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PhysOM: Physarum polycephalum Oriented Microstructures

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Figure 1: Our parameterized 3D anisotropic Physarum polycephalum model is able to generate foam-like cells and filaments ranging from coarse to dense cells (left side), to varying orientation and elongation magnitude (right side).

Abstract

Biological shapes possess facinating properties and behaviors that are the result of emergent mechanisms: they can evolve over time, dynamically adapt to changes in their environment, while also exhibiting interesting mechanical properties and aesthetic appeal. In this work, we bring and extend an existing biological-inspired model of the Physarum polycephalum, aka the blob, to the field of computer graphics, in order to design porous organic-like microstructures that resemble natural foam-like cells or filament-like patterns with variable local properties. In contrast to approaches based on static global optimization that provides only limited expressivity over the result, our method allows precise control over the local orientation of 3D patterns, relative cell extension, and precise infill of shapes with well defined boundaries. To this end, we extend the classical agent-based model for Physarum to fill an arbitrary domain with local anisotropic behavior. We further provide a detailed analysis of the model parameters, contributing to the understanding of the system behavior. The method is fast, parallelizable and scalable to large volumes, and compatible with user interaction, allowing a designer to guide the structure, erase parts, and observe its evolution in real-time. Overall, our method provides a versatile and efficient means of generating intricate organic microstructures that have potential applications in fields such as additive manufacturing, design, or biological representation and engineering.

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CCS Concepts

• Computing methodologies \rightarrow Shape modeling;

1. Introduction

Microstructure-made synthetic materials, which refer to smallscale structures filling the interior domain of a larger shape, have become increasingly popular in shape design. These materials offer desirable mechanical properties, including local stiffness, lightness, 5 and porosity [WWG21], that can be tailored by varying the type 6 of infill patterns. Furthermore, these microstructures exhibit visually compelling appearances, mimicking complex organic materi-8 als that may interest art-based design [MSDL17]. In this work, we 9 focus on microstructure exhibiting foam-looking patterns made of 10 oriented foam-like cells, or long filament organizations (see Fig. 1) and relate their parameters to mechanical and visually appealing characteristics. 13

crostructures has gained significant attention due to the efficiency of additive manufacturing to build them, in particular along the socalled topological optimization methods. However, designing these materials still presents several challenges. Firstly, creating intricate and delicate micro-structures that occupy relatively large volumes can result in laborious modeling, massive data structures, and prohibitively high computational costs associated with global optimization. Secondly, microstructure shapes can be highly diverse, which adds significant complexity to the design process and requires meticulous tailoring and parameterization for each object to achieve the expected behavior and appearance.

Through this work, we wish to highlight the power of adapted bioinspired morphogenesis approach to tackle such challenges. Indeed, real biological systems have evolved to optimize their struc-

¹⁴ In recent years, the study of complex shapes filled with mi-

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tures and functions through a variety of mechanisms, including 29 self-assembly, growth, and adaptation. Morphogenesis models in-30 spired by biological systems typically rely on local rules from which global characteristics emerge from an iterative process, possibly representing time evolution. A key interest of such a local 33 and time-evolving process relies upon its inherent adaptation to lo-34 cal conditions and its ability to generate organic-like microstruc-35 tures that follow a general pattern while exhibiting irregular local 36 variations. One may note that disordered porous structures found in nature are neither fully random nor perfectly ordered. Inter-38 estingly this structural disorder in organic architectured cellular 39 materials has been found to be beneficial for their damage toler-40 ance [vEYC*21]. These remarks led to our main motivation for 41 exploring biological-inspired microstructure generation. Instead of 42 aiming at a globally optimized shape model that would be hard to 43 control, we rather target locally adaptable behaviors that can give 44 rise to a global emerging structure, which remains controllable by 45 the end user at any time of the evolution, and provide a visual ap-46 pealing organic-looking disorder that enhances strength. 47

In this work, we show that the biological system designated as 48 Physarum polycephalum, also known as the blob [How31] can be 49 adapted to model our target microstructure for 3D shapes in a con-50 trollable way. This slime mold is a unicellular organism that has 51 gained attention in recent years as it has been found to exhibit re-52 markable capabilities for optimizing transport networks, and adapt-53 ing to changing environments. The Physarum polycephalum can 54 be represented by an agent-based models [Jon09] simulating vari-55 ous aspects of the organism behavior including its growth, forag-56 ing, and decision-making processes. Existing simulations of this 109 57 58 model already showed the natural emergence of global structures in 110 2D [Jon10] and in 3D [Jon15, EBPF20a, EBPF22] but were never 59 explored for oriented microstructures generation. We describe sev- 112 60 eral adaptations and/or simplifications to the existing models to ex-113 61 tend it to oriented organic-foam-looking emerging microstructure, 114 62 and provide insights into the choice of the model and simulation 115 63 parameters to converge toward the expected shape characteristics 116 64 effectively. Our technical contributions are twofolds. First, we ex- 117 65 tend the existing agent-based physarum models toward anisotropic 118 66 emerging infilled-pattern generation, and describe its efficient im-119 67 plementation. Second, we provide a comprehensive study on the 120 68 effect of the parameters in such an anisotropic context with a spe- 121 69 cific focus on providing explicit control over the averaged size and 122 70 elongation of microstructure cells. We demonstrate the applicabil-71 ity of our approach to various shape-design applications such as mechanical design and biology. Overall, we hope that such an ap-73 proach simulating the emergence of natural forms will help and 74 inspire designers from various domains in their process of creating 75 more complex and organic-looking shapes. 76

The rest of the article is organized as follows. We start with a re- 130 77 view of the related work (Sect. 2) and background on the particle-78 based model of the Physarum (Sect. 3). Section 4 presents an 79 overview of our generative method, followed by our technical con-80 tributions for 3D oriented emerging patterns (Sect. 5) and parame-81 ter analysis (Sect. 6). We then present our results (Sect. 7) before 135 82 concluding. 83

2. Related Work

In Section 2.1, we delve into the topic of microstructure infill for material design, as our method is closely tied to this field, particularly in terms of efficient fabrication using Additive Manufacturing. Furthermore, in Section 2.2, we address Bio-inspired Modeling and specifically focus on the Physarum polycephalum as a compelling model for our intended purpose.

2.1. Microstructures and shape infill for AM

The importance of internal fill patterns in additive manufacturing has a long research history in mechanical engineering [AAD98]. By avoiding dense interior fills, both printing time and material usage can be significantly reduced. As a result, the initial infill patterns were developed with the primary goal of reducing material density while also being quick to generate and effective in production.

Topology optimization, allowing the optimization of the distribution of material within a given domain subject to a set of constraints, has been successfully used in a large range of applications as described in these reviews [BS03, WSG21]. Porous material generation [WAWS18, SPG19] can, in particular, benefit from high resolution topology optimization [AALS17], but remain computationally prohibitive. Additional flexibility in the generated pattern has been proposed by the use of post-process applied on top of topology optimization result, such as the use of procedural functions [PT08, AGDP19], or reaction-diffusion [GSR22].

Periodic structures, originally proposed by Sigmund [Sig95], have also been widely explored to generate complex structures featuring, for instance, targeted rigidity [ZSCM17], deformation [TTZ*20], or balance [WWZW16]. Gradation of microstructure design can be achieved via the use of different tiles [PZM*15, GKW*19], or regularized deformation [GDAP20]. Although periodic microstructures are highly efficient, they depend on underlying regular grids, which make them challenging to apply to microstructure with spatially varying orientation and curved boundaries. Moreover, graded or oriented periodic microstructures seldom produce patterns with smooth connectivity at the interface between neighboring cells. This connectivity must thus be enforced by introducing additional geometrical constraints onto the generated patterns.

Close to our approach, stochastic processes provide design flexibility and progressive gradation since they are not restricted by a grid or subdivision rule. They have been recently used for foam-like microstructures [MDL16], and extended toward anisotropic [MSDL17] shapes, polyhedral cells [MHSL18], or lamellar structures [KTZK20]. Compared to fully periodic microstructures, the lack of a perfectly symmetric structure of stochastic process has been reported to exhibit greater resistance to fabrication imperfections [PVK*20, vEYC*21]. While 2D Voronoi cells [LS03, LCL*21] can relate to physarum cell look, the direct use of 3D Voronoi cells results in surfaces, generating membranes instead of filaments as observed in our case. Voronoi cell edges have been employed for foam modeling in previous works by Martinez et al. [MDL16, MSDL17], but we emphasize that these ap-

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proaches exhibit polygonal structures with straight edges, lacking 191 the organic, curved shapes that our method generates natively. 138 192

Our approach can be described as a stochastic morphogenetic 139 process and relies on the local emergence of non-periodic mi-140 crostructures spatially graded by orientation and density fields. 141 Compared to other stochastic approaches, it has the advantage of 142 relying only on local computation, thus avoiding computationally 143 expensive global optimization steps. In addition, the underlying 198 144 emerging process allows for generating flexible patterns that align 199 145 well with curved boundaries and can dynamically adapt to local 200 146 user modifications. 147

2.2. Bio-Inspired Modeling and Physarum polycephalum 148

Multi-scale structures are abundant in nature and can be observed 149 in various plant and animal organisms [Lak93, FW07]. Despite the 150 fact that their underlying conception processes are not always completely understood, they typically possess remarkable mechanical 152 properties. This has recently given rise to a new field known as "Bio-Inspired Modeling" or "Bio-Inspired Design," which is com-154 monly employed in the context of structural design for Additive 155 Manufacturing [DPBY*19, PMNWW19, ZWZ*20]. In contrast to 156 mathematical optimization, Bio-Inspired Modeling has its origins 157 in the field of morphogenesis and seeks to employ natural processes 158 to create optimized designs. Due to its interdisciplinary nature, it 159 is challenging to consider it as a singular design process. Rather, 160 it comprises various approaches that use different representations 161 and operate at different scales [CHK*05]. 162

The "many-headed" slime mold, known as Physarum poly- 220 163 cephalum or "blob", was initially misclassified as a fungus, but 221 164 is actually a giant unicellular protist with multiple nuclei. First 165 222 described by [How31], this organism has been extensively stud- 223 166 ied in various fields including cell biology, biochemistry, genet-167 ics, and more recently, physics. In recent years, it has gained in-168 224 terest due to its remarkable properties. Hence, despite lacking a 169 brain, the unicellular organism is capable of decision-making and 225 170 spatial memory of its environment [RLDB12, BL15]. Furthermore, ²²⁶ 171 227 akin to other self-organizing systems, the slime mold exhibits selfrepairing abilities [Ada13]. A seminal experiment conducted by Japanese researcher Toshiyuki Nakagaki [NYT00] demonstrated 229 174 that the slime mold can find the shortest route through a maze. 230 175 Since then, many studies have investigated its maze-solving ca-176 231 pability [NYT01, Nak01, ZZD11, NVSA17], and it has even been 177 232 shown to solve the Towers of Hanoi problem [RB13]. Its ability 233 178 to solve the Steiner tree problem $[TNT^*10]$ and thus to repro-179 duce real-world infrastructure networks [TTS*10] has also been 235 180 explored. 181 236

237 While several models have been proposed to explain the 182 238 Physarum behavior such as mathematical equations on flux 183 239 [TKN07] or Cellular Automaton [TS12], we direct our attention 184 240 to a particle-based model introduced by Jeff Jones [Jon09, Jon10], 185 241 described more precisely in Section 3, due to its ability to gener-186 242 ate intricate organic-looking structures. To the best of our knowl-187 243 edge, only a few works extended this concept to three dimen-188 244 sions. Jones [Jon15] himself described a brief exploration of three-189 dimensional environments, followed by applications toward design 245 190

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and architectures [MX17, DMM18], as well as for artistic application in the BioArt concepts [Sch13, Que20, Bar09, Seb23]. These approaches, however, provide only limited analysis of their model's parameters. Additionally, this early 3D variant [Jon15] relies on five samples for directional sensing (1 central, and 4 peripherals) which leads to artifacts when applied to the anisotropic case, as explained in Sect. 5.1.

The most recent extension of Jones' model in 3D, the so-called Monte Carlo Physarum Machine (MCPM), tailored for reconstructing and visualizing the cosmic web [BET*20, EBPF20a, EBPF20b, EBPF22], introduces stochastic updates helping to produce highly connected networks that align with cosmological data. This probabilistic model offers an elegant and robust formalization, and contains the deterministic Jones' model (called Max-PM by the authors, and relying on 1 central and 8 peripheral sensing directions) as its limit case. As stated in [EBPF22], Max-PM yields a clean, well-defined structure, which however does not consistently cover all the input cosmological data compared to the MCPM variant. Yet in our case, the property of providing clean and well-defined structure is very well suited for shape modeling, in contrast with the parameterized fuziness introduced by the MCPM variant. Nonetheless, we note that Ehrlich et al. [Ehr21, EHE22] have proposed an application to additive fabrication based on the MCPM model. If [Ehr21] limits the emergence of the pattern to a guiding mesh's surface, [EHE22] demonstrates the infilling of a 3D helmet, close to our volume-based pattern generation, but without the possibility to control the local orientation. We propose to build on the simple Max-PM model, with the addition of (i) rotation-invariant particles' sensor sampling for a fully 3D consistent model; (ii) a more comprehensive control over the local infill structure using anisotropic diffusion weights for precise oriented propagation behavior; (iii) associated to a pattern cells size analysis, suitable for both aesthetic application and additive manufacturing.

3. Background on Physarum Agent Based Model

Our methodology relies on an agent-based model of Physarum polycephalum, originally proposed by [Jon10] for 2D simulation. For the sake of clarity, we describe this existing model in this section.

As illustrated in Figure 2, the model represents the *Physarum* as a collection of agents $\mathcal{A} = \{a_k\}_{k \in [\![1,N]\!]}$ that interact with a density grid \mathcal{T} through a deposit/sensory behavior. Each agent a_k possesses a location p_k , a heading direction u_k , and three front sensors, each positioned at a distance r (Sensor Offset, denoted SO in the original paper) from the particle. The front sensor is oriented in the heading direction, and the positions of the left and right sensors are obtained through a rotation of $\pm \varphi$ (Sensor Angle or SA) around p_k . \mathcal{T} is stored as a 2D array of scalar values representing the marker (in biological contexts usually referred to as chemoattractant) emitted by the agents and is called the *trail map* as the density will evolve toward showing the trajectory trails of the agents through the simulation. By using its sensors, each agent can read the trail map ${\cal T}$ and adjust its movement accordingly. The simulated evolution of the model is obtained via discrete-time iterations, where both \mathcal{A} and \mathcal{T} are updated according to a six-step rule (see Fig. 2):

1. Sense. Each agent starts by reading the density in the cells of



Figure 2: Jones' Physarum model.

 \mathcal{T} corresponding to the positions of its three sensors. The agent 300 246 turns toward the direction of the sensor with the highest density 247 and takes a random one in case of equality. As a consequence, 248 the agents are sensible to the gradients of trail. During this sen-249 sor reading step only, \mathcal{T} can be weighted with another density 250 array, allowing to incorporate input data such as fixed attractive 251 areas.

- 253 2. Rotate. In case of change of direction, the heading vector of the 306 agent is then rotated from an angle α (Rotation Angle or RA). 307 254 Note that in general $\alpha \neq \varphi$. 255 308
- Move. The agent moves along its updated heading direction of 309 256 3. a distance h (Step Size or SS). 257
- **Deposit.** The agent located in p_k adds a small amount of density 311 4. 258 δ in the corresponding grid square of \mathcal{T} . 259 312

- 5. Diffuse. The trail diffuses according to the diffusion equation $\frac{\partial T}{\partial t} = d \Delta T$ during a short time frame.
- 6. Decay. A temporal decay is modeled by damping the diffusing values of \mathcal{T} by μ (typically 0.1) : $\mathcal{T} \to (1 - \mu)\mathcal{T}$.

Jones's original paper includes a collision detection step that guarantees the presence of at most one particle within each grid square. Measures of space availability (growth of the population of agents) and overcrowding (adaptation by population reduction) are also introduced later [Jon15]. These steps are essential for faithfully emulating the behavior of *Physarum polycephalum*, yet they do not crucially affect the patterns produced and we don't implement them in our approach. Additionally, avoiding this collision detection conveniently eliminates any sequential dependence, enabling enhanced computational parallelism. In practice, starting from a random distribution of agents, the emergence of patterns is observed after about a few hundred steps and leads to a relatively stable configuration around a few hundred steps. An exhaustive study of the effect of the different parameters $(r, h, \alpha, \varphi, agents population size s, de$ cay value μ ,...) on the pattern formation can be found in the founding paper [Jon10]. According to Jones, the transition from 2D to 3D sensors can be conceptualized as a shift from a flat anchor shape to a three-dimensional "grappling hook" shapes with four hooks. In our work, we expand upon this concept by adopting a 3D grappling hook shape with more hooks. Furthermore, we introduce the capability to confine pattern formation within a specific area and modify the diffusion equation to account for anisotropy. This modification enables the generation of oriented microstructures.

4. Method Overview

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Given a set of initial parameters (Fig. 3-left), the agent-based simulation updates the set of particles $\mathcal{A} = \{a_k\} = \{p_k, u_k\}$, where p_k represents their positions and u_k denotes their heading directions. Each particle can move and deposit a small amount of material density on the trail map \mathcal{T} , creating a network of patterns over time (Fig. 3-middle). The final shape surface is obtained by computing an isovalue of the *trail* \mathcal{T} (Fig. 3-right), typically using techniques such as marching cubes or dual contouring when a triangular mesh is required for visualization and manufacturing purposes.

In order to generate oriented microstructures that infill a guiding shape defined as a boundary surface \mathcal{B} , we introduce three specific input fields defined on a rectangular domain $\Omega \subset \mathbb{R}^3$, namely, an infill space field \mathcal{I} , a tensor field **D**, and a guiding trail \mathcal{T}^* .

Infill Space Field \mathcal{I} **.** The scalar field $\mathcal{I}(\mathbf{x} \in \Omega)$ is used to implicitly define the space where the microstructures grow as $\mathcal{D}_{\mathcal{I}} =$ $\{\mathbf{x} \in \Omega \mid \mathcal{I}(\mathbf{x}) > 0\}$. In practice, we consider \mathcal{I} to be a simple binary field (1 inside and 0 outside) or a signed distance function to the boundary surface.

Tensor field D. The tensor field $D(x \in \Omega)$ indicates the desired local orientation and local anisotropy of the microstructures. It can be derived from specific data available to mechanical engineering such as desired stress, or from a direct user's defined orientation field. **D** is used as a diffusion tensor in the **Diffuse** step and tends to align the cell-like and filament-like patterns with its principal direction, and its parameterization will be described in Sect. 6.1.

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Figure 3: Overview of our approach. [1] User defined constraints: Global shape domain with cell's local orientation and characteristics. [2] Expression of the constraints as models parameters: Infill domain (\mathcal{I}), anisotropic behavior (**D**), and boundary conditions (\mathcal{T}^*), that can be associated to spatially varying parameters to generate different local pattern aspects. Other simulation parameters are fixed. [3] The physarum simulation evolves through iterations leading to the emergence of microstructure patterns on the output Trail map \mathcal{T} stored in a grid structure. Interactive modifications from the user can be applied at any time, either via the change of the model parameters, or on the evolving Trail map. [4] A final mesh surface representing a porous structure can be extracted from the trail map using marching cube. Grid size for the Trail map \mathcal{T} : $370 \times 306 \times 534$.

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Guiding Trail \mathcal{T}^* . The scalar field $\mathcal{T}^*(\mathbf{x}) \in [0, 1]$ is used to en- 341 force the pattern formation in specific areas. The *guiding trail* op- 342 erates during the **Sense** step, we set the sensors to read an *effective density field*

$$\mathcal{T}^{\gamma} = (1 - \gamma) \, \mathcal{T}^{0} + \gamma \, \mathcal{T}^{*} \,, \qquad (1) \qquad {}^{343}_{344}$$

given as a linear interpolation between the normalized trail $T^0 =$ 345 $\frac{\mathcal{T}-\min\mathcal{T}}{\max\mathcal{T}-\min\mathcal{T}}$ in [0,1], and \mathcal{T}^* . The weight γ is a fixed value cho-346 318 sen in [0,1] which controls the influence of \mathcal{T}^* . Because of their 319 sensing behaviour, the agents are sensible to the local variations 320 347 of the *trail*, which means that attractive areas of \mathcal{T}^* are indicated 321 by its gradient toward its high value. Note that unlike T, the guiding trail \mathcal{T}^* does not evolve during the simulation, therefore fa-323 349 cilitating consistent pattern formation in specific areas. While we 324 350 can not give a general formulation for \mathcal{T}^* since it is provided 325 351 by the user as input, we can define it within a specific use case. 326 352 A common use of \mathcal{T}^* is 327

The control of the creation of an outer shell of thickness ε encouraging pattern formation along the curved boundary. Let us call d(\mathbf{x}, \mathcal{B}) the closest Eu-

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clidian distance between

x to the boundary \mathcal{B} , in



that case \mathcal{T}^* can typically be defined as a characteristic function

$$\mathcal{T}^*(\mathbf{x}) = \mathbb{1}_{\{\mathbf{x}, d(\mathbf{x}, \mathcal{B}) < \varepsilon\}} \tag{2} \quad \begin{array}{c} 363\\ 364 \end{array}$$

which creates a gradient of trail toward this area, illustrated Fig. 4.

While we call these three fields inputs, other parameters involved 367 in the model (sensor values, guiding influence, decay factor, etc) 368

can also vary spatially. We will call them grading parameters and 369

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provide a guideline in Sect. 6 to set their values depending on the expected pattern appearance.

5. Efficient 3D oriented Physarum Model

We describe in the following the specific modification we apply in the Physarum model in order to efficiently simulate its emergence in a 3D spatial environment in the anisotropic case.

5.1. Spatial sensing

The first essential element for 3D Physarum growth is to extend the agents' sensors into a 3D structure. Figure 5 showcases different possible layout for this. A natural extension consists in adding sensors associated with the rotation of the 2D basic layout around its heading direction. This leads to these n-legs grappling hooklooking shapes with 1 central sensor and *n* peripheral, that we call $\{1+n\}$ layout. Fig. 5 shows that *n* has to be sufficiently high (typically ≥ 8) to consistently sample the circle produced with the sensors and avoid any artifacts in the anisotropic case. Increasing nbeyond 10 has no real impact on the generated structures. Previous early attempts on 3D Physarum models [Jon15, MX17] (respectively $\{1+4\}$, $\{1+4\times 2\}$ layouts Fig. 5) used only four sampling directions which leads to directional artifacts in anisotropic representations due to the lack of angular sampling. Uniform sampling on a hemisphere could also be considered, but comparing the results given by the $\{1+8\}$ and the Unif. $\{9\}$ layouts shows that uniform distribution does not generate better aligned and clean cells. Moreover when the number of uniformly distributed sensors increases (Unif. {40} Fig. 5), the model changes of behavior and tend to always keep their heading direction aligned with the anisotropy direction leading to filament structures. As a result, the non-continuous sampling offset allows to form stable oriented



Figure 5: Resulting anisotropic structures for different 3D sensors layout, with parameters r = 3 and $\xi = 4$ (parameter introduced Section 6.1) generated on a $200 \times 200 \times 70$ grid. From left to right, the five first layouts $\{1+n\}$ are *n*-legs grappling hook-looking shape with increasing n, n = 4 being the layout proposed by [Jon15]. $\{1 + 4 \times 2\}$ layout comes from [MX17]. In the last two layouts Unif. $\{n\}, n$ sensors are uniformly distributed on the hemisphere using the Fibonacci Sphere.

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Figure 6: Agent and its sensors in 3D.

foam-cell structures, which is also why we do not rely on continu- 403 ous probabilistic distribution sampling such as in [EBPF20a]. In all 404 371 405

our examples we used the $\{1+10\}$ layout showed Figure 6.

5.2. Infill Space Restriction 373

To confine the pattern generation within a 3D shape described by 374 the infill space $\mathcal{D}_{\mathcal{I}}$, we add a positional constraint after the step 375 **Move**. In practice, we check at each frame the value of $\mathcal{I}(p_k)$ for $_{408}$ 376 377 each agent's position p_k . If $\mathcal{I}(p_k) \leq 0$, the position is outside of the 409 shape, then we project it back along its backward heading direction $_{410}$ 378 $-u_k \Delta t$, where Δt is the simulation time step. This yet simple 411 379 p_{l} approach proved to be sufficient to deal with boundaries without $_{412}$ 380 accumulation as the trail is diffused and is mixed with the guide 413 381 \mathcal{T}^* near the border \mathcal{B} . 382 414

5.3. Anisotropic patterns 383

The local pattern orientation can be controlled by the anisotropic 418 384 tensor **D** given as input. In the **Diffuse** step, the trail is diffused $_{419}$ 385 according to the anisotropic diffusion equation 386 420

$$\frac{\partial \mathcal{T}}{\partial t} = \nabla \cdot (\mathbf{D} \nabla \mathcal{T}) . \tag{3}$$

This modified equation results in patterns that align with the in-387 put tensor field. Specifically, the cells/filaments tend to follow the 388 direction indicated by the eigenvector of **D** corresponding to the 389 highest eigenvalue. 390

Care has to be taken when integrating anisotropic diffusion using discrete finite differences. Indeed, the use of the classical centered differences on a regular grid has greater accuracy along the main directions. While hardly visible for isotropic diffusion, this leads to strong artifacts when oriented patterns need to be generated in nonaxis-aligned directions. To address this issue, we propose a solution by generating at each frame a 3D rotation R randomly oriented on the unit sphere, and applied to both the trail map and the tensor field D before computing the Diffuse step. Once computed, the updated trail map is transformed back in applying the inverse rotation R^{-1} . By doing so, we can effectively handle any direction in a consistent manner while still utilizing the simplicity of a regular grid. It is important to note that this approach does not aim at improving numerical accuracy for the diffusion process but provides an orientation-invariant one, which is sufficient for our purpose.

5.4. Local block-based synthesis

Generating microstructures on a large-scale domain at once can be memory intensive. To avoid memory shortage in such case, we propose a per-block synthesis approach leveraging the local behavior of the physarum coupled with the guiding-trail input \mathcal{T}^* . As illustrated in Fig. 7, the global domain Ω can be split into blocks Ω_i with small overlapping areas on their boundary. The interior of the blocks is filled independently, one after the other, while ensuring a coherent boundary condition. This condition is ensured by copying the value of the trail of the filled neighbors block (Ω_1 in Fig. 7) on the overlapping domain into the guiding trail map \mathcal{T}^* of the block to be filled (Ω_2 in Fig. 7). This technique can scale to an arbitrary range and level of subdivision as long as the block size is sufficiently large compared to the typical length of a cell pattern. Therefore physarum structures can be generated on grids of any size using this approach. As shown Fig. 8, visual artifacts introduced by this process are minor if non-existent, as long as the



Figure 7: Per-block synthesis allows locally synthesizing a physarum structure and coupling it to nearby blocks via their common boundary.

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Figure 8: Physarum structure generated directly on the whole grid $_{453}$ (a) and using the local block approach shown Fig. 7 (b). Grid size : $_{454}$ $400 \times 200 \times 70.$ 455

423 fields vary continuously and infill structures are small compared to $\frac{423}{458}$ 424 the size of the blocks.

425 6. Parameters Study

Emerging morphogenesis models, similarly to Cellular Automata 426 or Reaction/Diffusion systems, often require a laborious process of 460 427 461 parameter selection in order to observe effective pattern formation. 428 This section proposes a comprehensive guideline describing how 462 429 to choose the main parameters depending on the expected pattern 463 430 464 formation. While we refer the reader to Jones [Jon10] for a general 431 465 study of the main behaviors generated using such Physarum simu-432 lation, we concentrate our analysis on the Anisotropic-Voronoi-like 466 433 behavior, which is more suitable for Additive Manufacturing. It is 467 434 worth noting that the introduction of anisotropy in the physarum 435 simulation increases its sensitivity to the parameters, resulting in a 469 436 470 narrower range of values where the patterns are consistently gen-437 erated. Additionally, a distinct behavior emerges with significant 471 438 relative anisotropy, as the microstructure patterns transition from 472 439 Voronoi-like cells to elongated filaments. 440 473

We focus on defining the parameter domain where cell or 474 filament patterns can be expected. And, for cell-like structure, to be able to control their appearance via a notion of averaged cell elongation ω , and a notion of cell area (2D) or cell volume (3D) σ (measured in pixel/voxel) using an adapted parameterization of the inputs. We define these average cell characteristics as: 477

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$$\boldsymbol{\omega} = \frac{\ell_{\parallel}}{\ell_{\perp}}, \quad \boldsymbol{\sigma} = \ell_{\parallel} \ \ell_{\perp}^{d-1}, \tag{4}$$

with d = 2 or 3 according to the dimension, and where ℓ_{\parallel} and ℓ_{\perp} are the averaged cell lengths in the most elongated, and least elongated directions.

While ℓ_{\parallel} and ℓ_{\perp} , and thus of ω and σ , can be directly computed by direct minimal and maximal length measurement over closed cells in 2D, their computation is not straightforward in 3D as the cells are not associated to bounded sub-domains. We propose a common computation methodology that can be applied to both two-dimensional and three-dimensional structures in relying on the apparent frequencies of the trail map.

Let us call

$$f_{\parallel} = \frac{L}{\ell_{\parallel}}, \text{ and } f_{\perp} = \frac{L}{\ell_{\perp}},$$
 (5)

the frequencies corresponding to the averaged cell length in the respective most and least elongated directions. Here, *L* designates the total length of the square/cubic domain. These two frequencies are the most apparent and, therefore, correspond to local peaks of magnitude in the spectrum of the trail map. To compute f_{\parallel} and f_{\perp} , we perform a Fast Fourier Transform on the output trail and consider a frequency as being locally dominant if its magnitude in the Fourier spectrum is higher than a certain percentage *p* of the local maximum magnitude. For values of *p* chosen between 25% and 75%, values of f_{\parallel} and f_{\perp} are translated by a similar offset, but their behavior remains the same. In all of our experiments, we chose p = 37% as this value gives elongation values that match a direct elongation computation in the 2D case for closed cells.

In the following, all numerical values are obtained using the parameters provided in the summary Table 1.

6.1. Diffusion ratio ξ and tensor parameterization

The diffusion tensor field **D** can be defined from specific data provided by the designer, such as desired mechanical properties, ori-



Figure 9: Patterns obtained for varying values of ξ and r.

Param.	Designation	Pattern affect	Values
ξ	Diffusion ratio	Elongation ω	[1,2.5]
		Fil. behavior $(r \le 2.5)$	[2.5, 4]
r	Sensor Offset	(ω, σ)	[2.5,6]
		Fil. behavior $(\xi \ge 2.5)$	[1.5, 2.5]
φ	Sensor Angle	Pattern aspect	27°
α	Rotation Angle	Pattern aspect	70°
h	Step Size	Pattern formation	1
μ	Decay	Scaling, edge thickn.	0.1
δ	Deposit	Trail scaling	0.01
γ	Guid. influence	Influence of \mathcal{T}^*	[0,1]
S	Pop. size	Pattern formation	[2,4]

Table 1: Summary of the Physarum parameters, respective in- $_{494}$ fluence, and typical values. The values of *r* and *h* are given in $_{495}$ pixel/voxel.

entation, stress, and so on, resulting in a generated structure in accordance with these properties. We propose to parameterize this diffusion tensor using, first, a normalized vector field $o(\mathbf{x}), \mathbf{x} \in \Omega$, 499 with $||o(\mathbf{x})|| = 1$ describing this main direction of diffusivity, and 500

thus the principal direction of the elongation of cells; and second, 501

a diffusion ratio $\xi(\mathbf{x}), \mathbf{x} \in \Omega$, guiding how much the cells should be elongated along this principle direction. In the following, we will assume that we restrict our model to elongated cells in a single direction in space, while assuming a transversal isotropy in the two remaining orthogonal directions

The 3 \times 3 diffusion tensor matrix D can be decomposed at any position $x\in\Omega$ as

$$\mathbf{D}(\mathbf{x}) = \mathbf{R}_o^I(\mathbf{x}) \, \mathbf{\Lambda}_{\boldsymbol{\xi}}^I(\mathbf{x}) \, \mathbf{\Lambda}_{\boldsymbol{\xi}}(\mathbf{x}) \, \mathbf{R}_o(\mathbf{x}) \, , \qquad (6)$$

where $\mathbf{R}_o(\mathbf{x})$ is a rotation matrix defining a local orientation frame for the diffusion, and $\mathbf{A}_{\xi}(\mathbf{x})$ is a diagonal matrix whose entries are the eigenvalues of $\mathbf{D}(\mathbf{x})$. The matrix $\mathbf{R}_o(\mathbf{x})$ can be defined from the directional vector $o(\mathbf{x})$ as $\mathbf{R}_o(\mathbf{x}) = (o(\mathbf{x}), o_1^{\perp}(\mathbf{x}), o_2^{\perp}(\mathbf{x}))$, where $(o_1^{\perp}, o_2^{\perp})$ can be arbitrarily oriented as long as they form an orthonormal frame. The matrix $\mathbf{A}_{\xi}(\mathbf{x})$ can be defined as

$$\mathbf{\Lambda}_{\boldsymbol{\xi}}(\mathbf{x}) = d_0 \operatorname{diag}\left(\boldsymbol{\xi}(\mathbf{x}), 1, 1\right) , \qquad (7)$$

where $d_0 \in \mathbb{R}$ is a diffusion magnitude, and $\xi(\mathbf{x}) \ge 1$ is the diffusion ratio.

 ξ has a direct influence on ω as increasing the diffusion ratio leads to longer cells in the principal direction provided by o. Both ξ and o are then considered as input parameters to control the result of the simulation. The value d_0 has a direct influence over the cell size

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Figure 10: Plots of $(\ell_{\parallel}, \ell_{\perp}, \omega, \sigma)$ as a function of (ξ, r) , in 2D and in 3D. Dashed color curves are the fitting curves for ℓ_{\parallel} and ℓ_{\perp} , and the resulting ones for ω and σ , at the extreme values where our proposed numerical model is valid for cells patterns (i.e. where ℓ_{\parallel} is defined, for $\xi \in [1, 4], r \in [3, 6]$). Values outside these intervals (darker blue curve of columns 1 and 3, leftmost part of columns 2 and 4) represent filament patterns. Black dotted curves for ω and σ are fitting curves with a more simple model provided equation 16.

since a large diffusivity leads to larger cells. However, the range 509 502 of possible values of d_0 is limited by the stability of the numerical 503 scheme to solve Eq. (3). Using an explicit scheme and a maximal 504 511 diffusion ratio $\xi \simeq 4$ leads to a value d_0 that cannot exceed 0.2 505 512 (assuming a discrete time step and spatial discretization set to 1). 506 513 507 Based on this limitation, we consider d_0 to be a constant and set it 514 508 to 0.1375.

6.2. Controlling cells characteristics

The second parameter to influence the cell size is the sensor offset r. At the opposite of d_0 , r is not restricted by numerical stability condition, and we therefore use it as a varying input variable. In the following, we analyze the influence of the two inputs (ξ, r) toward the cell characteristics $(\ell_{\parallel}, \ell_{\perp}, \omega, \sigma)$.

⁵¹⁵ We first illustrate in Fig. 9 the typical patterns obtained for vary-⁵¹⁶ ing values of ξ and *r*. While ξ acts primarily on the elongation of ⁵¹⁷ the patterns, *r* directly controls their global size. The dotted red line

delimits the two domains with different behaviors: the filament-like 560 518 pattern in the bottom-right, and foam-like patterns. The foam-like 561 519 patterns are obtained for moderate anisotropy relative to the cell 562 size. In this configuration, the cells are characterized by long edges 563 oriented along the main direction that increase with ξ , and a few 564 orthogonal edges which lead to this typical foam-cell appearance. 523 When the anisotropy becomes large ($\xi \ge 2.5$) while keeping a nar-524 row cell ($r \leq 2.5$), the edges orthogonal to the main anisotropic 525 567 direction are not able to emerge persistently anymore. As a result, 568 the generated patterns are then dominated by long edges propagat-527 569 ing along the principal direction that doesn't appear to be made of 528 570 cells but look like long filaments instead. 529

In Fig. 10, we provide a finer analysis of the relation between 530 $(\ell_{\parallel}, \ell_{\perp}, \omega, \sigma)$ plotted as functions of (r, ξ) . These curves can be 531 571 used as an abacus to finely set a prescribed cell size and elongation. Let us first consider the parametric subdomain where $r \in [3, 6]$, 533 $\in [1,4]$. This domain corresponds to foam-cell patterns where the 534 characteristic cell lengths can be well predicted by our inputs using 535 572 536 simple model. As we can see on the first line of the plots Fig. 10, 537 at the first order, the elongation length ℓ_{\parallel} can be approximated by having a linear dependence to ξ and to r independently. The trans-538 573 verse length ℓ_{\perp} can be approximated by having a linear dependance 539 574 to r, with a slope value that linearly decreases with ξ . We propose 540 the following numerical model that approximate this behavior as 575 541

$$\begin{cases} \ell_{\parallel}(\xi,r) &= a_{\parallel} \, \xi + b_{\parallel} \, r + c_{\parallel} \\ \ell_{\perp}(\xi,r) &= a_{\perp} \, \xi \, r + b_{\perp} \, r + c_{\perp} \,, \end{cases} \tag{8} \begin{array}{c} 576 \\ 577 \\ 578 \end{array}$$

⁵⁴² with an additional condition on the elongation:

$$\forall r, \quad \omega(1,r) = \frac{\ell_{\parallel}(1,r)}{\ell_{\perp}(1,r)} = 1 , \qquad (9)$$

describing the fact that the cells are not elongated in the isotropic

case. This gives a relation between the previous coefficients:

$$\begin{cases} a_{\parallel} + c_{\parallel} = c_{\perp} \\ b_{\parallel} = a_{\perp} + b_{\perp} , \end{cases}$$
(10)

which allows to eliminate two coefficients to re-write the expres sions under the form:

$$\begin{cases} \ell_{\parallel}(\xi,r) &= a_0 \,\xi + (a_1 + a_2) \,r + a_3 \\ \ell_{\perp}(\xi,r) &= (a_1 \,\xi + a_2) \,r + (a_0 + a_3) \,. \end{cases}$$
(11)

with values fitted using simple regression, for the 2D and the 3D cases, cases,

² 2D:
$$a_0 = 2.0, a_1 = -0.35, a_2 = 5.4, a_3 = 0$$

3D: $a_0 = 2.0, a_1 = -0.30, a_2 = 4.2, a_3 = 1.5$. (12) ⁵⁸⁸

590 The behavior is similar in 2D and 3D but the fitting coefficients 549 slightly differs between the dimension. Still, they remain close 550 592 enough to assimilate these numerical differences to the fact that 551 593 2D cells and 3D cells are not completely comparable as they define 552 slightly different structures and the 3D case exhibits wider degree 553 of freedom to measure ℓ_{\parallel} and ℓ_{\perp} . 554 594

Overall, characteristic lengths of foam cells behave in the same 595 way in 2D and 3D. ℓ_{\parallel} increases with ξ ($a_0 > 0$) and with r ($a_1 + 596$ $a_2 > 0$) which is to be expected. ℓ_{\perp} also increases with r ($a_1\xi + 597$ $a_2 > 0$) but it decreases with ξ ($a_1 < 0$) meaning the anisotropy 598 tends to bring the pattern closer in the transverse direction. For $r \le 599$ 3 and $\xi \ge 2.5$, the behavior changes to filament-like patterns. This change of behavior is also visualized on the plots, with a sudden increase of ℓ_{\parallel} values. Our model is still valid for ℓ_{\perp} because this length is still well defined in the filament behavior. These simple linear models are only first order approximation, and we may note for instance consistent local maxima for low ξ values (for ℓ_{\parallel}), that we neglect in such fitting.

 ω and σ can also be well approximated using our proposed model for ℓ_{\parallel} and ℓ_{\perp} (dashed color curves Fig. 10). To really understand how ξ and *r* influence these characteristics, we re-write their expressions differently:

$$\omega(\xi, r) = 1 + \frac{\xi - 1}{r} f(\xi, r) , \qquad (13)$$

with

$$f(\xi, r) = \frac{a_0 - a_1 r}{a_1 \xi + a_2 + \frac{a_0 + a_3}{r}},$$
(14)

and

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$$\sigma(\xi, r) = (a_1 + a_2)a_2^{d-1}r^d + g(\xi, r) , \qquad (15)$$

with $g(\xi, r)$ a rational polynomial that we do not fully develop and $d \in \{2, 3\}$ corresponding to the dimension (2D or 3D).

In practice we observe that we can approximate $f(\xi, r)$ to be linearly varying with ξ and with r independently, and that $g(\xi, r)$ can be approximated by a constant σ_0 as its variation is negligible in front of $(a_1 + a_2)a_2^{d-1}r^d$. Finally the cell elongation and area/volume can be approximated by:

$$\begin{cases}
\omega(\xi, r) = 1 + \frac{\xi - 1}{r} (\alpha_0 \xi + \alpha_1 r + \alpha_2) \\
\sigma(\xi, r) = (a_1 + a_2) a_2^{d-1} r^d + \sigma_0,
\end{cases}$$
(16)

with values fitted using simple regression, for the 2D and the 3D cases,

$$\begin{cases} 2D: \alpha_0 = 0.045, \ \alpha_1 = 0.08, \ \alpha_2 = 0.24, \ \sigma_0 = 50\\ 3D: \alpha_0 = 0.040, \ \alpha_1 = 0.11, \ \alpha_2 = 0.20, \ \sigma_0 = 1600 \end{cases} .$$

This corresponds to the plots of the black dotted curves for ω and σ Fig. 10. Under this form, this shows clearly that the elongation can be approximated by having a quadratic relation to ξ , which can be linked to the square of ξ found in the expression of $\mathbf{D}(\mathbf{x})$. The influence of ξ tends to diminish while *r* increases, as the invert of *r* appears in the expression, with the other coefficients being positive. This is also to be expected because as *r* increases, the size of pattern increases, but not the radius of diffusion, therefore anisotropic diffusion is less likely to affect larger patterns. The cell area/volume is governed by the quadratic/cubic term r^d in σ , which is also coherent with the expected influence of *r*.

6.3. Fixed parameters for patterns formation

Beyond ξ and *r* used as *grading parameters*, other simulation parameters may influence how well patterns are defined. We briefly review in the following the influence of these additional parameters and propose the appropriate values we have been considering in addition to their summary in Table 1.





Figure 11: *Physarum* 2D pattern with α continuously varying along the x axis and ϕ continuously varying along the y axis, for isotropic (left) and anisotropic cases (right).

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Rotation angle α and Sensor Angle ϕ have a significant influ- 632 600 ence on the appearance and formation of patterns. Jones [Jon10] 633 601 proposed the general rule of $\alpha = 2\varphi$, with typical value $\alpha = 45^{\circ}$, 602 $\varphi = 22.5^{\circ}$. We, however, note that the region where the foam-cell 603 635 pattern appears consistently is narrower in the anisotropic case, and 604 can lead to a mix of filament patterns when using $\alpha = 2\varphi$. This be-605 havior can be seen in Fig. 11 illustrating the patterns obtained along 606 a continuous variation of ϕ and α between $[0,\pi/2]$ in both isotropic 607 $(\xi = 1, \text{ left})$, and anisotropic case ($\xi = 4$, right). The square illus-608 trates the typical area providing foam-cells generation with con-640 609 sistent density in isotropic and anisotropic cases. This area corre-641 sponds to the ratio $\alpha = 2.6 \varphi$ and the typical considered values in 642 our case are respectively $\alpha = 70^{\circ}$, $\phi = 27^{\circ}$. 612 643

613 Pattern formation: Agent population, Step Size. In all our 645 simulation, the agents are initially uniformly distributed. The num-614 ber of agents as well as the step size both impact the possibility of the microstructure being generated. Too low or high-value results 646 in either non-connected structure or dense infill. We note that the sensibility of the simulation is higher when using large anisotropy, 647 618 and normalized them as follows. First, the total agent population 648 619 N is normalized by the ratio s = N/V, where V is the volume of 649 the infill space. We found that values between s = 2 and s = 4 give 650 621 satisfying results in all our cases. In fact, in 3D, satisfying results can be obtained with lower values i.e. for $s \ge 0.5$, which is very 652 convenient from a computational cost. We assume that s does not 653 624 need to be as high in 3D than in 2D because 3D shapes are much 654 625 more porous than 2D shapes, while similarly connected. Second, 655 the Step Size *h* is set to 1 pixel of the trail map image \mathcal{T} . 627 656

Scaling factors: Decay μ , Deposit δ . These two parameters do 658 not change the type of pattern, but only act as scaling factors. While 659 deposit δ simply scale the trail density value \mathcal{T} uniformly, the decay 660 μ slightly change at the extreme values near 0 and 1 the thickness 661 of the edges on the trail map. We consider in all our experiment $\mu = 0.1$ and $\delta = 0.01$.

Grid size. As parameters such as *r*, *h* and the diffusion radius are defined in pixels or voxels, the resolution of the underlying grid used for \mathcal{T} has a significant impact on the result. For a same set of parameters, if *L* designates the maximum length of the domain space Ω measured in pixel/voxel i.e. the resolution of the grid, the characteristic lengths of microstructures are equal to $\frac{\ell_{\parallel}}{L}$ and $\frac{\ell_{\perp}}{L}$. Hence a grid of higher resolution λL will return smaller microstructures by a factor $1/\lambda$, because the characteristics lengths do not vary in pixel/voxel.

Structure thickness. The thickness of cell edges/filaments can be slightly controlled in the final structure mesh by slightly shifting the isosurface used for the Marching Cube algorithm.

7. Results and Applications

We present the results of our pattern generation method using PhysOM. All the models are generated on a PC workstation with an Intel Xeon Gold 6148 CPU running at 2.40GHz, 128GB RAM and a GPU Nvidia Quadro RTX6000 with 24 GB memory. Our implementation is based on CuPy library in Python to store the 2D and 3D grids with operations performed in parallel on the GPU. The dimension of the voxel grid and the computational time associated with the full generation of our 3D examples are reported in Table 2. At the exception of Fig. 8, the examples were created from a single grid structure without the per-block optimization. The total time reported corresponds to the simulation for 200 steps. The core of the method relies on uncoupled particle simulation, where each particle update (steps Sense, Rotate, Move, Deposit) is "embarrassingly parallel". The diffusion step and the handling of the 3D texture and its rotation are implemented using trivial and non-optimal



Figure 12: Physarum 2D pattern on a 200 \times 200 grid at several steps on a simulation with the associated curves of porosity P, amount added ΔT_{+} , amount removed ΔT_{-} and variation ΔT . These last three quantities are scaled by a factor that allows them to be visualized on a single plot.

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algorithms in our case. A very detailed breakdown and fine com- 690 662 parison of timings with other approaches would not be relevant. 663 For instance, [EBPF22] reported measures implemented in C++ 664 692 with compute shader, while our 3D implementation uses Python. 665 693 The timings are indeed very different, and our approach could be 666 694 highly optimized with dedicated developments that are beyond the 667 scope of the current contribution. However, we have developed a 668 real-time 2D prototype relying on WebGL for the entire simula- 696 669 tion. This 2D version is obtained as an extension of a standard 697 physarum implementation proposed by Ghassaei [Gha22]. Please 671 refer to the attached video for a visual demonstration of the ani-672 mated outcomes. 673

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	Grid size (voxels)	Total time]
Teaser (Fig. 1)	$64 \times 64 \times 768 (3M)$	2 min	1
Owl (Fig. 3)	$369 \times 306 \times 534$ (60M)	20 min	1
Eiffel Tow. (Fig. 17a)	$666 \times 247 \times 247$ (40M)	15 min	1
Chair (Fig. 17b)	$256 \times 256 \times 256$ (16M)	6 min	1
Femur (Fig. 17c)	$400 \times 210 \times 265 (22M)$	8 min	1

706 Table 2: Grid dimension and timings of our 3D examples, for 200 steps of simulation.

7.1. Simulation convergence 674

The physarum simulation remains in permanent evolution over the 675 simulation steps which allows a designer to interact with the struc-714 ture by modifying the grading parameters locally or modifying the 677 715 trail map by erasing/adding some parts and letting the system adapt 678 716 at any time. However, for structure generation purposes, we may 679 consider that the system has reached a stable state when the general 680 type of patterns remains consistent over time. We defined this state 718 681 when the average density of the trail map reaches a stable state. Let 719 682 us call ΔT_+ the total amount of density added from Step 3 to Step 683 720 4, and ΔT_{-} removed by the decay process from Step 4 to Step 6. 684 We call $\Delta T = |\Delta T_+ - \Delta T_-|$, the net change of density. We con-685 722 sider that the simulation has converged when the relative variation 686 of $\Delta T/V_{\Omega}$ remains below a threshold during several steps. The evo-687 724 lution of these parameters along the simulation steps are illustrated 725 688 in Fig. 12-right. 689 726

7.2. Structural Design for Additive Manufacturing

A classical application of microstructure infill is the generation of porous material that can be 3D printed. We show in the following that our approach allows designing materials with varying stiffness and density/porosity in adapting the anisotropy, orientation, and cell area.

We introduce the material porosity P, i.e. the reverse of its averaged density, as

$$P = 1 - \frac{1}{V_{\mathcal{I}}} \sum_{\mathbf{x} \in \Omega} \mathcal{T}(\mathbf{x}) , \qquad (18)$$

where $V_{\mathcal{I}}$ is the volume of the infill shape domain $\mathcal{D}_{\mathcal{I}}$. The porosity tends to converge towards a constant value, as depicted in Figure 12-right, also confirming the attainment of a stable state.

To confirm the suitability of our generated structure for material design, we propose a study of its anisotropy-dependence deformation using the methodology used in density-based topology optimization [ACS*11]. We apply a continuous vertical compressive stress on generated structure, then simulate and measure the displacement of the global structure using an isotropic linear constitutive stress-strain relation. Each element of the grid $\mathbf{x} \in \Omega$ is associated to a Young's modulus:

$$E(\mathbf{x}) = E_{\min} + \mathcal{T}^{0}(\mathbf{x})(E_{0} - E_{\min})$$
 (19)

Here, E_0 represents the material's stiffness, and E_{\min} is a small stiffness assigned to void regions to prevent the stiffness matrix from becoming singular. \mathcal{T}^0 is the normalized trail in [0, 1]. The global stiffness matrix is denoted as $\mathbf{K} = (E(\mathbf{x})\mathbf{k}_0)_{\mathbf{x}\in\Omega}$, where \mathbf{k}_0 corresponds to the stiffness matrix for an element with a unit Young's modulus. The global displacement and force vectors at the grid nodes are represented by U and F, respectively. Computing the resulting displacement U caused by an applied force F involves solving the equation $\mathbf{K}\mathbf{U} = \mathbf{F}$, which can be expressed as $\mathbf{U} = \mathbf{K}^{-1}\mathbf{F}$.

Figure 13-a illustrates the results obtained in applying such constraint while varying the direction and magnitude of the anisotropy. From the isotropic case in the center, we increase the anisotropy, respectively, in the vertical direction on the left-side, and in the horizontal direction on the right-side. This experiment is performed at constant porosity value by selecting tailored values for (ξ, r) . To achieve this, the porosity is computed on small grid samples (200×200) for varying combinations of (ξ, r) , resulting in a density grid representation $P(\xi, r)$. By selecting an iso value of P, the

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(c) Deformation and displacement for a constant compressive stress along the vertical axis for varying anisotropy.

Figure 13: Structural simulations.

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desired values for (ξ, r) can be obtained. As expected, an increase 751 728 of anisotropy in the vertical direction provides stiffer material resistance, while a horizontal one will lead to a more deformable struc-729 730 ture. A similar test is conducted in 3D, as shown in Figure 13-c, using a cubic infill that demonstrates the same property. In Figure 13b, the anisotropy is held constant while progressively changing the 732 angle from vertical to horizontal, once again reinforcing the same conclusion. Finally, Figure 13-d illustrates the significant influence 734 of material porosity on the resulting displacement. Note that our simulations on 2D quad-meshes and 3D hex-meshes did not con-736 sider material nonlinearities, and the large displacement factor in visualization is deliberate. We found these simplifications accept-738 able for the purpose of validating the effects of the porosity and 739 anisotropy parameters. Indeed, to truly capture such large defor-740 mations, a simulation would need to consider geometrical nonlin-741 earities (e.g. buckling), material nonlinearities (e.g. plasticity), as 742 743 well as internal contacts.

Being able to link the parameters of the Physarum to the lo-744 757 cal porosity and anisotropy of the microstructures generated al-745 758 lows for tailored design for deformable shapes. A more in-depth 746 study could allow to map these parameters to material properties 747 760 or mechanical parameters such as the Young's modulus, similarly 748 to [MSDL17]. More generally, the effective constitutive law for any 749 761 set of Physarum parameters could be evaluated through homoge-762 750

(d) Deformation and displacement for a compression along the vertical axis for varying anisotropy and varying porosity.

nization, thus compiling a reverse mapping from desired mechanical properties to their appropriate generation parameters.



Figure 14: Printed cubic samples with respective varying (ξ, r) values. The numerical models were generated on a $200 \times 200 \times 200$ grid.

Based on these observations, our approach is particularly wellsuited for printing deformable structures. Figure 14 shows real pictures of printed cubic samples of 3D structures, illustrating both isotropic and anisotropic cases with various r values. These models, along with those showcased in Fig. 17, were generated using the most affordable SLA 3D printer available (Geeetech Alkaid) at the time of writing this paper, demonstrating the printability of such structures.

Using a flexible material, Fig. 15 demonstrates that the deformation of the structure is highly influenced by its anisotropy and the

alignment of the applied force. For a comparable applied pressure, 763

an anisotropic structure aligned with the force experiences less de-764

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formation than its isotropic counterpart. Additionally, the isotropic structure deforms less than the anisotropic structure perpendicular

766 to the applied force direction, aligning with our numerical simula-767 tions.



(a) Isotropic

(b) Anisotropic, F_{\parallel} (c) Anisotropic, F_{\perp}

Figure 15: Pressure applied on cubic samples (r = 6) for (a) the isotropic case, (b) the anisotropic case with anisotropy direction aligned with pressure direction, (c) the anisotropic case with anisotropy direction orthogonal to pressure direction.

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In Figure 16, we showcase a comparison of chairs where the top-769 left chair is generated from Martinez et al. [MSDL17], designed 770 using orthotropic k-nearest foams and inspired by the designs of Lilian van Daal [vD14]. The top-right chair in Figure 16 is obtained 800 through our PhysOM approach in 2D, employing a similar underly-801 ing orientation field. Note that our approach provides aesthetically 774 pleasing homogeneous organic features and well-defined borders 775 thanks to the use of the guiding trail \mathcal{T}^* . In Figure 16 (bottom row) 804 777 we show pictures of a 3D-printed model with deformation behavior also similar to the one shown in [MSDL17]. These models were 805 778 779 printed using an inexpensive FDM printer (Creality CR-10).

7.3. Art and Design 780

808 Our approach can also find relevant applications in art, design, and 781 animation. The natural and organic aesthetic of the shapes produced 782 by PhysOM can be of interest to creative designers and artists. Fig-783 811 ure 17a and 17b showcase our version of the Eiffel Tower and the 784 biomimicry chair inspired by Lilian van Daal [vD14]. The result-785 ing prints are presented Fig. 17e and 17f. It is worth noting that 786 these structures are dynamic and "alive" as long as the simulation 787 is running. This characteristic opens up various possibilities, such 788 as animation or the creation of unique designs. Two structures ex-789 tracted from the same simulation but at slightly different time steps 817 790 will appear very similar yet distinct, offering opportunities for pro-818 791 ducing diverse and individualized designs. 792 819

7.4. Biological representation 793

The organic pattern generated by our method can also be desir- 823 794 able for representing biological structures. It can be utilized to de-795 824 sign organic textures in 2D, as demonstrated by the dragonfly wing 796 825 showcased in Figure 18. Designers can sketch inputs such as the 826 797 infill space field or the orientation, specifying labeled areas for 827 798 anisotropy. Our method then automatically generates a coherent, 828 799



Figure 16: Top-left: Chair from Martinez et al. [MSDL17]. Topright: chair designed by PhysOM. Bottom Real 3D printed structure. Finger pressure is applied to the chair similarly to [MSDL17].

biologically-inspired texture that accurately represents the desired design.

This process can also be applied to 3D application, where, for instance, the patterns formed by Physarum organisms bear resemblance to the intricate network of trabeculae found in spongy bones, as illustrated in Figure 17c and Figure 17d.

8. Conclusion and Future Work

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In this work, we have presented a novel approach for generating intricate organic microstructures inspired by the behavior of the Physarum polycephalum slime mold. By extending the classical agent-based model of Physarum to a 3D domain with local anisotropic behavior, we have enabled the design of porous organic-like microstructures that resemble natural foam-like cells or filament-like patterns with variable local properties. Our method offers precise control over the local orientation of 3D patterns, relative cell extension and size (in 2D), and precise infill of 3D forms, all with well-defined boundaries.

The versatility and efficiency of our method make it suitable for a wide range of applications, including additive manufacturing, design, and biological representation and engineering. These microstructures, with their locally stiff or elastic, lighter, and porous properties, provide interesting mechanical characteristics that can be adjusted by varying the type of infill patterns. By leveraging the local rules and time-evolving processes inherent in biological systems, we have achieved locally adaptable behaviors that give rise to global emerging structures. This approach allows the end user to retain control over the evolution of the structure, ensuring a controllable and visually appealing organic-looking disorder that enhances strength. This visually appealing appearance of these mi-

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Figure 17: Examples of PhysOM model (first line), with their associated prints (second line).

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Figure 18: Synthesis of a Dragonfly wing patterns using our approach.

crostructures, mimicking complex organic materials, offer potential 829 for art-based design. We believe that our approach will inspire and 855 830 assist designers across diverse domains in their pursuit of creating 831 856 more complex and visually appealing shapes. Future research di-832 rections may involve exploring additional biological systems and 833 further refining the control mechanisms to enhance the capabilities 857 834 and versatility of our method. 835

While our approach for generating intricate organic microstruc- 859

tures using the Physarum polycephalum-inspired model shows promising results, there are certain limitations that should be acknowledged. Firstly, the current implementation focuses on infilling 3D domains with local anisotropic behavior, but it does not take into account external constraints or specific material properties. Future work could explore the integration of mechanical and material considerations to optimize the microstructures for specific applications. Additionally, the computational costs associated with simulating large-scale volumes and complex shapes could be a limitation, especially when real-time interactivity is desired. Developing more efficient algorithms or exploring parallel computing techniques could help address this issue. Furthermore, while our approach provides control over the evolving structure, there is still room for improvement in terms of providing intuitive and userfriendly design interfaces. Future research could investigate interactive design tools that enable users to manipulate and guide the evolution of the microstructures more effectively.

Conflict of Interest

None of the authors have a conflict of interest to disclose.

Data Availability

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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- [AAD98] ALEXANDER P., ALLEN S., DUTTA D.: Part orientation and
 build cost determination in layered manufacturing. *Computer-Aided De sign 30*, 5 (1998), 343–356. 2
- [AALS17] AAGE N., ANDREASSEN E., LAZAROV B. S., SIGMUND O.:
 Giga-voxel computational morphogenesis for structural design. *Nature* 929
 550, 7674 (2017), 84–86. 2 930
- [ACS*11] ANDREASSEN E., CLAUSEN A., SCHEVENELS M., 931
 LAZAROV B. S., SIGMUND O.: Efficient topology optimiza- 932
 tion in matlab using 88 lines of code. Structural and Mul- 933
 tidisciplinary Optimization 43, 1 (2011), 1–16. doi:https: 934
 //doi.org/10.1007/s00158-010-0594-7.12 935
- [Ada13] ADAMATZKY A.: Physarum wires: Self-growing self-repairing
 smart wires made from slime mould. *Biomedical Engineering Letters* 3
 (2013), 232–241. 3
 938
- [AGDP19] ALLAIRE G., GEOFFROY-DONDERS P., PANTZ O.: Topology optimization of modulated and oriented periodic microstructures 940 by the homogenization method. *Computers & Mathematics with Applications 78*, 7 (2019), 2197–2229. doi:https://doi.org/10. 1016/j.camwa.2018.08.007.2
- 880[Bar09]BARNETT H.: The physarum experiments, 2009. Accessed944881May 7, 2023. URL: https://heatherbarnett.co.uk/work/
the-physarum-experiments/. 3945
- [BET*20] BURCHETT J. N., ELEK O., TEJOS N., PROCHASKA J. X.,
 TRIPP T. M., BORDOLOI R., FORBES A. G.: Revealing the dark threads
 of the cosmic web. *The Astrophysical Journal Letters 891*, 2 (2020), L35.
 3
- [BL15] BEEKMAN M., LATTY T.: Brainless but multi-headed: decision
 making by the acellular slime mould physarum polycephalum. *Journal* of molecular biology 427, 23 (2015), 3734–3743. 3
- [BS03] BENDSOE M. P., SIGMUND O.: Topology optimization: theory, methods, and applications. Springer Science & Business Media, 2003.
 2
- [CHK*05] CHATURVEDI R., HUANG C., KAZMIERCZAK B., SCHNEI
 [DER T., IZAGUIRRE J. A., GLIMM T., HENTSCHEL H. G. E., GLAZIER
 J., NEWMAN S., ALBER M.: On multiscale approaches to three-
- 896
 dimensional modelling of morphogenesis. Journal of the Royal Soci 959

 897
 ety interface 2, 3 (2005), 237–253. doi:https://doi.org/10.
 960

 898
 1098/rsif.2005.0033.3
 961
- [DMM18] DINIZ N., MARIZZI C., MELENDEZ F.: Playing with growth 962
 patterns: Merging biotechnology and architectural design to sense envi-963
 ronmental toxins. In *Proceedings for 2018 ACSA Fall Conference "PLAY* with the rules" 11 13 Oct 2018, University of Wisconsin-Milwaukee, 965
 Milwaukee US (2018). 3
- [DPBY*19] DU PLESSIS A., BROECKHOVEN C., YADROITSAVA I., YADROITSEV I., HANDS C. H., KUNJU R., BHATE D.: Beautiful and functional: a review of biomimetic design in additive manufacturing. Additive Manufacturing 27 (2019), 408–427. doi:https: //doi.org/10.1016/j.addma.2019.03.033.3
- [EBPF20a] ELEK O., BURCHETT J. N., PROCHASKA J. X., FORBES
 A. G.: Monte carlo physarum machine: An agent-based model for reconstructing complex 3d transport networks. In *Artificial Life Confer- ence Proceedings 32* (2020), MIT Press One Rogers Street, Cambridge, 974
 MA 02142-1209, USA journals-info, pp. 263–265. 2, 3, 6
- [EBPF20b] ELEK O., BURCHETT J. N., PROCHASKA J. X., FORBES
 A. G.: Polyphorm: Structural analysis of cosmological datasets via in-977
- 4. O.: Folypholm: Structural analysis of cosmological datasets via in teractive physarum polycephalum visualization. *IEEE Transactions on Visualization and Computer Graphics* 27, 2 (2020), 806–816. 3
- [EBPF22] ELEK O., BURCHETT J. N., PROCHASKA J. X., FORBES 980
 A. G.: Monte carlo physarum machine: Characteristics of pattern formation in continuous stochastic transport networks. *Artificial Life* 28, 1 982
 (2022), 22–57. 2, 3, 12 983
- [EHE22] EHRLICH D., HAKIMSHAFAEI M., ELEK O.: Scaffolding gen-984
 eration using a 3d physarum polycephalum simulation. In *Proceedings of* 985

the 7th Annual ACM Symposium on Computational Fabrication (2022), pp. 1–2. 3

- [Ehr21] EHRLICH D.: Printing the polyphorm: Using 3d printing to manufacture biologically inspired rhizomatic structures. *Thesis, Univ. California Santa Cruz* (2021). 3
- [FW07] FRATZL P., WEINKAMER R.: Nature's hierarchical materials. Progress in materials Science 52, 8 (2007), 1263–1334. doi:https: //doi.org/10.1016/j.pmatsci.2007.06.001.3
- [GDAP20] GEOFFROY-DONDERS P., ALLAIRE G., PANTZ O.: 3-d topology optimization of modulated and oriented periodic microstructures by the homogenization method. *Journal of Computational Physics* 401 (2020), 108994. doi:https://doi.org/10.1016/j.jcp. 2019.108994.2
- [Gha22] GHASSAEI A.: Physarum transport network, 2022. URL: https://github.com/amandaghassaei/gpu-io. 12
- [GKW*19] GARNER E., KOLKEN H. M., WANG C. C., ZADPOOR A. A., WU J.: Compatibility in microstructural optimization for additive manufacturing. *Additive Manufacturing* 26 (2019), 65–75. 2
- [GSR22] GARNIER D.-H., SCHMIDT M.-P., ROHMER D.: Growth of oriented orthotropic structures with reaction/diffusion. *Structural and Multidisciplinary Optimization* 65, 11 (2022), 327. 2
- [How31] HOWARD F. L.: The life history of physarum polycephalum. American journal of botany (1931), 116–133. 2, 3
- [Jon09] JONES J.: Approximating the behaviours of physarum polycephalum for the construction and minimisation of synthetic transport networks. In Unconventional Computation: 8th International Conference, UC 2009, Ponta Delgada, Portugal, September 7-11, 2009. Proceedings 8 (2009), Springer, pp. 191–208. 2, 3
- [Jon10] JONES J.: Characteristics of pattern formation and evolution in approximations of physarum transport networks. *Artificial life 16*, 2 (2010), 127–153. 2, 3, 4, 7, 11
- [Jon15] JONES J.: From pattern formation to material computation: multi-agent modelling of Physarum Polycephalum, vol. 15. Springer, 2015. 2, 3, 4, 5, 6
- [KTZK20] KUMAR S., TAN S., ZHENG L., KOCHMANN D. M.: Inverse-designed spinodoid metamaterials. *npj Computational Materials* 6, 1 (2020), 73. 2
- [Lak93] LAKES R.: Materials with structural hierarchy. Nature 361, 6412 (1993), 511–515. doi:https://doi.org/10.1038/ 361511a0.3
- [LCL*21] LIU B., CHENG H., LIU M., CAO W., JIANG K.: Adaptive anisotropic porous structure design and modeling for 2.5 d mechanical parts. *Materials & Design 206* (2021), 109786. 2
- [LS03] LABELLE F., SHEWCHUK J. R.: Anisotropic voronoi diagrams and guaranteed-quality anisotropic mesh generation. In *Proceedings of* the nineteenth annual symposium on Computational geometry (2003), pp. 191–200. 2
- [MDL16] MARTÍNEZ J., DUMAS J., LEFEBVRE S.: Procedural voronoi foams for additive manufacturing. ACM Transactions on Graphics (TOG) 35, 4 (2016), 1–12. 2
- [MHSL18] MARTÍNEZ J., HORNUS S., SONG H., LEFEBVRE S.: Polyhedral voronoi diagrams for additive manufacturing. ACM Transactions on Graphics (TOG) 37, 4 (2018), 1–15. 2
- [MSDL17] MARTÍNEZ J., SONG H., DUMAS J., LEFEBVRE S.: Orthotropic k-nearest foams for additive manufacturing. ACM Transactions on Graphics (TOG) 36, 4 (2017), 1–12. 1, 2, 13, 14
- [MX17] MA Y., XU W.: Physarealm—a bio-inspired stigmergic algorithm tool for form-finding. In Protocols, Flows and Glitches, Proc. 22nd Int. Conf. Association for Computer-Aided Architectural Design Research in Asia (CAADRIA) 2017 (2017), pp. 499–509. 3, 5, 6
- [Nak01] NAKAGAKI T.: Smart behavior of true slime mold in a labyrinth. Research in Microbiology 152, 9 (2001), 767–770. 3

Author's version of article published in COMPUTER GRAPHICS Forum, May 2024.

16

⁸⁶⁰ **References**

- 986 [NVSA17] NTINAS V., VOURKAS I., SIRAKOULIS G. C., 1049
- 987 ADAMATZKY A. I.: Oscillation-based slime mould electronic cir- 1050
- cuit model for maze-solving computations. *IEEE Transactions on* 1051
 Circuits and Systems I: Regular Papers 64, 6 (2017), 1552–1563. 3
- [NYT00] NAKAGAKI T., YAMADA H., TÓTH Á.: Maze-solving by an 1053 amoeboid organism. *Nature* 407, 6803 (2000), 470–470. 3 1054
- [NYT01] NAKAGAKI T., YAMADA H., TOTH A.: Path finding by tube ¹⁰⁵⁵
 morphogenesis in an amoeboid organism. *Biophysical chemistry* 92, 1-2 1056
 (2001), 47–52. 3 1057
- [PMNWW19] PODROUŽEK J., MARCON M., NINČEVIĆ K., WAN-¹⁰⁵⁸
 WENDNER R.: Bio-inspired 3d infill patterns for additive manufacturing ¹⁰⁵⁹
 and structural applications. *Materials* 12, 3 (2019), 499. doi:https: ¹⁰⁶⁰
 //doi.org/10.3390/ma12030499.3
- [PT08] PANTZ O., TRABELSI K.: A post-treatment of the homogeniza tion method for shape optimization. *SIAM Journal on Control and Op timization 47*, 3 (2008), 1380–1398. doi:https://doi.org/10.
- 1002
 1137/070688900.2
 1065

 1003
 [PVK*20]
 PORTELA C. M., VIDYASAGAR A., KRÖDEL S., WEIS 1066
- SENBACH T., YEE D. W., GREER J. R., KOCHMANN D. M.: Ex-¹⁰⁶⁷
 treme mechanical resilience of self-assembled nanolabyrinthine materi-¹⁰⁶⁸
 als. *Proceedings of the National Academy of Sciences 117*, 11 (2020), 1069
- 1007 5686–5693. 2 1070 1008 [PZM*15] PANETTA J., ZHOU Q., MALOMO L., PIETRONI N., ¹⁰⁷¹
- $\begin{array}{c} \text{(IZM IS)} & \text{FARETRA S., ZHOC Q., MALOMO E., THEROM IN,} \\ \text{CIGNONI P., ZORIN D.: Elastic textures for additive fabrication. ACM <math>_{1072} \\ \text{Transactions on Graphics (TOG) 34, 4 (2015), 1-12. 2} \\ \end{array}$
- 1011
 [Que20]
 QUEFFURUS O.: Phys [art] um: pour une approche artistique de
 1074

 1012
 la culture de physarum polycephalum. 3
 1075
- [RB13] REID C. R., BEEKMAN M.: Solving the towers of hanoi-how an ¹⁰⁷⁶ amoeboid organism efficiently constructs transport networks. *Journal of* ¹⁰⁷⁷ *Experimental Biology* 216, 9 (2013), 1546–1551. 3
- [RLDB12] REID C. R., LATTY T., DUSSUTOUR A., BEEKMAN M.:
 [Number Stepson 1080]
 [Num Stepson 1080]
 [Num S
- 1020 [Sch13] SCHUBERT T.: Bodymetries, 2013. Accessed May 7, 1021 2023. URL: https://www.theresaschubert.com/works/ 1022 bodymetries/. 3
- 1023 [Seb23] SEBDON M.: Crtl, 2023. Accessed May 7, 2023. URL: https: 1024 //michaelsedbon.com/C-t-r-l. 3
- [Sig95] SIGMUND O.: Tailoring materials with prescribed elastic properties. *Mechanics of materials 20*, 4 (1995), 351–368. 2
- [SPG19] SCHMIDT M.-P., PEDERSEN C. B., GOUT C.: On structural topology optimization using graded porosity control. *Structural and Multidisciplinary Optimization 60*, 4 (2019), 1437–1453. doi:https: //doi.org/10.1007/s00158-019-02275-x. 2
- [TKN07] TERO A., KOBAYASHI R., NAKAGAKI T.: A mathematical model for adaptive transport network in path finding by true slime mold. *Journal of theoretical biology 244*, 4 (2007), 553–564. 3
- [TNT*10] TERO A., NAKAGAKI T., TOYABE K., YUMIKI K.,
 KOBAYASHI R.: A method inspired by physarum for solving the
 steiner problem. *International journal of unconventional Computing 6*,
 2 (2010). 3
- [TS12] TSOMPANAS M.-A. I., SIRAKOULIS G. C.: Modeling and hard ware implementation of an amoeba-like cellular automaton. *Bioinspira- tion & Biomimetics 7*, 3 (2012), 036013. 3
- [TTS*10] TERO A., TAKAGI S., SAIGUSA T., ITO K., BEBBER D. P.,
 FRICKER M. D., YUMIKI K., KOBAYASHI R., NAKAGAKI T.: Rules for biologically inspired adaptive network design. *Science 327*, 5964 (2010), 439–442. 3
- [TTZ*20] TRICARD T., TAVERNIER V., ZANNI C., MARTÍNEZ J., HUGRON P.-A., NEYRET F., LEFEBVRE S.: Freely orientable microstructures for designing deformable 3d prints. *ACM Trans. Graph.* 39, 6 (2020), 211–1. 2
 - Author's version of article published in COMPUTER GRAPHICS Forum, May 2024.

- [vD14] VAN DAAL L.: Biomimicry; 3d printed soft seat, 2014. Accessed June 8, 2023. URL: https://www.lilianvandaal.com/ biomimicry-3d-printed-soft-seat. 14
- [VEYC*21] VAN EGMOND D. A., YU B., CHOUKIR S., FU S., SINGH C. V., HIBBARD G., HATTON B. D., ET AL.: The benefits of structural disorder in natural cellular solids. arXiv preprint arXiv:2110.04607 (2021). 2
- [WAWS18] WU J., AAGE N., WESTERMANN R., SIGMUND O.: Infill optimization for additive manufacturing—approaching bone-like porous structures. *IEEE Transactions on Visualization and Computer Graphics* 24, 2 (2018), 1127–1140. doi:https://doi.org/10.1109/ TVCG.2017.2655523.2
- [WSG21] WU J., SIGMUND O., GROEN J. P.: Topology optimization of multi-scale structures: A review. *Structural and Multidisciplinary Optimization 63* (2021), 1455–1480. doi:https://doi.org/10. 1007/s00158-021-02881-8.2
- [WWG21] WU J., WANG W., GAO X.: Design and optimization of conforming lattice structures. *IEEE transactions on visualization and computer graphics* 27, 1 (2021), 43–56. doi:https://doi.org/10. 1109/TVCG.2019.2938946.1
- [WWZW16] WU J., WANG C. C., ZHANG X., WESTERMANN R.: Self-supporting rhombic infill structures for additive manufacturing. *Computer-Aided Design 80* (2016), 32–42. 2
- [ZSCM17] ZHU B., SKOURAS M., CHEN D., MATUSIK W.: Twoscale topology optimization with microstructures. ACM Transactions on Graphics (TOG) 36, 4 (2017), 1. 2
- [ZWZ*20] ZHANG Y., WANG Z., ZHANG Y., GOMES S., BERNARD A.: Bio-inspired generative design for support structure generation and optimization in additive manufacturing (am). *CIRP Annals 69*, 1 (2020), 117–120. doi:https://doi.org/10.1016/j.cirp. 2020.04.091.3
- [ZZD11] ZHANG Y. J., ZHANG Z. L., DENG Y.: An improved maze solving algorithm based on an amoeboid organism. In 2011 Chinese Control and Decision Conference (CCDC) (2011), IEEE, pp. 1440– 1443. 3