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The Legacy of Andrei Sakharov

J.P.Petit , H.Zejli

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Abstract : The fact that the standard model of cosmology Λ CDM cannot account for the presence of immense voids in the very large-scale structure, as well as the early birth of first-generation stars and galaxies, combined with the impossibility of explaining the absence of observations of primordial antimatter, means that we are forced to consider a paradigm shift, which is what the Janus model represents. This model has its origins in the one proposed by Andrei Sakharov in 1967. We retrace this genesis, focusing on the model's physical and mathematical coherence and show how the CPT symmetry of its model can be derived from topological assumptions.

1 - Introduction.

Between 1967 and 1980, A.Sakharov published articles ([1], [2], [3]) presenting a cosmological model with two universes, linked by an initial singularity, the Big Bang. The first is our own universe. The second is described by him as a twin universe. The time arrows of these two universes are antiparallel, and they are enantiomorphic, or mirrored. Through such a model, he proposed a possible answer to the lack of observation of primordial antimatter. For over half a century now, cosmology has been unable to conceal one of its greatest weaknesses. Not only has no answer been found to the fact that one particle of matter in a million escaped complete annihilation, but no explanation has yet been proposed for the absence of observation of the corresponding quantity of primordial antimatter. Sakharov's attention was focused on the violation of CP symmetry. To re-establish a generalized symmetry, he hypothesized that our universe would be endowed with a twin in which these violations would be reversed. Based on the fact that matter arises from the assembly of quarks and antimatter from the assembly of antiquarks, he hypothesized that the former reaction would have been faster than the latter in our sheet of universe, with the opposite situation in its twin. Thus, in our matter universe we would find a small remnant of matter, associated with an equivalent remnant of free-state antiquarks, with the opposite situation in this twin universe, which would therefore contain antimatter with a corresponding remnant of free-state quarks. However exotic this model may seem, we have to agree that it's the only answer we've been given to this loss of half the cosmic content. Thus it's only logical that we should try to examine the aspects of such a model.

2 – The significance of this inversion of time.

This T-symmetry refers to the inversion of the time coordinate. In 1970 [4], contributing to the development of symplectic geometry and its application to physics, mathematician J.M.Souriau provided the physical interpretation of this inversion of the time coordinate. The Gram matrix defining the Minkowski space is :

$$(1) \quad G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Its isometry group is the Poincaré group:

$$(2) \quad \begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix}$$

Where L is the matrix representing the Lorentz group, axiomatically defined by:

$$(3) \quad L^t G L = G$$

and where C is the quadrivector of space-time translations :

$$(4) \quad C = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

A very detailed commentary on the work can be found in reference [5]. The approach is based on the introduction of the space of motions as a dual of the Lie algebra of the group. This space of motions is thus a vector space of dimension 10, that of the group. Within these dimensions, one is identified with the energy E, of the motion, three others with its impulse, three with the spin (not quantized) and three others with a 3-vector qualified by Souriau as “passage f”. Together, they form a torsor :

(5)

$$M = \begin{pmatrix} 0 & -s_z & s_y & f_x \\ s_z & 0 & -s_x & f_y \\ -s_y & s_x & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{pmatrix} \quad P = \begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix}$$

$$\mu = (M, P) = \begin{pmatrix} M & -P \\ P^t & 0 \end{pmatrix}$$

with :

$$(6) \quad M = \begin{pmatrix} 0 & -s_z & s_y & f_x \\ s_z & 0 & -s_x & f_y \\ -s_y & s_x & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{pmatrix} \quad P = \begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix}$$

If we opt for a coordinate system accompanying motion, then the 3-vector f is zero. Seven parameters thus define a motion, defining a geodesic trajectory of Minkowski space. The action of the group on its motion space allows us to define all possible motions. This is written (equations (13.107) in chapter 13 of reference [4]).

$$(7) \quad M' = LM^{-1}L + C^{-1}P^{-1}L - LP^{-1}C$$

$$(8) \quad P' = LP$$

It's the second equation that sheds light on the physical significance of this inversion of the time coordinate. The Lorentz group has four related components:

- $\{L_n\}$ is the neutral component, which contains the neutral element of the group and reverses neither space nor time.

- $\{L_s\}$ reverses space, but not time. It is synonymous with P-symmetry.

Combining these two related components forms the orthochronous subgroup or restricted Lorentz group.

$$\{L_o\} = \{L_n\} \cup \{L_s\}$$

- $\{L_t\}$ reversing time but not space

- $\{L_{st}\}$ reversing both time and space.

By grouping these last two components together, we form the antichronous subset :

$$\{L_a\} = \{L_t\} \cup \{L_{st}\}$$

Finally, $\{L\} = \{L_o\} \cup \{L_a\}$ represents the complete Lorentz group.

The Poincaré group inherits this property. Classically, physics limits its use to its orthochronous components. Note that :

$$(9) \quad \{L_{st}\} = -\{L_n\}$$

$$(10) \quad \{L_t\} = -\{L_s\}$$

We can then present the complete Poincaré group as :

$$(11) \quad \begin{pmatrix} \lambda L_o & C \\ 0 & 1 \end{pmatrix} \quad \lambda = \pm 1$$

$\lambda = +1$ gives the restricted, orthochronous Poincaré group

$\lambda = -1$ gives the elements of the antichronous, time-reversing subgroup.

We can see that :

$$(12) \quad M' = L_o M {}^tL_o + \lambda C {}^tP {}^tL_o - \lambda L_o P {}^tC$$

$$(13) \quad P' = \lambda L_o P$$

This gives the result of reference [4]: time inversion is synonymous with energy inversion. This also leads to mass inversion [4].

$$(14) \quad m = \sqrt{P {}^t.P} \operatorname{sgn}(E)$$

Here we discover the physical interpretation of the model proposed by A. Sakharov: his second universe contains particles with negative energy and negative mass, when they have them.

3 – Introduction of electric charge.

This is synonymous with charge conjugation and matter-antimatter duality. The geometrical interpretation of this C-symmetry was given in 1964 by J.M.Souriau in chapter V of reference

[4]. The chapter is entitled "Relativity in 5 dimensions". The latter is then inscribed in Kaluza space, whose Gram matrix is :

$$(15) \quad \hat{G} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

This fifth dimension is of the space type. The associated dynamic group is :

$$(16) \quad \begin{pmatrix} \Lambda & \Gamma \\ 0 & 1 \end{pmatrix}$$

where Λ is the 5D analog of the Lorentz group and axiomatically defined by:

$$(17) \quad \Lambda^t \hat{G} \Lambda = \hat{G}$$

and where ζ , is the fifth dimension :

$$(18) \quad \Gamma = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \\ \Delta \zeta \end{pmatrix}$$

Whereas the Lorentz group is of dimension 6, this group is of dimension 10, bringing the number of dimensions of the isometry group of Kaluza space to 15. Adding a fifth dimension has the effect of endowing the moment space with additional dimensions, five in this case. In 2014 [7] the author restricted the extension of the group Λ under bounded space to :

$$(19) \quad \begin{pmatrix} 1 & 0 \\ 0 & \lambda L_o \end{pmatrix}$$

Translating this according to the group's action on 5D space gives :

$$(20) \quad \begin{pmatrix} 1 & 0 & \phi \\ 0 & \lambda L_o & C \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \zeta + \phi \\ \lambda L_o \xi + C \\ 1 \end{pmatrix}$$

This limits us to the subgroup of translations in this fifth dimension. The treatment by constructing the action of this new group on the dual of its Lie algebra endows the moment with an additional scalar, which can be identified with the electric charge q with :

$$(21) \quad q' = q$$

$$(22) \quad M' = L_0 M^t L_0 + \lambda C^t P^t L_0 - \lambda L_0 P^t C$$

$$(23) \quad P' = \lambda L_0 P$$

If we are satisfied with this subgroup, we obtain the constancy of this electric charge. But in reference [8], the author decides to rely on the orthochronous isometry subgroup, of dimension 15, with its two connected components. The action of the group reveals, in addition to the scalar q , a quadrivector Q . But this is the equivalent of the passage f , which appeared in the Poincaré group. Like the latter, it is cancelled out when we opt for a coordinate system linked to the particle. It is therefore not a characteristic of motion, a relevant attribute of momentum.

In 4D, the Lorentz group can be interpreted as the “group of hyperbolic rotations in 4D space”. As such, it preserves the length of the energy-impulse quadrivector P :

$$(24) \quad P^t G P = E^2 - p^2 c^2$$

In the same way, we can consider that the group represents “hyperbolic rotations in 5D”. Consider the pentavector:

$$(25) \quad \Pi = \begin{pmatrix} P \\ q \end{pmatrix} = \begin{pmatrix} E \\ p_x c \\ p_y c \\ p_{yc} \\ q \end{pmatrix}$$

The Group's action maintains its length:

$$(26) \quad \Pi^t \hat{G} \Pi = E^2 - p^2 c^2 - q^2$$

But in approach [8], these transformations no longer a priori preserve the electric charge q , which then becomes dependent on the chosen coordinate system. In reference [8], taking up the approach initiated in [6], the author opts for a closed fifth dimension, in which the radius of this “universe tube” becomes very small, of the order of Planck's length. He then rediscovers the invariance of electric charge and concludes [8], we quote

In this paper, we revisit the Kaluza-Klein theory from the perspective of the classification of elementary particles based on the coadjoint orbit method. The keystone conjecture is to consider the electric charge as an extra momentum on an equal footing with the mass and the linear momentum. We study the momentum map of the corresponding symmetry group \hat{G}_1 which conserves the hyperbolic metric. We show that the electric charge is not an invariant, *i.e.* it depends on the reference frame, which is in contradiction with the experimental observations. In other words, it is not the symmetry group of the Universe today as we know it. To avert this paradox, we scale the fifth coordinate and consider the limit when the cylinder radius ω vanishes. For the corresponding group \hat{G}_0 also of dimension 15, the charge is an invariant then independent of the frame of reference and the observer. On this ground, we propose a cosmological scenario in which the elementary particles of the early Universe are classified from the momenta of the group \hat{G}_1 , next the three former dimensions inflate quickly while the fifth one shrinks, leading to the 4D era in which as today the particles are characterized by the momenta of the group \hat{G}_0 . By this mechanism, the elementary particles can acquire electric charge as a by-product of the 4 + 1 symmetry breaking of the Universe. This work opens the way to the geometric quantization of charged elementary particles.

The expression for this characteristic dimension of this universal tube is given in [6] on page 412 :

$$(27) \quad \frac{\hbar}{e} \sqrt{\frac{\chi}{2\pi}}$$

χ being the Einstein constant taken equal to [6]:

$$(28) \quad \chi = -\frac{8\pi G}{c^2} = 1.856 \cdot 10^{-27} \text{ cm gr}^{-1}$$

By introducing numerical values, this characteristic length is $3.782 \cdot 10^{-32}$ cm. Dividing by 2π gives us the order of magnitude of Planck's length. In this view, the quantization of electric charge and its constancy are derived from the closure of the extra dimension associated with the decrease in the characteristic dimension associated with it.

This group refers to an extension of Poincaré's group, *i.e.* to a field-free, curvature-free universe. This construction of a five-dimensional relativity was suggested in 1964 in reference [6] and has been taken up again more recently in [8]. Note that it is in [6], page 413, that the link between charge conjugation and fifth-dimensional inversion is first mentioned.

By generalizing [9], we can envisage an extension of space-time to a space with 4 + p dimensions, all of which may see their characteristic dimensions reduced, like that of this fifth dimension, each of these collapses leading to the emergence and quantization of new quantum numbers, baryonic, leptonic, unique etc., the electric charge being only the first of these.

4 – Introduction of matter-antimatter symmetry.

The above technique can be extended and the letter ζ no longer designates one additional dimension, but p additional dimensions, according to :

$$(29) \quad \zeta = \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \dots \\ \zeta_p \end{pmatrix}$$

The Gramm matrix becomes :

$$(30) \quad \widehat{G} = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & -1 & 0 & 0 & \dots & 0 \\ 0 & 0 & -1 & 0 & \dots & 0 \\ 0 & 0 & 0 & -1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & -1 \end{pmatrix}$$

If we extend the group from the restricted Poincaré group and its orthochronous subgroup, the isometry group of this 4 + p-dimensional space is written as :

$$(31) \quad \begin{pmatrix} \widehat{\Lambda}_o & \widehat{\Gamma} \\ 0 & 1 \end{pmatrix}$$

Avec :

$$(32) \quad \widehat{\Lambda}_o^{-1} \widehat{G} \widehat{\Lambda}_o = \widehat{G}$$

The closure and shortening of the characteristic lengths associated with the additional p dimensions leads to quantization and constancy of the resulting quantum numbers. If the basic group is the orthochronous restricted Poincaré group, the number of related components remains limited to two. In passing, we can return to a simplified formulation where the group extension of is limited to p translations along p additional dimensions, ensuring the constancy of the p quantum charges that result:

$$(33) \quad \begin{pmatrix} 1 & 0 & \dots & 0 & 0 & \phi_1 \\ 0 & 1 & \dots & 0 & 0 & \phi_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 & \phi_p \\ 0 & 0 & \dots & 0 & L_o & C \\ 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \dots \\ \zeta_p \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \zeta_1 + \phi_1 \\ \zeta_2 + \phi_2 \\ \dots \\ \zeta_p + \phi_p \\ L_o \xi + C \\ 1 \end{pmatrix}$$

By introducing a charge symmetry, a C-symmetry, we double this number of related components by writing this group according to :

$$(34) \quad \begin{pmatrix} \mu & 0 & \dots & 0 & 0 & \phi_1 \\ 0 & \mu & \dots & 0 & 0 & \phi_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \mu & 0 & \phi_p \\ 0 & 0 & \dots & 0 & L_o & C \\ 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \dots \\ \zeta_p \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \mu\zeta_1 + \phi_1 \\ \mu\zeta_2 + \phi_2 \\ \dots \\ \mu\zeta_p + \phi_p \\ L_o\xi + C \\ 1 \end{pmatrix} \quad \mu = \pm 1$$

The corresponding group action is :

$$(35) \quad q_1' = \mu q_1 \quad ; \quad q_2' = \mu q_2 \quad ; \quad \dots \quad ; \quad q_p' = \mu q_p$$

$$(36) \quad M' = L_o M^t L_o + C^t P^t L_o - L_o P^t C$$

$$(37) \quad P' = L_o P$$

Symmetry $\mu = -1$ reverses all quantum charges.

5 – Group associated with A.Sakharov's model: the Janus group.

If we want to construct a group that translates the T-symmetry invoked by Sakharov, we'll replace L_o by λL_o with $\lambda = \pm 1$. But, as proposed in [7], we can translate what had already been proposed [1], we quote :

- All phenomena corresponding to $t < 0$ are, in this hypothesis, assumed to be CPT images of phenomena corresponding to $t > 0$.

§

$$(38) \quad \begin{pmatrix} \lambda\mu & 0 & \dots & 0 & 0 & \phi_1 \\ 0 & \lambda\mu & \dots & 0 & 0 & \phi_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda\mu & 0 & \phi_p \\ 0 & 0 & \dots & 0 & \lambda L_o & C \\ 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \dots \\ \zeta_p \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda\mu\zeta_1 + \phi_1 \\ \lambda\mu\zeta_2 + \phi_2 \\ \dots \\ \lambda\mu\zeta_p + \phi_p \\ \lambda L_o\xi + C \\ 1 \end{pmatrix} \quad \begin{matrix} \mu = \pm 1 \\ \lambda = \pm 1 \end{matrix}$$

The value $\lambda = -1$ is then synonymous with CPT-symmetry.

6 – Topology of the Janus Cosmological Model.

Andrei Sakharov therefore envisaged a pair of universes with CPT symmetry. We will show that this follows from topological assumptions. The idea behind Janus [10] is that the topology of the universe is that of the covering of a projective P^4 by a sphere S^4 .

Let's consider S^N spheres. The Euler Poincaré characteristic of these spheres is 2 if N is even and zero if N is odd. This has implications for tessellation. Spheres of odd dimensions can be equipped with regular tessellations, free of singularities. For example, the sphere S^3 can be paved with cubes. Odd-dimensional spheres, on the other hand, require two singularities. In the case of the S^2 sphere, these singularities are its poles. In the case of the S^4 sphere, these correspond to the Big Bang and Big Crunch singularities. By identifying diametrically opposed points, we can configure an S^N sphere as a two-sheet covering of a P^N projective. The Euler-Poincaré characteristic of the projective is half that of the sphere covering it. Odd-dimensional projectives are orientable. Odd-dimensional projectives are inorientable. When an even-dimensional sphere is configured as a covering for two projective sheets of the same dimension, the operation creates symmetries between adjacent sheets. We'll illustrate this point by taking the case of the S^2 sphere. This, by the way, will be presented as a didactic 2D image of a compact 4D space-time.

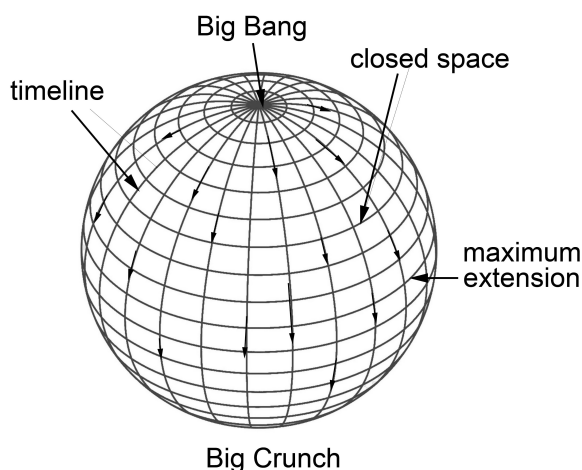


Fig.1 : 2D didactic image of a closed universe

Diametrically opposed, antipodal points can be brought into coincidence. The image is then that of a P^2 projective. The north and south poles, one representing the Big Bang and the other the Big Crunch, come into coincidence.

The following figure shows how this coincidence of antipodal regions generates T-symmetry. On the S^2 sphere, the direction of time is given by the orientation of the meridian curves. This orientation is shown on the left at the new state of maximum expansion, when space is identified with the sphere's equator. During this folding of the S^2 sphere, described in reference [11], the vicinity of this equator is configured as the two-sheet covering of a Möbius strip with three half-turns. See figure on the right.

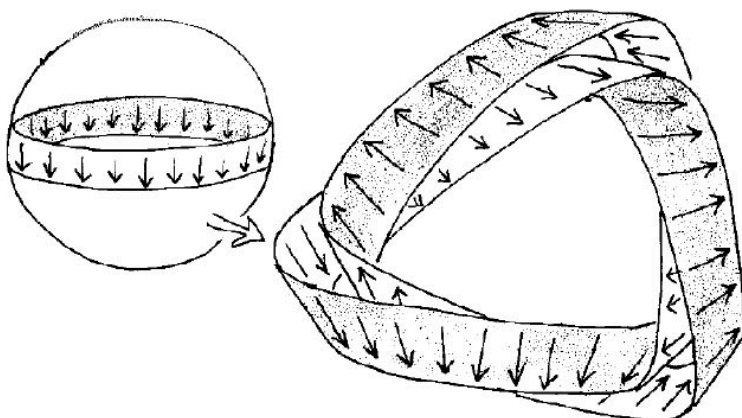


Fig.2 : How the coincidence of antipodal regions creates T-symmetry.

In the figure below, we evoke the appearance of T-symmetry by manipulating the vicinity of a meridian line. In addition, we evoke the possible elimination of the Big Bang - Big Crunch double singularity by replacing them with a tubular passage, which then gives this geometry the nature of the two-sheet lining of a Klein bottle.

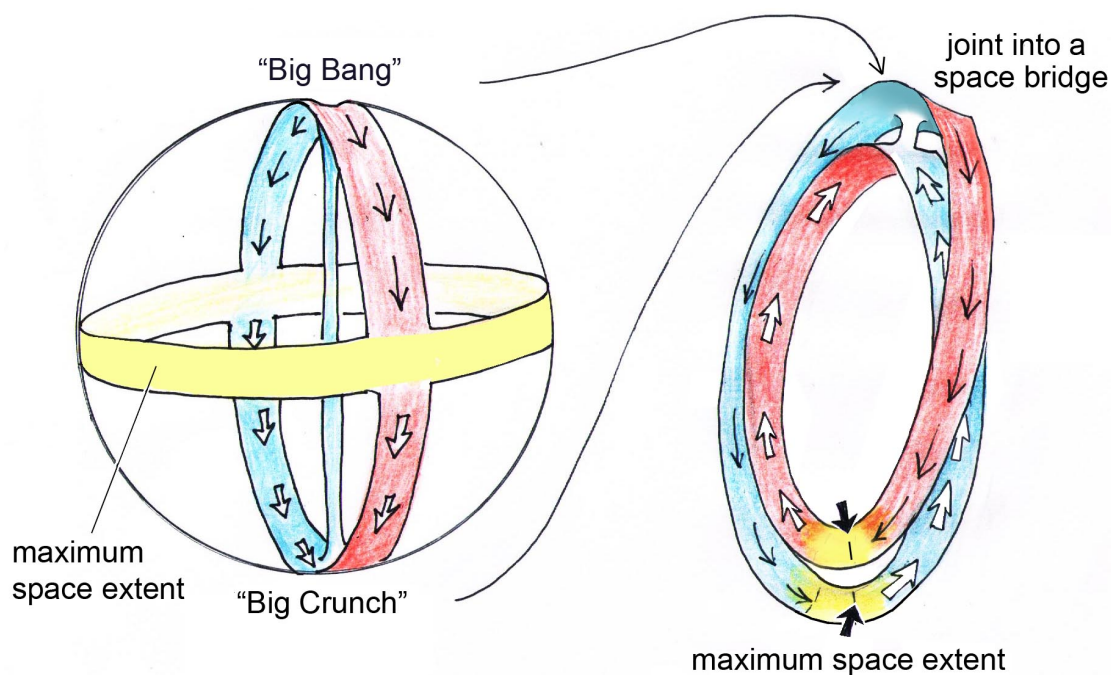


Fig.3 : Coincidence of antipodal regions on a sphere S^2 , according to the two-sheet coating of a half-turn Möbius strip, with the appearance of T-symmetry.

For enantiomorphy and P-symmetry to appear, the operation would have to be performed on a larger sphere. This aspect can be highlighted by considering the conjunction of antipodal regions in the vicinity of a meridian line, which is then configured according to the two-sheet covering of a half-turn Möbius strip. The following figure illustrates this enantiomorphic situation.

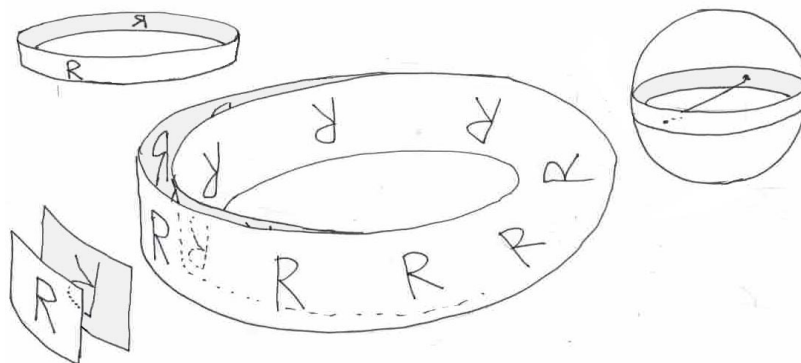


Fig.3 : P-symmetry as a consequence of contacting antipodal region neighborhoods on an S^2 sphere.

Configuring an S^4 sphere as a two-sheet covering of a P^4 projective creates this PT symmetry between adjacent sheets.

By bringing the antipodal points of even-dimensional spheres into coincidence, we locally create a two-sheet T-symmetrical configuration. In the case of the sphere S^2 , the image of the projective P^2 is its immersion in \mathbb{R}^3 , which corresponds to the surface described in 1903 by the German mathematician Werner Boy [12]. See next figure.

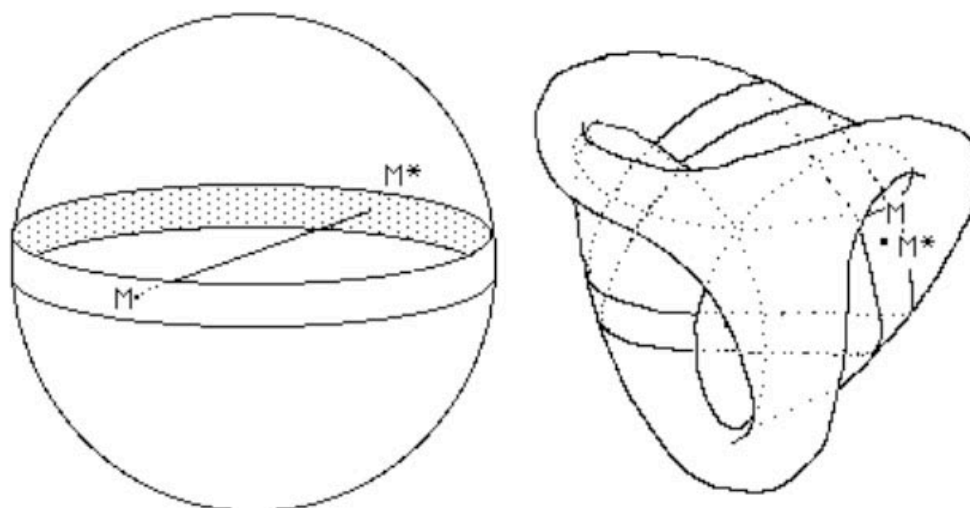


Fig.4 : Boy's surface, immersion of the P^2 projective in \mathbb{R}^3 .

Dans cette figure on montre comment la mise en coïncidence des points antipodaux de l'équateur de la sphère S^2 donne le revêtement à deux feuillets d'un ruban de Möbius à trois demi-tours.

In this section, we showed that the P and T symmetries invoked by A. Sakharov could be manifested as consequences of a purely topological structure, the covering of a P^4 projective. We then envisaged that the addition of the electric charge corresponded to the inscription of

movements in a space with an additional dimension, i.e. $4 + 1 = 5$. Under these conditions, we could envisage an S^5 sphere, with a P^5 projective. But then, not only is this S^5 sphere free of singularities, and thus no more Big Bangs and Big Crunches, but its two-sheet covering of an orientable P^5 projective doesn't enter into the desired symmetries.

We have seen that if the perimeter associated with the additional dimensions tends towards the Planck length, then the associated scalar becomes not only constant, but quantized to the values $\{-a, 0, +a\}$. We can then consider p additional dimensions, provided they are even. This leads us to spheres S^6, S^8, S^{10} . Under these conditions, the two singularities are brought into coincidence. Once again, this common singularity can be replaced by a tubular passage, giving the universe the topology of the decadicimensional equivalent of a Klein bottle.

7 – First attempt to introduce negative masses into the model.

Using dynamical group theory, we showed that this T-symmetry was synonymous with the introduction of negative masses into the cosmological model. A.Sakharov's primordial antimatter would therefore be endowed with negative mass. This first step is far from anecdotal since, if we neglect it, we admit to losing nothing less than half the universe from the outset. Is it then possible to introduce negative masses into the standard model of general relativity?

A first idea would be to consider that the field comes from two sources, represented by two tensors, the first referring to a positive mass content and the second to a negative mass content:

$$(39) \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi [T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)}]$$

We can then consider the metric solution corresponding to a region where the field is created, firstly by a positive mass content :

$$(40) \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi T_{\mu\nu}^{(+)}$$

Geodesics are given by a solution in the form of an external metric :

$$(41) \quad ds^2 = \left(1 - \frac{2GM^{(+)}}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM^{(+)}}{c^2 r}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

The geodesics evoke an attraction.

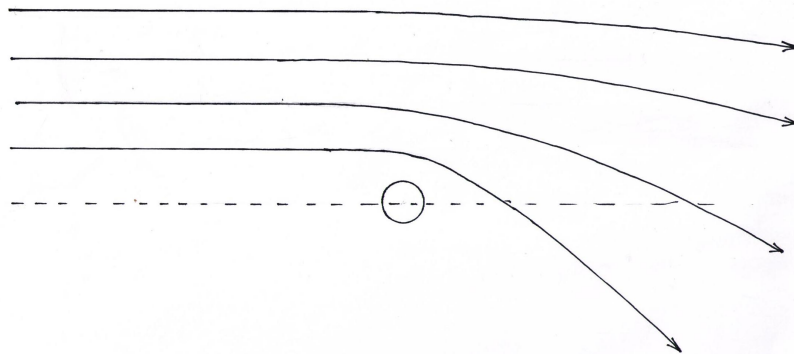


Fig.5 Geodesics created by a positive mass.

In this context, our single field equation provides only a single family of geodesics, which the test particles, with both positive and negative masses, must follow. We deduce that :

- *Positive masses attract both positive and negative masses.*

Now consider the field created by a negative mass $M^{(-)}$:

$$(42) \quad R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = \chi T_{\mu\nu}^{(-)}$$

The solution then corresponds to the metric :

$$(43) \quad ds^2 = \left(1 + \frac{2G|M^{(-)}|}{c^2 r} \right) c^2 dt^2 - \frac{dr^2}{1 + \frac{2G|M^{(-)}|}{c^2 r}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

The geodesics then represent a repulsion:

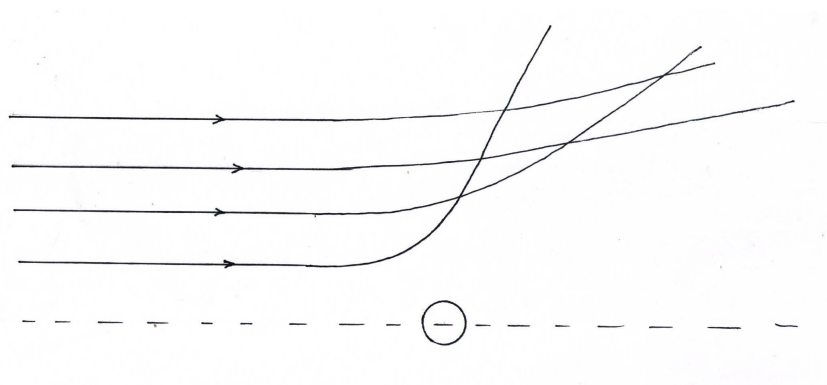


Fig.6 Geodesics created by negative mass.

We deduce that :

- *Negative masses repel both positive and negative masses.*

This result was illustrated in 1957 by H. Bondi [13]. If two masses of identical absolute values but opposite signs are brought together, the positive mass flees, pursued by the negative mass. Both then accelerate uniformly, but without any energy input, since the energy of the negative mass is itself negative. This phenomenon is known as runaway. What's more, this scheme violates the action-reaction principle. In 1957, the conclusion was reached that it was physically impossible to include negative masses in the cosmological model. This would only be possible at the price of a profound paradigmatic shift, not by denying the achievements of general relativity, but by considering its extension in a wider geometric context.

On en déduit que :

- Les masses négatives repoussent aussi bien les masses positives que les masses négatives.

8 – A paradigm shift to escape the crisis of today's cosmology.

In the mid-1970s, the excessive rotation speeds of stars in galaxies had already led specialists to propose the existence of dark matter, ensuring their cohesion. In 2011, the discovery that the cosmic expansion was accelerating was attributed to a new, unknown ingredient known as dark energy. Over the decades, all attempts to assign an identity to these new components ended in failure.

In 2017 [14] Hélène Courtois, Daniel Pomarède, Brent Tully and Yeudi Hoffman produced the first very-large-scale mapping of the universe, in a cube one and a half billion light-years across, with the Milky Way, our observation point, at the center. By subtracting the radial component of the velocity linked to the expansion motion, they indicate the trajectories followed by the masses. A dipolar structure appears. One formation, the Shapley attractor, comprising hundreds of thousands of galaxies, attracts galaxies to itself. But, symmetrically to this formation, 600 million light-years from the Milky Way, there is an immense void, some one hundred million light-years across, which, on the contrary, repels galaxies, and to which we give the name of dipole repeller. To date, no theory has been able to explain the existence of this vast void. While the idea of a gap in dark matter, positive and attractive, has been evoked, it doesn't hold water, as no mechanism has been found to give rise to it. Since 2017, several other such voids have been detected and located.

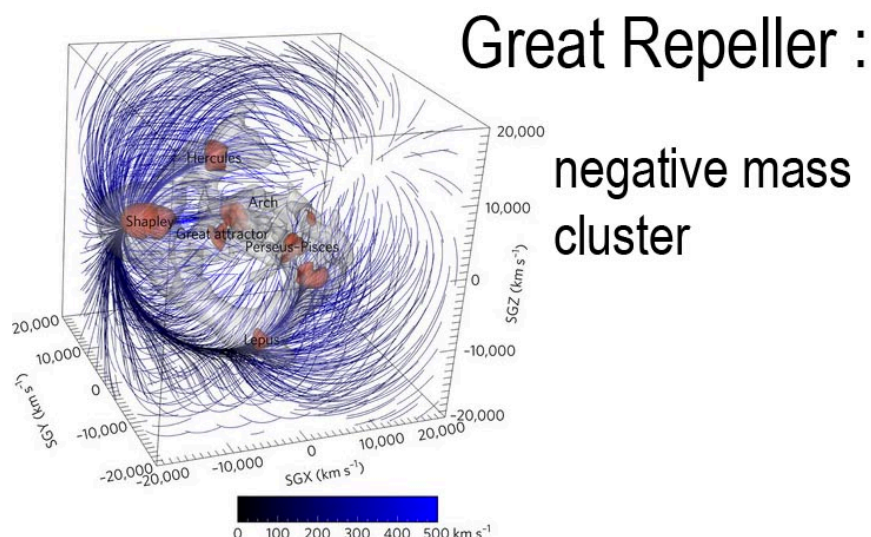


Fig.7 : The dipole repeller

The launch of the James Webb Space Telescope has only added to this crisis [15]. The Standard Model Λ CDM proposes a hierarchical mechanism for the birth of stars and galaxies. Gravitational instability appears as soon as matter and radiation are decoupled. The scenarios for the formation of both stars and galaxies in this model make use of the attributes conferred on hypothetical dark matter. But even with these parameters, it's impossible to imagine galaxies forming before a billion years. The Hubble Space Telescope was already able to obtain images in the near infrared. Early images of distant objects appeared to show groups of mini-galaxies. But the James Webb telescope showed that these objects were nothing other than HII regions belonging to barred spiral galaxies, fully formed, hosting old stars, only 500 million years old.

For decades, the Standard Model has relied on its ability to account for CMB fluctuations as gravito-acoustic oscillations, by adapting the numerous parameters relating to dark matter, dark energy and, in particular, the value of the Hubble constant. This desire to match observational data has resulted in a Hubble constant value of $67 \text{ kms}^{-1}\text{Mpc}^{-1}$. This is significantly lower than the value of $70 \text{ kms}^{-1}\text{Mpc}^{-1}$ deduced from direct observation of standard candles.

All these factors are creating a deep crisis within the specialist community, and some voices are beginning to be heard, suggesting the need to consider a profound paradigm shift. This is what the Janus cosmological model proposes.

Since we are unable to introduce negative masses into the general relativity model, let's consider a profound change of geometric paradigm, already evoked in the previous sections under the aspect of group theory and topology. The motion of positive masses, immersed in the gravitational field, takes place according to geodesics that we consider to be derived from a first metric $g_{\mu\nu}$. We will therefore describe the motion of negative masses using a second set of geodesics, derived from a second metric $\bar{g}_{\mu\nu}$. We thus have a manifold, whose points are marked by the coordinates $\{x^0, x^1, x^2, x^3\}$, equipped with a pair of metrics $(g_{\mu\nu}, \bar{g}_{\mu\nu})$.

We shall neglect the action of electromagnetic fields and consider only the field of gravity. From the metrics and we can construct Ricci tensors $R_{\mu\nu}$ and $\bar{R}_{\mu\nu}$ and their associated Ricci scalars R and \bar{R} .

9 – The Janus Cosmological Model.

To build this model, we'll start with the action.

$$(44) \quad J = \int_{D^4} \left[(R + S + \sigma) \sqrt{-g} + (\bar{R} + \bar{S} + \bar{\sigma}) \sqrt{-\bar{g}} \right] d^4x$$

Terms involving Ricci scalars are treated as classically, in the derivation of the Einstein equation from the Hilbert-Einstein action. It comes to:

$$(45) \quad \begin{aligned} \delta J = & \int_{D^4} \left[\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{-g}} \frac{\delta \sqrt{-g}}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g} S)}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \sigma)}{\delta g^{(+)\mu\nu}} \right] \sqrt{-g} \delta g^{\mu\nu} d^4x \\ & + \int_{D^4} \left[\frac{\delta \bar{R}}{\delta \bar{g}^{\mu\nu}} + \frac{\bar{R}}{\sqrt{-\bar{g}}} \frac{\delta \sqrt{-\bar{g}}}{\delta \bar{g}^{\mu\nu}} + \frac{1}{\sqrt{-\bar{g}}} \frac{\delta(\sqrt{-\bar{g}} \bar{S})}{\delta \bar{g}^{\mu\nu}} + \frac{1}{\sqrt{-\bar{g}}} \frac{\delta(\sqrt{-\bar{g}} \bar{\sigma})}{\delta \bar{g}^{\mu\nu}} \right] \sqrt{-\bar{g}} \delta \bar{g}^{\mu\nu} d^4x \end{aligned}$$

Posing:

$$(46) \quad \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g} S)}{\delta g^{\mu\nu}} = -\chi T_{\mu\nu}$$

$$(47) \quad \frac{1}{\sqrt{-\bar{g}}} \frac{\delta(\sqrt{-\bar{g}} \bar{S})}{\delta \bar{g}^{\mu\nu}} = -\chi \bar{T}_{\mu\nu}$$

$$(29) \quad \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \sigma)}{\delta g^{\mu\nu}} = \chi t_{\mu\nu}$$

$$(40) \quad \frac{1}{\sqrt{-\bar{g}}} \frac{\delta(\sqrt{-\bar{g}} \bar{\sigma})}{\delta \bar{g}^{\mu\nu}} = \chi \bar{t}_{\mu\nu}$$

Hence the system :

$$(49a) \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi \left[T_{\mu\nu} + \sqrt{\frac{\bar{g}}{g}} t_{\mu\nu} \right]$$

$$(49b) \quad \bar{R}_{\mu\nu} - \frac{1}{2} \bar{R} \bar{g}_{\mu\nu} = -\chi \left[\bar{T}_{\mu\nu} + \sqrt{\frac{\bar{g}}{\bar{g}}} \bar{t}_{\mu\nu} \right]$$

$T_{\mu\nu}$ is the source tensor of the field acting on positive masses and positive-energy photons. The term $\sqrt{\frac{\bar{g}}{\bar{g}}} t_{\mu\nu}$ is the source of this field attributed to the action of negative masses on these positive masses. The tensor $t_{\mu\nu}$ is therefore referred to as the interaction tensor. The terms $\bar{T}_{\mu\nu}$ and $\bar{t}_{\mu\nu}$ are the corresponding source elements of the second equation, $\bar{t}_{\mu\nu}$ being the second corresponding interaction tensor. Let's write this system of coupled equations in mixed notation:

$$(50a) \quad R_{\mu}^{\nu} - \frac{1}{2} R \delta_{\mu}^{\nu} = \chi \left[T_{\mu}^{\nu} + \sqrt{\frac{\bar{g}}{\bar{g}}} t_{\mu}^{\nu} \right]$$

$$(50b) \quad \bar{R}_{\mu}^{\nu} - \frac{1}{2} \bar{R} \delta_{\mu}^{\nu} = -\chi \left[\bar{T}_{\mu}^{\nu} + \sqrt{\frac{\bar{g}}{\bar{g}}} \bar{t}_{\mu}^{\nu} \right]$$

General relativity produces only a limited number of exact solutions. We'll follow the same logic.

10 – Construction of a time-dependent, homogeneous and isotropic solution.

Given the symmetry assumptions, the metrics then have the FLRW form. The variable x^0 is the common chronological coordinate (time marker).

$$(51a) \quad g_{\mu\nu} = dx^{02} - a^2 \left[\frac{du^2}{1 - ku^2} + u^2 d\theta^2 + u^2 \sin^2 \theta d\phi^2 \right]$$

$$(51b) \quad \bar{g}_{\mu\nu} = dx^{02} - \bar{a}^2 \left[\frac{du^2}{1 - \bar{k}u^2} + u^2 d\theta^2 + u^2 \sin^2 \theta d\phi^2 \right]$$

The determinants of the two metrics are :

$$(52) \quad g = -a^6 \sin^2 \theta \quad \bar{g} = -\bar{a}^6 \sin^2 \theta$$

As shown in reference [7] the treatment of the two equations leads to the compatibility relation :

$$(53) \quad \rho c^2 a^3 + \bar{\rho} \bar{c}^2 \bar{a}^3 = E = Cst$$

This translates into conservation of energy, extended to both populations. The exact solution, referring to two dust universes, corresponds to :

$$(54) \quad k = \bar{k} = -1$$

and :

$$(55a) \quad a^2 \frac{d^2 a}{dx^{o2}} = - \frac{4\pi G}{c^2} E$$

$$(55b) \quad \bar{a}^2 \frac{d^2 \bar{a}}{dx^{o2}} = + \frac{4\pi G}{c^2} E$$

A theoretical model is of no interest if it cannot be compared with observational data. The evolution of the positive species will correspond to an acceleration if the energy E of the system is negative. This provides a physical interpretation of the acceleration of the cosmic expansion ([16], [17]), which then follows from the fact that the energy content is predominantly negative. Numerical data have been successfully compared with observational data [18]. The corresponding curve is shown below.

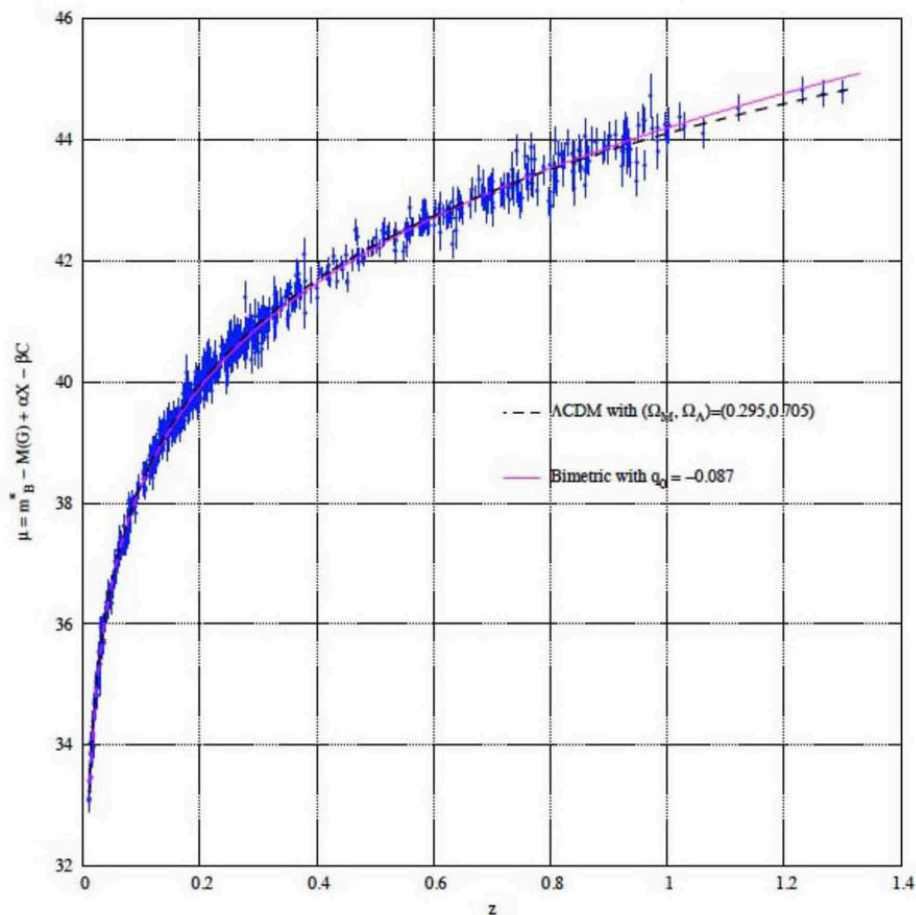


Fig.8 : Comparison of observed and theoretical magnitudes as a function of z.

To complete the model, we now need to provide exact stationary solutions. We'll restrict ourselves to $SO(3)$ symmetry.

11 – Lois d'interaction et conséquences au plan des observations.

From the metrics we note and we know how to construct covariant derivatives that we note:

$$(56) \quad \partial^\nu \quad \text{et} \quad \bar{\partial}^\nu$$

By construction we have :

$$(57) \quad \partial^\nu \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) = \bar{\partial}^\nu \left(\bar{R}_{\mu\nu} - \frac{1}{2} \bar{R} \bar{g}_{\mu\nu} \right) = 0$$

The corresponding covariant derivatives of the two second members must therefore also be zero, which corresponds to the Bianchi conditions, implying :

$$(58) \quad \partial^\nu T_{\mu\nu} = 0$$

$$(59) \quad \partial^\nu \left[\sqrt{\frac{g}{\bar{g}}} t_{\mu\nu} \right] = 0$$

$$(60) \quad \bar{\partial}^\nu \bar{T}_{\mu\nu} = 0$$

$$(61) \quad \bar{\partial}^\nu \left[\sqrt{\frac{\bar{g}}{g}} \bar{t}_{\mu\nu} \right] = 0$$

In stationary conditions, the square roots of the ratios of the determinants behave like constants, reflecting an "apparent mass effect". Conditions (59) and (61) can therefore be replaced by :

$$(62) \quad \partial^\nu t_{\mu\nu} = 0$$

$$(63) \quad \bar{\partial}^\nu \bar{t}_{\mu\nu} = 0$$

Let's write the system of equations in mixed notation, replacing the square roots, which have become constant, by the positive constants b^2 and \bar{b}^2 :

$$(64a) \quad R_\mu^\nu - \frac{1}{2} R \delta_\mu^\nu = \chi \left[T_\mu^\nu + b^2 t_\mu^\nu \right]$$

$$(64b) \quad \bar{R}_\mu^\nu - \frac{1}{2} \bar{R} \delta_\mu^\nu = -\chi \left[\bar{T}_\mu^\nu + \bar{b}^2 \bar{t}_\mu^\nu \right]$$

Using the Newtonian approximation, in both populations the non-zero tensor terms reduce to:

$$(65) \quad T_0^0 = \rho c^2 > 0 \quad t_0^0 = \bar{\rho} \bar{c}^2 < 0 \quad \bar{T}_0^0 = \bar{\rho} \bar{c}^2 < 0 \quad \bar{t}_0^0 = \rho c^2 > 0$$

In our system of coupled field equations, the presence of a minus sign in front of the second member of the second equation gives the following interaction laws:

- *Masses of the same sign attract each other*

- *Masses of opposite signs repel each other.*

We have thus eliminated the runaway effect.

The first conclusion to be drawn is that where one of the two types of mass is present, the other is absent, as immediately confirmed by simulations [19]. This is the case in the vicinity of the Sun, and under these conditions the first equation is identified with Einstein's 1915 equation. The model is therefore in line with all the classical local observational data of general relativity: Mercury's perihelion advance, deflection of light rays by the Sun. The model therefore does not invalidate that of general relativity, but presents itself as its extension, made essential to integrate the new observational data, which can no longer be managed by introducing the hypothetical components of dark matter and dark energy.

We have seen, in our construction of the unsteady solution, that negative energy dominates. The model is thus profoundly asymmetrical. Negative mass content will now replace the dark matter + dark energy pair. By the way, going back to the original idea, inspired by the work of Andrei Sakharov, this allows us to attribute a well-defined identity to these components. They are invisible, insofar as negative masses emit photons of negative energy that our optical instruments cannot capture. They are therefore simply copies of our own antimatter, assigned a negative mass. We then have a new distribution of contents:

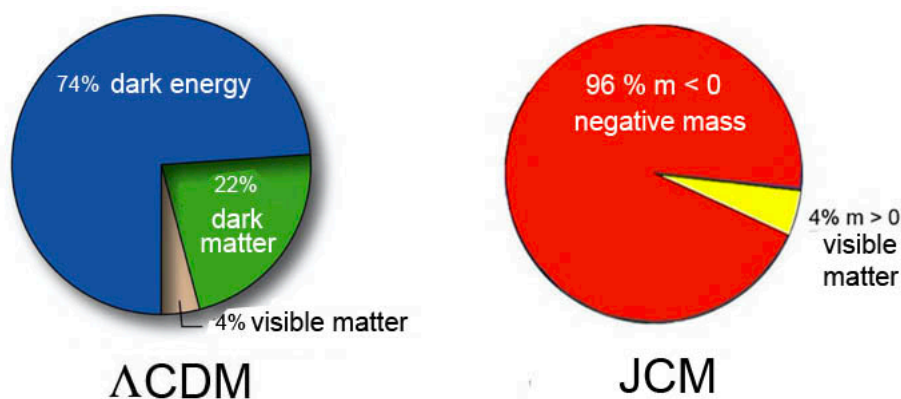


Fig.9 : Comparative contents of the Λ CDM and Janusus models :

At the moment of decoupling, when the gravitational instability can play its role (we must then speak of joint gravitational instabilities), the characteristic Jeans time is shorter for negative masses:

$$(66) \quad \bar{t}_j = \frac{1}{\sqrt{4\pi G |\bar{\rho}|}} \ll t_j = \frac{1}{\sqrt{4\pi G \rho}}$$

The result will be a regular distribution of negative-mass conglomerates of spheroidal antihydrogen and negative-mass antihelium. These will behave like immense negative-mass protostars. As soon as their temperature causes hydrogen reionization, their contraction will cease. These formations will then radiate in the red and infrared wavelengths. But their cooling time is then large compared to the age of the universe, which means that these objects will no longer evolve. The history of this negative world is totally different from our own. It will not give rise to stars, galaxies or planets. It will contain no atoms heavier than negative-mass antihelium. And there will be no life. And, as we'll see later: these negative formations are deliberately situated within the Newtonian approximation.

But there's another very important point. When these spheroidal conglomerates form, they confine the positive mass to the residual space, giving it a lacunar structure, comparable to joined soap bubbles. The negative mass is thus distributed in the form of thin plates, sandwiched between two negative conglomerates that exert a strong back pressure on it. The positive mass is thus violently compressed and heated. However, due to its plate-like arrangement, it can cool down very quickly through the emission of radiation..

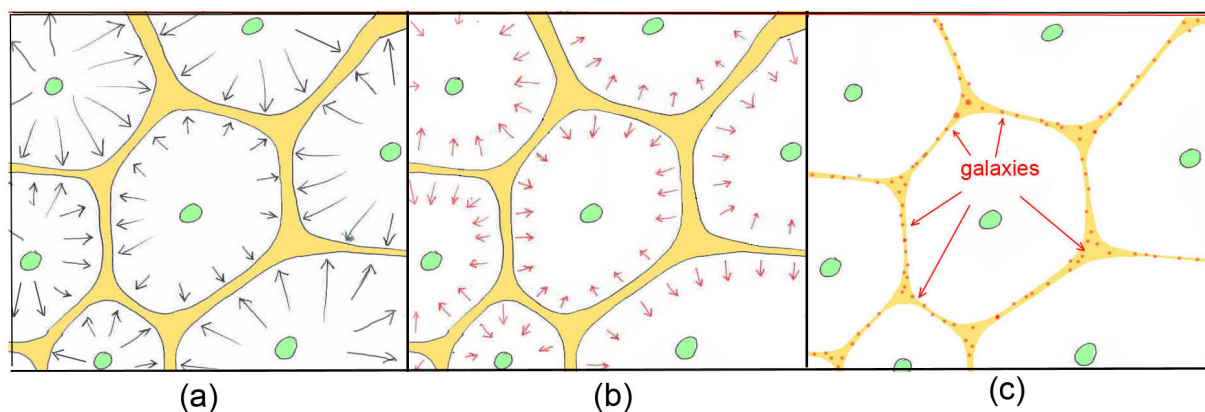


Fig.10 : Early rapid star and galaxy formation.

The result is a pattern of first-generation star and galaxy formation totally different from the standard one. This configuration had been the subject of simulations [19] since the first, heuristic, approach to the model, and the fact that objects all form within the first hundred million years was one of its predictions, largely confirmed by JWST data.

The lacunar structure, advocated as early as 1995, predicted the existence of large voids, which the discoveries of the dipole repeller and other similar large voids have also confirmed.

Once this lacunar structure has been formed, matter tends to concentrate along the segments common to three gaps, forming filaments. The nodes of this distribution will only develop into galaxy clusters.

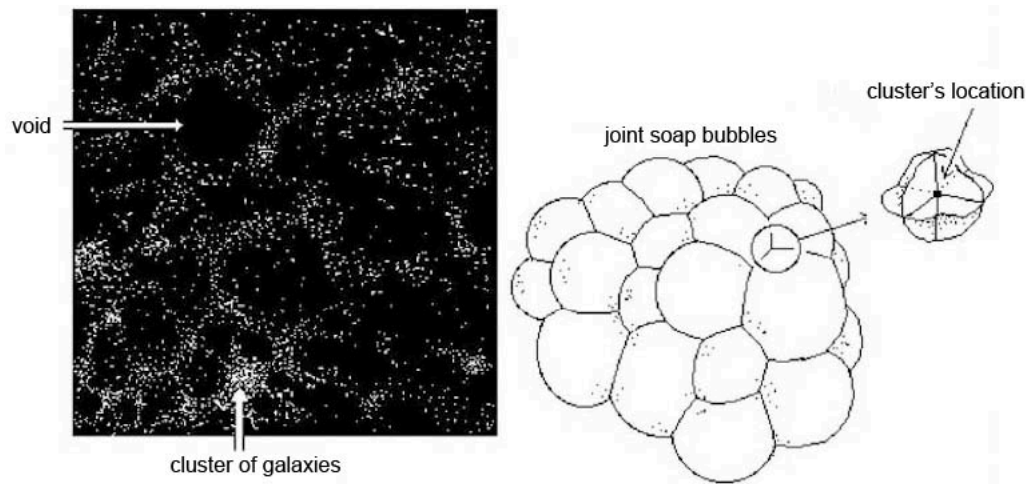


Fig.11 : Structure of positive mass in contiguous bubbles

12 – The problem of the model's mathematical consistency.

This is ensured in an isotropic, homogeneous and unsteady situation, the required condition being the generalized conservation of energy expressed by equation (53). We now turn to the case of stationary solutions, limiting ourselves to those that satisfy SO(3) symmetry. Bianchi conditions must then be satisfied, i.e. relations (58), (60), (62),(63).

First, we'll show the existence of asymptotic consistency in Newtonian approximation situations. Let's recall its various aspects.

- Velocities must be negligible compared to the speed of light. This is the case for velocities $\langle v \rangle$ and $\langle \bar{v} \rangle$ of thermal agitation in both media, which are involved in the definition of pressures and in both media. After decoupling :

$$(66) \quad \varepsilon p = \frac{\varepsilon \rho \langle v^2 \rangle}{3} \quad \varepsilon \bar{p} = \frac{\varepsilon \bar{\rho} \langle v^2 \rangle}{3}$$

- Curvature effects must be weak. We translate this by saying that the values of the radial coordinate must be large in front of the length characterizing curvature effects, i.e. the Schwarzschild radius :

$$(67) \quad \varepsilon R_s = \varepsilon \frac{2GM}{c^2} \ll r$$

12a – Newtonian approximation, field created by a positive mass M

SO(3) symmetry imposes the shapes of the two metrics:

$$(68a) \quad ds^2 = e^{\nu} dx^{\alpha 2} - e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

$$(68b) \quad d\bar{s}^2 = e^{\bar{\nu}} dx^{\alpha 2} - e^{\bar{\lambda}} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

The construction of a stationary solution then requires to calculate the functions :

$$\nu(r), \lambda(r), \bar{\nu}(r), \bar{\lambda}(r)$$

To locate this solution, we need to consider the shapes of the field source tensors:

$$(69) \quad T_{\mu}^{\nu}, t_{\mu}^{\nu}, \bar{T}_{\mu}^{\nu}, \bar{t}_{\mu}^{\nu}$$

Let's start by considering a situation where only positive mass is present. The tensors t_{μ}^{ν} and \bar{T}_{μ}^{ν} are then zero. The form of the tensor T_{μ}^{ν} is :

$$(70) \quad T_{\mu}^{\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -\varepsilon p & 0 & 0 \\ 0 & 0 & -\varepsilon p & 0 \\ 0 & 0 & 0 & -\varepsilon p \end{pmatrix}$$

As we're in the Newtonian approximation, ε is very small. With the introduction of the metric (68a) and the tensor (70) in the first field equation we are led to introduce the function $m(r)$ such that :

$$(71) \quad e^{-\lambda} = 1 - \frac{2m(r)}{r}$$

The classic calculation then leads to the relationship:

$$(72) \quad m(r) = 2 \frac{4\pi G r^3 \rho}{3c^2} \leq \frac{2GM}{c^2} = R_s$$

We then obtain the TOV equation (Tolman-Oppenheimer-Volkoff). Relation (72) places the small quantity in front of any quantity that will be neglected in the Newtonian approximation.:

$$(69) \quad \frac{dp}{dr} = - \frac{(\rho + \varepsilon p / c^2)(m + 4\pi \varepsilon G p r^3 / c^4) c^2}{r(r - 2\varepsilon G m / c^2)}$$

When ε tends to zero (or c tends to infinity) we get :

$$(70) \quad \frac{dp}{dr} = - \frac{\rho m c^2}{r^2} = - \frac{G\rho}{r^2} \frac{4 \pi r^3 \rho}{3}$$

The quantity $\frac{4 \pi r^3 \rho}{3}$ represents the amount of matter $\mu(r)$ contained inside a sphere of radius r . We know that the force of gravity exerted inside a mass of constant density is equivalent to that exerted by the mass located at the center of the sphere, and that the mass located outside this sphere gives a force of zero. So the quantity $-\frac{G\rho\mu(r)}{r^2}$ is the force of gravity, per unit volume, acting on the matter contained in an elementary volume around a point at distance r from the center. Thus the relation (70), which follows from the Newtonian approximation, expresses that the force of gravity balances the force of pressure. This is the classic Euler relationship.

To ensure the mathematical consistency of the system of two field equations, we therefore need to consider a form of the tensor t_{μ}^{ν} that gives back this same Euler relation when the Newtonian approximation is also applied to this solution. This is guaranteed with the form :

$$(71) \quad t_{\mu}^{\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & +\varepsilon p & 0 & 0 \\ 0 & 0 & +\varepsilon p & 0 \\ 0 & 0 & 0 & +\varepsilon p \end{pmatrix}$$

The calculation leads to the TOV equation equivalent of the second equation:

$$(72) \quad \frac{dp}{dr} = - \frac{(\rho - \varepsilon p / c^2)(m - 4\pi\varepsilon G p r^3 / c^4)c^2}{r(r + 2\varepsilon G m / c^2)}$$

This is not identical to equation (69) but, like it, identifies with the Euler equation when ε tending towards zero.

- We deduce that mathematical consistency is asymptotically assured when the conditions satisfy the Newtonian approximation.

It is then possible to construct the two metric solutions describing the geometry inside the sphere of radius r_0 containing matter of constant density ρ .

$$\text{Posing :} \quad \hat{R} = \sqrt{\frac{3c^2}{8\pi G \rho}} \quad r_s = \frac{2GM}{c^2} \text{ (Schwarzschild's length)}$$

I have used a lower-case letter r_s for the Schwarzschild radius to illustrate the fact that in the Newtonian approximation $r_s \ll r_0$. Conversely, for the characteristic radius associated with the interior metric I have used a capital letter \hat{R} , because then $\hat{R} \gg r_0$. The first interior metric corresponds to the classical solution :

$$(73a) \quad ds^2 = \left[\frac{3}{2} \sqrt{1 - \frac{r_0^2}{\hat{R}^2}} - \frac{1}{2} \sqrt{1 - \frac{r^2}{\hat{R}^2}} \right]^2 dx^{o2} - \frac{dr^2}{1 - \frac{r^2}{\hat{R}^2}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

The second is :

$$(73b) \quad d\bar{s}^2 = \left[\frac{3}{2} \sqrt{1 + \frac{r_0^2}{\hat{R}^2}} - \frac{1}{2} \sqrt{1 + \frac{r^2}{\hat{R}^2}} \right]^2 dx^{o2} - \frac{dr^2}{1 + \frac{r^2}{\hat{R}^2}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

These metrics connect with external metric solutions:

$$(74a) \quad ds^2 = \left(1 - \frac{r_s}{r} \right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

$$(74b) \quad d\bar{s}^2 = \left(1 + \frac{r_s}{r} \right) c^2 dt^2 - \frac{dr^2}{1 + \frac{r_s}{r}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

Let's now consider the case, still in the Newtonian approximation, where the geometry is determined by the presence of negative mass, corresponding to the dipole repeller..

12b – Newtonian approximation, field created by negative mass \bar{M}

We'll follow a similar procedure in every respect. The forms of the two metrics will remain (68a) and (68b), which simply reflect the assumption of SO(3) symmetry. The field equations become:

$$(75a) \quad R_\mu^v - \frac{1}{2} R \delta_\mu^v = \chi b^2 t_\mu^v$$

$$(75b) \quad \bar{R}_\mu^v - \frac{1}{2} \bar{R} \delta_\mu^v = -\chi \bar{T}_\mu^v$$

Our task is to shape the tensors \bar{T}_μ^v and t_μ^v in such a way that the geodesics derived from the inner and outer metrics reflect repulsion, and that the compatibility conditions of the two equations are satisfied when the Newtonian approximation is applied. This leads to the following ad hoc choices:

$$(76) \quad \bar{T}_{\mu}^{\nu} = \begin{pmatrix} \bar{\rho}\bar{c}^2 & 0 & 0 & 0 \\ 0 & +\varepsilon\bar{p} & 0 & 0 \\ 0 & 0 & +\varepsilon\bar{p} & 0 \\ 0 & 0 & 0 & +\varepsilon\bar{p} \end{pmatrix}$$

$$(77) \quad t_{\mu}^{\nu} = \begin{pmatrix} \bar{\rho}\bar{c}^2 & 0 & 0 & 0 \\ 0 & -\varepsilon\bar{p} & 0 & 0 \\ 0 & 0 & -\varepsilon\bar{p} & 0 \\ 0 & 0 & 0 & -\varepsilon\bar{p} \end{pmatrix}$$

The expressions are similar, but the signs of the pressure terms are reversed. This is due to the desire to find, in Newtonian approximation, the equivalent of Euler's equation in the object of negative mass, meaning that within it, pressure forms balance the force of gravity. For an observer made up of the same type of matter, this negative matter would behave in exactly the same way as our own. The change in sign is due to the presence of a minus sign in the second member of the second field equation.

Under these conditions, the second equation generates its own "TOV" equation:

$$(78) \quad \frac{d\bar{p}}{dr} = - \frac{(\bar{p} + \varepsilon\bar{p}/\bar{c}^2)(\bar{m} + 4\pi\varepsilon G \bar{p}r^3/\bar{c}^4)\bar{c}^2}{r(r - 2\varepsilon G \bar{m}/\bar{c}^2)}$$

With the first equation, we obtain:

$$(79) \quad \frac{d\bar{p}}{dr} = - \frac{(\bar{p} - \varepsilon\bar{p}/\bar{c}^2)(\bar{m} - 4\pi\varepsilon G \bar{p}r^3/\bar{c}^4)\bar{c}^2}{r(r + 2\varepsilon G \bar{m}/\bar{c}^2)}$$

Once again, by tending to zero or, what amounts to the same thing, by tending \bar{c} to infinity, these two equations can be identified with the Euler equation.

The asymptotic consistency of the Newtonian approximation is clearly restored.

Let's write :

$$(80) \quad \hat{R} = \sqrt{\frac{3\bar{c}^2}{8\pi G|\bar{\rho}|}} \quad \bar{r}_s = \frac{2G|\bar{M}|}{\bar{c}^2} \text{ (Schwarzschild's length)}$$

The inner metric of the positive sector becomes:

$$(81a) \quad ds^2 = \left[\frac{3}{2} \sqrt{1 + \frac{\bar{r}_0^2}{\hat{R}^2}} - \frac{1}{2} \sqrt{1 + \frac{r^2}{\hat{R}^2}} \right]^2 dx^{o2} - \frac{dr^2}{1 + \frac{r^2}{\hat{R}^2}} - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

That of negative sector :

$$(81b) \quad d\bar{s}^2 = \left[\frac{3}{2} \sqrt{1 - \frac{\bar{r}_0^2}{\bar{R}^2}} - \frac{1}{2} \sqrt{1 - \frac{r^2}{\bar{R}^2}} \right]^2 dx^{o2} - \frac{dr^2}{1 - \frac{r^2}{\bar{R}^2}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

These metrics connect with external metric solutions:

$$(82a) \quad ds^2 = \left(1 + \frac{\bar{r}_s}{r} \right) dx^{o2} - \frac{dr^2}{1 + \frac{\bar{r}_s}{r}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

$$(82b) \quad d\bar{s}^2 = \left(1 - \frac{\bar{r}_s}{r} \right) dx^{o2} - \frac{dr^2}{1 - \frac{\bar{r}_s}{r}} - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

13 – Dipole repeller prediction.

The Janus model is essentially falsifiable in Popper's sense. It predicted a large-scale twin structure with large voids. This has been confirmed. It predicted a very early birth of first-generation stars and galaxies. Also confirmed. A new prediction this time concerns the magnitude of sources located in the background of the large void. According to the model, the magnitude of the light emitted by these distant sources will be attenuated by negative lensing. This is a novel aspect, since it has been assumed that the two entities, positive and negative, interact only through antigravitation. Photons from these distant sources can then freely pass through the negative-mass conglomerates. This means that both external and internal geodesics must be used. The deflection effect of light rays will be greatest when they graze the surface of the object, with radius \bar{r} . This effect weakens as you move deeper into the object, becoming zero when the photons pass through its center..

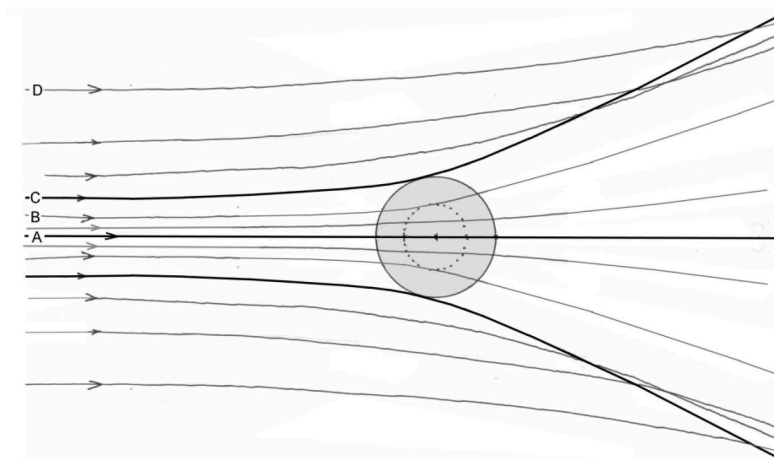


Fig.12 – Photon paths through the negative-mass object.

Eventually, we'll be able to map the magnitudes of objects in the background of the dipole repeller. Schematically, their luminosity will be attenuated in a ring-shaped formation.:

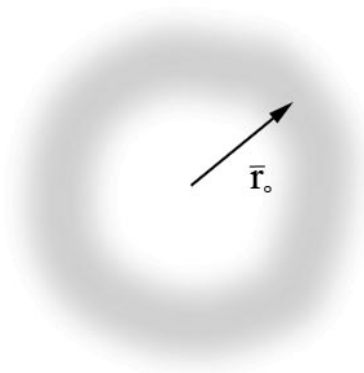


Fig.13 – Attenuation of the magnitude of objects in the background of the dipole repeller

This measurement will immediately give us the value of the radius \bar{r}_0 of this formation.

14- Beyond the Newtonian approximation..

These objects are absent in the negative world. In the positive world, objects that deviate from the Newtonian approximation are neutron stars and hypermassive objects located at the center of galaxies, which early images show to be the seat of a strong gravitational redshift effect, darkening their central part. These objects are a priori manageable using the classic pair of outer and inner metrics, taking rotation into account. It should be remembered that we are under no obligation to provide the form of the source tensor of the other sector, in this case an interaction tensor, whose form would be precisely imposed by the Bianchi condition. It's conceivable that one day someone will provide the exact form of this tensor.

But even in the absence of such an object, there is no a priori inconsistency.

15 – Conclusion.

The genesis of the Janus model spanned several decades. The starting point in 1965 was the attempt by the Russian Andrei Sakharov to provide the beginnings of an explanation for the absence of observations of primordial antimatter, which remains a major flaw in the Standard Model, which provides no explanation for such loss of half the universe's content. He therefore suggested a universe structure with two sectors, the second being T-symmetrical to our own. A few years later, in 1970, as an application of symplectic geometry, the mathematician Jean-Marie Souriau showed that this inversion of the time coordinate, this T-symmetry, is synonymous with the inversion of energy and mass. Taking this desire to reconstitute global symmetry a step further, Sakharov envisaged what he called a twin universe as CPT-symmetrical with our own. In this view, the invisible components of the

universe boil down to negative-mass antimatter. In 1994, we suggested that this universe structure corresponds to the two folds cover of a projective P^4 by a compact universe with the topology of an S^4 sphere. The two singularities of this spherical universe, the Big Bang and the Big Crunch, then coincide. By substituting a tubular structure, these singularities disappears. This configuration combines two PT-symmetrical sheets. These adjacent sectors are assumed to interact exclusively through gravity. The interaction between positive masses and negative masses in the other sector must therefore be considered. However, the introduction of negative masses is impossible within the framework of general relativity, as it would lead to interaction laws that are incompatible with physics. A bimetric model is therefore envisaged. A system of coupled field equations is then constructed from an action. Its form eliminates the unmanageable runaway effect. The model's interaction laws are such that masses of the same sign attract each other according to Newton's law, while masses of opposite signs repel each other according to anti-Newton's law. As these masses are mutually exclusive, the negative mass can be neglected in the vicinity of the Sun, and the first field equation is then identified with Einstein's equation. In this way, the model is consistent with local relativistic observations, such as the advance of Mercury's perihelion and the deflection of light rays by the Sun. The model is therefore an extension of the general relativity model. An exact, unsteady solution is constructed, revealing a generalized energy conservation law, extended to both sectors. Adaptation of the model to observations, revealing an acceleration of the expansion, then imposes a total dissymmetry between the two entities in presence. The vast majority of negative mass replaces the hypothetical components of dark matter and dark energy. The result is a distribution of 4% visible matter and 96% negative mass, invisible because it emits photons of negative energy that escape our observation instruments. This dissymmetry means that, after decoupling, the negative masses form a regular network of spheroidal conglomerates, while the positive mass, confined to the remaining space, adopts a patchy distribution. The model accounts for the existence of large voids, the dipole repeller having been the first of these. At the center of these large voids are invisible spheroidal conglomerates, behaving like gigantic protostars, with a cooling time greater than the age of the universe. These objects, which emit negative-energy photons corresponding to light in the red and infrared regions, do not evolve, nor do they give rise to stars, galaxies or atoms heavier than helium. Life is therefore absent from this negative sector, which is made up of a mixture of negative-mass antihydrogen and antihelium. The model also accounts for the very early birth of first-generation stars and galaxies, as recently demonstrated by the James Webb telescope. We then examine the question of the model's mathematical consistency, i.e. whether the Bianchi conditions are satisfied. We show that these can be asymptotically satisfied under conditions corresponding to the Newtonian approximation. We then settle the question of objects that don't fit into this approximation, essentially located on the positive side, by simply pointing out that we are under no obligation to provide the exact form of the interaction tensor, which is then determined by the zero divergence condition, and that the lack of definition of this tensor in no way invalidates the consistency of a non-linear solution.

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