariance function given by:

$$E[Z(\omega)Z(\omega^*)] = \sigma^2 R(\theta, \omega, \omega^*) \quad (2)$$

where  $E[.]$  is the expectation operator,  $\sigma^2$  represents the process variance and  $R(\theta, \omega, \omega^*) \in [0, 1]$  is a spatial correlation model depending on a scaling parameter  $\theta$  required to be optimized. As previously mentioned, the spatial correlation model is a monotonous function depending on the distance, positive or null, between the two frequency points  $\omega$ ,  $\omega^*$ . It is built such as two infinitely distant points that are not correlated while two identical points have a unitary correlation.

Some basic correlation functions are here used in the context of this study. Hence based on the obtained optimal parameters, the kriging predictor of a wavenumber  $k$  is determined by minimizing the constrained mean square error.

By using the Lagrange multipliers based method, from (Sacks 1989) the best linear unbiased predictor of the wavenumber  $k(\omega)$  can be ultimately expressed.

Hence, the previous predictor can be obtained for all wavenumbers which lets the obtaining the dispersion curve. It is worth mentioned, that in order to obtain the kriging predictors, it is necessary to fix beforehand the order of the regression, the type of spatial correlation model as well as the size  $N$  of the training set. A prior knowledge about the function to be approximated (differentiability and continuity, degree of nonlinearity, monotony, ...) can help and guide the choice of the order of regression and the type of spatial correlation and give an idea of the size of the learning set.

#### 4 APPLICATION AND RESULTS

In order to assess the efficiency of the kriging metamodel when used for the predicting and the estimating of the dispersion curves, a multi-degree of freedom beam-like waveguide is considered in this section. This waveguide structure is also considered in (Mencik 2005) as the basic element to define coupled waveguides.

A substructure of the beam-like waveguide is first described by a finite element model which is then exploited through the WFEM in order to obtain the dispersion curves. These are considered as the reference which will be used to evaluate the accuracy of the kriging based predictions. This accuracy is analyzed with respect to numerous parameters of the kriging predictor namely the regression order, the correlation function and the size of the learning data set. The main objective is to determine the parameters of kriging predictor that ensure suitable robustness properties for the accuracy of the predictions with respect to the complexity of the dispersion curves.

##### 4.1 Kriging model for the dispersion curve of multi-dof waveguide

As previously described, the wave modes present different patterns depending on the frequency parameter. Consequently, we can expect the necessity to define as many kriging predictors as there are wave modes. Each predictor would have its own optimal parameters in terms of the regression order, the correlation function and the sizes of the learning data sets in order to ensure accurate predictions of the wave modes.

Moreover, as the learning data samples are generated with a pseudo random process namely the Latin Hypercube Sampling (LHS) method, the generating process is repeated  $Q$  times in order to analyze the impact of the location of the generated learning

samples on the performances of the kriging metamodels. In fact, the main aim is to analyze the robustness of the kriging predictors with respect to the location of the learning samples.

The main performance which is considered to assess the kriging predictors of the wavenumbers is the square error (MSE) between the predicted and the reference value obtained by the WFEM, averaged on the size  $J$  of the testing sample data set.

##### 4.2 Kriging for the prediction of the real parts of the wave modes

The used kriging metamodels are built by considering the previously mentioned spatial correlation models, a zero order regression and one realization of each of the  $N$ -size learning data sets. It can be observed that the size  $N = 10$  of the learning data is insufficient to appropriately mimic the monotonous behaviour of the frequency-dependent real part of the wavenumber in particular when linear, spherical and spline correlation models are used. The Exponential and especially the Gauss correlation models already offer with these few samples a suitable overview about the function.

The real part of the corresponding wavenumber exhibits different patterns and they do not necessary monotonically evolves against the frequency. Indeed, the real part of the wavenumber associated to the third flexural mode presents two particular frequency segments namely  $[0, 5400[\text{Hz}]$  and  $]5400, 8000]$  Hz with a singularity around 5400 Hz which denotes a relatively abrupt change in the frequency-dependent evolution of the real part. Then, it is necessary to analyse the potential of kriging metamodels to be accurately predictive of this more complex behavior.

The kriging metamodels that are built by using single re-realizations of learning data sets with different size  $N = [10, 50, 100, 200, 500]$  have estimated the

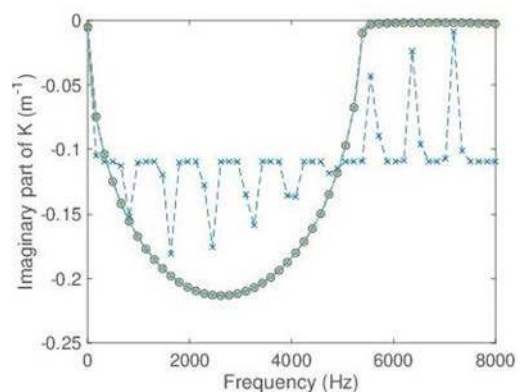


Figure 3. The 3rd flexural mode predicted by a zero-order regression based kriging metamodels with spline correlation function and different sizes  $N$ ;  $\times$  10 sample points,  $-$  50 sample points,  $\cdot$  100 sample points,  $o$  200 sample points,  $*$  500 sample points,  $-$  reference model.

wave-numbers the real parts of which are plotted in Figure 3.

It can be observed that the shape of the curve is correctly approximated by all the kriging predictors from  $N = 50$ . The difference between the different predictors is located in particular at the zone of occurrence of the singularity (near 5400Hz).

It appeared that the predictor with the Gauss correlation model has difficulties.

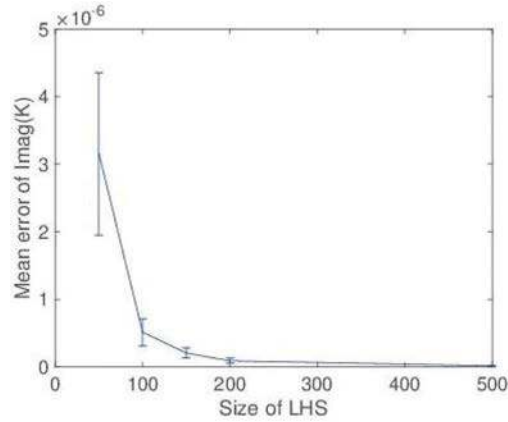


Figure 4. Evolution of the mean square error and the standard deviation - zero-order regression - spline correlation function versus the size of the LHS design on imaginary part of wavenumbers.

## 5 CONCLUSIONS

This paper has proposed the kriging metamodelling as an alternative for obtaining dispersion curves in complex waveguides in industrial applications.

This approach is proposed since using the WFEM still remains too costly when dealing with complex- The performed numerical simulations have shown an interesting potential for the kriging metamodelling approach since it has given suitable compromise between the computing costs and the accuracy of 245 predictions than the exclusive use of the WFEM. The choice of suitable spatial correlation model and the size of the learning data sample are the two parameters that condition this suitable compromise.

Some prior knowledge on the mathematical properties of the wavenumbers may help for an optimal choice of the spatial correlation model while the determining of a minimal size of the learning set remains a challenging issue.

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Table 1. Maximum and mean values, standard deviation of the prediction MSE for  $Q=50$  realization and  $N=150$  – imaginary part of flexural mode 3.

RegPoly 0					
	Spherical	Spline	Linear	Gauss	Exponential
Max MSE	2.13E-06	3.65E-07	2.29E-06	3.39E-05	2.29E-06
Mean MSE	8.02E-07	2.05E-07	8.48E-07	2.09E05	8.47E-07
STD	4.94E-07	7.36E-08	5.39E-07	9.00E-06	5.38E-07
RegPoly 1					
	Spherical	Spline	Linear	Gauss	Exponential
Max MSE	2.19E-06	3.62E-07	2.33E-06	3.10E-05	2.34E-06
Mean MSE	8.17E-07	2.03E-07	8.57E-07	2.21E05	8.59E-07
STD	5.11E-07	7.28E-08	5.48E-07	9.07E-06	5.51E-07
RegPoly 2					
	Spherical	Spline	Linear	Gauss	Exponential
Max MSE	1.75E-06	3.80E-07	1.87E-06	3.37E-05	1.88E-06
Mean MSE	6.88E-07	1.55E-07	7.29E-07	2.07E05	7.31E-07
STD	3.89E-07	7.17E-08	4.23E-07	8.66E-06	4.24E-07

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