

# Supportless 3D Printing of Shells: Adaptation of Ancient Vaulting Techniques to Digital Fabrication

Mahan Motamedi, Robin Oval, Paul Carneau, Olivier Baverel

**Abstract** With the improvement of the 3D printing industry, the interest in additive manufacturing of large-scale structures (AMLS) is rapidly increasing. Recent attempts of seeking solutions for 3D printing of large-scale buildings is the embodiment of the transition from current construction systems to automated robotic manufacturing workflows. The usage of formwork plays a crucial role in accelerating the progress of AMLS implementation in construction industries. Investigations of large-scale 3D printing of concrete structures are mostly related to robotics, material rheology and mechanics. Additionally, design and construction strategies for AMLS must be investigated for applications in architecture. This paper discusses solutions for supportless 3D printing of large-scale compression shells. The aid of special vault geometry and robotic trajectory generation comes from reverse engineering of ancient brick-laying techniques from worldwide-recognised vaulting precedents lacking formwork. Finally, strategies for the generation of robotic printing tool-path to span boundaries with variety of configurations with no temporary support is yielded and tested with the simulation of 1:20 scale construction practice by a 3Doodler Pro pen as extrusion head (child) and ABB IRB\_120 six-axis arm

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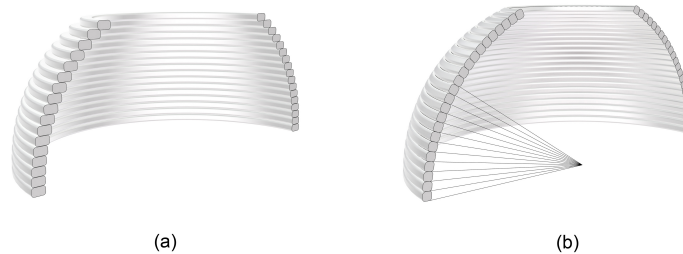
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(parent).

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## 1 Introduction

The term falsework is defined by itself as "false-work" or redundant work. This is a primary concern in the construction field to reduce or omit the need for formwork due to its cost and time-consuming aspects. Implementation of AMLS in the construction industry can significantly reduce the cost of formwork in the construction of concrete structures. Although the development of formworkless concrete printing method opens many thresholds into the future of construction technology; it is not entirely embedded in this industry. Nowadays most of our daily used products such as clothes, electrical devices, vehicles and furniture are fabricated using automated processes. Why should the buildings be excluded from this list? Construction, the way its done today is very wasteful and generates a lot of emissions. The process is pretty corruption prone, its very costly and is always over budget. In the last decade, a variety of methods encourage bridging the gap between design and construction. B. Khoshnevis suggested the pioneering method called Contour Crafting (CC) [5]. CC was limited to the vertical extrusion (2.5D printing) and obliged to utilisation of formwork for roofing. This fact is due to the uncontrollable surface contact between consecutive printing layers [2]. Fig1 shows how 2.5 D and 3D printing differ from each other. He also mentions another method for CC which is



**Fig. 1** (a) 2.5D printing- The height of the layers is constant, but the contact surfaces of the layers varies. (b) 3D printing- The contact surface is constant, but the layer height varies.

related to a formwork-free way of printing inspired by ancient mud vaults and dome construction. This topic was not followed up by researchers in this field. The majority of researches on 3D concrete printing (3DCP) focuses on the optimisation of the

cementitious paste mixture to enhance the workability, pumpability, extrudability and other mechanical properties of paste, in order to get more freedom of geometry design [7, 3, 8, 4]. However, 3DCP is a multidisciplinary domain comprising of material science, robotic science, mechanical engineering, civil engineering, and architecture. This paper follows the aforementioned precedents of in-situ formwork-free printing of adobe-like structures further by delving into the seminal history of worldwide formwork-less vaulting systems. This research will particularly focus on earth structures constructed in Iran between 248 BC- 1736 AD. A workflow is proposed, which provides the users the ability to generate from any configuration of convex polygonal boundaries a robotic path to produce a 3D printed structure without formwork. The proposed method is implemented with an extruder mounted on a six-axis robotic arm, gaining the full feature of 3D printing with maximum contact surface between each consecutive layer of printed material. The prototypes are printed with wooden strands due to special feature of fibre based material and their similarity to concrete and clay mixture in term of deposition. The method could be extended to other materials with the tuning of the printing parameters with respect to material characteristics. The prospect of this paper is to address a solution for a variety of formwork-less AMLS regardless printing material.

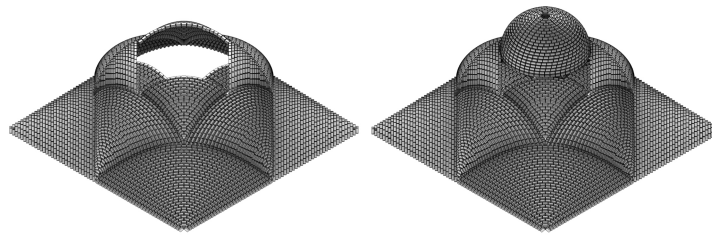
## 2 Background

### *Formworkless vaulting systems*

Unlike the Middle Age Gothic stone vaults and Roman masonry vaults, soil vault structures in the drylands had to be built formwork-less due to the lack of wood resources for carpenters. With the aid of special mortars, brick laying assemblies and geometries, masons built these structures with minimum equipment. Persian architecture history has contributed to the invention of sustainable types of vault structures. Masjed Jame Isfahan is considered as a collection of formwork-less vaulting techniques since 470 vaults were built with ten identified Persian vaulting techniques according to the classification in [6]. These vaults are primarily made of gypsum rib arches that act as the scaffolding, before becoming a part of the structure. The other major invention of the Persian vault system is the method for subdivision of square or rectangular boundaries into first smaller triangular parts and finally converting the initial boundary into circular boundary as the base for construction of a dome to fully cover long spans without any formwork. This paper mainly focuses on Filpoosh (Fig 2a) Method while mentioning two other method named as Potkaneh (Fig 2b) and Ahange-e-Oftadeh (Fig 2c).

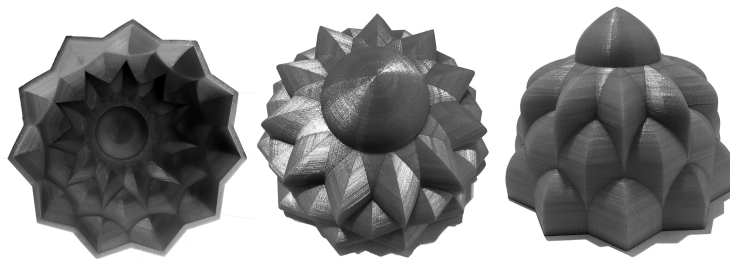
Fig 2a: Filpoosh is a method of converting rectangular based boundaries into the circular boundaries for construction of domes.

Fig 2b: Potkaneh is a method of converting polygonal boundaries into the circular boundaries for construction of domes



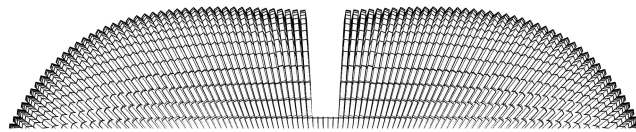
(a) Filpoosh Vaulting Method

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(b) Potkaneh Vaulting Method

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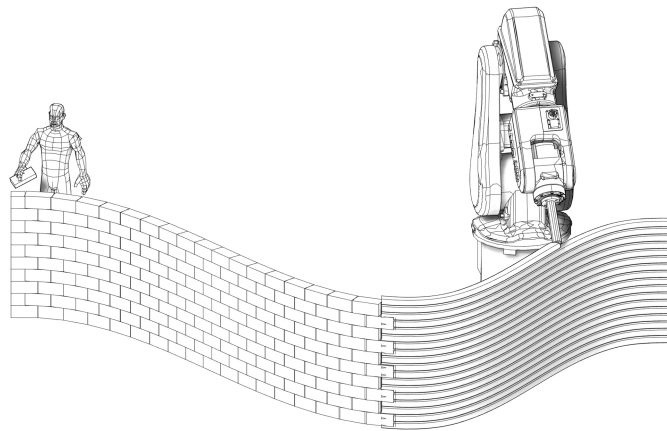
(c) Ahang-e-Oftadeh Vaulting Method

**Fig. 2** Three discreet methods of formwork-less vaulting recognised in Persian vaulting records.

Fig 2c: Ahange-e-Oftadeh is a method of covering rectangular based boundaries with inclined sequential layers of bricks (La-chasban brick laying method) without using any formwork. This type of vault is a derivative of Ahang vault with the difference that in Ahang vault, the first layer of the bricks will lean on the straight wall as support while in Ahange-e-Oftadeh vault the first layer of the bricks sits on the ground.

### 3 Computational design for 3D printing of a shell on a convex boundary

This section shows different types of Persian vaulting systems without formwork into a design workflow for 3D printing of shells with no support. The initial idea is to convert brick laying patterns into printing layers and follow their deposition order as its illustrated for the wall in Fig 3. Quad boundaries are investigated with a similar process that can be applied to other fully supported boundaries as shown in Fig 2a. This paper focuses examples on quad boundaries but the process can be executed for general convex polygon boundaries.



**Fig. 3** Transition from bricklaying by a human to the 3D printing of sequential layers by 6 axis robotic arm.

### 3.1 Patching a square

The initial idea for covering a quadrilateral boundary comes from the Filpoosh method shown in Fig 2a. Fig 4 shows the steps for covering a square boundary with squinches and pendentives and Finally the process of extracting robotic routes from squinches and pendentives as target geometries.

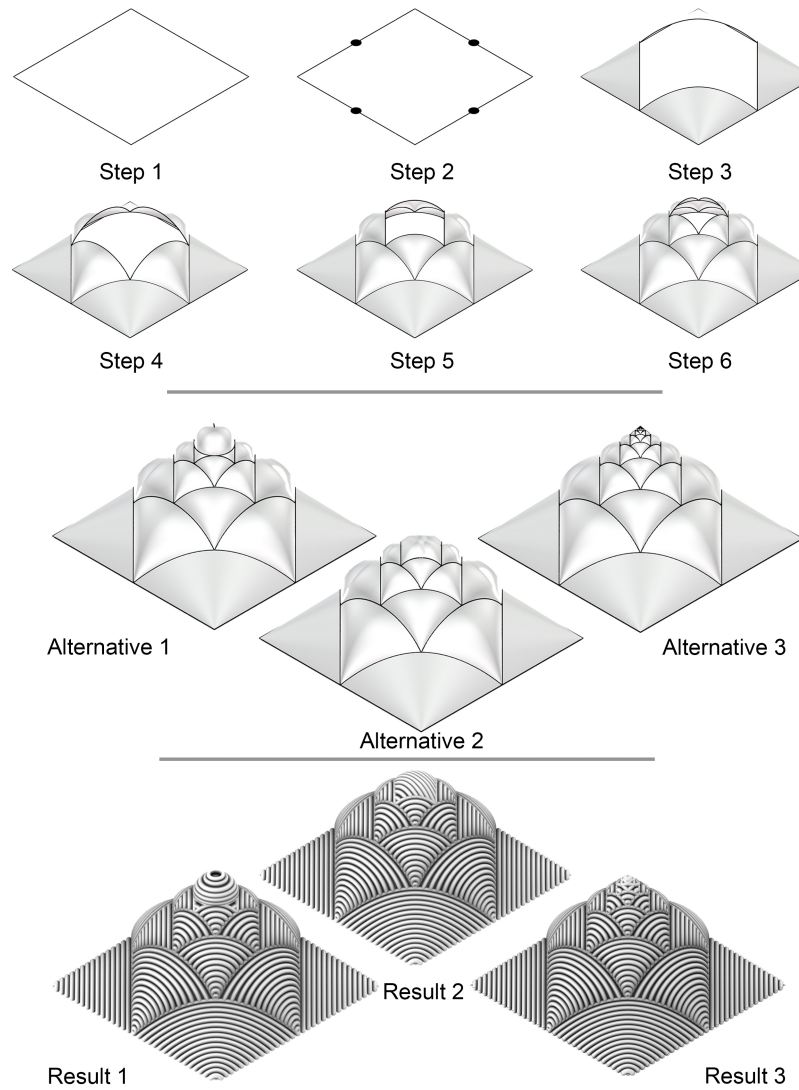
### 3.2 Patching a general quadrilateral

To have a strategy for spanning any convex quadrilateral configuration we need to describe and differentiate them more specifically. Therefore to analyse a large variety of quadrilaterals, a circle with four random points on it is used for generating multiple configurations of quadrilaterals. Sequentially, four random points are located on a circle and sorted in clockwise or counterclockwise order, to be connected as a polyline to generate each quad. This method was used to generate 250 different inscribed convex quadrilaterals as case study. Other possibilities can be quadrilaterals circumscribing a circle or any other kind of other convex quadrilaterals which are not the concern of this paper. A strategy for generating a vault for each quadrilateral is compulsory to classify quadrilaterals based on the numerical process used to cover the spans. An algorithm is developed based on skewness factor and evaluation of quads configurations to sort quadrilaterals. Fig 5 shows the result of some items from each type after filtering by developed algorithm.

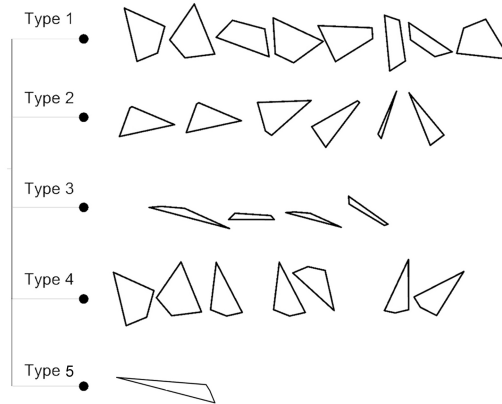
**Type 1.** In this type of quadrilaterals, after first subdivision by bimedians, all four generated triangles should have skewness less than 0.7. This number is obtained after tuning the parameters for sorting the quadrilaterals manually. These types of quads are closer to the configuration of square or rectangles with a low aspect ratio. quads in this type can directly follow the Filpoosh method for patching. The formula for calculating the skewness factor of triangles are shown below where  $\alpha$  represents angles of triangles generated from bimedians.

$$\max((\alpha_{\max} - 60)/120, (60 - \alpha_{\min})/60) \quad (1)$$

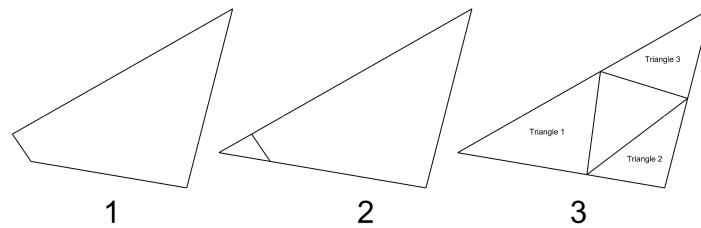
**Type 2.** These types of quads have often a segment which is smaller than the average of other sides length. In order to sort such quads the ratio of smallest segment length over the second smallest segment length after tuning the parameters is set to 0.45. Since these types of quads are very close to triangles with low skewness factor, the smallest segment can be eliminated as it is shown in Fig 6. One can replace the studying quad with a new triangle with first three largest segments. In order to avoid having unpredictable results in filtering process of this type of quads, a second condition is set as evaluation of the skewness of three generated triangles from the first subdivision of reconstructed triangle by its bimedians. This time the maximum skewness allowed for triangles after tuning the parameters is set to 0.85.



**Fig. 4** Steps to bridge a squared boundary by self supporting squinches and pendentives inspired by Filpoosh vaulting method.



**Fig. 5** Classification of convex inscribed quadrilaterals based on spanning method.



**Fig. 6** The process of eliminating the shortest segment and evaluation of three triangles generated by first subdivision with bimedians in quad type 2.

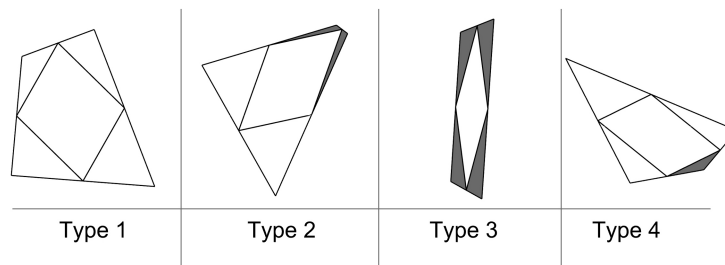
**Type 3.** These set of quads are close to stretched rectangles which have a small span length in comparison with their area. The strategy for covering the span of these types does not follow the main order of Filpoosh. For patching these types of quads we can use Ahange-e-Oftadeh vaulting method discussed in Fig 2c. In order to sort these types of quads the skewness of remained quads after filtering by type 1 and type 2 is evaluated and condition is set as, the quads with less than 0.55 of skewness factor fall into this type. The formula for calculating the skewness of inscribed quadrilaterals follows below equation where  $\alpha$  is the angles of a studying quad.

$$(\alpha_{\max} - 90)/90 \quad (2)$$

**Type 4.** These type of quadrilaterals are such as distorted kite quads. In order to sort these quads from the remained quadrilaterals after filtering by type 1, 2 and 3, the minimum amount for the proportion of each two facing angles of quadrilaterals evaluated. The condition is set to take quadrilaterals which have this proportion less



than 0.22. After filtering all generated quadrilaterals from Fig 5 by type 1,2,3 and 4, there will be some quadrilaterals remained which are sorted as type 5 shown in Fig 6. Since these quads are very close to highly skewed triangles, they can be patched such as a single squinch. However this classification is done numerically by setting conditions and rules in four steps, sometimes it's hard to decide a quad belongs to which type, specifically when a quad has the common features of two types at the same time. The main idea of spanning originates from The Filpoosh spanning observed in Persian vaulting systems without formwork for shape generation (which is directly used for quads in type 1). The other types are following the previously described method with some changes. They can not completely follow Filpoosh method due to skewed triangular areas obtained from the first division of quads as it is depicted in Fig 7.



**Fig. 7** The hatched triangular areas shown in the chart highlight the highly skewed triangle boundaries which are not proper for generating squinches. Quads type 2 will not have such boundaries after the first division by bimedians.

Fig 8 shows the five strategies used for bridging with squinches and pendentives.

### 3.3 Patching algorithm

Five bridging strategies are identified based on the configuration of quadrilaterals. In this section the algorithm for automating the process of patching for quadrilateral type 1 is explained in Fig 9 to make this process automated.

The following steps explain Fig 9:

- Step a. Take a quadrilateral;
- Step b. Extract the middle points and generate the bimedians (the lines connecting middle points of sides);
- Step c. Generate arches by connecting bimedians endpoints;
- Step d. Generate inscribed quadrilaterals by connecting middle points of the arches;

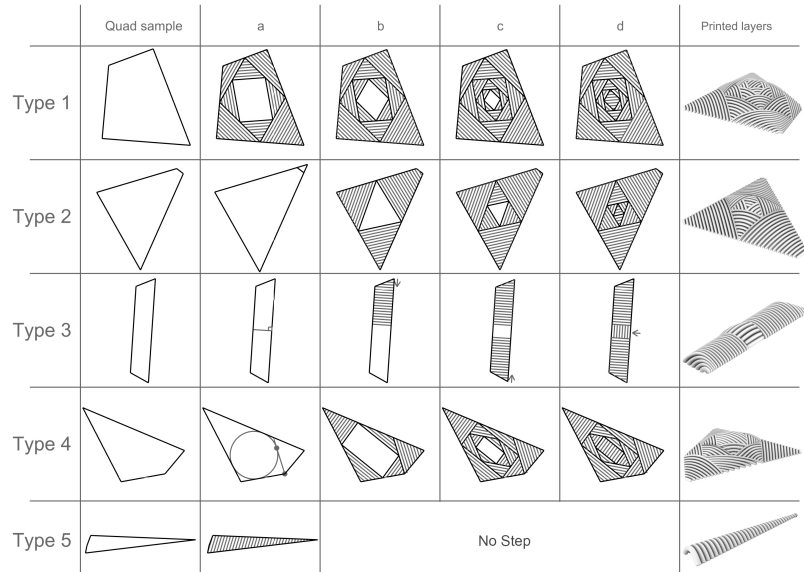


Fig. 8 Five discrete methodology for patching five types of quadrilaterals.

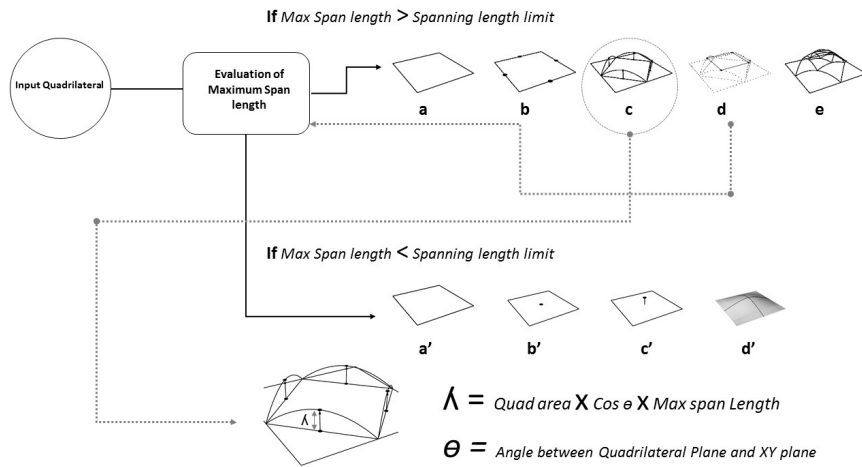


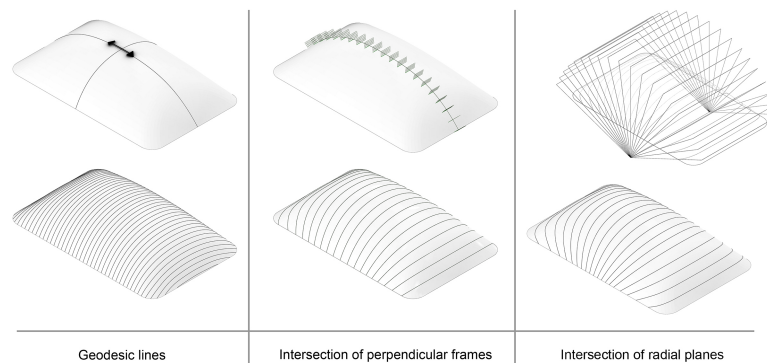
Fig. 9 Procedure for calculating target geometry for quadrilaterals of type one.

- Step e. Iterate the previous steps until the final quadrilateral with an eligible span length is achieved.
- Step a'. Take a quadrilateral;
- Step b'. Extract the quadrilateral centre point;
- Step c'. Raise the quadrilateral centroid by  $\lambda$  factor;
- Step d'. Generate the patch surface with the input quadrilateral and the raise centroid.

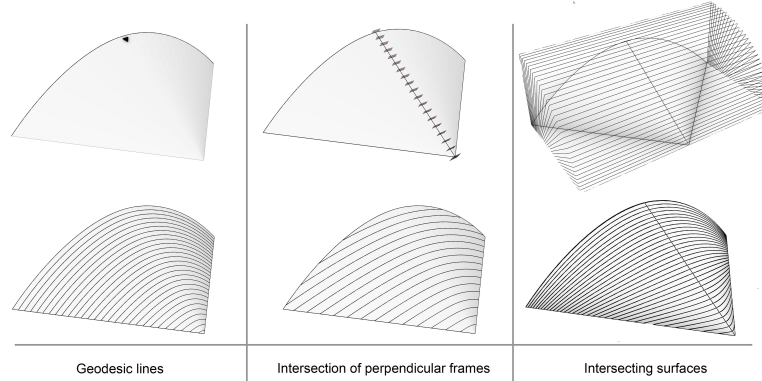
The  $\lambda$  factor determines the height and curvature intensity of the arches which shape the squinches and is specific for each quadrilateral based on its scale, maximum span length and orientation within space, if they are directly printed on a gridshell.

#### 4 Generation of robotic routes

After the creation of the target geometry, the next step is to seek a robotic tool-path. The robotic tool-path comprises robotic trajectories, robot motions settings, extrusion flow speed and other required parameters mostly determine by printing material. However there are different ways of generating robotic routes from target geometries, following 3D printing principles its always safer to ensure that the distance between layers is always constant. To ensure a constant distance between robotic routes on doubly curved surfaces such as Ahange-e-Oftadeh vault mentioned in Fig 4, the routes should be generated from geodesic coordinates on the target surface. The process of generating geodesic lines on target surfaces are inspired by [1]. Fig 10 and 11 show three different methods for generating robotic routes on two different surfaces.



**Fig. 10** Three methods of generating robotic routes on a target surface of Ahange-e-Oftadeh vault.



**Fig. 11** Three methods of generating robotic routes on a target surface of single squinch (part of Filpoosh).

### *Geodesic Routes*

This method will allow users to generate curves on general surfaces with constant distances of curves all over the surfaces. It becomes useful when the users want to ensure there is no convergence between layers and the thickness of the layers are always constant with respect to flow speed, robot speed and nozzle diameter.

### *Perpendicular Frames*

This method is more applicable to symmetric surfaces. It will generate the curves on surfaces by the intersection of perpendicular frames generated from the curve on the surface generated from intersection of surface and its symmetry plane. The result is a set of curves that are leaning sequentially towards each one. In this method, the degree of inclination of each layer is affected by the curvature of the surface and the users can only control the density of the layers. This method does not ensure the constant distance of generated curves on the surface.

### *Intersection of radial planes*

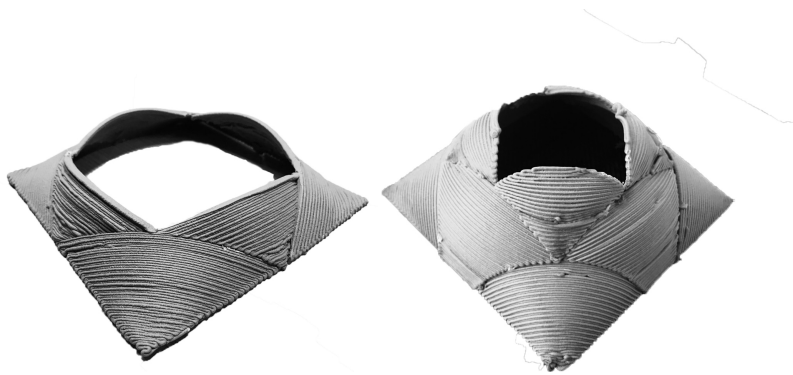
This method is applicable on symmetric surfaces. The radial arranged planes will rotate around the axis generated from a connecting line between two middle points of larger sides (end points of arc for squinch surface). The advantage of this method over the perpendicular frames method is that users have control over the amount of inclination between layers by pulling the rotation axis vertically up or down. As the rotation axis becomes further from the target surface, the amounts of layers inclinations decrease. This method and perpendicular frames method should be adjusted if the user wants to apply them to distorted asymmetric squinches for non-square quadrilateral boundaries.

## 5 Prototyping with ABB 120 and a 3Doodler Pro

Small-scale prototypes are produced to simulate the large-scale concrete or clay printing process at 1:20 scale. The ABB 120 miniature robotic-arm and the 3Doodler Pro as extrusion head are used with wooden strands as the extrusion paste even though the real scale using actual concrete and clay material are very different. Here we focus on geometry and tool-path. In order to mount the 3Doodler to the robot, an individual gripper was designed and fabricated. After running a primary test with a proper setting for the robot, speed 12.5 mm/s and doodler flow speed number 6; the nozzle temperature yielded was 200 °C.

### *Prototype of Filpoosh Vault*

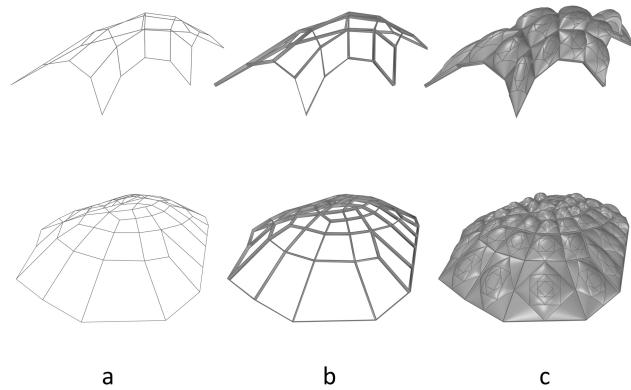
After solving robotic procedure and avoiding the clashes of extrusion head with the printed model, a prototype of the Filpoosh vault with the scale of 15 cm x 15 cm was tested successfully and printed with no temporary support thanks to squinches and pendentives as self supporting components. The thickness of printing layers after tuning the flow and robot speed parameters yielded as 1mm.



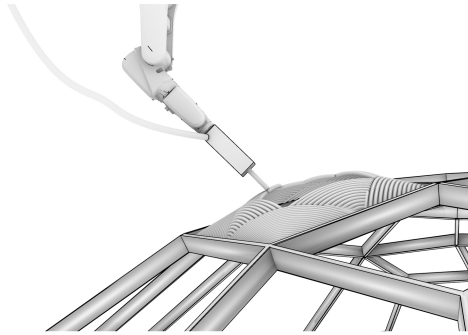
**Fig. 12** Filpoosh vault prototype printed with no temporary support.

## 6 Application to gridshells

The maximum scale of quadrilaterals which can be fully bridged by methods described in section 3.1 is mostly related to choice of the printing material. However, to cover larger spaces, this method can be used to fill the quadrilateral voids of gridshells. The algorithm described in Fig 9, make this procedure fully automated. Fig 13 shows the steps from taking networks of gridshell to the generation of target geometries for two gridshells.



**Fig. 13** Patching the quad voids of two funicular gridshells.



**Fig. 14** In-situ printing of a Filpoosh vault on voids of a gridshell.

## 7 Conclusion

This paper demonstrates a new set of workflow from generation of the shape to generation of the robotic trajectory in order to yield a method for supportless in-situ 3D printing of shells. The strategies of this proposal are briefly explained below:

- A.** Introduction to Filpoosh shape generation method which works on varieties of quadrilateral boundaries;
- B.** Classification of quadrilaterals based on their configuration and defining a discrete method of patching for each type;
- C.** Utilising three methods for generating robotic routes on target geometries. These set of procedures enhances the chance of achieving higher cantilevers by:
  - i. Maximising the contact surface between sequential layers of printing.

ii. Following the proper robotic order to generate self-supporting components (squinces and pendentives) up to the stage of covering the entire boundary.

iii. Yielding compression-dominant forms since the extrusion pastes such as concrete and clay do not have the high tensile capacity.

Further studies are regarding the examination of larger scale prototypes in concrete printing lab and understanding the constraints and opportunities, and later studying the possibility of application on gridshells in real scale.

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