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## **Closed cracks characterization in a steel sample applying a pump probe waves ultrasonic method**

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### ABSTRACT

Ultrasonic measurements are commonly used for crack sizing in industrial context, but due to potential partial crack closure, the crack depth can be underestimated. It could be of importance regarding the prediction of remaining service life and maintenance scheduling of industrial components. In the literature, nonlinear ultrasonic methods have proven efficient for this issue. Among them, a pump probe waves method is investigated here for possible use in an industrial context. In this paper, an industrial steel component with several partially closed fatigue cracks is studied. First, a three-point bending test associated with digital image correlation is performed with ultrasonic measurements to obtain reliable indications about the cracks closure state. A three-dimensional laser vibrometry experiment shows the possibility to open the crack using a pump wave. The pump probe waves method is conducted using a standard 45° shear wave transducer in echo mode as a probe. The results allow to discuss physical mechanism in play. It is also shown the possibility to detect the closed nature for all of the cracks even if the full opening state is not reached. Moreover, a coherent crack profile is found compared to mechanical test. The results are validated on a notched sample for which the nonlinear response is negligible. Industrial application as well as possible improvement are discussed.

## 1. Introduction

When an industrial component is subjected to mechanical fatigue, it is important to warrant its structural integrity to prevent any accident. With NDT (Non-Destructive Testing) and more specifically ultrasonic techniques, cracks can be detected and sized accurately in most cases. However, because of possible crack closure, the ultrasonic inspection can result in underestimation of the crack size with potential consequences in life remaining prediction and maintenance planification. From an ultrasonic standpoint, the closed part of the crack does not reflect waves, and the crack tip diffraction echo may not be detected or may be located at the separation between the closed and the open part of the crack [1–5].

To overcome this issue, multiple methods have been developed, among which the use of phased arrays transducers with specific protocols increasing the resolution of the resulting image like SAFT [6] and TFM [7,8] as well as nonlinear methods [9]. The latter involves physical phenomena such as friction, adhesion, hysteresis, etc., occurring at the crack lips during high amplitude wave solicitation. This class of methods has been proven more sensitive to closed cracks than regular linear techniques [10–12]. Various non-linear phenomena have been linked to the crack presence, like harmonic generation [12,13], sub-harmonic generation [11,14], or the nonlinearity of the relationship between the amplitude of the incident wave and the crack response [10,15].

Another kind of promising nonlinear technique relies on the interaction between a high amplitude low frequency wave (pump) and a high frequency wave (probe). The use of a pump wave to open and close the investigated crack modulates the amplitude of the probe wave as the size of the open part of the crack varies during the acquisition period [9,16,17]. Dynamic Acousto-Elastic Testing (DAET) is one of these methods, it is based on the analysis of the probe wave as a function of the pump amplitude. This method is mainly applied for characterization of rocks and concrete [18,19]. Nonetheless, recent studies show the influence of a single crack on DAET in metallic samples [20,21].

The aim of this paper is to study the efficiency of a pump probe wave method to characterize the closed portion of the cracks in industrial components. To validate the results, a 3-point bending test is performed to mechanically open the cracks in association with digital image correlation while the crack is probed with ultrasound. Experiments on a three-dimensional laser vibrometer show evidence of crack opening during pump excitation. The pump probe waves results are discussed, the limitations and potential improvements of the method are pointed out.

## 2. Crack opening methods

### a. Crack opening under quasi-static loading

The studied sample is an industrial steel pipe portion ( $300 \times 75 \times 40 \text{ mm}^3$ ) with 3 different cracks located in the centre of the upper surface (Fig.1a). A prior standard ultrasonic inspection using  $45^\circ$  shear wave transducers revealed cracks with an average depth of 2mm all around the pipe. However, a destructive study on samples from the same pipe reveals crack sizes ranging between 3 mm and 6 mm depth, meaning that the closed portion of the cracks ranges from 1 to 4mm.

In order to highlight the presence of the closed zone, a mechanical bending test is conducted with loads up to 24kN to prevent for crack propagation. During loading, a camera (PCO.edge 4Mpix) has also been placed close to the lateral part of the sample (red zone in Fig.1a) to image the crack during loading. Using an image correlation algorithm (Particle Image Velocimetry), a measure of the displacement gradient around the crack is obtained. It allows to image crack opening over time since gradients are

much greater near the crack than on the rest of the material (Fig.2). In addition, an angle beam 45°, 4MHz, shear wave transducer (Olympus, AM4S-8X9-45) is placed close to the side wall of the sample to monitor crack opening using the amplitude of the corner echo in rebound configuration (Fig.1b). A small echo is measured at approximately half of its time of flight (Fig.1b). It corresponds to the portion of the signal reflected directly by the lower surface of the sample due to its high roughness. This configuration is the only possible in industrial context with only a single side access. Moreover, the bottom face is not exploitable for flat transducers due to the circular shape of the pipe.

We note that the measurement is repeated 3 times for 8 different transducers locations (Fig.1c). For clarity purposes, only the location #8 is discussed because it corresponds to the zone imaged by the camera. The results from other points will serve for comparison purpose in the next sections.

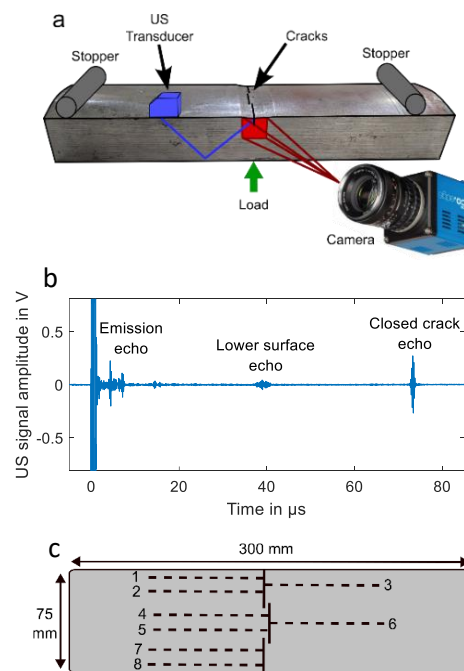


Figure 1: (a) Experimental setup for the bending test. The red zone is the area where strains are recorded using image correlation. (b) Example of recorded signal of the US transducer. (c) Location of the 8 different measurement points relative to crack position.

The results of crack imaging is given in Fig.2. The crack opening is well captured by the PIV algorithm. A threshold is chosen ( $\epsilon = 0.12\%$  in Fig.2) to measure the evolution of the depth of the open portion of the crack during loading (Fig.3b). The relative evolution of the ultrasonic corner echo amplitude  $\Delta A/A_0$  Fig.3a is also plotted in Fig.3b as a function of applied load. With these two measurements, the crack opening is well observed from the ultrasonic and visual point of view as a function of the loading until the crack is fully opened.

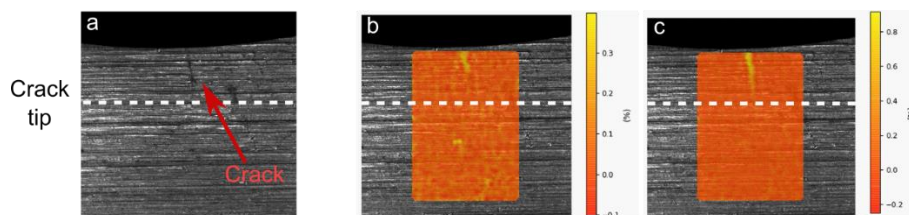


Figure 2: Imaging of the crack opening, (a) photo of the crack before the loading, (b) deformation around the crack for a 15 kN loading and (c) a 24 kN loading.

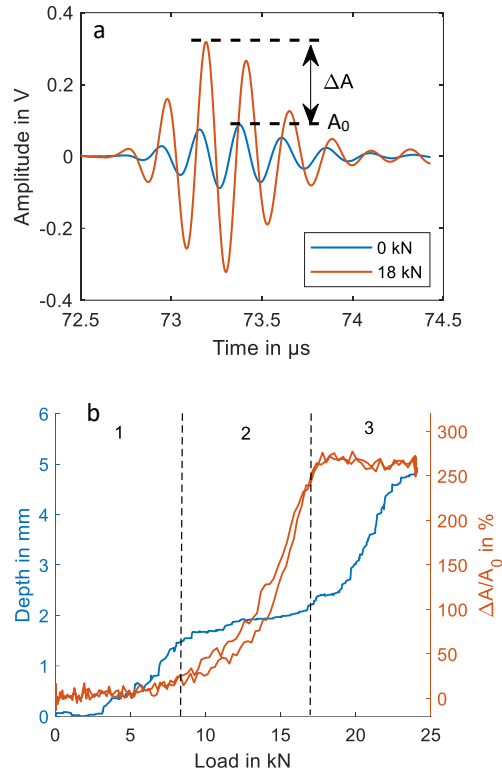


Figure 3: (a) Initial corner echo (0 kN) and corner echo at fully opened state (18 kN). (b) Evolution of the corner echo amplitude (from ultrasound) and of the crack open depth (from digital imaging) during loading

The first conclusion is that the crack does not propagate. It is shown by the presence of a plateau on ultrasonic amplitude from 16kN to 24kN and by the crack open depth which reaches the actual crack depth of 5mm. Moreover, the mechanical cycle is repeated 3 times, the same  $\Delta A/A_0$  plateau as well as the 5mm crack depth appear at the same load anytime.

The differences between ultrasound and digital imaging stem from the detection thresholds of these two methods. For an ultrasonic wave, a total reflection at an interface occurs when a layer of air of infinitesimal thickness between the two planes is sufficient. The theoretical value of the gap is of the order of a nanometer [22]. For the camera, a much greater displacement is required to be caught because its resolution is in the micrometer range. We can therefore consider that the crack opening will be detected much sooner by ultrasound than digital image correlation.

In zone 1 (Fig.3b), looking at DIC results, the opened part of the crack evolves fast while the ultrasonic amplitude does not evolve significantly. DIC highlights the displacements field evolution induced by the presence of the opened part, but as ultrasound does not catch significant changes, we can conclude that the depth does not evolve. Physically, in the closed part of the crack, residual stresses between the two walls prevent the crack from opening at low loads. In zone 2 (see Fig.3b), DIC reaches a kind of plateau at 2mm until the closed part starts opening (still from the DIC point of view) in phase 3 up to 5mm. From the ultrasonic point of view, due to higher sensitivity than DIC to opening state, the crack opening is caught in phase 2 (Fig.3b). Physically, the applied force becomes non-negligible compared to the residual stresses closing the crack. Then in zone 3 (Fig.3b), applied forces completely compensate for the closing forces. For this crack, the depth of the open part is around 2 mm, while the total depth is around 5 mm. This measurement is consistent with the results of previous destructive studies carried

out on similar cracked samples.

The evidence of a closed portion on the crack is made. However, for an industrial application, this setup is not applicable. In the following, a dynamic loading is employed on the sample to assess the possibility of open cracks in this configuration.

### b. Crack opening under dynamic loading

To mechanically open the sample cracks with a pump wave, the sample is driven in resonance conditions. As the cracks are located in the centre of the sample, the compressional mode is chosen to alternatively load in compression and tension state which should allow crack opening and closing.

For the pump wave, a 3.5 cm diameter ultrasonic cleaning transducer is glued on the lateral face of the sample (Fig.4) to drive the sample on its longitudinal eigen-mode (8.2 kHz). It is driven by a high voltage Tegam 2350 amplifier. The sample stands on a foam to prevent any mechanical interaction during resonance. To check that this pump wave is able to open the crack, a Polytec PSV-500 3D laser vibrometer is used to scan the cracked region (Fig.5a) during the pump wave excitation.

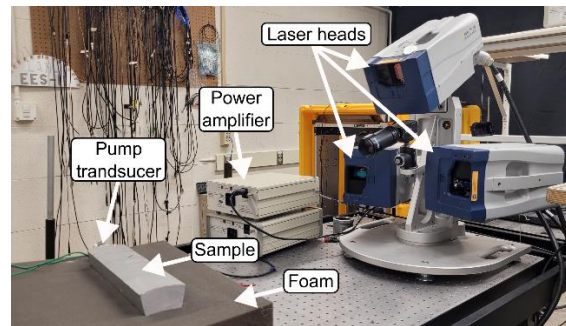


Figure 4 : 3D laser vibrometry experimental setup

This 3D laser vibrometer allows us to directly measure the amplitude of the displacement field in the scanned area (Fig.5a). At the highest possible amplitude, the displacements in the X direction at different depths is shown (Fig.5b-d). A displacement discontinuity is clearly visible starting from the surface displacements, then decreases up to a depth of 2.5mm. This discontinuity is the result of the crack opening which confirms the application of a sufficient opening stress generated by the pump. It is worth noticing that this opening state is equivalent to a 12 kN load on the bending experiment.

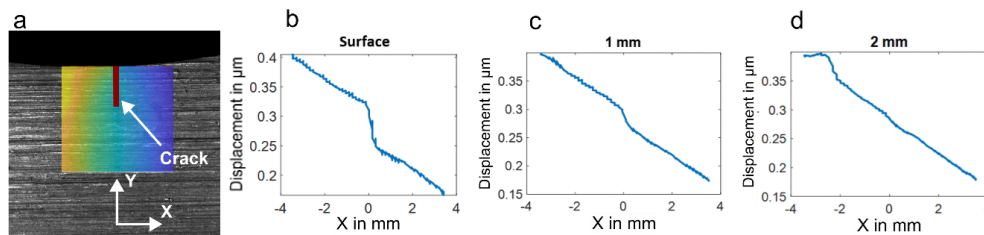


Figure 5: (a) Displacement in the X direction obtained with the 3D laser vibrometer around the observed crack, (b-d) Displacement profiles on the surface of the sample, at 1mm deep and 2 mm deep.

## 3. Pump probe waves experiment

### a. Experimental setup

The full experimental setup is shown in Fig.6. For the pump wave, the same protocol used in section 2.b is used. The probe wave is managed by the same transducer used in section 2.a (45° angle beam, 4MHz, shear wave transducer, Olympus, AM4S-8X9-45) driven in echo mode by a Sofranel 5055PR ultrasonic generator. Signal acquisition is achieved by a Teledyne LeCroy HDO4024 oscilloscope. The

experiment is synchronized by a function generator (Tektronix AFG31000) which allows to warrant the interaction of the probe wave with crack at known pump amplitude knowing the time of flight ( $73\mu\text{s}$ ). The pump transducer is fed by a 200ms sinusoidal tone burst at 8.2 kHz (longitudinal eigen-mode). The acquisition start when the resonance reaches the steady state regime (about 120ms in Fig.6). To improve the signal to noise ratio, the full acquisition is averaged 5 times.

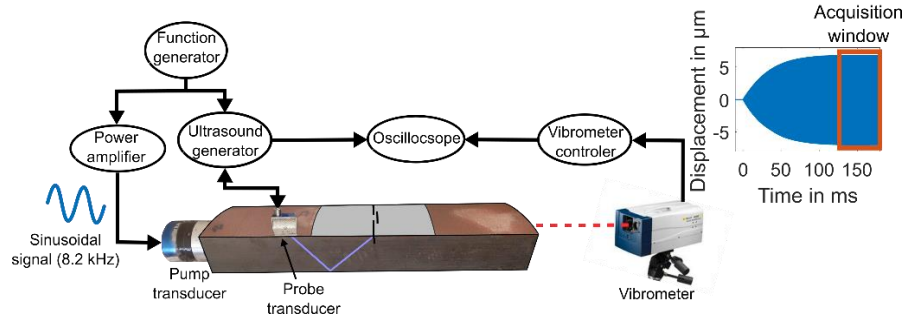


Figure 6: Experimental pump probe wave setup and pump amplitude measurement during the sample excitation.

Several measurements are achieved all along the cracks corresponding to the locations shown in Fig.1. The measurement campaign is carried out with two different pump amplitudes (amplitude of the displacement measured by the laser vibrometer in Fig.6). The goal of the first pump being to open the cracks as much as possible with the maximum reachable pump amplitude ( $7\mu\text{m}$  maximum displacement). Then, to observe the influence of the pump amplitude on the results, a weaker pump ( $3\mu\text{m}$ ) amplitude is generated. Like the bending test, the evolution of the corner echo amplitude ( $\Delta A/A_0$ ) is monitored during pump wave solicitation for every transducer location. The acquisition sequences are repeated 3 times with the transducer removed between each acquisition. For comparison purposes, a high amplitude acquisition on a 3-mm deep notch is also performed.

## b. Results

For clarity issues, in a first step, the results obtained on a specific position (number 3 in Fig.1b) are discussed here. In the following section, the 8 different transducer locations results are analyzed and compared to the bending test.

In Fig.7, the relative variation of the corner echo amplitude obtained with the  $7\mu\text{m}$  and  $3\mu\text{m}$  pump amplitude are compared to the measure performed on a notch. Under longitudinal resonance state, negative displacements relate compression state (the crack tends to close), positive ones indicates traction state (the crack tends to open). For the stronger pump, strong non-linearities and hysteresis occur. For a weaker pump, smaller variations are observed (10 times smaller) compared to the high amplitude pump. For comparison purposes, the results on the notch are 100 times and 10 times lower than the high and the low amplitude pump respectively. The very low amplitude variation induced by the notch validates our results on the cracks. It also reveals that the vibrational noise induced by the pump on the high frequency ultrasonic signals is negligible.

On the  $7\mu\text{m}$  pump curve, an asymmetric curve and a hysteresis is observed only in the positive pump values (Fig.7a) on the contrary to the  $3\mu\text{m}$  pump curve which exhibit a hysteresis in the entire pump cycle (Fig.7b).

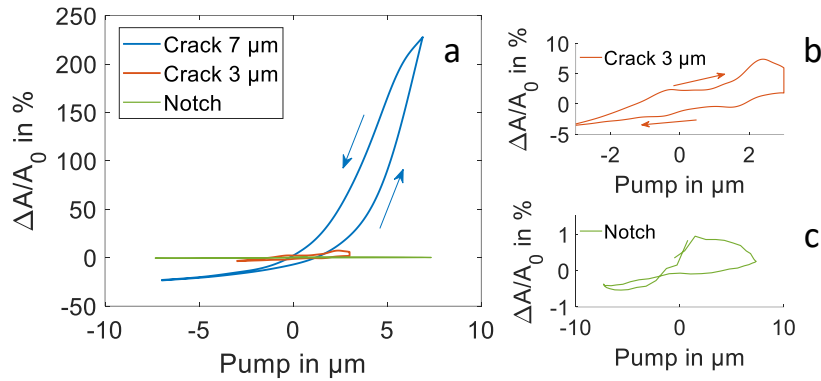


Figure 7: (a)  $\frac{\Delta A}{A_0}$  measured with the closed crack n°3 for a 7  $\mu\text{m}$  and 3  $\mu\text{m}$  pump and with a notch, (b) Zoom on  $\frac{\Delta A}{A_0}$  measured with the closed crack n°3 for a 3  $\mu\text{m}$ , (c) Zoom on  $\frac{\Delta A}{A_0}$  measured with a notch

### c. Comparison with the bending test

The maximum value of the relative amplitude variation of the corner echo for the bending test is compared to the 7  $\mu\text{m}$  pump wave excitation. The full set of data recorded and the standard deviation all along the sample are plotted as a function of the transducer location in the sample (according to Fig.1b) (Fig.8).

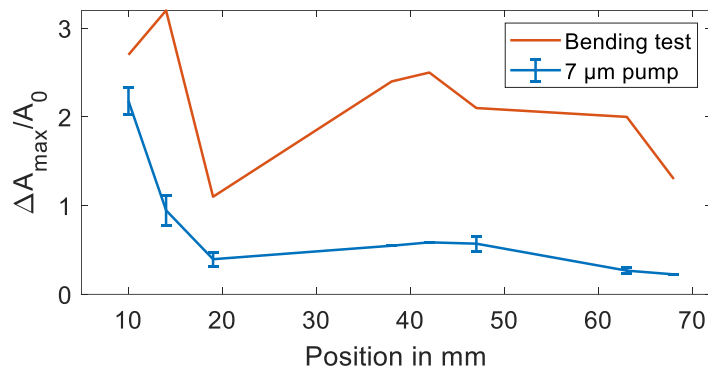


Figure 8: Maximal corner echo variations observed during the bending test and the 7  $\mu\text{m}$  pump excitation as a function of the transducer position.

It is noticeable that the trends of both curves are comparable with a decrease and small increase at 40mm. However, as it could be expected, the pump wave method does not allow to reach the same crack opening.

## 4. Discussion

Depending on the amplitude of the pump, different physical phenomena between the crack faces are to be considered [23]. The key parameter is to reach sufficient amplitude to break the contact between the crack faces during the solicitation to create a clapping mechanism [24]. This explains the threshold behavior observed between the low and high pump amplitudes (Fig.7).

When the clapping mechanism is activated (7  $\mu\text{m}$  pump amplitude), an intermittent contact appears between the crack faces. The pump solicitation keeps the contact for the negative displacements and breaks it for the positive ones. The asymmetry of the 7  $\mu\text{m}$  pump curves is then the result of the different physical phenomena in tension and compression states [25,26]. The hysteresis exhibited only for positive pump values is the result of the interaction forces between the crack faces (adhesion) [26,27] and the lower slope exhibited for negative pump values the result of the non-linear contact



between the crack lips [26,28].

The 3  $\mu\text{m}$  pump curves can be compared to the negative values part of the 7  $\mu\text{m}$  pump curve because the pump intensity is not enough to break the contact between the crack faces. The hysteresis loop seen on both positive and negative pump values is the result of the non-linear contact between the two crack faces [26,28] and possibly of friction forces during the excitation [29]. This difference in the physical phenomena at play explains the inversion of rotation direction in the hysteresis loop between the 7  $\mu\text{m}$  and the 3  $\mu\text{m}$  pump.

The  $\Delta A/A_0$  variations recorded on the notch are very low compared to real closed cracks. The slight variations in amplitude could be due to a slight sliding of the transducer during a pump cycle but it is still negligible compared to amplitude variations observed on closed cracks.

From the NDT point of view, even if the cracks are not fully opened, a clear difference between partially closed cracks and a notch is to be underlined without any doubt. Hence, the proposed method allows to detect the presence of a closed portion, it works at each measurement location. The shape of the closed crack along the sample is well retrieved from the evolution of  $\Delta A/A_0$  even if there is evidence of only partial opening of the crack compared to the mechanical test results. Moreover, the proposed angle beam transducer in echo mode method allows to greatly simplify the experimental apparatus needed to achieve a repeatable measurement. It also greatly simplifies the accessibility problematic in industrial context especially for single side access measurements. Even without laser to measure the pump amplitude, a threshold of about 2% variation in amplitude may be sufficient to detect the presence of a closed crack for our samples.

## 5. Conclusion

This study presents a method to characterize closed cracks using a pump probe waves based approach. The mechanical behavior of closed cracks during a quasi-static and a dynamic loading is studied. The quasi-static experiment shows the closed nature of the cracks thanks to the association of digital image correlation. The dynamic loading experiments shows evidence of crack opening during resonance of the sample thanks to a 3D laser scanning.

The pump probe wave method is conducted at low and high amplitudes. The results show that various physical phenomena are at play. An amplitude threshold is identified as a transition between classical nonlinear and clapping phenomena. The results on a notch with a significantly lower response validate these conclusions.

The experimental setup used a 45° shear wave angle beam transducer using the reflected corner echo. This industrial configuration simplifies the acquisition compared to usual through transmission methods in pump wave experiments, especially when the accessibility is an issue. It allows to detect all of the closed cracks in our sample. Moreover, a coherent crack profile is retrieved even if the response is lower than in fully opened state. These findings lead to a reliable NDT method to differentiate opened from closed cracks, thus to determine whether there is a risk of undersizing in an industrial context.

Possible improvements of the method rely in increasing the pump amplitude. However, gluing a transducer or attaching a shaker is often a difficulty in industrial context. Moreover, amplifying signal voltage in a sufficient way could also be difficult. Ongoing developments aim at optimizing a thermal solicitation to increase the local stresses around the crack to get closer to fully opening state.

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