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# Day-ahead lot-sizing under uncertainty: An application to green hydrogen production

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**Abstract.** This work investigates the short-term production planning of green hydrogen obtained through water electrolysis using electricity from a wind power source and a connection to the national electricity grid. Electricity consumption on the grid has to be declared a day ahead of production and cannot be adjusted afterwards, while future availability of the wind power source is uncertain. This production problem can be reduced to a two-stage stochastic lot-sizing model, and a cohesive framework is introduced to solve it efficiently. First, the innovative use of a variational auto-encoder to estimate the conditional wind power uncertainty and generate scenarios is investigated. Then, a time-efficient Benders decomposition approach is proposed, in which special features of our problem are exploited to speed up its resolution. Finally, a novel application of an adaptive partition-based approach and a stabilization method further improve the solving time of the decomposition scheme. A realistic simulation demonstrates the benefits of the presented framework.

**Keywords:** Lot-sizing · Data-driven stochastic programming · Benders decomposition · Adaptive partition · Stabilization method · Green hydrogen · Wind power uncertainty

## 1 Introduction

Hydrogen plays a crucial role in industrial processes such as glass and ammonia production, and is mainly generated from fossil fuels. Thus, green hydrogen production via water electrolysis powered by renewable energy sources is essential to decarbonize those industries, consuming only electricity and water in the process (see, *e.g.*, [1]). However, managing this electrolysis process poses challenges, such as ensuring that hydrogen hourly demand is met despite the uncertain availability of renewable energy sources. This work focuses on short-term electrolytic hydrogen production planning, considering a real-life case with a production site connected to a wind farm and the electricity grid. The wind farm supplies fluctuating renewable electricity at negligible cost, while grid electricity incurs a higher cost. Wind power forecasts, although imprecise, provide information on the future availability of this energy source. The purchase of electricity from

the grid has to be planned and declared a day ahead of production, *i.e.*, before the exact availability of wind power is known. Finally, on-site hydrogen storage offers some degree of flexibility for production planning, as it allows to produce hydrogen in advance.

The problem is similar to the widely studied single-item lot-sizing problem under uncertainty (see, *e.g.*, Brahim et al. [2]), and this work extends prior research to include uncertainties in power supply from a local wind farm. Existing works in unit-commitment address such uncertainties (see, *e.g.*, van Ackooij et al. [3]), but for complex problems where decomposition schemes for a large number of uncertainty scenarios are impractical. The use of Benders decomposition scheme is scarce in the lot-sizing literature. Adulyasak et al. [4] and Witthayapraphakorn et al. [5] apply it to lot-sizing problems under demand uncertainty, identifying a network flow structure in the dual second-stage mathematical model. In this work, a similar structure obtained from a novel problem approximation is leveraged to achieve a computational gain. Moreover, an adaptive partition-based approach as well as a stabilization method are subsequently introduced, which to the best of our knowledge have not yet been documented for lot-sizing problems. The estimation of the error distribution in wind power forecasting is also a well studied problem (see, *e.g.*, Pinson [6]). Cramer et al. [7] present a comparison of documented approaches for wind power scenario generation, and conclude that deep learning generative methods may offer more benefits for day-ahead stochastic production problems. In the present work, we use such a method called «Variational Auto-Encoder», known for the simplicity of its implementation (see, *e.g.*, Hernandez Capel and Dumas [8]), and illustrate its ability to generate reliable scenarios in realistic settings. Its use is innovative in its application to solving stochastic programming problems.

To the best of our knowledge, this work is the first one to propose a cohesive and time-efficient framework integrating stochastic programming and machine learning to solve a two-stage lot-sizing problem under uncertainty regarding the available energy source.

After a short problem description, a stochastic programming model with two decision stages is introduced in Section 2. Section 3 presents a probabilistic neural network used to generate wind power scenarios for uncertainty representation in the problem modeling. In Section 4, the problem is addressed by a Benders decomposition scheme, in which the mixed-integer sub-problems are approximately solved by a linear time algorithm, and the number of optimality cuts and iterations is reduced to improve computational performance. Realistic numerical experiments simulating the production site management over a year demonstrate the satisfactory results of the proposed approach in Section 5.

## 2 Problem description

### 2.1 Context and assumptions

This work is motivated by industrial use-cases such as the Bouin production plant of the Lhyfe company in France, or the renewable hydrogen demonstrator

of Siemens Gamesa in Zealand, Denmark. Hybrid production sites, connected to both renewable assets and the distribution grid, benefit from the cleanest possible energy from these renewable sources, while ensuring production despite intermittency thanks to the grid connection. Such a configuration enables the electrolytic hydrogen industry to take advantage of the ability of electrolyzers to adjust in near real-time to variations in energy supply and costs.

The cost of wind power is considered to be negligible compared to electricity market prices: we aim at using as much as possible the renewable energy source to produce hydrogen with the least greenhouse gas emissions. In practice, electricity prices from renewable assets are negotiated ahead of production and are of constant value. Note that a realistic representation can be retrieved from this model by subtracting the constant renewable cost from electricity market prices.

In practice, the limited hydrogen storage capacity is mainly used to reduce production costs by anticipating the lack of wind power or the increase in electricity market prices in the forthcoming hours. Consequently, the stored hydrogen is mostly produced and consumed during the same day, and the hydrogen available in storage at the beginning of a given day is almost always consumed during that day. Thus, the hydrogen stock level at the beginning of a day has an impact on the production plan for that day, but not on the production plan for the days beyond. In other words, there is little benefit to plan production more than two days in advance.

In the following, the uncertainty is only considered for the first day of the production planning, and the production of the second day is incorporated in the model in order to anticipate the impact of this uncertainty on future production costs. The two-day production planning problem is thus modeled as a two-stage stochastic program. Note that one may consider a longer planning horizon while using this approach, either by only considering uncertainty for the first day or by dynamically estimating future production costs, as described in Section 4.1.

The presented problem remains relatively general, as the proposed method can be adapted to fit a broad range of applications, some of which are discussed in Section 6. Therefore, this work aims at proposing a framework to address a wide variety of use-cases, rather than focusing on a single one.

## 2.2 Mathematical modeling

This work aims at minimizing the costs relative to the daily declaration of the grid electricity exploitation, for a production site equipped with a single electrolyzer. As discussed above, in our case, the production decisions relative to a given day do not affect the production plan beyond the following day. Therefore, the planning horizon extends over two days, each one divided into 24 time steps (hours), denoted  $T = \{1, 2, \dots, 24\}$ . In practice, production planning is carried out on the basis of a rolling horizon, *i.e.*, only first-day decisions are actually implemented, and second-day decisions are incorporated into the model to ensure that the future consequences of the first-day decisions are correctly evaluated.

At each period «day-hour»  $(d, t)$ , for  $d \in \{1, 2\}$  and  $t \in T$ , the site has to meet a hourly demand for hydrogen, noted  $q_{d,t}$ . In order to produce hydrogen, the

electrolyzer should be activated, which consumes a fixed amount of power  $p^{\text{on}}$ . It has a constant power-to-H<sub>2</sub> conversion efficiency, noted  $h$ , and can produce up to  $q^{\text{max}}$  kg of H<sub>2</sub> per time-step (corresponding to a maximum power consumption  $p^{\text{max}} = q^{\text{max}}/h$  per hour). The storage has a maximal capacity  $s^{\text{max}}$  and its initial stock level is noted  $s^0$ . It is assumed that the demand  $q_{d,t}$  at any hour  $t$  and day  $d$  is lower than the production capacity  $q^{\text{max}}$ .

The site is connected simultaneously to a wind farm and the national power grid. The purchase cost of wind power is negligible, but its availability at  $(d, t)$ , noted  $\tilde{w}_{d,t}$ , is limited, time-varying and uncertain. However, there is a deterministically known guaranteed minimum wind power at  $(d, t)$ , denoted by  $\bar{w}_{d,t}$ . The purchase cost of electricity from the grid at  $(d, t)$ ,  $c_{d,t}^g$ , is also assumed to be known with certainty. Any purchase from the grid has to be declared a day ahead of production, *i.e.*, before period  $(d, 1)$  for day  $d$  and, once declared, upward or downward adjustments of the electricity consumed during day  $d$  are forbidden.

A two-stage scenario-based stochastic programming approach is introduced to handle this problem. The first stage represents decisions to be made before the actual wind power availability is known. The second stage corresponds to decisions that can be postponed until after the realization of this uncertain parameter.

Recall that, once declared at the beginning of day  $d$ , the amount of grid electricity to be purchased cannot be adjusted during that day. This means that the demand might not be met on time if we rely on uncertain wind power, *i.e.*, on  $\tilde{w}_d - \bar{w}_d$ , to produce it. Thus, to avoid any shortage, the production of day  $d$ , intended to satisfy the demand of that same day, is not allowed to rely on wind power production beyond the minimum guaranteed output  $\bar{w}_d$ . Wind power production exceeding this output, *i.e.*,  $\tilde{w}_d - \bar{w}_d$ , is only exploited to produce hydrogen for storage, which is then used to meet future demand on day  $d + 1$ .

The first decision stage corresponds to setting up the production planning, and determining the day-ahead declaration of grid power purchase for day 1. The following variables are introduced for each period  $(1, t)$  of day 1. First, the binary variable  $z_{1,t}$  represents the active/inactive state of the electrolyzer. Then, the total power consumed for hydrogen production is denoted  $x_{1,t}$ . It decomposes into  $x_{1,t}^w$ , the amount of guaranteed wind power used, and  $x_{1,t}^g$ , the amount of power purchased from the grid. The stock of hydrogen at the end of the period  $(1, t)$  and available to meet hydrogen demand on day 1 is represented by  $s_{1,t}$ .

In this work, the wind power uncertainty during day 1 is incorporated into the modeling of the problem. This uncertainty is represented by a set  $\mathcal{S}$  of discrete scenarios of known probability distribution. A fixed scenario  $\xi \in \mathcal{S}$  corresponds to a potential realization  $w_{\xi,1,t}$  of the available wind power for all time-steps  $(1, t)$ ,  $t \in T$ . Note that the uncertainty related to day 2 is ignored: it has no significant impact on this two-day planning since it only affects the third day of production.

The second-stage decision variables concern both day 1 and day 2. Regarding day 1, a variable  $\tilde{s}_{\xi,1,t}$  is introduced for each time period  $(1, t)$  and scenario  $\xi \in \mathcal{S}$ , to represent an additional quantity of hydrogen produced using the real wind

power generated. This hydrogen is kept in stock, and it is only available to meet customer demand at the beginning of the second day, once its exact value is known. Regarding day 2, we use the same variables as the ones introduced for day 1, *i.e.*, variables  $z_{\xi,2,t}$ ,  $x_{\xi,2,t}$ ,  $x_{\xi,2,t}^w$ ,  $x_{\xi,2,t}^g$  and  $s_{\xi,2,t}$ . Note that these variables are now indexed by the scenario  $\xi$  and the index of day 2.

Before presenting the two-stage stochastic programming model, the set of constraints that the day-ahead production planning and declaration variables should respect is formally described for each day  $d$ . Given an initial stock  $s_d^{\text{in}}$  and a guaranteed minimum wind power  $\bar{w}_d$ , we have:

$$x_{d,t} = x_{d,t}^w + x_{d,t}^g, \quad \forall t \in T \quad (1)$$

$$x_{d,t} \leq z_{d,t} \cdot p^{\text{max}}, \quad \forall t \in T \quad (2)$$

$$x_{d,t}^w \leq \bar{w}_{d,t}, \quad \forall t \in T \quad (3)$$

$$s_{d,1} = s_d^{\text{in}} + (x_{d,1} - z_{d,1} \cdot p^{\text{on}}) \cdot h - q_{d,1}, \quad (4)$$

$$s_{d,t} = s_{d,t-1} + (x_{d,t} - z_{d,t} \cdot p^{\text{on}}) \cdot h - q_{d,t}, \quad \forall t \in T \setminus \{1\} \quad (5)$$

$$s_{d,t} \leq s^{\text{max}}, \quad \forall t \in T \quad (6)$$

$$x_{d,t}, x_{d,t}^w, x_{d,t}^g, s_{d,t} \geq 0, \quad \forall t \in T \quad (7)$$

$$z_{d,t} \in \{0, 1\}, \quad \forall t \in T \quad (8)$$

Constraints (1)-(2) compute the total power exploited by the electrolyzer and ensure that it complies with its activation/deactivation state. Constraints (3) limit the use of wind power to its minimum guaranteed quantity. Constraints (4)-(5) are the hydrogen stock balance equations which, together with the non-negativity requirements on the variables  $s_{d,t}$ , guarantee on-time demand satisfaction. Note that the quantity of hydrogen produced in  $(d, t)$  is calculated as  $(x_{d,t} - z_{d,t} \cdot p^{\text{on}}) \cdot h$  to account for the fixed activation power  $p^{\text{on}}$ . Inequalities (6) limit the amount of hydrogen stored to the maximum storage capacity. Let  $X_d(s_d^{\text{in}}, \bar{w}_d)$  be the set of all production plans  $(x_d, x_d^w, x_d^g, s_d, z_d)$  for day  $d$  complying with constraints (1)-(8), where each plan component is a  $|T|$ -dimensional vector.

Let the first-stage decision variables be denoted  $\chi = (x_1, x_1^w, x_1^g, s_1, z_1)$ , then the two-stage stochastic programming model can be formulated as follows:

$$\min \sum_{t \in T} c_{1,t}^g \cdot x_{1,t}^g + \mathbb{E}_{\xi \in \mathcal{S}} [\mathcal{Q}(\chi, \xi)] \quad (9)$$

$$\text{s.t. } \chi \in X_1(s^0, \bar{w}_1) \quad (10)$$

Objective function (9) aims at minimizing the grid power purchasing costs for day 1 and the expected value, over all scenarios, of the grid power purchasing costs for day 2. Here,  $\mathcal{Q}(\chi, \xi)$  returns the optimal value of the second-stage sub-problem, which computes the second-stage cost in the case where the first-stage decisions are equal to  $\chi$  and the scenario  $\xi$  realizes. This sub-problem involves the excess stock variables relative to day 1, and the day-ahead production planning and power purchase decisions relative to day 2 for this scenario. It is thus

formulated as:

$$\mathcal{Q}(\chi, \xi) := \min \sum_{t \in T} c_{2,t}^g \cdot x_{\xi,2,t}^g \quad (11)$$

$$\text{s.t. } \tilde{s}_{\xi,1,1} \leq (\tilde{w}_{\xi,1,1} \cdot z_{1,1} - x_{1,1}^w) \cdot h, \quad (12)$$

$$\tilde{s}_{\xi,1,t} - \tilde{s}_{\xi,1,t-1} \leq (\tilde{w}_{\xi,1,t} \cdot z_{1,t} - x_{1,t}^w) \cdot h, \quad \forall t \in T \setminus \{1\} \quad (13)$$

$$\tilde{s}_{\xi,1,1} \leq q^{\max} - (x_{1,1} - z_{1,1} \cdot p^{\text{on}}) \cdot h, \quad (14)$$

$$\tilde{s}_{\xi,1,t} - \tilde{s}_{\xi,1,t-1} \leq q^{\max} - (x_{1,t} - z_{1,t} \cdot p^{\text{on}}) \cdot h, \quad \forall t \in T \setminus \{1\} \quad (15)$$

$$\tilde{s}_{\xi,1,t} \leq s^{\max} - s_{1,t}, \quad \forall t \in T \quad (16)$$

$$\tilde{s}_{\xi,1,t} \geq 0, \quad \forall t \in T \quad (17)$$

$$(x_{\xi,2}, x_{\xi,2}^w, x_{\xi,2}^g, s_{\xi,2}, z_{\xi,2}) \in X_2(s_{1,24} + \tilde{s}_{\xi,1,24}, \bar{w}_2) \quad (18)$$

Inequalities (12)-(17) deal with the management of the hydrogen produced during day 1, using the generated wind power, and available to meet demand only at the beginning of day 2. Note that constraints (12)-(15) directly represent the ability to generate additional stock according to the additional available wind power and production capacity. They can thus be considered as inventory balance equations. Finally, inequalities (16) ensure that the maximum hydrogen storage capacity is respected, and constraint (18) reflects the need to build a feasible production planning for day 2. Note also that the initial incoming stock for day 2,  $s_{1,24} + \tilde{s}_{\xi,1,24}$ , depends on the scenario  $\xi$ , *i.e.*, on wind power realization during day 1.

Note that the only coupling between the first and second stages comes from the final stock of day 1,  $s_{1,24} + \tilde{s}_{\xi,1,24}$ , being the incoming stock of day 2. Therefore, the recourse  $\mathcal{Q}(\chi, \xi)$  is relatively complete. The second-stage problem is feasible for any first-stage decision  $\chi$  and any scenario  $\xi$ , since a production plan may be found for any incoming stock value.

### 3 Uncertainty modeling

Establishing a wind forecast error distribution is a complex problem (see, *e.g.*, Pinson [6]). First, the uncertainty depends on the weather conditions: here the wind power forecast is already known, and we consider that the error distribution depends on the forecast itself. Furthermore, these errors are strongly correlated over time and depend on the overall shape taken by the forecast across the whole horizon. Fortunately, error models can be identified and learnt from recurrent patterns from past forecasts that resemble those being studied. For instance, in the case of sudden increases in wind power, forecasts may often correctly identify the time periods of such events, but not their exact amplitude.

State-of-the-art approaches for scenario-based probabilistic energy forecasts include Gaussian copula, auto-regressive models and more recently, deep learning generative methods. Cramer et al. [7] compare these approaches to generate wind power scenarios for day-ahead stochastic production problems. The authors conclude that deep learning generative methods may present more benefits, using

a «Normalizing Flow» technique. In the present work, a similar approach is chosen, in the form of a probabilistic neural network called «Variational Auto-Encoder» (see, *e.g.*, Kingma and Welling [9]). As shown by Hernandez Capel and Dumas [8], this method might provide poorer results for complex tasks but it is far simpler to implement. It is, however, sufficient for the relatively simple task of deriving scenarios from a forecast input, rather than from weather data alone.

This machine learning method first transforms a time series representing the forecast, used as input parameter, into the mean and variance of a multivariate normal distribution by the use of a traditional, non-linear neural network. Samples obtained from this distribution are then transformed back into output parameters in a similar fashion, thus obtaining the desired wind power forecast scenarios. The overall process corresponds to a single, probabilistic neural network trained in a unified manner. The neural networks before and after sampling are trained jointly to recognize forecast error patterns across the training dataset.

This method is trained on a dataset comparing predicted and actual wind power. Through scenario sampling, it constructs a discrete approximation of the error distribution for any given forecast, based on the observed errors of similar forecasts in the historical data. In this case study, the forecast error distribution is estimated for the first day of the planning horizon. A comparison of actual wind power with the forecasted and scenario-sampled ones is illustrated below. The neural network may fail to correctly estimate the distribution, if the situation at hand has not yet been observed in the historical data upon which it has been trained.

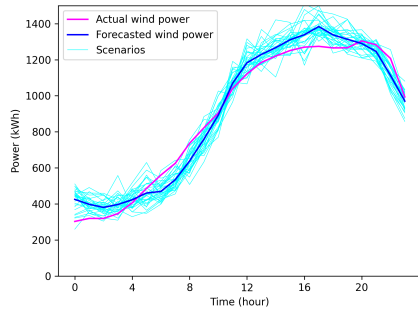


Fig. 1. First example of test data

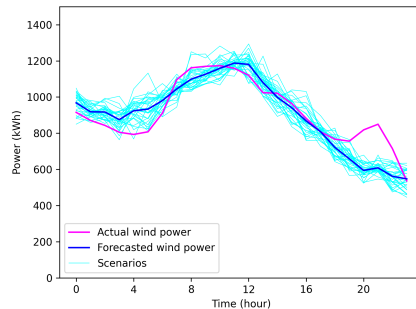


Fig. 2. Second example of test data

## 4 Time-efficient Benders decomposition algorithm

A large number  $|\mathcal{S}|$  of scenarios are needed to accurately represent the uncertainty distribution. As the size of the mixed-integer linear problem is broadly proportional to  $|\mathcal{S}|$ , solving it directly as a whole leads to significant numerical



difficulties. However, it has a block-decomposable structure that makes it suitable for a Benders decomposition approach (see, *e.g.*, Rahmaniani et al. [10]).

In our case, the Benders decomposition approach relies on the formulation (9)-(10) involving only the first-stage decision variables  $\chi = (x_1, x_1^w, x_1^g, s_1, z_1)$  and an approximation of a closed-form expression of the function  $\mathcal{Q}(\cdot, \xi)$  for each scenario  $\xi \in \mathcal{S}$ , denoted hereafter  $Q_\xi$  for simplicity. The objective of a Benders decomposition algorithm thus consists in iteratively constructing such approximations taking the form of a convex piece-wise linear function.

At iteration  $j$  of the Benders decomposition algorithm, we therefore first solve the following relaxation of problem (9)-(10), which is called the master problem:

$$\min \sum_{t \in T} c_{1,t}^g \cdot x_{1,t}^g + \mathbb{E}_{\xi \in \mathcal{S}} [Q_\xi] \quad (19)$$

$$\text{s.t. } \chi \in X_1(s^0, \bar{w}_1), \quad (20)$$

$$Q_\xi \geq \pi_m^\xi \chi + \rho_m^\xi, \quad \forall m \in \{1, \dots, j-1\}, \forall \xi \in \mathcal{S} \quad (21)$$

Once the first-stage decisions  $\chi^j$  are made for iteration  $j$ , the  $|\mathcal{S}|$  single-scenario sub-problems  $Q(\chi^j, \xi)$  are solved, and their solution is used to generate a new optimality cut of the form (21) which is added to the formulation of the master problem (19)-(21) in order to improve the current under-approximation of function  $\mathcal{Q}(\cdot, \xi)$ . This decomposition algorithm thus iteratively solves the master problem and a sequence of single-scenario sub-problems until convergence.

However, such an approach requires to repeatedly solve a large number of single-scenario sub-problems, and a master problem whose size gradually increases due to the iterative addition of optimality cuts. Thus, to be numerically tractable, the single-scenario sub-problems and the master problem have to be solved in a reasonable time. Both issues are discussed in the following.

#### 4.1 Approximate resolution of the second-stage sub-problems

Each sub-problem  $\mathcal{Q}(\chi, \xi)$  is a mixed-integer linear program involving binary activation variables. A Benders decomposition approach, based on the exact resolution of a set of mixed-integer linear sub-problems at each iteration, is likely to require prohibitive computation times. Instead, a particular feature of our problem is exploited to speed up the resolution of each sub-problem  $\mathcal{Q}(\chi, \xi)$  at each iteration. Namely, the set of constraints  $X_2(s_{1,24} + \tilde{s}_{\xi,1,24}, \bar{w}_2)$ , relative to the production planning of day 2, depends only on the final stock of day 1 and is therefore scenario-independent. Consequently,  $\mathcal{Q}(\chi, \xi)$  can be reformulated as follows:

$$\mathcal{Q}(\chi, \xi) := \min \Gamma(s_{1,24} + \tilde{s}_{\xi,1,24}) \quad (22)$$

$$\text{s.t. (12)-(17)} \quad (23)$$

Here, the function  $\Gamma(\cdot)$  gives the value of the second-day production cost as a function of the final stock level at the end of day 1. Note that the function  $\Gamma(\cdot)$  depends only on the stock level but neither on the scenario nor on other variables.

Moreover,  $\Gamma(\cdot)$  is a non-increasing function that would be convex without the binary activation variables. Indeed, the greater the input stock at the beginning of day 2, the less hydrogen production is necessary during that day, and the lower the corresponding production cost. In addition, this input stock is used to avoid production from the most expensive production hours to the cheapest ones. Hence the incremental decrease in production cost on day 2 tends to become smaller as the level of incoming stock increases, with occasional variations due to the electrolyzer activation state.

The resolution time of each sub-problem  $\mathcal{Q}(\chi, \xi)$  is therefore reduced by solving an approximate continuous problem  $\widehat{\mathcal{Q}}(\chi, \xi)$ , whose model relies on a piece-wise linear and convex approximation  $\widehat{\Gamma}(\cdot)$  of the function  $\Gamma(\cdot)$ . To construct this approximation as a pre-optimization step, we start by decomposing the interval  $[0, s^{\max}]$  into a set  $N = \{1, \dots, n\}$  of sub-intervals. Let  $\delta_i$  be the width of interval  $i \in N$ . We then calculate the value of  $\Gamma(\cdot)$  at each corresponding break-point in the approximation by solving the second-day production problem for each of these values. Finally, we construct the convex hull  $\widehat{\Gamma}(\cdot)$  of the resulting image points to obtain  $\widehat{\Gamma}(0), \widehat{\Gamma}(\delta_1), \widehat{\Gamma}(\delta_1 + \delta_2), \dots, \widehat{\Gamma}(\sum_{i=1}^n \delta_i)$ .

The slope of the piece-wise linear approximation of  $\widehat{\Gamma}(\cdot)$  over interval  $i \in N$  is given by  $\kappa_i = \left( \widehat{\Gamma}(\sum_{j=1}^{i-1} \delta_j) - \widehat{\Gamma}(\sum_{j=1}^i \delta_j) \right) / \delta_i$ . Since this convex piece-wise linear function appears in the objective of a minimization problem, it can be expressed by introducing a set of continuous variables  $w_i, i \in N$ :

$$\widehat{\mathcal{Q}}(\chi, \xi) := \min \widehat{\Gamma}(0) - \sum_{i \in N} w_i \cdot \kappa_i \quad (24)$$

$$\text{s.t. (12)-(17),} \quad (25)$$

$$\sum_{i \in N} w_i \leq \tilde{s}_{\xi, 1, 24} + s_{1, 24}, \quad (26)$$

$$0 \leq w_i \leq \delta_i, \quad \forall i \in N \quad (27)$$

Let  $a, b$  and  $c$  be the non-negative, constant right-hand side vectors of constraints (12)-(13), (14)-(15) and (16). The dual of the approximated sub-problem (24)-(27) is formulated as follows:

$$\max \widehat{\Gamma}(0) - \left( \sum_{t \in T} [a_t \cdot \alpha_t + b_t \cdot \beta_t + c_t \cdot \eta_t] + s_{1, 24} \cdot \lambda + \sum_{i \in N} \delta_i \cdot \mu_i \right) \quad (28)$$

$$\text{s.t. } \alpha_t + \beta_t + \eta_t \geq \alpha_{t+1} + \beta_{t+1}, \quad \forall t \in T \setminus \{24\} \quad (29)$$

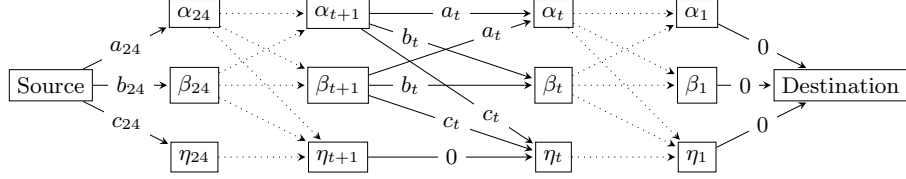
$$\alpha_{24} + \beta_{24} + \eta_{24} \geq \lambda, \quad (30)$$

$$\lambda + \mu_i \geq \kappa_i, \quad \forall i \in N \quad (31)$$

$$\alpha_t, \beta_t, \eta_t, \lambda, \mu_i \geq 0, \quad \forall t \in T, \forall i \in N \quad (32)$$

Here,  $\alpha, \beta, \eta$  and  $\mu$  are respectively the dual vectors of variables for constraints (12)-(13), (14)-(15), (16) and (27), while  $\lambda$  is the dual variable for constraint (26). This problem may be solved in linear time. Suppose that for a given scenario  $\xi \in \mathcal{S}$ , the optimal solution  $\lambda^\xi$  to this problem is known. Indeed, the optimal

values  $(\alpha^\xi, \beta^\xi, \eta^\xi)$  at time-step  $t$  depend on those at time-step  $t + 1$ , and the values at time-step 24 depend on  $\lambda^\xi$ . One may use dynamic programming to deduce these values, by solving a shortest path problem on the directed acyclic graph represented below. There, each variable whose corresponding node is part of the shortest path has its optimal value equal to  $\lambda^\xi$ , and any other variable have its value equal to zero.



**Fig. 3.** Directed acyclic graph for the second-stage approximate dual problem

As for the optimal values  $\mu^\xi$ , they are directly deduced so that:

$$\mu_i^\xi = \max\{0, \kappa_i - \lambda^\xi\}, \quad \forall i \in N \quad (33)$$

Hence, since the optimal values  $(\alpha^\xi, \beta^\xi, \eta^\xi)$  depend linearly on the optimal value of  $\lambda$ , the only possible extreme points of this latter variable belong to the set  $\{0, \kappa_1, \dots, \kappa_n\}$ , and the variable  $\lambda$  is of optimal value in this set. Thus, to solve this approximate sub-problem, it is sufficient to solve the aforementioned shortest path problem once, and then enumerate the possible values of the variable  $\lambda$ . For a solution value  $\mathcal{P}$  of the shortest path problem, the approximate dual sub-problem is equivalent to:

$$\min \left\{ (\mathcal{P} + s_{1,24}) \cdot \lambda + \sum_{i \in N} \delta_i \cdot \max\{0, \kappa_i - \lambda\} : \lambda \in \{0, \kappa_1, \dots, \kappa_n\} \right\} \quad (34)$$

Handling the sub-problem requires both this enumeration and solving the shortest path problem, and is therefore of linear complexity  $\mathcal{O}(|N| + |T|)$ . Note that the set of optimal solutions is finite, since the number of paths in Figure 3, and the possible optimal values of  $\lambda$ , are finite. An optimal solution  $(\alpha^\xi, \beta^\xi, \eta^\xi, \lambda^\xi, \mu^\xi)$  of the approximate dual sub-problem is finally used to infer parameters  $(\pi^\xi, \rho^\xi)$  like so:

$$\rho^\xi = \widehat{F}(0) - \left( \sum_{t \in T} [q^{\max} \cdot \beta_t^\xi + s^{\max} \cdot \eta_t^\xi] + \sum_{i \in N} \delta_i \cdot \mu_i^\xi \right) \quad (35)$$

$$\begin{aligned} \pi^\xi \cdot \chi &= \sum_{t \in T} \left[ s_{1,t} \cdot \eta_t^\xi - (\widetilde{w}_{\xi,1,t} \cdot z_{1,t} - x_{1,t}^w) \cdot h \cdot \alpha_t^\xi \right] \\ &+ \sum_{t \in T} \left[ (x_{1,t} - z_{1,t} \cdot p^{\text{on}}) \cdot h \cdot \beta_t^\xi \right] - s_{1,24} \cdot \lambda^\xi \end{aligned} \quad (36)$$

The resulting optimality cut is then added to the set of constraints (21) in the master problem.

Therefore the second-stage problem can be solved with good precision in an efficient manner, as guaranteed by its linear complexity. Although the network flow structure of the second-stage problem was already known for two-stage stochastic lot-sizing problems, this particular application accounting for the presented approximation can be considered to be an original contribution.

## 4.2 Accelerating the Benders decomposition scheme

This sub-section tackles the difficulties met when solving the master problem once a large number of optimality cuts has been generated. It aims at reducing the number of generated optimality cuts at each iteration, and the number of iterations to achieve convergence.

First, it is possible to exploit scenarios with identical second-stage dual solutions to reduce the number of generated optimality cuts. An adaptive partition-based Benders decomposition is implemented (see, *e.g.*, Song and Luedtke [11]). At each iteration  $j$  we solve the master problem, as well as the second-stage dual problem for each scenario  $\xi \in \mathcal{S}$ , and retrieve the corresponding second-stage dual solution. We use this information to define a partition of the scenario set  $\mathcal{S}$  into  $K \leq |\mathcal{S}|$  non-empty subsets  $S_k^j$ ,  $k \in \{1, \dots, K\}$ , gathering scenarios of identical dual solution. Then, we replace the optimality cuts presented in (21) and (35)-(36) by a single one per non-empty subset  $S_k^j \neq \emptyset$  of this partition:

$$\mathbb{E}_{\xi \in S_k^j} [Q_\xi] \geq \mathbb{E}_{\xi \in S_k^j} [\pi_j^\xi \chi + \rho_j^\xi] \quad (37)$$

This approximate optimality cut remains precise: all scenarios in that set share very similar parameters  $(\pi_j^\xi, \rho_j^\xi)$  since they all have an identical dual solution  $(\alpha^k, \beta^k, \eta^k, \lambda^k, \mu^k)$ . Thus, the right-hand term of the approximate optimality cut (37) is expressed as:

$$\begin{aligned} \mathbb{E}_{\xi \in S_k^j} [\rho_j^k] &= \widehat{\Gamma}(0) - \left( \sum_{t \in T} [q^{\max} \cdot \beta_t^k + s^{\max} \cdot \eta_t^k] + \sum_{i \in N} \delta_i \cdot \mu_i^k \right) \quad (38) \\ \mathbb{E}_{\xi \in S_k^j} [\pi_j^\xi \cdot \chi] &= \sum_{t \in T} \left[ s_{1,t} \cdot \eta_t^k - \left( \mathbb{E}_{\xi \in S_k^j} [\tilde{w}_{\xi,1,t}] \cdot z_{1,t} - x_{1,t}^w \right) \cdot h \cdot \alpha_t^k \right] \\ &\quad + \sum_{t \in T} \left[ (x_{1,t} - z_{1,t} \cdot p^{\text{on}}) \cdot h \cdot \beta_t^k \right] - s_{1,24} \cdot \lambda^k \quad (39) \end{aligned}$$

In other words, all scenarios in that set share an identical optimality cut except for the available wind power  $\tilde{w}$  that depends on the considered scenario. Rather than generating multiple and extremely similar optimality cuts, a single one is added to the model with an available wind power approximated by its average value across that set. This approach enables a significant reduction in the number of optimality cuts added to the master problem. In fact, scenarios

of different values having the same impact on certain first-stage decisions are bundled together into a single constraint.

In practice, the partition-based acceleration improves convergence in the early iterations of the decomposition scheme, with a fast decrease of the precision gap. Yet because of the mentioned approximation on the available uncertain wind power, once a small gap is achieved, the scheme could take more time than expected to converge. Thus one may consider dialing down the partitioning according to the observed gap at each iteration, for example using partial refinement as presented by Song and Luedtke [11].

Finally, a trust-region stabilization method is introduced to reduce the number of iterations of the decomposition scheme (see, *e.g.*, van Ackooij et al. [12]). It forces the master problem solution of the next iteration to lie in the neighborhood of the previous one. Such method improves the performance of the decomposition scheme by decreasing its instability, *i.e.*, the fact that two successive first-stage decisions can be «very far apart». It is here more specifically applied to the binary activation variable  $z$ : at iteration  $j + 1$ , there can be no more than  $C$  changes in the variable values by adding the following constraint to the master problem:

$$\sum_{\{t:z_{1,t}^j=1\}} (1 - z_{1,t}) + \sum_{\{t:z_{1,t}^j=0\}} z_{1,t} \leq C \quad (40)$$

In practice, the optimal production plan is mainly dependent on the choice to activate the production unit at each period. Thus, one may limit the change in activation variable values from a planning to another to effectively force them to share a similar shape, reducing the instability of the decomposition scheme in the process.

These acceleration methods result in a master problem of reduced size that has to be solved for a smaller number of iterations, without losing the finite-optimal convergence of the Benders decomposition algorithm. To the best of our knowledge, none were used until now to improve a Benders decomposition scheme applied to a stochastic lot-sizing problem.

## 5 Numerical experiments

This section assesses the performance of the stochastic programming approach to reduce the production overcosts induced by wind power forecast uncertainty.

To that end, a production site is considered with the following parameter values:  $p^{\text{on}} = 200$  kWh,  $p^{\text{max}} = 1000$  kWh,  $q^{\text{max}} = 15$  kg,  $h = 0.015$  kg/kWh,  $s^{\text{max}} = 70$  kg and  $q = 9$  kg. We simulate the day-to-day operation of this site over a rolling one-year period. Electricity prices of the national grid network correspond to those of France in the year 2016. Actual and forecasted wind power values are obtained from realistic, open-access data by Pan et al. [13]. The comparison between actual and predicted wind power is carried out over two years: the scenario generation method is trained on the first year and the simulation is performed on the second year.

For each day of the simulated year, we calculate the production plan provided by the stochastic model (9)-(10), the one provided by a deterministic oracle model having full knowledge of the future wind power availability, and the one provided by a deterministic naive model using the forecast but not considering any uncertainty. The stochastic model is evaluated for  $|\mathcal{S}| = 1000$  scenarios, a stabilization parameter  $C = 3$  and  $|N| = 10$  pieces approximating the function  $\Gamma(\cdot)$ . For each model, the decisions relative to the first day are implemented, and the actual production cost and resulting storage value are recorded.

Table 1 displays the simulation results, comparing the overcosts related to the wind power forecast error over the year. This overcost is defined as a percentage of the theoretical minimum costs indicated by the oracle model. The stochastic model performs better than the naive one with 0.8% less overcosts in total for the year, which represents 21% of the loss caused by the uncertainty.

**Table 1.** Overcosts related to uncertainty as part (%) of the theoretical optimum

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.	Year average
Naive	3.1	4.9	3.5	3.4	4.3	<b>3.7</b>	4.	4.	3.8	3.6	3.8	3.7	3.8
Stochastic	<b>1.9</b>	<b>2.2</b>	<b>2.9</b>	<b>2.8</b>	<b>3.1</b>	3.9	<b>2.8</b>	4.	<b>2.6</b>	<b>3.5</b>	<b>3.1</b>	<b>3.6</b>	<b>3.</b>

We also demonstrate the benefits of the partition-based and stabilized decomposition in reducing the runtime of the stochastic model. The day-by-day runtime across the simulation over a year for  $|\mathcal{S}| = 1000$  is observed to compare the decomposition scheme enhanced with one of these methods, both or none. The computational experiments were conducted on a CentOS Linux machine with 16 GB of RAM and an Intel Core CPU i7-8565U processor at 1.80 GHz. All the mixed-integer linear programs were solved with the commercial solver IBM CPLEX 12.10, for which a single thread was used. The algorithms were realized in Python, and CPLEX was called via the framework of Pyomo. The cumulative runtime over the simulation for each method is shown in Figure 4.

Using the stabilization and partition-based methods jointly in the decomposition scheme appears to be most beneficial in reducing the runtime. Overall when using both acceleration methods, the solving time is reduced by two third compared to the standard decomposition scheme. Take note that although the partition-based decomposition scheme alone doesn't exhibit significantly better performance compared to the standard decomposition scheme, it becomes highly beneficial when integrated with the stabilization scheme, reducing by a third the average solving time compared to a scheme with stabilization only.

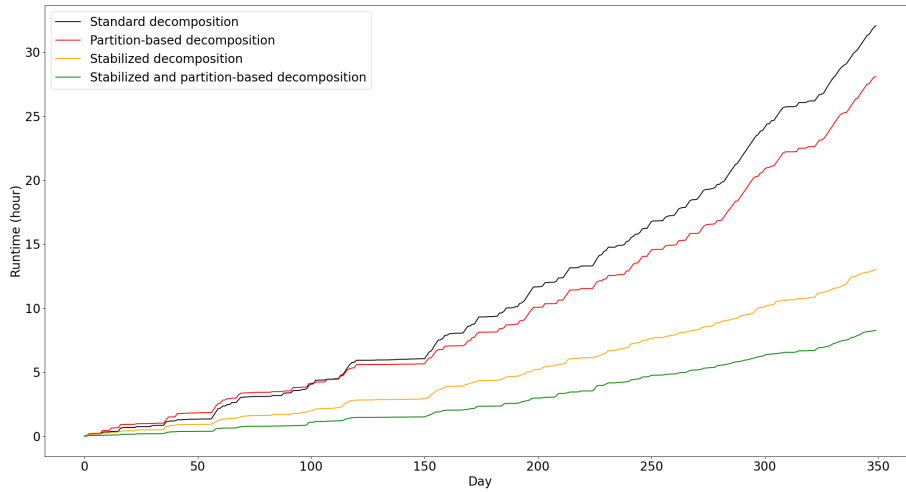


Fig. 4. Cumulative runtime comparison of the stochastic models

The presented numerical experiments evaluate the proposed stochastic approach in a real-life setting and for a wide variety of electricity grid prices and wind power availability. They demonstrate the method’s ability to improve production performance within a reasonable timeframe, for a broad range of practical cases. The data used to carry out this simulation (grid prices, wind power, scenarios) is available for reproducibility purposes<sup>4</sup>.

## 6 Conclusion

This work addresses day-ahead production planning with uncertain availability of a wind power source and a historical dataset of forecast error. It is modelled as a two-stage stochastic lot-sizing problem, solved by a Benders decomposition scheme with an innovative approximation of the second-stage model and the novel use of acceleration methods. An accurate method for generating wind power scenarios and a time-efficient resolution provide a significant reduction in production costs, as demonstrated by a realistic simulation of the production site.

Possible extensions to more realistic models could be achieved from this work. The presented problem is solved in a similar fashion when accounting for other uncertain energy sources (*e.g.* photovoltaic), or for the deterministic participation to other energy markets (*e.g.* intra-day market). The electrolysis production efficiency could be more precisely represented by a piecewise-linear concave approximation with continuous representation (see, *e.g.*, Baumhof et al. [14]), with minor changes to the network flow model of the second-stage approximate dual

<sup>4</sup> <https://github.com/VSpitzer/Stochastic-day-ahead-lot-sizing.git>

problem. We may also extend the work to consider a set of parallel hydrogen production units. Finally, demand uncertainty might bear strong resemblance to the studied uncertain power availability, as both seem equivalent to a variation in production capacity.

Further research work would include the consideration of uncertainty relative to day-ahead electricity grid prices, as it mainly impacts the first-stage decision. In addition, other stabilization methods could further improve the time efficiency of the decomposition scheme.

## References

1. Global Hydrogen Review 2023. In: International Energy Agency (IEA), Paris (2023)
2. Brahim, N., Absi, N., Gendreau M., Dauzère-Pérès, S., Nordli, A.: Single-item dynamic lot-sizing problems: An updated survey. *European Journal of Operational Research* **263**(3), 838–863 (2017)
3. van Ackooij, W., Danti Lopez, I., Frangioni, A., Lacalandra, F., Tahanan, M.: Large-scale unit commitment under uncertainty: an updated literature survey. *Annals of Operations Research* **271**(1), 11–85 (2018)
4. Adulyasak, Y., Cordeau, J.-F., Jans, R.: Benders decomposition for production routing under demand uncertainty. *Operations Research* **63**(4), 851–867 (2015)
5. Witthayapraphakorn, A., Charnsethikul, P.: Benders decomposition with special purpose method for the sub problem in lot sizing problem under uncertain demand. *Operations Research Perspectives* **6**, 100096 (2019)
6. Pinson, P.: Estimation of the uncertainty in wind power forecasting. Ph.D. Thesis, École des Mines de Paris (2006)
7. Cramer, E, Paeleke, L., Mitsos A., Dahmen, M.: Normalizing flow-based day-ahead wind power scenario generation for profitable and reliable delivery commitments by wind farm operators. *Computers & Chemical Engineering* **166**, 107923 (2022)
8. Hernandez Capel, E., Dumas, J.: Denoising diffusion probabilistic models for probabilistic energy forecasting. In: *IEEE PowerTech 2023 Conference*, Belgrade, Serbia, June 25-29 (2023)
9. Kingma, D.P., Welling, M.: Auto-encoding variational Bayes. In: *2nd International Conference on Learning Representations (ICLR 2014)*, Banff, Canada, April 14-16 (2014)
10. Rahmaniani, R., Crainic, T.G., Gendreau, M., Rei, W.: The Benders decomposition algorithm: A literature review. *European Journal of Operational Research* **259**(3), 801–817 (2017)
11. Song, Y., Luedtke, J.: An adaptive partition-based approach for solving two-stage stochastic programs with fixed recourse. *SIAM Journal on Optimization* **25**(3), 1344–1367 (2015)
12. van Ackooij, W., Frangioni, A., de Oliveira, W.: Inexact stabilized Benders’ decomposition approaches with application to chance-constrained problems with finite support. *Computational Optimization and Applications* **65**(3), 637–669 (2016)
13. Pan, K., Matusz, K., Lallanne, C.: Predicting excess wind electricity in Ireland: Machine learning against climate change. In: *Towards Data Science*, June 3 (2021)
14. Baumhof, M.T., Raheli, E., Johnsen, A.G., Kazempour, J.: Optimization of hybrid power plants: when is a detailed electrolyzer model necessary? In: *IEEE PowerTech 2023 Conference*, Belgrade, Serbia, June 25-29 (2023)