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# Reactive Cooperation of Multi-Vehicle System for Efficient Intersection Crossing based on PIDP Speed Space Assessment 

Shan $\mathrm{He}^{1}$ and Lounis Adouane ${ }^{1}$


#### Abstract

This paper proposes a Dynamic Collision-Free Cooperative decision-making method based on the Predicted InterDistance Profile (DCFC-PIDP) for Connected Autonomous Vehicles (CAVs), aimed at solving congestion, and increasing the efficiency and safety of the transport system. Under this framework, this paper studies a typical urban driving scenario: unsignalized intersection. The method employs a greedy algorithm to explore with an appropriate metrics, the main possible scenarios, involving CAVs with a risk of collision and find the suboptimal feasible solution. The decision-making cost function is constructed by considering the risk of collisions and the passing efficiency of CAVs. Finally, the effectiveness and feasibility of the proposed decision-making framework are verified under statistic studies.


## I. INTRODUCTION

As the number of vehicles in cities continues to rise, urban traffic congestion is becoming increasingly severe, leading to significant unnecessary waiting time, fuel wasting, air pollution, and a higher likelihood of urban traffic accidents. Further, in current traffic systems, traffic signal control, especially at intersections, is a primary method for alleviating congestion and enhancing vehicular safety. However, the application of inefficient traffic signal controls indirectly reduces travel efficiency, contributing thus to congestion. With the advancement of intelligent transportation systems and autonomous driving technologies, traffic signal control becomes unnecessary for Connected Autonomous Vehicles (CAVs) [1] [2]. Vehicles do not necessary to stop completely and wait for green lights, thus reducing traffic delays and increasing capacity. CAVs can share information including vehicle movement, driving intentions, and status, laying the foundation for more efficient, safer, and economical decisionmaking and planning. Consequently, CAVs represent a new trend in solving traffic congestion and collision problems at unsignalized intersections.

Currently, the main decision-making methods for CAVs at unsignalized intersections can be categorized into two types: centralized and distributed. In the centralized model, all CAVs entering a controlled area should receive control commands from a central controller. The central traffic management system receives information from all the CAVs near the intersection and makes decisions based on the given target: less congestion, less fuel consumption, or more safety. The central traffic management system then manages the scheduling of all vehicles in the control area based on the optimal passing sequence [3]. The Automated Intersection

[^0]Management (AIM) system is the most renowned [4] and the scheduling system is reservation-based in the time field. To ensure safety and efficiency at unsignalized intersections, the Alternately Iterative Descent Method (AIDM) is applied to assign the best entry time for each CAV [5]. The centralized methods have generally a very good efficiency in terms of crossing time, mainly induced by the quasi-optimality of the obtained solutions which take into account all the information (with high accuracy) of the studied system.

Compared with the centralized model, the distributed model needs less information on the state and dynamic of the CAVs. CAVs can collaborate without the aid of a central traffic manager. Its computational is much less than the centralized model but it has more risks. The authors in [6] proposed a new cooperative decision-making strategy with the combination of Monte Carlo Tree Search (MCTS) and heuristic for CAVs at unsignalized intersections, which maintains a good balance between performance and computational flexibility. The Probability Collective Algorithm (PC) [7] was applied to traffic management at unsignalized intersections in [8]. The shared speed space is applied to do the probability analysis for CAVs coordination. It was extended to Epsilon-PC in [9]. As a pure stochastic method, the PC and Epsilon-PC algorithms have long execution times, which are generally not suitable for dynamic environments.

This paper proposes a space field method, which could be categorized as centralized, since the CAV shares all the information according to an infrastructure blackboard [10] [11], which uses an appropriate use of PIDP (Predicted InterDistance Profile) [12] metrics to efficiently and reactively converge the CAV velocities towards a timely suboptimal solution without collision to do the collision check in the space field and decision-making in the speed field. Decisionmaking methods for collision detection and rapid reduction of collision risks are elaborated in section II. A greedy algorithm that explores all possible speed space cases and a communication model for real-time updates of the optimized speeds are introduced in section III. Simulation experiments under two different cases are conducted in section IV. Section V gives the conclusion of the paper with some prospects.

## II. RELATED WORKS

## A. Predicted Inter-Distance Profile

The Predicted Inter-Distance Profile (PIDP) metrics is used in [13], [14], [15], and [8] for the evaluation and execution of overtaking maneuvers on highways and roundabouts crossing, depict how the distance between two vehicles or a


Fig. 1: $P I D P$ curves and the corresponding $m P I D P$ and $e P I D P$
vehicle and an obstacle will change over a future time frame. If the information of the ego-vehicle and other vehicle (path, speed profiles, etc.) is known, and assuming these factors remain constant during a certain time horizon of prediction, the evolution of the inter-vehicle distance between them is feasible to be projected. In the dynamic environment, PIDP will be recalculated every time at the end of the control process and when the speed policy changes.

The safety distance should be declared if PIDP is applied to check the collision relationship between two vehicles or vehicles and an obstacle. The safety distance $d_{\text {safety }}$ is the minimum distance between two vehicles without collision, it is defined as:

$$
\begin{equation*}
d_{\text {safety }}=r_{i}+r_{j}+\text { Margin } \tag{1}
\end{equation*}
$$

Where $r_{i}$ is the safety radius of vehicle $i$, Margin is a certain distance to guarantee to take into account the different uncertainties linked to the communication system as well as the capacity of maximum braking of the vehicles.
$m P I D P$ (cf. Figure 1) is the minimum value of the PIDP curve, which is the shortest distance between the two vehicles in the future. If $m P I D P$ is smaller than the safety distance $d_{\text {safety }}$, it means that if neither speed nor path changes, the two vehicles have the risk of collision. For convenience, we define $e P I D P$ (cf. Figure 1) as the difference between $m P I D P$ and $d_{\text {safety }}$ :

$$
\begin{equation*}
e P I D P=m P I D P-d_{\text {safety }} \tag{2}
\end{equation*}
$$

If $e P I D P$ is positive, it indicates no collision risk, and if it is negative, it signifies the presence of collision risk in the future. An example is shown in the Figure 1.

## B. Proposed MRMCO-PIDP Decision-Making Algorithm

At the intersection, in cases where the collision type is not a rear-end collision of vehicles coming from the same direction, both accelerating to pass the intersection quickly and decelerating to allow another vehicle to pass


Fig. 2: $P I D P$-based decision-making: (a) is the speed profile of the initial speed profile, increased speed profile, and decreased speed profile; (b) is the PIDP curves corresponding to (a).
the intersection are viable solutions. However, the goal is to find a sub-optimal solution in intersection crossing time to avoid the collision and try to find it as soon as possible.

The Multi-Risk Management Cooperative Optimization approach based on the PIDP (MRMCO-PIDP) is a method for rapidly identifying strategies to avoid collisions in scenarios where there is a risk of collision. It is an extension of the PIDP method, capable not only of addressing collision risk avoidance between two vehicles but also expandable to multi-vehicle and multi-risk management scenarios.

When the vehicle has the risk of collision with others, it will select the increased speed profile or the decreased speed profile based on the current speed profile, and calculate the $P I D P$ curves according to the conflicted vehicle. The decision to accelerate or decelerate is determined based on the numerical values of both obtained $e P I D P$. An example of the MRMCO-PIDP is shown in Figure 2, the superscript of "in" and "de" are the abbreviation of "increased speed" and
"decreased speed". The simplest scenario, the collision risk management between two vehicles, is discussed first, with multi-vehicle collision risk management to be introduced later:

1) If $e P I D P^{i n}>0>e P I D P^{d e}$, it means the increased speed profile doesn't have collision but the decreased still has the collision, so, select the increased one is better.
2) If $e P I D P^{d e}>0>e P I D P^{i n}$, the decreased speed profile is the better choice, the reason as the same as the previous one.
3) If both of them are positive, it is selected the speed with a smaller absolute value of $e P I D P$ because it has a lower cost in terms of modification of the initial vehicle state.
4) If both of them are negative, we select the speed profile with a smaller absolute value of $e P I D P$ too, because it has the larger minimum distance between other conflicted vehicles, even if they still have the risk of collision.
5) If they have the same value, the decreased speed profile should be selected to prevent unknown risks.
This method has been extended to multi-vehicle risk management. Consider a scenario in which most of vehicles in the intersection have the risk of collision with more than one vehicle. In this case, a special situation output by the previous method should be dealt with: the output of PIDPbased risk management between a CAV and the first collision vehicle is to accelerate, but the output of the second or the third collision vehicle is to decelerate.

The extension is designed to deal with this problem. In this case, each vehicle should make a balance between accelerating and decelerating. The final decision of vehicle $i$ is represented as Decision $_{i}$ :

$$
\begin{gather*}
\text { Decision }_{i}=\sum_{j \in C_{i}}\left(\left|e P I D P_{i j}^{o p t}\right| * s_{i j}\right)  \tag{3}\\
s_{i j}= \begin{cases}1 & \text { if } e P I D P_{i j}^{o p t}=e P I D P_{i j}^{i n} \\
-1 & \text { if } e P I D P_{i j}^{o p t}=e P I D P_{i j}^{d e}\end{cases} \tag{4}
\end{gather*}
$$

$e P I D P_{i j}^{o p t}$ is the optimized $e P I D P$ between vehicle $i$ and vehicle $j$ through the introduced method. $s_{i j}$ indicates the motion of $e P I D P_{i j}^{o p t}$, if the optimized speed profile is the increased speed, $s_{i j}=1$, otherwise $s_{i j}=-1 . C_{i}$ is the set of vehicles that have the risk of collision with vehicle $i$. If Decision $_{i}$ is positive, the current favorite strategy will be replaced by the increased speed profile, otherwise, it is replaced by the decreased speed profile.

After we know that the vehicle should accelerate or decelerate, the next process is calculating the change in speed:

$$
\begin{equation*}
\Delta v_{i}=K_{p} * \sum_{j \in C_{i}} e P I D P_{i j}^{n e} \tag{5}
\end{equation*}
$$

$\Delta v_{i}$ is the change in speed, $K_{p}$ is the positive proportional coefficient. $e P I D P_{i j}^{n e}$ is the $e P I D P$ between vehicle $i$ and $j$ with the negative value, which is calculated by the current


Fig. 3: Flow chart of DCFC-PIDP
speed profile, rather than the increased or decreased speed profile.

## III. PROPOSED METHOD

The proposed Dynamic Collision-Free Cooperative decision-making method based on the Predicted InterDistance Profile (DCFC-PIDP) is a greedy cooperative algorithm at unsignalized intersections which is applied in real-time working of the CAV. The flow chart of the DCFC-PIDP is shown in Figure 3 and this method is repeated every sampling time when a possible conflict could exist between the CAVs.

The communication and intersection model is introduced in section III-A as the working scenario of the proposed method. The method to generate speed profiles is introduced in section III-B and III-C. The method to build the objective function to select the best strategy is introduced in section III-D. The main optimization process is introduced in section III-E.

## A. Communication and Intersection Model

The communication model between infrastructure and CAVs is shown in Figure 4. The decision-making part is done in the intersection infrastructure (cf. Figure 5). CAVs update their dynamic as given commands as soon as a new best strategy is reached.

The model of the unsignalized intersection is shown in Figure 5. It consists of 4 areas:


Fig. 4: Communication model between CAVs and the intersection infrastructure


Fig. 5: General Intersection Model

1) The Core area is the center of the intersection, vehicles interact in this area.
2) The Action area is a small area near the center of the intersection, vehicles in this area have the highest priority and only participate in the communication process, their strategy will not be changed.
3) The Decision-making area is a large area near the intersection, vehicles in this area are included in the optimization list, and the optimization process works at each sampling time.
4) The Buffer area is the area for vehicles that are going to enter the intersection to build communication with the infrastructure of the intersection.

## B. Speed Space Assessment Method

In the proposed DCFC-PIDP method, we are going to explore all the possible cases during the working of the negotiation algorithm. As we discussed in the section II-B, whether accelerating or decelerating, the risk of collision can be reduced.


Fig. 6: Greedy Speed Space exploration

The exploration focuses on the target speed, which is the end of the planned speed profile. We assume that it has three different actions during the optimization process: accelerate, decelerate, and keep the current target speed. New speed profiles are always generated based on the current target speed. As shown in Figure 6, the "Acc Target Speed" and "Dec Target Speed" are decided by:

$$
\left\{\begin{array}{l}
v_{a c c}^{t a r}=v_{c u r}^{t a r}+\Delta v  \tag{6}\\
v_{d e c}^{t a r}=v_{c u r}^{t a r}-\Delta v
\end{array}\right.
$$

where: $v_{a c c}^{t a r}$ is the accelerated target speed, $v_{d e c}^{t a r}$ is the decelerated target speed, $v_{\text {cur }}^{t a r}$ is the current target speed. $\Delta v$ is the gap speed. Different from the speed profile in Figure 6, the speed profiles in the simulation are smooth curves generated by the quintic spline method that satisfies the dynamic constraints.

## C. Collision Checking based Gap Speed Generation

Before we get the "Acc Target Speed" and "Dec Target Speed", the first step is checking whether the current favorite speed profiles have the risk of collision. The reason is that for a certain CAV, if there is a collision risk at its current favorite speed, then finding a speed profile without collision risk has the highest priority, the method is introduced in Section II-B. For a CAV, if its current favorite speed profile doesn't have the risk, the $\Delta v$ is calculated as:

$$
\begin{equation*}
\Delta v=a_{\max } * \Delta T \tag{7}
\end{equation*}
$$

where $a_{\max }$ is the maximum acceleration of the CAVs, $\Delta T$ is the sampling time of CAVs.

If its current favorite speed profile has the risk, the method to calculate $\Delta v$ is shown in equation (5).

## D. Objective Function

The objective function is applied to find the best speed profile strategy from all the possible cases. To enhance the functionality of the objective function, it is essential to revisit the primary objectives of the method proposed:

1) Collision-free decision-making with multi-risk in CAVs.
2) CAVs take the least total time to pass the intersection. The objective function is written as:

$$
\begin{equation*}
J(S)=J_{\text {Coil }}+J_{\text {Time }} \tag{8}
\end{equation*}
$$

For the Collision-free part, the method we apply is linked to the use of PIDP, which is introduced in section II-A.

$$
\begin{align*}
J_{\text {Coill }} & =W_{\text {Dist }} \sum_{i=1}^{N_{v}} \sum_{j \in V_{i}} e P I D P_{i j}^{p o} \\
& +W_{\text {Penalty }} \sum_{i=1}^{N_{v}} \sum_{j \in C_{i}}\left(-e P I D P_{i j}^{n e}\right) \tag{9}
\end{align*}
$$

where $V_{i}$ is the set of vehicles that intersect with the path of vehicle $i$. $e P I D P_{i j}^{p o}$ is the $e P I D P$ between vehicle $i$ and $j$ when its value is positive. $e P I D P_{i j}^{n e}$ is the $e P I D P$ between vehicle $i$ and $j$ when its value is negative. $W_{\text {dist }}$ and $W_{\text {Penatly }}$ are positive coefficients. $W_{\text {Penalty }}$ is a value that tends to infinity, which is a penalty function when vehicles have negative $e P I D P$.

For the least total passing time part, the predicted exiting time and average speed are applied. The cost function is written as:

$$
\begin{align*}
J_{\text {Time }} & =W_{S p d} \sum_{i=1}^{N_{v}} \int_{t_{i}^{\text {enter }}}^{t_{i}^{e x i t}}\left(v_{\max }-v_{i}(t)\right) \\
& +W_{T} \sum_{i=1}^{N_{v}}\left(t_{i}^{e x i t}-t_{i}^{e n t e r}\right) \tag{10}
\end{align*}
$$

where $t_{i}^{\text {enter }}$ is the time of vehicle $i$ enters the decisionmaking area, $t_{i}^{e x i t}$ is the predicted time of vehicle $i$ exits the action area. $v_{\max }$ is the maximum legal speed near the intersection, $v_{i}(t)$ is the speed profile of vehicle $i . W_{S p d}$ and $W_{T}$ are positive coefficients.

## E. Optimization Process

The optimization process is shown as the Algorithm 1. We note that the time complexity of this algorithm is $3^{N_{o p t}}, N_{o p t}$ is the number of CAVs in the optimization, and 3 means each vehicle has 3 different actions for each situation.

In the Algorithm 1, if the current best strategy doesn't have a better cost than the current best solution, it means the better strategy cannot be found at that moment. But, as time goes by and CAVs continue to move, possible better strategy could be found in the near future, since the state of the CAV is dynamic.

## IV. SIMULATION RESULTS

The goal of the simulation has been to make decisions for CAVs around the unsignalized intersection to avoid collision and pass the intersection as fast as possible. The main parameters considered in the tested scenario are summarized in Table I. The initial position of each CAV and the model of intersection in the simulation are shown in Figure 7. All the experiments were run by a program developed in MATLAB

```
Algorithm 1 Greedy sub-optimal optimization process
    while Set of vehicles inside the buffer area is not empty
    do
        Input CAVs and their planned trajectory
        Check the risk of collision based on PIDP
        Generate \(\Delta v\) for each CAV in the decision-making
        area and corresponding "Acc", "Dec", and "Keep"
        speed profiles
    5: Generate all situations resulting from all possible com-
        binations of behaviors of these CAVs and calculate
        their cost \(J\).
        Select the current best cost \(J_{\text {cur }}\) from all the possible
        cases and corresponding strategy \(S_{c u r}\)
        if \(J_{\text {cur }}<J_{\text {best }}\) then
            \(S_{\text {best }} \leftarrow S_{\text {cur }}\)
            \(J_{\text {best }} \leftarrow J_{\text {cur }}\)
            Output the new best strategy \(S_{\text {best }}\) to CAVs
        end if
        Wait for the next sampling time
    end while
```

with a computer of Core $\mathrm{i} 7-12700 \mathrm{H}, 2.30 \mathrm{GHz}$, and 16 GB RAM.

Four vehicles including left-turn and straight maneuvers at an intersection were set in the simulation scenario. Some simulation's results are given in Figure 8. Videos can be found in https://youtu.be/7WgIzRovx-0

Figure 8(a) depicts the curves of the target speed and the real-time speed of CAVs over time. Figure 8(b) is the sum of cost of strategies over time. Figure 8(c) is the curve of the acceleration over time. Figure 8(d) is the minimum $e P I D P$ of each CAV. If ePIDP is a positive value, it indicates that the current strategy of the CAVs is free from collision risk. Figure 8(e) is the predicted exit time for each CAV, and average exiting time. Predicted exit time is the predicted time to exit the core area in the intersection if CAVs follow the current best strategy.

It is important to emphasize that the given value of $W_{\text {Penalty }}$ is very large in the table I, the cost curve in Figure

TABLE I: Parameters and initial states

| Parameters | Value | Parameters | Value |
| :---: | :---: | :---: | :---: |
| $\left[x_{1}, y_{1}\right]$ | $[2.5,-34][\mathrm{m}]$ | $a_{\max }$ | $3\left[\mathrm{~m} / \mathrm{s}^{2}\right]$ |
| $\left[x_{2}, y_{2}\right]$ | $[28,2.5][\mathrm{m}]$ | $v_{\max }$ | $10[\mathrm{~m} / \mathrm{s}]$ |
| $\left[x_{3}, y_{3}\right]$ | $[34,2.5][\mathrm{m}]$ | $W_{S p d}$ | 0.5 |
| $\left[x_{4}, y_{4}\right]$ | $[-30,-2.5][\mathrm{m}]$ | $W_{T}$ | 0.5 |
| $v_{1}$ | $3[\mathrm{~m} / \mathrm{s}]$ | $W_{\text {Penalty }}$ | 1000 |
| $v_{2}$ | $4[\mathrm{~m} / \mathrm{s}]$ | $W_{\text {Dist }}$ | 1 |
| $v_{3}$ | $3[\mathrm{~m} / \mathrm{s}]$ | $K_{p}$ | 0.5 |
| $v_{4}$ | $4[\mathrm{~m} / \mathrm{s}]$ | Margin | $0.2[\mathrm{~m}]$ |
| $T_{\text {Horizon }}$ | $10[\mathrm{~s}]$ | $\Delta T$ | $0.01[\mathrm{~s}]$ |

TABLE II: Initial Speed

| Parameters | Value | Parameters | Value |
| :---: | :---: | :---: | :---: |
| $v_{1}$ | $9[m / s]$ | $v_{3}$ | $9[\mathrm{~m} / \mathrm{s}]$ |
| $v_{2}$ | $9[\mathrm{~m} / \mathrm{s}]$ | $v_{4}$ | $9[\mathrm{~m} / \mathrm{s}]$ |



Fig. 7: Scenario with risk of collision

8(b) is much less than the given value, which means it found a feasible solution very quickly at the beginning, the job in the coming horizon is to find a better solution. Local optimal solutions will be replaced by possible better solutions in the future as time goes by and vehicles continue to move. We noticed that the average exiting time of the proposed method is reduced by $39.38 \%$ compared to the exiting time of the initial speed.

Considering the CAVs entered the intersection at very low initial speeds in the above simulation, we reset the CAVs' speeds only to allow them to enter the intersection at an initial speed close to the maximum and then do the optimization again. The new inital speeds are given in Table II. Results with different initial speeds are shown in Figure 9.

If CAVs have larger initial speeds when entering the decision-making area of the unsignalized intersection, the optimization time left for the infrastructure will be shorter, and it is more difficult to do the speed planning. As shown in Figure 9(a), because CAVs' speeds are close to the maximum legal speed, the method to avoid collision is to decelerate first, then reaccelerate. In this simulation, even if the initial speeds are close to the maximum speed, the average exiting time of the proposed method is still reduced by $7.73 \%$ compared to the exiting time of the initial speed, and the risk of collision has been solved.

## V. CONCLUSIONS

This paper proposed a Dynamic Collision-Free Cooperative decision-making method based on the Predicted InterDistance Profile (DCFC-PIDP). The greedy algorithm is used to explore all the possible cases for CAVs and appropriate metrics based on PIDP has been used as away to converge toward the best strategy, which will be accepted and updated


Fig. 8: Performance indicators of the achieved simulation: (a) speed profile and target speed, (b) cost, (c) acceleration, (d) minimum ePIDP, and (e) average crossing time


Fig. 9: Performance indicators of the achieved simulation with initial speeds close to maximum speed
in real time. The proposed reactive approach is scalable and guarantee the convergence to the suboptimal solution. In future works, the proposed method will be extended from a single intersection to multiple adjacent intersections.

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