



HAL
open science

On the Philosophical Standpoint of a Recent Mathematical Color Perception Model

Filippo Pelucchi, Michel Berthier, Edoardo Provenzi

► **To cite this version:**

Filippo Pelucchi, Michel Berthier, Edoardo Provenzi. On the Philosophical Standpoint of a Recent Mathematical Color Perception Model. *Foundations of Science*, 2024, 10.1007/s10699-024-09951-2 . hal-04575499

HAL Id: hal-04575499

<https://hal.science/hal-04575499>

Submitted on 15 May 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

On the philosophical standpoint of a recent mathematical color perception model

Filippo Pelucchi (a), Michel Berthier (b) and Edoardo Provenzi (c)

(a) Università degli Studi di Milano, Dipartimento di Filosofia, Via Festa del Perdono, 7, 20122 Milano, Italy. Email: filippo.pelucchi@studenti.unimi.it

(b) Laboratoire MIA, Bâtiment Pascal, Pôle Sciences et Technologie, Université de La Rochelle, 23, Avenue A. Einstein, BP 33060, 17031 La Rochelle cedex, France. Email: michel.berthier@univ-lr.fr

(c) Université de Bordeaux, CNRS, Bordeaux INP, IMB, UMR 5251, F-33400, 351 Cours de la Libération, Talence, France. Email: edoardo.provenzi@math.u-bordeaux.fr

Abstract

The problem of explaining color perception has fascinated painters, philosophers and scientists throughout the history. In many cases, the ideas and discoveries about color perception in one of these categories influenced the others, thus resulting in one of the most remarkable cross-fertilization of human thought. At the end of the nineteenth century, two models stood out as the most convincing ones: Young-Helmholtz's trichromacy on one side, and Hering's opponency on the other side. The former was mainly supported by painters and scientists, although with some noticeable exceptions as, e.g., Otto Runge, while the majority of philosophers supported the latter. These two apparently incompatible models were proven to be two complementary parts of the hugely complex chain of events which leads to human color perception. Recently, a rigorous mathematical theory able to incorporate both trichromacy and opponency has been developed thanks to the use of the language and tools of quantum information. In this paper, we discuss the placement of this model within the philosophical theories about color.

1. Introduction

Since the Hellenic era, a multitude of artists, philosophers and scientists dedicated a lot of effort in trying to understand how to describe colors. Without any mean to verify their claims, several ideological branches spread out. It was only in the nineteenth and twentieth century that technological advances allowed physicists and biologists to develop strategies to actually test the descriptions and predictions of the various color theories.

A pivotal figure in understanding color in the nineteenth century was certainly the Scottish physicist James Clerk Maxwell, who both clarified the nature of light as an electromagnetic wave and indirectly verified Thomas Young's bold hypothesis on the existence of three photoreceptors with different sensitivities in humans' retina. Basil Mahon described Maxwell's work with these relevant words: "Some of the greatest leaps in science have come when two sets of apparently different phenomena are explained by a single new theory. This was one such leap: at a stroke, he had united the old science of optics with the much newer one of electromagnetism" (Mahon:2003, page 109). Alongside those by Maxwell, the most important contributions to color science in the nineteenth century came from von Helmholtz and Grassmann (see, e.g., von Helmholtz (1852), Grassmann (1853), Peruzzi & Roberti (2023)).

In the twentieth century, the advent of electron microscopy allowed biologists and neuroscientists to better understand the stages of color vision that follow light absorption. What became evident is that retinal neurons, in particular ganglion cells, intertwine

photoreceptors signals in a way that confirmed another bold hypothesis, that of Ewald Hering, on color opposition.

Thus, two seemingly antagonist theories, the so-called Young-Helmholtz trichromatic one and Hering's opponent model, see e.g. (Turner, 1994), were found to be two complementary pieces of a hugely complicated chain of neurophysiological events that leads to human color vision. This chain involves both the eyes and the brain in the so-called visual zones, whose behavior is, for the most part, still unknown.

In spite of these advances in our knowledge about color perception, the philosophical debate regarding the interpretation of color and what is its most suitable description is still open. In this paper we discuss the philosophical standpoint of a very recent mathematical theory of color perception based on a quantum paradigm. In order to better understand the place of this theory inside the jumble of color perception models, we first review the major directions of thought about color description. This will allow us to point out similarities and differences between this novel quantum-inspired color theory and the philosophical threads already existing.

2. Philosophical theories of color

Colors are the subject of a vast and impressive body of empirical research and theory, but also a very attractive argument in different branches of philosophy. Colors are taken to be phenomenal properties (philosophy of mind), secondary qualities, universals, or particulars (metaphysics) and objects of a priori knowledge and of empirical knowledge (epistemology). Of course, they are the essence of philosophy color, which deals, among other things, with the following questions: What is color? Are colors physical properties of the objects we perceive? Is there only one way to perceive color? Does color exist without a visual system? In the last forty years, the debate has been mainly centered on two opposite sides: color realism and color irrealism.

Color realists claim that color is (i) identical to certain physical properties of the objects we perceive and (ii) mind (or perceiver) independent (e.g., Tye 2000, McLaughlin 2003), while others identify colors with intrinsic microphysical properties (e.g., Jackson & Pargetter 1987, Lewis, 1997). Instead, color irrealists hold color is a subjective property, meaning that it is a product of the brain and mind/perceiver dependent (e.g. Boghossian & Velleman (1989), Averill (2005), Maund (2006, 2011)).

The general idea is that the commonsense conception of color is just wrong. Objects are not colored in any sense we believe them to be. Note that this view has also support from prominent color scientists like Zeki, Kuehni and Chrismont. The world presents itself as colored and if it really is not colored, we are owed at least an explanation of how we could have been so wrong. For while the world itself has no color, there are undeniably experiences of color. Those experiences may be caused by physical objects, but the qualities of pain and pleasure are in us, not in the objects. So too we may say for color. In thus locating color within perceptual experience, we make it perceiver dependent, and thus, in some sense, cease to view color as part of the objective world. Now, as for color realism, there are different version of color irrealism.

In fact, among color irrealists we can find color projectivists (Averill 2005, Averill & Hazlett 2011). The basic idea is that colors are subjective qualities "projected" onto physical objects and light-sources - qualities which visual experiences represent objects as having. We "project" colors that are "in us" onto objects in our environment. On some projectivist accounts, colors are identified with properties instantiated by elements of perceiver's visual

systems, resulting in a systematic illusion of the instantiation of color properties by objects around us (Maund 2006, 2011).

In order to better understand and illustrate the differences between these two points of view, we shall now briefly mention a few historical contributions from supporters of both color realism and irrealism, as well as some other views that differ somewhat from these positions.

2.1 Color Realism

According to color realism, the physical objects which populate our world (e.g., oranges, stars, trees, mountains and so on) are colored. These objects possess physical properties, i.e., colors, which allow them to appear in specific ways to any sort of being able to perceive colors (humans and animals). Some famous endorsers of color realism are Aristotle, John Locke, Isaac Newton and, more recently, Alex Byrne and David Hilbert.

Aristotle. He claimed that colors are qualities which are external, particular, and remote from the perceiver. According to Aristotle, visual perception becomes possible when colors of external particulars are at a visible distance and there is a transparent medium between the perceiver and the remote objects. Light is the state that this medium is in when it is actually transparent, and this is due to the presence of a fiery substance (i.e., the element of Fire). Color is, according to Aristotle, *the power to affect the character of the illuminated medium*, by altering the presence of the fiery substance within it. Colors are not bodies but powers or states of objects, which implies that they are at best indirectly located, inheriting their location from the objects in which they inhere (Kalderon:2015, p. 51-52).

John Locke. The idea of colors as powers proposed by Aristotle was used by Locke to argue for a different form of color realism, namely *dispositionalism*. Locke argued that colors are “real qualities in the subject”, meaning by “the subject” not the person seeing a physical object but the physical object which is seen (Locke:1689[1977], 2.8.10). These “real qualities” are merely *powers* (dispositions) to produce sensations in our minds. For example, he held that the green color of an emerald is its power to produce the sensation of green in the mind of a sentient being. Regarding the nature of these powers, Locke defined them as active, where an active power “is an ability to cause a certain kind of change” (Hyman:2006).

Isaac Newton. He advocated that colors are permanent qualities of light, able to maintain their individuality. He came to this conclusion after his celebrated double prisms experiment in which the first prism was used to create a rainbow effect from daylight and the second one to show that the colors that appeared were not further decomposed, but only deflected, thus leading to his hypothesis that these colors were irreducible and eternal: “The colors of homogeneal lights, I call primary, homogeneal and simple” (Newton:1704[2017], Definition VIII in the first book, part 1).

Although this description of colors supports realism, it is worth noting that Newton also seemed to suggest in the following excerpt from *Opticks* that colors are more subjective experiences rather than inherent properties of light: “For the Rays, to speak properly, are not colored. In them, there is nothing else than a certain power and disposition to stir up a sensation of this or that color.” This is confirmed in Query 13, where he asked about the possible causes for the emergence of the sensations of color: “Do not several sorts of rays make vibrations of several brightnesses, which, according to their brightnesses, excite sensations of several colours?”.

Alex Byrne and David Hilbert. One of the most important contemporary versions of color realism is due to (Byrne&Hilbert2003;2021) and (Hilbert:1987). The main idea behind their account is that a color is assimilated to a specific physical property called *surface spectral reflectance* (SSR), which describes the absorption curve of a surface as a function of light

(Byrne & Hilbert 2021). This is actually how the most basic digital image formation models are conceived. The core idea, as other forms of color realism, is that color has three defining properties, it is identified with a specific physical property, it is mind-independent (i.e., it exists whether there is a perceiver or not) and it belongs to the object, not to the perceiver. The realist view of Byrne and Hilbert is probably the most influential and popular among philosophers of color. However, it is not as widely shared by color scientists, especially those concerned with psychological or neurobiological aspects of color vision (Byrne & Hilbert 2003).

2.2 Color Irrealism

According to color irrealism, objects are not really colored, but they *seem* to be colored. On one version of this view, as Barry Maund remarks, “even though the color of physical objects, such as peaches and foxes and trees and lips and glasses of wine, and skies, etc., would seem to be their most visually conspicuous of features, these objects do not have the colors they appear to have” (Maund:2011, p.1).

Some famous thinkers who endorsed color irrealism are Democritus, Galileo, Descartes, Goethe, Schopenhauer, Russell and Hardin.

Democritus. Consider this famous passage from Democritus taken from the book (Guthrie:1962): “Colors, sweetness, bitterness, exist by convention. In reality, there are atoms and the void.” In Democritus’ view, colors do not really exist. The objects which populate our world are not really colored but we believe it to be the case. This does not mean that Democritus (nor any irrealist about color) did not have his own theory of color. In fact, Democritus distinguished a set of basic “colors” - bright, dark, red and green - and described them in terms of the shapes and structures they are composed of, as claimed by Theophrastus *De Sensibus*. All the phenomena of nature, including color experiences, are to be explained in terms of how atoms of different kinds are distributed in different ways throughout the void. The atoms themselves have no color but are limited to properties that have come to be called the *primary qualities*: shape, size, motion, position, solidity (Pasnau:2007).

Galileo Galilei. In his 1623 book *Il Saggiatore*, Galileo explicitly considered colors as *secondary qualities*: “colors have their residence only in the sensitive body, without it they are only names”.

John Locke. He shared the distinction between primaries and secondary qualities and interpreted colors, sounds, etc., as secondary: “such qualities which in truth are nothing in the objects themselves but power to produce various sensations in us by their primary qualities, i.e. by the bulk, figure, texture, and motion of their insensible parts, as colours, sounds, tastes, &c. These I call secondary qualities” (Locke:1689).

René Descartes. As Galileo, Descartes believed that there are no colors in the physical world, as we ordinarily and naively understand them to be. In Descartes’ *Principles of Philosophy* we find this passage: “It is clear then that when we say we perceive colors in objects, it is really just the same as saying that we perceived in objects something as to whose nature we are ignorant, but which produces in us a very clear and vivid sensation, what we call the sensation of color”, (Descartes:1644 [1988]: paragraph 70).

Descartes observed that in the bodies that we define as colored, colors are nothing other than the various ways in which the bodies receive light and reflect it to our eyes. To understand why Descartes claimed that colors do not belong to objects “in themselves”, we need to look at his writings on physiology. In the *Optics* (Descartes:1637 [1897] Section 6, 130) he declared: “Regarding light and color [...] we must suppose our soul to be of such a nature that what makes it have the sensation of light is the force of the movements taking

place in the regions of the brain where the optic nerve-fibers originate, and what makes it have the sensation of color is the manner of these movements. But in all this there need be no resemblance between the idea which the soul conceives and the movements which cause these ideas”.

Johann Wolfgang von Goethe. He clearly and forcefully emphasized the difference that should be made between physical and physiological colors. Regarding physical colors, “The colors in question are thus produced in the eye through such external given causes or are merely reflected to the eye when by whatever means they are already given produced without us”, whereas physiological colors “belong altogether, or in a great degree, to the subject - to the eye itself” (Goethe:1970).

Arthur Schopenhauer. Strongly influenced by Goethe, Schopenhauer was a strong antagonist of what he calls *Newton's false doctrine* in which “colors are parts of a ray of light dispersed by refraction”, (Schopenhauer, *On vision and colors*, p. 75). He identified Goethe's physical colors, the divided ray of light, as the primary causes for the appearance of color sensations, and physiological colors as their effects: “The object is red means that it produces the color red in the eye” and “physiological colors are divided activity of the retina” (Schopenhauer:2010, p. 58).

Bertrand Russell. He not only embraced color irrealism, but he also incisively pointed out that the empirical color perception laws cannot be investigated without making reference to an experimental environment and an observing apparatus: “When, in ordinary life, we speak of the color of the table, we only mean the sort of color which it will seem to have to a normal spectator from an ordinary point of view under usual conditions of light. But the other colors which appear under other conditions have just as good a right to be considered real; and therefore, to avoid favoritism, we are compelled to deny that, in itself, the table has any one particular color”, (Russell:1912).

Probably inspired by his work on the theory of relativity and quantum mechanics, Weyl has expressed a very similar point of view: “The structure of our scientific cognition of the world is decisively determined by the fact that this world does not exist in itself but is merely encountered by us as an object in the correlative variance of subject and object”, (Weyl:1934).

C. L. Hardin. He proposed a peculiar version of color irrealism, which is known in the literature as *color eliminativism*: “Colored objects are illusions” (Hardin:1988, p. 111). Hardin claims: “We are to be eliminativists with respect to color as a property of objects” (Hardin:1988, p.112). One of his arguments in favor of color eliminativism is the following: we have no good reason to believe that there are non-physical bearers of color phenomena. We also have no good reason for claiming that color is exactly one physical property and not another. Colored objects would have to be physical or non-physical, so we have no good reason to believe that there are colored objects. Physical objects are not colored (i.e., they are neither reddish nor yellowish nor bluish nor greenish). Colored objects are illusions, but not unfounded illusions. We are normally in chromatic perceptual states, and these are neural states. Because perceptions of color differences and perceptions of boundaries are closely intertwined neural processes, we see colors and shapes together. Roughly speaking, as color goes, so goes visual shape. Consequently, “there are no visual shapes in the ultimate sense, just as there are no colors” (Hardin:1988, p.111). From this, Hardin concludes that color realism (especially, color physicalism as proposed, e.g., by Byrne and Hilbert) ultimately fails, “it fails because nothing in the domain of objects, properties and processes beyond our skins is both causally connected with our color experiences and models the essential characteristics of colors (Hardin:1984).

We find it pertinent to end this section with the description of color given by the painter Paul Klee in the quote “color is the place where our brain and the universe meet”.

2.3 Empiricism and nativism

During the last decades of the nineteenth century, there was a significant controversy in the field of physiological optics. This controversy has usually been characterized as a disagreement between two positions: empiricism and nativism. Each party had an important protagonist: Hermann von Helmholtz for empiricism and Ewald Hering for nativism. Empiricism gave more importance to the experiential history of the individual as fundamental in many aspects of our vision. Nativism, instead, intended to explain most visual perception phenomena with innate physiological features of our sense organs. The breakpoint in the controversy between empiricism and nativism was thought to turn on how much the eyes need, or depend, on a mind (or consciousness) to see. This corresponds, roughly, to the debate between color realism and color irrealism on the exact role assumed by the perceiver.

Hermann von Helmholtz. He conceived color sensations as signs of the external world that we then interpret and organize thanks to experience and memory “Those undulations which strongly affect our eyes, and which we call light, excite the impression of different colors, according to the length of the waves” (Helmholtz:1868, page 238) and (Helmholtz, 1868[1995], p. 177). In that sense, an observer has to make unconscious inferences [*unbewusster Schlüsse*], as Helmholtz called them, in order to organize her/his visual perception. These inferences are unconscious processes of association (or induction) using past sensations. (Helmholtz, 1866[1962] v. 3, p.26-27). Accordingly, Helmholtz makes a fine distinction between sensation and perception. Sensation occurs when external agents [stimuli] act on the sensory nerves, provided the nerves are alive and connected to the brain, which is the organ of consciousness (Helmholtz, 1866[1962], v. 2, p.1). Perception is the meaning attributed to sensations, a meaning which is inferred unconsciously from experience. “The sensations aroused by light in the nervous mechanism of vision enable us to form conceptions as to the existence, form and position of external objects. These ideas are called visual perceptions.” (Helmholtz, 1866[1962], v. 3, p.1). For Helmholtz, “[t]he fundamental thesis of the empirical theory is: The sensations of the senses are tokens for our consciousness, it being left to our intelligence to learn how to comprehend their meaning.” (Helmholtz, 1866[1962], v. 3, p. 533).

Ewald Hering. His theory is based upon an analysis of visual perception. Experience indicates that a conscious subject needs four unique colors in order to characterize perception: blue, yellow, red and green. Usually, two of these hues, but never more, constitute a color sensation, e.g., orange contains red and yellow while blue and green are the components of turquoise. The central idea in Hering's concept was that red and green are *opposite hues* because they are never elicited simultaneously by a color stimulus; the same is true for blue and yellow. In Hering's words: “The color circle shows how the four primary chromatic colors are interconnected by a series of transitional intermediate hues so that we distinguish a red-yellow, a yellow-green, a green-blue, and a blue-red series of intermediate chromatic colors or hues. There is, on the other hand, no series of red-green or yellow-blue intermediate hues, and thus no color whose hue seems similar or related simultaneously to primary red and primary green, or at the same time to primary yellow and primary blue. No color is clearly reddish as well as greenish, no color is both yellowish and bluish at the same time; redness and greenness as well as yellowness and blueness are mutually exclusive. From a color that is somewhat reddish we can arrive at a more or less greenish color through a continuous chromatic color sequence only by a detour through primary yellow or primary blue, and from a somewhat yellowish color to a bluish one there is a

continuous series of hues only by way of primary red or primary green" (Hering:1878, p. 49).

Hering also postulated that the perception of opponent colors is mediated by opponent processes involving certain elements of the nervous system. Today, we know that there are neurons that respond in a qualitatively different way to spectral stimuli of different frequencies, as correctly anticipated by Hering (Baumann:1992).

Hering observed the phenomenon of color by minimizing (as much as possible) the role of experience, postulating innate physiological mechanisms. His claim is a drastic change of paradigm with respect to all previous beliefs about color: "To do this we must at first disregard altogether the causes and conditions of their arousal. For a systematic grouping of colors, the only thing that matters is color itself. Neither the qualitative (frequency) nor quantitative (amplitude) physical properties of the radiations are relevant" (Hering:1878, p. 25).

More generally, nativists believe that the scope of what Helmholtz calls "perception" is not as wide as the empiricists defend: there is a lot of our visual experience that is directly given through sensation, thanks to the physiological features of our sensory nerves, and not learned.

3. A quantum-like theory of color perception

Wittgenstein's approach to color perception provides an important source of motivations for the quantum-like model that we will describe in subsection 3.2. This is why, before dealing with it, we introduce some compelling Wittgenstein's remarks.

3.1 Wittgenstein's' phenomenological and grammatical approach

In *Philosophical Remarks*, Ludwig Wittgenstein expressed his idea about a phenomenological theory of color by declaring: "What I need is a psychological or rather phenomenological color theory, not a physical and equally not a physiological one", (Wittgenstein:1975, p.218).

In proposing his theory, Wittgenstein adopted Goethe's phenomenological approach, which favored what is sometimes called "the natural image situation," i.e., the color image as it appears to perceivers outside laboratories. Furthermore, the theory should only be made up of what is actually perceptible and in which no hypothetical objects – e.g., electromagnetic waves, photoreceptors and all that – occur (Wittgenstein:1975, p.218). In *Remarks on Color*, the point is expressed more broadly: "As I mean it [a proposition about color], it can't be a proposition of physics", (Wittgenstein:1977, p.3).

Wittgenstein's next step was to identify phenomenology with grammar. His idea is that the regular structure of the possibilities of experience (i.e., phenomenology) cannot be distinguished from the regular structure of what can be meaningfully said (grammar). A phenomenological investigation of color involves the construction of a symbolism which represents colors perspicuously. This means that there must be a perfect correspondence between representation and what is represented (Noe:1994).

In "Some Remarks on Logical Form" (Wittgenstein:1929), Wittgenstein discussed the possible ways of analyzing color ascriptions (e.g., 'the lemon is yellow'). What is it that is ascribed when we say that something is "yellow"? Wittgenstein seemed to argue that color

ascriptions are elementary, i.e., irreducible to simpler parts. To back up his argument, he proposed the following reasoning: if we could identify color with the n wavelengths (expressed in discrete units of nanometers) that appear in the spectral composition of its stimulus, then there would be a problem: since any n includes also $n-1$, $n-2$, and so on, then when we would say (as an “elementary” proposition) that a color is described by n wavelengths, then it would also be described by any lower number of such wavelengths. This goes against what we commonly experience, hence Wittgenstein concluded that color ascriptions are elementary, i.e., irreducible to simpler parts.

However, the understanding of color may require that behind it there is something with even greater logical finesse: the geometry of color or the arithmetic of color (Westphal :2016). Wittgenstein claimed that “the geometry of visual space is the syntax of the propositions about objects in visual space”. In particular, he found the Newtonian concept of “pure color” particularly hard to grasp: “The word whose meaning is not clear is pure or saturated. How do we learn its meaning? How can we tell if people mean the same thing by it?” (Wittgenstein:1977).

He characterizes the phenomenological investigation of, e.g., visual space or color, as a kind of geometrical investigation of a grammatical structure – where the structure of the domain of grammar concerns the description of visual experience. Wittgenstein's thought here is that the grammatical relations of the between color-terms can be represented via the color space. The points are the pure colors, and the space in between corresponds to permissible transitions.

We are now going to show that, within the quantum-like model of color perception, the concept of pure color, together with all the other color attributes, can be defined in a mathematical vocabulary that satisfies the demand for linguistic rigor formulated by Wittgenstein.

3.2 The quantum-like model

According to Hardin, “Virtually all of the published philosophical discussions of colors [...] were written before 1970. As a result, a gamut of philosophers ranging from Armstrong to Wittgenstein have written in ignorance of important physiological links between the physical and phenomenal aspects of color”, and “it is time for a new look at color, taken from the perspective of the opponent process theory” (Hardin:1984).

Hardin’s wish should be accomplished together with the reconciliation between physical and physiological colors, which is a difficult task because, as mentioned before, physical colors are supposed to describe color sensations from the sole point of view of causes that create these sensations, while in Hering’s view physiological colors disregard a priori the causes and conditions of color sensation arousal.

In what follows, we explain, starting from an axiomatic approach, how both these tasks can be achieved by adopting a quantum information point of view about color. Before giving an overview of the quantum model, we underline that, from a philosophical point of view, the quantum interpretation of the microscopic world bears a strict resemblance with color irrealism, as remarkably pointed out by Dirac: “Science is concerned only with observable things and that we can observe an object only by letting it interact with some outside influence. An act of observation is thus necessary accompanied by some disturbance of the object observed”, and also: “Questions about what decides the photon’s direction of

polarization when it does go through cannot be investigated by experiment and should be regarded as outside the domain of science”, (Dirac:1930).

A quantum theory of color perception has been proposed and developed in a series of papers (Berthier & Provenzi:2019), (Berthier:2020), (Provenzi:2020), (Berthier & Provenzi:2021), (Berthier & Garcin & Prencipe & Provenzi:2021), (Berthier & Provenzi:2022), (Berthier & Prencipe & Provenzi:2022). The model is characterized by a high degree of mathematical abstraction and the interested reader can consult the papers just quoted for more details. Here we will avoid the mathematical details and present only the key information about the theory that permit to discuss its philosophical position with respect to the color theories discussed in section 2.

To clarify the context in which our study takes place, it may be useful to recall the key concepts involved when studying quantum systems. A system is by definition a setting where one can perform measurements giving rise to quantitative results in conditions that are as isolated as possible from external influences. The observables of the system are the objects of measurements, and the states of the system are the ways in which this one can be prepared for the measurement of its observables. The result of a measurement is the expectation value of an observable in a given state, i.e. the average result of multiple measures of the observable conducted in the physical system prepared in the same state.

We outline now the main mathematical steps of the reasoning that leads to the quantum-like model of color perception. The starting point is the remarkable work about color space performed by Schrödinger (Schrödinger:1920). His idea was to endow the set of all color sensations, indicated from now on with C , with a system of mathematical properties and to study C not as a simple set, but as a mathematically structured space. Clearly, more properties we require C to satisfy, less possibilities we have. In theory, if we demand the right amount and type of properties we can end up with a single possibility. Of course, these properties must be coherent with the largely accepted features of color perception established by experiments. Schrödinger selected the features exhibited by Newton, Maxwell, Grassmann and Helmholtz and added one feature himself. The result, in mathematical terms, is that C is a 3-dimensional regular convex cone which, roughly speaking, can be imagined as a single cone with a vertex, in which any two points inside it can be connected with a line segment entirely contained in C . However, these properties are not sufficient to determine C precisely. Moreover, they do not take into account at all color opposition.

H.L. Resnikoff (Resnikoff:1974) completed, mathematically, the program initiated by Schrödinger by adding a final request on C , called homogeneity, which can be described as the fact that no color is special with respect to the others. With the full set of requests, Resnikoff obtained a remarkable classification: there exist only two non-equivalent models for C , the first turned out to be the one established by the CIE (Commission International de l'Éclairage) and the second was a completely novel color space which showed much more possible to combine the analysis of these two options through the use of the so-called Jordan algebras, which were introduced in 1932 by Jordan in an attempt to provide an algebraic formalism for quantum observables.

The crucial step that allows to recast Resnikoff's second option in a quantum-like framework, which Resnikoff missed, consists in applying Koecher-Vinberg theorem which states precisely that C is equivalent to the set of positive observables of a so-called spin factor. The corresponding system describes a “rebit”, a quantum system over the real numbers which can be measured in only two states. Thus, starting from Schrödinger's and Resnikoff's contributions, based only on the trichromatic aspect of color vision, a purely mathematical derivation leads to investigate color vision from the point of view of the rebit system.

The reconciliation between trichromacy and opponency is provided by the description of the state space of the rebit, or equivalently of the set of its density matrices which is represented by the unit disk of the real plane. Density matrices can describe both pure and mixed states and a fine descriptor for the degree of purity or mixedness of a state is the von Neumann entropy. This quantity is the expectation gain of information after an experiment is performed on a system prepared in a given state. If we already know everything about the state, then no experiment can provide us any further information, hence the von Neumann entropy will be zero and the state will be called “pure”. On the contrary, the maximally mixed state is the one for which any experiment gives the highest possible degree of information on the system. Between pure states and the maximally mixed state we find states with intermediate and monotonically increasing von Neumann entropy. Thus, this paramount important mathematical concept is what permits to characterize purity in the quantum-like color perception theory.

The “Bloch representation” of density matrices permits to write a generic density matrix as the sum of the density matrix describing the chromatic state with maximal von Neumann entropy, and two other terms in which there appear the difference between density matrices parameterized by antipodal points of the disk. The first density matrix is associated to the achromatic color sensation, while antipodal density matrices can be rigorously proven to be associated to orthogonal, mutually incompatible, chromatic states. It is thus natural to consider these states as opponent in the sense intended by Hering.

As a consequence, the trichromatic nature of color leads to a mathematical description of the color space C that, when interpreted using a quantum paradigm, leads to an equivalent description which turns out to be precisely adherent to Hering’s opponent theory. It is important to underline that Hering’s opposition emerges naturally from the sole quantum interpretation, without any ad hoc manipulation.

The only other analysis we know of in which Hering’s description of perceived color is treated from a purely theoretical point of view is contained in the very interesting paper of Buchsbaum and Gottshalk (Buchsbaum & Gottshalk:1983). In that contribution, the authors construct a chromatic covariance matrix from the L,M,S output of the cones and, in order to be able to perform explicit calculations on it, they introduce a Dirac-delta covariance function, which, as the authors observe, is a realistic hypothesis only if the visual stimuli are monochromatic. With this choice, the chromatic covariance matrix has all positive entries and this allows them to use the celebrated Perron-Frobenius theorem to assure that the eigenvectors of the chromatic covariance matrix are divided in two classes: one eigenvector has all positive components, while the other two show opponency. In spite of the elegance of this analysis, it is important to underline that signals coming from real scenes are broadband, so there is no theoretical guarantee that the chromatic covariance matrix will have all positive entries in that case. For more information about these issues, the interested reader can consult for instance (Provenzi & Delon & Gousseau & Mazin:2016).

To understand how physical and physiological colors can be linked another quantum concept must be introduced: the “effect”. Physicists worked with (classical) states for centuries before the development of quantum mechanics, instead the concept of effect is a relatively young one, having been introduced in the late sixties of the twentieth century. Without entering in complicated details, we can say that effects are the mathematical translation of the probabilistic nature of measurement results in quantum theories. The difference between preparations (states) and measurements (effects) has been very

incisively described in (Auletta & Fortunato & Parisi:2009) as follows: “A measurement is a process through which we ask the system about the value of a certain observable, the measurement outcome. Obviously, the answer of the system is not controllable by the experimenter. In other words, a preparation is determinative, while a measurement is interrogative”.

The crucial point is that the space C has an intrinsic property, called “self-duality” which implies that neither states nor effects can alone account for a precise description of perceived colors, but they must always be considered in “duality”. In other words, within the quantum color model, perceived colors are the results of measurements, represented by effects, on suitable preparations of a visual scene, represented by states. This is how physical and physiological colors are linked together in the quantum color model.

Let us finally mention that this model permits to obtain rigorous and well-sounded mathematical definitions for the usual perceptual attributes associated to a perceived color: brightness, lightness, colorfulness, chroma, saturation and hue, and to precisely reformulate the so-called lightness constancy phenomenon using the quantum information measurements describing color perception. This is a first hint of the coherency of the proposed system of definitions, see (Berthier & Prencipe & Provenzi, 2022) for the details.

4. Conclusion

The mathematical model of color perception described in section 3 permitted to build a coherent system of definitions of color sensations in (Berthier & Prencipe & Provenzi, 2022). This provides an answer to Wittgenstein’s concerns about the importance of grammar and language in color description.

Furthermore, in this model a color sensation is described as the result of a perceptual measurement. The elements involved in this measurement process are, on one side, the visual stimulus coming from the preparation (in the quantum sense) of a visual scene and, on the other side, the human visual system of an observer, considered as a (perceptual) measurement apparatus. In this sense, this model could, in principle, be compatible with color relationalism, a view defended for example by Cohen (2009), with the idea that colors are defined by relations between perceivers and physical properties. Moreover, the model is also coherent with Hardin’s viewpoint since the chromatic opposition, on which the model relies, is a property of the measurement apparatus and not of the stimulus.

The point of view of the quantum model of color perception is hardly compatible with a purely realistic interpretation of color. It is much better suited to modern versions of color irrealism, such as Maund’s illusionism and Averill’s projectivism and especially, as it was slightly suggested in section 3.2., Hardin’s eliminativism.

Besides the need to consider both states and effects, or equivalently, to consider both the visual scenes and the observers, and the duality or coupling between them, we would like to emphasize, following Jordan, that “Observations not only disturb what has to be measured, they produce it... We ourselves produce the results of measurements” (Mermin:1985). Colors are thus produced by the observer as a result of measurement. Furthermore, Dirac’s quote above: “Questions about what decides the photon’s direction of polarization when it does go through cannot be investigated by experiment and should be regarded as outside the domain of science” (Dirac:1930), reformulated in our context as “Questions about what decides the color an object takes cannot be investigated by experiment and should be regarded as outside the domain of science” tends to confirm that color science is first and foremost, and only, the science of observed colors.

As Niels Bohr once expressed “Nothing exists until it is measured”. The quantum paradigm shifts the question of describing colored sensations into the more general and widely debated problem of understanding the mechanisms underlying the act of measurement.

Acknowledgement

We would like to thank the anonymous reviewers for their valuable comments and suggestions, which helped us to improve the preliminary version of this paper.

Ethical Approval

Not applicable.

Funding

No funding has been received for this work.

Availability of data and materials

Not applicable.

References

- Auletta, Gennaro & Fortunato, Mauro & Parisi, Giorgio (2009). *Quantum mechanics*. Cambridge university press.
- Averill, Edward Wilson (2005). Toward a Projectivist Account of Color. *Journal of Philosophy* 102 (5): 217-234.
- Averill, E., & Hazlett, A. (2011). Color objectivism and color projectivism. *Philosophical Psychology* 24 (6):751-765.
- Baumann C. (1992). “Ewald Herings Gegenfarben. Geschichte einer Idee” [Ewald Hering's opponent colors. History of an idea]. *Ophthalmologie*. 89(3): 249-52.
- Berthier, Michel & Provenzi, Edoardo (2019). When geometry meets psycho-physics and quantum mechanics: Modern perspectives on the space of perceived colors. *Lecture Notes in Computer Science*, p. 621-630, Springer Berlin-Heidelberg.
- Berthier, Michel (2020). Geometry of color perception. Part 2: perceived colors from real quantum states and Hering's rebit. *The Journal of Mathematical Neuroscience*, vol. 10(1):1-25.
- Berthier, Michel & Garcin, Valérie & Prencipe, Nicoletta & Provenzi, Edoardo (2021). The relativity of color perception. *Journal of Mathematical Psychology*. Vol 103,102562.
- Berthier, Michel & Provenzi, Edoardo (2021). From Riemannian trichromacy to quantum color opponency via hyperbolicity. *Journal of Mathematical Imaging and Vision*, vol 63(6):681–688. Springer.
- Berthier, Michel & Provenzi, Edoardo (2021). The Quantum Nature of Color Perception: Uncertainty Relations for Chromatic Opposition. *Journal of Imaging*, vol 7(40). MDPI.

- Berthier, Michel & Provenzi, Edoardo (2022). Quantum measurement and colour perception: theory and applications. *Proceedings of the Royal Society A*. Vol. 478, 2258, 20210508.
- Berthier, Michel & Prencipe, Nicoletta & Provenzi, Edoardo (2022). A Quantum Information-Based Refoundation of Color Perception Concepts. *SIAM Journal on Imaging Sciences*. Vol.15(4), 1944-1976.
- Boghossian, Paul A. & Velleman, J. David (1989). Color as a secondary quality. *Mind* 98 (January): 81-103.
- Buchsbaum, G., Gottschalk, A. (1983). Trichromacy, opponent colours coding and optimum colour information transmission in the retina. *Proc. Royal Society of London B* 220: 89-113.
- Byrne, Alex & Hilbert, David R. (2003). Color realism and color science. *Behavioral and Brain Sciences* 26 (1): 3-21.
- Byrne, Alex & Hilbert, David R. (2021). Objectivist reductionism. In Fiona Macpherson & Derek Brown (eds.), *Routledge Handbook of the Philosophy of Color*. London: Routledge.
- Cohen, Jonathan D. (2009). *The Red and the Real: An Essay on Color Ontology*. Oxford, GB: Oxford University Press UK.
- Descartes, René (1644) [1988], *Principles of Philosophy*, reprinted in John Cottingham, Robert Stoothoff, Dugald Murdoch, and Anthony Kenny (eds.), *Descartes: Selected Philosophical Writings*, Cambridge: Cambridge University Press, pp. 160–212.
- Descartes, René (1637), "Optics", in Adam, Ch. & Tannery, Paul (1897). *Oeuvres de Descartes*. J. Vrin.
- Dirac, Paul Adrien Maurice (1930). *The principles of quantum mechanics*. In No. 27 in international series of monographs on physics (4th ed.). Oxford: Oxford University Press.
- Goethe, Johann Wolfgang von (1970). *Theory of color*. The MIT Press.
- Grassmann, Hermann (1853). Zur theorie der farbenmischung. *Annalen Der Physik*, 165(5), 69–84.
- Guthrie, W. K. C. (1962). *A History of Greek Philosophy*. Cambridge: Cambridge University Press.
- Hering Ewald (1878), *Outline of a theory of the light sense*, p.49. Translated by Leo M. Hurvich and Dorothea Jameson, Harvard University Press, Cambridge, Massachusetts, 1964.
- Hardin, Clyde Laurence (1984). A new look at color. *American Philosophical Quarterly* 21 (2): 125-133.
- Hardin, Clyde Laurence (1988). *Color for Philosophers: Unweaving the Rainbow*. Hackett.
- Helmholtz, Hermann von. (1868). The recent progress of the theory of vision. Trans. by E. Atkinson, New York: D. Appleton and Company, 1873: 127-203.

- Helmholtz, Hermann von. *Treatise on Physiological Optics*, Vol. 1 (1855), Vol. 2 (1860), Vol. 3 (1866). Trans. James P. Croke, Mineola (New York): Dover Publications Inc, 1962 and repr. 2005.
- Helmholtz, Hermann von. *Treatise on Physiological Optics*, Vol. 1 (1855), Vol. 2 (1860), Vol. 3 (1866). Trans. James P. Croke, Mineola (New York): Dover Publications Inc, 1962 and repr. 2005.
- Hering, Ewald (1878). *Outlines of a theory of light sense*. Trans. by Leo M. Hurvich and Dorothea Jameson, Cambridge (Mass): Harvard University Press, 1872/1964.
- Hilbert, David R. (1987). *Color and Color Perception: A Study in Anthropocentric Realism*. Csl Press.
- Hyman, John (2006). *The Objective Eye: Color, Form, and Reality in the Theory of Art*. University of Chicago Press.
- Jackson, Frank & Pargetter, Robert (1987). An objectivist's guide to subjectivism about color. *Revue Internationale de Philosophie* 41 (1):127-141.
- Kalderon, Mark Eli (2015). *Form without Matter: Empedocles and Aristotle on Color Perception*. Oxford University Press.
- Lewis, David, (1997). Naming the colors. *Australasian Journal of Philosophy* 75: 325–42.
- Locke, John (1689). *An Essay concerning Human Understanding*. Edited by R. Woolhouse. London: Penguin.
- Maund, Barry (2006). The Illusory Theory of Colors: An Anti-Realist Theory. *Dialectica* 60 (3):245-268.
- Maund, Barry (2011). Colour eliminativism. In Lawrence Nolan (ed.), *Primary and Secondary Qualities: The Historical and Ongoing Debate*. Oxford University Press.
- Mahon, Basil (2003). *The man who changed everything*. Wiley.
- McLaughlin, Brian P. (2003). Color, consciousness, and color consciousness. In Quentin Smith & Aleksandar Jokic (eds.), *Consciousness: New Philosophical Perspectives*. Oxford University Press: 97-154.
- Mermin, David (1985). Is the moon there when nobody looks? Reality and the quantum theory. *Physics Today*, April 1985: 38-47.
- Newton, Isaac (1704). *Opticks*. Pantianos Classics 2017 edition.
- Noë, Robert Alva (1994). Wittgenstein, phenomenology and what it makes sense to say. *Philosophy and Phenomenological Research* 54 (1):1-42.
- Pasnau, Robert (2007). Democritus and secondary qualities. *Archiv für Geschichte der Philosophie* 89 (2): 99-121.
- Peruzzi, Giulio and Roberti, Valentina (2023). Helmholtz and the geometry of color space: gestation and development of Helmholtz's line element, *Archive for History of Exact Sciences*, 77, 2, 201-220.

- Provenzi, E., Delon, J., Gousseau, Y., & Mazin, B. (2016). On the second order spatiochromatic structure of natural images. *Vision research*, 120, 22-38.
- Provenzi, Edoardo (2020). Geometry of color perception. Part 1: Structures and metrics of a homogeneous color space. *The Journal of Mathematical Neuroscience*. Vol.10(1), 1-19. Springer.
- Resnikoff, Howard L. (1974). Differential geometry and color perception. *Journal of Mathematical Biology*, Vol.1, 97-131.
- Russell, Bertrand (1912). *The problems of philosophy*. Enhanced media 2016.
- Schopenhauer, Arthur (2010). *On vision and colors*. Princeton Architectural Press.
- Schrödinger, Erwin (1920). Outline of a theory of colour measurement for daylight vision. *Annalen der Physik*. Vol.6(4), p. 397-456; 481-520.
- Tye, M. (2000). *Consciousness, Color, and Content*. Cambridge MA: MIT Press.
- Turner, Steven (1994). *In the Eye's Mind: Vision and the Helmholtz-Hering Controversy*. Princeton University Press.
- von Helmholtz, Hermann (1852). Über die Theorie der zusammengesetzten Farben. *Annalen der Physik*, 163(9), 45-66.
- Westphal, Jonathan (2016). Wittgenstein on Color. In *A Companion to Wittgenstein* (eds H.-J. Glock and J. Hyman).
- Weyl, Hermann (1934). *Mind and nature, selected writings on philosophy, mathematics, and physics*. Princeton University Press 2009.
- Wittgenstein, Ludwig (1929). Some Remarks on Logical Form. *Aristotelian Society*, Supplementary Volume 9 (1): 162 - 171.
- Wittgenstein, Ludwig (1975). *Philosophical Remarks*. Oxford, England: University of Chicago Press.
- Wittgenstein, Ludwig (1977). *Remarks on colour*. Blackwell publishing.