



HAL
open science

A literature review of dense crowd simulation

Huu-Tu Dang, Benoit Gaudou, Nicolas Verstaevel

► **To cite this version:**

Huu-Tu Dang, Benoit Gaudou, Nicolas Verstaevel. A literature review of dense crowd simulation. *Simulation Modelling Practice and Theory*, 2024, 134, pp.102955. 10.1016/j.simpat.2024.102955. hal-04573057

HAL Id: hal-04573057

<https://hal.science/hal-04573057v1>

Submitted on 13 Sep 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License



A literature review of dense crowd simulation

Huu-Tu Dang*, Benoit Gaudou, Nicolas Verstaevl

UMR 5505 IRIT, Université Toulouse Capitole, Toulouse, France

ARTICLE INFO

Keywords:

Crowd simulation
High density
Multi-level modeling

ABSTRACT

Pedestrian modeling and simulation has become an interesting topic that has gained scientists' interest over the past few decades. While this field has yielded significant achievements in various applications, questions have been raised regarding the applicability of simulations in high-density scenarios. To this purpose, this paper comprehensively reviews pedestrian simulation models specifically focused on high-density situations. The review examines a total of 116 articles and categorizes their approaches for modeling pedestrian behaviors to different decisional levels. The strengths and limitations of these modeling approaches are compared and evaluated using different criteria for dense crowd simulations, such as their ability to simulate common emergent behaviors in crowded situations, performance, validation, and capacity to integrate into high-level modeling. Finally, the review provides potential directions for future research and development of dense crowd simulations.

1. Introduction

Over the past few decades, pedestrian modeling and simulation has become an interesting subject that has captured the attention of scientists. The field has achieved remarkable advancements and has found applications in various domains such as urban planning [1], computer graphics [1,2], evacuation [3], safety science [4], etc. In the early 2000s, several influential models were proposed to simulate pedestrian locomotion, ranging from microscopic [5–7] to macroscopic models [8,9]. Inspired by these pioneering approaches, researchers have been putting a lot of work into developing new techniques as well as variation models to simulate pedestrians in more complex situations, especially in high-density scenarios. Such scenarios are characterized by restricted movement of most pedestrians due to limited available space and more complex interactions among individuals [10]. For example, during mass-gathering events such as festivals or crowded metro stations at peak times, pedestrians tend to move slower and need a strategic plan to navigate through dense crowds [11].

Modeling techniques encounter considerable challenges when they are used to simulate pedestrians in crowded environments. Compared to regular situations, pedestrians behave differently in dense scenarios [12]. Pedestrians' behaviors are strongly influenced by their level of personal space, which is severely constrained by neighbors in dense situations. Consequently, new patterns of pedestrian behavior emerge in response to dynamic environments. In addition, the validation of dense crowd models is highly challenging due to several reasons, such as the fact that the majority of models were calibrated for low-density situations [13]. Empirical data for high-density scenarios is still missing, primarily because of the difficulties in accurately extracting dense crowd data from recordings of real-world events [14]. Additionally, laboratory data obtained through controlled experiments frequently concentrates on investigating single behaviors or phenomena in high-density settings [12].

While numerous reviews on pedestrian modeling exist in the literature, they lack a focus on dense crowds. The details of these reviews are described in the next section. In comparison, our review provides the following novel contributions: (1) a comprehensive

* Corresponding author.

E-mail address: huu-tu.dang@ut-capitole.fr (H.-T. Dang).

<https://doi.org/10.1016/j.simpat.2024.102955>

Received 3 October 2023; Received in revised form 3 May 2024; Accepted 7 May 2024

Available online 11 May 2024

1569-190X/© 2024 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

analysis in methodologies of pedestrian simulation models ranging from classical approaches to their subsequent modifications for simulating different characteristics of dense crowds at different decisional levels, (2) a comparison of these models across multiple criteria, including model performance, adaptability to high-density situations, and extensibility to multi-level modeling, (3) an assessment of advantages and disadvantages of these models in application to dense crowd simulation, and (4) an outline of open research questions alongside proposed future directions for simulating dense crowds.

The remainder of this paper is organized as follows. Section 2 provides a review of related works and discusses the current gap in comprehensive analyses focused on high-density studies. In Section 3, the main concepts are introduced to encompass the most common phenomena typically observed in crowded environments. Section 4 describes the methodology for collecting related articles and classifying them according to the proposed criteria. A statistical overview of the results is then provided to summarize recent trends in dense crowd simulation research. In Section 5, existing methods for simulating dense crowds at different levels of behavior [15] are examined and compared, highlighting the advantages, drawbacks, and potential development directions for each method. Section 6 discusses future directions that could be developed in modeling pedestrians in high-density scenarios. Finally, the paper ends with a conclusion and provides some perspectives in Section 7.

2. Related work

Many reviews have been carried out to characterize and synthesize pedestrian modeling techniques. A critical assessment of modeling techniques for pedestrian behavior in metropolitan zones was proposed by Papadimitriou et al. [16] focusing on describing route-choice and crossing behavior. The study brought attention to the fact that there was a deficiency in modeling approaches that were capable of simulating pedestrian behaviors over whole trips in urban settings. Nicola and Christian [17] have provided a detailed analysis of different mathematical models for macroscopic-scale simulation of crowd dynamics. In their review, Zhou et al. [18] have presented abstract criteria based on model capability and performance to evaluate crowd modeling techniques. A short history of pedestrian modeling has been introduced in the review presented by Helbing and Johansson [19]. The study has explored the challenges of modeling the self-organized behavior of pedestrians, as well as the panic that happens during evacuations, and the way that social force-based models could tackle these difficulties. A study has been conducted by Cheng et al. [20] with the purpose of summarizing microscopic approaches that could reproduce the group behavior of pedestrians. Ijaz et al. [21] have carried out a survey to analyze hybrid techniques used for crowd simulation. Surveys on experimental data and empirical methods could be found in [12,22]. The majority of collected empirical data is found to have been obtained under normal circumstances, while only a smaller portion originates from emergency and panic scenarios. Almoaid et al. [23] have performed a survey to investigate modeling techniques that have been applied to simulate pedestrians in the Hajj event — the biggest mass gathering event in the world. The result reveals that the microscopic methods were mostly used to address this problem, and the maximum number of simulated pilgrims was often much lower than the actual number of participants at the event. Yang et al. [24] have conducted a survey to update pedestrian modeling techniques in the last decade. Musse et al. [25] have presented the evolution of crowd simulation across different periods through a huge set of articles collected over the past three decades. Similarly, van Toll and Pettre [26] have examined the advancements of the principal approaches that have emerged or expanded within the last decade, with a particular focus on the microscopic paradigms. A comprehensive review of data collection methods for pedestrian behaviors was presented by Feng et al. [14]. Lemonari et al. [27] have examined modeling techniques and available authoring tools and software for various aspects of authorable crowd simulation, including high-level behaviors such as strategy and path planning as well as low-level behaviors like local avoidance, animation, and visualization. Zhong et al. [28] have conducted a survey of various calibration and validation techniques for simulation models using empirical pedestrian datasets. Korbmacher and Tordeux [29] performed a detailed comparative analysis of deep learning algorithms and traditional knowledge-based models used in simulating and predicting pedestrian trajectories. Their key finding pointed out that the deep learning algorithms surpassed the traditional models in terms of distance-based accuracy. Basu et al. [30] have conducted a systematic review to analyze various factors influencing pedestrian route choice behavior. They have classified these factors into three groups, including pedestrian socio-demographic characteristics, environmental information, and trip characteristics. Recently, Alexandre and Fadratul [31] investigated the impact of social groups on global crowd dynamics and reviewed various techniques to model these groups.

Although numerous reviews and surveys have been conducted to review pedestrian modeling techniques as mentioned above, **most of them focus on the characteristics of simulation models in normal situations rather than high-density scenarios.** Recent literature lacks systematic reviews published since 2021 that concentrate specifically on modeling methodologies for simulating dense crowd characteristics. For more details, Musse et al. [25] provided a historical overview of new methods emerging in each period over more than three decades without detailing specific techniques. Van Toll and Pettre [26] provided a comprehensive and valuable review on popular microscopic model techniques for crowd simulation at various density scenarios; yet there is a need for further quantitative comparison and evaluation of the performance of these algorithms. Korbmacher and Tordeux [29] mainly focused on comparing application domains, technical aspects, and distance-error accuracy between knowledge-based models and neural network models. Zhong et al. [28] only focused their survey on calibration and validation methods for crowd models, as well as a deep discussion of what these models learn from empirical datasets. Basu et al. [30] mainly investigate the modeling of route choice high-level of pedestrians, modeling local movement and interactions have not been considered in this review. While Nicolas and Hassan [31] reviewed the influence of social groups in different crowd densities and methods to model such groups, it is important to recognize that social groups represent only one dimension within the broader scope of modeling dense crowd dynamics. To the best of our knowledge, the comprehensive review from Dorine et al. [13] conducted in 2013 is the only one addressing both low- and high-density crowd simulations. However, there is a need to examine a larger number of recent

articles, as well as the detailed capabilities and performance of these models in dense situations. To this purpose, we have conducted a review of recent trends and techniques used for dense crowd simulation. As various well-known pedestrian simulation approaches have emerged in the previous two decades, our review primarily focuses on publications from January 1, 2000 to December 31, 2023.

3. General concepts

This section presents the primary concepts used in dense crowd simulations and highlights their importance. A glossary of other related terminologies that are typically used can be found in [13,19,32].

Density level

In pedestrian modeling and simulation, *density level* refers to the number of pedestrians within a space unit. A space becomes high-density when the number of pedestrians per space unit exceeds a certain threshold. While there are various definitions of high-density for pedestrian crowds, this contribution defines high-density based on pedestrian movement conditions. More specifically, density is defined as high if it exceeds 2.0 ped/m^2 . This definition aligns with that in [10], where such densities correspond to restrictions for all pedestrian movements. However, it is crucial to acknowledge that different crowd phenomena emerge at different high-density levels. For instance, crowd flow starts to decrease at densities higher than 2.8 ped/m^2 , stop-and-go waves occur at densities above 4.3 ped/m^2 [33], and crowd turbulence arises at densities surpassing 8.5 ped/m^2 [33]. Therefore, modeling emergent phenomena corresponding to each density interval is also essential to improve the realism of simulations. In some real-world scenarios, exceptionally high density can reach up to 9 ped/m^2 [34].

Environment

Environment refers to the spatial settings where pedestrian agents are located and navigate at each simulation step [35]. The environment can be two-dimensional, ranging from simple corridors to shapefiles or more complex like three-dimensional spaces.

Physical interaction

Physical interactions refer to physical contact or collisions that occur between pedestrians. This behavior is more likely to happen in high-density crowds where individuals are close to each other. Physical interactions can have a significant impact on pedestrian behavior and crowds, especially in extremely high density, which can lead to crowd turbulence [34].

Following behavior

The following behavior, also known as leader–follower behavior, is the tendency of pedestrians to follow others in front of them. In dense scenarios, individuals often lack sufficient personal space to walk freely at their preferred velocity and do not have a clear path to their destination, which may require them to follow their predecessors to navigate through the crowd [36].

Group behavior

Group behavior refers to pedestrians' behaviors when they are walking in social groups (e.g., friends or family). The formation and dynamics of these groups can be influenced by factors such as the number of group members and the surrounding environment [37]. It is essential to model group behaviors in crowd simulation, especially in mass-gathering real-world events as most participants are in groups [20,37].

Replan

A *replan* behavior of pedestrians refers to the ability of pedestrians to adjust their planned route in response to changes in their surrounding environment such as crowd dynamics. In crowded environments, individuals may need to replan their route in order to avoid potentially congested areas as the shortest path may not be the fastest path [38].

Multi-level behavior

A pedestrian's behavior includes several levels depending on their objective, motivation, and environment. This paper uses Hoogendoorn's description for categorizing pedestrian behaviors [15]:

- **Strategic level:** pedestrians determine a list of activities (or targets) and when they want to perform these activities.
- **Tactical level:** pedestrians choose a path to the predefined destinations based on information about the environment.
- **Operational level:** pedestrians adjust their local movements, such as collision avoidance, to adapt to the surrounding area.

The three levels of pedestrian behaviors are important for providing a comprehensive understanding of pedestrian behaviors as they take into account both long- and short-term decision-making, not just only immediate reactions to the environment.

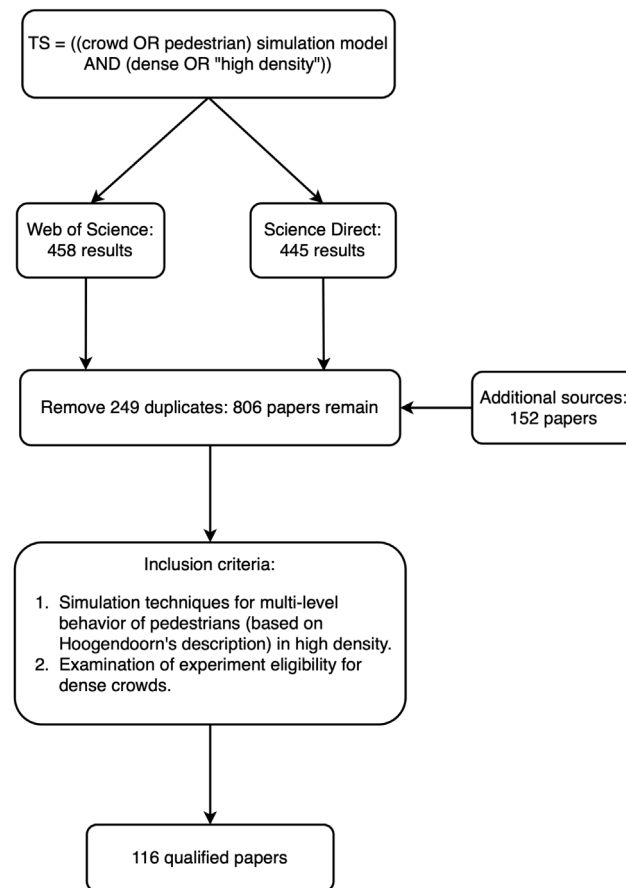


Fig. 1. Diagram for the systematic review on dense crowd simulation.

4. Methodology

4.1. Literature collection

To gather a comprehensive collection of articles, a systematic review was conducted by searching the Web of Science and Science Direct databases. A set of relevant keywords, “TS = ((crowd OR pedestrian) simulation model AND (dense OR high-density))” (as seen in Fig. 1), was strategically selected to reflect our primary focus on high-density simulation of crowds. The terms “crowd OR pedestrian” were used to ensure the capture of all possible studies about pedestrian crowds, while the inclusion of “simulation model” focused on the use of models to simulate pedestrian behaviors. Lastly, the terms “dense OR high density” were used to pinpoint the particular context of high-density situations, whereas the “AND” operator ensured every key aspect of our focus was presented in the resulting literature. The selected keywords were queried in January 2023 to find their appearance in the titles and abstracts of journal articles and proceeding papers published from 2000 to 2022. This yielded 458 results in the Web of Science and 445 results in Science Direct. During the review process, an additional 152 articles were identified from the references of the original articles as well as recent publications in 2023. After removing 249 duplicates, the screening method was performed on a total of 806 articles.

To be included in our review, each article must meet specific criteria for assessing the quality of the modeling techniques used to simulate pedestrian behavior in crowded environments. The assessment was based on Hoogendoorn’s description of three levels of pedestrian behavior [15]. Each article must describe modeling techniques that pertain to at least one of the three levels described above, with a particular emphasis on the characteristics of dense crowds. Additionally, experiments must be able to specify frequent phenomena of dense crowds, such as high levels of density, physical interactions of pedestrians, and density-dependent behavior, among others. Our inclusion criteria eventually matched 116 articles; however, the reference list contains a greater number of studies to provide additional context, include other review articles, and support our arguments.

Table 1
Categories of modeling techniques for crowd simulation.

Levels of behavior	Modeling techniques
Strategic	–
Tactical	Graph-based [39–43] Navigation mesh [44] Potential field [45–48]
Operational	Forced-based models [5,49] Velocity Obstacle models [50,51] Cellular Automata models [6,7] Agent-based models [52,53] Data-driven models [54,55] Macroscopic models [8,9,56] Hybrid models [57–59]

4.2. Results

After the collection process, each paper was categorized into one of the three modeling levels [15]. Table 1 presents the mainstream modeling techniques for each level. Graph-based [39–43], navigation mesh [44] and potential field [45–48] are the main approaches for the tactical level modeling, while Social Force model (SFM) [5,49], Cellular Automaton (CA) model [6,7], Velocity Obstacle (VO) model [50,51], Agent-based model (ABM) [52,53], data-driven model [54,55], macroscopic model [8,9,56], and hybrid model [57–59] are normally used to simulate pedestrian crowd at the operational level.

The majority of papers explore modeling techniques for the operational level of pedestrians, whereas only a few articles concentrate on strategic level modeling (as shown in Fig. 2a). It is important to note that while an article might relate to multiple levels of modeling including strategic, tactical, and operational, its primary contribution is typically associated with one level. The reason for this highlight is that collision avoidance is a key challenge in crowd simulation and it belongs to the operational level.

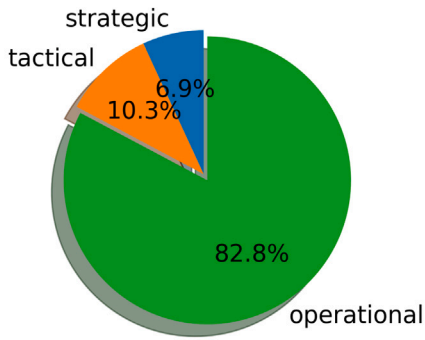
Furthermore, the experiments from the finalized papers were extracted and classified into several main types: evacuation [32], bottleneck [13,32], bidirectional [13], Hajj-Tawaf [4], and crossing [13] (as shown in Fig. 2b). The most popular experiments are simulations of pedestrians during evacuation and at bottlenecks, whereas the modeling of dense crowds at the Hajj-Tawaf [4] – a mass gathering religious event – presents an intriguing real-world case study.

Fig. 2d shows the number of publications per year for the finalized articles dataset. The number of publications related to high-density pedestrian simulation has generally exhibited an increasing trend over the past two decades, with a significant increase during the second half of this period. One of the primary factors contributing to this trend is the emergence of numerous well-known deep learning models in recent years for predicting human trajectory with better accuracy compared to other simulation models [60]. Fig. 2c draws the word clouds from the abstract and author keywords. The terms “pedestrian”, “crowd simulation”, and “dynamic” are frequently used along with “high density” and “evacuation” which indicate a focus on dense crowd simulation.

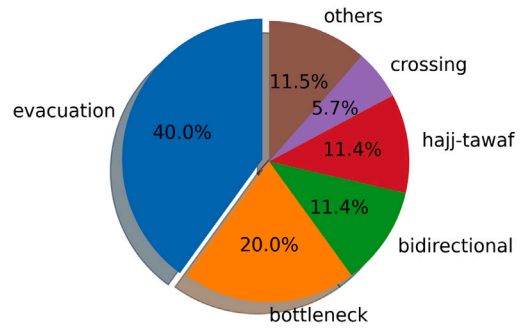
To examine the variations in the usage of the simulation techniques over time, the timeline was divided into two periods: 2000 to 2011 and 2012 to 2023. As illustrated in Fig. 2e,f, the number of articles in all three modeling levels has increased during the latter period. However, the operational level of modeling shows a minor increase in the number of articles for VO models in the second half period. In contrast, other operational level models, including SFM, CA model, ABM, macroscopic model, and hybrid model, exhibit a consistent upward trend. The emergence of data-driven models for human trajectory prediction and simulation is a relatively recent development, supported by advances in artificial intelligence, computational resources, and the availability of extensive datasets.

5. Analysis of pedestrian simulation models

In complex environments with many obstacles, pedestrians can have behaviors at three levels [15] depending on their objective. The global architecture is shown in Fig. 3. First, at the strategic level, pedestrians decide which destinations they want to target. Next, pedestrians will engage in tactical behavior which entails combining their knowledge of the environment with their predetermined targets in order to establish either a global or a local route that leads them closer to their target. Path planning models are implemented in order to determine the most efficient way for pedestrians. The information about the selected route will be used for the operational level in the form of a set of waypoints or desired velocities. At the operational level, simulation models are used to handle local interactions and compute the exact positions of pedestrians at the next time step. Pedestrians can also alter their initial plans such as their goal and guiding path as a way to adapt to the dynamic environment while they perform their operational behavior. In the rest of this section, strategic level models are first reviewed and compared (Section 5.1), followed by the tactical level models (Section 5.2), operational level models (Section 5.3), and finally a comparison of operational level models (Section 5.4).



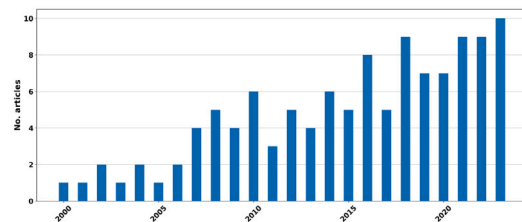
(a) Classification of paper contributions to three levels of modeling [15].



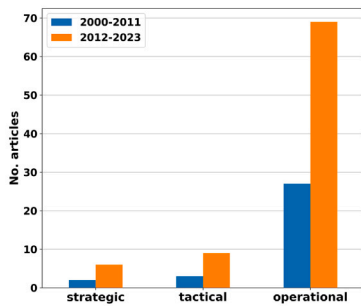
(b) Classification by experiment types.



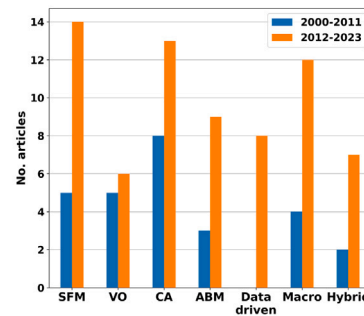
(c) Top keywords.



(d) Number of yearly publications.



(e) Multi-level modeling.



(f) Operational level models.

Fig. 2. Analysis results on the collected articles.

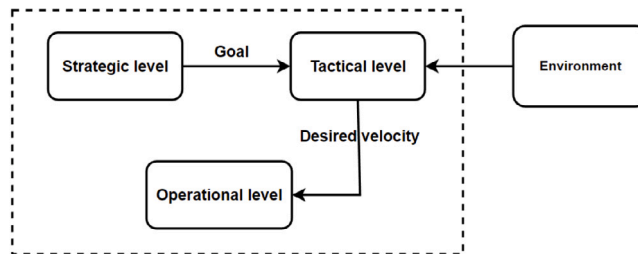


Fig. 3. The classical Perception-Decision-Action schema.

5.1. Strategic level modeling

Strategic level modeling, which is also known as *goal selection* [61], refers to the process of selecting a target for pedestrians. Generally, pedestrians choose a target based on their psychological interests. While crowd simulation has been extensively studied and with great success in various domains, modeling of strategic behavior has not been examined to a large extent by recent studies (as shown in Fig. 2a). One of the key reasons for this is that academics have focused primarily on modeling pedestrian behavior at the tactical and operational levels, which covers most pedestrian activities. Additionally, the lack of empirical data makes it challenging to calibrate input parameters and validate existing models [62]. In addition, this level has been extensively studied independently from the pedestrian modeling.

Strategic level modeling is mainly studied in the context of emergency evacuations. During an evacuation, evacuees typically determine an exit to escape as quickly and safely as possible. The strategic level models can be used to simulate goal selection behavior by specifying a safety goal for each individual. These models include exit choice models [62,63], interest function models [64], logit-based discrete choice models [65,66], genetic-algorithm models [67], and game theory-based models [68], leader-follower exit-choice models [69].

5.2. Tactical level modeling

Tactical level modeling, often referred as *path planning* [61], is the process by which pedestrians devise a plan to move from their current location to their predetermined targets based on environmental data. In simple settings, pedestrians are likely to go straight to their goals while still handling local interactions with other neighbors. Hence, the simulation of crowds in some specific scenarios, such as unidirectional flow, bidirectional flow, or bottlenecks, can be addressed by performing only operational level models. However, in a large-complex environment with many obstacles, utilizing solely operational level models can lead to pedestrians becoming locally stuck in particular areas. In addition, it is assumed that every pedestrian possesses a cognitive map of the environment [70], which they may possibly use to compute a path toward their destinations. Pedestrian routes can serve as a navigation strategy in simulations to guide pedestrians toward their targets by specifying a set of desired velocities or local waypoints along the direction of each path segment (see Fig. 3).

Path planning has been successfully applied to robotics navigation [71] and agent navigation in virtual environments. In general, the work can be categorized into two classes: global path planning and local path planning. Global path planning involves utilizing the whole map of the environment to determine a complete route from the starting location to the destination. Graph-based approaches [72,73] represent the connectivity of the environment in terms of a graph, where vertices correspond to important locations or decision points, and edges represent the possible paths between these locations. On the other hand, navigation mesh approaches [74] divide the environment into connected, traversable regions, forming a continuous mesh that represents the walkable areas. Both methods require search algorithms, normally shortest path algorithms, to find a route for each pedestrian. In contrast, the local path planning method generates a local path by collecting information from the immediate surroundings of the pedestrian's current location, such as potential fields [75], without relying on the complete map of the environment.

In low-density situations, static characteristics (e.g., static obstacles, walkable space, etc.) are typically used to formulate a pedestrian route while disregarding the presence of crowds. However, when adapting the path planning algorithms to dense environments, additional information is required to account for dynamic crowd factors. In crowded environments, relying solely on spatial subdivisions of static environments is insufficient for navigating through densely populated areas. The tendency of many pedestrians to select the same shortest paths to the destination will result in potential congestion and cause an increase in travel time (faster-is-slower effect [19,38]). To address this, factors such as flow dynamics, local pedestrian interactions, or crowd density must also be taken into account to determine more accurate and efficient navigation strategies for pedestrians in crowded environments [15]. Similar works [76–78] were also applied to robotics navigation in crowded environments.

In global path-planning approaches, information derived from crowd dynamics is typically used to actively update graph weights, enabling search algorithms to find adaptive routes that respond to changing environmental conditions. Stubenschrott et al. [42] have added a local density factor to estimate perceived travel time along the graph's edge, while Sud et al. [41] have used a density factor with a continuous update to the weights of the roadmap. Guy et al. [79] have proposed the Principle of Least Effort for pedestrians and updated the average speed of pedestrian flow along the edge to the graph weight in order for pedestrians to calculate the most energy-efficient route. Stüvel et al. [80] have added clearance factor to Voronoi edges (as shown in Fig. 4a) to allow pedestrians to select the optimal route through dense crowds. Liu et al. [43] have introduced emotions in individuals' and crowds' decisions as well as a perilous field to Delaunay triangulation representations to simulate crowd evacuation. Van Toll et al. [44] have used medial axis and Explicit Corridor Map to partition the environment to navigation mesh (see Fig. 4b) and added density information to navigation mesh so density value is updated each time pedestrians enter a new mesh. Using the global guidance map to represent the entire environment in large environments such as a city would result in execution time issues, particularly during the search process of paths. In these cases, using hierarchical graph [39,40] will reduce the calculation cost of the planning process. In dynamic environments where the density value can vary rapidly, it is unrealistic for the agent to take an initial route and stick to it during the entire journey. In dense situations, pedestrians may update more frequently their path [81,82]. When the next intended area becomes more crowded, pedestrians might search for other alternatives to avoid congested areas. To simulate this phenomenon, Stubenschrott et al. [42] have examined the existence of superior routes when pedestrians arrive at the next node with at least one alternative, whereas pedestrians in [44] regularly update their plans after a specific quantity of time.

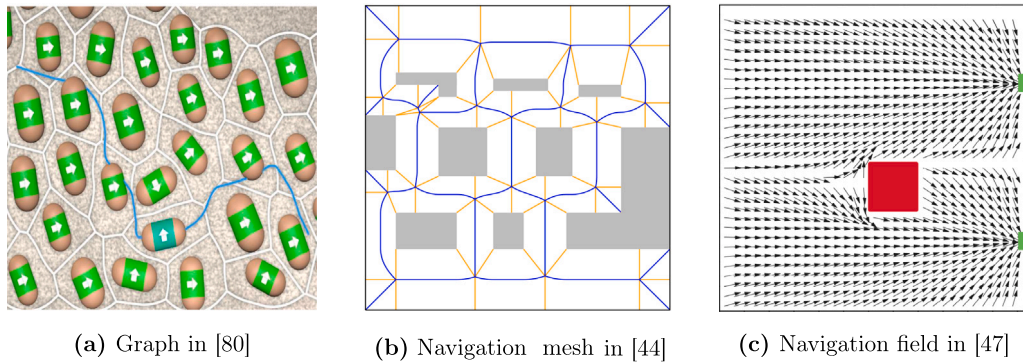


Fig. 4. Tactical navigation maps: (a) Graph created using Voronoi edges (white lines); (b) Navigation mesh constructed via the medial axis (blue lines) and orange lines that connect the medial axis vertices to the nearest obstacles; (c) Navigation field represents the desired walking direction of pedestrians across different local areas.

The local path design, on the other hand, takes into account the fact that pedestrians only have a restricted view of their surroundings and limited memory. In crowded environments, the local route is constructed from static variables (e.g., obstacles, walls, ...) and crowd dynamics which are both reflected in pedestrians' visibility maps. Zhang et al. [45] proposed the perceived potential field which is the extension of the potential field in [75] to describe pedestrians with poor visibility. Jiang et al. [47] suggested a local dynamic navigation field (see Fig. 4c) by solving an Eikonal-type equation on rectangular grids. Sun and Liu [48] proposed a local density navigation field and an equipotential field coupled with the SFM to simulate crowd evacuation. Hoogendoorn et al. [46] considered density-gradient dependent terms as an important factor in choosing local route choice; pedestrians are governed to avoid high-density areas to lower the overall walking cost.

Table 2 presents the characteristics of the tactical level models for high-density pedestrian simulations. Overall, the results indicate that the capability of tactical level models is affected by several factors, including the complexity of the environment, the number of simulated agents, and the modeling algorithm itself. Global path planning algorithms are generally advantageous in providing optimal paths in large-scale environments where the complete environmental map is available. These algorithms are commonly used to determine initial routes for a large number of agents, as such routes can be pre-computed based on static environmental data. However, they can become computationally expensive when used to model the replan behavior of pedestrians in dynamic environments, especially for large-crowd simulations.

On the other hand, local path planning algorithms are usually used for scenarios in which pedestrians have incomplete knowledge of the environment or when the environment is highly dynamic. The advantage of these methods is their ability to compute short-term, reactive paths that respond to local environmental changes. Although this can lead to more realistic and adaptive pedestrian behavior, it requires more computational effort due to the frequent updates needed to reflect the most recent local changes. The choice between global or local path planning algorithms for dense crowd simulations requires careful consideration of the simulated environment's requirements, desired realism in pedestrian behavior, and available computational resources to effectively balance performance and realism.

5.3. Operational level modeling

In pedestrian simulations, operational level models are designed to compute the velocity for each pedestrian in the following steps of their movement. These models can incorporate information from the tactical level, such as a path, through the use of local targets or preferred velocities. Numerous pedestrian simulation models have been developed and employed at the operational level. Fig. 5 presents a hierarchical classification of operational level models. These models are initially divided by their simulation scale into three categories: macroscopic, microscopic, and hybrid models. Macroscopic models focus on describing the overall dynamics of crowds, whereas microscopic models concentrate on modeling the behavior and interactions of individuals. Hybrid models typically combine the two approaches to take advantage of their strengths. Within microscopic models, there are theory-driven models and data-driven models. Theory-driven models, including Social Force Models (SFMs), Velocity Obstacle (VO) models, Cellular Automata (CA) models, and Agent-Based Models (ABMs), are based on foundational principles and hypotheses and require calibration of model parameters, while data-driven models learn pedestrian behaviors from empirical datasets.

Dense crowd simulations pose numerous challenges that must be addressed to model pedestrian behavior in crowded environments accurately. One crucial concern is whether operational level models, which have been demonstrated to simulate realistically pedestrian phenomena and behavior in low-density situations, can maintain their applicability in high-density scenarios. In addition, a major challenge involves adapting operational level models to capture the emergence of new behaviors and phenomena in dense situations such as group behavior, following behavior, replan behavior, and physical interactions between pedestrians, as elaborated in detail in Section 3. Another challenge is that the operational level models must generate meaningful results applicable to real-world situations. This requires calibration and validation of the models as critical aspects of dense crowd simulation in order to

Table 2

Comparison of tactical level models (CT: Construction time, MUT: Map update time, PST: Path search time, NA: Number of agents, RT: Run time).

Articles	Class	Environment	CT (ms)	MUT (ms)	PST (ms)	NA	RT (fps)
Lamarche et al. (2004) [39]	Graph	City: 1.3 km × 1.3 km, 2600 obstacles	–	–	25(2000) ^a	2000	10
Paris et al. (2006) [40]	Graph	Lazare train station, France	–	11	–	2000	7
Sud et al. (2008) [41]	Graph	Tradeshov: 511 booths	–	5.5	7(1000)	1000	22
Stubenschrott et al. (2014) [42]	Graph	U2 subway station, Austria	–	–	–	–	–
Stuvel et al. (2017) [80]	Graph	Small room	–	–	–	20–30	–
Liu et al. (2018) [43]	Graph	Big maze	150	–	–	2000	≈17
Van Toll et al. (2012) [44]	Nav mesh	City 500 m × 500 m	403	2	0.3(1)	20K	34 ^b
Zhang et al. (2012) [75]	Potential field	Room 16 m × 10 m	–	–	–	600	–
Jian et al. (2014) [45]	Potential field	Corridor with 90° corner	–	–	–	100–200	–
Hoogendoorn et al. (2015) [46]	Potential field	Room 30 m × 30 m	–	–	–	100–300	–
Jiang et al. (2020) [47]	Potential field	Room 18 m × 18 m, 1 obstacle 3 m × 3 m	–	–	–	100–500	–
Sun et al. (2021) [48]	Potential field	Complex office 25 m × 30 m	–	–	–	100–500	–

^a 25 ms on 2000 agents.

^b Without path search.

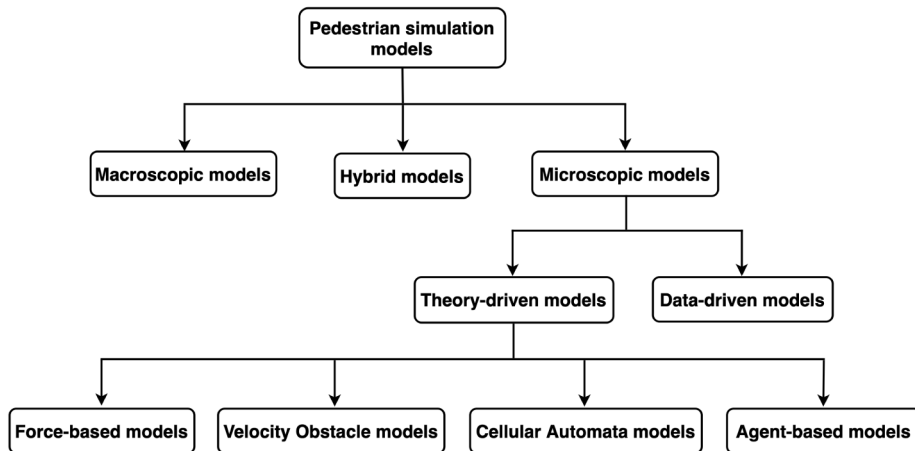


Fig. 5. Hierarchical classification of operational level models.

ensure the accuracy and reliability of the models in simulating pedestrian behaviors. However, acquiring accurate data in high-density scenarios can be challenging due to difficulties in capturing numerous complex interactions and precise measurements of pedestrian movements [14].

This section presents the classical approach for each of the operational level models mentioned above, discussing their respective methodologies and assumptions. Subsequently, the application of these models within the context of dense crowd simulations is explored, highlighting the potential problems and limitations that may arise when dealing with high-density scenarios. Finally, various improvements and adaptations are examined and evaluated for each approach, showcasing how they can be tailored to better address the complexities and challenges of simulating dense pedestrian environments.

Force-based models

Helbing et al. introduced the Social Force Model (SFM) [5,49], which characterizes pedestrian motion as being governed by the concept of social forces. These forces originate from both internal factors, such as an individual's attraction to a personal goal, and external factors, such as repulsion from neighbors and walls, and serve as the impetus for pedestrian movement. The SFM is formulated based on Newton's second law:

$$m_i \frac{dv_i}{dt} = m_i \frac{v_i^0(t) e_i^0(t) - v_i(t)}{\tau_i} + \sum_{j \neq i} \mathbf{f}_{ij} + \sum_W \mathbf{f}_{iW}, \quad (1)$$

where m_i , v_i^0 are the mass and preferred speed while v_i and e_i^0 are vectors of current velocity and desired direction of pedestrian i , respectively. The first term of Eq. (1) describes the acceleration from current velocity to the preferred velocity within the reaction time τ_i , whereas \mathbf{f}_{ij} and \mathbf{f}_{iW} are vectors describing interaction forces with other pedestrians and walls, respectively. The SFM has been demonstrated to realistically reproduce self-organization phenomena such as lane formation and arc-shaped clogging at bottlenecks [5,49].

To model emerging behavior in densely populated environments, additional forces need to be incorporated into Eq. (1). Helbing et al. [5] designed sliding friction force and a counteracting body compression force when pedestrians have physical contact with other pedestrians or walls. Nuria et al. [83] proposed a pushing force for panicked people along with falling behavior for pedestrians who receive a sufficient amount of combined pushing pressure. Yu and Johansson [33] modified the repulsion force to have potent reactions on pedestrians in extremely dense areas. As a consequence of this, erratic displacements are spread across a larger region, reproducing crowd turbulence. Mohamed et al. [84] designed new forms of forces to simulate crowds at mass-gathering events such as the Hajj-Tawaf pilgrimage. An anticipatory force was established by Karamouzas et al. [85] in order to realistically generate collective behavior that is less vulnerable to crowd density. Xu et al. [86] coupled the forced-based model with an emotional contagion model to simulate the propagation of emotions through a dense crowd. Subramanian et al. [87] modified the SFM to simulate the collective behavior of exclusive groups navigating in the chain-like formation through dense crowds. Similarly, Song et al. [88] introduced an enhanced SFM that incorporates a forward-leaning force to simulate the domino effect resulting from physical interactions of pedestrians in a queue.

However, in high-density scenarios, repulsive forces between pedestrians intensify as interpersonal distances decrease. Hence, sudden and significant changes in these repulsive forces can cause unrealistic oscillatory behaviors [89]. Moreover, the conventional values for these parameters cause pedestrians to maintain a certain distance from other neighbors and obstacles, which makes it challenging to reach an extremely high density of crowds. Numerous studies have been conducted to calibrate the SFM's parameters for more realistic dense crowd simulations. Moonsoo et al. [90] utilized aggregate data from videos to calibrate SFM parameters by a maximum likelihood estimation method for both low-density and high-density scenarios. Their study found that the effect of the force toward the destination becomes less significant as the crowd density increases. Similarly, Haghani and Sarvi [91] applied a multi-directional search algorithm to calibrate the SFM parameters to simulate realistic pedestrian flows at narrow exits. Their research identified relaxation time and friction force parameters as the most important factors in producing accurate pedestrian outflows. Shuaib [92] proposed a 6-order polynomial function of local density to set the value of the anisotropic angular parameter in [5]. Likewise, Narang et al. [93] proposed density-dependent filters for SFM agents to compute a preferred velocity in the first term of Eq. (1) to fit the empirical fundamental diagram [34,94–99]. Sticco et al. [100] modified the friction coefficient in [5] to control pedestrian flow in an extremely congested zone.

The realism of SFM in dense simulations has also been improved through the implementation of several other enhancements. A field of vision [83,84,101] has been established in order to obtain information regarding neighbors who actually affect the movement of pedestrians. In densely populated areas, pedestrians begin to oscillate due to the significant and rapidly varying forces they experience. Nuria et al. [83] reduced oscillations by applying stopping rules when agents are suffering repulsion forces from neighbors in the opposite direction of their preferred direction and are not in panic mode. Better representations of the human body, such as spheropolygons [102], three-circle shape [103], and elliptical shape [104] are implemented to simulate heterogeneous crowds.

Velocity Obstacle models

The Velocity Obstacle (VO) model [105], proposed by Paolo and Zvi Shiller in 1998, describes the avoidance maneuver of robots in dynamic environments by constructing geometric space, VO_B^A , for potential-collision velocity:

$$VO_B^A(\mathbf{v}_B) = \{\mathbf{v} \mid \lambda(\mathbf{p}_A, \mathbf{v} - \mathbf{v}_B) \cap B \oplus -A \neq \emptyset\}, \quad (2)$$

where A and B are disc-shaped agents moving in the two-dimensional plane, with their centers located at \mathbf{p}_A and \mathbf{p}_B , and their velocities denoted by \mathbf{v}_A and \mathbf{v}_B , respectively. $\lambda(\mathbf{p}, \mathbf{v})$ is the ray initiating from point \mathbf{p} and extending in the direction of \mathbf{v} :

$$\lambda(\mathbf{p}, \mathbf{v}) = \{\mathbf{p} + t\mathbf{v} \mid t \geq 0\}. \quad (3)$$

Additionally, $A \oplus B$ denotes the Minkowski sum of the geometries of agents A and B, and $-A$ presents the reflection in the geometry of agent A:

$$A \oplus B = \{\mathbf{a} + \mathbf{b} \mid \mathbf{a} \in A, \mathbf{b} \in B\}, \quad -A = \{-\mathbf{a} \mid \mathbf{a} \in A\}. \quad (4)$$

Agent A would collide with agent B at some time in the future if a ray emanating from agent A's position \mathbf{p}_A in the direction $\mathbf{v}_A - \mathbf{v}_B$ (relative velocities of agents A and B) intersects the Minkowski sum of B and $-A$ located at \mathbf{p}_B . $VO_B^A(\mathbf{v}_B)$ (see Fig. 6a), or velocity

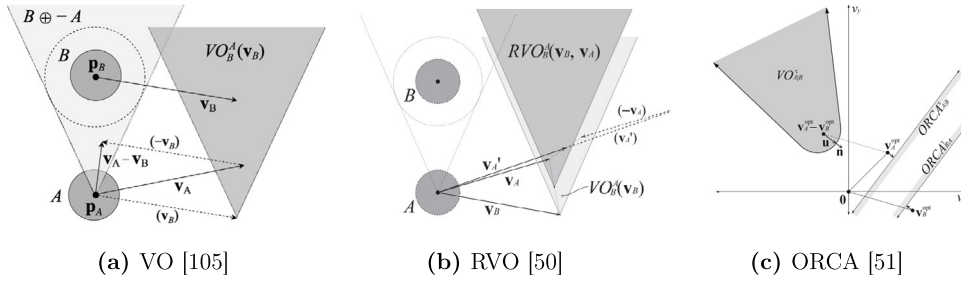


Fig. 6. Velocity obstacle models [50,51,105].

obstacle of A induced by B, is the set of velocities that, if agent A uses them to move, will eventually result in a collision with agent B. The avoidance velocity is selected from the feasible velocity set so that this new velocity is outside all velocity obstacles created by neighboring agents and obstacles.

However, this avoidance behavior does not take into account anticipation of the future movements of other agents, hence it is prone to cause oscillations in the trajectory (see III. C in [50] of for a detailed illustration). Van den Berg et al. have proposed the Reciprocal Velocity Obstacle (RVO) [50] (as shown in Fig. 6b) to overcome this issue, presuming that other moving agents will similarly make equal attempts and reasoning to prevent collisions:

$$RVO_B^A(\mathbf{v}_B, \mathbf{v}_A) = \{\mathbf{v} \mid 2\mathbf{v} - \mathbf{v}_A \in VO_B^A(\mathbf{v}_B)\}. \quad (5)$$

RVO has been shown to generate collision-free and non-oscillation paths for each agent. On top of the success of RVO, Van den Berg et al. further developed Optimal Reciprocal Collision Avoidance (ORCA) [51] (see Fig. 6c) for choosing an optimal velocity for avoiding multiple agents simultaneously.

$$ORCA_{A|B}^\tau = \{\mathbf{v} \mid (\mathbf{v} - (\mathbf{v}_A + \frac{1}{2}\mathbf{u})) \cdot \mathbf{n} \geq 0\}, \quad (6)$$

where \mathbf{u} is defined as the vector from $\mathbf{v}_A - \mathbf{v}_B$ to the nearest point on the boundary of the velocity obstacle (as illustrated in Fig. 6c), while \mathbf{n} denotes the normalized vector of \mathbf{u} . Consequently, \mathbf{u} represents the minimum required change in relative velocity between agents A and B to ensure collision-free navigation within a time span of τ seconds.

In crowded environments, the increasing number of neighbors can cause velocity obstacles to occupy the entire set of admissible velocities. Hence, finding a collision-free velocity outside of the velocity obstacles is not guaranteed. In such cases, the RVO algorithm [50,70] facilitates the selection of new velocities within the reciprocal velocity obstacles using a penalty function that maximizes the time-to-collision with neighboring agents and minimizes the difference with the preferred velocity. Similarly, ORCA [51] chooses a velocity within the optimal reciprocal collision avoidance area induced by as many neighbors as possible. To improve dense crowd simulations, various adaptations of velocity obstacle formulations have been developed, including truncating the velocity obstacle cone to a limited range and employing parallelism [106], combining reciprocal velocity obstacle and velocity obstacle for oscillation-free navigation [107], and using an elliptical representation of agents to better approximate human body shapes [108].

However, the new velocity is selected from the admissible velocity set to be as close as possible to the preferred velocity. Consequently, even in densely packed environments, the agent still has a propensity to choose high-magnitude velocities. This leads to the conclusion that the relation between velocity and density does not conform to the empirical fundamental diagram [34,94–99]. This issue can be remedied by incorporating physiological and psychological constraints, which allow for a more precise determination of the agent's preferred velocity to be taken into account such as the density-filter [93,109], relationship of the stride-walking speed and personal space [110].

While the VO-based approaches primarily focus on collision avoidance, a crowded environment can lead to a wide variety of other pedestrian responses. For example, physical contact is a typical occurrence that can be observed. To address this, Kim et al. [111,112] proposed a method for integrating physical interactions and velocity obstacle formulation by incorporating various force descriptions to construct force constraints within velocity obstacle space. The methodology is detailed as follows:

$$\mathbf{v}_A^f = \mathbf{v}_A + \frac{\mathbf{f}_A}{m_A} \Delta t, \quad (7)$$

$$FC_A = \{\mathbf{v} \mid (\mathbf{v} - \mathbf{v}_A^f) \cdot \hat{\mathbf{f}}_A \geq 0\}, \quad (8)$$

$$PV_A = FC_A \cap \bigcap_{B \neq A} ORCA_{A|B}. \quad (9)$$

where \mathbf{f}_A represents the total force exerted on agent A, m_A is its mass, and Δt is the simulation time step. FC_A is the force constraint induced by the total force \mathbf{f}_A . Geometrically, FC_A is a half-plane whose boundary runs across \mathbf{v}_A^f in a line that is perpendicular to the normalized force $\hat{\mathbf{f}}_A$ of \mathbf{f}_A . There are a number of velocities in this half-plane whose differences with current velocity \mathbf{v}_A are equal to or larger than the minimal velocity changes generated by the force \mathbf{f}_A . Agent A is now able to choose its new velocity in

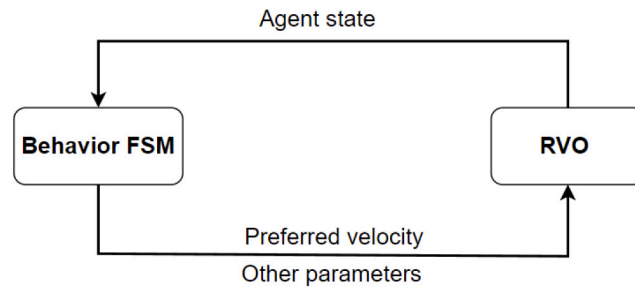


Fig. 7. Combination of FSM and RVO in [113].

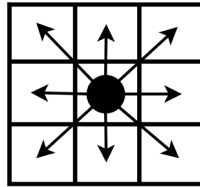


Fig. 8. Possible moving directions in [6].

the set of permitted velocities PV_A , which is the intersection of FC_A and $ORCA_A$. By adding a force constraint for velocity selection in crowded environments, their model was able to reproduce realistic physical interactions among agents.

[112,113] have combined a Finite State Machine (FSM) with the RVO to represent high-level decision-making behaviors of pedestrians during specific events such as the Hajj-Tawaf pilgrimage. The FSM consists of states corresponding to distinct behaviors triggered by temporal, spatial, and the agent's state conditions. The FSM first updates the agent parameters before performing local collision avoidance behavior (with RVO) (see Fig. 7 for illustration). Likewise, Sudkhot et al. [114] proposed a framework that integrates the Belief-Desire-Intension (BDI) model for high-level path planning and the RVO for low-level collision avoidance for faster and smoother navigation of agents. Through long-range collision anticipation, Golas et al. [115] developed density-sensitive behavior for distant crowded groups. This enables agents to steer out of the congested zone earlier, which generates smoother agent trajectories.

Cellular Automaton models

The pioneering works of using Cellular Automaton (CA) models for pedestrian simulation were introduced by Burstedde et al. [6] in 2001 and Kirchner and Andreas [7] in 2002. In these models, the environment is represented as a grid discretized into cells with a size of $40\text{ cm} \times 40\text{ cm}$. Each cell can accommodate at most one pedestrian. At every simulation step, pedestrians choose a neighboring cell based on the transition probability computed from the concept of *floor field*. The floor field can be understood implicitly as the additional grid of cells located beneath the grid of cells occupied by pedestrians [6]. The floor field has two components: *static floor field* and *dynamic floor field*. The static floor field, determined by the distance between cells and the destination, provides a general direction toward desired objectives, whereas the dynamic floor field, inspired by chemotaxis, represents the virtual trail left by pedestrians as they walk. This virtual footprint can be highly dispersed or decaying. The static and dynamic floor fields are used to update the transition matrix which determines the probability of moving to empty adjacent cells (Moore neighbors, as shown in Fig. 8). The simulation results produce realistic outcomes, such as the appearance of lane formation or a semi-circle at the bottleneck.

The CA model with a typical cell size of $40\text{ cm} \times 40\text{ cm}$ can simulate various types of crowds, but it is limited to a maximum density of 6.25 ped/m^2 , which is insufficient for simulating real-world scenarios with extremely high densities reaching values of $8\text{--}9\text{ ped/m}^2$ [34]. Additionally, representing a pedestrian as a single cell helps resolve local collisions during movement; however, the movement direction and speed are restricted to a limited range of feasible values at any given time. To increase the ability to simulate higher levels of density and enable more flexible movement, several proposed works approximate the environment with a finer level of discretization. By using smaller cell sizes, pedestrian shape can be represented by multiple cells, such as triangular cell [116], group of 3×3 cells [117,118], group of 4×4 cells [119], group of 20 cells [120], group of 39 cells [121] (as seen in Fig. 9).

Furthermore, employing multiple smaller cells to represent pedestrian bodies leads to a wider range of mobility options. The traditional CA models [6,7] utilize the Moore or Von Neumann neighborhood, which restricts the number of possible pedestrian directions to either four or eight. Several studies have increased the choice of directions to enable smoother pedestrian movement. For instance, Jingwei et al. [116] used triangular cells to represent pedestrian bodies, facilitating movement in 14 directions through connections between each cell and its 14 adjacent cells. Claudio et al. [122] introduced extra positions at the midpoint of each cell edge, allowing pedestrians to move to the middle of the next cell edge if the total number of pedestrians in that cell is below a

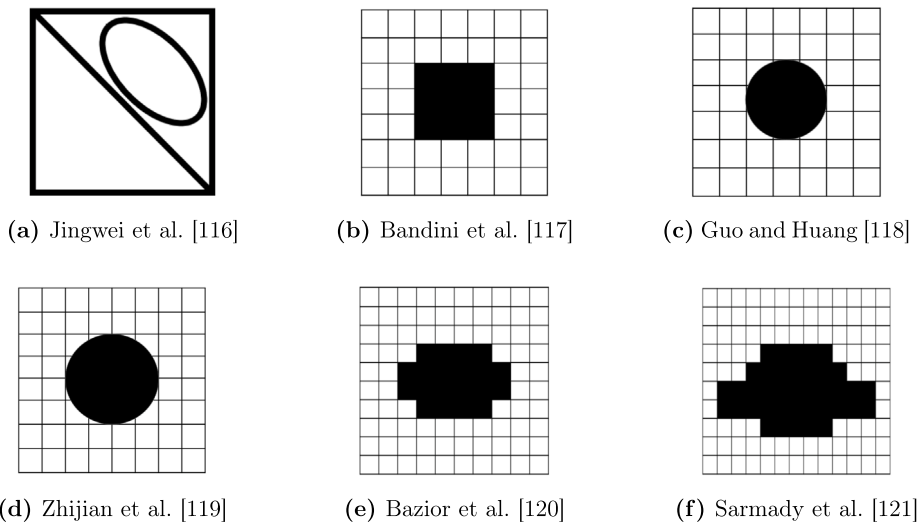


Fig. 9. Finer representations for pedestrian shape.

specified threshold. Huo et al. [123] proposed an extended Moore neighborhood, which allows pedestrians to move two cells in one time step, to simulate the stampede behavior of pedestrians during evacuation. Zhang et al. [124] proposed movement rules in the CA model for pedestrians walking in pairs with different decision-making patterns to reproduce small group behaviors at the metro station.

In the CA approaches, emerging behavior of pedestrians in high-density scenarios can be addressed by modifying floor fields to account for physical interactions [118,125], group behaviors [126], or by incorporating new floor fields such as force floor field [127,128] for physical interactions, local view floor field [129] for replanning behavior, anticipation floor field [122,130] for future collision anticipation, interplay floor field [131] for social distance among agents, environmental floor fields such as wall floor field [122,131] and water flood field [132]. These floor fields contribute to the transition matrix to determine the likelihood of selecting adjacent cells in subsequent steps.

Agent-based models

Agent-based modeling is a bottom-up method in which agents or individuals are treated as unique and independent entities that can locally interact with other individuals as well as with their environment [133]. In the context of crowd simulation, an agent is a rule-based object that represents an individual pedestrian. Agent-based paradigms can be used to simulate heterogeneous crowds and can also be coupled together with various types of models [134]. Although agent-based models offer more flexibility in terms of modeling behavior, modelers still have to deal with the computational cost trade-off.

It is important to note that there is some ambiguity among the different definitions of agent-based methods. Hence, SFM, CA, and VO methods can occasionally be considered as part of the agent-based approach because pedestrians are considered individually (and not aggregated in a flow). In this article, we will classify agent-based models only the ones in which pedestrian agents have autonomous decision making. More specifically, agents in ABMs normally possess more complex cognitive processes that enable them to perceive and evaluate their surroundings. Based on these perceptions, their behavior varies accordingly to adapt to the environment rather than being strictly predetermined. For instance, in the context of egress from a building, pedestrian agents may select the nearest exit under low-density conditions; however, in high-density situations, they may prioritize the fastest one or simply follow a leader.

Shao and Terzopoulos [52] have proposed a comprehensive architecture combining perceptual, behavioral, and cognitive components to simulate pedestrians with sophisticated behaviors in complex environments. In order to specify agents' decision-making behavior, Zhou et al. [135] have embedded three different fuzzy inference systems for the decision-making process at each level of pedestrian behavior [15]. Information from agents' perceptions, as well as their previous experience and knowledge, was used as input to three fuzzy logic inferences. The outputs from these inferences were merged to determine the turning angle and speed for the next step. Sharma et al. [136] have developed a neural network to learn and predict the adaptive behavior of pedestrians during evacuation. In their research, a fuzzy logic model was integrated to infer agents' speeds based on their levels of emotion and stress.

In addition, agents are able to perceive information from their surroundings. The information that agents observe can have a significant impact on their local actions, particularly in dense situations. Zheng et al. [98] have demonstrated that the impact of front neighbors in crowded areas is much stronger than lateral neighbors. The traditional approaches [5,50,51] take into account the local interactions between all pedestrians located within a specific radius. Although this method offers benefits in terms of implementation and computational speed, simulation can be more realistic in high-density regions by excluding neighbors with

whom it does not actually interact. Some studies have designed improved visual fields for agents, allowing them to perceive only the agents they actually interact with, such as a rectangle [83] or the shape of a cone [52,135,137,138].

In crowded scenarios, people frequently cannot walk at their preferred velocity due to limited personal space. Consequently, psychological factors, such as discomfort, may significantly influence decision-making, leading to the emergence of distinct behaviors in congested areas [19]. One of the most prominent behaviors in these situations is the leader–follower behavior. Pedestrians may tend to follow frontal pedestrians who are traveling in similar directions in order to lessen the amount of effort required to navigate through dense areas [139]. To model this behavior, Samuel et al. [53] conducted an experiment in one-dimensional traffic to develop and evaluate a simulation model that captures pedestrian following behavior, in which pedestrian accelerations depend on the relative motion characteristics of the agents in front of it. Godoy et al. [139] based on the evaluation of agents' most similar and most constrained neighbors in a dense crowd to optimize their next action. Liang et al. [140] clustered crowds into different groups depending on their targets and velocities. Pedestrians who are close to the target and do not find another individual to follow will assume the group leadership. The rest of the group will follow the leader while keeping collision-free with inter-group members. Likewise, Liu et al. [141] have considered several criteria, such as environmental similarity, exit distance, and navigation knowledge, to choose a leader who would guide the group during the evacuation process. Mingbi et al. [142] have presented a role-dependent model to simulate leader–followers in a high-density crowd, where one of the criteria for choosing a leader is that the leader is a pedestrian whose current speed is greater than the average speed of the crowd. Li et al. [143] developed a double-layer decision model to simulate the detour decisions of pedestrians when encountering crowded areas ahead. Xie et al. [144] have proposed an information-theoretic method to simulate the spontaneous leader–follower behavior of pedestrians during crowd evacuation.

Data-driven models

The data explosion and advances in computational capabilities over the past few decades have enabled neural network approaches to become a powerful tool for identifying hidden relationships in large datasets by replicating interconnected neuron functions found in the human brain. As universal function approximators, neural networks have been shown to address complex problems and are widely used in various domains, including computer vision, autonomous vehicles, and business analytics.

The neural network approach for human trajectory prediction and knowledge-based models in pedestrian modeling represent two distinctive methodologies. The knowledge-based models are grounded in predefined rules derived from empirical observations and studies, primarily aimed at simulating pedestrian behaviors, with trajectory predictions emerging as a by-product [29]. In contrast, neural network approaches learn patterns and relationships directly from large datasets and primarily focus on predicting trajectories based on these learned patterns.

The neural network models use pedestrian past positions as input, along with other contextual information such as environmental features or neighboring dynamics, for a specified number of time steps to predict future trajectories for a given pedestrian. The most prominent work to predict pedestrian trajectories is the Social-LSTM (Long Short-Term Memory) proposed by Alahi et al. [54]. In the Social-LSTM, social pooling is introduced to aggregate the hidden states of neighbors in a scene, considering local interactions while making a prediction. Another well-known neural network for pedestrian trajectory prediction is the Social-GAN (Generative Adversarial Networks) model proposed by Agrim et al. [55]. The architecture in the Social-GAN consists of an RNN (Recurrent Neural Network) Encoder–Decoder generator and an RNN-based encoder discriminator that takes into account the effects of all pedestrians in the scene. Numerous variants of LSTM and GAN networks have been developed in order to gain more accurate predictions or adapt to a variety of environments [145–148].

The neural network approaches have achieved considerable success in predicting human trajectories in low-density conditions [149]. However, they still present challenges for researchers in dense situations. The primary objective of neural network techniques is to predict pedestrian trajectories as accurately as possible. Hence, average displacement error (ADE) or final displacement error (FDE) are commonly used as training functions and evaluation metrics. When the density becomes higher, other important metrics, e.g. the number of collisions [149,150], must be taken into account. This is because a loss function that focuses solely on minimizing distance error could still produce unrealistic behaviors like high collision rates [151]. To address this, several works have been conducted to improve the loss function for crowded scenarios by introducing additional loss terms such as time-to-collision loss [152] or interaction energy loss [151] into the loss function along with traditional distance-error loss. Moreover, the prediction time in neural network approaches is substantially longer than knowledge-based models, ranging from 3.0 to 5.0 seconds [29]. This time horizon of prediction must be thoroughly investigated in dense scenarios, as the number of interactions increases in high-density situations. Finally, low-density datasets are usually utilized to train neural networks because of the limited availability of high-density datasets.

Macroscopic models

Macroscopic models are a class of models for crowd simulation using a set of partial differential equations. These models treat pedestrian flow similarly as a continuum without considering individual differences.

One common method is to model pedestrian flow as fluid-like dynamics, where the continuity equation [153] is used to ensure the conservation of flow:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (10)$$

where ρ and \mathbf{v} are the density and velocity of the flow of pedestrians, respectively. This equation has been widely used in various macroscopic models for crowd simulation. Hughes has proposed a continuum theory [8,154] based on the continuity equation, which incorporated physical and psychological factors of pedestrians into the modeling of pedestrian flow:

$$\frac{\partial \rho}{\partial t} - \nabla \cdot (\rho g(\rho) f^2(\rho) \nabla \phi) = 0, \quad (11)$$

where $f(\rho)$, $g(\rho)$ are density-dependent factors influencing individual speed and discomfort, respectively, while $\nabla\phi$ geometrically describes the primary direction of individual movement at each location in the environment. Narain et al. [56] introduced a unilateral incompressibility constraint for density in Eq. (10) to simulate pedestrian flow at maximum density, which cannot be denser and compressible when density reaches a maximum level:

$$\rho \leq \rho_{\max} = \frac{2\alpha}{\sqrt{3}d_{\min}^2}, \quad (12)$$

where ρ_{\max} is the maximum density which is computed from the constant factor α and minimum distance d_{\min} between pedestrians. Jiang et al. [155,156] have added an equation specifying a relaxation term and an anticipation term to address traffic instability in crowd flows:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + c^2(\rho) \frac{\nabla \rho}{\rho} = \frac{U_e(\rho) \vec{v} - \mathbf{v}}{\tau}, \quad (13)$$

where c , τ , U_e , \vec{v} are the sonic speed, relaxation time, equilibrium value, and desired direction, respectively.

On the other hand, Treuille et al. [9] have proposed a continuum model in which global navigation is handled by dynamic potential field by eikonal equation (Eq. (14)):

$$\|\nabla\phi(\mathbf{x})\| = C, \quad (14)$$

$$\mathbf{v} = -f(\mathbf{x}, \theta) \frac{\nabla\phi(\mathbf{x})}{\|\nabla\phi(\mathbf{x})\|}, \quad (15)$$

where ϕ , C are potential function and unit cost in the direction $\nabla\phi$, respectively. The cost value to the goals is based on distance, time, and discomfort factors. The pedestrian velocity is determined by the direction opposite to the gradient of function in Eq. (14), while the magnitude depends on the speed field $f(\mathbf{x}, \theta)$ evaluated at the position \mathbf{x} with the moving direction θ . Likewise, Jiang et al. [155] minimized the instantaneous total walking cost from origin to destination using an instantaneous equilibrium travel cost and discomfort field. Recently, crowd simulation models [157–159] derived from Smoothed Particle Hydrodynamics [160] have been developed to simulate crowds at extremely high densities.

Macroscopic models are commonly used for high-density situations due to their assumption of the continuum of pedestrian flow. These models focus on the aggregation of crowd information rather than micro-level interactions between individuals. They can be used to address various macroscopic phenomena of crowds under extreme density such as shockwaves [157,159,161–163], panic propagation [164–167], crowding force [161,165]. Macroscopic models are an effective approach for large-scale and high-density crowd simulation due to their computational efficiency and ability to handle large numbers of individuals.

Hybrid models

Hybrid modeling typically combines macroscopic and microscopic models to benefit from both [168]. This approach provides a balance between the level of resolution detail and simulation efficiency. Hybrid models are commonly used in scenarios that demand a comprehensive understanding of both the overall dynamics of large crowds and the specific behaviors of individuals. The principal challenge of hybrid modeling lies in the interactive communications between macroscopic and microscopic models, notably the consistent transition of simulation results between these scales. The transition includes the aggregation of individuals under statistics from the microscopic model to the continuous medium of the macroscopic model and the disaggregation of the macroscopic information like crowd density distribution back into individuals for microscopic simulation.

There are two main types of combined models: region-based models and auto-switch models. Region-based models [57–59,169–172] divide the environment into distinct regions, with macroscopic and microscopic simulations conducted simultaneously in different regions. The choice of an appropriate simulation model applied to each region is based on expert knowledge or simulation state. For example, crowded anticipated areas can be simulated using macroscopic models, while lower-density areas use microscopic models. However, as crowd dynamics change over time, it can be adapted in simulation by dynamically switching between models for a region based on conditions such as density thresholds [170,171]. The transition criteria and synchronization mechanism are activated when pedestrians traverse from one region to another. Boundary cells [57,170,171], which are the extra cells adjacent to the region of macroscopic simulation, can be used to improve synchronization consistency.

On the other hand, auto-switch models [173,174] incorporate the two kinds of models in a way that allows them to be interchangeable, with only one being executed during simulation. The conversion between macroscopic and microscopic simulations is governed by predefined triggers, which are normally specific events or crowd characteristics.

5.4. Comparison of operational level models

In this section, a comprehensive evaluation framework is proposed to assess and compare the various operational level models applied in high-density situations, which based on several key criteria. One criterion considers examining the capability to simulate emergent behaviors and phenomena in crowded scenarios, such as physical interactions, group behavior, following behavior, and replan behavior. Another criterion evaluates the performance of these models based on quantitative factors such as duration of time step, maximum density, number of agents, and computational time. Additionally, a third criterion focuses on the validation of these models with respect to fitting the empirical Fundamental Diagram (FD) [34,94–99] or to real-world data. Lastly, the compatibility of each operational level model with multi-level behavior modeling, including both strategic and tactical levels, is assessed under the multi-level modeling criterion.

The comparison of the various operational level models is detailed in [Table 3](#) and analyzed in the following section. The characteristics of these models are assessed using the following syntax:

- ✓: The model is able to simulate the corresponding characteristic.
- ×: The model cannot address the corresponding characteristic.
- -: Unknown.

Behavior capability

Simulating emergent behaviors and phenomena in high-density situations is crucial for accurately capturing realistic crowd dynamics. In these environments, pedestrians exhibit a variety of behaviors and phenomena, including physical interactions, following behavior, group formations, and adaptation to their surroundings, as explained in [Section 3](#). These factors significantly influence crowd movement patterns and can contribute to potential hazards in congested areas.

As indicated in [Table 3](#), the SFMs have been used to simulate a broader range of emergent behaviors of pedestrians in high-density scenarios compared to other modeling approaches. Furthermore, only the classical model of SFMs demonstrates the capability to simulate emergent behaviors of pedestrians in such scenarios, while the classical models of the other approaches are inadequate for capturing these emergent behaviors without adaptations.

Different modeling approaches are adapted to simulate pedestrian emergent behaviors in high-density situations. With SFMs, emergent behaviors are addressed by extending [Eq. \(1\)](#) with supplementary force components. However, high densities may cause intense and rapidly fluctuating forces, leading to unrealistic phenomena such as shaking [\[89\]](#). Consequently, it is essential to undertake parameter calibration and validation against empirical data to ensure the model's realism and adaptability to specific contexts.

For CA models, the original cell size of 40 cm x 40 cm [\[6\]](#), which governs collision-free movements, must be reduced to model physical interactions. Additionally, the transition matrix governing pedestrian movement in the CA models needs to account for the influence of new floor fields. The addition of new floor fields can also increase the model complexity and require non-trivial modifications to the existing codebase. Moreover, the accuracy of the results may depend on the accuracy of the new floor fields used in the model.

With ABMs, simulating emergent behaviors in high densities involves introducing new variables that affect the agent's behavior and decision-making process. This approach pre-defines desired emergent behaviors that need to be simulated and may become computationally expensive when simulating large numbers of agents with complex decision-making processes.

On the other hand, the capabilities of VO models and macroscopic models to simulate pedestrian emergent behaviors are notably limited in crowded situations. For VO models, a new velocity obstacle space and velocity constraints must be defined to address new behaviors. Meanwhile, macroscopic models usually take a higher-level macroscopic perspective focused on overall crowding forces rather than emergent individual behaviors in dense crowds. The ability of hybrid models to simulate emergent behaviors relies substantially on the submodels incorporated in the hybrid framework.

Data-driven models currently lack the ability to address high-density emergent behaviors, indicating a need for further research and development in this area [\[149\]](#). Recent deep learning models in data-driven approaches are primarily designed and trained for predicting human trajectories in low-density scenarios [\[149\]](#). Therefore, predicting human behavior in high-density scenarios requires the introduction of new network architectures and training functions. However, these models may lack interpretability and generalization to new situations, compounded by a shortage of available high-density datasets for pedestrian trajectories and interactions.

Model performance

The performance of the operational level models is evaluated through quantitative factors such as duration of time step, maximum density, number of agents, and computational time. Models with lower sensitivity to time step changes can produce more robust and stable results. Additionally, smaller time steps can yield smoother motion; however, excessively small ones can render the simulation computationally. Maximum density can be used as a benchmark to assess the ability to simulate dense crowds. The scalability and efficiency of the models are evaluated by examining the relationship between computational time and the number of simulated agents.

As presented in [Table 3](#), SFMs, ABMs, and macroscopic models usually use small time steps (0.01–0.1 s) to ensure stability. On the other hand, VO models exhibit the ability to adapt to a broader range of time steps ranging from 0.01 s to 0.25 s. The time steps for CA models and data-driven models are mainly determined by the cell size and the frame rate of the training data, typically 0.3 s and 0.4 s, respectively. For hybrid models, the time step synchronization between macro and micro models in the hybrid framework remains questionable.

In terms of density levels, SFMs, VO models, and ABMs have been used to simulate densities of 2–6 ped/m², as shown in [Table 3](#). These models may exhibit unrealistic behaviors, such as oscillation [\[83\]](#) or stuck phenomena [\[142\]](#), at extremely high densities (6–8 ped/m²) and thus require careful calibration of parameters to simulate pedestrian dynamics under such extreme conditions accurately. In contrast, crowd densities simulated by CA models and macroscopic models can reach 6–8 ped/m². The reason that CA models can address such extremely high densities is due to their intrinsic characteristics. CA models enable more efficient use of space by discretizing the environment into small cells for pedestrians. Additionally, CA models do not consider the concept of personal space or inter-agent repulsion and collision avoidance. Therefore, in theory, pedestrians can occupy very confined spaces as long as the transition rules allow the movement.

Regarding simulation speed and scalability, SFMs, CA models, and ABMs are typically employed for small-scale simulations (100–500 agents) and medium-scale simulations (500–5K agents). On the other hand, VO models, macroscopic models, and hybrid models are capable of simulating large-scale scenarios with 5K–100K agents while maintaining efficient computational times. At present, data-driven models primarily focus on predicting the behavior of a single primary pedestrian.

Validation

Validation of operational level models is a critical step in verifying the reliability and accuracy of the models in simulating crowd dynamics. This process involves comparing the model's output to empirical FD [34,94–99] and real-world datasets. Empirical fundamental diagrams, which represent the fundamental relationships between density, flow, and speed, provide a theoretical reference to general patterns of pedestrian movement for validating the operational level models. Comparing the model's output to real-world datasets allows for a more specific validation, as it directly evaluates the model's performance in replicating actual pedestrian interactions behaviors and phenomena.

As shown in Table 3, ABMs and data-driven models are among the most validated approaches, as they often incorporate empirical data and real-world scenarios into their approach development. In contrast, macroscopic models and hybrid models have seen relatively limited validation in the literature. The traditional approaches of SFMs, VO models, and CA models have not been extensively validated using either empirical fundamental diagrams or comparisons to real-world datasets. However, recent studies have begun to address this issue by incorporating validation components in their models, making these models more realistic and reliable in dense situations. This trend emphasizes the growing importance of rigorous validation efforts in operational level modeling to ensure the accuracy and reliability of the models across a diverse range of crowd dynamics and situations.

Multi-level modeling

The capacity to incorporate high-level modeling at strategic and tactical levels is crucial in achieving a comprehensive representation of pedestrian behavior. This incorporation enhances the operational level models' adaptability to effectively simulate across a broad range of environments and scenarios.

Among the operational level models, CA models and macroscopic models inherently integrate tactical navigation through mechanisms such as static floor fields or potential fields. SFMs, VO models, ABMs, and hybrid models, on the other hand, can be readily integrated with high-level modeling at both strategic and tactical levels through a set of preferred targets and velocities. However, data-driven models currently face limitations in incorporating high-level modeling, as they are primarily used for predicting short time horizons.

Based on the evaluation and comparison of various operational level models presented above, an assessment of their abilities, advantages, and disadvantages in addressing specific characteristics is summarized in Table 4 using the following criteria:

- ++: The approach easily addresses the corresponding characteristic with minor or no modifications.
- + : The approach can handle the corresponding characteristic but requires some adaptations.
- - : The approach addresses the corresponding characteristic only with significant modifications.
- --: The approach cannot address the corresponding characteristic without a complete redesign.

In summary, specific operational level models have been used to simulate pedestrian behaviors and movements in crowded contexts. SFMs are commonly applied to simulate pedestrian emergent behaviors as well as to simulate medium-high density scenarios (2–6 ped/m²). With appropriate calibration, these models can address crowds with densities of 6–8 ped/m². CA models can handle densities of 2–8 ped/m², while ABMs typically focus on modeling decision-making behaviors. SFMs, CA models, and ABMs are often applied to small to medium-scale simulations (100 to 5K agents). VO models provide flexibility over various time steps, with the capability to simulate large-scale crowds (5K–100K agents) at densities of 2–6 ped/m². On the other hand, the effectiveness of data-driven models in high-density situations is restricted due to the lack of high-density datasets. These models primarily focus on predicting the behavior of a primary pedestrian in a scene. Macroscopic models are used for high-density and large-scale scenarios that aim to understand aggregate information and macroscopic phenomena of crowds. Lastly, hybrid models are often used to simulate situations that require representing both high-level crowd dynamics and individual behaviors.

6. Future directions

In this section, several future perspectives for dense crowd simulation are proposed based on our assessments of simulation models for pedestrian simulations. Future research directions, including high-density data collection, deep learning perspectives, integration of different modeling levels, and multi-scale simulation, are proposed and discussed.

High-density data collection

In order to make simulation models more realistic in crowded scenarios, it is important to calibrate and validate them using high-density datasets. However, the majority of currently available datasets are based on low-density situations: ETH [179], UCY [180]. Only a few high-density datasets have been conducted through laboratory experiments in Julich [181], limiting their applicability to real-world dense crowd situations. Therefore, more research is needed to collect high-density datasets, especially in real-world environments.

Table 3

Comparison of operational level models (PI: Physical Interaction, GB: Group behavior, F: Following, R: Replan, TS: Time step (s), MD: Max density (ped/m²), NA: Number of agents, RT: Run time (fps), S: Strategic, T: Tactical, O: Operational).

Articles	Class	Behavior capability				Model performance				Validation		Multi-level		
		PI	GB	F	R	TS	MD	NA	RT	Fit FD	Real data	S	T	O
Helbing et al. (2000) [5]	SFM	✓	✓	×	×	–	–	200	–	–	–	×	×	✓
Lakoba and Kaup (2005) [89]	SFM	✓	✓	×	×	<0.0625	–	100	–	–	–	×	×	✓
Yu and Johansson (2007) [33]	SFM	✓	✓	×	×	–	9	–	–	–	–	×	×	✓
Nuria et al. (2007) [83]	SFM	✓	✓	✓	✓	–	–	1800	25	–	–	×	✓	✓
Guy et al. (2010) [79]	SFM	×	×	×	✓	–	4.0	10K	15.1	–	✓	×	✓	✓
Moonsoo et al. (2013) [90]	SFM	×	×	✓	×	–	–	–	–	–	✓	×	×	✓
Shuaib (2014) [92]	SFM	✓	✓	×	×	0.001	4.5	100–400	–	✓	–	×	×	✓
Best et al. (2014) [109]	SFM	×	×	×	×	0.0625	3.96	–	–	✓	✓	×	✓	✓
Mohamed et al. (2015) [84]	SFM	✓	✓	✓	×	–	8	–	–	✓	–	×	✓	✓
Karamouzias et al. (2017) [85]	SFM	×	✓	×	×	0.01–0.4	–	5K	4.2	–	–	×	✓	✓
Jingni et al. (2019) [103]	SFM	✓	×	×	×	–	8.6	–	–	✓	✓	×	×	✓
Haghani and Sarvi (2019) [91]	SFM	✓	✓	×	×	–	3	200	–	–	✓	×	×	✓
Sticco et al. (2020) [100]	SFM	✓	×	×	×	0.05	9	50–400	–	✓	–	×	×	✓
Xu et al. (2020) [86]	SFM	✓	×	×	×	–	–	60–250	–	–	–	×	×	✓
Kolivand et al. (2021) [175]	SFM	✓	✓	✓	×	–	–	4500	–	–	✓	×	×	✓
Wang et al. (2022) [101]	SFM	✓	✓	×	✓	–	≈5.3	–	–	✓	–	×	✓	✓
Subramanian et al. (2022) [87]	SFM	✓	✓	✓	×	–	–	–	–	✓	–	×	×	✓
Zanlungo et al. (2023) [104]	SFM	✓	✓	×	×	0.05	3	–	–	–	✓	×	×	✓
Song et al. (2023) [88]	SFM	✓	×	×	×	0.01	9	–	–	–	✓	×	×	✓
Van den Berg et al. (2008) [50]	VO	×	×	×	×	0.25	–	1000	12.5	–	–	×	×	✓
Van den Berg et al. (2008) [70]	VO	×	×	×	×	0.25	–	20K	2(6) ^a	–	–	×	✓	✓
Guy et al. (2009) [106]	VO	×	×	×	×	–	–	10K	302(32)	–	–	×	✓	✓
Van den Berg et al. (2011) [51]	VO	×	×	×	×	0.25	–	5K	42.4(8)	–	–	×	×	✓
Cutis et al. (2011) [113]	VO	×	×	×	×	0.1	7.3	35K	26	–	–	×	✓	✓
Kim et al. (2013) [111]	VO	✓	×	×	×	–	–	1600	229.6	–	–	×	✓	✓
Golas et al. (2013) [115]	VO	×	✓	×	✓	0.01	3.5(5.5) ^b	1000	192.3	✓	–	×	×	✓

(continued on next page)

Table 3 (continued).

Articles	Class	Behavior capability				Model performance				Validation		Multi-level		
		PI	GB	F	R	TS	MD	NA	RT	Fit FD	Real data	S	T	O
Kim et al. (2015) [112]	VO	✓	×	×	×	0.01–0.2	7.4	35K	5.7	–	–	×	✓	✓
Narang et al. (2015) [93]	VO	×	×	×	×	–	≈6.2	980	111.49	✓	✓	×	✓	✓
Narang et al. (2017) [108]	VO	×	×	×	×	–	2.5	200	185.2	–	✓	×	✓	✓
Sudkhot et al. (2023) [114]	VO	×	×	×	✓	0.25	–	40K	0.16	–	–	×	✓	✓
Burstedde et al. (2001) [6]	CA	×	×	×	×	0.3	(6.25)	–	–	–	–	×	✓	✓
Kirchner and Andreas (2002) [7]	CA	×	×	×	×	0.3	(6.25)	1116	–	–	–	×	✓	✓
Kirchner et al. (2003) [125]	CA	✓	×	×	×	0.3	(6.25)	–	–	–	–	×	✓	✓
Henein et al. (2004) [127]	CA	✓	×	×	×	0.3	(6.25)	–	–	–	–	×	✓	✓
Henein and White (2007) [128]	CA	✓	×	×	×	0.3	(6.25)	–	–	–	–	×	✓	✓
Guo et al. (2008) [118]	CA	✓	×	×	×	0.1	(6.25)	240	–	–	–	×	✓	✓
Ma et al. (2010) [176]	CA	×	×	✓	×	0.3	5.5(6.25)	–	–	✓	–	×	✓	✓
Zeng et al. (2011) [129]	CA	×	×	×	✓	0.3	(6.25)	200–300	–	–	–	×	✓	✓
Suma et al. (2012) [130]	CA	✓	×	×	✓	0.3	(6.25)	–	–	–	–	×	✓	✓
Bandini et al. (2014) [117]	CA	✓	×	×	×	0.33	7(12.5)	100–300	–	✓	–	×	✓	✓
Claudio et al. (2016) [122]	CA	✓	×	×	✓	–	6(10)	–	–	✓	✓	×	✓	✓
Lu et al. (2017) [126]	CA	×	✓	✓	×	0.3	(6.25)	750	–	–	✓	×	✓	✓
Jingwei et al. (2018) [116]	CA	×	×	×	×	–	8(8)	943	–	–	–	×	✓	✓
Fu et al. (2018) [119]	CA	✓	×	×	×	0.05–0.2	6(6.25)	1000	–	✓	✓	×	✓	✓
Zheng et al. (2019) [132]	CA	✓	×	×	×	1.0	(4.0)	300–800	–	–	–	×	✓	✓
Bazior et al. (2020) [120]	CA	✓	×	×	×	0.06–0.1	5.28(10.4)	–	–	–	–	×	✓	✓
Zhang et al. (2021) [177]	CA	×	×	×	✓	0.3	6.25(6.25)	–	–	✓	–	×	✓	✓
Huo et al. (2022) [123]	CA	✓	×	×	×	0.4	4.0(6.25)	180–600	–	–	–	×	✓	✓
Sarmady et al. (2022) [121]	CA	×	×	×	×	0.025	≈5.8(8)	–	–	✓	–	×	✓	✓
Zhang et al. (2023) [124]	CA	×	✓	×	×	0.33	1.77(6.25)	100	–	–	✓	×	✓	✓
Porzycki and Wąs (2023) [131]	CA	✓	×	✓	×	0.025 - 0.2	5	100–200	169.49	–	✓	×	✓	✓
Shao and Terzopoulos (2007) [52]	ABM	×	✓	✓	✓	–	–	1400	30	–	✓	✓	✓	✓
Robin et al. (2009) [137]	ABM	×	×	✓	×	–	–	–	–	–	✓	×	×	✓
Ondvrej et al. (2010) [138]	ABM	×	×	✓	×	–	–	200	25	–	–	×	×	✓

(continued on next page)

Deep learning perspectives

Despite the promising results obtained from the increasing amount of research on the deep learning approach for predicting human trajectories [29], the prediction of pedestrian trajectories in dense crowds remains a challenging problem. In high-density situations, pedestrian movements become more complex due to the increase in the number of neighbors and interactions. Furthermore, the majority of existing datasets for human trajectory prediction are collected in low-density situations, leading to deep

Table 3 (continued).

Articles	Class	Behavior capability				Model performance				Validation		Multi-level		
		PI	GB	F	R	TS	MD	NA	RT	Fit FD	Real data	S	T	O
Samuel et al. (2012) [53]	ABM	×	×	✓	×	0.1	1.9	–	–	✓	✓	×	×	✓
Zhou et al. (2016) [135]	ABM	×	×	×	×	–	3	50–200	–	✓	–	✓	✓	✓
Godoy et al. (2016) [139]	ABM	×	×	✓	×	0.025	–	300	–	–	–	×	×	✓
Liang et al. (2016) [140]	ABM	×	✓	✓	×	–	–	100	161.3	–	–	×	×	✓
Liu et al. (2018) [141]	ABM	×	✓	✓	×	–	–	100–500	–	–	–	×	✓	✓
Mingbi et al. (2018) [142]	ABM	×	×	✓	×	–	–	–	–	✓	✓	×	×	✓
Sharma et al. (2018) [136]	ABM	×	✓	×	×	–	–	35–75	–	✓	–	✓	✓	✓
Li et al. (2021) [143]	ABM	✓	✓	×	✓	–	≈1.1	8–64	–	–	✓	×	✓	✓
Xie et al. (2022) [144]	ABM	✓	✓	✓	×	–	–	200	–	–	✓	×	×	✓
Alahi et al. (2016) [54]	Data-driven	×	×	×	×	0.4	–	–	–	–	✓	×	×	✓
Lee et al. (2017) [145]	Data-driven	×	×	×	×	0.4	–	–	–	–	✓	×	×	✓
Agrim et al. (2018) [55]	Data-driven	×	×	×	×	0.4	–	–	–	–	✓	×	×	✓
Anirudh et al. (2018) [146]	Data-driven	×	×	×	×	0.4	–	–	–	–	✓	×	×	✓
Amir et al. (2019) [147]	Data-driven	×	×	×	×	0.4	–	–	–	–	✓	×	×	✓
Zhu et al. (2022) [148]	Data-driven	×	×	×	×	0.4	–	–	–	–	✓	×	×	✓
Korbmacher et al. (2023) [152]	Data-driven	×	×	×	×	0.4	–	–	–	–	✓	×	×	✓
Dang et al. (2023) [151]	Data-driven	×	×	×	×	0.4	–	–	–	–	✓	×	×	✓
Hughes (2002) [8]	Macro	×	×	×	✓	–	5.0	–	–	–	–	×	✓	✓
Treuille et al. (2006) [9]	Macro	×	×	×	✓	–	–	10K	12	–	–	×	✓	✓
Narain et al. (2009) [56]	Macro	×	×	×	×	–	–	25K	11.35	–	–	×	✓	✓
Jiang et al. (2010) [155]	Macro	×	×	×	✓	–	5.0(10)	–	–	–	–	×	✓	✓
Jiang et al. (2015) [156]	Macro	×	×	×	✓	–	1.8(10)	–	–	–	–	×	✓	✓
Zhao et al. (2019) [164]	Macro	×	×	×	×	–	> 7	2000	–	–	–	×	×	✓
Jebrane et al. (2019) [161]	Macro	✓	×	×	×	0.01	7(8.5)	–	–	–	–	×	×	✓
van Toll et al. (2020) [157]	Macro	✓	×	×	×	0.02	> 8.0	30K	51.9	–	–	×	✓	✓
Yuan et al. (2020) [158]	Macro	×	×	×	×	0.0001	4	–	–	–	–	×	✓	✓
van Toll et al. (2021) [159]	Macro	✓	×	×	×	0.02	7.23(8)	30K	53.85	–	–	×	✓	✓
Liang et al. (2021) [165]	Macro	✓	×	×	✓	–	≈ 6.0(7)	–	–	✓	–	×	✓	✓
Chen et al. (2021) [162]	Macro	✓	×	×	×	–	–	2688	24.2	–	–	×	✓	✓
Zhou et al. (2022) [163]	Macro	×	×	×	×	–	–	–	–	–	–	×	✓	✓

(continued on next page)

learning models being trained and tested on these datasets. To overcome this issue, one prospective solution is to use simulation models to generate synthetic data for training deep learning models. The combination of simulated and real-world data could help improve the accuracy of the models and generalize predictions in various scenarios, including high-density situations. Another area for future research is the development of new neural network architectures and training functions, particularly deep reinforcement

Table 3 (continued).

Articles	Class	Behavior capability				Model performance				Validation		Multi-level		
		PI	GB	F	R	TS	MD	NA	RT	Fit FD	Real data	S	T	O
Jiang et al. (2022) [178]	Macro	×	×	×	×	–	8(10)	3600	–	–	–	×	✓	✓
Li et al. (2023) [166]	Macro	×	×	×	×	–	4(7)	–	–	–	✓	×	✓	✓
Zhu et al. (2023) [167]	Macro	×	×	×	×	–	≈10	2000	–	–	✓	×	✓	✓
Xiong et al. (2009) [173]	Hybrid	×	×	✓	✓	–	≈4	4000	–	–	–	×	✓	✓
Xiong et al. (2010) [57]	Hybrid	×	×	×	×	–	≈2	–	–	–	–	×	×	✓
Anh et al. (2012) [58]	Hybrid	×	×	×	×	–	–	–	–	–	–	×	✓	✓
Xiong et al. (2013) [169]	Hybrid	×	×	×	×	0.5	≈6	1000	≈5.7	–	–	×	✓	✓
Rabiaa et al. (2016) [170]	Hybrid	×	×	×	×	–	–	–	–	–	–	×	✓	✓
Göttlich et al. (2018) [174]	Hybrid	✓	✓	×	×	0.0025	–	200	–	–	✓	×	×	✓
Yang et al. (2019) [59]	Hybrid	✓	✓	×	×	–	≈5(5.4)	–	–	–	–	×	×	✓
Biedermann et al. (2021) [171]	Hybrid	×	×	×	×	0.01	–	1000	–	–	–	×	×	✓
Dang et al. (2023) [172]	Hybrid	✓	✓	×	×	0.1	8	15 000	≈1.4	–	–	×	✓	✓

^a 2 fps on 6 cores.

^b Max simulated and theoretical density: 3.5 and 5.5 ped/m².

learning architectures that can better adapt to the challenges of predicting dense crowd behavior. Lastly, new evaluation metrics designed for dense crowd scenarios, instead of traditional distance-based metrics like ADE and FDE, are needed to assess the performance of these models accurately.

Integration of different modeling levels

In environments with complex geometries and obstacles, using only operational level models can result in pedestrians getting stuck in specific areas. Significant efforts have been made in creating the Menge framework [61], a crowd simulator that aggregates various modeling algorithms for different levels of pedestrian behavior. Nevertheless, this framework permits only a single modeling technique to be applied at each level for all pedestrians in the simulation.

Furthermore, most existing models focus on simulating a single pedestrian’s behavior, which limits their ability to fully address all phenomena in complex and varying-density environments. Pedestrian behavior also significantly varies between low-density and high-density settings, necessitating the use of combinations of different models at each level to simulate a wide range of phenomena. To address this, a general architecture that allows agents to change their model at each level of behavior dynamically could be a solution. The transition rules between different models must then be determined based on specific information about the environment and crowd dynamics.

Multi-scale simulation

In real-world mass-gathering events, pedestrians often travel not only individually but also in groups with their friends or family. Depending on the occasion, the percentage of participants in groups can range from 40% to 70% [20,37]. Besides, pedestrian flows emerge in crowded areas where people have the same local targets and similar velocities. Therefore, simulations of crowds in real-world dense events should be multi-scale, including simulations of individuals, groups, and flows. Hybrid modeling, which combines macroscopic, mesoscopic, and microscopic models, is an appropriate approach to handle these situations. Hybrid models could deal with the trade-off between detailed simulation of individuals and computational efficiency. However, choosing which models to apply and when and where to apply them must be determined depending on specific environments and simulation objectives.

Mechanisms for the aggregation and disaggregation of crowds are required for hybrid models to have a consistent transition between scale levels. Different entities in simulations, including individuals, groups, and flows, can be aggregated or disaggregated depending on specific conditions in environments and crowd dynamics (as seen in Fig. 10). Aggregation conditions of individuals to a group or flow can be considered by proximity metrics, which can be based on the similarity of local targets and velocity. Disaggregation from flow to individuals or from groups to individuals may occur when there is a difference in individual goals, instant velocities, and decisions. Furthermore, the dynamic identification of group and flow formation must be studied to have realistic simulations that can be better applied in a variety of scenarios.

Table 4

The assessments, advantages, and disadvantages of operational level models (PI: Physical Interaction, GB: Group behavior, F: Following, R: Replan, TSA: Time step adaptivity, D: Density, SS: Simulation speed, S: Scalability, IHM: Integration of high-level modeling, AHM: Ability of high-level modeling).

Models	Behavior Capability				Model performance				Multi-level	
	PI	GB	F	R	TSA	D	SS	S	IHM	AHM
SFMs	++	++	++	+	-	+	+	-	--	++
Advantages	Easy implementation by adding new types of forces to Newton's second Eq. (1) to describe new behavior				Stable with the time step of 0.01–0.1 s; Medium high density (2–6) ped/m ² ; Small scale (100–500 agents) and medium scale (500–5K agents) simulations				Easy to be integrated with high-level modeling technique	
Disadvantages	Need parameter calibration to avoid unrealistic behavior at high densities				Sensitive by large time step 0.2 s–0.4 s; Computational with large-scale simulation					
VO models	-	+	+	+	+	+	++	++	--	++
Advantages	Defining a new velocity obstacle space and new velocity constraints for the new behavior				Flexibility to various time step (0.01–0.25s); Density from 2–6 ped/m ² ; Capable of parallel running; Large-scale simulation (5K–100K agents)				Easy to be integrated with high-level modeling technique	
Disadvantages					Stuck phenomena in extremely high density					
CA models	-	-	-	-	--	++	-	-	++	-
Advantages	Adding new floor field for new behavior				Density of 2–8 ped/m ² ; Small and medium scale simulations					
Disadvantages	Increasing the model's complexity and requiring moderate code revisions				Time step depends on cell size; Computational with large-scale simulation				Tactical level modeling must be described in the form of floor field	
ABMs	+	+	+	++	+	+	-	-	-	++
Advantages	Behavior-oriented modeling				Small and medium scale simulations				Easy to be integrated with high-level modeling technique	
Disadvantages	Computational when simulating large numbers of agents with complex decision-making processes				Time step depends on modeling technique; Computational with large-scale simulation					
Data-driven models	--	--	--	--	--	--	-	-	--	-
Advantages										
Disadvantages	Lack of high-density datasets; New network architecture and new training functions must be introduced				Time step depends on frame rate in the training data; Prediction focuses on single primary pedestrian				Can not be integrated to high-level modeling	
Macroscopic models	-	-	-	+	-	++	++	++	++	--
Advantages					Small time step					
Disadvantages	Focusing on aggregate macroscopic phenomena information and of crowds				Sensitive by large time step				Tactical level modeling must be described in the form of potential field	
Hybrid models	+	+	+	+	-	+	+	+	+	+
Advantages	Dependent on the micro model in the hybrid framework								Easy to be integrated with high-level modeling to micro model	
Disadvantages					Time step synchronization between macro and micro models					

7. Conclusion

This paper presents a comprehensive review of pedestrian simulation models in high-density situations. A total of 116 articles over the past two decades have been selected and classified into three decision-making levels of pedestrian behaviors including strategic, tactical, and operational. The classification reveals that the majority of articles explore the operational level, where the main focus is on simulating interactions with neighboring pedestrians and obstacles. The most popular modeling techniques at each level are analyzed thoroughly to identify their strengths and limitations when applied in crowded scenarios, while less common particular methods are not covered within our review. Additionally, the models are evaluated and compared using multiple criteria tailored for dense crowd simulations, taking into account their ability to simulate common emergent behaviors in crowded situations, model performance, validation, and capacity to integrate into high-level modeling. The results provide critical insights for researchers and practitioners to select appropriate models for simulating crowds in high-density scenarios by considering different factors such as the desired level of detail of simulations, density level, computational efficiency, and scalability. Furthermore, our review highlights several potential future research directions. The importance of high-density data collection, deep learning perspectives, integration

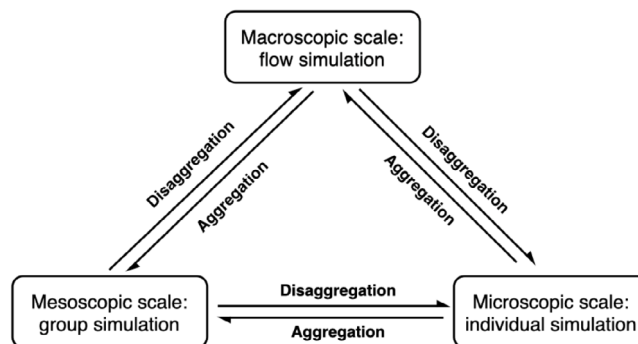


Fig. 10. Multi-scale simulation.

of different modeling levels, and multi-scale simulations are emphasized to enhance the realism, accuracy, and applicability of crowd simulation models. Our future research looks into data collection in real-world mass-gathering events and developing a modular framework to facilitate the integration of simulation models at different levels based on the characteristics of the environment and crowd dynamics.

Data availability

No data was used for the research described in the article.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the author(s) used GPT-4 in order to improve readability and language. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

Acknowledgments

The authors acknowledge the Franco-German research project MADRAS funded in France by the Agence Nationale de la Recherche (ANR, French National Research Agency), grant number ANR-20-CE92-0033, and in Germany by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation), grant number 446168800.

References

- [1] D. Thalmann, S.R. Musse, *Crowd Simulation*, Springer Science & Business Media, 2012.
- [2] M.-L. Xu, H. Jiang, X.-G. Jin, Z. Deng, Crowd simulation and its applications: Recent advances, *J. Comput. Sci. Tech.* 29 (5) (2014) 799–811.
- [3] H. Vermuyten, J. Beliën, L. De Boeck, G. Reniers, T. Wauters, A review of optimisation models for pedestrian evacuation and design problems, *Saf. Sci.* 87 (2016) 167–178.
- [4] H. Gayathri, P. Aparna, A. Verma, A review of studies on understanding crowd dynamics in the context of crowd safety in mass religious gatherings, *Int. J. Disaster Risk Reduct.* 25 (2017) 82–91.
- [5] D. Helbing, I. Farkas, T. Vicsek, Simulating dynamic features of escape panic, *Nature* 407 (2000) 487–490.
- [6] C. Burstedde, K. Klauck, A. Schadschneider, J. Zittartz, Simulation of pedestrian dynamics using a two-dimensional cellular automaton, *Phys. A* 295 (2001) 507–525.
- [7] A. Kirchner, A. Schadschneider, Simulation of evacuation processes using a bionics-inspired cellular automaton model for pedestrian dynamics, *Phys. A* 312 (2002) 260–276.
- [8] R. Hughes, A continuum theory for the flow of pedestrians, *Transp. Res. B* 36 (2002) 507–535.
- [9] A. Treuille, S. Cooper, Z. Popovic, Continuum crowds, *ACM Trans. Graph.* 25 (2006) 1160–1168.
- [10] F. Martínez-Gil, M. Lozano, I. García-Fernández, F. Fernández, Modeling, evaluation, and scale on artificial pedestrians: A literature review, *ACM Comput. Surv.* 50 (5) (2017) 1–35.
- [11] P. Karthika, V. Kedar, A. Verma, A walk accessibility-based approach to assess crowd management in mass religious gatherings, *J. Transp. Geogr.* 104 (2022) 103443.
- [12] M. Haghani, M. Sarvi, Crowd behaviour and motion: Empirical methods, *Transp. Res. Part B: Methodol.* 107 (2018) 253–294.
- [13] D.C. Duives, W. Daamen, S.P. Hoogendoorn, State-of-the-art crowd motion simulation models, *Transp. Res. Part C: Emerg. Technol.* 37 (2013) 193–209.
- [14] Y. Feng, D. Duives, W. Daamen, S. Hoogendoorn, Data collection methods for studying pedestrian behaviour: A systematic review, *Build. Environ.* 187 (2021) 107329.
- [15] S.P. Hoogendoorn, P.H. Bovy, Pedestrian route-choice and activity scheduling theory and models, *Transp. Res. B* 38 (2) (2004) 169–190.
- [16] E. Papadimitriou, G. Yannis, J. Golias, A critical assessment of pedestrian behaviour models, *Transp. Res. Part F: Traffic Psychol. Behav.* 12 (3) (2009) 242–255.
- [17] N. Bellomo, C. Dogbe, On the modelling crowd dynamics from scaling to hyperbolic macroscopic models, *Math. Models Methods Appl. Sci.* 18 (supp01) (2008) 1317–1345.
- [18] S. Zhou, D. Chen, W. Cai, L. Luo, M.Y.H. Low, F. Tian, V.S.-H. Tay, D.W.S. Ong, B.D. Hamilton, Crowd modeling and simulation technologies, *ACM Trans. Model. Comput. Simul. (TOMACS)* 20 (4) (2010) 1–35.

- [19] D. Helbing, A. Johansson, Pedestrian, crowd and evacuation dynamics, in: *Encyclopedia of Complexity and Systems Science*, Springer New York, New York, NY, 2009, pp. 6476–6495.
- [20] L. Cheng, R. Yarlagadda, C. Fookes, P. Yarlagadda, A review of pedestrian group dynamics and methodologies in modelling pedestrian group behaviours, *World J. Mech. Eng.* 1 (1) (2014) 1–13.
- [21] K. Ijaz, S. Sohail, S. Hashish, A survey of latest approaches for crowd simulation and modeling using hybrid techniques, in: *17th UKSIMAMSS International Conference on Modelling and Simulation*, 2015, pp. 111–116.
- [22] X. Shi, Z. Ye, N. Shiwakoti, O. Grembek, A state-of-the-art review on empirical data collection for external governed pedestrians complex movement, *J. Adv. Transp.* 2018 (2018).
- [23] A. Owaidah, D. Olaru, M. Bennamoun, F. Sohel, N. Khan, Review of modelling and simulating crowds at mass gathering events: Hajj as a case study, *J. Artif. Soc. Soc. Simul.* 22 (2) (2019).
- [24] S. Yang, T. Li, X. Gong, B. Peng, J. Hu, A review on crowd simulation and modeling, *Graph. Models* 111 (2020) 101081.
- [25] S.R. Musse, V.J. Cassol, D. Thalmann, A history of crowd simulation: the past, evolution, and new perspectives, *Vis. Comput.* 37 (12, SI) (2021) 3077–3092.
- [26] W. Van Toll, J. Pettre, Algorithms for microscopic crowd simulation: Advancements in the 2010s, *Comput. Graph. Forum* 40 (2) (2021) 731–754.
- [27] M. Lemonari, R. Blanco, P. Charalambous, N. Pelechano, M. Avraamides, J. Pettré, Y. Chrysanthou, Authoring virtual crowds: A survey, in: *Computer Graphics Forum*, vol. 41, Wiley Online Library, 2022, pp. 677–701.
- [28] J. Zhong, D. Li, Z. Huang, C. Lu, W. Cai, Data-driven crowd modeling techniques: A survey, *ACM Trans. Model. Comput. Simul. (TOMACS)* 32 (1) (2022) 1–33.
- [29] R. Korbmayer, A. Tordeux, Review of pedestrian trajectory prediction methods: Comparing deep learning and knowledge-based approaches, *IEEE Trans. Intell. Transp. Syst.* 23 (12) (2022) 24126–24144.
- [30] N. Basu, M.M. Haque, M. King, M. Kamruzzaman, O. Oviedo-Trespalacios, A systematic review of the factors associated with pedestrian route choice, *Transp. Rev.* 42 (5) (2022) 672–694.
- [31] A. Nicolas, F.H. Hassan, Social groups in pedestrian crowds: review of their influence on the dynamics and their modelling, *Transportmetrica A: Transp. Sci.* 19 (1) (2023) 1970651.
- [32] J. Adrian, N. Bode, M. Amos, M. Baratchi, M. Beermann, M. Boltes, A. Corbetta, G. Dezechache, J. Drury, Z. Fu, et al., A glossary for research on human crowd dynamics, *Collect. Dyn.* 4 (2019) 1–13.
- [33] W. Yu, A. Johansson, Modeling crowd turbulence by many-particle simulations, *Phys. Rev. E* 76 (4) (2007) 046105.
- [34] D. Helbing, A. Johansson, H. Al-Abideen, Dynamics of crowd disasters: An empirical study, *Phys. Rev. E, Stat., Nonlinear, Soft Matter Phys.* 75 (2007) 046109.
- [35] M. Wooldridge, *An Introduction to Multiagent Systems*, John Wiley & Sons, 2009.
- [36] S.P. Hoogendoorn, W. Daamen, Pedestrian behavior at bottlenecks, *Transp. Sci.* 39 (2) (2005) 147–159.
- [37] M. Moussaïd, N. Perozo, S. Garnier, D. Helbing, G. Theraulaz, The walking behaviour of pedestrian social groups and its impact on crowd dynamics, *PLoS One* 5 (4) (2010) e10047.
- [38] A. Garcimartín, I. Zuriguel, M. Pastor, C. Martín-Gómez, D. Parisi, Experimental evidence of the “faster is slower” effect, *Transp. Res. Procedia* 6 (2014) 760–767.
- [39] F. Lamarche, S. Donikian, Crowd of virtual humans: A new approach for real time navigation in complex and structured environments, in: *Computer Graphics Forum*, (3) Wiley Online Library, 2004, pp. 509–518.
- [40] S. Paris, S. Donikian, N. Bonvalet, Environmental abstraction and path planning techniques for realistic crowd simulation, *Comput. Animat. Virtual Worlds* 17 (3–4) (2006) 325–335.
- [41] A. Sud, R. Gayle, E. Andersen, S. Guy, M. Lin, D. Manocha, Real-time navigation of independent agents using adaptive roadmaps, in: *ACM SIGGRAPH 2008 Classes*, 2008, pp. 1–10.
- [42] M. Stubenschrott, C. Kogler, T. Matyus, S. Seer, A dynamic pedestrian route choice model validated in a high density subway station, *Transp. Res. Procedia* 2 (2014) 376–384.
- [43] B. Liu, Z. Liu, D. Sun, C. Bi, An evacuation route model of crowd based on emotion and geodesic, *Math. Probl. Eng.* 2018 (2018).
- [44] W.G. Van Toll, A.F. Cook, IV, R. Geraerts, Real-time density-based crowd simulation, *Comput. Animat. Virtual Worlds* 23 (1) (2012) 59–69.
- [45] X.-X. Jian, S. Wong, P. Zhang, K. Choi, H. Li, X. Zhang, Perceived cost potential field cellular automata model with an aggregated force field for pedestrian dynamics, *Transp. Res. Part C: Emerg. Technol.* 42 (2014) 200–210.
- [46] S.P. Hoogendoorn, F. van Wageningen-Kessels, W. Daamen, D.C. Duives, M. Sarvi, Continuum theory for pedestrian traffic flow: Local route choice modelling and its implications, *Transp. Res. Procedia* 7 (2015) 381–397.
- [47] Y. Jiang, B. Chen, X. Li, Z. Ding, Dynamic navigation field in the social force model for pedestrian evacuation, *Appl. Math. Model.* 80 (2020) 815–826.
- [48] Y. Sun, H. Liu, Crowd evacuation simulation method combining the density field and social force model, *Phys. A* 566 (2021) 125652.
- [49] D. Helbing, P. Molnar, Social force model for pedestrian dynamics, *Phys. Rev. E* 51 (1995).
- [50] J. van den Berg, M. Lin, D. Manocha, Reciprocal velocity obstacles for real-time multi-agent navigation, in: *Proceedings of 2008 IEEE International Conference on Robotics and Automation*, Pasadena, CA, USA, May 19–23, 2008, 2008, pp. 1928–1935.
- [51] J. van den Berg, S. Guy, M. Lin, D. Manocha, Reciprocal n-body collision avoidance, 70, 2011, pp. 3–19.
- [52] W. Shao, D. Terzopoulos, Autonomous pedestrians, *Graph. Models* 69 (5–6) (2007) 246–274.
- [53] S. Lemerrier, A. Jelic, R. Kulpa, J. Hua, J. Fehrenbach, P. Degond, C. Appert-Rolland, S. Donikian, J. Pettre, Realistic following behaviors for crowd simulation, *Comput. Graph. Forum* 31 (2012) 489–498.
- [54] A. Alahi, K. Goel, V. Ramanathan, A. Robicquet, L. Fei-Fei, S. Savarese, Social lstm: Human trajectory prediction in crowded spaces, in: *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 2016, pp. 961–971.
- [55] A. Gupta, J. Johnson, L. Fei-Fei, S. Savarese, A. Alahi, Social gan: Socially acceptable trajectories with generative adversarial networks, in: *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 2018, pp. 2255–2264.
- [56] R. Narain, A. Golas, S. Curtis, M.C. Lin, Aggregate dynamics for dense crowd simulation, in: *ACM SIGGRAPH Asia 2009 Papers*, 2009, pp. 1–8.
- [57] M. Xiong, M. Lees, W. Cai, S. Zhou, M.Y.H. Low, Hybrid modelling of crowd simulation, *Procedia Comput. Sci.* 1 (1) (2010) 57–65, ICCS 2010.
- [58] N.T.N. Anh, Z.J. Daniel, N.H. Du, A. Drogoul, V.D. An, A hybrid macro-micro pedestrians evacuation model to speed up simulation in road networks, in: *Advanced Agent Technology: AAMAS 2011 Workshops, AMPLE, AOSE, ARMS, DOCM 3 AS, ITMAS*, Taipei, Taiwan, May 2–6, 2011. Revised Selected Papers 10, Springer, 2012, pp. 371–383.
- [59] X. Yang, Q. Wang, Crowd hybrid model for pedestrian dynamic prediction in a corridor, *IEEE Access* 7 (2019) 95264–95273.
- [60] R. Korbmayer, A. Tordeux, Review of pedestrian trajectory prediction methods: Comparing deep learning and knowledge-based approaches, *IEEE Trans. Intell. Transp. Syst.* 23 (12) (2022) 24126–24144.
- [61] S. Curtis, A. Best, D. Manocha, Menge: A modular framework for simulating crowd movement, *Collect. Dyn.* 1 (2016) 1–40.
- [62] M. Haghani, M. Sarvi, Human exit choice in crowded built environments: Investigating underlying behavioural differences between normal egress and emergency evacuations, *Fire Saf. J.* 85 (2016) 1–9.
- [63] A. Edrisi, B. Lahoorpoor, R. Lovreglio, Simulating metro station evacuation using three agent-based exit choice models, *Case Stud. Transp. Policy* 9 (3) (2021) 1261–1272.

- [64] P.M. Kielar, A. Borrmann, Modeling pedestrians' interest in locations: A concept to improve simulations of pedestrian destination choice, *Simul. Model. Pract. Theory* 61 (2016) 47–62.
- [65] H.-J. Huang, R.-Y. Guo, Static floor field and exit choice for pedestrian evacuation in rooms with internal obstacles and multiple exits, *Phys. Rev. E* 78 (2) (2008) 021131.
- [66] G. Ren-Yong, H. Hai-Jun, Logit-based exit choice model of evacuation in rooms with internal obstacles and multiple exits, *Chin. Phys. B* 19 (3) (2010) 030501.
- [67] A. Abdelghany, K. Abdelghany, H. Mahmassani, W. Alhalabi, Modeling framework for optimal evacuation of large-scale crowded pedestrian facilities, *European J. Oper. Res.* 237 (3) (2014) 1105–1118.
- [68] W.-L. Wang, F.-F. Wan, S.-M. Lo, Game theory model of exit selection in pedestrian evacuation considering visual range and choice firmness, *Chin. Phys. B* 29 (8) (2020) 084502.
- [69] M.A. Lopez-Carmona, A.P. Garcia, Adaptive cell-based evacuation systems for leader-follower crowd evacuation, *Transp. Res. Part C: Emerg. Technol.* 140 (2022) 103699.
- [70] J. van den Berg, S. Patil, D. Manocha, M. Lin, Interactive navigation of individual agents in crowded environments, in: *Interactive 3D Graphics and Games, (I3D)*, 2008.
- [71] B. Patle, A. Pandey, D. Parhi, A. Jagadeesh, et al., A review: On path planning strategies for navigation of mobile robot, *Def. Technol.* 15 (4) (2019) 582–606.
- [72] R. Gayle, A. Sud, M.C. Lin, D. Manocha, Reactive deformation roadmaps: motion planning of multiple robots in dynamic environments, in: *2007 IEEE/RSJ International Conference on Intelligent Robots and Systems, IEEE*, 2007, pp. 3777–3783.
- [73] J.-C. Latombe, *Robot Motion Planning*, vol. 124, Springer Science & Business Media, 2012.
- [74] R. Oliva, N. Pelechano, Automatic generation of suboptimal navmeshes, in: *International Conference on Motion in Games*, Springer, 2011, pp. 328–339.
- [75] P. Zhang, X.-X. Jian, S. Wong, K. Choi, Potential field cellular automata model for pedestrian flow, *Phys. Rev. E* 85 (2) (2012) 021119.
- [76] P. Henry, C. Vollmer, B. Ferris, D. Fox, Learning to navigate through crowded environments, in: *2010 IEEE International Conference on Robotics and Automation, IEEE*, 2010, pp. 981–986.
- [77] P. Trautman, J. Ma, R.M. Murray, A. Krause, Robot navigation in dense human crowds: Statistical models and experimental studies of human-robot cooperation, *Int. J. Robotics Res.* 34 (3) (2015) 335–356.
- [78] C. Cao, P. Trautman, S. Iba, Dynamic channel: A planning framework for crowd navigation, in: *2019 International Conference on Robotics and Automation, ICRA, IEEE*, 2019, pp. 5551–5557.
- [79] S.J. Guy, J. Chhugani, S. Curtis, P. Dubey, M.C. Lin, D. Manocha, Pedestrians: A least-effort approach to crowd simulation, in: *Symposium on Computer Animation*, 2010, pp. 119–128.
- [80] S.A. Stüvel, N. Magnenat-Thalmann, D. Thalmann, A.F.v.d. Stappen, A. Egges, Torso crowds, *IEEE Trans. Vis. Comput. Graphics* 23 (7) (2017) 1823–1837.
- [81] W. Liao, A.U. Kemloh Wagoum, N.W. Bode, Route choice in pedestrians: determinants for initial choices and revising decisions, *J. R. Soc. Interface* 14 (127) (2017) 20160684.
- [82] A. Gabbana, F. Toschi, P. Ross, A. Haans, A. Corbetta, Fluctuations in pedestrian dynamics routing choices, *PNAS Nexus* 1 (4) (2022) pgac169.
- [83] N. Pelechano, J. Allbeck, N. Badler, Controlling individual agents in high-density crowd simulation, in: *Proc. Symp. Computer Animation*, vol. 2007, 2007, pp. 99–108.
- [84] M. Dridi, Simulation of high density pedestrian flow: A microscopic model, *Open J. Model. Simul. (OJMSI)* 3 (2015) 81–95.
- [85] I. Karamouzas, N. Sohre, R. Narain, S.J. Guy, Implicit crowds: Optimization integrator for robust crowd simulation, *ACM Trans. Graph.* 36 (4) (2017) 1–13.
- [86] T. Xu, D. Shi, J. Chen, T. Li, P. Lin, J. Ma, Dynamics of emotional contagion in dense pedestrian crowds, *Phys. Lett. A* 384 (3) (2020) 126080.
- [87] G.H. Subramanian, N. Choubey, A. Verma, Modelling and simulating serpentine group behaviour in crowds using modified social force model, *Phys. A* 604 (2022) 127674.
- [88] Y. Song, X. Hu, L. Shen, W. Weng, Modeling domino effect along the queue using an improved social force model, *Phys. A* 625 (2023) 129008.
- [89] T.I. Lakoba, D.J. Kaup, N.M. Finkelstein, Modifications of the Helbing-Molnar-Farkas-Vicsek social force model for pedestrian evolution, *Simulation* 81 (5) (2005) 339–352.
- [90] M. Ko, T. Kim, K. Sohn, Calibrating a social-force-based pedestrian walking model based on maximum likelihood estimation, *Transportation* 40 (1) (2013) 91–107.
- [91] M. Haghani, M. Sarvi, Simulating pedestrian flow through narrow exits, *Phys. Lett. A* 383 (2–3) (2019) 110–120.
- [92] M.M. Shuaib, Preserving socially expected crowd density in front of an exit for the reproduction of experimental data by modeling pedestrians' rear perception, *J. Stat. Mech. Theory Exp.* 2014 (10) (2014) P10037.
- [93] S. Narang, A. Best, S. Curtis, D. Manocha, Generating pedestrian trajectories consistent with the fundamental diagram based on physiological and psychological factors, *PLoS One* 10 (2015) e0117856.
- [94] V.M. Predtechenskii, A.I. Milinskii, *Planning for Foot Traffic Flow in Buildings*, Amerind Publishing, New Dehli, 1978.
- [95] M. Mōri, H. Tsukaguchi, A new method for evaluation of level of service in pedestrian facilities, *Transp. Res. Part A: General* 21 (3) (1987) 223–234.
- [96] A. Seyfried, B. Steffen, W. Klingsch, M. Boltes, The fundamental diagram of pedestrian movement revisited, *J. Stat. Mech. Theory Exp.* 10 (2005).
- [97] J. Zhang, W. Klingsch, A. Schadschneider, A. Seyfried, Transitions in pedestrian fundamental diagrams of straight corridors and T-junctions, *J. Stat. Mech. Theory Exp.* 2011 (2011) P06004.
- [98] Z. Fang, S. Lo, J.-a. Lu, On the relationship between crowd density and movement velocity, *Fire Saf. J. - Fire Saf. J.* 38 (2003) 271–283.
- [99] U. Weidmann, *Transporttechnik der Fußgänger: transporttechnische eigenschaften des fußgängerverkehrs, literaturauswertung, IVT Schriftenr.* 90 (1993).
- [100] I.M. Sticco, G.A. Frank, F.E. Cornes, C.O. Dorso, A re-examination of the role of friction in the original social force model, *Saf. Sci.* 121 (2020) 42–53.
- [101] W.-L. Wang, H.-C. Li, J.-Y. Rong, Q.-Q. Fan, X. Han, B.-H. Cong, A modified heuristics-based model for simulating realistic pedestrian movement behavior, *Chin. Phys. B* 31 (9) (2022) 094501.
- [102] F. Alonso-Marroquin, J. Busch, C. Chiew, C. Lozano, Á. Ramírez-Gómez, Simulation of counterflow pedestrian dynamics using spheropolygons, *Phys. Rev. E* 90 (6) (2014) 063305.
- [103] J. Song, F. Chen, Y. Zhu, N. Zhang, W. Liu, K. Du, Experiment calibrated simulation modeling of crowding forces in high density crowd, *IEEE Access* 7 (2019) 100162–100173.
- [104] F. Zanlungo, C. Feliciani, Z. Yücel, K. Nishinari, T. Kanda, Macroscopic and microscopic dynamics of a pedestrian cross-flow: Part II, modelling, *Saf. Sci.* 158 (2023) 105969.
- [105] P. Fiorini, Z. Shiller, Motion planning in dynamic environments using velocity obstacles, *Int. J. Robotics Res.* 17 (1998) 760.
- [106] S. Guy, J. Chhugani, C. Kim, N. Satish, M. Lin, D. Manocha, P. Dubey, ClearPath: Highly parallel collision avoidance for multi-agent simulation, in: *Eurographics/ ACM SIGGRAPH Symposium on Computer Animation*, 2009, pp. 177–187.
- [107] J. Snape, J. van den Berg, S.J. Guy, D. Manocha, The hybrid reciprocal velocity obstacle, *IEEE Trans. Robot.* 27 (4) (2011) 696–706.
- [108] S. Narang, A. Best, D. Manocha, Interactive simulation of local interactions in dense crowds using elliptical agents, *J. Stat. Mech. Theory Exp.* 2017 (2017) 033403.
- [109] A. Best, S. Narang, S. Curtis, D. Manocha, DenseSense: Interactive crowd simulation using density-dependent filters, in: *Symposium on Computer Animation*, 2014, pp. 97–102.

- [110] S. Curtis, D. Manocha, Pedestrian simulation using geometric reasoning in velocity space, ISBN: 978-3-319-02446-2, 2014, pp. 875–890.
- [111] S. Kim, S. Guy, D. Manocha, Velocity-based modeling of physical interactions in multi-agent simulations, in: Proceedings - SCA 2013: 12th ACM SIGGRAPH / Eurographics Symposium on Computer Animation, 2013.
- [112] S. Kim, S.J. Guy, K. Hillesland, B. Zafar, A.A.-A. Gutub, D. Manocha, Velocity-based modeling of physical interactions in dense crowds, *Vis. Comput.* 31 (5) (2015) 541–555.
- [113] S. Curtis, S. Guy, B. Zafar, D. Manocha, Virtual tawaf: A case study in simulating the behavior of dense, heterogeneous crowds, in: 2011 IEEE International Conference on Computer Vision Workshops, (ICCV Workshops), 2011, pp. 128–135.
- [114] P. Sudkhot, K. Wong, C. Sombathheera, Collision avoidance and path planning in crowd simulation, *ICIC Express Lett.* 17 (2023) 13.
- [115] A. Golas, R. Narain, S. Curtis, M. Lin, Hybrid long-range collision avoidance for crowd simulation, *IEEE Trans. Vis. Comput. Graph.* 20 (2013).
- [116] J. Ji, L. Lu, Z. Jin, S. Wei, L. Ni, A cellular automata model for high-density crowd evacuation using triangle grids, *Phys. A* (2018).
- [117] S. Bandini, M. Mondini, G. Vizzari, Modelling negative interactions among pedestrians in high density situations, *Transp. Res. Part C-Emerg. Technol.* 40 (2014) 251–270.
- [118] R.-Y. Guo, H.-J. Huang, A modified floor field cellular automata model for pedestrian evacuation simulation, *J. Phys. A* 41 (38) (2008) 385104.
- [119] Z. Fu, Q. Jia, J. Chen, J. Ma, K. Han, L. Luo, A fine discrete field cellular automaton for pedestrian dynamics integrating pedestrian heterogeneity, anisotropy, and time-dependent characteristics, *Transp. Res. Part C: Emerg. Technol.* 91 (2018) 37–61.
- [120] G. Bazio, D. Pałka, J. Waś, Using cellular automata to model high density pedestrian dynamics, ISBN: 978-3-030-50370-3, 2020, pp. 486–498.
- [121] S. Sarmady, F. Haron, A.Z. Talib, Simulation of pedestrian movements using a fine grid cellular automata model, *IAES Int. J. Artif. Intell.* 11 (4) (2022) 1197.
- [122] C. Feliciani, K. Nishinari, An improved cellular automata model to simulate the behavior of high density crowd and validation by experimental data, *Phys. A* 451 (2016).
- [123] F. Huo, Y. Li, C. Li, Y. Ma, An extended model describing pedestrian evacuation considering pedestrian crowding and stampede behavior, *Phys. A* 604 (2022) 127907.
- [124] Q. Zhang, J. Qu, Y. Han, Pedestrian small group behaviour and evacuation dynamics on metro station platform, *J. Rail Transp. Plan. Manag.* 26 (2023) 100387.
- [125] A. Kirchner, K. Nishinari, A. Schadschneider, Friction effects and clogging in a cellular automaton model for pedestrian dynamics, *Phys. Rev. E* 67 (5) (2003) 056122.
- [126] L. Lu, C.-Y. Chan, J. Wang, W. Wang, A study of pedestrian group behaviors in crowd evacuation based on an extended floor field cellular automaton model, *Transp. Res. Part C: Emerg. Technol.* 81 (2017) 317–329.
- [127] C.M. Henein, T. White, Agent-based modelling of forces in crowds, in: International Workshop on Multi-Agent Systems and Agent-Based Simulation, Springer, 2004, pp. 173–184.
- [128] C.M. Henein, T. White, Macroscopic effects of microscopic forces between agents in crowd models, *Physica A: Stat. Mech. Appl.* 373 (2007) 694–712.
- [129] J.-B. Zeng, B. Leng, Z. Xiong, Z. Qin, Pedestrian dynamics in a two-dimensional complex scenario using a local view floor field model, *Internat. J. Modern Phys. C* 22 (08) (2011) 775–803.
- [130] Y. Suma, D. Yanagisawa, K. Nishinari, Anticipation effect in pedestrian dynamics: Modeling and experiments, *Phys. A* 391 (1–2) (2012) 248–263.
- [131] J. Porzycki, J. Waś, Modeling spatial patterns in a moving crowd of people using data-driven approach—A concept of interplay floor field, *Saf. Sci.* 167 (2023) 106266.
- [132] Y. Zheng, X.-G. Li, B. Jia, R. Jiang, Simulation of pedestrians' evacuation dynamics with underground flood spreading based on cellular automaton, *Simul. Model. Pract. Theory* 94 (2019) 149–161.
- [133] C.W. Reynolds, Flocks, herds and schools: A distributed behavioral model, in: Proceedings of the 14th Annual Conference on Computer Graphics and Interactive Techniques, 1987, pp. 25–34.
- [134] D. Helbing, Agent-based modeling, in: *Social Self-Organization*, Springer, 2012, pp. 25–70.
- [135] M. Zhou, H. Dong, D. Wen, X. Yao, X. Sun, Modeling of crowd evacuation with assailants via a fuzzy logic approach, *IEEE Trans. Intell. Transp. Syst.* 17 (9) (2016) 2395–2407.
- [136] S. Sharma, K. Ogunlana, D. Scribner, J. Grynovicki, Modeling human behavior during emergency evacuation using intelligent agents: A multi-agent simulation approach, *Inf. Syst. Front.* 20 (4) (2018) 741–757.
- [137] T. Robin, G. Antonini, M. Bierlaire, J. Cruz, Specification, estimation and validation of a pedestrian walking behavior model, *Transp. Res. B* 43 (1) (2009) 36–56.
- [138] J. Ondřej, J. Pettré, A.-H. Olivier, S. Donikian, A synthetic-vision based steering approach for crowd simulation, *ACM Trans. Graph.* 29 (4) (2010) 1–9.
- [139] J.E. Godoy, I. Karamouzas, S.J. Guy, M. Gini, Implicit coordination in crowded multi-agent navigation, in: Thirtieth AAAI Conference on Artificial Intelligence, 2016, pp. 2487–2493.
- [140] L. He, J. Pan, W. Wang, D. Manocha, Proxemic group behaviors using reciprocal multi-agent navigation, in: 2016 IEEE International Conference on Robotics and Automation, ICRA, IEEE, 2016, pp. 292–297.
- [141] H. Liu, B. Liu, H. Zhang, L. Li, X. Qin, G. Zhang, Crowd evacuation simulation approach based on navigation knowledge and two-layer control mechanism, *Inform. Sci.* 436 (2018) 247–267.
- [142] M. Zhao, J. Zhong, W. Cai, A role-dependent data-driven approach for high-density crowd behavior modeling, *ACM Trans. Model. Comput. Simul. (TOMACS)* 28 (4) (2018) 1–25.
- [143] M. Li, P. Shu, Y. Xiao, P. Wang, Modeling detour decision combined the tactical and operational layer based on perceived density, *Phys. A* 574 (2021) 126021.
- [144] W. Xie, E.W.M. Lee, Y.Y. Lee, Simulation of spontaneous leader–follower behaviour in crowd evacuation, *Autom. Constr.* 134 (2022) 104100.
- [145] N. Lee, W. Choi, P. Vernaza, C.B. Choy, P.H. Torr, M. Chandraker, Desire: Distant future prediction in dynamic scenes with interacting agents, in: Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 2017, pp. 336–345.
- [146] A. Vemula, K. Muelling, J. Oh, Social attention: Modeling attention in human crowds, in: 2018 IEEE International Conference on Robotics and Automation, ICRA, IEEE, 2018, pp. 4601–4607.
- [147] A. Sadeghian, V. Kosaraju, A. Sadeghian, N. Hirose, H. Rezatofighi, S. Savarese, Sophie: An attentive gan for predicting paths compliant to social and physical constraints, in: Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, 2019, pp. 1349–1358.
- [148] K. Zhu, B. Li, W. Zhe, T. Zhang, Collision avoidance among dense heterogeneous agents using deep reinforcement learning, *IEEE Robot. Autom. Lett.* 8 (1) (2022) 57–64.
- [149] R. Korbmacher, T. Dang-Huu, A. Tordeux, N. Verstaevl, B. Gaudou, Differences in pedestrian trajectory predictions for high-and low-density situations, in: 14th International Conference on Traffic and Granular Flow, (TGF) 2022, Springer, 2022, in press.
- [150] P. Kothari, S. Kreiss, A. Alahi, Human trajectory forecasting in crowds: A deep learning perspective, *IEEE Trans. Intell. Transp. Syst.* 23 (7) (2021) 7386–7400.
- [151] H.-T. Dang, R. Korbmacher, A. Tordeux, B. Gaudou, N. Verstaevl, TTC-SLSTM: Human trajectory prediction using time-to-collision interaction energy, in: 2023 15th International Conference on Knowledge and Systems Engineering, KSE, 2023, pp. 1–6.
- [152] R. Korbmacher, H.-T. Dang, A. Tordeux, Predicting pedestrian trajectories at different densities: A multi-criteria empirical analysis, *Phys. A* (2023) 129440.

- [153] B. Andersson, R. Andersson, L. Håkansson, M. Mortensen, R. Sudiyo, B. Van Wachem, *Computational Fluid Dynamics for Engineers*, Cambridge University Press, 2011.
- [154] R.L. Hughes, The flow of human crowds, *Annu. Rev. Fluid Mech.* 35 (1) (2003) 169–182.
- [155] Y.-q. Jiang, P. Zhang, S. Wong, R.-x. Liu, A higher-order macroscopic model for pedestrian flows, *Phys. A* 389 (21) (2010) 4623–4635.
- [156] Y.-Q. Jiang, S.-G. Zhou, F.-B. Tian, A higher-order macroscopic model for bi-direction pedestrian flow, *Phys. A* 425 (2015) 69–78.
- [157] W. Van Toll, C. Braga, B. Solenthaler, J. Pettré, Extreme-density crowd simulation: Combining agents with smoothed particle hydrodynamics, in: *Proceedings of the 13th ACM SIGGRAPH Conference on Motion, Interaction and Games*, 2020, pp. 1–10.
- [158] Y. Yuan, B. Goñi-Ros, H.H. Bui, W. Daamen, H.L. Vu, S.P. Hoogendoorn, Macroscopic pedestrian flow simulation using smoothed particle hydrodynamics (SPH), *Transp. Res. Part C: Emerg. Technol.* 111 (2020) 334–351.
- [159] W. Van Toll, T. Chatagnon, C. Braga, B. Solenthaler, J. Pettré, SPH crowds: Agent-based crowd simulation up to extreme densities using fluid dynamics, *Comput. Graph.* (2021).
- [160] R.A. Gingold, J.J. Monaghan, Smoothed particle hydrodynamics: theory and application to non-spherical stars, *Mon. Not. R. Astron. Soc.* 181 (3) (1977) 375–389.
- [161] A. Jebrane, P. Argoul, A. Hakim, M. El Rhabi, Estimating contact forces and pressure in a dense crowd: Microscopic and macroscopic models, *Appl. Math. Model.* 74 (2019) 409–421.
- [162] Q. Chen, G. Luo, Y. Tong, X. Jin, Z. Deng, A linear wave propagation-based simulation model for dense and polarized crowds, *Comput. Animat. Virtual Worlds* 32 (1) (2021) e1977.
- [163] J. Zhou, S. Chen, C. Ma, S. Dong, Stability analysis of pedestrian traffic flow in horizontal channels: A numerical simulation method, *Phys. A* 587 (2022) 126528.
- [164] R. Zhao, Q. Hu, Q. Liu, C. Li, D. Dong, Y. Ma, Panic propagation dynamics of high-density crowd based on information entropy and aw-rasclé model, *IEEE Trans. Intell. Transp. Syst.* 21 (10) (2019) 4425–4434.
- [165] H. Liang, J. Du, S.C. Wong, A continuum model for pedestrian flow with explicit consideration of crowd force and panic effects, *Transp. Res. Part B: Methodol.* 149 (2021) 100–117.
- [166] C. Li, R. Zhao, C. Han, R. Arifur, Y. Ma, Q. Liu, Dynamic disturbance propagation model of pedestrian panic behaviors and Lyapunov-based crowd stability analysis, *Appl. Sci.* 13 (21) (2023) 11762.
- [167] W. Zhu, R. Zhao, H. Zhang, C. Li, P. Jia, Y. Ma, D. Wang, M. Li, Panic-pressure conversion model from microscopic pedestrian movement to macroscopic crowd flow, *J. Comput. Nonlinear Dyn.* 18 (12) (2023).
- [168] N. Siyam, O. Alqaryouti, S. Abdallah, Research issues in agent-based simulation for pedestrians evacuation, *IEEE Access* 8 (2019) 134435–134455.
- [169] M. Xiong, S. Tang, D. Zhao, A hybrid model for simulating crowd evacuation, *New Gener. Comput.* 31 (2013) 211–235.
- [170] C. Rabiaa, C. Foudil, Toward a hybrid approach for crowd simulation, *Int. J. Adv. Comput. Sci. Appl.* 7 (1) (2016).
- [171] D.H. Biedermann, J. Clever, A. Borrmann, A generic and density-sensitive method for multi-scale pedestrian dynamics, *Autom. Constr.* 122 (2021) 103489.
- [172] H.-T. Dang, B. Gaudou, N. Verstaavel, A multi-level density-based crowd simulation architecture, in: *International Conference on Practical Applications of Agents and Multi-Agent Systems*, Springer, 2023, pp. 64–75.
- [173] M. Xiong, W. Cai, S. Zhou, M.Y.H. Low, F. Tian, D. Chen, D.W.S. Ong, B.D. Hamilton, A case study of multi-resolution modeling for crowd simulation, in: *Proceedings of the 2009 Spring Simulation Multiconference*, 2009, pp. 1–8.
- [174] S. Göttlich, M. Pfirsching, A micro-macro hybrid model with application for material and pedestrian flow, *Cogent Math. Stat.* 5 (1) (2018) 1476049.
- [175] H. Kolivand, M.S. Rahim, M.S. Sunar, A.Z.A. Fata, C. Wren, An integration of enhanced social force and crowd control models for high-density crowd simulation, *Neural Comput. Appl.* 33 (2021) 6095–6117.
- [176] J. Ma, W.-g. Song, J. Zhang, S.-m. Lo, G.-x. Liao, K-nearest-neighbor interaction induced self-organized pedestrian counter flow, *Phys. A* 389 (10) (2010) 2101–2117.
- [177] X. Zhang, P. Zhang, M. Zhong, A dual adaptive cellular automaton model based on a composite field and pedestrian heterogeneity, *Phys. A* 583 (2021) 126334.
- [178] Y.-Q. Jiang, Y.-G. Hu, X. Huang, Modeling pedestrian flow through a bottleneck based on a second-order continuum model, *Phys. A* 608 (2022) 128272.
- [179] S. Pellegrini, A. Ess, K. Schindler, L. Van Gool, You'll never walk alone: Modeling social behavior for multi-target tracking, in: *2009 IEEE 12th International Conference on Computer Vision*, IEEE, 2009, pp. 261–268.
- [180] A. Lerner, Y. Chrysanthou, D. Lischinski, *Crowds by example*, in: *Computer Graphics Forum*, Wiley Online Library, 2007, pp. 655–664.
- [181] S. Cao, A. Seyfried, J. Zhang, S. Holl, W. Song, Fundamental diagrams for multidirectional pedestrian flows, *J. Stat. Mech. Theory Exp.* 2017 (3) (2017) 033404.