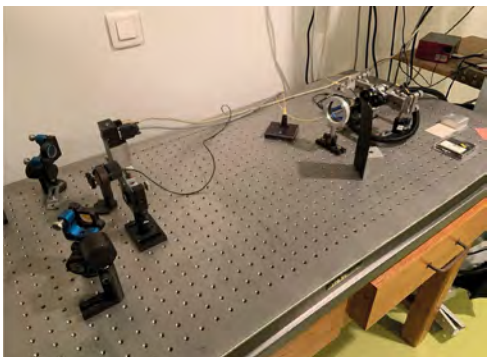
Two-photon interference and the Hong-Ou-Mandel effect

Benjamin Vest, Lionel Jacubowiez

Two-photon interference and the Hong-Ou-Mandel effect

Benjamin VEST, Lionel JACUBOWIEZ,
Université Paris-Saclay, Institut d'Optique Graduate School, CNRS, Laboratoire Charles Fabry, 91127 Palaiseau, France
***benjamin.vest@institutoptique.fr**



The Hong-Ou-Mandel (HOM) experiment is a landmark in quantum optics. A labwork version of this famous two-photon interference effect was developed at Institut d'Optique for students in engineering and MSc tracks. The setup enables the observation of the iconic HOM "dip" and the measurement of photon indistinguishability.

The HOM labwork setup @ Institut d'Optique.

https://doi.org/10.1051/photon/202312524

The Hong-Ou-Mandel experiment (HOM), performed in 1987 [1], is the first experiment reporting the observation of a two-photon quantum interference, occurring when two indistinguishable photons are sent simultaneously on the two input ports of a beamsplitter (BS). The photon distribution at the exit is properly astounding: indeed, the two indistinguishable photons leave the setup by the same output port and are never split by the BS ! This gregarious behavior is called photon bunching, and strongly contradicts the "classical" behavior: independent particles with a 50% chance of being either transmitted or reflected are expected to pick different output ports 50% of the time.

The HOM effect is exploited to provide a quantitative measurement of particles indistinguishability, an important requirement to create complex multi-particle superposition states, a common resource in quantum information. The HOM effect extends well beyond the exclusive case of photons: indeed, experiments have been notably performed to investigate the same effect with other quantum particles [2,3].

This experiment is now routinely proposed and performed as a Labwork session in the LEnsE (Laboratoire d'Enseignement Expérimental) of Institut d'Optique as both an illustration of the quantum weirdness and standard experimental protocol in the field of quantum technologies.

Classical description of a lossless beamsplitter

The HOM experiment relies on the concept of a beamsplitter (BS): a device that splits incoming light into a reflected and a transmitted wave. In classical electrodynamics, input and output fields are related via complex reflection and transmission coefficients r and t. We can write using matrix formalism:

$$\begin{pmatrix} E_c \\ E_d \end{pmatrix} = U \begin{pmatrix} E_a \\ E_b \end{pmatrix} \text{ (Eq. 1)}$$

$$\text{with U} = \begin{pmatrix} t & r \\ r & t \end{pmatrix}, \text{ the BS matrix}$$

The energy conservation condition writes $|E_c|^2 + |E_d|^2 = |E_a|^2 + |E_b|^2$. By multiplying Eq. 1 by its conjugate, we immediately get $U^\dagger U = I$. It is said that U is unitary, and this condition leads to constraints on the coefficients:

$$|r|^2 + |t|^2 = 1$$

$$rt^* + r^*t = 2Re(rt^*) = 0 \text{ or } r = \pm it$$

The first condition can be directly related to energy conservation. The second condition shows that the absence of losses impose a phase relation between the reflection and transmission coefficients. The HOM effect being an interference effect, this second condition plays a crucial role in the experiment.

Lossless beamsplitter in quantum optics

While the energy of a classical wave is a continuous quantity that splits into two output fields, the behavior of a photon, is different. It can be either reflected (with probability $|r|^2$) or transmitted (resp. $|t|^2$), but its energy is never divided into smaller parts.

This result is predicted by the quantum optics formalism: we replace classical fields by operators.

We introduce $\hat{a}$ and $\hat{a}^\dagger$ the annihilation et creation operators associated to the electric field $\hat{E}_a$ of input port a of the BS. We also introduce the number operator $\hat{n}_a = \hat{a}^\dagger \hat{a}$, so that the field hamiltonian writes $\hat{H}_a = \hbar\omega(\hat{n}_a + 1/2)$ Its eigenvectors are the Fock states : for example, $|n\rangle$represents a field state with n photons.

Similarly one introduces $\hat{b}, \hat{b}^\dagger$ and $\hat{n}_b$ for input port b and so on for output ports c and d of the BS.

The transformation induced by the BS can be expressed using relations between input port and output port operators. They are all encompassed by the same matrix U used in the previous part:

$$\begin{pmatrix}\hat{c}\\ \hat{d}\end{pmatrix} = U\begin{pmatrix}\hat{a}\\ \hat{b}\end{pmatrix}$$

The unitarity of U leads to a new formulation of energy conservation

$$\hat{c}^\dagger\hat{c} + \hat{d}^\dagger\hat{d} = \hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b} \text{ or } \hat{n}_c + \hat{n}_d = \hat{n}_a + \hat{n}_b$$

This time interpreted as a conservation of the number of photons. More generally, one can show that operators are related via [4]:

$$\begin{aligned}\hat{a}^\dagger &= t\hat{c}^\dagger + r\hat{d}^\dagger\\ \hat{b}^\dagger &= r\hat{c}^\dagger + t\hat{d}^\dagger\end{aligned}$$

One photon on a beamsplitter

Let's start with a simple situation: a single photon is sent on input port a. This state can be expressed using $\hat{a}^\dagger$:

$$|1_a, 0_b\rangle = \hat{a}^\dagger|0,0\rangle$$

The previous relation enables us to rewrite the same state in the output space of the system:

$$|1_a, 0_b\rangle = \hat{a}^\dagger|0,0\rangle = (t\hat{c}^\dagger + r\hat{d}^\dagger)|0,0\rangle = t|1_c, 0_d\rangle + r|0_c, 1_d\rangle$$

We get a quantum superposition state. It explicitly shows that the photon can be either measured on output port c with probability $|t|^2$ OR on output port d with probability $|r|^2$. For a stream of single photons reaching the BS, two detectors placed at each of the BS output never click simultaneously. This measurement is an observation of photon anti-bunching, and an illustration of the particle-like behavior of photons [5].

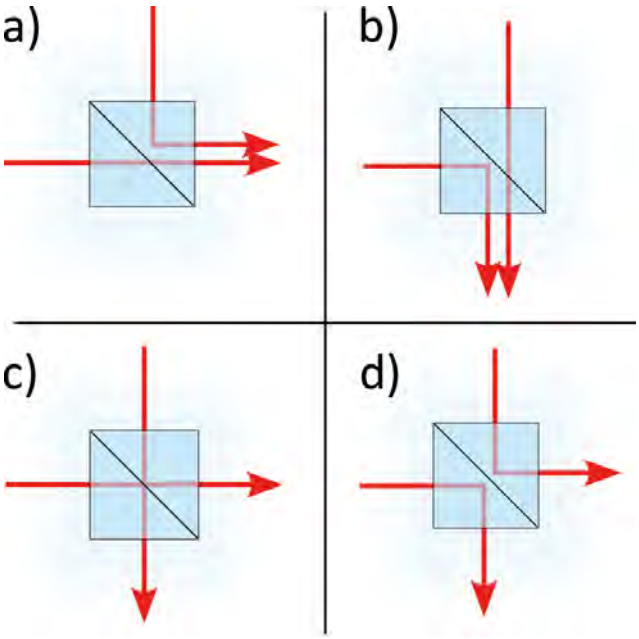


Figure 1. Four classical scenarios for a pair of photons impinging on a beamsplitter.

Two photons on a beamsplitter

In an HOM setup, two indistinguishable photons are sent at the two input ports of a lossless BS. Expanding the way of thought of the previous paragraph, we state that each photon is either reflected or transmitted, leading to four different scenarios (see Fig 1):

a) Photon *a* is transmitted and photon *b* is reflected ; both photons exit port *c*

b) Photon *a* is reflected and photon *b* is transmitted ; both photons exit port *d*

c) Both photons are transmitted ; photon *a* exits port *c* and photon *b* exits port *d*

d) Both photons are reflected ; photon *a* exits port *d* and photon *b* follows exits port *c*

We use creation operators and the vacuum states to write the quantum state corresponding to this situation :

$$|\Psi_{HOM}\rangle = \hat{a}^\dagger \hat{b}^\dagger |0,0\rangle$$
$$|1_a, 1_b\rangle = \hat{a}^\dagger \hat{b}^\dagger |0_a, 0_b\rangle$$

In the output space:

$$(t\hat{c}^\dagger + r\hat{d}^\dagger)(r\hat{c}^\dagger + t\hat{d}^\dagger)|0,0\rangle =$$
$$(tr\hat{c}^\dagger\hat{c}^\dagger + rt\hat{d}^\dagger\hat{d}^\dagger + t^2\hat{c}^\dagger\hat{d}^\dagger + r^2\hat{d}^\dagger\hat{c}^\dagger)|0,0\rangle = tr|2_c, 0_d\rangle + rt|0_c, 2_d\rangle +$$
$$t^2|1_c, 1_d\rangle + r^2|1_c, 1_d\rangle$$

and we can identify 4 terms for the 4 different scenarios discussed above.

The calculation is not over yet ! We have candidly "ommitted" to factorize the last two terms: indeed, $t^2|1_c, 1_d\rangle + r^2|1_c, 1_d\rangle$ can be written $(t^2 + r^2)|1_c, 1_d\rangle$.

This trivial operation hides a subtlety : this operation is acceptable provided that the output quantum state obtained when both photons are reflected is exactly the same as the quantum state obtained when both photons are transmitted. In other words, we considered that these two scenarios are *indistinguishable*.

Indistinguishability means here that there is absolutely no way, experimentally *or even in principle*, to perform any type of measurement that would allow us to tell, for a given photon pair, which one of the two scenarios (double transmission or double reflection) occurred in practice. This indistinguishability criteria requires:

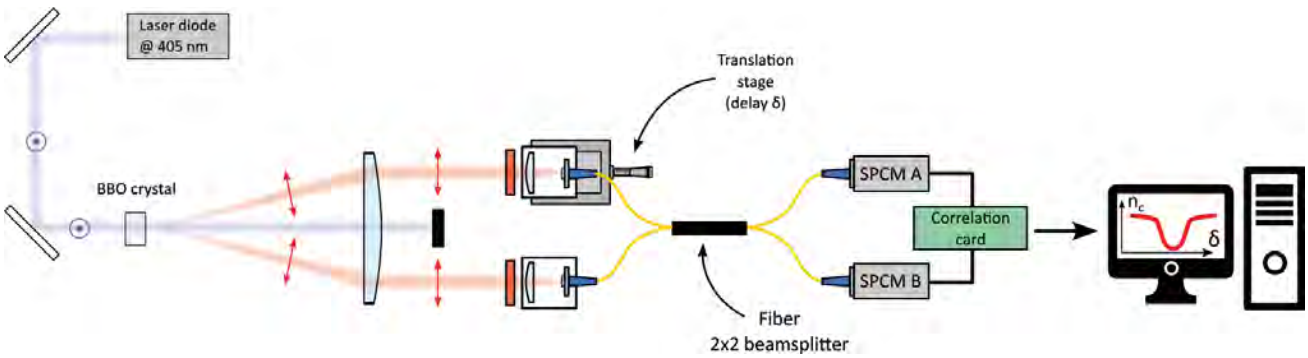
a) That the intrinsic properties of the photons are the same (polarization, energy/frequency)

b) That the photons cannot be distinguished spatially (in other words, a photon from input *b* experiencing transmission and a photon on input *a* experiencing reflection end up populating the same spatial mode of output *c*).

c) That the photons cannot be distinguished temporally (both photons must reach the BS simultanously, so that there is no timing information available to identify one photon or the other.)

These criteria being considered as fulfilled, the final step is now to measure the probability of observing each one of the 4 different scenarios listed. Let us start with the probability of

Figure 2. Two-photon interference setup as implemented in the LEnsE at Institut d'Optique.



measuring state $|2_c, 0_d\rangle$:

$$P(2_c, 0_d) = |\langle 2_c, 0_d|\Psi_{HOM}\rangle|^2 = |t|^2\,|r|^2$$

By symmetry, we have $P(2_c, 0_d) = P(0_c, 0_d)$. Finally, we derive the probability of observing the photons exiting distinct output ports:

$$P(1_c, 1_d) = |\langle 1_c, 1_d|\Psi_{HOM}\rangle|^2 = |t^2 + r^2|^2$$

This expression can look rather familiar in the context of an interference experiment: the probability can be interpreted as the interference between two complex probability amplitudes r^2 and t^2.
We now compute exactly the results for a balanced lossless BS. The coefficients can be chosen as $r = 1/\sqrt{2}$ and $t = i/\sqrt{2}$ and we get $P(2_c, 0_d) = P(0_c, 2_d) = 1/2$ and $P(1_c, 1_d) = 0$!
In other words, the amplitudes associated to $|1_c, 1_d\rangle$ interfere destructively *for a balanced lossless BS* and the photons *never* exit the system using two different output ports. They are always travelling together, in output c 50% of the time or in output d 50% of the time. The total output quantum state now writes:

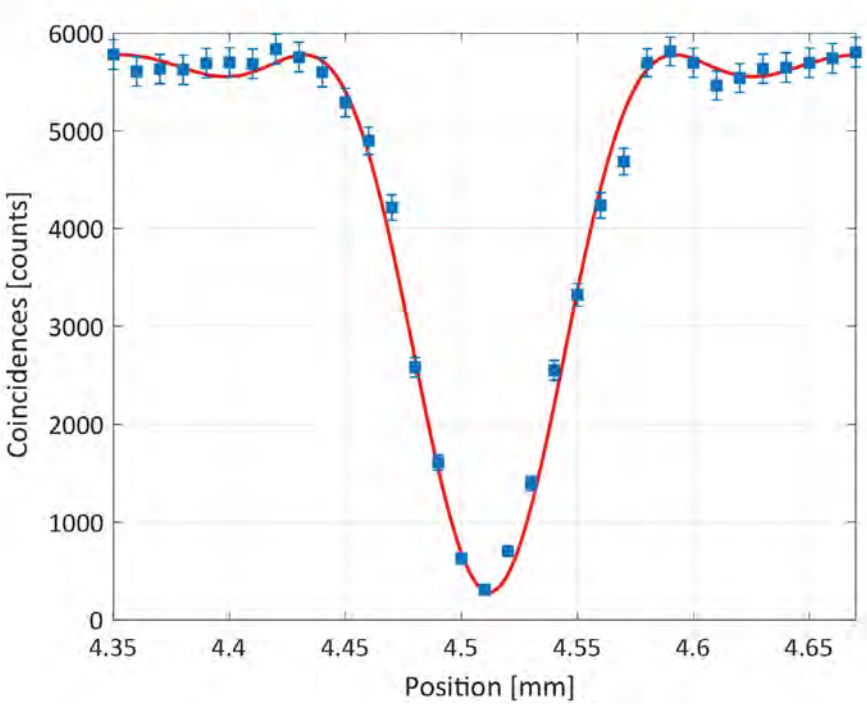
$$|\Psi_{HOM}\rangle = \frac{|2_c, 0_d\rangle + |0_c, 2_d\rangle}{\sqrt{2}}$$

(and the trained eye identifies an entangled state!)
As for every interference effect, the relative phase (here between coefficients r and t) is essential ; it stems here from the energy conservation. The quantum weirdness of the HOM effect has some connection with entanglement, in the sense that it deals with multiple-particle states, that have striking non-classical behaviors.

Experiment: coincidence measurement

The experimental protocole of the HOM effect relies on the observation of state $|1_c, 1_d\rangle$ **via coincidence measurements**.

Figure 3. Measured HOM dip (number of coincident counts in 10s as a function of the position of the translation stage).



One uses a pair of single photon detectors, placed symmetrically at each of the BS outputs: if the two photons involved in one run of an experiment exit by two different output ports, both single photon detectors will register detection events quasi-simultaneously. Following this idea, the correlation (hence the product) of the detector signals is linked to the probability of measuring $|1_c, 1_d\rangle$.

Two main regimes can be identified:

a) When both photons reach the BS with significant relative optical delays, the situation is all classical and no interference occurs: $P(1_c, 1_d)$ is non-zero and coincidence counts are registered at an average rate related to the pair generation rate.

b) For vanishing optical delays, the HOM interference effect progressively builds up. Eventually, all detection events correspond to photon pairs exiting via the same output: the coincidence rate decreases as we get $P(1_c, 1_d) = 0$.

A HOM experiment can therefore consist in a plot of the coincidence rate measured between two detectors as a function of the delay between the two incoming photons.

Hardware implementation

The practical implementation involves the generation of indistinguishable photon pairs using a type I single photon parametric down-conversion (SPDC) effect in a BBO non-linear crystal. This process and its symmetrical version (second harmonic generation SHG) are routinely used with wavelengths at 405 nm and 810 nm, that display several advantages: cheap GaN laser diode emitting in the 100 mW range are easily found, and the 810 nm wavelength matches the high sensitivity range of silicon detectors.

BBO crystals for SHG and SPDC are usually cut so that their optical axis is oriented at 29.2° with respect to the input face. When pumped at 405 nm at normal incidence and in ordinary polarization, one gets two non-collinear beams at 810 nm exiting the output face along a cone with an apex angle of 3°.

The SPDC process is broadband and emits photons in various directions. The rest of the setup must therefore be designed to select photons from a same pair and ensure intrinsic photon indistinguishability. Spatial mode indistinguishability can be conveniently achieved using single mode fibers and a fiber BS instead of a free-space setup.

Therefore, we place a 500 mm-focal length doublet to collimate the output mode of the crystal and we use mounted collimators to couple the photons in a 2×2 single-mode, polarization-maintaining fused fiber splitter. We get rid of the stray light by using band-pass filters of 10 nm spectral width around 810 nm.

One of the collimators is mounted on a translation stage with a 15mm range and a 10µm resolution in the direction of the incident beam to set a variable delay between the two photons. The two outputs of the fiber splitter are directly connected to two single-photon counting modules (SPCM-AQ4C from Perkin Elmer) with a dark count rate of 300Hz. The signal is sent to an FPGA board (Altera DE2), programmed to detect simultaneous events on two channels receiving 25 ns TTL pulses from the detector channels. We visualize the raw detector count and coincidence rates with a Labview interface. An experiment run consists of recording the coincidence rate as a function of the translation stage position, convertible into a path delay.

Results

The width of the HOM dip reveals the length of the two wavepackets and the shape of the dip is related to the temporal wavepacket, that means to the spectra. Assuming that the photons had a gaussian spectrum, the data is fitted with the product of a gaussian with a sinc function. We inferred a visibility of 95.4% and a full width at half maximum (FWHM) of 56 ± 8µm, in good agreement with the ~ 60µm expected for Fourier-transform-limited photons with a 10nm bandwidth. The dip depth shows that the photons of each pair are nearly indistinguishable and demonstrates that this simple setup can achieve a good degree of control over their parameters.

Conclusion

The LEnsE implementation of the HOM experiment is a robust setup operating since 2015 [6] in the optical engineering track and M2 tracks of IOGS and Université Paris-Saclay. The objective of IOGS is to further expand the scope of these quantum labworks to establish a full and versatile quantum photonics experimental platform adressing different physical platforms and enabling training on standard characterization procedures: this will include antibunching measurements for the characterization of a solid-state single photon source, quantum key distribution, NV center magnetometry and a progressive refinement of our historical Bell [7] and HOM setups.

REFERENCES

[1] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987).

[2] R. Lopes, A. Imanaliev, A. Aspect, M. Cheneau, D. Boiron, and C. I. Westbrook, Nature **520**, 66 (2015).

[3] B. Vest, M.-C. C. Dheur, É. Devaux, A. Baron, E. Rousseau, J.-P. P. Hugonin, J.-J. J. Greffet, G. Messin, and F. Marquier, Science **356**, 1373 (2017).

[4] J. Jeffers, J. Mod. Opt. **47**, 10 (2000).

[5] P. Grangier, G. Roger, and A. Aspect, Europhys. Lett. **1**, 173 (1986).

[6] A. Ourjoumtsev, M. C. Dheur, T. Avignon, and L. Jacubowiez, Eur. J. Phys. **36**, 065034 (2015).

[7] B. Vest and L. Jacubowiez, Photoniques **113**, 26 (2022).